Can Storage Arbitrage Explain Commodity Price Dynamics?[1]

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ABSTRACT

The nature of commodity price behavior remains in substantial dispute. Previous empirical tests of the standard storage model have concluded that storage arbitrage cannot explain the high serial correlation of prices. We introduce a Maximum Likelihood estimator that, unlike available approaches, makes full use of the predictions of the model about the most striking feature of commodity prices, the skewness revealed in their asymmetric dynamics, displaying occasional spikes. Our results for sugar establish the empirical relevance of storage in determining its price behavior. The dynamics of commodity prices can be quite different from those of the shocks that drive them.

Subject headings: Commodity Price Dynamics, Storage, Speculation, Sugar, Maximum Likelihood Estimation, Non-linear Dynamic Models.

1. Introduction

Recent sharp increases in volatility of commodity prices have been the subject of international discussions and policy proposals (FAO, 2009; Council of The European Union, 2010). The controversies surrounding these discussions highlight the fact that economic interpretation of commodity price behavior remains in dispute among economists. Authoritative observers have attributed recent price spikes to diverse factors such as income growth in China, India and other emerging economies (Krugman, 2010) or to financial influences such as low interest rates (Frankel, 2009), large short run movements of international financial assets, or bubbles (Calvo, 2008; Caballero, Farhi, and Gourinchas, 2008; Timmer, 2009; Baffes and Haniotis, 2010).

There is substantial agreement on some stylized facts about commodity price behavior. Many annual series of commodity prices are highly correlated, and characterized by episodes of sharp price spikes, followed by precipitous falls, interspersed by longer intervals of less extreme variation. The price fluctuations are asymmetric; there are no steep troughs to match the spikes. The standard model of annual price behavior of storable commodities in the tradition of Gustafson (1958) (see Samuelson 1971; Scheinkman and Schechtman 1983; Stokey and Lucas 1989, chapter 10) is consistent with these stylized facts. In the model, storage arbitrage induces positive serial correlation in prices and implies that the price distribution is skewed, due to the higher sensitivity of price to shocks in net supply when discretionary stocks are zero (i.e., during "stockouts") and prices are high. Though it was

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useful for analysis of commodity policies (Johnson and Sumner, 1976; Gardner, 1979), the model presented so many analytical, numerical and econometric challenges that it remained untested for more than three decades, until the path-breaking work of Deaton and Laroque (1992, 1995, 1996).[2] Their overall conclusion, summarized in Deaton (2010), was strongly negative. In particular, they found the standard model to be incapable of replicating the observed high serial correlation of commodity prices. Coleman (2009) and Roberts and Schlenker (2010), though not full tests of the storage model, present more positive evidence regarding the role of storage arbitrage in determining key features of commodity price behavior.

Cafiero et al. (forthcoming), after improving the numerical accuracy of the implementation of the PML estimator of Deaton and Laroque (1995, 1996), obtain more reasonable estimates of storage costs and find a lower frequency of stockouts, which in turn imply levels of autocorrelation much closer to those measured on the price data series. PML, however, has the limitation that it fails to exploit the most striking aspect of the price data, captured by the Gustafson model, namely the obvious skewness evident in the occasional price spikes exhibited by commodity price series.

In this paper we introduce a Maximum Likelihood (ML) estimator for the commodity storage model with stockouts, based on prices only, that exploits information about higher moments of prices. While it imposes no additional assumptions on the model, it possesses small sample properties markedly superior to those of PML.[3]

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[2]They avoided continuing problems with the reliability of time series of production, consumption and stocks, by using only price data.

[3]To our knowledge, this is the first time ML estimation has been implemented for a storage model with stockouts. We address technical issues related to existence of the equilibrium and continuity of the likelihood function in sections 2 and 3 below. The ML estimations

Application to a series of international sugar prices yields excellent results.[4] The estimated level of the cutoff price at which discretionary stocks go to zero is higher than that obtained by PML, the implied frequency of stockouts is lower, and price correlations, skewness and kurtosis implied by the model closely match those seen in the annual sugar price data. We find the price of sugar to be highly responsive to small changes in consumption. Storage offers a substantial buffer against price spikes. But when inventories are not available to buffer the effects of negative supply shocks on consumption, prices must increase sharply to induce the consumption changes needed to clear the market.

Our results are important in establishing the empirical relevance of storage as a factor determining observed price behavior. We show why, under the truth of the estimated model for sugar, production shocks are not necessarily aligned with price spikes; the same production shock can give rise to very different price responses, depending on whether or not there are sufficient stocks to buffer its impact.

2. The model

We adopt the standard model of storage arbitrage. We assume all agents are competitive and have rational expectations. Storers are risk neutral and have a constant

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presented by Miranda and Glauber (1993) and Miranda and Rui (1999) avoid the possibility of discontinuities in the likelihood surface caused by stockouts by using storage cost functions that ensure that stocks are always positive.

