Can Math Education Research Improve the Teaching of Abstract Algebra?

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Teaching matters. It is arguably the most important factor affecting student learning. Efforts to improve teaching have led to reform initiatives being proposed and tested throughout the college mathematics curriculum. Abstract algebra specifically has been the subject of such reform, including new curricula and pedagogies, since at least the 1960s, yet there is little evidence that these change initiatives have widely influenced the way abstract algebra is taught. We conducted a survey of abstract algebra instructors to investigate typical teaching practices and, more specifically, faculty knowledge, goals, and orientation towards teaching and learning. Results revealed that a majority of respondents appear quite content to lecture. Even among those who indicated a willingness to consider a change of pedagogical strategy, there is very little usage of existing reform materials or interaction with pedagogical research results. There appears to be an impermeable barrier between the pedagogical researchers' findings and recommendations and practitioners who might implement them.

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Research Questions

There is essentially no research that helps us understand why some mathematicians adopt reform practices in their teaching and some do not [3]. There has been little research attempting to explore these issues from the perspective of the instructors who are the ones being asked to change practice. We investigated the following research questions: (1) What pedagogical practices do abstract algebra professors report using in their classrooms and why? (2) What encouragement and constraints on their use of nonlecture practices do they perceive?

Methods and Data Analysis

To create an instrument designed to measure the knowledge, goals, and teaching/learning orientation of mathematicians, we adapted questions from both Henderson and Dancy's physics education survey [1] and the Characteristics of Successful Programs in College Calculus survey (see `www.maa.org/cspcc` for more information about the CSPCC project). In addition to basic demographic information, the survey[1] questions asked the professors to rate the

[1] *Survey available at* `pcrg.gse.rutgers.edu/algebra-survey`.

importance of various sources of information and to list factors that influenced their teaching decisions. In an attempt to understand their beliefs about teaching and learning, we asked them to describe and characterize their classroom practices, including the motivation behind those choices. Finally, we asked questions designed to test the claim, found in the education literature, that instructors are reluctant to change and, if such resistance was identified, to elucidate the reasons why. Survey requests were sent to departmental administrators at approximately two hundred institutions, targeting instructors who teach undergraduate abstract algebra. Our intention was to survey instructors at Master's- and PhD-granting institutions; however, a small portion of our respondents (9 percent) did come from schools that offer only a Bachelor's degree in mathematics. In total, we received 131 completed surveys. On the whole, the respondents (92 percent tenure-stream faculty) had significant experience both with teaching in general (81 percent reporting 6+ years) and with abstract algebra specifically and were most likely to be teaching an undergraduate groups-first course designed for a mixed (i.e., education, physics, engineering majors commingled with pure math majors) audience. (See Figure 1.)

After compiling the demographic information, we focused our attention on instructor satisfaction in order to determine if any impetus for change existed. To address the first research question, we examined the self-reported teaching practices of the respondents by asking how frequently per class period they engaged in various practices, e.g., using visual or physical representations of groups, having students discuss or work together on problems, having students question one another. Allowable responses were: zero times, one or two times, three or more times. We compared these responses to instructors' self-reported satisfaction with outcomes and their extent of agreement with a series of statements designed to measure teaching/learning orientation. Some examples of those statements are: *I think lecture is the best way to teach; I think students learn better when they struggle with the ideas prior to me explaining the material to them; I think that all students can learn advanced mathematics.* Respondents indicated their level of agreement on a four-point scale.

In our discussion, we highlight areas where the respondents appear to hold beliefs that should lead to certain pedagogical actions but they themselves do not report engaging in those actions. To address the second research question, we categorize instructor reports on implementation of nonlecture reform practices in terms of perceived constraints and viable supports, and we compare these with those cited in the literature.

