

# Can Prospect Theory Explain the Disposition Effect? A New Perspective on Reference Points\*

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## Abstract

There is a recent debate on whether prospect theory can explain the disposition effect. Using both theory and simulation, this paper shows that prospect theory often predicts the disposition effect when lagged expected final wealth is the reference point, regardless of whether the reference point is updated or not. When initial wealth is the reference point, however, there is often no disposition effect. Reference point adjustment weakens the disposition effect, leads to more aggressive initial stock purchase strategies and predict history-dependence in stock holding. These findings also provide a explanation for why market experience reduces behavioral biases.

**Keywords:** disposition effect, prospect theory, loss aversion, reference point, expectations

**JEL Classification Codes:** G02, D03

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# 1 Introduction

The disposition effect refers to the propensity of certain investors to sell stocks that have risen in value rather than stocks that have fallen in value since purchase (Shefrin and Statman, 1985; Odean, 1998; Weber and Camerer, 1998). The disposition effect poses a challenge to standard models which based on expected-utility maximizing investors.<sup>1</sup> Prospect theory, as developed by Kahneman and Tversky (1979), has been the most popular theory used to explain the disposition effect. This theory assumes that people derive utility based on gains and losses relative to a reference point. The popular explanation, however, often relies on one of prospect theory's assumptions called diminishing sensitivity, which assumes concave utility function above the reference point and convex utility function below the reference point (an S-shaped value function).

With the implicit assumption of purchase price or initial wealth as the reference point, risk aversion in the gains domain (as implied by diminishing sensitivity) explains more sales of stocks that earn positive profits. However, the core element of prospect theory is loss aversion, which assumes that losses relative to the reference point hurt an individual more than equal-sized gains cause satisfaction.<sup>2</sup> Surprisingly, Barberis and Xiong (henceforth BX) (2009) suggest that loss aversion often leads to more sales of stocks that make negative profits. This effect dominates that of diminishing sensitivity so that, taken together, prospect theory tends to predict a pattern opposite the disposition effect. While BX carefully examined the robustness of their conclusion in many directions, they did not focus on the role of the reference point. This paper takes this task for the first time to formally analyze the implications of reference point on the disposition effect.

The empirical measure of the disposition effect naturally benchmarks on the average purchase price to define winning and losing stocks. Accordingly, most studies also assume the reference point in utility to be the initial wealth.<sup>3</sup> While this is consistent with the traditional status quo assumption of reference points in behavioral economics, the reference point to judge psychological gains and losses may deviate from average purchase price. For instance, an investor expecting

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<sup>1</sup>Odean (1998) explicitly considers expected-utility explanations for asymmetry across winners and losers based on richer specifications of the investor's problem, finding that portfolio rebalancing, transaction costs, taxes, and rationally anticipated mean reversion cannot explain observed asymmetries. Weber and Camerer (1998) also find that incorrect beliefs concerning mean reversion cannot explain the disposition effect.

<sup>2</sup>The third feature of prospect theory, nonlinear probability weighting, roughly assumes that investors systematically over-weigh small probabilities and under-weigh large probabilities. For simplicity's sake, the literature on the disposition effect often does not discuss this feature, nor does this paper.

<sup>3</sup>One notable exception is Health et al. (1999), who analyze the option of exercising decisions, wherein there is no natural purchase price to rely on. They find that historical high price plays an important role in driving decisions. In BX (2009), the assumed reference point is wealth from investing in risk-free assets, which is essentially a status quo assumption which takes interest rate into account.

to earn 5% of returns from an investment portfolio may experience a 3% return as a loss of 2% rather than a gain of 3%. Inspired by Köszegi and Rabin's (henceforth KR) (2006, 2007, 2009) new reference-dependent models that endogenize reference points as rational expectations, many recent behavioral economics studies provide supporting empirical evidence for an expectations reference point. This paper, however, explores for the first time the implications of expectations reference points on investors' trading behaviors.

We build a dynamic model of individual trading behavior, assuming loss aversion and diminishing sensitivity, that formally considers rational expectations as an alternative reference point. Our model follows BX's setup (2009) as closely as possible for comparison purposes while following the spirit of KR's (2009) preferred personal equilibrium to endogenize the reference point as lagged rational expectations. We analyze three specifications of the expectations reference point: initial expected final wealth (EC) as the constant reference point as well as both one-period-lagged (L1) and two-period-lagged (L2) expected final wealth as variable reference points. All specifications provide strong intuition (confirmed by our simulation results) that loss aversion successfully predicts the disposition effect when lagged expected final wealth defines the reference point. Since the effect of loss aversion is dominant in our model, prospect theory as a whole can thus often predict the disposition effect using an expectations reference point. In reaching this conclusion, the disposition effect is measured, as before, by defining gains and losses relative to the average purchase price, only the reference point in the utility function is varied.

The intuition is simple: The kink generated by loss aversion implies a discontinuous change in the marginal utility around the reference point. This sharp change leads to behavior demonstrating excessive risk aversion (Rabin, 2000) which in a stock trading setting predicts the bunching of sales around the reference point. When stock trading profits are either too low or high relative to the reference point (wherein the probability of crossing the reference point in the future is low) the effect of the kink vanishes: Investors become less risk averse and are thus more likely to hold a stock. Given this relationship, the location of the reference point changes investors' risk attitudes. BX (2009) made an important observation that the stocks in question must have positive expected returns for loss-averse investors to purchase them in the first place. Such returns distribution tends to generate large trading gains and small trading losses relative to average purchase price. When initial wealth is the reference point, loss aversion predicts a relatively smaller stock position in the domain of trading losses since generated trading gains are, on average, farther away from the reference point than trading losses. This paper, in contrast, shows that when the reference point is expected final wealth (which is typically higher than initial wealth) most trading gains are on average closer to the reference point than trading losses. Loss aversion therefore implies a strong bunching of sales at winning stocks rather than losing stocks, thus creating the disposition effect.

Besides the disposition effect, our model also generates novel predictions. We show that the adjustment in reference point leads to a weaker disposition effect, more aggressive shareholding strategy during the initial period, and history-dependence on optimal stock holding. Since market experience can allow investors to admit gains or losses more easily and adjust their reference point more quickly, these results provide a reasonable explanation for how market experience reduces behavioral bias (List, 2003; Feng and Seasholes, 2005; Da Costa et al., 2013). When the reference point adjustment is slow, investors in our models also tend to sell stocks with small trading gains/losses more often than those with large trading gains/losses, a fact demonstrated in Table III of Odean (1998) yet not well-explained by alternative theories. When the reference point is adjusted quickly, however, a reverse pattern appears tending to predict the findings in Ben-David and Hirshleifer (2012). Our model may thus be able to reconcile these two contradicting empirical findings.

A robustness check suggests that using the alternative equilibrium concept in disappointment-aversion models (Loomes and Sugden, 1986; Bell, 1985; Gul, 1991) and assuming current expected final wealth (no lag) as the reference point both lead to no stock holding in the first place. Other simplifications relative to KR's assumptions in their new reference-dependent model (e.g. stochastic reference point, the consumption utility in addition to gain-loss utility, and the length of the evaluation period), however, do not substantially affect the model's major predictions on the disposition effect.

In addition to BX (2009), several theoretical studies on prospect theory and the disposition effect assume initial wealth as the reference point. Kaustia (2010) as well as Hens and Vleck (2011) used the same partial equilibrium approach as BX and reached similar conclusions. Li and Yang (2013) characterized the predictions of prospect theory in a general equilibrium framework, finding that diminishing sensitivity predicts the disposition effect, price momentum, reduced return volatility, and a positive return-volume correlation; loss aversion generally predicts the opposite. When stock returns are negatively skewed, however, loss aversion may actually predict the disposition effect.

BX (2009, 2012) developed an alternative explanation of the disposition effect based on realization utility, posing a distinction between paper and realized gains by assuming that additional gain-loss utility occurs at the moment of sale. The optimal solution is characterized by a threshold strategy which makes investors sell stocks once they reach a certain liquidation point in gain. Combined with positive time discounting, realization utility in their model predicts the disposition effect among a wide range of other predictions. Ingersoll and Jin (2012) also assumed realization utility, but added diminishing sensitivity to their model, which predicts the disposition effect as a dynamic result. McQueen and Vorkink (2004) investigated the effects of loss aversion and

changing risk aversion on asset prices, with risk aversion and attention to news depending on past investment performance.<sup>4</sup>

This paper contributes to the broad literature on expectations-based reference points by applying expectations reference point to the disposition effect for the first time. KR's theoretical work has inspired several laboratory and empirical studies outside finance. For the classical endowment effect, researchers have mixed evidence: Ericson and Fuster (2011) reported evidence consistent with the expectations-based reference point, while Heffetz and List (2014) found that varying the probability of exchange opportunity did not affect a subject's tendency to trade (yet subjects tended to keep their assigned goods when those goods varied). They therefore conclude that the assignment, rather than expectations, is what matters. For labor supply, both Abeler et al.'s (2011) as well as Gill and Prowse's (2012) experimental studies revealed that subjects exerted greater effort when they had higher expected payments. Assuming rational expectations, Crawford and Meng (2011) showed that New York cab drivers' labor supply decisions are strongly affected by income and target hours proxied by past sample average. Risk attitudes are also shown to be affected by expectations-based reference point. Using the data from a popular game show, Post et al. (2008) found that participants were less risk averse following unexpected big gains and losses while more risk averse following small gains and losses. Sprenger (2015) showed that using stochastic expectations as the reference point makes subjects generally less risk averse. Song's (2015) experimental study further showed that an expectations-based reference point exists and adjusts relatively quickly to the resolution of uncertainty.

This paper also contributes to the literature on market experience and behavior biases. List (2003) was the first to show in a field experiment that market experience reduces the endowment effect. Camerer et al. (1996) also showed that cab drivers' targeting behavior was less pronounced among more experienced drivers. Feng and Seasholes (2005) found that individual sophistication and trading experience together eliminate the tendency to hold losses. In a laboratory study, Da Costa et al. (2013) showed that undergraduate students demonstrated a stronger disposition effect than experienced investors. The literature, however, has been vague about the underlying mechanism of such an effect: This paper thus provides a reasonable channel through a reference point adjustment lens.

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<sup>4</sup>There are several other studies on the disposition effect that do not involve loss aversion. For instance, Ben-David and Hirshleifer (2011) tested whether a simple preference for realizing winners exists as characterized by a discontinuous drop in the probability of holding a stock at purchase price. Their empirical analysis suggests that the magnitude of the discontinuity is too small to explain the calculated disposition effect within the same data. They therefore propose a model based on speculative motive to explain the disposition effect. Chang et al. (2016) combined real trading data with a laboratory experiment to show that the delegated portfolio demonstrates an anti-disposition effect, proposing that cognitive dissonance is one source of the disposition effect.

The remainder of the paper is organized as follows: Section 2 builds a dynamic model of prospect theory preferences that accommodate rational expectations as the reference point. Section 3 simulates the model's predictions and presents the simulation results. Section 4 discusses the simulation results. Section 5 performs the robustness check of the models' main conclusions. Section 6 concludes the paper.

