Can "Big Bath" and Earnings Smoothing Co-exist as Equilibrium Financial Reporting Strategies?

MICHAEL KIRSCHENHEITER* AND NAHUM D. MELUMAD*

ABSTRACT

We study a model of financial reporting where investors infer the precision of reported earnings. Reporting a larger earnings surprise reduces the inferred earnings precision, dampening the impact on firm value of reporting higher earnings, and providing a natural demand for smoother earnings. We show that for sufficiently "bad" news, the manager under-reports earnings by the maximum, preferring to take a "big bath" in the current period in order to report higher future earnings. If the news is "good," the manager smoothes earnings, with the amount of smoothing depending on the level of cashflows observed. He either over-reports or partially under-reports for slightly good news, and gradually increases his under-reporting as the news gets better, until he is under-reporting the maximum amount for sufficiently good news. This result holds both when investors are "naïve" and ignore management's ability to manipulate earnings, or "sophisticated" and correctly infer management's disclosure strategy.

1. *Introduction*

The discussion of earnings management in both the academic and popular business press often connote some misdeed by management, which suggests earnings are manipulated by management to the detriment of investors. Some regulators also hold this view. Even though reported earnings may reflect manipulation, it has been argued that the market rewards management for smoother earnings. Another claim is that managers sometimes under-report earnings by a large amount for sufficiently bad earnings news, a behavior known as taking a big bath. While explanations have been offered for managers' wish to smooth earnings, no theory has been presented to explain why he would wish to take a big bath, nor have these two phenomena been addressed in the same model. The research objective of this paper is to develop such a model and to analyze whether there exist equilibrium reporting strategies that support the claims discussed above.

Our study offers a single model in which both earnings smoothing and the big bath phenomena are part of an equilibrium reporting strategy. Our findings suggest that these practices—sometimes criticized by regulators—may be the natural responses on the part of a manager wishing to maximize the value of his company. Our study also provides a theoretical explanation for a number of empirically documented phenomena regarding reported earnings and their relationship to firm prices.

We assume that cashflows in any period are composed of a transitory portion that occurs only in the current period, and a permanent portion that repeats each period. In our model, reported earnings affect firm value in two ways. First, ceteris paribus, investors infer a higher level of permanent cashflows from a higher level of reported earnings. For this reason, higher reported earnings increase the value of the firm. Second, investors infer that earnings have higher precision if the reported earnings surprise is smaller (i.e., reported earnings are closer to expected earnings). Reporting a larger earnings surprise reduces the inferred precision of the reported earnings, dampening the increase in firm value from reporting higher earnings. This dual role causes the manager to wish to smooth earnings.

Our main result shows that when the reporting environment permits discretion, an optimal disclosure policy exists in which the manager either takes a big bath or smoothes earnings. For sufficiently "bad" news (i.e., for sufficiently low levels of cashflows), the manager under-reports earnings by the maximum amount possible, preferring to take a big bath in the current period in order to report higher earnings in the future. If the news is not sufficiently bad, the manager smoothes earnings; the amount of smoothing depends on the level of cashflows observed. The manager over-reports for lower levels of observed cashflows and decreases the over-reporting as the news gets better, until he is under-reporting the maximum amount for news that is sufficiently good. This result is robust to different pricing structures and to different assumptions about the investor's ability to infer the

manager's strategy (i.e., whether we assume investors are "sophisticated" or "naïve").

As mentioned above, our study offers theoretical support for a number of empirically documented phenomena. Using a numerical example, we illustrate that under the credible disclosure equilibrium that we derive, reported earnings surprises shift away from the zero surprise level and concentrate at a slightly higher level. The example also shows that, under this credible disclosure strategy, the distribution of prices will have a fatter lower tail and a thinner upper tail than if no discretionary earnings were reported. Last, under this credible equilibrium, investors set prices having correctly inferred the manager's strategy. These prices may appear distorted compared to how one would expect them to appear if investors were to set prices based on naïve inferences.

2. Related Literature

The question of why management would smooth reported earnings has generated numerous studies. Dye [1988] suggests two reasons for management to manipulate earnings: an external demand to increase the firm's stock price, and an internal demand relating to optimal contracting. Earnings management plays a role in the latter case since managerial agents may be unable to communicate all relevant information to their principal (top management or board of directors). In simple terms, earnings that can be manipulated allow the agent to communicate this information.[1]

Models that study smoothing behavior as a signal include Ronen and Sadan [1981] and Chaney and Lewis [1995].[2] To make smoother earnings a credible signal of superior performance in these models, the smoothing behavior must be costly. In Ronen and Sadan, false signaling is costly due to actions by auditors, legal liability or SEC enforcement. Chaney and Lewis use taxes to separate high- and low-type firms: for high-type firms, the cost in additional corporate taxes from over-reporting earnings is more than offset by being identified as a high-type firm. Low-type firms choose a tax minimizing strategy, as the benefit of being identified as a high-type firm is exceeded by the additional expected tax penalty from over-reporting.

Share price maximization may also prompt earnings smoothing if smoothing behavior raises the expected cashflow to investors. Trueman and Titman

---

[1] In an early work, Lambert [1984] studied optimal contracting with risk-averse managers. He showed that real income smoothing could be generated by rational behavior that was not dependent on myopia or a misunderstanding by any party. A number of other studies (for example Dye and Verrecchia [1995] and Demski [1996]) analyze earnings smoothing due to discretionary reporting from an optimal contract design perspective. We assume that the compensation contracts are exogenously set; studies focusing on optimal contracting complement our work.

[2] See Hunt, Moyer and Shevlin [1996] for discussions of empirical work supporting the argument that smoothing behavior may add to the informativeness of accounting earnings. See Kirschenheiter and Melumad [2002] for a theoretical study that supports this argument.

[1988] investigate this alternative explanation using another signaling setting. They argue that managers smooth earnings to convince potential debtholders that earnings have lower volatility, and hence represent a reduced risk. Since debt can be raised at lower cost, smoothing increases the expected cashflow to shareholders. Trueman and Titman assume that the ability to smooth differs across firms and that it is management's private information. In their model, exogenous smoothing costs arise from higher taxes and auditor costs.

Fudenberg and Tirole [1995] develop a model where contracting considerations, rather than share price maximization, drive earnings smoothing. Their model makes two key assumptions. First, the manager has "incumbency rents." These rents are lost if the manager loses his job and the principal is unable to compensate the manager for this potential loss. (This is referred to as infinite risk aversion.) Second, the value of information decays, so that more recent information is given greater weight in assessing and rewarding performance. The main result of their analysis is that income smoothing is optimal behavior. Income smoothing occurs because the manager boosts the income reported in bad times (to raise the probability of keeping his job) and lowers income reported in good times (to take advantage of the impact of information decay).

Our study contributes to the literature in three ways. First, we eschew both the contracting and the signaling approaches and base our approach on the models of uncertain precision used by Penno [1996] and Subramanyam [1996]. In our model, smoothing behavior occurs because the manager wishes to increase the inferred precision of the earnings report. Second, our approach avoids some of the restrictive assumptions made by other research. Unlike Chaney and Lewis or Ronen and Sadan, our model does not require exogenous costs to support smoothing. Unlike Trueman and Titman, we show smoothing can arise even in the absence of debt. Unlike Fudenberg and Tirole, the smoothing in our model arises without assuming either incumbency rents or information decay. Third, our model can be used to interpret recent empirical work on earnings management, as we describe next.

Besides showing that both taking a big bath and smoothing earnings can be part of a single equilibrium disclosure strategy, our model supports a number of "stylized facts." These "facts" are identified by recent empirical work but not specifically addressed by the earlier theoretical studies. First, a number of studies have documented that firms avoid reporting small negative earnings surprises while they also show that a larger percentage of firms report small positive earnings surprises (see for example Burgstahler and Dichev [1997], Abarbanell and Lehavy [2000], and DeFond and Park [2000]). Second, Abarbanell and Lehavy find evidence that the distribution of prices will have a fat left-hand tail and a thin right-hand tail. Third, the change in price as a function of earnings surprises appears steeper surrounding the zero earnings surprise level and this phenomenon is more pronounced for firms with higher price/earnings ratios. This effect has been described as a "torpedo" effect of missing the earnings

estimate (see Skinner and Sloan [2000]). As we argue explicitly in section 4.3, the equilibrium credible disclosure strategy that we derive under monotonic pricing is consistent with each of these results. Further we show that the structure of the equilibrium disclosure strategy is robust. For example, it is very similar whether we assume pricing is monotonic, or non-monotonic, or whether investors are sophisticated and are able to infer the disclosure strategy of management, or if they are naïve and are assumed to believe no earnings management is occurring. The fact that the structure of the disclosure strategies is unaffected by the assumed sophistication of investors suggests that we may not be able to use the empirical results discussed above to conclude whether or not investors are naïve.

3. Model

3.1 CASHFLOW, EARNINGS AND INFERRED CASHFLOW VARIABLES

In our model, the firm has an infinite life. To capture the finite time horizon of management, we suppose that a risk neutral manager operates the firm for two periods. Investors value the firm based on inferences they make about future cashflows conditional on the earnings reported by management. For periods $n = 1$ or 2, we assume total cashflow in period n, denoted as Y_n, is the sum of two random variables: permanent cashflow, X, which repeats every period, and transitory cashflow, T_n, which flows only in period n, so that $Y_n = X + T_n$. The variable X is normally distributed with mean μ and precision h_X, i.e., $X \sim N(\mu, 1/h_X)$. The T_n variables are independent, identically and normally distributed, with zero mean and an unknown precision. This means that $Y_n \sim N(\mu, 1/H_Y)$, where the mean μ is known, but the precision H_Y is unknown. We assume H_Y is itself a random variable having realization h_Y.[3]

The risk neutral manager has discretion in reporting earnings, denoted by $\delta \in [\delta_L, \delta_U]$ with $\delta_L < 0 < \delta_U$. The risk neutrality assumption insures that any demand for earnings smoothing that arises will not arise because of the manager's attempt to optimally shift consumption risk between periods. We assume that the discretionary earnings reported in the first period, δ, reverse in the subsequent period, so the manager cannot over or under-report in aggregate. This captures the timing dimension of accrual accounting. Formally, this assumption means that a manager who reports $m_1 = y_1 + \delta$ in

---

[3] Our modeling borrows heavily from prior literature, especially Verrecchia [1983] and Kirschenheiter [1997] for modeling tri-variate normality and Subramanyam [1996] for modeling uncertain precision. Regarding the notation, we use upper case letters to denote random variables and lower case letters to denote their realizations. Also, we assume h_X is known for convenience and tractability. In a more general setting, both precisions will be unknown. We assert our results will be qualitatively unaffected by relaxing this simplifying assumption. Also, without loss of generality, we use as our primitives the random variables Y_n instead of the variables T_n. Although using T_n might seem more natural, we find this approach preferable because Y_n is always observable and because this approach simplifies the exposition.

period one will be restricted to reporting $m_2 = y_2 - \delta$ in period two. The function $\psi(y_1): \Re \to [\delta_L, \delta_U]$ is the manager's disclosure strategy.

We distinguish between cashflows and earnings. We call the y_n "cashflow" to indicate this is what the manager observes. The m_n are called "earnings" to indicate that these are reports issued by a manager. We assume that a manager's choice of δ has no impact on cashflows generated by the firm.[4]

Investors never directly observe the cashflow variables, however they use the reported earnings to infer the level of discretionary earnings reported. We denote the disclosure strategy inferred by shareholders based on re-ported earnings as $\psi^S(m_1): \Re \to [\delta_L, \delta_U]$, which may be either a function or a correspondence.[5] We use the symbol " ˆ " to denote the level of cashflow inferred by investors. Hence, if the manager observes y_1, chooses $\psi(y_1) = \delta$, and reports $m_1 = y_1 + \delta$, and if investors infer $\psi^S(m_1) = \delta^S$, then investors infer cashflows of $\bar{y}_1 = m_1 - \delta^S = y_1 + \delta - \delta^S$ for period one. We use the nota-tion "Δ" before the cashflow and earnings variables to denote the deviation from the mean, e.g., $\Delta x = x - \mu$ and $\Delta y_n = y_n - \mu$. We sometimes refer to the deviation from the mean in the first period as the "surprise" amount.

3.2 PRICING, MANAGERIAL PAYOFF AND EQUILIBRIUM

Investors value the firm based on the net present value of the expected cashflows. At date $n = 0$, the value of the firm, P_0, is the expected present value of an infinite stream of cashflows of X in each period, discounted by a factor $1 + r$. Hence, $P_0 = (1/r)E[X] = M\mu$, where $M \equiv 1/r$. In subsequent periods, investors will value the firm based on the inferred cashflows condi-tioned on the reported earnings. The inferred cashflows, $\bar{y}_1$ and $\bar{y}_2$, depend on actual cashflows, y_1 and y_2, the discretionary strategy of management, $\psi(y_1)$, and the discretionary strategy inferred by shareholders, $\psi^S(m_1)$. We convey this by denoting the price at dates 1 and 2 as $P_1(y_1; \psi, \psi^S)$ and $P_2(y_1, y_2; \psi, \psi^S)$, respectively. To value the firm at dates 1 and 2, investors also must estimate the precision variable, H_Y. Let $f(h_Y)$, $f(h_Y \mid y_1)$, etc., be the marginal and conditional distributions for the precision variable and

---

[4] Since every actual "cashflow" measure may also be "manipulated" by management, the distinction between "cashflow" and "earnings" cannot be taken literally. One may interpret the discretionary amount, δ, as all accruals for period 1 or as the discretionary accrual amount. In the latter case, $y_n(m_n)$ would represent the earnings before (after) discretionary accruals. This amount may reflect the manager's choice of non-cash expense, such as bad debt or excess and obsolete inventory expense. Alternatively, it may reflect decisions to accelerate or delay a shipment, a purchase, a payment or a receipt, insofar as these decisions have no effect on value creation by the firm, but do affect the earnings reported.