[4]The choice of sugar price is not accidental. The deflated price series presented in figure 5 does not show the strong time trend exhibited by price series of other food staples. In the context of this stochastic dynamic model, the presence of a significant trend raises additional problems of modeling and estimation that have yet to be addressed in this literature.

discount rate $r > 0$. The cost of storing $x_t \geq 0$ units of discretionary stocks[5] from time t to time $t+1$, paid at time t, is kx_t, with $k \geq 0$, and these stocks deteriorate at rate d, with $0 \leq d < 1$. Supply shocks ω_t are i.i.d.. The state variable z_t is total available supply at time t, $z_t \equiv \omega_t + (1-d)x_{t-1}$, and $z_t \in Z$, where Z is a subset of $\mathbb{R}$. Consumption, c_t, has inverse demand $F : Z \to (-\infty, \infty]$. F is continuous, strictly decreasing, with $EF(\omega) \in \mathbb{R}$, $\left(\frac{1-d}{1+r}\right) EF(\omega) - k > 0$, and $\lim_{c\to\infty} F(c) \leq 0$, where E denotes the expectation taken with respect to the random variable $\omega \equiv \omega_t$.

Previous PML implementations of the model specify the distribution of the shocks as normal,[6] and we maintain that assumption in our empirical implementation. Since we could not find a proof of the existence of the equilibrium for the model, in this section we present such a proof, to furnish the basis for our empirical procedure.[7] Note also that the possibility of an additive component in the marginal cost of storage implies that the market does not necessarily clear at a non-negative price for all admissible specifications of F. To ensure that equilibrium prices are non-negative, we assume free disposal.

A Stationary Rational Expectations Equilibrium, (SREE), is a price function $f : Z \to (-\infty, \infty]$, which describes the current price p_t as a function of the state z_t, and satisfies for all $z_t \in Z$,

$$p_t = f(z_t) = \max\left\{ \left(\frac{1-d}{1+r}\right) E_t f\big(\omega_{t+1} + (1-d)x_t\big) - k, F(z_t) \right\}$$

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[5]We normalize non-discretionary stocks at zero.

[6]See Deaton and Laroque 1995, pps.S14 and S17 and Deaton 2010, p.8.

[7]With our proof we address a loose end in Deaton and Laroque (1992, p.11): "[E]ven though, in theory, these distributions have an unbounded support, in practice we truncate the distributions at five standard deviations from the mean, so that the theorems of the previous section can be directly applied."

where

$$x_t = \begin{cases} z_t - F^{-1}\big(f(z_t)\big), & \text{if} \quad z_t < z^* \equiv \inf\{z : f(z) = 0\} \\ z^* - F^{-1}(0), & \text{if} \quad z_t \geq z^*, \end{cases}$$

and E_t denotes the expectation taken with respect to the random variable ω_{t+1}. Since the ω_t's are i.i.d., f is the solution to the functional equation

$$f(z) = \max\left\{ \left(\frac{1-d}{1+r}\right) Ef\big(\omega + (1-d)x(z)\big) - k, F(z) \right\}, \tag{1}$$

and

$$x(z) = \begin{cases} z - F^{-1}\big(f(z)\big), & \text{if} \quad z < z^* \\ z^* - F^{-1}(0), & \text{if} \quad z \geq z^*. \end{cases} \tag{2}$$

Existence and uniqueness of the SREE, as well as some properties are given by the following Theorem:

Theorem. *There is a unique stationary rational expectations equilibrium f in the class of non-negative continuous non-increasing functions. Furthermore, if $p^* \equiv \left(\frac{1-d}{1+r}\right) Ef(\omega) - k$, then:*

$$\begin{aligned} f(z) &= F(z), && \textit{for} \quad z \leq F^{-1}(p^*), \\ f(z) &> \max\{F(z), 0\}, && \textit{for} \quad F^{-1}(p^*) < z < z^*, \\ f(z) &= 0, && \textit{for} \quad z \geq z^*. \end{aligned}$$

f is strictly decreasing whenever it is strictly positive. The equilibrium level of inventories, $x(z)$, is strictly increasing for $z \in [F^{-1}(p^*), z^*)$.

Proof. See Appendix A. □

The inverse market demand f is a function of available supply z. It is coincident with the inverse consumption demand F for price above p^*. Below p^* the market demand includes the demand for consumption and the demand for stocks x to be carried over to the next period, and F is strictly steeper than f. In this model, the positive probability of stockouts implies that there may be a kink in the market demand at p^*.

3. Maximum Likelihood Estimation

In this section we present our ML estimator in a setting that includes the normal distribution for harvest as a special case. Given the SREE function f, the model implicitly defines a mapping from harvests ω_t to prices p_t, conditional on the previous price p_{t-1}. For positive prices:

$$\begin{aligned} p_t|p_{t-1} &= f\left(z_t\right) \\ &= f\left[\omega_t + (1-d)\, x_{t-1}\right] \\ &= f\left[\omega_t + (1-d)\left(z_{t-1} - F^{-1}\left(f\left(z_{t-1}\right)\right)\right)\right] \\ &= f\left[\omega_t + (1-d)\left(f^{-1}\left(p_{t-1}\right) - F^{-1}\left(p_{t-1}\right)\right)\right] \end{aligned}$$

We assume that the distribution of ω_t is absolutely continuous with respect to the Lebesgue measure. The fact that the inverse price function f^{-1} has bounded derivative for prices away from zero implies that $p_t|p_{t-1}$ has a density l.