## 2 A Model of Prospect Theory Preferences with Expectations as the Reference Point

### 2.1 Model Setup

We consider a dynamic model of asset allocation between a risk-free asset and a stock assuming loss aversion and diminishing sensitivity. Our model is exactly the same as that in BX (2009) excepting that the reference point in our model is endogenously determined by rational expectations (whereas it is exogenously given in BX). In particular, we consider a portfolio choice setting with  $T$  dates,  $t = 0, 1, \dots, T - 1$ . There are two assets: a risk-free asset, which earns a gross return of  $R_f = 1$  in each period, and a risky asset, the price of which evolves along a binomial tree. Hence, the risky asset's gross return from  $t$  to  $t + 1$ ,  $R_{t,t+1}$  is distributed according to the following:

$$R_{t,t+1} = \begin{cases} R_u > R_f & \text{with probability } \frac{1}{2}; \\ R_d < R_f & \text{with probability } \frac{1}{2}; \end{cases} \quad \text{i.i.d. across periods.} \quad (1)$$

We use  $u$  to denote an event wherein the gross return is  $R_u$ , and  $d$  to denote an event wherein the gross return is  $R_d$ . We assume  $\frac{1}{2}R_u + \frac{1}{2}R_d$  to be strictly larger than  $R_f = 1$ , so expected stock return exceeds the risk-free rate.

We study the trading behavior of investors with prospect theory preferences. Specifically, at any date  $t = 0, 1, \dots, T - 1$ , the investor makes the optimal decision by comparing their final wealth  $W_t^T$  to their reference point  $W_t^R$ . Let  $\Delta W_t = W_t^T - W_t^R$ , denoting the expected gain/loss relative to their reference point. Following Tversky and Kahneman (1992) and BX (2009), we consider a specific form for the utility function which demonstrates loss aversion and diminishing sensitivity:

$$v(\Delta W) = \begin{cases} \Delta W^\alpha & \text{for } \Delta W \geq 0; \\ -\lambda(-\Delta W)^\alpha & \text{for } \Delta W < 0. \end{cases} \quad (2)$$

We have set the parameter values such that  $\alpha = 0.88$  and  $\lambda = 2.25$  following Tversky and Kahneman (1992).  $\lambda$  is the loss-aversion coefficient while  $\lambda > 1$  implies loss aversion (i.e. the investor hates losses more than he enjoys gains).  $\alpha$  governs the curvature of the value function above and below the reference point.  $\alpha = 0.88$  implies diminishing sensitivity, i.e. the investor's value function is concave above and convex below the reference point.

At each date from  $t = 0$  to  $t = T - 1$ , the investor must decide how to split his wealth between the risk-free and risky assets. Let  $x_t$  be the number of shares of the risky asset he holds at date  $t$ , and let  $h_t = \{x_0, \dots, x_{t-1}, R_{0,1}, \dots, R_{t-1,t}\}$  represent the realized history at the beginning of date  $t$ .  $x_t$  is chosen based on previous history  $h_t$ , while the choice of  $x_t$  determines  $W_{t+1}$  per the following budget constraint:

$$W_{t+1} = W_t R_f + x_t P_t (R_{t,t+1} - R_f). \quad (3)$$

Given  $\{x_\tau\}_{\tau=t}^{T-1}$ , we can therefore further derive  $W_t^T$  for a realization of  $R_t^T = \{R_{t,t+1}, \dots, R_{T-1,T}\}$  based on Equation (3):

$$W_t^T = (W_T | h_t, R_t^T) = W_t R_f^{T-t} + \sum_{\tau=t}^{T-1} x_\tau P_\tau R_f^{T-\tau-1} (R_{\tau,\tau+1} - R_f). \quad (4)$$

At any history  $h_t$ , the investor's decision problem is to maximize the expected gain-loss utility in the final period as follows:

$$\max_{x_t} E[v(\Delta W_t)] = E[v(W_t^T - W_t^R)], \quad (5)$$

Subject to Equation (4), the requirement that  $x_\tau$  is optimal at every history  $h_\tau$ , and a non-negativity of wealth constraint  $W_T \geq 0$ .

In the tradition of Benartzi and Thaler (1995) as well as BX (2009), this model imposes the assumption called "narrow framing" or "mental accounting" (Thaler and Johnson, 1990). The  $T$  trading dates together form an "evaluation period", which is the horizon for evaluating investment performance and deriving gain-loss utility. Benartzi and Thaler (1995) empirically estimate an evaluation period of approximately one year, which this paper follows for its simulations. By analyzing the choice between a risk-free asset and stock, our model also implicitly assumes that investors evaluate gains and losses stock-by-stock (Barberis, Huang and Thaler, 2006; BX, 2009). Barberis and Huang (2001) supports this assumption, showing that treating the trading decision as if investors consider each stock separately better fits the empirical data.

## 2.2 Fixed Reference Point

### A. Model Analysis

We first consider a case where the reference point is fixed over time, i.e. the date-0 expectation of final wealth:

$$W_t^R = \bar{W} = EW_T, \forall t.$$

Given any fixed reference point  $\bar{W}$ , we can rewrite investors' dynamic optimization problem as a static problem wherein investors directly choose their wealth in different possible states at the final date, as shown in Cox and Huang (1989) and BX (2009).

Following BX (2009), we rank the  $t + 1$  nodes at any date  $t$  as  $j = 1, \dots, t + 1$ , where  $j = 1$  corresponds to the highest node in the tree at that date and  $j = t + 1$  to the lowest. We also use  $P_{t,j} = P_0 R_u^{t-j+1} R_d^{j-1}$  to denote risky asset price in node  $j$  at date  $t$ ;  $\pi_{t,j}$  to denote the ex-ante probability of reaching that node; and  $q_{t,j}$  to denote the state price density for that node.

We thus rewrite investors' dynamic optimization problem for a fixed reference point  $\bar{W}$  as follows:

$$\max_{\{W_{T,j}\}_{j=1,\dots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(W_{T,j} - \bar{W}), \quad (6)$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} W_{T,j} = W_0, \quad (7)$$

and a non-negativity wealth constraint

$$W_{T,j} \geq 0, \quad j = 1, \dots, T + 1. \quad (8)$$

The above property enables us to focus on final period wealth allocation when defining our equilibrium.

**Definition 1.** We call  $\{\bar{W}^*, \{W_{T,j}^*\}_{j=1,\dots,T+1}\}$  a rational expectation equilibrium if the following two conditions are satisfied:

1. Given  $\bar{W}^*$ ,  $\{W_{T,j}^*\}_{j=1,\dots,T+1}$  is the solution of the above optimization problem (6)-(8).
2.  $\bar{W}^*$  is determined by rational expectation:

$$\bar{W}^* = \sum_{j=1}^{T+1} \pi_{T,j} W_{T,j}^*. \quad (9)$$



Definition 1 defines the rational expectation equilibrium and the reference point corresponding to the transformed static problem. We base this definition upon KR's definition of personal equilibrium in their new reference-dependent model (2006). Personal equilibrium imposes internal consistency in the sense that fixing the reference point, the resulting optimal solution also generates the reference point itself. In particular, this solution requires two steps: (1) fixing the reference point and deriving an optimal solution and (2) ensuring the optimal solution generates the reference point. There may exist multiple equilibria due to the self-fulfilling property of rational expectations. Following KR's preferred personal equilibrium, we further refine our definition of equilibrium by focusing on the most efficient equilibrium in the presence of multiple equilibria. One imagines that when the investor is able to select the reference point at date 0, he always would want to select the reference point yielding the highest expected utility.

For an arbitrary  $\bar{W}$ , let  $V(\bar{W})$  be the optimal expected utility by solving the maximization problem (6)-(8). The following proposition characterizes the reference point for any candidate equilibrium. The proofs of the following and all subsequent results can be found in the Appendix.

**Proposition 1.** *For  $k = 1, \dots, T + 1$ , denote*

$$\bar{W}(k) = \frac{\sum_{i=1}^k W_0 \pi_{T,i} q_{T,i}^{-\frac{1}{1-\alpha}}}{(1 - \sum_{i=1}^k \pi_{T,i}) \sum_{i=1}^k \pi_{T,i} q_{T,i}^{-\frac{\alpha}{1-\alpha}} + \sum_{i=1}^k \pi_{T,i} q_{T,i}^{-\frac{1}{1-\alpha}} \sum_{i=1}^k \pi_{T,i} q_{T,i}}, \quad (10)$$

and

$$U(k) = \left( \sum_{i=1}^k \pi_{T,i} q_{T,i}^{-\frac{\alpha}{1-\alpha}} \right)^{1-\alpha} \left( W_0 - \bar{W}(k) \sum_{i=1}^k \pi_{T,i} q_{T,i} \right)^\alpha - \lambda \bar{W}(k)^\alpha \sum_{i=k+1}^{T+1} \pi_{T,i}. \quad (11)$$

*In any rational expectation equilibrium, the equilibrium reference point  $\bar{W}$  must be some  $\bar{W}(k)$ , and  $\bar{W}(k)$  can be an equilibrium reference point if  $U(k) = V(\bar{W}(k))$ .*

We base our construction of rational expectation equilibrium on BX's (2009) conclusion, which shows that an investor's optimal policy is to use a "threshold" strategy. In this strategy, the investor allocates a wealth greater than the reference level  $\bar{W}^*$  upon the  $k$  date  $T$  nodes that offer the highest prices for risky assets as well as a wealth level of 0 for the remaining date nodes. For the reference point  $\bar{W}(k)$ , we first calculate the optimal levels of final wealth following conditions (6)-(8) given this threshold strategy. We then use condition (9) to pin down the expression of  $\bar{W}(k)$ , and this gives us (10). Equation (11) is then the optimal expected utility given the reference point expressed in (10). Based on these results, we can calculate the rational expectation equilibrium using the following procedures: First, for every  $k : 1 \leq k \leq T + 1$ , we calculate the corresponding  $\bar{W}(k)$  using (10). Second, we consider that Equation (10) alone does not guarantee that the calculated  $\bar{W}(k)$  is indeed a rational expectation reference point: If  $k^*$  leads to a rational

expectation equilibrium, we must ensure that, given  $\bar{W}(k^*)$  as reference point, the investor has no incentive to deviate to another threshold value  $k$ , i.e. the optimal expected utility given that a reference point of  $\bar{W}(k^*)$  (denoted by  $V(\text{bar}W(k^*))$ ) exactly equals the expected utility generated by threshold  $k^*$  (denoted by  $U(k^*)$ ). Third, in the case of multiple equilibria, we further derive the most efficient equilibrium that generates the highest expected gain-loss utility.

Following BX (2009), if  $k^*$  is indeed an equilibrium threshold strategy then the optimal wealth allocation  $W_{T,j}$  in node  $j$  at final date  $T$  is given by the following:

$$W_{T,j} = \bar{W}(k^*) + q_{T,j}^{-\frac{1}{1-\alpha}} \frac{W_0 - \bar{W}(k^*) \sum_{i=1}^{k^*} \pi_{T,i} q_{T,i}}{\sum_{i=1}^{k^*} \pi_{T,i} q_{T,i}^{-\frac{\alpha}{1-\alpha}}}$$

for  $j \leq k^*$ ; and  $W_{T,j} = 0$  otherwise.

