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Can $\iota(1440)$ Be a Pseudoscalar Glueball Which Appreciably Mixes with $\eta'(958)$?

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We have studied the η-η'-ι mixing by using the Gell-Mann-Oakes-Renner type approach to the chiral $U(3)\times U(3)$ and also $U(4)\times U(4)$ algebras involving anomaly and found that η'-ι mixing could be appreciable. The model also predicted (by using PCAC and also sometimes a simple quark counting argument) that while the rate of $\iota\to\gamma\gamma$ is relatively small, $\Gamma(\iota\to\rho\gamma)$ will be rather large$\simeq$1 MeV. The η-η'-ι mixing has also been studied by us using the method of "asymptotic flavor $SU(3)$ symmetry plus the constraint algebras involving the generators of underlying symmetry groups of QCD". Essentially the same conclusion as derived in the first approach has been obtained for the structures of η-η'-ι mixing. In this paper, we study the $\iota\to\gamma\gamma$ and $\iota\to\rho\gamma$ decays in the second approach without using quark counting argument. We find a result which is compatible (at least in flavor $SU(3)$ symmetry studied) with that of the first approach. We conclude that a part of the present experimental situation can be understood with the presence of pseudoscalar glueball $\iota(1440)$ which mixes rather appreciably with the η'. Critical experiments for the model are also discussed.

§ 1. Introduction

It has already been some time since an argument was made that $\iota(1440)$ observed[1] in the radiative decay of J/ψ could be a $J^{PC}=0^{-+}$ glueball.[2] Even if $\iota(1440)$ indeed has a genetic origin in glueball, it can also acquire $q\bar{q}$-components through the mixing with the 0^{-+} $q\bar{q}$-mesons. At present the following three problems prevent us from reaching a definite conclusion about $\iota(1440)$:

(1) In the subsequent decays of ι originated from the reaction $J/\psi\to\iota\gamma$, why is the $\iota\to\eta\pi\pi$ mode suppressed relative to the $\iota\to K\bar{K}\pi$, if the latter proceeds through $\iota\to\delta\pi\to(K\bar{K})\pi$?[3]

(2) Recently the width of $\iota\to\gamma\gamma$ is found to be small.[4),5)] Does this fact contradict the rather large width[6] of $\iota\to\rho\gamma$ decays inferred from the $J/\psi\to\rho\gamma\gamma$ decay and exclude an appreciable $q\bar{q}$ contamination in $\iota(1440)$?

(3) In the recent study of the reaction $\pi^- p\to K^+\bar{K}^0\pi^- n$, the bump at 1420 MeV in the $(K^+\bar{K}^0\pi^-)$ system is reported[7] to be mainly 0^{-+}. It decays into $\delta\pi$ and $\bar{K}^*K+K^*\bar{K}$ systems, with the latter process dominating. One may be tempted to identify this bump with the ι found in the $J/\psi\to\iota\gamma$ decay. However, the ι has been reported[3] to decay mainly into $K\bar{K}\pi$ through the $\delta\pi$ channel.

As to the problem (1), several theoretical attempts have been put forward; chiral Lagrangian method,[8] destructive interference between the $\iota\to\delta\pi\to(\eta\pi)\pi$ and $\iota\to\eta\varepsilon$

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(700)$\to\eta(\pi\pi)$ amplitudes,[9] the effect of $K\bar{K}$ threshold on the δ (presently $a_0(980)$) propagator[10] and the $K\bar{K}$ molecule interpretation of the $S^*(f_0(975))$ and δ scalar mesons,[11] etc. In the present approach the amplitude $\iota\to a_2(1320)\pi\to(\eta\pi)\pi$ and $\iota\to\kappa\bar{K}+\bar{\kappa}K$ can also interfere with the $\iota\to\delta\pi\to(\eta\pi)\pi$.

For the πp reactions related to the problem (3), various results have been obtained by various groups. In the π^+p production processes,[12] the state with $M=1420$ MeV and $J^{PC}=1^{++}$ is observed not only in the final $K\bar{K}\pi$ system but also in the $\eta\pi\pi$ system, while the signal for the $J^{PC}=0^{-+}$ state is not significant. However, in the π^-p production processes,[13] the opposite result has been observed, i.e., 0^{-+} state has been seen in both the $K\bar{K}\pi$ and $\eta\pi\pi$ systems. Also in the $p\bar{p}$ production processes,[14] presumably the same 0^{-+} state has been observed in the $K\bar{K}\pi$ channel. From these experiments, it seems that there exist at least two states (0^{-+} and 1^{++}) in the mass range 1420—1450 MeV. The 0^{-+} state may be identified with the iota found in the $J/\psi\to\iota\gamma$ decay and the 1^{++} state with $E(1420)$. We are also certainly tempted to identify the 0^{-+} 1420 MeV $K^+\bar{K}^0\pi^-$ bump observed[7] in the reaction $\pi^-p\to K^+\bar{K}^0\pi^-n$ with the iota. However, as mentioned in (3) above, this may not be tenable, if the bump decays into the final $K\bar{K}\pi$ state through the $K^*\bar{K}+\bar{K}^*K$ channel rather than the $\delta\pi$ channel as reported in Ref. 7).