[5] We are required to address numerous technical complications in the process of deriving the price equations and defining the equilibrium later in this section. To simplify the presentation, we relegate the discussion of these complications to appendix A. Also, throughout the paper we write ψ^S as a function of m_1, and not of (m_1, m_2). The second period earnings report affects the investors' inference only when he does not infer the cashflows amount perfectly from the first period report, as we explain in the proof of theorem 1. We address this situation when we define consistent prices in appendix A.

define

$$E[H_Y \mid m_1, y^S] \equiv \int\limits_{h_Y \in H_Y \subseteq [0,\infty)} h_Y f(h_Y \mid m_1, y^S) \, dh_Y.$$

The explicit equations for these prices are given as shown below (see appendix A for their derivation).

$$P_0 = M\mu,$$

$$P_1(y_1; \psi, \psi^S) = P_0 + \Delta \bar{y}_1 \left((M-1) \frac{E[H_Y \mid m_1, \psi^S]}{h_X} + 1 \right), \quad \text{and}$$

$$P_2(y_1, y_2; \psi, \psi^S) = P_0 + (\Delta \bar{y}_1 + \Delta \bar{y}_2) \int\limits_{H_Y} \frac{((M-1)h_Y + h_X)}{h_Y + h_X}$$

$$\times f(h_Y \mid m_1, m_2, \psi^S) \, dh_Y.$$

Let P denote the price vector, i.e. $P \equiv (P_0, P_1, P_2)$. As noted earlier, the prices are functions of the observed cash flow realizations, y_1 and y_2, parameterized on the manager's disclosure strategy, ψ, and the investors' inferred disclosure strategy, ψ^S. We indicate that the expectations in the price equations are based on the investors' information by writing the relevant distributions as conditional distributions conditioned on the reported earnings, m_1 and m_2, and the investors' inferred disclosure strategy, ψ^S. For example, we use $E[H_Y \mid m_1, \psi^S]$ to represent the expected precision conditioned on the investors' information. The expected precisions in the pricing equations above differ from the expected precision that is relevant to the manager's decision. We discuss this point in more detail when we discuss the manager's payoff function, which we do next.

The risk neutral manager chooses his disclosure strategy, ψ, to maximize his expected payoff, given his information at date 1. We assume that his payoff is a linear function of the change in price between dates 0 and 2, which we denote as $W(y_1, y_2; \psi, \psi^S) = P_2(y_1, y_2; \psi, \psi^S) - P_0$. This means that given an inferred disclosure strategy ψ^S and having observed y_1, the manager chooses δ to maximize the expected payoff, which we denote as $EW(y_1; \psi, \psi^S)$.[6] This expectation is given as follows:

$$EW(y_1; \psi, \psi^S) = \int\limits_{\hat{Y}_2} W(y_1, y_2; \psi, \psi^S) f(\bar{y}_2 \mid y_1, \psi, \psi^S) \, d\bar{y}_2$$

$$= \int\limits_{H_Y} \Delta y_1 \left(\frac{(M-1)h_Y}{h_X} + 1 \right) f(h_Y \mid y_1, \psi, \psi^S) \, dh_Y$$

$$= \Delta y_1 \left(\frac{(M-1)E[H_Y \mid y_1, \psi, \psi^S]}{h_X} + 1 \right)$$

---

[6] To capture the fact that management has a finite employment contract, we do not allow for the possibility of deferred compensation.

The equation's simplicity follows primarily from the normality assumption; normality also implies the manager's expectation for period two exhibits mean reversion. More specifically, the manager expects the second period cashflow surprise to have the same sign, but a lower magnitude, than the first period cashflow surprise. While simple in appearance, the derivation of $EW(y_1; \psi, \psi^S)$, involves a number of subtleties that deserve elaboration.

First, this manager's objective function depends on the precision that a manager expects investors to expect. This expectation is based on the manager's information about the investors' information, and we denote it as $E[H_Y \mid y_1, \psi, \psi^S]$. As mentioned at the end of the discussion of the pricing, it is critical to distinguish $E[H_Y \mid y_1, \psi, \psi^S]$ from $E[H_Y \mid m_1, \psi^S]$. Recall $E[H_Y \mid m_1, \psi^S]$ is the expected precision conditioned on the investors' information and this determines the price at date 1. We assume that there are no arbitrage opportunities, so that the price at date one equals the investors expectation of the second period price, or $P_1 = E[P_2 \mid \text{investors' information}]$. However the expected payoff, $EW(y_1; \psi, \psi^S)$, is based on the manager's additional information concerning his disclosure strategy. If the manager has the same information as the investors, the expected payoff will equal the actual first period price change, so that $EW(y_1; \psi, \psi^S) = P_1(y_1; \psi, \psi^S) - P_0$ will hold. For example, this will be true when there is no discretion (i.e., if $\psi \equiv 0$ and $\psi^S \equiv 0$). However, in general this will not hold.

Second, when there is discretion, discretionary earnings affect expected payoff only through the expected precision, or only through $E[H_Y \mid y_1, \psi, \psi^S]$. The manager wishes to either maximize or minimize $E[H_Y \mid y_1, \psi, \psi^S]$ depending on whether the news is good or bad, that is, whether Δy_1 is positive or negative. If the news is good, then $EW(y_1; \psi, \psi^S)$ is positive, and the manager maximizes this value by maximizing $E[H_Y \mid y_1, \psi, \psi^S]$. For bad news, $EW(y_1; \psi, \psi^S)$ is negative, and the manager maximizes his expected payoff by minimizing the expectation, $E[H_Y \mid y_1, \psi, \psi^S]$. While we focus on the case in which the manager knows only y_1, other situations are possible. Regardless of the manager's information, an "optimal" disclosure strategy for the manager is defined as one that maximizes his expected payoff, conditional on his information and given the inferred disclosure strategy and the price equations.

Finally, an equilibrium is a triple, (ψ, ψ^S, P), where ψ is chosen to optimize the manager's expected payoff given ψ^S and the vector of prices, P.[7] We consider two types of equilibrium: credible and naïve, and distinguish between them based on the investors' inferred disclosure strategy. For example, if the manager chooses the discretionary earnings amount $\psi(y_1) = \delta$, then the investors observe $m_1 = (y_1 + \delta)$. We assume that for each m_1, the investors form a set of inferred discretionary earnings, denoted as $\psi^S(m_1)$. While this set may be a singleton, in general this is not so, as the manager

---

[7] The formal definition of an equilibrium is a triple, (ψ, ψ^S, P), where ψ is optimal for a complete ψ^S and consistent prices, P. Again, the formal details for this definition are relegated to appendix A.

may report m_1 for multiple values of y_1. A credible equilibrium requires investors to correctly infer the manager's strategy. Formally this means that we require that for each y_1, the realization of $\psi(y_1)$ is an element in the set $\psi^S(m_1)$. For a naïve equilibrium, we require that $\psi^S \equiv 0$. We refer to a disclosure strategy, ψ, which supports a credible (naïve) equilibrium as a credible (naïve) disclosure strategy. The definition of a credible equilibrium is standard in the literature (see Dye [1986] or Kirschenheiter [1997]). Studying the naïve equilibrium is valuable because it captures the intuition behind arguments concerning "unsophisticated" investors. Also, our results show that surprising similarities exist between naïve and credible disclosure strategies.

4. Results

4.1 PRELIMINARY (NO SMOOTHING) RESULTS

We start by observing that, in our model, both the distinction between permanent and transitory cashflows and the assumption of uncertain precision are crucial for management to have a desire to smooth earnings. First, if we assume that permanent and transitory earnings are capitalized in the same manner, the manager will have no incentive to smooth earnings.

OBSERVATION 1. There is no demand for earnings smoothing if all earnings are transitory.

This simple observation says that if permanent cashflows last only one period (so that permanent and temporary cashflows are capitalized in the same manner), the manager will never benefit from shifting earnings between the periods. This holds regardless of whether the precision is certain or uncertain, and it demonstrates the importance of distinguishing between permanent and transitory cashflows in order to generate a demand for smoothing in our setting. Next, we consider the case of certain precision.

OBSERVATION 2. With known precision, the manager is indifferent about his choice of the level of reported discretionary earnings.

Observation 2 provides the standard result that, when reported earnings do not affect the inferred precision, the manager again does not benefit from shifting earnings between periods. With known precision, the effect on his payoff of reporting discretionary earnings in the first period is exactly offset by the reversal of these earnings in the second period. Hence, unknown or random precision is necessary for smoothing to occur in our setting.

Next, we address the pricing when the precision is a random variable. The manager will choose his disclosure strategy to maximize his expected payoff, $EW(y_1; \psi, \psi^S)$. Thus, the manager's choice of a disclosure strategy depends on the pricing structure, and, in turn, the pricing structure depends on the disclosure strategy. To establish an equilibrium with discretion, we first specify the pricing structure, and then show that the conjectured pricing

supports the equilibrium. We distinguish between two possible alternative pricing structures, specified by conditions A1 and A2.

CONDITION A1 (MONOTONIC PRICING). The price equation, $P_1(y_1; \psi, \psi^S)$, is strictly increasing in Δy_1.

CONDITION A2 (NON-MONOTONIC PRICING). $P_1(y_1; \psi, \psi^S)$, is concave for $\Delta y_1 \geq 0$ (reaching its maximum at $\Delta y_1 = \Delta y_{max}$), is convex for $\Delta y_1 \leq 0$ (reaching a minimum at $\Delta y_1 = \Delta y_{min}$), and is increasing at $\Delta y_1 = 0$.

We find the pricing structures presented in conditions A1 and A2 interesting for two reasons. First, the structures represented by conditions A1 and A2 reflect the predominant viewpoints concerning the structure of pricing. Many empirical studies assume a monotonic relation between earnings surprise and change in price. Under condition A1, the objective function is strictly increasing in cashflow surprise: i.e., no optimum exists, consistent with the predominant view. While most studies assume that prices are monotonic in earnings surprises, some (see Freeman and Tse [1992] and Subramanyam [1996]) suggest that a large earnings surprise may reduce the precision of the reported earnings so much that managers may wish to report lower earnings for sufficiently high cashflow surprises. Under condition A2, two levels of cashflow surprise exist, one above zero that maximizes the objective function without discretion, and one below zero that minimizes this function. Hence, condition A2 captures the pricing structure suggested in this more recent work. Our second reason for focusing on the pricing given in conditions A1 and A2 is related to the first. While pricing may have a structure other than that described by these conditions, our results may be extended in a straightforward fashion to cover many of the alternative pricing structures. Hence these two cases form convenient bench-marks for our analysis.

To summarize, observations 1 and 2 show that having uncertain precision and distinguishing permanent from transitory cashflows are necessary conditions for smoothing to occur. Also, having noted that the manager's choice of disclosure strategy will depend on the pricing structure, we introduced two different pricing structures as benchmarks. We next analyze the equilibrium disclosure strategies when both uncertain precision and a distinction between permanent and transitory cashflows exist.

4.2 EQUILIBRIA WITH MONOTONIC PRICING

Given the observations presented in the previous section and given the dependence of the disclosure strategies on the pricing structure, one might expect a fully separating equilibrium to exist under monotonic pricing. If so, it may be that smoothing behavior cannot support an equilibrium. We show in lemma 1 that this argument fails, and that there does not exist a fully separating credible equilibrium.

LEMMA 1.   *If Condition A1 holds for $P_1(y_1; \psi \equiv 0, \psi^S \equiv 0)$, then there does not exist a fully separating credible equilibrium in pure strategies where $P_1(y_1; \psi^*, \psi^{S*})$ is characterized by condition A1.*

There exist pure strategies that support a separating equilibrium if the news is either sufficiently bad or sufficiently good, but the separating equilibria break down when news is slightly bad. In particular, as the proof of lemma 1 shows, there does not exist a separating strategy for cashflow surprises in an interval slightly below zero. The lack of a separating strategy in this interval is due to the manager's optimizing behavior when he observes these small negative cashflow surprises. Recall that a manager observing bad news will choose his disclosure strategy to minimize the inferred precision of the earnings report (i.e., minimize $E[H_Y | y_1, \psi, \psi^S]$). He does this by reporting in a way that will ensure that the absolute value of the earnings surprise in the two periods is maximized. For bad news sufficiently close to zero, the manager finds it beneficial to report differently from what investors expect.

Since the non-existence of a fully separating equilibrium is somewhat surprising, we provide the following simple example to illustrate why the fully separating equilibrium fails. Suppose we have the model parameters $\mu = 0$, $\delta_U = 1$, $\delta_L = -1$, and $h_X = 1$, and suppose $E[H_Y|y_1] = 0.4$ at $y_1 = -0.5$, so that when the manager observes $y_1 = -0.5$, then $E[Y_2|y_1] = y_1$ $(\frac{E[H_Y|y_1]}{h_X}) = -0.2$. As explained in the previous paragraph, the manager wishes to minimize $E[H_Y | y_1, \psi, \psi^S]$, which he does by reporting the level of discretionary earnings that he expects will maximize the sum of the absolute deviations from the mean for the two periods. This means that for an inferred discretion amount δ^S, the manager wishes to choose δ to maximize

$$(\bar{y}_1)^2 + (\bar{y}_2)^2 = (y_1 + \delta - \delta^S)^2 + (E[Y_2 | y_1] - \delta + \delta^S)^2$$
$$= (-0.5 + \delta - \delta^S)^2 + (-0.2 + \delta - \delta^S)^2.$$

To show that each separating strategy fails, we need to show that the manager will always wish to report a discretion amount that is different than the amount inferred by the investors.