The optimal share holdings  $x_{t,j}$  are given by

$$x_{t,j} = \frac{W_{t+1,j} - W_{t+1,j+1}}{P_0(R_u^{t-j+2} R_d^{j-1} - R_u^{t-j+1} R_d^j)},$$

where we can calculate the intermediate wealth allocations by working backwards from date  $T$  using

$$W_{t,j} = \frac{\frac{1}{2} W_{t+1,j} q_{t+1,j} + \frac{1}{2} W_{t+1,j+1} q_{t+1,j+1}}{q_{t,j}},$$

for all

$$0 \leq t \leq T - 1, \quad 1 \leq j \leq t + 1.$$

The key issue is determining whether the investor wants to purchase a stock at date 0: Notice we can view BX (2009) as a special case within our model, as their model's reference point  $W_0 R_f^T = W_0$  is exactly the same as  $\bar{W}(T + 1)$ , as defined by equation (10) (due to the facts that  $\sum_{i=1}^{T+1} \pi_{T,i} = 1$  and  $\sum_{i=1}^{T+1} \pi_{T,i} q_{T,i} = 1$ ).<sup>5</sup> We thus immediately derive the following corollary:

**Corollary 1.** *The range of the expected stock returns leading to no stock purchase is the same as BX (2009), which assumes  $\bar{W} = W_0$ .*

This argument has two parts. First, if it is optimal to not buy stock when the reference point is  $\bar{W} = W_0$ , then  $\bar{W} = W_0$  is also part of the rational expectation equilibrium. If other rational expectation equilibria involving stock purchase exist, the equilibrium with  $\bar{W} = W_0$  is the most efficient. This is because an expected utility without stock purchase is zero, while those with stock

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<sup>5</sup>While BX's model also assumes a reference point of wealth from risk-free asset investment, their simulation assumes that  $R_f = 1$ .

purchase are negative due to gains and losses being roughly symmetric relative to their expected values. This leads to negative expected gain-loss utility due to loss aversion. Second, when it is optimal for the investor to purchase shares under the reference point  $\bar{W} = W_0$ ,  $\bar{W} = W_0$  cannot be an equilibrium reference point. We must thus determine the reference point endogenously by rational expectation and the equilibrium involve the stock purchase.

The above argument leads to the question of why a loss-averse investor would wish to purchase that stock in the first place: Purchasing the stock will lead to roughly symmetric outcomes relative to the expectations reference point thus the expected utility tends to be negative due to loss aversion. By not purchasing the stock, the investor incurs no gains or losses, This argument, however, implicitly assumes that in calculating rational expectation equilibrium, when the action is not purchasing stock the reference point changes to  $\bar{W} = W_0$ . Our solution follows personal equilibrium in assuming that the reference point remains the same when deriving an optimal action. In other words, if purchasing the stock falls within a rational expectation equilibrium, the reference point is always the expected final wealth as generated by the decision to purchase the stock. As a result, not purchasing the stock would generate a sure loss, while purchasing the stock will at the very least generate possible gains. Not purchasing the stock may be therefore be suboptimal.<sup>6</sup> This argument differs from the previous paragraph, as it pertains to deriving rational expectation equilibrium, while the previous paragraph compares different rational expectation equilibria.

## B. An Example

When rational expectations endogenously determine the reference point, BX's (2009) result that prospect theory often predicts the opposite of the disposition effect no longer holds true. To illustrate this basic intuition, we present a simple numerical example with  $T = 2$  and the parameter values exactly the same as the corresponding example in BX (2009). In particular, we set  $\mu$ , the annual gross expected return, as 1.1 and  $\sigma$ , the standard deviation of stock return, as 0.3. This values for  $\mu$  and  $\sigma$  imply  $(R_u, R_d) = (1.25, 0.85)$ . We set other parameters as  $W_0 = 40$ ,  $P_0 = 40$ , and  $R_f = 1$  with an evaluation period of one year. As shown by BX (2009), when the reference point is  $W_0 = 40$  the optimal trading strategy is given by

$$(x_0, x_u, x_d) = (4.0, 5.05, 3.06).$$

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<sup>6</sup>The disappointment-aversion models in the literature assume that the reference point changes with the action when deriving an optimal solution (Loomes and Sugden, 1986; Bell, 1985; Gul, 1991). The choice-acclimating personal equilibrium in KR (2007) exhibits a similar feature. We favor our personal equilibrium approach not only because it is internally consistent but also because it predicts stock purchase in the first place, while existing disappointment-aversion models predict no stock purchase. For more detail, see our robustness check in Section 5.

Initially, an investor buys 4.0 shares of the risky asset. After a good stock return at date 1, he increases his position to 5.05 shares. After a poor stock return at date 1, he decreases his position to 3.06 shares. There is thus no disposition effect under the reference point  $W_0$ .

When rational expectations (as defined in our model) determines the reference point, however, the new reference point is calculated at  $\bar{W} = 52.8$ . The optimal trading strategy under this new reference point thus changes to

$$(x_0, x_u, x_d) = (3.26, 2.51, 3.93).$$

Initially, an investor buys 3.26 shares of the risky asset. After a good stock return at date 1, he decreases his position to 2.51 shares. After a poor stock return at date 1, he increases his position to 3.93 shares. The investor thus demonstrates strong disposition effect.

What is the intuition for this result? Figures 1 and 2 illustrate the reason: For both figures,  $x$ -axis is the change in wealth relative to initial wealth (not the reference point) which represents trading gains and losses. For instance,  $\Delta W_u$  is the change in wealth relative to  $W_0$  after realizing a good return at date 1, while  $\Delta W_{ud}$  is the change in wealth relative to  $W_0$  following a good return at date 1 and a bad return at date 2. The  $y$ -axis is gain-loss utility.

Fig. 1 illustrates a case wherein initial wealth is the reference point, thus the kink occurs at 0. As BX (2009) has clearly shown, if an investor is currently in the losses/gains domain relative to the reference point, he will take a stock position such that the highest/lowest wealth leaves him at or near the reference point. Crucially, if the investor has a wealth farther from the reference point in either direction, then he needs to take a larger stock position in order for the highest (in the case of losses) or lowest (in the case of gains) wealth to reach the reference point. This non-monotonic trading pattern mainly results from loss aversion, which generates a sharp change in marginal utility, which in turn implies first-order risk aversion. This makes the investor extremely risk averse around the reference point, demanding fewer shares of the stock. When current wealth is far away from the reference point, however, loss aversion diminishes, thus the investor demands more shares. Diminishing sensitivity complicates this intuition a little bit, but its effect is only second order.

In this example,  $\Delta W_u = 40$  and  $\Delta W_d = -24$ . The trading gain at date 1 is therefore farther from the reference point than the trading loss, so the investor tends to demand more shares of the stock if there is a trading gain, a pattern opposite the disposition effect.

Compared to Fig. 1, the key change in Fig. 2 is that the kink moves from 0 to 12.8 due to the change in the reference point. In this case,  $\Delta W_u = 32$  and  $\Delta W_d = -20$ . Since trading loss is farther away than trading gain from the reference point 12.8, the investor tends to demand more shares of the stock if there is a trading loss, a pattern consistent with the disposition effect.

[Fig. 1 inserted here]

[Fig. 2 inserted here]

In general, the reference point defined as rational expectations is higher than the status quo reference point if the expected stock returns are positive. As a result, trading gains are generally closer to the reference point than trading losses. Since loss-averse investors tend to demand more shares if they're farther from the reference point, they tend to demand more shares if there are trading losses.

This example also shows the intuition on why investors wish to purchase a stock in the first place when rational expectations define the reference point. In fixing the reference point to  $\bar{W} = 52.8$ , not purchasing stock generates a sure loss of 12.8, while purchasing stock leads to a lottery in final wealth of  $(0.25, 103.6; 0.5, 53.6; 0.25, 0)$ . Expected gain-loss utility is  $-21$  for the former and  $-16$  for the latter case, so purchasing stock is optimal at date 0 under a rational expectation equilibrium. It's simple to check that not purchasing the stock at date 0 is not a rational expectation equilibrium, because given the reference point  $\bar{W} = 40$ , purchasing the stock generates an expected gain-loss utility of 1, while purchasing no stock generates neither gains nor losses.

## 2.3 Variable Reference Point

### A. Model Analysis

One may think that a fixed reference point model is unrealistic: As time goes by, an investor can adjust the expectations and thus his reference point. In this section, we analyze the variable reference point model. The ability to adjust a reference point correlates to the investor's level of sophistication: The more experienced the investor, the more likely he can quickly adjust the reference point. Studying the adaption of the reference point therefore relates to studies on the effects of market experience. Previous studies have shown that behavioral bias, including the disposition effect, reduces when people have greater experience (List, 2003; Feng and Seasholes, 2005; Da Costa et al., 2013). The literature has yet, however, to formalize the specific channel of this reduction. The adjustment in reference point provides a reasonable channel to explain this effect.

At date  $t$ , there are  $2^t$  different histories. We denote the reference point at a certain history  $h_t$ ,  $W^R(h_t)$  to be  $\bar{W}(h_t)$ . Similar to the fixed reference point model, we define our concept of equilibrium as follows:

**Definition 2.** We call  $\{\bar{W}^*(h_t), W^*(h_t), x^*(h_t)\}$  a rational expectation equilibrium if the following three conditions are satisfied:

1. At any history  $h_t$ , given  $\{\bar{W}^*(h_t), W^*(h_t)\}$ ,  $x^*(h_t)$  is the solution of the optimization problem (5).
2.  $W^*(h_t)$  is the current wealth that is updated according to Equation (3).
3.  $\bar{W}^*(h_t)$  is determined by a specific rational expectation condition, which we will explain later.

In this case, the investor's dynamic optimization problem is path dependent; hence, we can no longer treat the dynamic problem as a static problem and must take a backward procedure to solve it. We make the following assumption on parameter values:

**Assumption 1.** Define  $g = (R_u - R_f)/(R_f - R_d) = (R_u - 1)/(1 - R_d)$ . Then  $g < \lambda$ .

Since we let  $\lambda = 2.25$ , the above assumption holds for all numerical examples considered in this paper. We base the following proposition on BX (2006), the working paper version of BX (2009), and characterizes the optimal decision at  $T - 1$ . We omit the proof, which appears in BX (2006).

**Proposition 2.** If Assumption 1 holds, the investor's optimal date  $T - 1$  share holdings are given by:<sup>7</sup>

$$x_{T-1}(\Delta W) = \begin{cases} \min \left\{ x_L(\Delta W), \frac{W_{T-1}}{P_{T-1}(1-R_d)} \right\} & \text{if } \Delta W < 0 \\ x_G(\Delta W) & \text{if } \Delta W \geq 0; \end{cases} \quad (12)$$

where  $\Delta W = W_{T-1} - W_{T-1}^R$ ,

$$x_L(\Delta W) = \frac{-\Delta W}{P_{T-1}} \left( \frac{R_u - R_d}{\left( \frac{R_u - 1}{\lambda(1-R_d)} \right)^{\frac{1}{1-\alpha}} + 1} - (1 - R_d) \right)^{-1}, \quad (13)$$

and

$$x_G(\Delta W) = \frac{\Delta W}{P_{T-1}} \left( \frac{R_u - R_d}{\left( \frac{R_u - 1}{(1-R_d)} \right)^{\frac{1}{1-\alpha}} + 1} + (1 - R_d) \right)^{-1}. \quad (14)$$

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<sup>7</sup>BX (2006) only require the condition  $g < \lambda^{1/\alpha}$  to be satisfied. This condition holds under Assumption 1, since  $\alpha = 0.88$ .

Proposition 2 establishes the same trading pattern as the aforementioned numerical example in Section 2.2. Due to loss aversion, the optimal stock holding at date  $T - 1$  increases with the distance of current wealth to the reference point. In Definition 2, we don't fully specify our rational expectation condition, as there are many different assumptions about the reference point to cover within this condition. For instance, depending on how quickly the investor can adapt to the previous gains/losses and adjust their reference point, the reference point can be the current rational expectations of final wealth or rational expectations with different lags.

We denote  $n$ -period-lagged reference point as  $\bar{W}^*(h_t) = E[W_T | h_{\max\{t-n, 0\}}]$  for an integer  $n \geq 0$ . We first show that if current final wealth is the reference point, i.e.  $n = 0$  and  $\bar{W}^*(h_t) = E[W_T | h_t]$ , then the investor will not purchase stock in a rational expectation equilibrium.