In this connection, recent Mark III analysis[15] of the processes, $J/\psi\to\iota\gamma$ and $\iota\to K\bar{K}\pi$, is also very interesting. Apparently iota shape could not be fitted by a single Breit-Wigner resonance. However, it can be fitted by two Breit-Wigner resonances and the high mass region (>1.45 GeV) seems to have $K^*\bar{K}+\bar{K}^*K$. In any case further more precise experiments are eagerly awaited.

As for the problem (2), it becomes quite serious recently since the high statistics TPC/2γ experiment yields[5] $\Gamma(\iota\to\gamma\gamma)B(\iota\to K\bar{K}\pi)<1.6$ keV. Theoretical problems arise from the calculation of the rates of $\iota\to2\gamma$ and $\iota\to\rho\gamma$ decays based on the vector-meson dominance model[16](VDM) and the Bag model.[17] A conclusion has been drawn that the $\rho\gamma$ system in the decay $J/\psi\to(\rho\gamma)\rho$ cannot be associated with $\iota(1440)$ and also that ι cannot contain an appreciable $q\bar{q}$-component. However, the result may be dependent upon the model used to calculate the rates of $\iota\to2\gamma$ and $\rho\gamma$.

In contrast, in the Gell-Mann-Oakes-Renner type approach to the chiral $U(3)\times U(3)$[18] and $U(4)\times U(4)$[19] algebras involving anomaly in which we have studied the η-η'-ι mixing from the input masses of pseudoscalar mesons, we have previously derived a result which does not necessarily contradict the experimentally observed small $\Gamma(\iota\to\gamma\gamma)$ and also the rather large $\Gamma(\iota\to\rho\gamma)$ of order of MeV. However, in this approach the presence of rather appreciable ι-η' mixing is indicated. Essentially the same result was also obtained[20] for the η-η'-ι mixing from the distinct approach of "asymptotic $SU_f(3)$ flavor symmetry plus the constraint algebras involving the generators (i.e., vector and axial-vector charges) of underlying symmetries of QCD". Recently, Milton also obtained[21] a similar conclusion for the rate of $\iota\to\gamma\gamma$ from the method of anomalous chiral Ward identities.

In this paper, we reexamine the problems, particularly the problems (2), from our second approach (in flavor asymptotic $SU(3)$ symmetry) mentioned above and show that our previous conclusions from our first approach remain essentially unchanged. We argue that $\iota(1440)$ has a genetic origin in glueball but can mix rather appreciably

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with η' (at least in the present framework of $SU_f(3)$ symmetry) and can provide a main cause for the *non-ideal* behavior of the ground-state $q\bar{q}$ PS mesons. However, $\iota(1440)$ has a very small octet component so that its decay branching ratio into $K^*\bar{K}+\bar{K}^*K$ is shown to be very small. Therefore, the establishment of the presence of genuine sizable $\iota\to K^*\bar{K}+\bar{K}^*K$ mode can exclude the model under discussion.

§ 2. Rates of the $\iota\to\gamma\gamma$ and $\iota\to\rho\gamma$ decays in the framework of "asymptotic flavor $SU(3)$ symmetry plus constraint algebras of QCD"