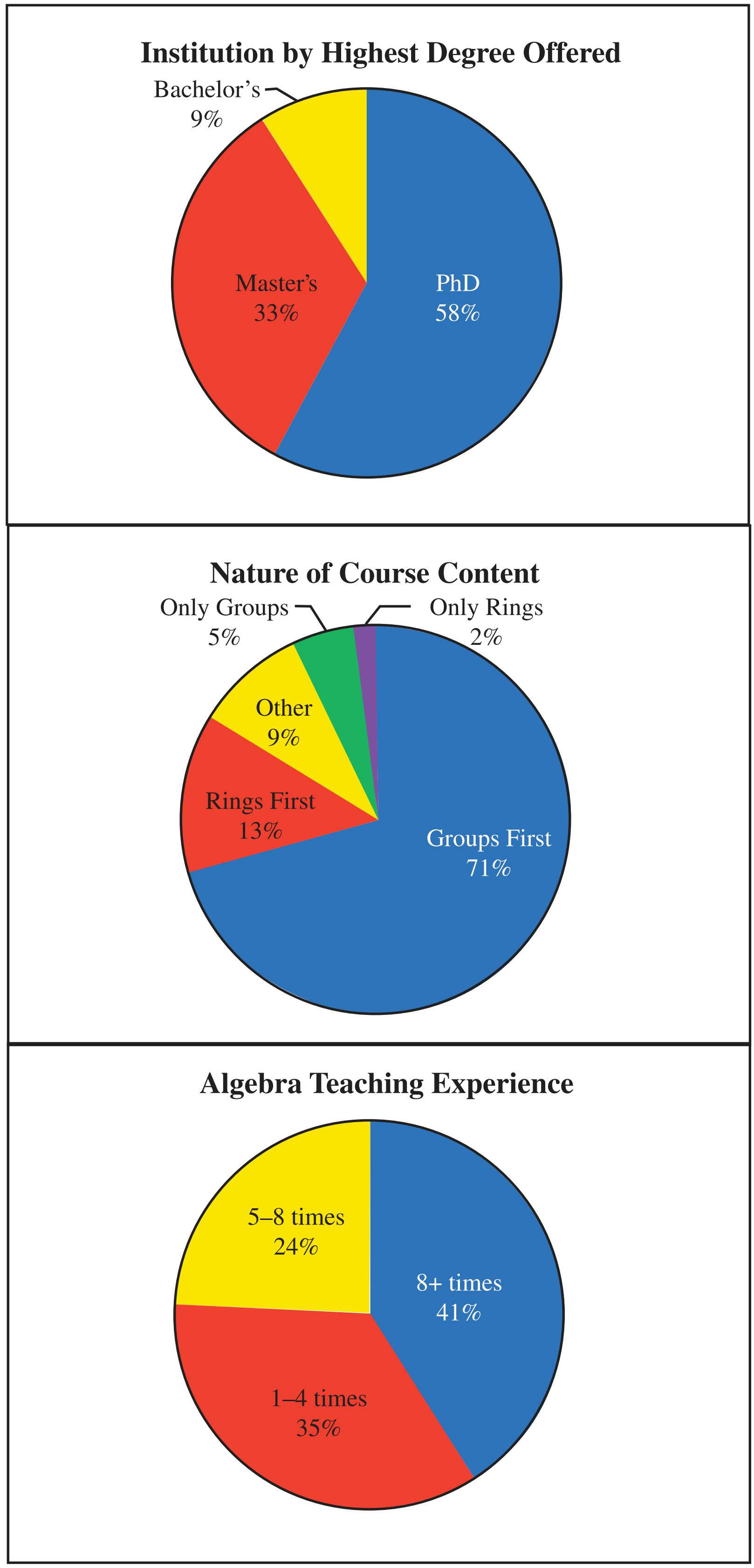

Figure 1. Information about Survey Respondents.

Results

Satisfaction

When measuring satisfaction, several dimensions were considered. For this report, we choose to discuss two in particular: textbook and student learning outcomes. The questions, asked separately in open-ended format (*How satisfied are you with your textbook/students' learning? Please give some explanation.*), were analyzed and categorized by the research team in terms of level of satisfaction: *Satisfied, Mixed, Dissatisfied.* Collectively, 87.6 percent of respondents indicated that they were satisfied with the textbook they used. Instructor comments indicated that the satisfactory rating stemmed from the breadth, depth, and sequencing of content. Even amongst the satisfied, however, complaints about pricing and frequency of new editions were rampant.

When reporting on satisfaction with student learning outcomes, an overwhelming majority of the classifiable responses fell into the *Mixed* (44 of 89) or *Satisfied* (23) categories; fewer than one-quarter gave responses we categorized as *Dissatisfied* (22). The responses were organized by domain (student engagement, student preparation, student performance, student understanding, curriculum issues) and level of satisfaction, allowing us to look for common themes. In summary, instructors that we interpreted as reporting *Mixed* satisfaction indicated (unsurprisingly) that students learned most of the important content and worked reasonably hard. The courses might be in need of a little reorganization or supplemental materials, but major pedagogical overhauls were considered neither warranted nor desired. The comments of the instructors we characterized as *Dissatisfied* were complaints about the unsatisfactory work ethic, motivation, and ability of the students. In contrast, the *satisfied* instructors were less likely to mention the students; rather, instructors who reported high levels of satisfaction were the most likely to comment on the format and curriculum of their courses, with nearly 40 percent (9/23) of them indicating belief that their course was different from most traditional abstract algebra courses due to the use of some form of inquiry-based learning (increased use of examples, student research, Modified Moore Method, etc.).