**Corollary 2.** *If  $\bar{W}^*(h_t) = E[W_T | h_t]$ , the investor does not buy any stock in a rational expectation equilibrium.*

We use our model with  $T = 2$  to discuss the intuition. At date 1, when current expected final wealth is the reference point, there are only two possible outcomes of final wealth in calculating the reference point. Assume that the reference point involves holding some stock shares. Given positive expected stock returns, expected final wealth will be higher than current wealth. As shown in BX (2009) and Proposition 2, the optimal strategy is thus to gamble until the better final outcome reaches the reference point, then both high and low final wealth are below the reference point so expected final wealth cannot equal the given reference point. Purchasing stock is thus not a rational expectation equilibrium. When the reference point is one-period-lagged expected final wealth, on the other hand, the investor at date 1 calculates the expected value of four possible levels of final wealth to obtain the reference point. In deriving the optimal strategy at each of the two nodes at date 1, however, the investor only considers two outcomes at date 2. The contradiction mentioned for current expected final wealth as the reference point does not necessarily exist.

The above corollary implies the need for using lagged rational expectations as reference points for any reasonable result. KR (2009) also uses this approach, and there is an empirical reason for such choice. Using data from a popular game show Deal or No Deal, Post et al. (2008) estimated the reference point adjustment and showed that reference point does not fully adjust to current state, but is rather affected by lagged expected values. In an experimental stock market, Baucells et al. (2011) elicited subjects' reference point and found that the average of intermediate prices plays important roles. Using a similar design, Arkes et al. (2008) suggested that the reference point adapts slower to the most recent price when there is a loss rather than a gain. These studies indicate that the reference point is not very likely to immediately adjust fully to the current price in stock trading.

In the following discussion, we explore two specifications within the lagged expectation reference point via  $n = 1$  and  $n = 2$ . Once we specify all reference points, we can then solve backwards for investor’s dynamic optimization. For example, given Proposition 2’s depiction of the investor’s optimal date  $T - 1$  share holdings, using a one-period-lagged expectations reference point at date  $T - 2$  the investor decides how to allocate wealth at history  $h_{T-1} = \{h_{T-2}, u\}$  and  $h_{T-1} = \{h_{T-2}, d\}$  to maximize their expected utility through the reference point  $\bar{W}(h_{T-2})$ . Similarly, we can solve the problems at date  $t \leq T - 3$ , assuming all future share holdings are optimal. Finally, we impose rational expectation conditions to pin down all reference points.

## B. An Example

We use a numerical example with  $T = 3$  to illustrate the rational expectation equilibrium with a variable reference point. We assume a reference point of one-period-lagged expected final wealth with the same parameters as Part B in Section 2.1. Table 1 reports the realized returns (top-left panel), stock price (top-right panel), optimal share held (bottom-left panel) and wealth (bottom-right panel) at each note. We include the reference point determining the optimal share holding at each note in the bracket of the bottom-right panel.

This example shows a clear disposition effect, especially at date 2. Following a good return realization at date 1, the investor changes his stock position from 1.47 to 1.14 after a good return at date 2, while increasing his position to 1.78 following a bad return at date 2. The pattern is the same after a bad return at date 1. Although the investor sells some shares at date 1 regardless of a good or bad return, he tends to sell more shares after a good return. These simple statistics imply a strong disposition effect. Unsurprisingly, the reference point also adjusts based on historical realized returns. After a good return at date 1, the reference point increases from 49.54 to 72.61, while it decreases from 49.54 to 26.48 following a bad return at date 1.

[Table 1 inserted here]

## 3 Simulation

The numerical examples in the previous section are merely illustrative. To provide systematic evidence on how the reference point affects the disposition effect, this section formally simulates the trading pattern of loss-averse investors using different specifications of the reference point. We consider the following four specifications as reference points:



- SQ: BX's (2009) status quo wealth as the reference point
- EC: Initial expected final wealth as the constant reference point
- L1: One-period-lagged expected final wealth as the variable reference point
- L2: Two-period-lagged expected final wealth as the variable reference point

For comparison purposes, our simulation uses BX's (2009) specifications: BX (2009) simulated the selling versus holding decision of 10,000 loss-averse investors, each trading four stocks  $T$  times over an evaluation period of one year. Each investor has an initial wealth of 40 to allocate to each stock. All stocks start with an initial price of 40, and the stock return follows a binomial distribution with an annual gross return of  $\mu$  ranging from 1.03 to 1.13. Standard deviation is  $\sigma$  of 0.3. Investors have a loss aversion coefficient fixed at  $\lambda = 2.25$  and a curvature of value function  $\alpha = 0.88$ . We take the risk-free rate to be  $R_f = 1$ . Assuming that the price will go up or down with equal probability, the values of  $R_u$  and  $R_d$  relate to  $\mu$  and  $\sigma$  as follows:

$$R_u = \mu^{1/T} + \sqrt{(\mu^2 + \sigma^2)^{1/T} - (\mu^2)^{1/T}},$$

$$R_d = \mu^{1/T} - \sqrt{(\mu^2 + \sigma^2)^{1/T} - (\mu^2)^{1/T}}.$$

The simulation has three stages. Stage 1 solves for the most efficient rational expectation equilibrium. The first two specifications (SQ and EC) are relatively easy, as we can transform a dynamic investment decision problem into a static problem to obtain closed-form solutions (see Proposition 1). The latter two cases, L1 and L2, are much more complicated: We must first guess the reference point at each history, then solve backwards for the dynamic investment decision problem. We reach equilibrium when the guessed reference points match the one-period-lagged or two-period-lagged expected final wealth.<sup>8</sup> Considering the complicated nature of the simulation process for the latter two cases, we report only the cases that  $T = 2, 3, 4$ . These cases are sufficient to illustrate the main predictions of the model. Stage 2 generates 40,000 realizations of the price sequence based on the given distribution and number of trading dates within a year. For each price sequence, we can easily obtain the optimal stock position and corresponding reference point based

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<sup>8</sup>Since we do not have closed-form solutions for the L1 and L2 cases, it's also difficult to claim with certainty that our simulated results are indeed the most efficient equilibrium. We tried our best to solve this problem during the simulations: For example, we started from an initial guess with all reference points being  $W_0$ , and also tried other initial guesses to see whether we achieved similar results. Finally, we compared our L1 and L2 results with those of the SQ and EC cases to see whether the results strongly differed. Considering these efforts, we are quite confident that our simulation's results are the most efficient rational expectation equilibrium.

on the results in Stage 1. Stage 3 further calculates the statistics to show the model’s predictions, which we describe in greater detail below.

### 3.1 The Disposition Effect

Odean (1998) constructed the classical measurement for the disposition effect: He first calculates trading gains and losses by comparing current to average purchase price, then defines the proportion of gains realized (PGR) as the number of realized gains divided by the number of realized and paper gains. He defines the proportion of losses realized (PLR) as the number of realized losses divided by the number of realized and paper losses. If  $PLR < PGR$ , a disposition effect exists. Simple numerical examples can show that the  $PGR/PLR$  ratio is a more robust measure of the disposition effect than difference ( $PGR-PLR$ ) as the ratio is not affected by such confounding factors as portfolio size and trading frequency.<sup>9</sup> This paper therefore reports ratio, as well as PGR and PLR measures.

Table 2 reports the simulated  $PGR/PLR$  ratio and the PGR and PLR values in the bracket for SQ and EC. The left three columns take the status quo assumption by assuming a reference point of initial wealth (SQ), while the right three columns assume an expectation reference point, i.e. initial expected final wealth (EC).

[Table 2 inserted here]

Our first case, SQ, which assumes BX’s (2009) status quo wealth as the reference point, successfully replicates BX’s (2009) result. The  $PGR/PLR$  ratio is smaller than 1 in all cases with returns larger than 1.08, suggesting essentially *no*, or even the *opposite*, of the disposition effect.

When assuming initial expected final wealth as the reference point, interestingly, the  $PGR/PLR$  ratio is larger than 1 in most cases, indicating a strong disposition effect. In all cases, when  $T = 2$  and  $T = 3$ , almost all gains are realized ( $PGR=1.00$  or  $PGR=.80$ ), but no losses ( $PLR=0.00$ ), suggesting an infinite degree of disposition effect. When  $T = 4$ , there is a strong disposition effect for returns ranging from 1.06 to 1.10: As returns increase, the disposition effect generally disappears.

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<sup>9</sup>We would like to thank an anonymous referee for providing an example illustrating this point: Assume three investors have the same tendency for a disposition effect, in the sense that their ratios of hazard rates for selling winners divided by losers is the same. Investors A and B have the same turnover rates, but different size portfolios. Investors A and C have the same size portfolios, but different turnover rates. Using numerical examples, one can show that the calculated difference  $PGR-PLR$  differs significantly across the three investors, yet the  $PGR/PLR$  ratio is the same.

*Prediction 1: (Constant expectations reference point) Assuming initial expected final wealth as a constant reference point generates the disposition effect in most cases.*

As BX (2009) points out, simulated loss-averse investors are only willing to accept stocks with expected returns much higher than the risk-free rate. When initial expected wealth becomes the reference point, the region in which investors would like to purchase the stock initially remains the same. This is consistent with the theoretical result in Corollary 1. We also observe that, as the number of trading opportunities within a year increases, investors tend to accept lower stock returns at date 0. This is because more trading opportunities smooth the stock risk from the perspective of the initial date.

Table 3 reports the same PGR and PLR statistics for the two variable reference point specifications. The left two columns report a reference point of one-period-lagged expected final wealth (L1) for  $T = 3$  and  $T = 4$ , while the right column reports two-period-lagged expected final wealth (L2) for  $T = 4$ .

[Table 3 inserted here]

Table 3 reports a substantial disposition effect in many cases. Assuming expectations as the reference point, our model tends to give a robust prediction of the disposition effect, regardless of whether the reference point is variable or not. Furthermore, we can see that (compared to the case of EC) an adjusted reference point weakens the magnitude of the disposition effect: PGR/PLR is smaller in Table 3 than the corresponding cases in EC of Table 2. To have a more accurate measure, as the expected stock returns increase, let us define the switching return as the first return not demonstrating the disposition effect. For  $T = 4$ , the switching return is 1.10 in specification L1, 1.11 in specification L2, and 1.11 in specification EC: Clearly, the quicker the reference point adapts to price realization, the less likely we can observe the disposition effect. The existence of the disposition effect also depends on the number of trading opportunities  $T$ . To see this, notice that the switching return is 1.13 in model L1 when  $T = 3$ , and 1.10 when  $T = 4$ . Clearly, as the number of trading opportunities  $T$  increases, fewer cases demonstrate the disposition effect.

*Prediction 2: (Variable expectations reference point) When investors update their reference point as one-period-lagged (L1) or two-period-lagged (L2) expected final wealth, the disposition effect still exists in many cases. The magnitude of the disposition effect, however, is weaker compared to a constant expectations reference point (EC). In general, the quicker the reference point adapts to price realization, the less likely we can observe the disposition effect.*

### 3.2 Difference in Initial Position

An initial purchase decision is very important in BX's (2009) argument for why prospect theory predicts mostly the opposite of the disposition effect. In this section, we explore the predictions of initial position under different specifications. Table 4 reports the case  $T = 4$  for all four specifications; all other case patterns are similar.