We now discuss the problem of the rates of the $\iota\to\gamma\gamma$ and $\iota\to\rho\gamma$ decays from our second approach, which is based on the method of "asymptotic flavor symmetry plus the constraint algebras, involving the generators (i.e., vector and axial-vector charges V_α and A_α) of underlying symmetry groups of QCD". Instead of chiral algebras involving anomalies used in Refs. 18) and 19), we use the well-known chiral $SU_f(3)_L\times SU_f(3)_R$ algebras which are valid in QCD. Flavor symmetry breaking in QCD can be characterized by the presence of the so-called "exotic" commutators, $[\dot{V}_\alpha, V_\beta]=[\dot{V}_\alpha, A_\beta]=0$, where $\dot{V}_\alpha=i[H, V_\alpha]$ and H is the total Hamiltonian. (α, β) stands for the "exotic" combination of physical flavor indices. The hypothesis of asymptotic flavor symmetry implies that the linear relation $|P^A\rangle=\sum_a R^{Aa}|P^a\rangle$, where $|P^A\rangle$ represent the physical states ($A=\pi, K, \eta, \eta'$ and ι), while $|P^a\rangle$ the hypothetical representation states ($a=0, 1, \cdots, G_0$), can be exactly valid but *only* when the state under consideration has infinite momentum[20] ($\boldsymbol{p}\to\infty$). This requires that the creation and annihilation operators of physical (i.e., "in" or "out") particles do transform *linearly* under flavor transformation generated by V_α but only in the infinite momentum limit, provided that the particle mixing is also taken into account in the same limit. We limit ourselves to the case of $SU_f(3)$ in this paper. According to asymptotic $SU_f(3)$ symmetry, the annihilation operators of physical η, η' and ι-meson, $a_\eta(\boldsymbol{p})$, $a_{\eta'}(\boldsymbol{p})$ and $a_\iota(\boldsymbol{p})$, are related *linearly*, but *only* in the limit $\boldsymbol{p}\to\infty$, to the annihilation operators of hypothetical representation states η_8, η_0 and G_0 (G_0 denotes bare glueball), $a_8(\boldsymbol{p})$, $a_0(\boldsymbol{p})$ and $a_G(\boldsymbol{p})$, as follows:

$$\begin{pmatrix} a_\eta \\ a_{\eta'} \\ a_\iota \end{pmatrix}=\begin{pmatrix} \alpha_8 & \alpha_0 & \alpha_G \\ \beta_8 & \beta_0 & \beta_G \\ \gamma_8 & \gamma_0 & \gamma_G \end{pmatrix}\begin{pmatrix} a_8 \\ a_0 \\ a_G \end{pmatrix}, \qquad \boldsymbol{p}\to\infty . \tag{2·1}$$

The α's denote the mixing parameters. In Ref. 22), we have parametrized the α's in terms of mixing angles θ_1, θ_2 and θ_3 defined by

$$\begin{aligned} &\alpha_8=c_1c_2\,, && \alpha_0=-c_1s_2s_3-s_1c_3\,, && \alpha_G=-c_1s_2c_3+s_1s_3\,, \\ &\beta_8=s_1c_2\,, && \beta_0=-s_1s_2s_3+c_1c_3\,, && \beta_G=-s_1s_2c_3-c_1s_3\,, \\ &\gamma_8=s_2\,, && \gamma_0=c_2s_3\,, && \gamma_G=c_2c_3\,, \end{aligned} \tag{2·2}$$

where $c_i\equiv\cos\theta_i$ and $s_i\equiv\sin\theta_i$ ($i=1, 2, 3$). In the same literature, we have determined the probable range of the values of θ_1, θ_2 and θ_3, which are compatible with the constraints obtained by realizing the above-mentioned constraint algebras in our

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asymptotic limit. For details see Ref. 22). The typical values of θ_i may be given by

$$\theta_1=-8.52°\,,\quad \theta_2=3.52° \quad \text{and} \quad \theta_3=-21.98°\,, \tag{2·3}$$

which expresses Eq. (2·1) as follows (in the limit $\boldsymbol{p}\to\infty$):

$$\begin{pmatrix} a_\eta \\ a_{\eta'} \\ a_\iota \end{pmatrix}=\begin{pmatrix} 0.987 & 0.160 & -0.001 \\ -0.148 & 0.917 & 0.379 \\ 0.062 & -0.374 & 0.926 \end{pmatrix}\begin{pmatrix} a_8 \\ a_0 \\ a_G \end{pmatrix}$$

$$=\begin{pmatrix} 0.701 & -0.714 & -0.001 \\ 0.661 & 0.648 & 0.379 \\ -0.270 & -0.266 & 0.926 \end{pmatrix}\begin{pmatrix} N \\ S \\ G_0 \end{pmatrix}, \tag{2·4}$$

where $N=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $S=s\bar{s}$. As seen, the $q\bar{q}$ structure of η, η' and ι found above is remarkably compatible with the one in Ref. 18) obtained from the very different algebras involving anomalies using $\boldsymbol{p}\to 0$ limit.

To discuss $\iota\to\rho\gamma$ decay, we pick up the following charge-charge density algebras (which are valid in the framework of L_{QCD} plus L_{EM}) as the useful "constraint" algebras,

$$[j_0^{\text{EM}}(x)\,,\quad V_{K^0}]=0\,, \tag{2·5a}$$

$$[j_0^{\text{EM}}(x)\,,\quad \dot{V}_{K^0}]=0\,, \tag{2·5b}$$

where $V_{K^0}=V_6+iV_7$.