First, suppose $\delta^S < -0.15$, so that investors infer the manager is under-reporting by an amount greater than 0.15 in absolute value. In this case, the manager wishes to maximally over-report, since for $\delta' < \delta_U$ we have

$$(-0.5 + \delta_U - \delta^S)^2 + (-0.2 - \delta_U + \delta^S)^2 > (-0.5 + \delta' - \delta^S)^2 + (-0.2 - \delta' + \delta^S)^2.$$

Next, suppose $\delta^S > -0.15$, so that investors infer the manager is over-reporting or under-reporting an amount that is less than 0.15 in absolute value. Now the manager wishes to maximally under-report since for $\delta' > \delta_L$ we have

$$(-0.5 + \delta_L - \delta^S)^2 + (-0.2 - \delta_L + \delta^S)^2 > (-0.5 + \delta' - \delta^S)^2 + (-0.2 - \delta' + \delta^S)^2.$$

Finally, suppose $\delta^S = -0.15$. In this case the manager again will deviate, but now he is indifferent between under and over-reporting the maximum. We can see this by calculating the value of the above maximization equation

at the two extremes. At $\delta_L = -1$ we have

$$(-0.5 + \delta_L - \delta^S)^2 + (-0.2 + \delta_L - \delta^S)^2$$
$$= (-0.5 - 1 + 0.15)^2 + (-0.2 + 1 - 0.15)^2$$
$$= (-1.35)^2 + (-0.65)^2$$

and at $\delta_U = 1$ we have

$$(-0.5 + \delta_U - \delta^S)^2 + (-0.2 - \delta_U + \delta^S)^2$$
$$= (-0.5 + 1 + 0.15)^2 + (-0.2 - 1 - 0.15)^2$$
$$= (-0.65)^2 + (-1.35)^2.$$

To summarize, the example shows the manager always deviates. He will over-report the maximum if $\delta^S < -0.15$, he will under-report the maximum if $\delta^S > -0.15$, and if $\delta^S = -0.15$, he is indifferent between over or under-reporting the maximum.

While no fully separating equilibrium exists, there do exist multiple partially separating equilibria among which we need to choose. We focus on searching for equilibria that result in maximum separation, that is, equilibria in which the set of cashflows that are perfectly inferred by the investors is maximized. We have chosen this focus as it seems the natural bench-mark when assuming investors are sophisticated. Further, it offers a natural balance to the case when investors are assumed to be naïve, which we also analyze. From the proof of lemma 1, we know that separation fails for small negative cashflow surprises, so we seek a partially separating equilibrium in which the manager pools when he observes these cashflow surprises, and separates otherwise. Theorem 1 characterizes one such partially separating, credible equilibrium under monotonic pricing.

THEOREM 1.   *If pricing is monotonic (i.e., condition A1 holds) in the setting without discretion (i.e., for $P_1(y_1; \psi \equiv 0, \psi^S \equiv 0)$), then there exists a partially separating, credible equilibrium, (ψ^*, ψ^{S*}, P), in which the price equation with discretion, $P_1(y_1; \psi^*, \psi^{S*})$, is characterized by A1. The credible disclosure strategy involves maximum under-reporting either for sufficiently good news or sufficiently bad news, and it involves pooling for cashflow surprises in an interval around zero. There are N pooled reports, where N is an integer determined by the limits of the available discretion, δ_U and δ_L.*

Theorem 1 characterizes a credible disclosure strategy in which the manager takes a big bath if he observes sufficiently bad news and reports smoothed earnings otherwise. The intuition behind theorem 1 is most easily understood by considering the manager's behavior under two cases: when he observes news that is slightly good or slightly bad and when he observes news that is either very good or very bad.

First, for slightly good or slightly bad news (observed cashflows in the interval around $\Delta y_1 = 0$), the managers chooses to pool under the credible equilibrium of theorem 1. As in most pooling equilibria, the manager prefers to pool because his payoff will be lower under any alternative feasible report.

The payoff would be lower because we employ the standard assumption and assume the investors infer the worst when the manager deviates. For slightly good news, he pools to smooth the earnings reported in each period, thereby raising the inferred precision. For slightly bad news, he pools to avoid the inference of lower earnings, to lower the inferred precision, or both.

Second, suppose the news is very good or very bad (observed cashflows outside the interval around $\Delta y_1 = 0$). If the observed cashflow surprises are sufficiently positive or negative, theorem 1 shows that the manager under-reports the maximum amount, but he does so for different reasons. Recall that the manager expects the cashflow surprise in period 2 to have the same sign, but a lower magnitude, than the first period cashflow surprise. If he observes a negative surprise at date 1, he expects the cashflow at date 2 to be negative, but less so than for the surprise at date 1. For sufficiently negative cashflow surprises, the manager under-reports to increase the disparity between the reports in each period, hoping in this manner to introduce noise so as to lower the inferred precision of the earnings report. The cut-off below which the manager takes a big bath depends on the level of available discretion, being larger when there is more discretion available. For positive cashflows, the manager expects the cashflow at date 2 to again be positive, but to be less positive than the cashflow observed at date 1. He wishes to under-report and perfectly smooth the earnings, in order to increase the inferred precision of the earnings report, but is constrained by his available discretion. Hence, he chooses to under-report the maximum amount. Finally, we wish to emphasize that for very good and very bad news, the manager prefers to adhere to his chosen strategy even though the investors perfectly infer the observed cashflow.

Next, we extend the analysis to the situation where investors are naïve, that is we assume $\psi^S(\bullet) \equiv 0$.[8] We sometimes refer to such investors as exhibiting functional fixation. Results derived assuming naïve or functionally fixated investors are often dismissed as unimportant, since they are expected to differ from the results associated with sophisticated investors. Interestingly, this is not so in the current model. In fact, the form of the naïve and credible disclosure strategies are very similar, as corollary 1 indicates.

COROLLARY 1.   *Let the conditions of theorem 1 hold but assume $\psi^S(\bullet) \equiv 0$. Then there exists a unique naïve equilibrium, $(\psi^*, \psi^S \equiv 0, P)$, where $P_1(y_1; \psi^*, \psi^S)$ is characterized by condition A1. Specifically, there exists a cashflow realization Δy_L, such that the unique naïve strategy is given as follows:*

$$\psi^*(y_1) = -\frac{1}{2}\Delta y_1 \left(1 - \frac{E[H_Y \mid y_1]}{h_X}\right) \text{ if } \Delta y_1 \in [0, \Delta y_L] \text{ and } \psi^*(y_1)$$

$$= \delta_L \text{ otherwise.}$$

---

[8] Investors need not be genuinely naïve in their inferences for this analysis to be of interest; it suffices that managers believe investors are naïve. Casual evidence is consistent with the assumption that some managers believe the investing public is naïve or myopic in their response to earnings reports.

Corollary 1 shows that the unique naïve equilibrium is supported by a disclosure strategy in which the manager takes a big bath for bad news and smoothes good news. The unique naïve strategy under-reports the maximum amount for observed cashflows that are sufficiently large in absolute value, either negative or positive. Further, the manager smoothes his reported earnings for slightly good news by reporting $\Delta m_1 = (\Delta y_1/2)/ (1 + E[H_Y \mid y_1]/h_X)$ in the range $\Delta y_1 \in [0, \Delta y_L]$.

In comparing the credible equilibrium of theorem 1 to the naïve equilibrium of corollary 1, one sees that the unique naïve equilibrium disclosure strategy has a structure that closely resembles the structure of the credible disclosure strategy of theorem 1. This similarity in structure suggests that we may need to reconsider how we interpret the role investors' sophistication plays in our research. First, the empirical findings discussed in the next section are sometimes interpreted as support for the argument that investors are naïve. This interpretation presumes that the strategies will differ depending on whether or not investors are sophisticated: our results question the validity of this interpretation. Second, analyzing an equilibrium assuming naïve investors may offer more research insights than had previously been thought. It is often argued that, if investors are naïve, managers will exercise their discretion in a manner that significantly distorts the earnings reports, resulting in reported earnings that are quite different from those which would be reported were investors not so naïve. The similarity in the structure of the disclosure strategies derived above suggests that the naïveté of the investors may not distort the optimal disclosure policy as much as is sometimes alleged.

Earlier we argued that the credible disclosure strategy we derive helped to explain a number of empirical phenomena. This argument was presented in general terms; in the next section, we present this argument in greater detail.

4.3 RELATION TO EMPIRICAL RESEARCH

In this section, we analyze a numerical example to clarify the effect on reported earnings and pricing of the manager's strategy in a credible equilibrium under monotonic pricing. Let the model parameters be $\delta_U = 1$ and $\delta_L = -1$, so that $(\delta_U - \delta_L) = 2$. Figure 1a shows the manager's disclosure choices when he follows the disclosure strategy given in theorem 1. He under-reports the maximum for observed cashflows less than -2 and greater than 2. Between -2 and 0, he chooses a gradually decreasing discretion amount, first over-reporting then under-reporting. He follows a similar strategy for observed cashflows in the interval from 0 to 2. This leads to reported earnings as shown in figure 1b.

Figures 1a and 1b show that, under the credible equilibrium disclosure strategy from theorem 1, reported earnings are concentrated at earnings surprise levels of $+1$, with no earnings surprises reported in the interval $(-1, 1)$. This illustrates that the model provides partial theoretical support for a popularly held view that firms avoid reporting small negative earnings surprises instead concentrating their earnings reports in the small positive earnings surprise category. (See Burgstahler and Dichev [1997], Abarbanell
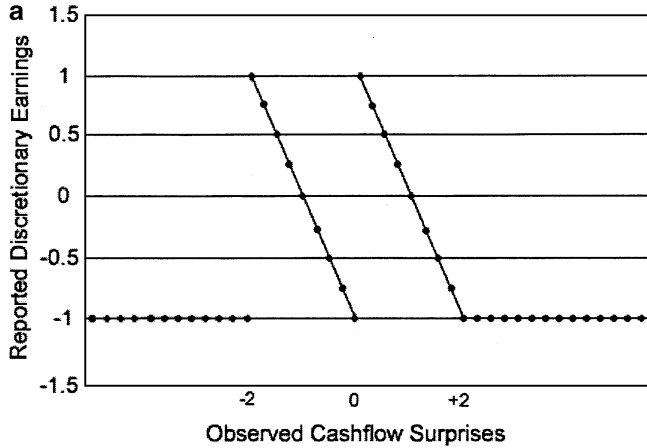
FIG. 1a.—Example of a credible equilibrium disclosure strategy as specified in theorem 1 when the model parameters are $\delta_U = 1$ and $\delta_L = -1$, so that $(\delta_U - \delta_L) = 2$. Figure 1a shows the manager under-reports the maximum for observed cashflows less than -2 and greater than 2. Between -2 and 0, he chooses a gradually decreasing discretion amount, first over-reporting then under-reporting. He follows a similar strategy for observed cashflows in the interval from 0 to 2.

and Lehavy [2000], and DeFond and Park [2000] for empirical work relating to these points.) Also, under-reporting for extreme levels of earnings surprises, as occurs in the credible equilibrium of theorem 1, will cause the left-hand tail of the distribution of prices to grow and the right-hand tail to

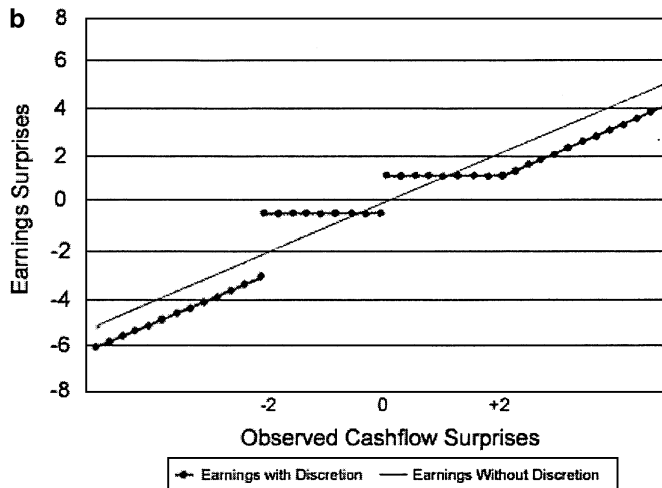FIG. 1b.—Example of earnings reported without discretion compared to earnings reported under the credible equilibrium disclosure strategy as specified in theorem 1 when the model parameters are $\delta_U = 1$ and $\delta_L = -1$, so that $(\delta_U - \delta_L) = 2$. Figure 1b shows the manager under-reports the maximum when he is reporting earnings sufficiently high or sufficiently low, and reports pooled earnings otherwise.

shrink. (See Abarbanell and Lehavy [2000] for empirical work supporting this result.)

The support of the popular viewpoint provided by the model is not unqualified. First, there may be multiple levels of pooling. Second, in the example above there is a concentration of earnings reported at earnings surprise level of -1, with no earnings surprises reported in the interval $(-3,-1)$. We hope to address these issues in future research by relaxing some of the assumptions of the model.