[Table 4 inserted here]

Compared to a status quo reference point (SQ), an initial expected final wealth (EC) reference point generally leads investors to become more conservative in purchasing stock shares in their initial decisions. This is because initial expected final wealth (when purchasing stock) is higher than the status quo wealth, so more prices will be coded as generating losses hence reducing the attractiveness of the stock. When investors adjust the reference point based on historical realized returns, however, they become more aggressive and initially demand more shares compared to the case SQ and EC. This is because in bad cases, investors will adjust the reference point down to code some bad returns as gains. On average, there will be fewer big losses and gains relative to the reference point compared to cases with a constant reference point. Since losses affect the level of utility more than gains due to loss aversion, stocks become more attractive using a variable reference point. Comparing the results of L1 and L2, we can also see that the quicker the reference point is adjusted the more attractive the stock, thus the more shares investors demand at the initial date.

*Prediction 3: (the initial position) With a constant reference point, using initial expected final wealth as the reference point (EC) leads to fewer shares in the initial position compared to status quo wealth (SQ). With a variable reference point, investors tend to demand more shares at the initial date than with a constant reference point: The quicker the adjustment, the more shares investors initially demand.*

### 3.3 History-Dependent Effect With Variable Reference Point

With an updated expected final wealth reference point, price history not only affects current wealth but also the reference point, thus there is a natural history-dependent effect in the trading patterns we explore in this section. A binary returns distribution enables us to naturally define good and bad history using realized returns at date 1. Specifically, if the return at date 1 is  $R_u$ , we classify all realized prices following this node as having good history; otherwise, a bad history.

For a constant reference point, as long as the current price is the same, we can show that the optimal stock position will remain the same regardless of which path leads to current price. When investors adjust the reference point, however, the optimal stock position will differ following different histories, even though the current price is the same (since price history affects the reference point level). Specifically, let  $P_{2,ij} = P_0 R_i R_j$  with  $i = u, d; j = u, d$  denote the price at date 2 following a return  $R_i$  at date 1 and  $R_j$  at date 2. Let  $x_{2,ij}$  denote the corresponding optimal stock position following each price history. The following prices are the same:  $P_{2,du} = P_{2,ud}$ . Therefore, with a constant reference point,  $x_{2,du} = x_{2,ud}$ . For variable reference points, however this is not the case. Table 5 reports the levels of  $x_{2,du}$  and  $x_{2,ud}$ , assuming investors update their reference points as one-period-lagged expected final wealth. These two positions share the same current price, but differ in price histories.

[Table 5 inserted here]

For both  $T = 3$  and  $T = 4$ , and when the expected stock returns are low, given the same current price, investors tend to hold less shares on the stock following good history. This pattern is reversed when the expected stock returns are high, i.e. investors take a larger position following good history.

For this pattern's intuition, at price  $P_{2,ud}$  a good return at date 1 generates a high reference point relative to the current state at date 2 (with a bad return at date 2), placing current wealth in the losses domain. For a similar reason, at price  $P_{2,du}$  current wealth following bad history falls in the gains domain relative to its reference point. Calculating distance from the reference point in each case suggests that when expected stock returns are low, the reference point following a good history is not too high so losses tend to be closer than gains to the reference point, thus investors tend to hold a smaller position following a good price history. As expected stock returns become higher, investors become more aggressive and take a larger position. When a good return is realized at date 1, with the larger initial stock position, the reference point is very high: This creates larger losses, leading to a larger stock position in the losses domain.

*Prediction 4: (History-dependent effect on the optimal position) With a variable reference point and given the same current price, the optimal shares held is smaller following a good price history for small expected stock returns. When expected stock returns increase, the pattern is reversed.*

### 3.4 Returns on Paper Gains/Losses and Realized Gains/Losses

Odean's (1998) Table III observes that returns on realized gains/losses are smaller in absolute values than returns on paper gains/losses, where gains and losses are defined relative to the average

purchase price. This pattern implies that investors are more likely to realize small winners and losers than big ones, a fact seldom explained by existing literature. To explore our model’s prediction in this regard, Table 6 reports the simulated returns on paper and realized gains/losses under various specifications.

Panel A reports that, for two cases with a constant reference point, regardless of whether the reference point is status quo or initial expected wealth, the average returns on realized gains/losses are smaller in absolute value than returns on paper gains/losses in most cases (except for very high expected stock returns). Panel B demonstrates cases with a variable reference point, wherein we see the same pattern with a slow adjustment (L2). When the adjustment is relatively quick (L1), however, there is no substantial difference between paper and realized gains/losses. For expected returns ranging from 1.06 to 1.08, paper gains/losses are even smaller than realized ones in absolute value.

The intuition for this pattern comes from the prediction of loss aversion. Since small trading gains and losses (relative to average purchase price) are on average closer to the reference point than big ones, loss aversion predicts more sales of the stocks at small trading gains and losses. While this is why the status quo reference point can also generate such a pattern, a quickly adjusting reference point can change its location so much so that small trading gains and losses are not necessarily close to the reference point at any given time.

*Prediction 5: (The returns on paper and realized gains/losses) In specifications SQ, EC, and L2, realized gains/losses are smaller than paper ones in absolute values, a pattern consistent with Odean’s (1998) empirical finding. This pattern is not present or even reversed, however, in specification L1 when the reference point is updated relatively quickly.*

[Table 6 inserted here]

## 4 Discussion of Simulation Results

Our simulation results clearly show that an expected final wealth reference point (whether it be initial expectations as a constant reference point or lagged expectations as a variable reference point) can generate the disposition effect in most cases. Besides successfully predicting the disposition effect, our model also explores a new dimension of reference point adjustment to generate new predictions. Predictions 2 to 5 offer new predictions aside from the disposition effect, which distinguishes the current model from other alternative theories on the disposition effect.

Prediction 5 has already found empirical support from Odean’s (1998) study, although our other predictions require new empirical tests.

An adjusting reference point can naturally relate to another important topic in the literature: market experience and bias reduction. List (2003) and others (e.g. Feng and Seasholes, 2005; Da Costa et al., 2013) have shown that market experience can significantly reduce behavioral biases. The channel of such reduction, however, remains unclear. This paper thus suggests an adjusting reference point as a reasonable channel. As investors become more experienced, they may realize that it is irrational to stick to their initial expected or status quo wealth in judging gains and losses. Our results show that the quicker investors adjust their reference point, the less likely they demonstrate the disposition effect. Our model also predicts that reference point adjustment leads investors to be less conservative and demand more shares of stocks at the initial date. These predictions are reasonable characteristics of the experienced investors’ behavior.

Our prediction 5 requires more discussion. Despite Odean’s (1998) empirical finding that the average return of realized gains/losses is smaller in absolute value than paper ones, Ben-David and Hirshleifer (2012) find that the probability of selling monotonically increases for positive returns, i.e. investors are more likely to realize big than small winners. A substantial heterogeneity in trading behavior possibly explains these different results, as Table 5 implies: Our predictions are largely consistent with Odean when the reference point is not quickly adjusted, yet model L1’s predictions support Ben-David and Hirshleifer when expected stock returns are from 1.08 to 1.11 ( $T = 3$ , not reported here) and from 1.06 to 1.08 ( $T = 4$ ). When investors quickly update the reference point, it’s possible that big winners fall closer to the updated reference point than small winners, hence investors choose to realize the big winners. Assuming such heterogeneity, the pattern that ultimately appears depends upon whether most investors update their reference points relatively quickly as well as which subsample the estimation focuses on. Therefore our model may provides a potential channel to reconcile these contradictory empirical findings in the literature based on reference point adjustment.

A number of authors in the literature have suggested a model of realization utility to explain the disposition effect (e.g. BX, 2012), wherein investors derive utility from *realizing* gains and losses instead of final total wealth (as assumed in our model). We would like to emphasize that our model and the model of realization utility are, in fact, complementary: In reality, investors probably derive utility from both realizing gains and losses and final total wealth. Our model aims to understand to what extent disposition effect can be explained using a standard prospect theory model. It is possible our model cannot fully explain the reality, and other models such as realization utility are needed to increase the explanatory power.

Our model differs from BX (2012), however, in terms of predictions. BX’s model (2012) predicts

a strong disposition effect in the sense that unless forced to sell at a loss by a liquidity shock, the investor only sells stocks while trading at a price higher than the original purchase. In other words, selling at a loss is solely triggered by exogenous liquidity shock in BX's (2012) model and selling at a loss also implies a complete exit of the market. This result comes from their model's basic setting: Investors derive utility from realizing gains and losses, and are hence reluctant to sell stocks under losses based on their active decisions. In our model, however, selling at a loss can be an endogenous decision by the investor and there can be partial sales.<sup>10</sup>

## 5 Robustness Check

### 5.1 Alternative Modeling Approaches

In our definition of equilibrium, the investor takes the reference point as given when making an investment decisions. Another modeling approach, however, is to let investors determine both the reference point and their decision at the same time, which falls similar to the equilibrium in such disappointment-aversion models as Loomes and Sugden (1986); Bell (1985); and Gul (1991); as well as the choice-acclimating personal equilibrium in KR (2007). Specifically, in this alternative approach the decision problem with the fixed reference point case can be written as:

$$\max_{\bar{W}, \{W_{T,j}\}_{j=1, \dots, T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(W_{T,j} - \bar{W}), \quad (15)$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} W_{T,j} = W_0, \quad (16)$$

a non-negativity of wealth constraint

$$W_{T,j} \geq 0, \quad j = 1, \dots, T + 1, \quad (17)$$

and the rational expectation constraint

$$\bar{W} = \sum_{j=1}^{T+1} \pi_{T,j} W_{T,j}. \quad (18)$$

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<sup>10</sup>If we replace BX's (2012) exogenous liquidity shock with wealth constraint in our model, then BX's (2012) results tend to imply that selling only occurs when the investor's wealth constraint is binding, hence their model never realizes small losses. In contrast, our model allows for the realization of small losses, and a slow-adjusting reference point makes the likelihood of realizing small losses larger than big losses.



**Proposition 3.** *The optimal solution to maximization problem (15)-(18) must satisfy  $\bar{W}^* = W_0 R_f^T$  and the investor does not buy any stock in equilibrium.*

The proof of the above proposition is similar to that of Corollary 1 and is hence omitted in this section. Obviously,

$$\bar{W}^* = W_{T,1}^* = \dots = W_{T,T+1}^* = W_0 R_f^T$$

satisfies constraints (16)-(18) and gives an expected utility of zero. Other combinations of

$$\{\bar{W}, \{W_{T,j}\}_{j=1,\dots,T+1}\}$$

satisfying constraints (16)-(18), in contrast, cannot yield higher expected utility under the gain-loss utility function.

The intuition has been discussed previously. When the reference point and optimal stock holding change at the same time, purchasing stock is always suboptimal: This is because when purchasing stock (i.e. the reference point is the expected final wealth) the distribution of gains and losses is roughly symmetric relative to expected value, implying a negative expected gain-loss utility due to loss aversion. Not purchasing stock (i.e. the reference point is the initial wealth) generates neither gains nor losses.

## 5.2 Exogenous Initial Reference Point

One may argue that the initial reference point can be exogenous (rather than an endogenous expectation of final wealth). KR (2009) also discusses a case wherein the decision maker's initial reference point is not entirely rational. We consider a two-period model where the exogenous date-0 reference point is set to  $\bar{W}_0$ , and the date-1 reference point is the expectation of final wealth. We thus write our equilibrium definition as follows:

**Definition 3.** *For a given  $\bar{W}_0$ , we consider  $\{\bar{W}_1^*, W^*(h_t), x^*(h_t)\}$  a rational expectation equilibrium if it satisfies the following two conditions:*

1. *At any history,  $h_t$ ,  $x^*$  is the solution of the optimization problem (5).*
2.  *$W^*(h_t)$  is the current updated wealth according to Equation (3).*
3.  *$\bar{W}_1^*$  is determined by rational expectation condition  $\bar{W}_1^* = E[W_2|h_0]$ .*

We consider two possible initial reference points:  $\bar{W}_0 = W_0$  and  $\bar{W}_0 = 1.5W_0$ . Interestingly, we find no presence of the disposition effect when  $\bar{W}_0 = W_0$ , while there is a disposition effect when  $\bar{W}_0 = 1.5W_0$ . Specifying the initial reference point thus tends to affect the disposition effect.