We sandwich Eqs. (2·5a) and (2·5b) between the states $\langle\rho^0(\boldsymbol{p})|$ and $|\bar{K}^0(\boldsymbol{k})\rangle$ with $\boldsymbol{p}\to\infty$ and $\boldsymbol{k}\to\infty$ and use asymptotic $SU_f(3)$ symmetry.[23] We then obtain two sum rules,

$$-g_{\rho^0\pi^0\gamma}+\sqrt{3}\alpha_8 g_{\rho^0\eta\gamma}+\sqrt{3}\beta_8 g_{\rho^0\eta'\gamma}+\sqrt{3}\gamma_8 g_{\rho^0\iota\gamma}+g_{\bar{K}^{*0}\bar{K}^0\gamma}=0\,, \tag{2·6}$$

$$-(m_\pi^2-m_K^2)g_{\rho^0\pi^0\gamma}+\sqrt{3}(m_\eta^2-m_K^2)\alpha_8 g_{\rho^0\eta\gamma}+\sqrt{3}(m_{\eta'}^2-m_K^2)\beta_8 g_{\rho^0\eta'\gamma}$$
$$+\sqrt{3}(m_\iota^2-m_K^2)\gamma_8 g_{\rho^0\iota\gamma}+(m_\rho^2-m_{K^*}^2)g_{\bar{K}^{*0}\bar{K}^0\gamma}=0\,. \tag{2·7}$$

These are the modifications of the old sum rules discussed by Matsuda and Oneda[25] and also Oneda et al.[26] in the presence of $\iota(1440)$. The g's are defined by

$$\sqrt{4p_0k_0}\,\langle V(\boldsymbol{p})|J_\mu^{\text{EM}}(0)|P(\boldsymbol{k})\rangle=g_{VP\gamma}((p-k)^2)\varepsilon_{\mu\nu\rho\sigma}e_V{}^\nu(\boldsymbol{p})p^\rho k^\sigma\,, \tag{2·8}$$

where e_V is the polarization four-vector of V.

Let us, for example, eliminate $g_{\rho^0\eta\gamma}$ which is least well-known from the above two sum rules to obtain

$$-(m_\eta^2-m_\pi^2)g_{\rho^0\pi^0\gamma}+(m_\eta^2-m_{\eta'}^2)\sqrt{3}\beta_8 g_{\rho^0\eta'\gamma}+(m_\eta^2-m_\iota^2)\sqrt{3}\gamma_8 g_{\rho^0\iota\gamma}$$
$$-(m_\eta^2-m_K^2-m_\rho^2+m_{K^*}^2)g_{\bar{K}^{*0}\bar{K}^0\gamma}=0\,. \tag{2·9}$$

In Eq. (2·7) we can see explicitly that the term involving the $\iota\to\rho^0\gamma$ decay coupling constant $g_{\rho^0\iota\gamma}$ contributes rather significantly. In the absence of ι (i.e., $\gamma_8=\gamma_0=\alpha_G=\beta_G=0$, $\gamma_G=1$, $\alpha_8=\beta_0=\cos\theta$, $\alpha_0=-\beta_8=-\sin\theta$ and $\theta\simeq-10°$), Eqs. (2·6) and (2·7) become

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$$\frac{g_{\rho^0\eta'\gamma}}{g_{\rho^0\pi^0\gamma}}=\frac{-(m_\eta{}^2-m_K{}^2-m_\rho{}^2+m_{K^*}^2)g_{\bar{K}^{*0}\bar{K}^0\gamma}/g_{\rho^0\pi^0\gamma}+(m_\eta{}^2-m_\pi{}^2)}{(m_\eta{}^2-m_{\eta'}^2)\sqrt{3}\sin\theta}\,. \qquad (2\cdot10)$$