While the groups did vary widely in typical responses, it was interesting to note that there were two common themes that emerged across all levels of satisfaction. The first observation was a general frustration with students' lack of prerequisite proof skills and poor proof-writing ability. The other common opinion was that it was both difficult and inappropriate to design and teach a course for different constituencies (most often cited was the commingling of math and math education majors). The consensus was that neither population was being adequately served by teaching them simultaneously. A surprising finding of our research was that despite this mixed sense of satisfaction with student learning outcomes, the overall grade distribution was actually quite agreeable. Of those instructors surveyed, the combined passing rate of the students was a whopping 87.82 percent (33.37 percent A, 33.85 percent B, 20.55 percent C), with only 12.18 percent receiving D/F/W grades.

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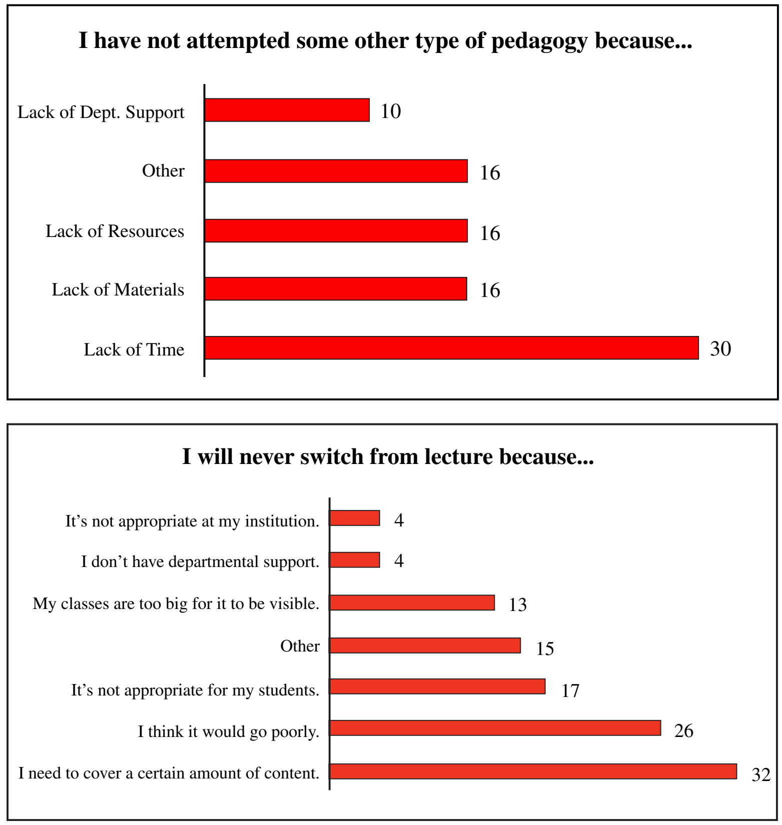

Figure 2. Perceived constraints on the use of nonlecture practices.