Next, we investigate the extent to which pricing is "distorted" in this equilibrium. Empirical research usually relates reported earnings to observed prices. However, investors may be inferring a different permanent level of cashflow than what is reported by management and investors may be setting prices using this inferred level of cashflow. Even if the market is pricing the firms efficiently, the relationship between the reported earnings and observed prices may appear "distorted" since the researcher is relating prices to reported earnings and not to the inferred cashflow amount that is being used by investors. Figures 2a and 2b illustrate how this apparent pricing distortion would appear in our model.
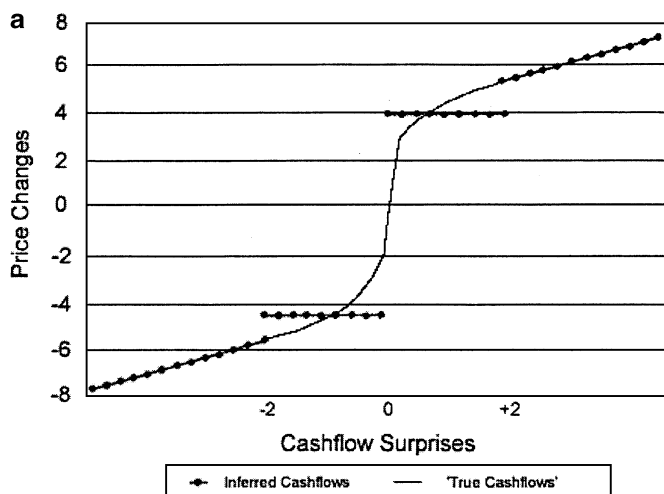

FIG. 2a.—Example of pricing based on earnings reported without discretion (called 'True' Cashflows) compared to pricing based on the Inferred Cashflows assuming that $M = 10$, $h_X = 1$ and that the prior distribution of H_Y is exponential. Pricing under Inferred Cashflows is based on cashflows inferred when earnings are reported under the credible equilibrium disclosure strategy as specified in theorem 1 for model parameters $\delta_U = 1$ and $\delta_L = -1$, so that $(\delta_U - \delta_L) = 2$. Under the credible strategy of theorem 1, investors perfectly infer the cashflow amounts that are sufficiently high or low. Figure 2a shows this occurs in this example when inferred cashflows fall below -2 or above $+2$. Otherwise reported earnings are pooled, and the price is set based on the average inferred cashflows. Figure 2a shows that under the conditions of the example, two pools exist; one for cashflows between -2 and 0 and a second between 0 and $+2$.
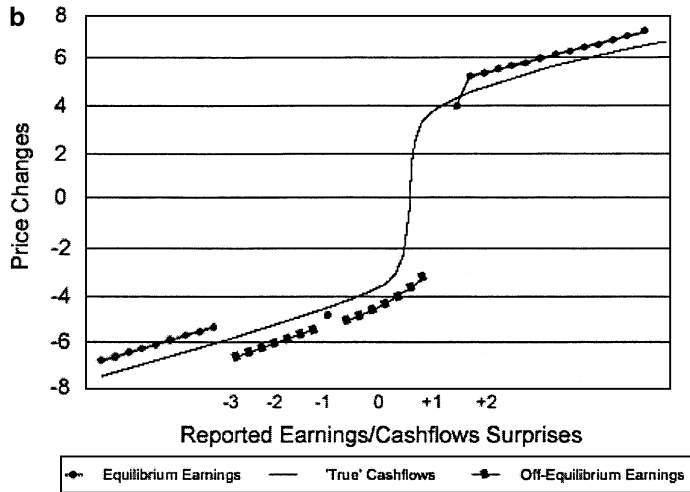
FIG. 2b.—Example comparing pricing that would have occurred had earnings been reported without discretion to pricing that would have occurred had earnings been reported under the credible equilibrium disclosure strategy as specified in theorem 1 assuming that $h_X = 1$ and the prior distribution of H_Y is exponential. As in the previous figures, figure 2b is based on model parameters $\delta_U = 1$ and $\delta_L = -1$, so that $(\delta_U - \delta_L) = 2$. Figure 2b shows that for reported earnings that are sufficiently high or low, specifically above $+1$ or below -3 in this example, prices based on reported earnings with discretion are higher than those based on "true" cashflows, reflecting the adjustments made by investors for discretionary earnings. Figure 2b also shows the prices for the off-equilibrium earnings reports (between -3 and -1 and between -1 and $+1$) as well as the prices for the pooled earnings reports (at -1 and $+1$). Finally, we see that for intermediate levels of reported earnings, the graph of prices is first relatively flat and then has a relatively steep slope, again reflecting the investors' inferences concerning the amount of discretionary earnings being reported.

Figure 2a compares the pricing that would result if the manager reported honestly and the investors knew this (labeled as the pricing under true cashflows) to the pricing that would result if the credible equilibrium of theorem 1 held. In such a credible equilibrium, the investors infer the cashflows perfectly for sufficiently good and sufficiently bad news. Continuing with the above example, this occurs for cashflows below -2 and above 2 (that is, reported earnings below -3 and above 1). However, since the manager pools in the interval $[-2, 0)$, the investors assign an average price to these inferred cashflows. The same happens for the cashflows in the interval $[0, 2]$. As figure 2a shows, investors perfectly infer the true cashflows from the reported earnings unless the true cashflows are in the interval $[-2, 2]$. Hence, for all reports—except those in the pooling areas surrounding zero earnings surprise—investors perfectly invert the reports and the manager's discretion does not distort the pricing.

The true cashflows and the investors' inferred cashflows shown in figure 2a are never observed by outsiders. What is observed is the pricing of the reported earnings. Continuing with the example, the pricing of reported

earnings is shown in figure 2b, again, with the pricing of "true" cashflows included as a benchmark. Figure 2b illustrates two points. First, the pricing based on reported earnings appears distorted relative to the price that one would expect if cashflows were reported without discretion. However, this cannot be interpreted to mean that mispricing exists. As was seen in figure 2a, cashflows are perfectly inferred for most reports. Second, the slope of the graph based on reported earnings is steeper than the one that is based on the true cashflows in the interval from slightly negative earnings surprise to the slightly positive earnings surprise. Hence, the penalty for missing the target earnings is exaggerated for prices based on reported earnings with discretion when compared to the penalty that is expected for the prices set assuming earnings are reported without discretion. Further, this result is more pronounced for firms with higher price earnings ratios (higher M in our model), consistent with recent empirical arguments (see for example Skinner and Sloan [2000]).

4.4 EQUILIBRIA WITH NON-MONOTONIC PRICING

While it seems more likely that pricing will be monotonic, non-monotonic pricing is possible (as shown by Subramanyam [1996]). To address this possibility, we consider a price equation that has the non-monotonic pricing structure specified in condition A2. Theorem 2 illustrates that both smoothing and taking a "big bath" continue to characterize an optimal disclosure policy.

THEOREM 2. *Suppose pricing is non-monotonic (condition A2 holds) in the setting of reporting without discretion (i.e., for $P_1(y_1; \psi \equiv 0, \psi^S \equiv 0)$). Then, in the setting with discretion, there exists a non-empty interval of cashflow surprise realizations, ΔY_A, such that for each $\Delta y_A \in \Delta Y_A$, there exists a credible equilibrium, where $P_1(y_1; \psi^*, \psi^{S*})$ is characterized by the non-monotonic pricing of condition A2. The credible equilibrium disclosure strategy involves maximum under-reporting either for sufficiently good news or sufficiently bad news, and smooth reporting for intermediate levels.*

Theorem 2 shows that there exists a set of credible disclosure strategies and it characterizes this set in terms of an interval of cashflow surprise realizations, ΔY_A. Each cashflow surprise in this set, $\Delta y_A \in \Delta Y_A$, determines a separate credible disclosure strategy. The structure of these strategies is very similar; in all of them the manager takes a big bath for sufficiently bad news and smoothes good news, pooling at report Δy_A when possible. The smoothing includes both over and under-reporting, depending on the level of the cashflows that the manager observes. The strategies differ only in the choice of the report, Δy_A, that acts as the pooled level of reported earnings.

Each credible disclosure strategy is composed of four distinct areas of disclosure, depending on the level of cashflow surprise. For cashflow surprise realizations below the minimal level denoted by Δy_{min}, the manager under-reports the maximal amount and takes a "big bath." For levels of cashflow surprise above the minimal level, the disclosure strategy is based on a focal

level of cashflow surprise, Δy_A, from the set ΔY_A. For levels immediately above the minimal level, he over-reports the maximum amount until he reaches the point where he can report the level, Δy_A. For cashflow surprise realizations in the interval $[\Delta y_A - \delta_U, \Delta y_A - \delta_L,]$ the manager is able to report Δy_A, and he does so. For yet higher realizations, the manager chooses to under-report the maximum amount.

Next, we extend the analysis to consider naïve investors, and find the resulting "naïve" equilibrium. Similar to the case of monotonic pricing, the form of the naïve and credible disclosure strategies are very similar, as corollary 2 indicates.

COROLLARY 2.   *Let the conditions of theorem* 2 *hold but assume* $\psi^S \equiv 0$. *Then there exists a cashflow surprise realization,* $\Delta y_B < 0$, *that determines the unique naïve equilibrium,* $(\psi^*, \psi^S \equiv 0, P)$, *where the price equation* $P_1(y_1; \psi^*, \psi^S \equiv 0)$ *is characterized by condition A2. Similar to the credible strategy, the naïve strategy involves maximum under-reporting for either sufficiently good news or sufficiently bad news, and smooth reporting for intermediate levels. Furthermore, this equilibrium also is credible if and only if* $\Delta y_{min} = \Delta y_B$ *and* $\Delta y_{max} \in \Delta Y_A$, *where* ΔY_A *is as defined in theorem* 2 *and* Δy_{max} *and* Δy_{min} *are the maximum and minimum levels of cashflows, as defined in condition A2.*

Corollary 2 shows that a naïve equilibrium exists where pricing meets condition A2. In this equilibrium, the disclosure strategy involves taking a big bath and smoothing depending on whether the news is bad or good, just as it did for the credible equilibria of theorem 2. This equilibrium is determined by a cut-off level, $\Delta y_B < 0$, at which point the manager is just indifferent between choosing δ_L or δ_U. Not only does the equilibrium strategy under functional fixation take the same form as the credible disclosure strategies, the naïve disclosure strategy may itself be credible.

Theorem 2 and corollary 2 clarify how the pricing structure specified by condition A2 affects the equilibrium disclosure strategies. The point at which the manager switches from taking a big bath to maximally over-reporting and the interval over which smoothing occurs both depend on the curvature of the price change equations (in particular, on Δy_{min}). These results, together with theorem 1 and corollary 1, demonstrate the robustness of our result. Specifically, they demonstrate that the structure of the disclosure policy is robust to differing price structures and differing assumptions on investor beliefs.

5. *Summary*

We model earnings smoothing and the taking of "big baths" as equilibrium behavior. Our model assumes that investors do not know the precision of the reported earnings. Hence, reported earnings are used to make inferences about both the true level of the long-run earnings stream and about the precision of the earnings announcement. As in other settings, the manager wishes to report higher earnings in order to convey higher long-run earnings. However, if the news is good, he will wish to report smaller

earnings surprises in order to raise the inferred precision of his earnings report. It is this latter incentive that drives the demand for smoothing. Also, if the news is bad, the manager will wish to introduce additional noise into his report, in order to reduce the inferred precision of the report. He does this by taking a "big bath" in the current period, thus enabling him to shift the discretionary income into subsequent periods.

Our results show that disclosure strategies that involve taking a big bath for bad news and smoothing good news are robust strategies. In particular, we show that these strategies may be optimal either when investors are naïve and assume no manipulation by management or when investors are sophisticated and correctly infer the disclosure strategy being adopted. We also show the same structure for the disclosure strategy when prices are monotonic or non-monotonic. Under monotonic pricing, we show that no pure strategy credible equilibrium exists, but we derive a partially separating credible equilibrium, where the pooling occurs at a report slightly above the expected level of earnings. Using a numerical example, we illustrate that the credible equilibrium strategy exhibits a number of characteristics that are consistent with recent empirical research. More specifically, we show that no earnings are reported slightly below the expected level, earnings reports are concentrated at a level slightly above the expected level, and the distribution of prices have fatter left tails and thinner right tails than are expected in the absence of discretion. Further, we show that the equilibrium disclosure strategy results in pricing in which the change in pricing due to missing the earnings target seems to be amplified. These pricing results hold even though investors perfectly infer the underlying cashflows over a large portion of the range of reported earnings.

The model developed in this paper offers a number of avenues for future research. First, we assume that the manager only observes the first period cashflows. The model may be extended to consider how the disclosure strategy would change if the manager had private information, either about the permanent level of earnings or about the second period earnings. Second, we assume that the available discretion is exogenously given. In practice, it seems likely that the available discretion will depend on prior reporting choices. A more difficult question concerns how the disclosure policy is related to reporting choices made sequential over many periods. These and other questions suggest that the model of smoothing based on uncertain precision may be a fruitful path for future research.

While, undoubtedly, some companies have misused accounting flexibility, our work suggests an alternative, more benign interpretation for smoothing behavior. (For discussions of recent empirical work supporting the argument that smoothing behavior may add to the informativeness of accounting earnings, see Hunt, Moyer, and Shevlin [1996].) In contrast to interpreting smoothing as an abuse of flexibility in reporting, we argue that rational managers, who try to maximize the value of their firms, may be using their reporting discretion, within the confines of acceptable accounting and legal requirements, to maximize the value of the companies they manage.