If we use the experimental values[27)]

$$\Gamma(\rho^0\to\pi^0\gamma)=0.070\pm0.008\text{ MeV}\,,$$

$$\Gamma(\bar{K}^{*0}\to\bar{K}^0\gamma)=0.118\pm0.010\text{ MeV}\,, \qquad (2\cdot11)$$

and the meson masses involved we then obtain with an appropriate choice of the phases of the couplings,

$$\text{right-hand side of Eq. }(2\cdot10)\simeq3.58\,. \qquad (2\cdot12)$$

However, the use of the experimental value[27)]

$$\Gamma(\eta'\to\rho^0\gamma)=0.072\pm0.010\text{ MeV}\,, \qquad (2\cdot13)$$

then implies,

$$\text{left-hand side of Eq. }(2\cdot10)\simeq1.90\pm0.17\,, \qquad (2\cdot14)$$

which deviates from the value of Eq. (2·12) and may thus call for the presence of ι, i.e., the $g_{\rho^0\iota\gamma}$ terms in Eqs. (2·6) and (2·7). By using the values of mixing parameters given by Eq. (2·4) we can determine $g_{\rho^0\iota\gamma}$ from Eqs. (2·6) and (2·7) to obtain

$$\Gamma(\iota\to\rho^0\gamma)=2.8\pm0.7\text{ MeV}\,, \qquad (2\cdot15)$$

which agrees fairly well with the experimental value

$$\begin{aligned}\Gamma(\iota\to\rho\gamma)=2\text{ MeV}&\quad\text{Crystal Ball}\,,\\ 1\text{ MeV}&\quad\text{Mark II}\,,\\ 0.5\text{ MeV}&\quad\text{DM II}\,.\end{aligned} \qquad (2\cdot16)$$

We here note that our present method uses a much more *dynamical* information than that obtained by mere (asymptotic) flavor $SU(3)$ symmetry. As demonstrated in Eq. (2·7) (which is obtained from the algebra involving Eq. (2·5b)), we are dealing with the explicit *interplay* between the masses of mesons and coupling constants. As stated in Ref. 25), a well-satisfied mass formula $m_\rho{}^2-m_\pi{}^2=m_{K^*}^2-m_K{}^2$ and a mass-mixing-angle-coupling constant interplay, $g_{\phi\pi\gamma}/g_{\omega\pi\gamma}=-\tan\theta_{\omega\phi}(m_\rho{}^2-m_\omega{}^2)/(m_\rho{}^2-m_\phi{}^2)$ have been demonstrated in the similar sum rules obtained from Eqs. (2·5a) and (2·5 b). Therefore, we have a considerable confidence in the method used. However, for the present problems involving η and η', the neglected possible mixings with the η_c and the radially excited PS-meson states could produce some small corrections. Equation (2·7) is certainly more sensitive than Eq. (2·6) to these mixings, since large masses will come into play. Nevertheless, Eq. (2·15) seems to imply strongly that $\Gamma(\iota\to\rho\gamma)$ is of the order of MeV and not of the order of keV.

We may also make an estimate of $\rho^0\to\eta\gamma$ decay in the presence of $\iota\rho^0\gamma$ coupling. If we consider the sum rule obtained by eliminating $g_{\rho^0\iota\gamma}$ from Eqs. (2·6) and (2·7) and use experimental input Eqs. (2·11) and (2·13) together with the mixing parameters

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given by Eq. (2·4), we obtain

$$\Gamma(\rho^0\to\eta\gamma)=0.037\pm0.005\ \mathrm{MeV}\,. \tag{2·17}$$

The particle data group has not yet given the value of $\Gamma(\rho^0\to\eta\gamma)$. However, our value (2·17) agrees within a factor 2 with the value recently cited by Hitlin,[3]

$$\Gamma(\rho^0\to\eta\gamma)=0.0725\pm0.014\ \mathrm{MeV}\,. \tag{2·18}$$

We next study the $\iota\to\gamma\gamma$ decay using the method used in Ref. 25) a long time ago. We now sandwich the constraint algebras, Eqs. (2·5a) and (2·5b), between the states $\langle\gamma(\boldsymbol{q})|$ and $|\bar{K}^0(\boldsymbol{p})\rangle$ and $\boldsymbol{q}\to\infty$ and $\boldsymbol{p}\to\infty$ with $s=(p-q)^2=0$. We then obtain two sum rules

$$\begin{aligned}&F^{\pi^0}-\sqrt{3}\alpha_8F^{\eta}-\sqrt{3}\beta_8F^{\eta'}-\sqrt{3}\gamma_8F^{\iota}\\&\quad=n\sqrt{2}\langle\gamma(\boldsymbol{q})|V_{K^0}|n(\boldsymbol{q})\rangle\langle n(\boldsymbol{q})|j_\mu{}^{E}(0)|\bar{K}^0(\boldsymbol{p})\rangle\,,\end{aligned} \tag{2·19}$$