### 5.3 Evaluation Period

Following Benartzi and Thaler (1995) as well as BX (2009), we assume an evaluation period of one year. To give an example of how different evaluation periods affect the disposition effect, we simulate a case with an evaluation period of half a year and  $T = 3$ . In all specifications of the reference point, investors require an annual expected stock return of at least 1.11 to purchase stock, suggesting that the shorter the evaluation period, the more risk averse the investor. With fixed initial expected final wealth as the reference point, there still remains a substantial disposition effect with an infinite PGR/PLR ratio, the same as the result in Table 2. Surprisingly, using one-period-lagged expected final wealth as the variable reference point creates a much stronger disposition effect than the corresponding case in Table 2: PGR/PLR is consistently around 1.2 for annual expected stock returns equal or higher than 1.11, while there is no disposition effect in Table 2 when the annual expected stock return is higher than 1.11. In general, a shorter evaluation period makes investors more risk averse in their initial choice of the stock and leads to a stronger disposition effect after the stock purchase. This result is consistent with Benartzi and Thaler (1995) who find that narrow framing is a behavioral bias reinforcing the effect of loss aversion. Barberis et al. (2006) also show that loss aversion requires narrow framing to explain first-order risk aversion in purchasing small gambles.

### 5.4 Other Simplifications Relative to KR's New Reference-Dependent Model

KR (2006) develop a more general version of the reference-dependent model in which total utility is a weighted average of consumption and gain-loss utilities; consumption utility is also the unit for gain-loss comparison. This more general version keeps the essential feature of loss aversion while incorporating the effect of standard consumption utility. Strictly following this specification does not lead to qualitatively different conclusions for the purposes of this paper, as it always implied the same effect of loss aversion on risk attitude.

KR's (2006) original specification also uses a stochastic reference point, and the utility is a probability-weighted average of gain-loss utilities relative to different outcomes of that reference point. Masatioglu and Raymond (2016) show that whether the reference point is stochastic or deterministic has important implications to an individual's risk attitude. Sprenger (2010) also shows in a lab experiment that stochastic reference points generally make investors less risk averse. The prediction that stock holding monotonically increases in distance to the reference point is essentially preserved, however, except that now the distance is also a probability-weighted average of the distances to all possible outcomes. This does not affect the main predictions of our model.

## 6 Conclusion

This paper proposes a formal model based on the central feature of loss aversion in prospect theory to explain the disposition effect. The predictions of loss aversion on individual selling patterns can lead to the disposition effect in most cases with a reference point of lagged expected final wealth (for both constant initial expectation or updated). Our model also predicts that the disposition effect is more likely to be present when updating the reference point doesn't occur quickly enough; when the expected stock return is low; and when stock trading is infrequent. Besides the disposition effect, our model also makes interesting new predictions. Adjusting the reference point predicts a history-dependent effect on stock holding even when the current price remains the same. The quicker investors are able to adjust their reference points, the less conservative they are in initial purchase decisions. When reference point adjustment is slow, our model's investors are more likely to realize small wins and losses as opposed to big ones, an observation consistent with Odean's (1998) findings.

It is important to clarify that defining the reference point as lagged expected final wealth is a sufficient but unnecessary condition for loss aversion to generate the disposition effect. In theory, any reference point sufficiently higher than initial wealth can lead to the disposition effect. Given the abundant empirical evidence on expectations reference points outside of finance, however, assuming an expectations-based reference point seems a reasonable first step. While this paper assumes that expectations are rational, the theoretical framework is flexible enough to accommodate several forms of expectations that potentially deviate from rational expectations. For instance, investors may linearly extrapolate from their past returns to form new expectations, or demonstrate overoptimistic beliefs and representative biases. Building more realistic expectation formations into our model will further improve its explanatory power.

As the first attempt to introduce expectations reference points into investors' trading behaviors, this paper focuses on the disposition effect and its related individual trading patterns. The idea that expectations can affect the reference point has several important implications for asset prices and deserves a systematic and careful examination in future research.

# Appendix: Omitted Proofs

## A.1 Proof of Proposition 1

*Proof.* The proof proceeds in two steps. First, we state several key properties of the solutions to the optimization problem (6)-(8). Second, based on the established properties, we solve the rational expectation equilibrium from Definition 1.

Define  $\hat{w}_{T,j} = W_{T,j} - \bar{W}$  to be the investor's gain/loss relative to that reference point, and then the optimization problem (6)-(8) can be rewritten as

$$\max_{\{\hat{w}_{T,j}\}_{j=1,\dots,T+1}} \sum_{j=1}^{T+1} \pi_{T,j} v(\hat{w}_{T,j}), \quad (19)$$

subject to the budget constraint

$$\sum_{j=1}^{T+1} \pi_{T,j} q_{T,j} \hat{w}_{T,j} = W_0 - \bar{W}, \quad (20)$$

and a nonnegativity of wealth constraint

$$\hat{w}_{T,j} \geq -\bar{W}, \quad j = 1, \dots, T + 1. \quad (21)$$

The following properties have already been proved by BX (2009), and hence we just state these properties without proving them.

**Lemma 1.** *There exists at least one solution to the optimization problem (19)-(21).*

**Lemma 2.** *If the investor's optimal gain/loss  $\hat{w}_{T,j}$  is different from zero at the end of some path, then it is different from zero at the end of all paths.*

**Lemma 3.** *If the optimal allocation is nonzero, there must be at least one path at the end of which the gain/loss is  $-\bar{W}$ , so that the investor is wealth constrained.*

**Lemma 4.** *It is not optimal to have a path with an unconstrained negative allocation  $\hat{w}_{T,j} < 0$  (i.e.,  $-\bar{W} < \hat{w}_{T,j} < 0$ ).*

**Lemma 5.** *Any path with a constrained negative wealth allocation must have a state price density not lower than that of any path with a positive allocation.*

The above results imply that if  $\hat{w}_{T,j}$  is not zero for all  $j$ , then  $\hat{w}_{T,j}$  can be either strictly positive or constrained negative ( $-\bar{W}$ ). Moreover, the optimal wealth allocation has a threshold property. There is a threshold in the state price density such that paths with a state price density higher

than that threshold have a constrained negative allocation, while paths with a state price density lower than that threshold have a strictly positive allocation.<sup>11</sup> Since we have already ranked the nodes according to the price/state price density, the above threshold property implies that there is a threshold  $k$  in the node such that

$$\hat{w}_{T,j} = \begin{cases} > 0 & \text{if } j \leq k; \\ -\bar{W} & \text{if } j > k. \end{cases} \quad (22)$$

From Equation (20), we obtain

$$\sum_{j=1}^k \pi_{T,j} q_{T,j} \hat{w}_{T,j} = W_0 - \sum_{j=1}^k \pi_{T,j} q_{T,j} \bar{W}. \quad (23)$$

For  $i, j \leq k$ , the first order conditions imply

$$\frac{v'(\hat{w}_{T,i})}{v'(\hat{w}_{T,j})} = \frac{q_i}{q_j} \Rightarrow \frac{\hat{w}_{T,j}}{\hat{w}_{T,i}} = \left(\frac{q_i}{q_j}\right)^{1/(1-\alpha)}. \quad (24)$$

Plugging Equation (24) into Equation (23) yields

$$\hat{w}_{T,i} = \frac{W_0 - \sum_{j=1}^k \pi_{T,j} q_{T,j} \bar{W}}{\sum_{j=1}^k \pi_{T,j} q_{T,j}^{-\alpha/(1-\alpha)}} q_{T,i}^{-1/(1-\alpha)}. \quad (25)$$

Finally, the rational expectation condition (9) requires that

$$\sum_{i=1}^k \pi_{T,i} (\hat{w}_{T,i} + \bar{W}) = \bar{W}. \quad (26)$$

Equation (26) is a linear equation about  $\bar{W}$ . Solving this equation yields Equation (10).

There are two remarks on the above proof. First, if the investor's optimal gain/loss  $\hat{w}_{T,j}$  is zero at the end of all paths, then by Equation (23), we must have  $\bar{W} = W_0$ . And this can be viewed as a special case of Equation (10) by letting  $k = T + 1$ . Second, Equation (10) only derives the equilibrium reference point in a candidate equilibrium. To be an equilibrium, the investor's optimal threshold strategy has to be  $k$  under the reference point  $\bar{W}(k)$ , which is true when  $U(k) = V(\bar{W}(k))$ .  $\square$

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<sup>11</sup>As argued by BX (2009), this threshold property crucially depends on the assumption that the probabilities of events  $u$  and  $d$  in each period are the same, and may fail if this assumption does not hold.

## A.2 Proof of Corollary 1

*Proof.* The key of the proof is to show that  $\bar{W} = W_0$  can be a rational expectation equilibrium reference point if and only if it is optimal not to buy any stock when the reference point is  $\bar{W} = W_0$ . First, if it is optimal not to buy any stock when the reference point is  $\bar{W} = W_0$ , then it is straightforward to verify that

$$\{\bar{W}^* = W_0, \{W_{T,j}^* = W_0\}_{j=1, \dots, T=1}\}$$

constitutes a rational expectation equilibrium from Definition 1. Second, if it is optimal to buy some stock when the reference point is  $\bar{W} = W_0$ , then the rational expectation condition in Definition 1 cannot be satisfied because the expected stock return exceeds the risk-free rate.

If there exists a rational expectation equilibrium with reference point  $\bar{W} = W_0$ , then this equilibrium is also the most efficient rational expectation equilibrium. Obviously, under the rational expectation equilibrium

$$\{\bar{W}^* = W_0, \{W_{T,j}^* = W_0\}_{j=1, \dots, T=1}\},$$

the date-0 expected utility is zero. Suppose that there exists another equilibrium  $\{\bar{W}^*, \{W_{T,j}^*\}_{j=1, \dots, T+1}\}$  with date-0 expected utility

$$\sum_{j=1}^{T+1} \pi_{T,j} v(W_{T,j}^* - \bar{W}^*).$$

Due to loss aversion and risk aversion, the above expression is less than  $v(\sum_{j=1}^{T+1} \pi_{T,j} W_{T,j}^* - \bar{W}^*) = 0$ . Therefore, the equilibrium

$$\{\bar{W}^* = W_0, \{W_{T,j}^* = W_0\}_{j=1, \dots, T=1}\}$$

is the most efficient rational expectation equilibrium.  $\square$

## A.3 Proof of Corollary 2

*Proof.* Suppose that at date  $T - 1$ , the reference points satisfy  $\bar{W}^*(h_{T-1}) = E[W_T | h_{T-1}]$ . We want to show that  $W_{T-1} = \bar{W}_{T-1}^*$  in any rational expectation equilibrium. Suppose not, and  $W_{T-1} > \bar{W}_{T-1}^*$ . From Proposition 2, we can calculate  $W_T(u)$  and  $W_T(d)$  as follows:

$$W_T(u) = \bar{W}_{T-1}^* + 2 \frac{W_{T-1} - \bar{W}_{T-1}^*}{q_u^{-\frac{\alpha}{1-\alpha}} + q_d^{-\frac{\alpha}{1-\alpha}}} q_u^{-\frac{1}{1-\alpha}} \quad \text{and} \quad W_T(d) = \bar{W}_{T-1}^* + 2 \frac{W_{T-1} - \bar{W}_{T-1}^*}{q_u^{-\frac{\alpha}{1-\alpha}} + q_d^{-\frac{\alpha}{1-\alpha}}} q_d^{-\frac{1}{1-\alpha}},$$

where

$$q_u = \frac{2(R_f - R_d)}{R_f(R_u - R_d)} \quad \text{and} \quad q_d = \frac{2(R_u - R_f)}{R_f(R_u - R_d)}.$$

Obviously, both  $W_T(u)$  and  $W_T(d)$  are strictly larger than  $\bar{W}_{T-1}^*$ . Hence,

$$E_{T-1}[W_T] = \frac{1}{2}W_T(u) + \frac{1}{2}W_T(d) > \bar{W}_{T-1}^*.$$

Contradiction!