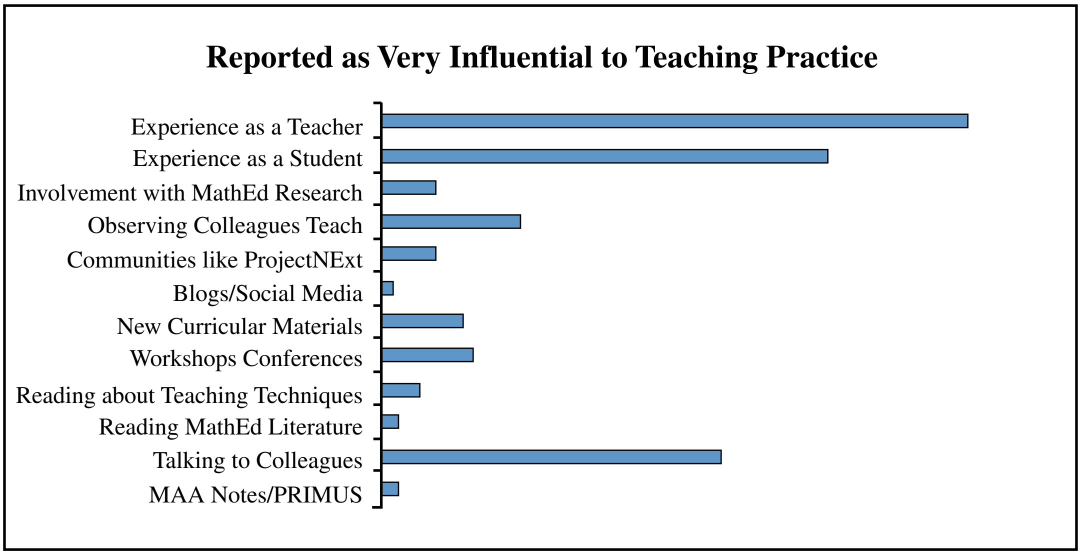

Figure 3. Resources reported as Very Influential by respondents.

Teaching Methods

Lecture was the most common pedagogical practice, with 85 percent of respondents claiming that they currently lecture to teach abstract algebra. This includes the 8 percent of instructors who report returning to lecture after trying some other method. Of the 23 percent who either now or in the past used nonlecture pedagogy and curricular materials, most (fifteen respondents) created it themselves without formal support (typically drawing on a mixture of texts and problem-sets). There were only two respondents who cited use of a particular established curriculum (*Teaching Abstract Algebra for Understanding*, Larsen, 2013; *Learning Abstract Algebra with ISETL*, Dubinsky and Leron, 1994). The others used their own experiences with inquiry-based classes, collaboration with other instructors practicing IBL, or participation in the Academy of Inquiry-Based Learning as a guide to develop their materials and shape their practice.

Of the 85 percent who are currently teaching with lecture, 56 percent of them say that they would consider teaching with nonlecture practices (the remaining 44 percent say they would never do so). The reasons instructors provided for not yet attempting other pedagogy and the explanations offered for why they would never change their habits can be seen in Figure 2.

In short, the two main themes in the comments related to the effort and support needed to revise and teach such a class and concerns about covering the appropriate amount of material. Of the thirty-two instructors who stated coverage as a reason not to adopt a nonlecture format, twenty-three of them answered in the negative when asked: *Do you feel pressure from your department to cover a fixed set of material in your abstract algebra course?* It appears, therefore, that concerns about coverage might be internally situated rather than stemming from an actual source of external pressure.

One of the most interesting findings was the apparent contradiction that emerged when comparing the responses to the following prompts. 82 percent of respondents agreed with the statement *Lecture is the best way to teach.* However, 56 percent agreed (and 26 percent more slightly agreed) with the statement *I think students learn better when they do mathematical work (in addition to taking notes and attending to the lecture) in class.* This result suggests that faculty support the use of nonlecture class activities, yet when asked what students do in class besides taking notes (given a list of options), the only things that instructors claimed that students did in class, even at a rate of once per month, was doing calculations, working with examples, or working with applications. Moreover, 63 percent reported that students never spent time working on mathematics problems in class. So it would appear that what instructors think is best for student learning (students doing mathematical work in class) is not happening with any frequency. Thus, we argue that there exists a mismatch between beliefs about student learning and actual teaching practice. One could argue that this mismatch might be explained by a perceived lack of time to make adjustments to their teaching practices on the part of the instructors; however, the data indicate otherwise. When asked if they believe that they would have time to plan and redesign their courses in a way that would be supported and valued in terms of formal review, nearly all (100/129) respondents reported this as a possibility (42 *yes* and 58 *maybe*). Therefore, in general, it does not appear to be the case that time

Photo by Lawrence Academy.

Photo by David Plaxco, 2014.

Tim Fukawa-Connelly (top) and Estrella Johnson (bottom) teaching.

constraints alone account for the discrepancy between how instructors say they want to teach and how they actually teach.