$$\begin{aligned}&(m_\pi{}^2-m_K{}^2)F^{\pi^0}-\sqrt{3}(m_\eta{}^2-m_K{}^2)\alpha_8F^{\eta}-\sqrt{3}(m_{\eta'}{}^2-m_K{}^2)\beta_8F^{\eta'}-\sqrt{3}(m_\iota{}^2-m_K{}^2)\gamma_8F^{\iota}-\delta\\&\quad=\sum_n\sqrt{2}\langle\gamma(\boldsymbol{q})|\dot{V}_{K^0}|n(\boldsymbol{q})\rangle\langle n(\boldsymbol{q})|j_\mu{}^{\mathrm{EM}}(0)|\bar{K}^0(\boldsymbol{p})\rangle\,,\end{aligned} \tag{2·20}$$

where F^P are defined as

$$\sqrt{4p_0k_0}\,\langle\gamma(\boldsymbol{p},e)|J_\mu{}^{\mathrm{EM}}(0)|P(\boldsymbol{k})\rangle=F^P\varepsilon_{\mu\nu\rho\sigma}e^\nu p^\rho k^\sigma\,. \tag{2·21}$$

In Eq. (2·20), δ represents the possible non-negligible contribution of η_c,

$$\delta\equiv\sqrt{3}(m_{\eta_c}^2-m_K{}^2)R^{\eta_c8}F^{\eta_c}\,, \tag{2·22}$$

where R^{η_c8} denotes the fraction of octet component in η_c state. If we examine the right-hand side of Eq. (2·19) by using field-current identity, the main contribution to the intermediate states $n(\boldsymbol{q})$ will come from the zero mass $\bar{K}^{*0}$-meson state with momentum $\boldsymbol{q}$ and it is given by

$$\begin{aligned}&\sqrt{2}\langle\gamma(\boldsymbol{q})|V_{K^0}|\text{zero mass }\bar{K}^{*0}(\boldsymbol{q})\rangle\langle\text{zero mass }\bar{K}^{*0}(\boldsymbol{q})|j_\mu{}^{\mathrm{EM}}|\bar{K}^0(\boldsymbol{p})\rangle\\&\quad=X\{F^{\pi^0}/2-(3\sqrt{3}/2)\alpha_8F^{\eta}-(3\sqrt{3}/4)\beta_8F^{\eta'}\}\,,\end{aligned} \tag{2·23}$$

where X stands for the right-hand side of Eq. (2·19), which is the correction term for $SU(3)$ breaking.[25] On the other hand, the right-hand side of Eq. (2·20) vanishes, since, as mentioned above, the intermediate $|n(\boldsymbol{q})\rangle$ state is the extrapolated $|\bar{K}^{*0}\rangle$ state with mass zero so that $\langle\gamma(\boldsymbol{q})|\dot{V}_{K^0}|n(\boldsymbol{q})\rangle=0$. Therefore, Eq. (2·20) becomes

$$\begin{aligned}&(m_\pi{}^2-m_K{}^2)F^{\pi^0}-\sqrt{3}(m_\eta{}^2-m_K{}^2)\alpha_8F^{\eta}-\sqrt{3}(m_{\eta'}^2-m_K{}^2)\beta_8F^{\eta'}-\sqrt{3}(m_\iota{}^2-m_K{}^2)\gamma_8F^{\iota}\\&\quad=\delta\,.\end{aligned} \tag{2·24}$$

If we eliminate F^{π^0} from Eqs. (2·23) and (2·24), we then obtain to the first order in X

$$\begin{aligned}&\{m_\eta{}^2-m_\pi{}^2+(m_\pi{}^2-m_K{}^2)X\}\alpha_8F^{\eta}+\{m_{\eta'}^2-m_\pi{}^2+(m_\pi{}^2-m_K{}^2)X/4\}\beta_8F^{\eta'}\\&\quad+\{m_\iota{}^2-m_\pi{}^2+(m_\pi{}^2-m_K{}^2)X/2\}\gamma_8F^{\iota}=-\delta/\sqrt{3}\,.\end{aligned} \tag{2·25}$$

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From these sum rules, the experimental rates of $\eta\to\gamma\gamma$ and $\eta'\to\gamma\gamma$ decays

$$\begin{aligned}\Gamma(\eta\to\gamma\gamma)&=0.41\pm0.06\,\text{keV} \quad \text{for Ref. 27)},\\ &=0.560\pm0.04\,\text{keV} \quad \text{for Ref. 4)},\\ \Gamma(\eta'\to\gamma\gamma)&=4.6\pm0.8\,\text{keV} \quad \text{for Ref. 27)},\\ &=4.3\pm0.1\,\text{keV} \quad \text{for Ref. 4)},\end{aligned} \tag{2·26}$$