Now suppose that  $W_{T-1} < \bar{W}_{T-1}^*$ . Still from Proposition 2, we can calculate  $W_T(u)$  and  $W_T(d)$  as follows:

$$W_T(u) = \min\left\{\bar{W}_{T-1}^* + 2\frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u}, 2W_{T-1}/q_u\right\}$$

and

$$W_T(d) = \max\left\{\bar{W}_{T-1}^* - 2\frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u} \times \left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)}, 0\right\}.$$

If

$$W_T(u) = \bar{W}_{T-1}^* + 2\frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u}$$

and

$$W_T(d) = \bar{W}_{T-1}^* - 2\frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u} \times \left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)},$$

$E_{T-1}[W_T] < \bar{W}_{T-1}^*$  under Assumption 1 since

$$\frac{q_u}{q_d} = \frac{R_f - R_d}{R_u - R_f} = \frac{1}{g}.$$

This also contradicts the requirement that  $\bar{W}_{T-1}^* = E_{T-1}[W_T]$ .

Another possibility is that  $W_T(u) = 2W_{T-1}/q_u$  and  $W_T(d) = 0$ . Then  $\bar{W}_{T-1}^* = E_{T-1}[W_T]$  implies that

$$\frac{1}{2}W_T(u) + \frac{1}{2}W_T(d) = \bar{W}_{T-1}^* \Rightarrow \bar{W}_{T-1}^* = W_{T-1}/q_u.$$

But then

$$\begin{aligned} & \bar{W}_{T-1}^* + 2\frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u} \\ &= W_{T-1}/q_u + 2\frac{W_{T-1}/q_u - W_{T-1}}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u} \\ &= W_{T-1}/q_u \left[ 1 + \frac{2(1 - q_u)}{q_d\left(\frac{\lambda q_u}{q_d}\right)^{1/(1-\alpha)} - q_u} \right]. \end{aligned}$$

Under Assumption 1,  $\frac{\lambda q_u}{q_d} > 1$  and hence

$$\frac{2(1 - q_u)}{q_d \left( \frac{\lambda q_u}{q_d} \right)^{1/(1-\alpha)} - q_u} < \frac{2(1 - q_u)}{q_d - q_u} = 1.$$

The last equation is true because  $q_d + q_u = 2$ . Therefore, when  $\bar{W}_{T-1}^* = W_{T-1}/q_u$ ,

$$\bar{W}_{T-1}^* + 2 \frac{\bar{W}_{T-1}^* - W_{T-1}}{q_d \left( \frac{\lambda q_u}{q_d} \right)^{1/(1-\alpha)} - q_u} < 2W_{T-1}/q_u.$$

But this contradicts with the assumption that  $W_T(u) = 2W_{T-1}/q_u$ . This leaves the only possibility that  $W_{T-1} = \bar{W}_{T-1}^*$ . Therefore, in any rational expectation equilibrium, the investor does not buy any stock at date  $T - 1$ .

At date  $T - 2$ , since the investor does not buy any stock at date  $T - 1$ , the same logic implies that  $W_{T-2}$  equals to the reference point  $\bar{W}_{T-2}^*$  in any rational expectation equilibrium. By induction, we conclude that the investor does not buy any stock at all in a rational expectation equilibrium.  $\square$



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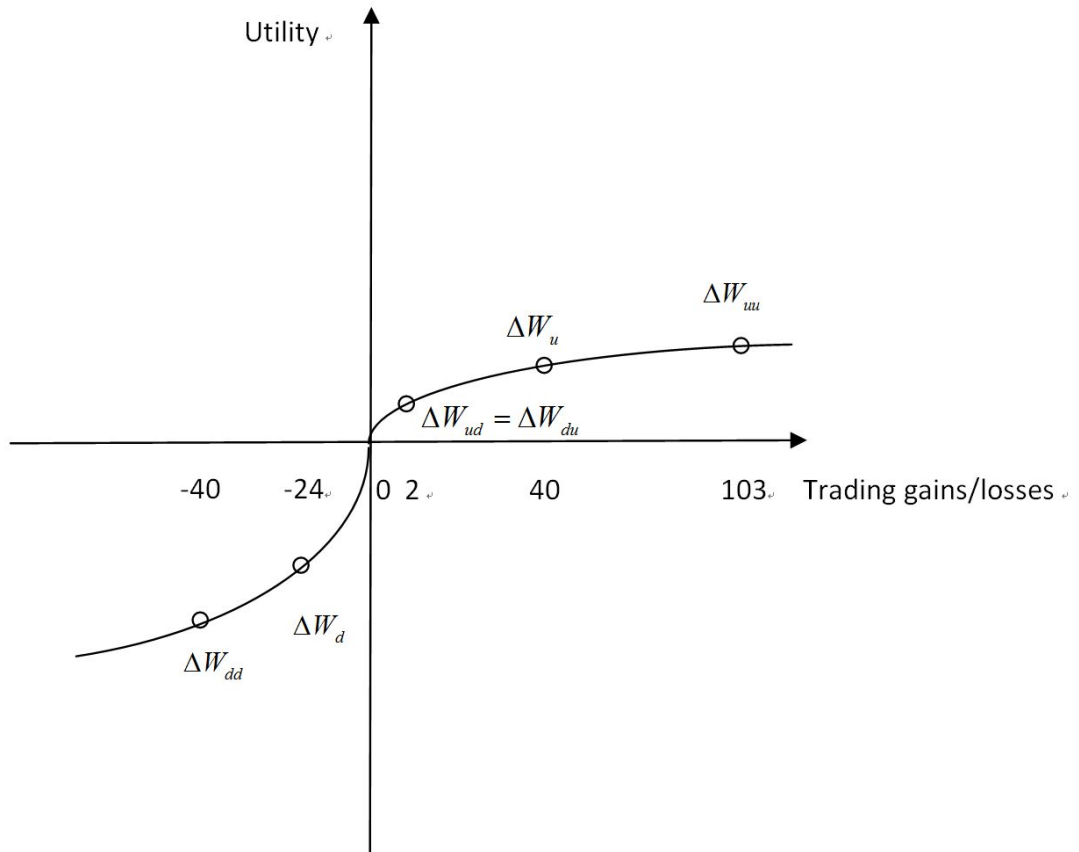
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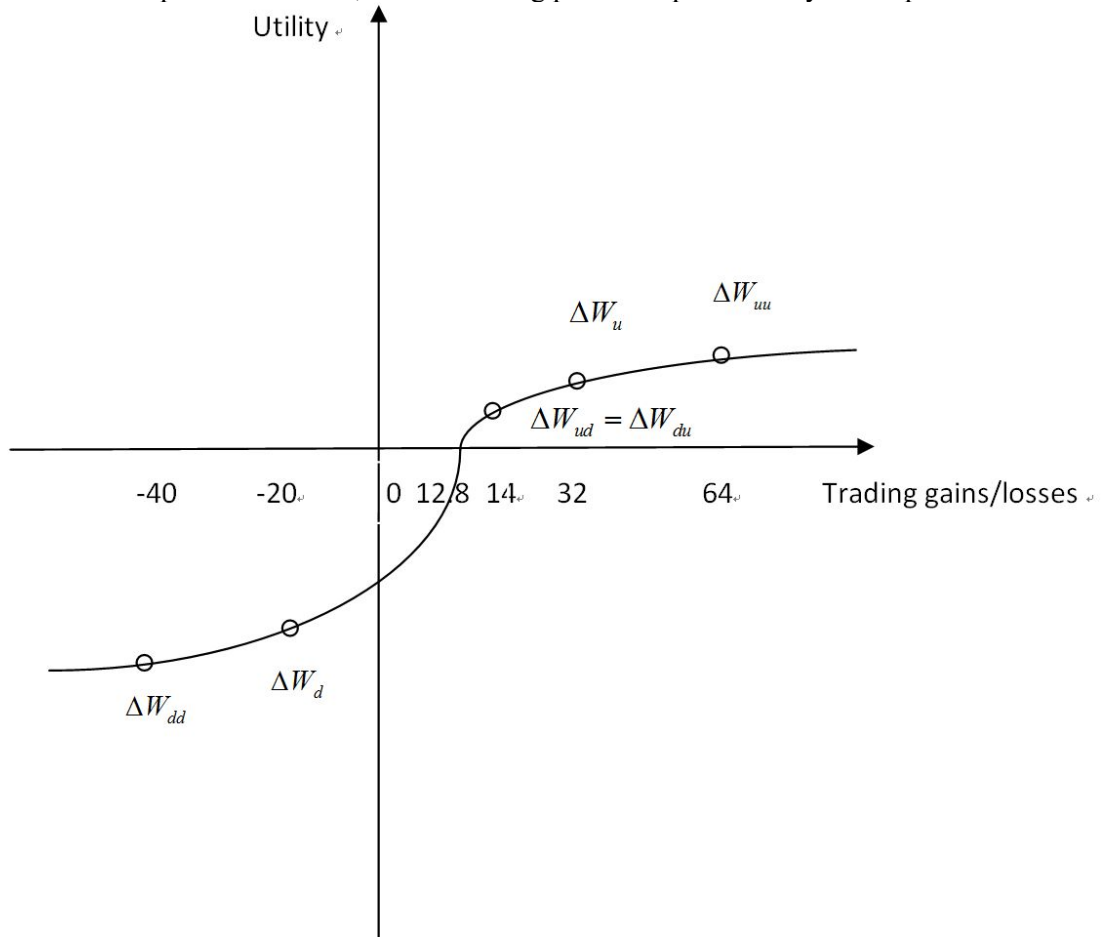
**Fig. 1. The Gain-loss Utility When Reference Point is the Initial Wealth**

The x-axis is the change in wealth relative to *the initial wealth* (not the reference point) that represents trading gains and losses. For instance,  $\Delta W_u$  is the change in wealth relative to the initial wealth in date 1 after  $R_1 = R_u = 1.25$  is realized, and  $\Delta W_{ud}$  is the change in wealth relative to the initial wealth in date 2 after  $R_1 = R_u = 1.25, R_2 = R_d = 0.85$  are realized. The y-axis is the gain-loss utility. In this numerical example the reference point is at 0, and the trading pattern implies the opposite of the disposition effect.



**Fig. 2. The Gain-loss Utility When Reference Point is the Initial Expected Final Wealth**

The x-axis is the change in wealth relative to *the initial wealth* (not the reference point) that represents trading gains and losses. For instance,  $\Delta W_u$  is the change in wealth relative to the initial wealth in date 1 after  $R_1 = R_u = 1.25$  is realized, and  $\Delta W_{ud}$  is the change in wealth relative to the initial wealth in date 2 after  $R_1 = R_u = 1.25, R_2 = R_d = 0.85$  are realized. The y-axis is the gain-loss utility. In this numerical example the reference point occurs at 4, and the trading pattern implies exactly the disposition effect.