APPENDIX A— DETAILED DERIVATION OF PRICE EQUATIONS AND DEFINITIONS OF EQUILIBRIUM

A.1 Derivation of the Expectations

We assume that the permanent cashflow amount, X, is normally distributed with mean μ and precision h_X, which is written as $X \sim N(\mu, 1/h_X)$. The random variable, $Y_n = X + T_n$, denotes the total cashflow in period $n = 1, 2$, where $T_n \sim N(0, 1/h_T)$ is the amount of transitory cashflows in period n. Hence, for period $n = 1$ or 2, we have $Y_n \sim N(\mu, 1/h_Y)$ where $1/h_Y = 1/h_X + 1/h_T$, so that $0 \le h_Y < h_X$ holds. For Section A.1 we assume that h_Y is known. We use "Δ" to denote the deviation from the mean. For example, $\Delta x = x - \mu$, while $\Delta y_n = y_n - \mu$ and $\Delta m_n = m_n - \mu$ for period $n = 1$ or 2. The assumption that transitory cashflows are independent, identically distributed and uncorrelated with the permanent cashflows simplifies the analysis. First, it implies that the co-variance between permanent and total cashflows in period n, denoted as σ_{Xn}, equals one over the precision of permanent cashflows, $1/h_X$, since

$$\sigma_{Xn} = E[(\Delta X)(\Delta Y_n)] = E[(\Delta X)(\Delta X + T_n)] = E[(\Delta X)^2 + (\Delta X)T_n] = 1/h_X.$$

Second, it implies that the covariance between the total cashflow in periods one and two, denoted as σ_{12}, also equals $1/h_X$, since

$$\sigma_{12} = E[(\Delta Y_1)(\Delta Y_2)] = E[(\Delta X + T_1)(\Delta X + T_2)]$$
$$= E[(\Delta X)^2 + (\Delta X)(T_1 + T_2) + T_1 T_2] = 1/h_X.$$

Third, this implies that the expectation of the permanent cashflow surprise based on period one cashflow, $E[\Delta X \mid y_1]$, equals the expected second period cashflow based on the first period cashflow, $E[\Delta Y_2 \mid y_1]$, since

$$E[\Delta X \mid y_1] = \Delta y_1 \sigma_{X1} h_Y = \Delta y_1 h_Y / h_X = \Delta y_1 \sigma_{12} h_Y = E[\Delta Y_2 \mid y_1].$$

Expected permanent cashflow surprise based on two period cashflow, denoted as $E[\Delta X \mid y_1, y_2]$, is

$$E[\Delta X \mid y_1, y_2] = \left(\frac{\sigma_{X1}/h_Y - \sigma_{X2}\sigma_{12}}{(1/h_Y)^2 - (\sigma_{12})^2} \right) \Delta y_1 + \left(\frac{\sigma_{X2}/h_Y - \sigma_{X1}\sigma_{12}}{(1/h_Y)^2 - (\sigma_{12})^2} \right) \Delta y_2.$$

Substituting for σ_{X1}, σ_{X2} and σ_{12} from above and rearranging gives

$$E[\Delta X \mid y_1, y_2] = \frac{\left(\frac{1/h_Y - 1/h_X}{h_X} \right)}{(1/h_Y)^2 - (1/h_Y)^2} (\Delta y_1 + \Delta y_2)$$
$$= \left(\frac{1/h_X}{1/h_Y + 1/h_X} \right) (\Delta y_1 + \Delta y_2) = \frac{h_Y}{h_Y + h_X} (\Delta y_1 + \Delta y_2).$$

To summarize, we have $E[\Delta X \mid y_1] = \frac{\Delta y_1 h_Y}{h_X}$ and $E[\Delta X \mid y_1, y_2] = (\Delta y_1 + \Delta y_2) \frac{h_Y}{h_Y + h_X}$. Hence, assuming the investors know h_Y, the investors'

expectations are given as follows:

$$\Delta \hat{y}_1 \equiv E[\Delta Y_1 \mid m_1, \psi^S] = \Delta m_1 - \delta^S \quad \text{and}$$

$$E[\Delta X \mid m_1, \psi^S] = (\Delta m_1 - \delta^S) \frac{h_Y}{h_X} = \Delta \hat{y}_1 \frac{h_Y}{h_X},$$

$$\Delta \hat{y}_2 \equiv E[\Delta Y_2 \mid m_1, m_2, \psi^S] = \Delta m_2 + \delta^S \quad \text{and}$$

$$E[\Delta X \mid m_1, m_2, \psi^S] = \frac{(\Delta \hat{y}_1 + \Delta \hat{y}_2) h_Y}{h_Y + h_X}.$$

As described above (at the end of section 3.1 in the body of the paper), $\Delta \hat{y}_1$ and $\Delta \hat{y}_2$ denote the first and second period cashflow surprise inferred by the investor. These expectations are useful in the derivation of the price and objective functions when the precision is unknown.

A.2 Derivation of the Price Equations

For most of this section, we continue with the assumption that h_Y is known, relaxing it at the end. Denoting firm value at date n as P_n, we have, for date $n = 0$, that the price is the expected present value of an infinite stream of cashflows of X in each period, discounted by a factor $R = 1 + r$. This is given as

$$P_0 = E[R^{-1}x + R^{-2}x + R^{-3}x + \cdots] = \frac{E[x]}{r} = M\mu,$$

where $M \equiv 1/r$. Two features complicate the derivation of the price equations. First, price reflects the dividend policy. Second, it depends on the inferences made by investors based on their information. The investors know their inferred disclosure strategy, ψ^S. Also, at date 1 they observe m_1 and at date 2, they know m_1 and m_2, so the price at each of these dates will depend on how investors use this information. Using these two features, the ex dividend prices at dates 1 and 2 are given as follows:

$$P_1(y_1; \psi, \psi^S) = ME[X \mid m_1, \psi^S] + E[Y_1 \mid m_1, \psi^S]$$

$$- \text{dividend in period one, and}$$

$$P_2(y_1, y_2; \psi, \psi^S) = ME[X \mid m_1, m_2, \psi^S] + E[Y_1 + Y_2 \mid m_1, m_2, \psi^S]$$

$$- \text{dividend in periods 1 and 2.}$$

At each date, price is composed of three elements. First, investors price the permanent portion of earnings based on the available information; this is the first expectation in each equation. Second, they price the contemporaneous cashflow amount, again based on their available information; this is the second expectation in each equation. Third, price is reduced for the dividends. The role of dividends requires additional explanation.

The price set by investors will reflect the rate of return that investors expect that the firm will earn on any difference between the dividends and the expected permanent cashflow component. This additional complication

will not affect our analysis, but it would complicate the presentation. We avoid this additional complication by assuming that dividends equal the expected permanent cashflow component. This assumption is consistent with assuming that dividends convey no additional information other than what is included in the earnings reports. Hence, setting period 1 dividends equal to $E[X \mid m_1, \psi^S]$, the ex dividend price at date 1 becomes

$$
\begin{aligned}
P_1(y_1; \psi, \psi^S) &= (M-1) E[X \mid m_1, \psi^S] + E[Y_1 \mid m_1, \psi^S] \\
&= P_0 + (M-1)(E[X \mid m_1, \psi^S] - \mu) + E[Y_1 \mid m_1, \psi^S] - \mu \\
&= P_0 + (M-1) E[\Delta X \mid m_1, \psi^S] + E[\Delta Y_1 \mid m_1, \psi^S] \\
&= P_0 + \Delta \hat{y}_1 \left(\frac{(M-1) h_Y}{h_X} + 1 \right).
\end{aligned}
$$

The final equality in this expression uses the expectations derived in the previous section. The price in period 1 is based on the first period reported earnings surprise Δm_1. The change in price from date 0 to date 1 is composed of two portions: the permanent surprise portion, $E[\Delta X \mid m_1, \psi^S]$, which is capitalized by the factor $(M-1)$, and the portion inferred to be the transitory surprise, $E[\Delta Y_1 \mid m_1, \psi^S]$, which is capitalized by a factor of \$1. The simplicity of this price equation derives primarily from the normality assumption.

For period 2, the net dividend is $2E[X \mid m_1, m_2, \psi^S] - E[X \mid m_1, \psi^S]$, so the accumulated dividends equal $2E[X \mid m_1, m_2, \psi^S]$. Hence, the ex dividend price at date 2 can be written as follows:

$$
\begin{aligned}
P_2(y_1, y_2 : \psi, \psi^S) &= M E[X \mid m_1, m_2, \psi^S] + E[Y_1 + Y_2 \mid m_1, m_2, \psi^S] \\
&\quad - 2E[X \mid m_1, m_2, \psi^S] \\
&= P_0 + (M-2) E[\Delta X \mid m_1, m_2, \psi^S] \\
&\quad + E[\Delta Y_1 \Delta Y_2 \mid m_1, m_2, \psi^S] \\
&= P_0 + (\Delta \hat{y}_1 + \Delta \hat{y}_2) \left(\frac{(M-2) h_Y}{h_Y + h_X} + 1 \right) \\
&= P_0 + (\Delta \hat{y}_1 + \Delta \hat{y}_2) \left(\frac{(M-1) h_Y + h_X}{h_Y + h_X} \right).
\end{aligned}
$$

The final equality in this expression again uses the expectations derived earlier in section A.1, as well as the equality $\Delta y_1 + \Delta y_2 = \Delta \hat{y}_1 + \Delta \hat{y}_2$. Just as for the price at date 1, the price at date 2 is composed of two parts, a permanent surprise portion, $E[\Delta X \mid m_1, m_2, \psi^S]$, capitalized by a factor of $(M-2)$, and the portion inferred to be a transitory surprise, $E[\Delta Y_1 + \Delta Y_2 \mid m_1, m_2, \psi^S]$, which is capitalized by a factor of \$1.

To this point, we had assumed h_Y was known; next, we show the price equation with unknown precision. Let $f(h_Y)$, $f(h_Y \mid m_1, \psi^S)$, etc., represent the marginal and conditional distributions for the precision variable and define $E[H_1 \mid m_1, \psi^S] = \int_{H_Y} h_Y f(h_Y \mid m_1, \psi^S) dh_Y$. With unknown h_Y, the initial price remains the same, so that $P_0 = M\mu$. However, the prices at dates 1 and 2 change to reflect the investors' inference of the expected variance.

These prices maybe summarized as follows:

$$P_1(y_1, \psi, \psi^S) = P_0 + \Delta y_1 \int_{H_Y} \left(\frac{(M-1)h_Y}{h_X} + 1 \right) f(h_Y \mid m_1, \psi^S) \, dh_Y$$

$$= P_0 + \Delta \bar{y}_1 \left((M-1) \left(\frac{E[H_Y \mid m_1, \psi^S]}{h_X} \right) + 1 \right)$$

$$P_2(y_1, y_2; \psi, \psi^S) = P_0 + (\Delta y_1 + \Delta y_2) \int_{H_Y} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right)$$

$$\times f(h_Y \mid m_1, m_2, \psi^S) \, dh_Y.$$

To insure that these price equations are well defined, additional structure must be created. However, first we derive the manager's objective function assuming that these price equations exist.

A.3 Derivation of the Manager's Payoff Function

The manager wishes to choose δ to maximize his payoff, given his information at date 1. We assume that his payoff is the change in price from date 0 to date 2, so that the payoff he wishes to maximize is given by the expression $W(y_1, y_2; \psi, \psi^S) = P_2(y_1, y_2; \psi, \psi^S) - P_0$. While our results will hold under a more complicated payoff function, we focus on this simple function to better address our research objectives. We wish to characterize equilibria that involve smoothing behavior that is not driven by myopic behavior by the manager, as would be the case if we were to assume that he wished to maximize his payoff in a single, exogenously determined period. Also, we wish to address the situation in which the manager's demand for smoothing is not driven by an exogenously imposed restriction on the manager's ability to borrow or lend. Assuming that the manager wishes to maximize the second period price, regardless of the level of price at date 1, allows us to address the case where these aspects are nullified.

Continuing with the analysis of the manager's optimization problem: If the manager knows y_1, then he knows $\bar{y}_1$, and he wishes to maximize his expected payoff, which is written as

$$EW(y_1; \psi, \psi^S) = \int_{\bar{y}_2} \left(\int_{H_2} (P_2(y_1, y_2; \psi, \psi^S) - P_0) f(h_Y \mid \bar{y}_1, \bar{y}_2) \, dh_Y \right)$$

$$\times f(\bar{y}_2 \mid y_1, \psi, \psi^S) \, d\bar{y}_2$$

$$= \int_{\bar{y}_2} (\Delta y_1 + \Delta y_2) \left(\int_{H_2} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right) \right.$$

$$\left. \times f(h_Y \mid \bar{y}_1, \bar{y}_2, \psi^S) \, dh_Y \right) f(\bar{y}_2 \mid y_1, \psi, \psi^S) \, d\bar{y}_2.$$

Since

$$f(h_Y \mid y_1, \bar{y}_2) \, f(\bar{y}_2 \mid y_1, \psi, \psi^S) = f(h_Y, \bar{y}_2 \mid y_1, \psi, \psi^S)$$
$$= f(\bar{y}_2 \mid h_Y, y_1, \psi, \psi^S) \, f(h_Y \mid y_1, \psi, \psi^S),$$

substituting back into the equation and integrating first with respect to $\bar{y}_2$ and then h_Y gives

$$EW(y_1; \psi, \psi^S) = \int_{H_Y} \left(\Delta y_1 + \frac{\Delta y_1 h_y}{h_X} \right) \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right)$$

$$\times f(h_Y \mid \bar{y}_1, E[\bar{y}_2 \mid y_1, \psi, \psi^S]) \, dh_Y$$

$$= \Delta y_1 \int_{H_Y} \left(\frac{(M-1)h_y}{h_X} + 1 \right) f(h_Y \mid \bar{y}_1, E[\bar{y}_2 \mid y_1, \psi, \psi^S]) \, dh_Y$$

$$= \Delta y_1 \left(\frac{(M-1)E[H_y \mid y_1, \psi, \psi^S]}{h_X} + 1 \right).$$

We write the expectation $E[H_Y \mid \bar{y}_1, E[\bar{y}_2 \mid y_1, \psi, \psi^S]]$ in abbreviated form as $E[H_Y \mid y_1, \psi, \psi^S]$. This is the precision that the manager expects the investor to use in pricing in period 2. It is based on two reports, $\bar{y}_1$ and $E[\bar{y}_2 \mid y_1, \psi, \psi^S]$, and reflects the manager's better information, specifically his knowledge of ψ. The manager does not know the actual realization h_Y, so he forms his expectation of $\Delta \bar{y}_2$ based on his expectation of the precision, so that $E[\Delta \bar{y}_2 \mid y_1, \psi, \psi^S] = \Delta y_1 E[H_Y \mid y_1] / h_X - \delta + \delta^S$. Distinguishing among the different expectations will be critical in deriving some of the results. The role of these expectations is discussed further in the proofs of lemma 1 and theorem 1 in appendix B below.