and the values of α_8, β_8 and γ_8 given by Eq. (2·16), we obtain

$$\begin{aligned}\Gamma(\iota\to\gamma\gamma)&=0.6 - 4.0\,\text{keV} \quad \text{for Ref. 27)},\\ &=5.1 - 7.6\,\text{keV} \quad \text{for Ref. 4)},\end{aligned} \tag{2·27}$$

by using $X\simeq0.1$ and $\delta\simeq0.0$. If the contribution of η_c, i.e., δ in Eq. (2·22) is crudely estimated, we obtain an estimate $\delta/\sqrt{3}F^{\eta'}<0.02\,\text{GeV}^2$ from the experiment $\Gamma(\eta_c\to\gamma\gamma)<6.6\,\text{keV}$[27] and the value $R^{\eta_c 8}\simeq0.01$ obtained in Ref. 19). Then the range of predictions given in Eq. (2·27) is modified to $\Gamma(\iota\to\gamma\gamma)=0.1-6.9\,\text{keV}$ for Ref. 27) and 2.7 -11.5 keV for Ref. 4). In the absence of $\iota(1440)$, the sum rule predicted,[25] $\Gamma(\eta\to\gamma\gamma)\simeq0.5\,\text{keV}$ and $\Gamma(\eta'\to\gamma\gamma)\simeq5.5\,\text{keV}$. The first value of Eq. (2·27) based on the experimental input of Ref. 27) (Particle data group 86) may be said to be consistent with the present value of experimental value

$$\begin{aligned}\Gamma(\iota\to\gamma\gamma)B(\iota\to K\bar{K}\pi)&<2.2\,\text{keV (95\% c.l.)},\\ &<2.0\,\text{keV (95\% c.l.), Ref. 4)}.\end{aligned} \tag{2·28}$$

The second value of Eq. (2·27) based on the input from Ref. 4) probably a little too large. However, as seen from our procedure used, an error of $\simeq10\%$ at many places may build up a fairly sizable error. The η_c contribution can also reduce the expected values (2·27) as stated above.

Thus our result shows that the smallness of $\gamma(\iota\to\gamma\gamma)$, i.e.,$\simeq1$ keV, does not necessarily implies small ι-η' mixing. However, if $\Gamma(\iota\to\gamma\gamma)$ turns out to be $\lesssim0.1$ keV, it will at least exclude the present $SU_f(3)$ model.

§ 3. Comments and experimental tests of the model

In § 2 we have argued that the rate of $\iota\to\gamma\gamma$ decay, computed from the sum rule Eq. (2·24) derived from the approach which is based on QCD constraint algebras, can be compatible with the present experimental values given by Eq. (2·28). The similar result was also obtained recently by using chiral Ward identity[21] in which ι-η' mixing can also be appreciable as in the present approaches. Thus the smallness of the $\iota\to\gamma\gamma$ rate *around* 1 keV does not necessarily imply the smallness of ι-η' mixing, if we compute the rate of $\iota\to\gamma\gamma$ decay without recourse to the assumption of vector meson dominance, etc. However, $\Gamma(\iota\to\gamma\gamma)$ turns out to be $\lesssim0.1$ keV, the possibility of large ι-η' mixing will be excluded at least in the present model discussed in § 2.

In the approach of "asymptotic flavor $SU(3)$ symmetry plus the constraint algebras involving the generators of underlying symmetry groups of QCD" used in § 2,

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the presence of "exotic" commutators, $[\dot{V}_\alpha, V_\beta]=[\dot{V}_\alpha, A_\beta]=0$, etc, requires that the canonical mass formula for each $SU_f(3)$ nonet is Schwinger's nonet mass relation,[24] so long as we consider *only* the intra-multiplet (i.e., nonet) mixing. For the PS-meson nonet, this relation predicts the mass of the ninth meson η' around 1.6 GeV in terms of the mass values of π, K and η. It is, therefore, natural (at least, in the framework of $SU_f(3)$) to attribute the deviation of the η' mass from the nonet value 1.6 GeV to the presence of a pseudoscalar glueball with a mass close to 1.6 GeV, which mixes rather appreciably with the $I=0$ $q\bar{q}$ PS-mesons. From this point of view, $\iota(1440)$ is perhaps an ideal candidate and one may think that the unusual behavior of ground-state PS-mesons is due to the ι-η-η' mixing. The large violation of quark-line rule in the processes involving η and η' can then be attributed to this mixing. One caution is that the inclusion of the effect of η_c in this game may be important, since η_c is certainly not a pure $c\bar{c}$-state. A study of ι-η-η'-η_c mixing in the framework of asymptotic flavor $SU(4)$ symmetry is being undertaken.[28]

We may add here some relevant remarks on the problems (1) and (3) mentioned in § 1.