**Table 1. An Three-period Example when Reference Point is One-period-lagged Expected Final Wealth**

This table reports the result of the most efficient rational expectation equilibrium for a three-period example assuming that reference point is the one-period-lagged expected final wealth. The top-left panel shows return realization at each node in the tree. The top-right panel shows the corresponding price at each node. The bottom-left and bottom-right panels report, for each node, the optimal number of shares held in the risky asset and the optimal wealth, respectively. Reference point determining the optimal share holding at each node is included in the bracket of the bottom-right panel. The investor's initial wealth is \$40, the net risk-free rate is zero, and the initial price, annual expected return, and annual standard deviation of the risky asset are \$40, 1.1, and 0.3, respectively.

Return realizations			Prices		
		uuu			67.90
	uu			56.92	
		uud			49.62
	u	udu	47.72		49.62
		ud		41.59	
1.00		udd			36.26
		duu	40.00		49.62
	du			41.59	
	d	dud		34.87	36.26
		ddu			36.26
	dd			30.39	
		ddd			26.49
Risky Asset Shares Held			Wealth		
		-			94.00
	1.14			81.60	
		-		(72.61)	73.20
	1.47		68.00		73.60
		-	(49.54)	59.20	
	1.78			(72.61)	49.60
3.64		-	40.00		
		-	(49.54)		51.20
	1.79			36.80	
	2.31			(26.48)	24.00
		-	(49.54)		27.20
	2.81			10.80	
		-		(26.48)	0

**Table 2. The Simulated PGR and PLR for the Specification SQ and EC**

This table reports the proportion of gains realized (PGR), the proportion of losses realized (PLR) and the ratio (PGR/PLR). PGR and PLR are from 10,000 investors' simulated selling versus holding decision. Each investor trades four stocks. The PGR/PLR ratio is presented for different values of gross expected returns over the year ( $\mu$ ) and different number of trading date within the year ( $T$ ). The left three columns present the case in which reference point is the initial wealth; The right three columns report the case in which initial expected final wealth is the constant reference point. Other parameter values are set to be  $\sigma = 0.3$ ,  $\lambda = 2.25$ ,  $\alpha = 0.88$ ,  $R_f = 1$ ,  $W_0 = 40$ , and  $P_0 = 40$ . The dash suggests that the investors are not willing to purchase that stock in the first place.

$\mu$	(SQ) BX's (2009) status quo wealth as the reference point			(EC) Initial expected final wealth as the constant reference point		
	$T=2$	$T=3$	$T=4$	$T=2$	$T=3$	$T=4$
1.03	-	-	-	-	-	-
1.04	-	-	-	-	-	-
1.05	-	-	-	-	-	-
1.06	-	-	3.00 (0.67/0.22)	-	-	2.97 (0.67/0.22)
1.07	-	-	3.00 (0.67/0.22)	-	-	3.00 (0.67/0.22)
1.08	-	0.61 (0.80/0.00)	0.60 (0.67/0.22)	-	0.61 (0.80/0.00)	0.60 (0.67/0.22)
1.09	-	0.60 (0.40/0.67)	0.60 (0.40/0.67)	-	0.60 (0.80/0.00)	0.60 (0.67/0.22)
1.10	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.60 (0.40/0.67)	0.00 (1.00/0.00)	0.60 (0.80/0.00)	0.60 (0.67/0.22)
1.11	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.60 (0.40/0.67)	0.00 (1.00/0.00)	0.60 (0.80/0.00)	0.71 (0.50/0.70)
1.12	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.33 (0.25/0.75)	0.00 (1.00/0.00)	0.60 (0.80/0.00)	0.51 (0.36/0.70)
1.13	0.00 (0.00/1.00)	0.60 (0.40/0.67)	0.33 (0.25/0.75)	0.00 (1.00/0.00)	0.60 (0.80/0.00)	0.51 (0.36/0.70)



**Table 3. The Simulated PGR and PLR for the Specification L1 and L2**

This table reports the proportion of gains realized (PGR), the proportion of losses realized (PLR) and the ratio (PGR/PLR). PGR and PLR are from 10,000 investors' simulated selling versus holding decision. Each investor trades four stocks. The PGR/PLR ratio is presented for different values of gross expected returns over the year ( $\mu$ ) and different number of trading date within the year ( $T$ ). The left three columns present the case in which reference point is one-period-lagged expected final wealth; The right three columns report the case in which the reference point is two-period-lagged expected final wealth. Other parameter values are set to be  $\sigma = 0.3$ ,  $\lambda = 2.25$ ,  $\alpha = 0.88$ ,  $R_f = 1$ ,  $W_0 = 40$ , and  $P_0 = 40$ . The dash suggests that the investors are not willing to purchase that stock in the first place.

$\mu$	(L1) One-period-lagged expected final wealth as the variable reference point		(L2) Two-period-lagged expected final wealth as the variable reference point
	$T = 3$	$T = 4$	$T = 4$
1.03	-	-	-
1.04	-	-	-
1.05	-	-	-
1.06	-	3.10 (0.93/0.30)	6.57 (0.73/0.11)
1.07	-	2.62 (0.79/0.30)	. (0.67/0.00)
1.08	-	2.63 (0.79/0.30)	1.28 (0.64/0.50)
1.09	1.20 (0.80/0.67)	6.34 (0.63/0.10)	1.40 (0.38/0.27)
1.10	1.20 (0.80/0.67)	0.75 (0.50/0.67)	1.40 (0.38/0.27)
1.11	1.20 (0.80/0.67)	0.61 (0.43/0.70)	0.61 (0.39/0.64)
1.12	1.20 (0.80/0.67)	0.72 (0.43/0.60)	0.50 (0.33/0.67)
1.13	0.67 (0.50/0.74)	0.53 (0.43/0.80)	0.35 (0.28/0.80)

**Table 4. The Optimal Stock Position Following Good and Bad Price Histories**

This table reports the levels of  $x_{2,ud}$  and  $x_{2,du}$  assuming that investors update their reference points as one-period-lagged expected final wealth. These two positions share the same current price but different price histories. If the return at date 1 is  $R_u$ , all the price realizations following this node is classified as having the good history; otherwise the bad history. The results are from 10,000 investors' simulated selling versus holding decision. Each investor trades four stocks. The results presented for different values of gross expected returns over the year ( $\mu$ ) and different number of trading dates within the year. Other parameter values are set to be  $\sigma = 0.3$ ,  $\lambda = 2.25$ ,  $\alpha = 0.88$ ,  $R_f = 1$ ,  $W_0 = 40$ , and  $P_0 = 40$ . The dash suggests that the investors are not willing to purchase that stock in the first place.

$\mu$	$T = 3$		$T = 4$	
	Good History ( $P_{2,ud}$ )	Bad History ( $P_{2,du}$ )	Good History ( $P_{2,ud}$ )	Bad History ( $P_{2,du}$ )
1.03	-	-	-	-
1.04	-	-	-	-
1.05	-	-	-	-
1.06	-	-	0.52	0.89
1.07	-	-	0.60	1.05
1.08	1.54	1.71	0.61	0.98
1.09	1.67	1.75	0.64	0.94
1.10	1.78	1.79	2.48	2.13
1.11	1.87	1.83	2.77	2.78
1.12	4.69	2.20	4.83	2.93
1.13	5.44	3.57	4.62	2.60

**Table 5. The Initial Position**

This table reports the initial positions in different specifications of the reference point. The initial positions are from 10,000 investors' simulated stock trading decisions. Each investor trades four stocks. The initial positions are presented for different values of gross expected returns over the year ( $\mu$ ) and  $T = 4$ . The cases with other values of  $T$  have similar pattern. Other parameter values are set to be  $\sigma = 0.3$ ,  $\lambda = 2.25$ ,  $\alpha = 0.88$ ,  $R_f = 1$ ,  $W_0 = 40$ , and  $P_0 = 40$ .

$\mu$	SQ	EC	L1	L2
1.03	0	0	0	0
1.04	0	0	0	0
1.05	0	0	0	0
1.06	1.09	0.89	1.90	0.87
1.07	1.22	0.94	2.18	1.33
1.08	1.36	0.99	2.12	1.60
1.09	1.51	1.03	2.05	2.36
1.10	1.66	1.08	2.19	2.59
1.11	1.82	4.08	3.08	2.75
1.12	5.81	4.21	4.37	4.85
1.13	6.18	4.34	5.11	7.40

**Table 6. The Paper Gains/Losses and Realized Gains/Losses**

This table reports the paper gains and losses and the realized gains and losses from 10,000 investors' simulated selling versus holding decision. Each investor trades four stocks. The results are presented for different values of gross expected returns over the year ( $\mu$ ) and  $T = 4$ . The cases with other values of  $T$  have similar pattern. Other parameter values are set to be  $\sigma = 0.3$ ,  $\lambda = 2.25$ ,  $\alpha = 0.88$ ,  $R_f = 1$ ,  $W_0 = 40$ , and  $P_0 = 40$  throughout this simulation. The dash suggests that the investors are not willing to purchase that stock in the first place.

$\mu$	Paper gains	Realized gains	Paper losses	Realized losses	Paper gains	Realized gains	Paper losses	Realized losses
<b>Panel A: Constant reference point</b>								
	(SQ) BX's (2009) status quo wealth as the reference point				(EC) Initial expected final wealth as the constant reference point			
1.03	-	-	-	-	-	-	-	-
1.04	-	-	-	-	-	-	-	-
1.05	-	-	-	-	-	-	-	-
1.06	0.15	0.08	-0.15	-0.11	0.16	0.09	-0.15	-0.10
1.07	0.15	0.08	-0.15	-0.10	0.16	0.09	-0.14	-0.09
1.08	0.15	0.08	-0.15	-0.10	0.16	0.09	-0.14	-0.08
1.09	0.15	0.02	-0.15	-0.11	0.17	0.09	-0.13	-0.08
1.10	0.14	0.02	-0.15	-0.11	0.17	0.10	-0.13	-0.07
1.11	0.14	0.02	-0.14	-0.10	0.17	0.16	-0.14	-0.17
1.12	0.16	0.12	-0.12	-0.13	0.16	0.09	-0.13	-0.16
1.13	0.16	0.12	-0.12	-0.13	0.16	0.10	-0.13	-0.16
<b>Panel B: Adjusted reference point</b>								
	(L1) One-period-lagged expected final wealth as the variable reference point				(L2) Two-period-lagged expected final wealth as the variable reference point			
1.03	-	-	-	-	-	-	-	-
1.04	-	-	-	-	-	-	-	-
1.05	-	-	-	-	-	-	-	-
1.06	0.17	0.17	-0.16	-0.18	0.16	0.12	-0.14	-0.06
1.07	0.18	0.21	-0.15	-0.17	0.15	0.08	-0.15	.
1.08	0.18	0.21	-0.15	-0.17	0.16	0.09	-0.14	-0.10
1.09	0.19	0.15	-0.14	-0.06	0.17	0.07	-0.13	-0.09
1.10	0.19	0.17	-0.12	-0.07	0.17	0.07	-0.12	-0.09
1.11	0.18	0.18	-0.13	-0.12	0.17	0.07	-0.12	-0.10
1.12	0.18	0.18	-0.13	-0.13	0.16	0.07	-0.14	-0.12
1.13	0.17	0.16	-0.13	-0.14	0.18	0.11	-0.13	-0.15