Having defined the manager's payoff, we now return to pricing under managerial discretion. Specifically, we define complete inferred disclosure strategies and consistent pricing.

A.4 *Definition of Complete Inferred Disclosure Strategies and Consistent Prices*

In general the investors will use the earnings reported in both periods 1 and 2 when they infer the amount of discretion that is reported by the manager. This means that in general, the inferred disclosure strategy of the investors is a mapping $\psi^S(m_1, m_2): \Re \times \Re \to [\delta_L, \delta_U]$. As we discussed in footnote 5 in the text, for most of the reports issued in the credible equilibrium we study, the investors will perfectly infer the cashflow amount. Hence, in these situations, we can, without loss of generality, suppress the second argument in this function. Since the second period earnings report is used by the investors to infer the manager's discretion only when multiple cashflow amounts are associated with the first period earnings report, this is the

only time we write $\psi^S(m_1, m_2)$ instead of just $\psi^S(m_1)$. As we explain below, this is exactly the situation that requires the introduction of the definition of consistent prices.

There are two problems that we must resolve in order to have well defined prices in our model. To understand these problems, consider the general definition of an inferred disclosure strategy of the investors given above, that is the mapping $\psi^S(m_1, m_2)$: $\Re \times \Re \rightarrow [\delta_L, \delta_U]$. If this mapping is a function whose domain is the entire real line, pricing with discretion is straightforward. This was implicitly done in the price equations derived at the end of section A.2 above. However, problems arise either if this mapping is not defined over the entire set of possible earnings reports (i.e., if the domain of $\psi^S(\bullet)$ is a strict subset of $\mathbb{R}$) or if this mapping is not a function but a correspondence. The first problem occurs if some earnings amounts are never expected to be reported in equilibrium. In this case, we must specify how the firm would be valued for such off-equilibrium reported earnings. The second problem occurs if investors may infer multiple levels of cashflow from a single earnings report; in this case we must specify how the different cashflow inferences should be aggregated to value the firm.

We address both of these problems by introducing additional structure to the model. First, we use pricing without discretion to specify a complete inferred disclosure strategy, that is, an inferred disclosure strategy that is defined over all the possible earnings reports. Second, we describe the pricing under arbitrary disclosure strategies, in which multiple cashflow realizations may be inferred from a single earnings report. We do this by assuming that price equals the expected price, with the expectation taken over the set of feasible inferred cashflow amounts.

For the first problem, suppose there is an earnings report that the investor expects never to observe, i.e., the domain of $\psi^S(\bullet)$ is a strict subset of $\mathbb{R}$, the set of real numbers. This means there exists a pair of reports, (m_1, m_2), such that the discretion is the null set, or $\psi^S(m_1, m_2) = \{\varnothing\}$. For investors with inferred disclosure strategy $\psi^S(\bullet)$, denote the set of reporting pairs that he never expects to see as $M_{null}(\psi^S) = \{(m_1, m_2) \mid \psi^S(m_1, m_2) = \{\varnothing\}\}$. To specify how the investors react to such a report, we extend the inferred strategy to a mapping that has the entire set of real numbers as its domain. We call this inferred disclosure strategy complete. We do this by requiring investors to infer the discretionary amount to be that value which is both feasible and which results in the lowest price change. Formally, a complete inferred disclosure strategy is given as follows:

DEFINITION OF A COMPLETE INFERRED DISCLOSURE STRATEGY. Consider any inferred disclosure strategy, $\psi^{S*}(m_1, m_2)$: $\Re \times \Re \rightarrow [\delta_L, \delta_U]$ and define $M_{null}(\psi^S) = \{(m_1, m_2) \mid \psi^S(m_1, m_2) = \{\varnothing\}\}$. We will define $\psi^S(m_1, m_2)$ as the complete inferred disclosure strategy derived from $\psi^{S*}(m_1, m_2)$, where $\psi^S(m_1, m_2) = \psi^{S*}(m_1, m_2)$ if $(m_1, m_2) \notin M_{null}(\psi^S)$, and if $(m_1, m_2) \in M_{null}(\psi^S)$, then we have $\psi^S = \delta$, where $\delta = \arg \min_{\delta' \varepsilon [\delta_L, \delta_U]} \{P(y_1) \mid y_1 = m_1 - \delta'\}$.

Every inferred disclosure strategy gives rise to a unique complete disclosure strategy. In subsequent analysis we consider only complete inferred disclosure strategies.

Next, suppose $\psi^S(m_1, m_2)$ is a correspondence, with investors inferring that more than one level of discretion is possible when they observe earnings report m_1. Hence, a single earnings report may produce multiple inferred cashflows, and correspondingly, multiple possible prices. In this case, we assume that the investors value the firm based on the expected value of the inferred cashflow, with the expectation taken over the set of possible inferred cashflows. Let $D^S(m_1, m_2) = \{\delta^S \,|\, \delta^S = \psi^S(m_1, m_2)\}$ be the set of discretionary amounts inferred by the investors when they observe m_1 and m_2. Refer to a price equation as consistent if, when the set $D^S(m_1, m_2)$ is not a singleton, the price equals the expected value of the price equation evaluated over the set of cashflows inferred from the set $D^S(m_1, m_2)$. Formally, the definition of a consistent price equation is given as follows:

DEFINITION OF A CONSISTENT PRICE EQUATION. Given a disclosure strategy $\psi(\bullet)$ and a complete inferred disclosure strategy $\psi^S(\bullet)$, the price equation $P_2(y_1, y_2; \psi, \psi^S)$ is called consistent if for each cashflow realization $y_1 \in \mathbb{R}$, where $D^S(m_1, m_2) = \{\delta^S \,|\, \delta^S = \psi^S(m_1, m_2)\}$, the following holds:

$$P_2(y_1, y_2; \psi, \psi^S) = E[P_2(y_1, y_2; \psi, \psi^S) \,|\, \forall \delta^S \in D^S(m_1, m_2)].$$

As we discussed in the first paragraph of section A.4, the definition of consistent prices is the only situation where investors actually use the information in the second period report to infer the level of discretionary income being reported by the manager. Hence, this is the only time we need to write the function as $\psi^S(m_1, m_2)$ instead of just $\psi^S(m_1)$. In our subsequent analysis, and in the discussion in the text, we suppress the argument m_2 when discussing the inferred disclosure strategy.

Given a disclosure strategy and a complete inferred disclosure strategy, the vector of consistent prices is well defined. Next we define the equilibria.

A.5 Definition of Equilibrium

First, we define the optimal disclosure strategy in terms of complete inferred disclosure strategies and consistent prices as the strategy that maximizes the manager's objective function.

DEFINITION OF AN OPTIMAL DISCLOSURE STRATEGY. A disclosure strategy, ψ, is optimal with respect to the investors' inferred disclosure strategy, ψ^S, and a vector of prices, P, if for any ψ' and for each y_1, the following holds:

$$E W(y_1; \psi, \psi^S) \geq E W(y_1; \psi', \psi^S).$$

Finally, we define naïve and credible equilibria in terms of completed inferred disclosure strategies, consistent prices and optimal disclosure strategies. Our definition of "credible" equilibrium, which is standard in the

accounting literature, is equivalent to a "Bayesian Nash" equilibrium, which is a standard term used in economics.

DEFINITION OF AN EQUILIBRIUM. An equilibrium is a triple, (ψ, ψ^S, P) where ψ is an optimal disclosure strategy with respect to the investors' complete inferred disclosure strategy, ψ^S, and a consistent price vector, P. Further, call this a "naïve" equilibrium if $\psi^S \equiv 0$, and call it a credible equilibrium, if for each $y_1 \in \mathbb{R}$, $m_1 = y_1 + \psi(y_1)$ implies $y_1 = m_1 - \delta^S$ for some $\delta^S \in D^S(m_1) = \{\delta^S \mid \delta^S = \psi^S(m_1)\}$.

APPENDIX B—PROOFS OF LEMMA 1, THEOREMS 1 AND 2, AND COROLLARIES 1 AND 2

LEMMA 1.   *If condition A1 holds for $P_1(y_1; \psi \equiv 0, \psi^S \equiv 0)$, then there does not exist a fully separating credible equilibrium in pure strategies where $P_1(y_1; \psi^*, \psi^{S*})$ is characterized by condition A1.*

PROOF OF LEMMA 1. We prove lemma 1 in three steps. In step 1, we derive the manager's optimal disclosure strategy when the manager knows the total cashflows in both period 1 and 2 in order to show that the optimal strategy is linear in both these variables. In step 2, we derive the optimal strategy when the manager knows only the first period cashflow, showing that this strategy depends on whether Δy_1 is positive or negative. While the proof of lemma 1 relies only on the manager's optimal strategy when Δy_1 is negative, his strategy for positive Δy_1 is used in the proof of theorem 1 and corollary 1. In step 3, we show the non-credibility of pure strategy equilibria. First though, we derive the first order condition for the manager's optimization problem.

If the manager knows both y_1 and y_2, he would wish to choose $\psi(y_1, y_2) = \delta$ in order to maximize the increase in the second period price, so that his optimization problem is given as follows:

$$\max_{\delta \in [\delta_L, \delta_U]} (P_2(y_1, y_2; \psi, \psi^S) - P_0)$$

$$= \max_{\delta \in [\delta_L, \delta_U]} \left((\Delta y_1 + \Delta y_2) \int_{H_Y} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right) f(h_Y \mid \bar{y}_1, \bar{y}_2) \, dh_Y \right).$$

Rational expectations by investors require that $y_1 = m_1 - \delta^S$ and $y_2 = m_2 + \delta^S$ both hold, which means that $f(h_Y \mid \bar{y}_1, \bar{y}_2) = f(h_Y \mid y_1, y_2)$ will also hold. It is clear that the manager wishes to choose discretion to maximize or minimize the integral when $(\Delta y_1 + \Delta y_2) > 0$ or when $(\Delta y_1 + \Delta y_2) < 0$, respectively. If $(\Delta y_1 + \Delta y_2)$ is positive, the optimal disclosure strategy solves the point-wise maximization problem. I.e., the strategy is the ψ function such that the first order condition (FOC), $\partial E[H_Y \mid \Delta y_1, \Delta y_2]/\partial \delta = 0$, holds for each pair of realizations, $(\Delta y_1, \Delta y_2)$. We obtain this FOC using an approach analogous to that used by Subramanyam [1996]. First, rewrite the

conditional distribution as

$$f(h_Y \mid \Delta \bar{y}_1, \bar{y}_2) = \frac{f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y)}{\int_{H_Y} f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y) dh_Y}.$$

Substituting into the integral and taking the derivative we have

$$0 = \int\limits_{H_Y} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right) \frac{\partial f(h_Y \mid \bar{y}_1, \bar{y}_2)}{\partial \delta} dh_Y$$

$$= \frac{\int_{H_Y} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right) \frac{\partial f(\bar{y}_1, \bar{y}_2 \mid h_Y)}{\partial \delta} f(h_Y) dh_Y \int_{H_Y} f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y) dh_Y}{\left(\int_{H_Y} f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y) dh_Y \right)^2}$$

$$- \frac{\int_{H_Y} \left(\frac{(M-1)h_Y + h_X}{h_Y + h_X} \right) f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y) dh_Y \int_{H_Y} \frac{f(\bar{y}_1, \bar{y}_2 \mid h_Y)}{\partial \delta} f(h_Y) dh_Y}{\left(\int_{H_Y} f(\bar{y}_1, \bar{y}_2 \mid h_Y) f(h_Y) dh_Y \right)^2}.$$

This equation holds when $\frac{\partial f(\bar{y}_1, \bar{y}_2 \mid h_Y)}{\partial \delta} = 0$, and again we have $\frac{\partial f(\bar{y}_1, \bar{y}_2 \mid h_Y)}{\partial \delta} = \frac{\partial f(y_1, y_2 \mid h_Y)}{\partial \delta}$ where $y_1 = m_1 - \delta^S$ and $y_2 = m_2 + \delta^S$. By assumption, $f(y_1, y_2 \mid h_Y)$ is the density of a bivariate normal distribution, so letting $\rho = h_Y / h_X$, we have

$$f(\Delta y_1, \Delta y_2 \mid h_Y) = \frac{h_Y}{2\Pi\sqrt{1 - \rho^2}}$$

$$\times \mathbf{exp}\left(-\frac{h_Y}{2(1 - \rho^2)} ((\Delta y_1)^2 - 2\rho \Delta y_1 \Delta y_2 + (\Delta y_2)^2) \right).$$

Solving for the FOC we have

$$\frac{\partial f(\Delta y_1, \Delta y_2 \mid h_Y)}{\partial \delta} = -\frac{h_Y}{2(1 - \rho^2)} (2\Delta y_1 - 2\rho \Delta y_2 + 2\rho \Delta y_1 - 2\Delta y_2)$$

$$\times f(\Delta y_1, \Delta y_2 \mid h_Y)$$

$$= -\frac{h_Y}{2(1 - \rho^2)} 2(1 + \rho)(\Delta y_1 - \Delta y_2) f(\Delta y_1, \Delta y_2 \mid h_Y)$$

$$= -(\Delta y_1 - \Delta y_2) \frac{h_Y}{1 - \rho} f(\Delta y_1, \Delta y_2 \mid h_Y)$$

$$= -(\Delta y_1 - \Delta y_2 + 2(\delta - \delta^S)) \frac{h_Y}{1 - \rho} f(\Delta y_1, \Delta y_2 \mid h_Y),$$

where the final equality used $y_1 = m_1 - \delta^S$ and $y_2 = m_2 + \delta^S$. This gives the optimal disclosure strategy when the manager knows both y_1 and y_2 as $\psi(y_1, y_2) = \delta^S - \frac{y_1 - y_2}{2}$.