Influences on Instruction

When asked to identify the primary influences on their teaching practice (*How influential are the following on your teaching? Very/Somewhat/Not at all*), the respondents overwhelmingly identified three sources of inspiration. In decreasing order of significance, the participants reported that their experiences as a teacher (84 percent) and experiences as a student (64 percent) were far and away the most significant. Participants also reported that talking to colleagues about how to teach specific content was important (49 percent). Little importance was assigned to the normal means that grant-supported projects use to disseminate new teaching ideas: Project NExT (8 percent), MathFest, MAA mini-courses or other workshops (13 percent), or publications about teaching such as the MAA Notes series or PRIMUS (2 percent). From these numbers, it appears that most mathematicians have few influences on their teaching that are external to their own universities. To be fair, it should be noted that we do not know the distribution of those individuals who read the literature and attend professional development opportunities. Unless there exist mechanisms of which we are unaware, it appears that the majority of math departments might be closed to outside influences on teaching.

This lack of outside influences is likely to be especially prohibitive for the 59 of 106 respondents who would consider trying something other than lecture but have not because they haven't had time to redesign their course (30/59), haven't found materials that they like (16/59), or don't know where to start (16/59). So, it would appear that the very resources designed to alleviate some of these challenges are failing to meet that objective. Again, looking only at the 59 of 106 participants who state that they would consider not lecturing, only one finds PRIMUS or the MAA Notes series *very influential*; only 1 finds mathematics education research literature *very influential*; only 6 find talks, workshops, or conferences about teaching (e.g., MathFest mini-courses) *very influential*; and only 4 find participating in communities like Project NExT *very influential*. It is our belief that this is not because the materials themselves are not useful, but rather that those who most need them are not utilizing them.

Conclusions

There are four major findings that we highlight. First, lecture is the predominant mode of instruction (97/126), and even those who have tried other pedagogies appear to switch back to lecturing at surprisingly high rates (10/29). Moreover, given the significant amount of time, money, and energy spent developing, testing, promoting, and training mathematicians to use new curricula and pedagogies, there is almost no uptake. Those using nontraditional materials are far more likely to have developed their own materials than to have adopted NSF-supported curricula.

The second major finding relates to the factors that influence pedagogical decisions. In decreasing order of significance, the participants reported that their experiences as a teacher and student were far and away the most significant influence, followed by talking to colleagues about how to teach specific content; the least significant source of influence was grant-supported distribution methods such as publications and workshops. If mathematicians essentially give no weight to the traditional means of dissemination of new pedagogical ideas and techniques (and evidence of their effectiveness), reformers have few means of promoting change other than individual conversation. This alone suggests why reforming undergraduate mathematics, and abstract algebra in particular, is difficult.

Third, while faculty claim they have the ability to change their courses, the reported satisfaction levels indicate they do not have the desire to do so. Furthermore, the majority of dissatisfaction stems from perceived problems with the students and not the course materials. Given the strong content focus and high belief in the efficacy of (and preference for) lecture, it appears that as a collective, the abstract algebra teaching faculty has little interest in adopting new pedagogical approaches at this time.

We propose two concurrent research directions. First, we need to better explore the reasons that mathematicians appear to strongly believe in their current practice, the types of evidence that they hold as dispositive, and what means of dissemination of new approaches achieve meaningful penetration. Second, we need to further explore the types of changes to the practice of lecture that mathematicians would adopt. There appears to be a conflict between the stated goals of policy boards and national organizations and the way that faculty, on the ground, think about their courses. Math educators are responding to the claims of the stated goals of changing undergraduate courses to include more student-active work, but if mathematicians have different perceived needs, as our work shows, these new ideas won't gain traction. Thus, we want to have a conversation about what is understood as practical and feasible in the eyes of those charged with delivering the instruction.

Finally, for us and mathematics education researchers generally, we wonder how best to propose new strategies about teaching and how to receive feedback from the mathematical community as to their interest and feasibility. Basically, if the only people that mathematics instructors ever talk to are their colleagues, it is a closed circle with no obvious entry point for new ideas. As an example, a major source of dissatisfaction revealed in this survey was instructor frustration with students' poor proof-writing abilities, an area that has received significant attention from mathematics education researchers and has produced practical suggestions for improving proof comprehension. These ideas are heavily researched and, given the comprehensive pedagogical supports available, often do not require the extensive time commitment often incorrectly assumed of nontraditional methodology. But without open communication between researchers and practitioners, the validity and viability of these ideas go unappreciated. We mathematics education researchers have spent significant time, literally decades, trying to understand how students learn mathematics in general and specific content areas in particular. Help us to help you. If you are dissatisfied with your current practice or results, if you are frustrated by lecture-dominated classes, if you are looking for inspiration—we might have the answer. All you have to do is ask!

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