(a) $\iota\to K^*\bar{K}+\bar{K}^*K$

As mentioned at the end of § 1, in the present model $\iota(1440)$ has a very small octet component as seen from Eq. (2·4), so that we obtain the very small rate, $\Gamma(\iota\to K^*\bar{K}+\bar{K}^*K)\simeq 0.79$ MeV. Therefore, if $K^*\bar{K}+\bar{K}^*K$ mode is found to be an appreciable fraction of $\iota\to K\bar{K}\pi$, ι cannot be identified with the 0^{-+} object being considered by us. Of course, the $K^*\bar{K}+\bar{K}^*K$ mode may come from the radially excited 0^{-+} state which might exist in the same mass region around 1440 MeV. Confirmation of the presence of $K^*\bar{K}+\bar{K}^*K$ mode is certainly very interesting.

(b) *Possible intermediate states to the processes of $\iota\to\eta\pi\pi$ and $\iota\to K\bar{K}\pi$*

One may elaborate a little about the possible mechanisms which lead to $\iota\to\eta\pi\pi$ and $K\bar{K}\pi$. In addition to the process $\iota\to a_0(980)\pi\to(\eta\pi)\pi$ and $\iota\to a_0(980)\pi\to(K\bar{K})\pi$, one can also consider another competing processe $\iota\to a_2(1320)\pi\to(\eta\pi)\pi$ and $\iota\to a_2(1320)\pi\to K\bar{K}\pi$. One can estimate the $\iota\to a_0(980)\pi$ and $\iota\to a_2(1320)\pi$ couplings, $g_{\iota a_0\pi}$ and $g_{\iota a_2\pi}$, as follows. As discussed in Ref. 22), in the present theoretical framework we have general predictions on the asymptotic matrix elements of the axial-vector charge $A_\pi(A_{\pi^+}=A_1+iA_2$, etc.) involving η, η' and ι. For example, we have

$$\frac{\langle\iota(\boldsymbol{p})|A_{\pi^+}|a_0^-(980)\rangle}{\langle\eta(\boldsymbol{p})|A_{\pi^+}|a_0^-(980)\rangle}=\frac{\langle\iota(\boldsymbol{p})|A_{\pi^+}|a_2^-(1320)\rangle}{\langle\eta(\boldsymbol{p})|A_{\pi^+}|a_2^-(1320)\rangle}\equiv\frac{R_3}{R_1}\qquad \boldsymbol{p}\to\infty\,. \tag{3·1}$$

According to the argument of Ref. 22) updated, $|R_3/R_1|$ is constrained as

$$|R_3/R_1|\simeq 0.38\ -\ 0.47\,. \tag{3·2}$$

By using PCAC for A_π in the $\boldsymbol{p}\to\infty$ limit (which yields a hard-pion extrapolation) we can rewrite (3·1) (for derivation, see Ref. 24), § 4.3) as

$$\frac{(m_{a_0}^2-\eta^2)g_{a_0\iota\pi}}{(m_{a_0}^2-\iota^2)g_{a_0\eta\pi}}=\frac{(m_{a_2}^2-\iota^2)g_{a_2\iota\pi}}{(m_{a_2}^2-\eta^2)g_{a_2\eta\pi}}=\frac{R_3}{R_1}\,, \tag{3·3}$$

which implies

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$$\frac{g_{a_0 \iota \pi}}{g_{a_0 \eta \pi}} \simeq 1.67 \frac{R_3}{R_1}, \tag{3·4}$$

$$\frac{g_{a_2 \iota \pi}}{g_{a_2 \eta \pi}} \simeq 4.27 \frac{R_3}{R_1}. \tag{3·5}$$

The estimate $|R_3/R_1|$ given by (3·2), in fact, yields

$$\frac{\Gamma(\iota \to \delta\pi)}{\Gamma(\delta \to \eta\pi)} \simeq 0.66 - 1.02. \tag{3·6}$$

The results (3·4) and (3·5) suggest that the contribution of $\iota \to a_2\pi \to (\eta\pi)\pi$ and $\iota \to a_2\pi \to (K\bar{K})\pi$ may be appreciable. As mentioned in Ref. 22), via PCAC the coupling strengths of ι-κ-K and ι-K^{**}-K ($\kappa \equiv K_0^*(1350)$ and $K^{**} \equiv K_2^*(1430)$) are comparable with that of ι-a_0-π and their contributions to the $\iota \to K\bar{K}\pi$ decays may also be appreciable.

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