Moving to step two, next suppose the manager knows only the first period cashflow, y_1. In this case he replaces y_2, in the above equation with his expectation of y_2. More specifically, the manager wishes to maximize the

expected objective function that is written as

$$EW(y_1; \psi, \psi^S) = \int_{H_Y} \Delta y_1 \left(\frac{(M-1)h_Y}{h_X} + 1 \right) f(h_Y \mid y_1, E[y_2 \mid y_1]) \, dh_Y$$

$$= \Delta y_1 \left(\frac{(M-1)E[H_Y \mid y_1, E[y_2 \mid y_1]]}{h_X} + 1 \right).$$

Without discretion, we have $f(h_Y \mid y_1, E[y_2 \mid y_1]) = f(h_Y \mid y_1)$, so that the expected second period price equals the actual first period price, as discussed at the end of section 3.2 in the body of the paper.

Once again, the manager chooses δ to maximize (or minimize) the expected precision given his information, $E[H_Y \mid y_1, E[y_2 \mid y_1]]$, as Δy_1 is positive (or negative). As we showed in step 1, the optimal strategy when the manager knows y_2 is linear in y_2. Since the manager is risk neutral, we can substitute the manager's expectation for the second period total cashflow, so that the optimal strategy when the manager knows only the first period cashflow is given as follows:

$$\psi(y_1, E[y_2 \mid y_1]) = \delta^S - \frac{y_1 - E[y_2 \mid y_1]}{2} = \delta^S - \frac{y_1(1-\rho)}{2}.$$

This means that given h_y and δ^S, then, for each y_1, the manager will choose discretion δ^* to solve the equation $(\Delta y_1(1-\rho) + 2(\delta^* - \delta^S)) = 0$.

We assume that, in equilibrium, the manager will know δ^S, but we assume that, instead of h_Y, the manager uses the expectation $E[H_Y \mid y_1]$. This means the manager replaces $\rho = h_Y / h_X$ with ρ_1 where we let $\rho_1 = E[H_Y \mid y_1]/h_X$. Hence, the FOC for sophisticated investors is $(\Delta y_1(1-\rho_1) + 2(\delta^* - \delta^S)) = 0$, so that, when Δy_1 is positive, the optimal discretionary disclosure amount is $\delta^* = -\Delta y_1(1-\rho_1)/2 + \delta^S$. This implies that the optimal level of inferred cashflow is $\Delta \bar{y}_1 = \Delta y_1 + \delta^* - \delta^S = \Delta y_1(1+\rho_1)/2$. Substitute $\delta^S = 0$ to give the FOC as $-\Delta y_1(1-\rho_1)/2 = \delta^*$ for naïve investors. This means $\Delta \hat{y}_1 = \Delta y_1 + \delta^* = \Delta y_1(1+\rho_1)/2$ is the optimal level of inferred cashflows for naïve investors, just as it was for the sophisticated investors. Next consider the manager's strategy when Δy_1 is negative. Recall that for negative Δy_1, the manager wishes to choose δ to minimize $E[H_Y \mid y_1, E[y_2 \mid y_1]]$. When $\Delta y_1 \leq 0$, the manager wishes to minimize the expected precision. Since $E[H_Y \mid y_1, E[y_2 \mid y_1]]$ is strictly decreasing in the sum $(\Delta \hat{y}_1)^2 + (E[\Delta \hat{y}_2 \mid y_1, \psi, \psi^S])^2$, the manager chooses δ to maximize the sum

$$(\Delta \bar{y}_1)^2 + \left(E[\Delta \bar{y}_2 \mid y_1, \psi, \psi^S] \right)^2 = (\Delta y_1 + \delta - \delta^S)^2 + (\Delta y_1 \rho_1 - \delta + \delta^S)^2.$$

Given δ^S, the manager maximizes this sum by under-reporting the maximal amount when $(\Delta y_1 - \delta^S) < (\Delta y_1 \rho_1 + \delta^S)$ and maximally over-reporting when this inequality reverses.

For step 3, we show the non-existence of a credible pure strategy. First, if $\delta^S \in [0, \delta_U]$, the manager chooses δ_L for all $\Delta y_1 < 0$. This implies the strategies $\delta^* = \delta^S \in [0, \delta_U]$ are not credible. Second, if $\delta^S \in [\delta_L, 0]$, he

chooses δ_U when $0 > \Delta y_1 (1-\rho_1) > 2\delta^S$, which means that the strategies $\delta^* = \delta^S \in [\delta_L, 0]$ are not credible. Hence, no pure strategy credible equilibrium exists, as stated in lemma 1.

<div align="right">Q.E.D. on lemma 1.</div>

THEOREM 1. *There exists a partial pooling credible equilibrium, (ψ^*, ψ^{S*}, P), where the manager maximally under-reports if the news is sufficiently good or bad and pools otherwise. More specifically, let y_L denote the cashflow realization that solves the equation $\Delta y_L (1 - E[H_Y \mid y_L]/h_X) = -2\delta_L$. Then the disclosure strategy, $\psi^*(y_1)$, is given as follows:*

If $\Delta y_1 \geq (\delta_U - \delta_L)$ or if $\Delta y_1 \leq -\Delta y_L$ then $\psi^*(y_1) = \delta_L$; otherwise the manager smoothes, reporting one of $N+1$ reports, where N is the integer that solves $N(\delta_U - \delta_L) \geq \Delta y_L \geq (N-1)(\delta_U - \delta_L)$. The disclosure policy for the $N+1$ different pooled reports is given as follows:

$$1) \quad \delta = -\Delta y_1 - ((N-1)\delta_U - N\delta_L)$$
$$\text{if } \Delta y_1 \in (-\Delta y_L, -(N-1)(\delta_U - \delta_L),);$$

$$2) \quad \delta = -\Delta y_1 - ((N-2)\delta_U - (N-1)\delta_L)$$
$$\text{if } \Delta y_1 \in [-(N-1)(\delta_U - \delta_L), -(N-2)(\delta_U - \delta_L));$$

$$\ldots$$

$$N-1) \quad \delta = -\Delta y_1 - (\delta_U - 2\delta_L)$$
$$\text{if } \Delta y_1 \in [-2(\delta_U - \delta_L), -(\delta_U - \delta_L));$$

$$N) \quad \delta = -\Delta y_1 + \delta_L$$
$$\text{if } \Delta y_1 \in [-(\delta_U - \delta_L), 0);$$

$$N+1) \quad \delta = -\Delta y_1 + \delta_U$$
$$\text{if } \Delta y_1 \in [0, (\delta_U - \delta_L)).$$

PROOF OF THEOREM 1. As in the proof of lemma 1, the manager's choice of disclosure strategy depends on whether Δy_1 is positive or negative. First, we show that for sufficiently good news and sufficiently bad news, under-reporting the maximum is a credible strategy. Second, we show that the pooling strategy of theorem 1 is credible by showing that the manager would not wish to deviate.

We know from the proof of lemma 1 that for $\Delta y_1 > 0$, and in particular for $\Delta y_1 \geq (\delta_U - \delta_L)$, the manager wishes to report $\delta^* = -\Delta y_1 (1 - \rho_1)/2 + \delta^S$. Hence, if $\delta^S = \delta_L$, the manager wishes to choose $\delta^* < \delta_L$. However, he is constrained from doing so, hence, $\delta^* = \delta_L$ is credible for any positive Δy_1, particularly for $\Delta y_1 \geq (\delta_U - \delta_L)$. Next, consider the case of bad news. From lemma 1, the manager wishes to under-report the maximal amount if $(\Delta y_1 - \delta^S) \leq (\Delta y_1 \rho_1 + \delta^S)$, that is, as long as $\Delta y_1 \leq 2\delta^S/(1 - \rho_1)$. So, for sufficiently bad news (i.e. for $\Delta y_1 \leq -\Delta y_L$ when $\delta^S = \delta_L$), $\delta^* = \delta_L$ is again a credible disclosure strategy.

We complete the proof by showing the manager's pooling strategy is credible. First, we show that the manager will not wish to deviate from the inferred disclosure strategy of theorem 1. Second, we show that the equilibrium is robust to investors updating using the second period earnings reports.

Consider the cashflows in the pooling range, i.e., for Δy_1 in the interval $-\Delta y_L \leq \Delta y_1 \leq (\delta_U - \delta_L)$. For most of these cashflows, in particular for cashflows in the interval $\Delta y_1 \in (\Delta y_L, 0)$, any deviation from the specified scheme results in the manager issuing off-equilibrium earnings reports. The manager would never wish to report off-equilibrium earnings, since these, by definition, provide the minimal expected payoff. Hence, he will not deviate for these cashflows in the pooling area below zero. For the remaining cashflows, that is, for the cashflows in the interval $\Delta y_1 \in [0, (\delta_U - \delta_L))$, the manager can deviate to report an equilibrium report by over-reporting. However, if he did so, the investors would infer that he is under-reporting by the maximal amount, which would cause the manager to wish to under-report, not over-report. If the manager wishes to under-report, he must either under-report an off-equilibrium report, or under-report by issuing the pooling report. Hence, he does not wish to deviate if he observes cashflows in this interval either. Since no deviation is desired, the disclosure strategy is credible for the pooling reports, as long as the investors cannot disentangle the pooling. This leads to the last step in the proof.

Throughout the proof, we have assumed the investors do not use the information of the second period report to infer the level of discretion chosen by the manager. Up to this point in the proof, this was clearly the case, as the investors were able to perfectly infer the manager's discretion without this information. However, if a pooling report is issued, the additional information from the second period earnings report will be used by the investors to infer the level of discretion being chosen by the manager. This is done and is used by the investors in setting prices, as is shown formally in the definition of consistent prices. However, since every level of second period earnings may be reported for each level of first period report, the amount of discretion cannot be inferred perfectly. While the information is used to set the price and is reflected in the manager's expected payoff, it will not affect the arguments presented in the preceding paragraph. Hence, the manager will continue to refrain from deviating, so that the strategy is still credible, despite the investors' use of the second period earnings in their inferences. This completes the proof of the theorem.

Q.E.D on theorem 1.

COROLLARY 1.   *Let the conditions of theorem 1 hold but assume $\psi^S(\bullet) \equiv 0$. Then $(\psi^*, \psi^S \equiv 0, P)$ is the unique naïve equilibrium, where $P_1(y_1; \psi^*, \psi^S)$ is characterized by Condition A1. Specifically, the manager chooses $\psi^*(y_1) = -\Delta y_1(1 - \rho_1)/2$ if $\Delta y_1 \in [0, \Delta y_L]$, where Δy_L is as defined in theorem 1, and $\psi^*(y_1) = \delta_L$ otherwise.*

PROOF OF COROLLARY 1. From the proof of lemma 1, we know that for negative Δy_1, the manager wishes to maximally under-report, so that $\delta^* = \delta_L$

is optimal. For Δy_1 positive, the manager wishes to report $\Delta m_1 = \Delta y_1 (1 + \rho_1)/2$, so that, if feasible, $\delta^* = -\Delta y_1(1 - \rho_1)/2$ is optimal. This is feasible and so it is chosen for $\Delta y_1 \in [0, \Delta y_L]$. However, for $\Delta y_1 > \Delta y_L$, the manager is constrained in his disclosure choice, so that $\delta^* = \delta_L$ is optimal.

<div align="right">Q.E.D. on Corollary 1.</div>

THEOREM 2. *If condition A2 holds for $P_1(y_1; \psi \equiv 0, \psi^S \equiv 0)$, then there exists a non-empty interval of realizations, ΔY_A, such that for each $\Delta y_A \in \Delta Y_A$, there exists a credible equilibrium, (ψ^*, ψ^{S*}, P), where $P_1(y_1; \psi^*, \psi^{S*})$ is characterized by condition A2 and where the manager chooses $\psi^*(y_1)$ as follows: He chooses $\psi^*(y_1) = -\Delta y_1 + \Delta y_A$ if $\Delta y_1 \in [\Delta y_A - \delta_U, \Delta y_A - \delta_L]$, chooses $\psi^*(y_1) = \delta_U$ if $\Delta y_1 \in [\Delta y_{min}, \Delta y_A - \delta_U]$, and chooses $\psi^*(y_1) = \delta_L$ if $\Delta y_1 \leq \Delta y_{min}$ or if $\Delta y_1 \geq \Delta y_A - \delta_L$, where Δy_{min} denotes the minimal level of cashflows, as defined in A2.*

PROOF OF THEOREM 2. First, we need additional notation. Denote the first period pricing equation without discretion as $P_1(y_1)$, so that $P_1(y_1) \equiv P_1(y_1; \psi = \psi^S \equiv 0)$, and denote Δy_{min} as the cashflow surprise which minimizes $P_1(y)$, consistent with condition $A2$. For an arbitrary cashflow surprise, Δy_1^*, let $EP_1(y_1^*)$ denote the average price of all cashflows that could generate Δy_1^* as the report, i.e., define $EP_1(y_1^*)$ as $EP_1(y_1^*) \equiv E[P_1(y) \mid \Delta y_1 \in [\Delta y_1^* - \delta_U, \Delta y_1^* - \delta_L]]$. Further, let ΔY_A denote the interval $\Delta Y_A = [\Delta y_{A,L}, \Delta y_{A,U}]$ where $y_{A,L}$ solves $EP_1(y_{A,L}) = P_1(y_{A,L} - \delta_L)$, and $y_{A,U}$ solves $EP_1(y_{A,U}) = P_1(y_{A,U} - \delta_U)$.

To prove the credibility of the disclosure strategies specified in theorem 2, we first show ΔY_A is non-empty by identifying a specific cashflow realization in the interval ΔY_A. We show the strategy identified in the theorem is credible for this realization, then we show this same argument applies to all realizations in the interval ΔY_A.

First, by assumption, $P_1(y)$ reaches a maximum, which means there exists a level of cashflow surprise, greater than zero and denoted as Δy_A^*, which solves $P_1(y_A^* - \delta_L) = P_1(y_A^* - \delta_U)$. By the consistency of P_1 the manager is compensated based on the expected value of the cashflow surprises inferred from his report. This means that, under the strategy specified in the theorem, he is paid $EP_1(y_A^*) \equiv E[P_1(y) \mid y_1 \in [\Delta y_A^* - \delta_U, \Delta y_A^* - \delta_L]]$, where $EP_1(y_A^*) > P_1(y_A^* - \delta_L) = P_1(y_A^* - \delta_U)$. Hence, $\Delta y_A^* \in \Delta Y_A$, so ΔY_A is non-empty.

Next, recall that for every $\Delta y_A \in \Delta Y_A$, the manager chooses $\delta^* = -\Delta y_1 + \Delta y_A$ and reports $\Delta m_1 = \Delta y_A$ for $\Delta y_1 \in [\Delta y_A - \delta_U, \Delta y_A - \delta_L]$. Also, $\delta^* = \delta_L$ for $\Delta y_1 > \Delta y_A^* - \delta_L$, while for $\Delta y_1 < \Delta y_A^* - \delta_U$, the manager chooses disclosure $\delta^* = \delta_L$ for $\Delta y_1 \leq \Delta y_{min}$ and $\delta^* = \delta_U$ for $\Delta y_1 \geq \Delta y_{min}$. We prove the credibility of strategy $\psi^*(\bullet)$ in two steps, first by analyzing it for $\Delta y_1 \geq \Delta y_A^* - \delta_U$ and second, for $\Delta y_1 < \Delta y_A^* - \delta_U$.

First consider $\Delta y_1 \geq \Delta y_A^* - \delta_U$. By construction, $\Delta y_{max} < \Delta y_A^* - \delta_L$ implies $P_1(y)$ is decreasing on $\Delta y_1 > \Delta y_A^* - \delta_L$ so choosing $\delta^* = \delta_L$ is credible for these observations. For $\Delta y_1 \in [\Delta y_A^* - \delta_U, \Delta y_A^* - \delta_L]$, the disclosures are

also credible. For $\Delta m_1 > \Delta y_A^*$, investors infer cashflow surprise of $\Delta m_1 - \delta_L$ and the manager is paid $P_1(\Delta m_1 - \delta_L) < P_1(\Delta y_A^* - \delta_L) < E P_1(y_A^*)$. For reports $\Delta m_1 < \Delta y_A^*$, the investors infer cashflow surprise of $\Delta m_1 - \delta_U$, and the manager is paid $P_1(m_1 - \delta_U) < P_1(y_A^* - \delta_U) < E P_1(y_A^*)$.

For $\Delta y_1 < \Delta y_A^* - \delta_U$, $\delta^* = \delta_L$ for $\Delta y_1 \leq \Delta y_{min}$ and $\delta^* = \delta_U$ for $\Delta y_1 \geq \Delta y_{min}$. If $\Delta y_1 = \Delta y_{min}$, then the manager could choose $\delta \in (\delta_L, \delta_U)$, i.e. an interior disclosure, but these are off equilibrium reports, and by the completeness of ψ^S, the manager is compensated $P_1(y_{min})$ anyway. If $\Delta y_1 < \Delta y_{min}$, then the manager could choose $\delta \in (\delta_L, \delta_U]$, in which case either the report is an off equilibrium report, and the manager receives $P_1(y_{min}) < P_1(y_1)$, or the investor infers the manager is reporting δ_L, and the manager receives $P_1(y_1 + \delta - \delta_L) < P_1(y_1)$. An analogous analysis shows $\delta^* = \delta_U$ for $\Delta y_A + \delta_L > \Delta y_1 \geq \Delta y_{min}$, completing the proof of the credibility of ψ^* based on Δy_A^*.

Finally, since these arguments proving credibility apply to any cashflow realization, Δy_1^*, as long as $E P_1(y_1^*) > P_1(y_1^* - \delta_L)$ and $E P_1(y_1^*) > P_1(y_A^* - \delta_U)$ both hold, it suffices to show that these inequalities hold for $\Delta y_A \in [\Delta y_{A,L}, \Delta y_{A,U}]$. First, start at Δy_A^* and consider the gradually increasing cashflow surprises in the interval $\Delta y_A \in [\Delta y_A^*, \Delta y_{A,U}]$. $E P_1(y_A)$ may initially increase, but eventually it will decrease, due to $P_1(\cdot)$ being concave. Further, $P_1(y_A - \delta_U)$ increases and $P_1(y_A - \delta_L)$ decreases until the upper endpoint of this range, $\Delta y_{A,U}$, is reached, at which point $E P_1(y_{A,U}) = P_1(y_{A,U} - \delta_U) > P_1(y_{A,U} - \delta_L)$ holds. This demonstrates the credibility of the strategies based on $\Delta y_A \in [\Delta y_A^*, \Delta y_{A,U}]$. Repeat the procedure for decreasing cashflow surprises starting at Δy_A^* in the range $\Delta y_A \in [\Delta y_{A,L}, \Delta y_A^*]$. Again $E P_1(y_A)$ will decrease, and now $P_1(y_A - \delta_U)$ decreases and $P_1(y_A - \delta_L)$ increases until the lower endpoint of this interval, $\Delta y_{A,L}$, is reached, at which point $E P_1(y_{A,L}) = P_1(y_{A,L} - \delta_L) > P_1(y_{A,L} - \delta_U)$ holds. Hence, the strategies based on $\Delta y_A \in [\Delta y_{A,L}, \Delta y_{A,}^*]$ are also credible, completing the proof of the theorem.

Q.E.D. on theorem 2.

COROLLARY 2.   *Let the conditions of theorem 2 hold, but assume $\psi^S \equiv 0$. Then there exists a level of cashflow surprise, $\Delta y_B < 0$, that determines the unique naïve equilibrium, (ψ^*, ψ^S, P), and under which $P_1(y_1; \psi^*, \psi^S)$ is characterized by condition A2. The equilibrium strategy has $\psi^*(y_1) = -\Delta y_1 + \Delta y_{max}$ for $\Delta y_1 \in [\Delta y_{max} - \delta_U, \Delta y_{max} - \delta_L]$, $\psi^*(y_1) = \delta_U$ if $\Delta y_1 \in [\Delta y_B, \Delta y_{max} - \delta_U]$, and $\psi^*(y_1) = \delta_L$ if $\Delta y_1 \leq \Delta y_B$ or if $\Delta y_1 \geq \Delta y_{max} - \delta_L$, where Δy_{max} is the maximal level of cashflows defined in condition A2. Furthermore, ψ^* is also a credible disclosure strategy if and only if $\Delta y_{min} = \Delta y_B$ and $\Delta y_{max} \in \Delta Y_A$, where ΔY_A is as defined in theorem 2.*

PROOF OF COROLLARY 2.   As in the proof of theorem 2, let $P_1(y_1) \equiv P_1(y_1; \psi = \psi^S \equiv 0)$. Assuming $P_1(y_1)$ is maximized at $\Delta y_{max} > 0$, the manager would always report Δy_{max} if the choice set was unbounded. However, $\delta \in [\delta_L, \delta_U]$ means he reports Δy_{max} for $\Delta y_{max} - \delta_U \leq \Delta y_1 \leq \Delta y_{max} - \delta_L$. By assumption, $P_1(y)$ is strictly decreasing in reported earnings on

$\Delta y_1 \leq \Delta y_{max} - \delta_L$, so for these values of Δy_1 the manager reports δ_L. For $\Delta y_1 < \Delta y_{max} - \delta_U$, the optimal disclosure is more complicated.

By assumption, $P_1(y)$ reaches a minimum, which means there exists a level of cashflow surprise, less than zero and denoted as Δy_B, which solves $P_1(y_B + \delta_L) = P_1(y_B + \delta_U)$. Assumption A2 also insures that, for $\Delta y_1 < \Delta y_B$, $P_1(y_1 + \delta_L) > P_1(y_1 + \delta_U)$, prompting the manager to choose $\delta^* = \delta_L$ for these observations. Finally consider $\Delta y_{max} - \delta_U \geq \Delta y_1 \geq \Delta y_B$. First we know that $\Delta y_B < \Delta y_{max} - \delta_U$, since $P_1(y_B + \delta_U) < P_1(y_{max})$ follows from Δy_{max} being the maximum. Since $P_1(y)$ is increasing on $\Delta y_{max} \geq \Delta y_1 \geq \Delta y_B$, $\delta^* = \delta_U$ is optimal for cashflow surprises $\Delta y_B \leq \Delta y_1 \leq \Delta y_{max} - \delta_U$, proving that ψ^* is optimal if $\psi^S \equiv 0$.

Next consider the necessary and sufficient conditions for ψ^* to be credible. Sufficiency follows immediately from the proof of theorem 2. Necessity is proven by supposing, in turn, that each of the conditions does not hold for ψ^*, and then showing that ψ^* is not credible. First, suppose $\Delta y_B < \Delta y_{min}$ (an analogous approach works for $\Delta y_B > \Delta y_{min}$). Then, since under ψ^*, the manager chooses $\delta^* = \delta_U$ for $\Delta y_1 \in (\Delta y_B, \Delta y_{min})$, the investors invert the report and the manager is paid $P_1(y_1)$. The manager could have chosen $\delta' = \delta_U + \Delta y_B - \Delta y_1 < \delta_U$, in which case he would have received $P_1(y_B) > P_1(y_1)$, proving ψ^* is not credible. Second, suppose $\Delta y_{max} \notin \Delta Y_A$. Then, from the definition of ΔY_A it must be the case that either $E P_1(y_{max}) < P_1(y_{max} - \delta_L)$ or that $E P_1(y_{max}) < P_1(y_{max} - \delta_U)$, where similar to theorem 2, we use $E P_1(y_{max}) \equiv E[P_1(y_1) \mid y_1 \in [\Delta y_{max} - \delta_U, \Delta y_{max} - \delta_L]]$. Suppose $E P_1(y_{max}) < P_1(y_{max} - \delta_L)$ holds; then there exists a cashflow level, $\Delta y_1^* = \Delta y_{max} - \delta_L + \varepsilon$, for some $\varepsilon > 0$, where the manager would deviate from ψ^*, preferring $\delta^* = 0$, and again proving ψ^* is not credible. This completes the proof of corollary 2.

<div align="right">Q.E.D. on Corollary 2.</div>

REFERENCES

ABARBANELL, J., AND R. LEHAVY. "Can Stock Recommendations Predict Earnings Management and Analysts' Earnings Forecast Errors?" Working paper, Haas School of Business, University of California, Berkeley, September 2000.

BURGSTAHLER, D., AND I. DICHEV. "Earnings Management to Avoid Earning Decreases and Losses." *Journal of Accounting and Economics* 24 (1997): 99–126.

CHANEY, P. K., AND C. M. LEWIS. "Earnings Management and Firm Valuation Under Asymmetric Information." *Journal of Corporate Finance: Contracting, Governance and Organization* 1 (1995): 319–45.

DEFOND, M., AND C. PARK. "Earnings Surprises Expressed in Cents Per Share: Stock Price Effects and Implications for Accrual Management." Working paper, Leventhal School of Accounting, University of Southern California, August 2000.

DEGROOT, M. *Optimal Statistical Decisions.* New York: McGraw Hill, 1970.

DEMSKI, J. S. "Endogenous Performance Smoothing." Working paper, University of Florida, July 1996.

DYE, R. A. "Proprietary and Nonproprietary Disclosures." *Journal of Business* (1986): 331–66.

———. "Earnings Management in an Overlapping Generations Model." *Journal of Accounting Research* (Autumn 1988): 195–235.

———, AND R. E. VERRECCHIA. "Discretion vs. Uniformity: Choices Among GAAP." *The Accounting Review* (July 1995): 389–416.

FREEMAN, R., AND S. TSE. "A Nonlinear Model of Price Responses to Unexpected Earnings." *Journal of Accounting Research* (Autumn 1992): 185–209.

FUDENBERG, D., AND J. TIROLE. "A Theory of Income and Dividend Smoothing Based on Incumbency Rents." *Journal of Political Economy* 103 (1) (1995): 75–93.

HUNT, A.; S. MOYER; AND T. SHEVLIN. "Earnings Volatility, Earnings Management, and Equity Value." Working paper, University of Washington, 1996.

KIRSCHENHEITER, M. "Information Quality and Correlated Signals." *Journal of Accounting Research* 35 (1) (Spring 1997): 43–59.

———, AND N. MELUMAD. "Earnings Quality and Smoothing" Working paper, Columbia University, Graduate School of Business, 2002.

LAMBERT, R. A. "Income Smoothing As Rational Equilibrium Behavior." *The Accounting Review* 59 (October 1984): 604–18.

PENNO, M. "Unobservable Precision Choices in Financial Reporting." *Journal of Accounting Research* (Spring 1996): 141–9.

RONEN, J., AND S. SADAN. *Smoothing Income Numbers: Objectives, Means, and Implications.* Reading, MA: Addison-Wesley, 1981.

SCHIPPER, K. "Earnings Management." *Accounting Horizons* 3 (1989): 91–106.

SKINNER, D., AND R. SLOAN. "Earnings Surprises, Growth Expectations, and Stock Returns or Don't Let an Earnings Torpedo Sink Your Portfolio." Working paper, University of Michigan Business School, 1996.

SUBRAMANYAM, K. R. "Uncertain Precision and Price Relations to Information." *The Accounting Review* (April 1996): 207–20.

TRUEMAN, B., AND S. TITMAN. "An Explanation of Accounting Income Smoothing." *Journal of Accounting Research* (Supplement 1988): 127–39.

VERRECCHIA, R. E. "Discretionary Disclosure." *Journal of Accounting and Economics* (December 1983): 447–73.