For a vector of parameters θ and a sample of positive prices $p_t,\ t = 0, 1, \cdots, T$, the likelihood function is:

$$L(\theta|p_0, \cdots, p_T) = \prod_{t=1}^{T} l(p_t|p_{t-1}) \tag{3}$$

Given a sample of positive prices, none of which coincides with a kink in the market demand, we can write the following expression for L:

$$L(\theta|p_0, \cdots, p_T) = \prod_{t=1}^{T} \phi(\omega_t)|J_t| = \prod_{t=1}^{T} \phi[f^{-1}(p_t) - (1-d)(f^{-1}(p_{t-1}) - F^{-1}(p_{t-1}))]|J_t| \tag{4}$$

where ϕ is the density of ω_t, and $J_t = \dfrac{df^{-1}}{dp_t}(p_t)$ is the Jacobian of the mapping $p_t \mapsto \omega_t$. Note that the probability that a price realization equals p^* is zero.

In implementing the ML procedure, the first step is to find the price function f that solves the storage model. Other than in very restrictive cases, no closed form solution is known for the functional equation defined by (1) and (2). The solution, however, can be approximated through numerical methods.

We approximate the equilibrium price function f with a cubic spline, f^{sp}.[8] The search for f^{sp} follows an iterative procedure based on (1) and (2). This requires approximating the expectations with respect to the distribution of the harvests. To do so we use a quadrature formula with nodes $\{\omega_s\}_{s=1}^S$ and weights $\{\pi_s\}_{s=1}^S$. The n-th iteration is:

$$f^{\mathrm{sp}}_{<n+1>}(z) = \max\left\{ \left(\frac{1-d}{1+r}\right) \sum_{s=1}^{S} f^{\mathrm{sp}}_{<n>}\big(\omega_s + (1-d)x_{<n>}\big) \cdot \pi_s - k,\ F(z) \right\} \tag{5}$$

where

$$x_{<n>} = \begin{cases} z - F^{-1}(f^{\mathrm{sp}}_{<n>}(z)), & \text{if} \quad z < z^*_{<n>} \equiv \inf\{z : f^{\mathrm{sp}}_{<n>}(z) = 0\} \\ z^*_{<n>} - F^{-1}(0), & \text{if} \quad z \geq z^*_{<n>}. \end{cases}$$

The first iteration uses a guess $f^{\mathrm{sp}}_{<1>}$ on the right hand side of (5). This guess implies a value of $z^*_{<1>}$. Conditional on $f^{\mathrm{sp}}_{<1>}$, we evaluate $f^{\mathrm{sp}}_{<2>}$ on an equally spaced grid of 500 points over a suitable range of z. Iterations continue until the maximum difference between $f^{\mathrm{sp}}_{<n+1>}$ and $f^{\mathrm{sp}}_{<n>}$ evaluated at each grid point is within a given small tolerance, and we take f^{sp} to be the last guess, implying $z^* \equiv \inf\{z : f^{\mathrm{sp}}(z) = 0\}$. Recognizing free disposal and

[8] For a discussion of function approximation, see Judd (1998, Chapter 6). For an application to the storage model, see Miranda (1985).

the possibility of stockouts, we define the solution as:

$$\hat{f} \equiv \begin{cases} F(z), & \text{if} \quad z \leq F^{-1}(p^*), \\ f^{\mathrm{sp}}(z), & \text{if} \quad F^{-1}(p^*) < z \leq z^*, \\ 0, & \text{if} \quad z^* < z, \end{cases}$$

where $p^* = \left(\frac{1-d}{1+r}\right) \sum_{s=1}^{S} f^{\mathrm{sp}}\left(\omega_s\right)\pi_s - k$.[9]

We assume a linear inverse consumption demand $F(c_t) = a + bc_t$ with $b < 0$, and a normal harvest distribution with mean μ and standard deviation σ. To identify the parameters we estimate, we set the mean and standard deviation of the unobserved harvests at $\mu = 0$ and $\sigma = 1$.[10]

To approximate expectations taken under a normal distribution with zero mean and unit standard deviation, we use a Gauss-Hermite quadrature. (See Judd, 1998, section 7.2.)[11]

We approximate the logarithm of (3) as:

$$\begin{aligned} \ln \hat{L}(\theta | p_0, \ldots, p_T) \quad = \quad & -\left(\frac{T}{2}\right) \ln(2\pi) + \sum_{t=1}^{T} \ln |\hat{J}(p_t)| \\ & -\frac{1}{2} \sum_{t=1}^{T} \left[\hat{f}^{-1}(p_t|\theta) - (1-d)\left(\hat{f}^{-1}(p_{t-1}|\theta) - F^{-1}(p_{t-1}|a,b)\right)\right]^2 \end{aligned} \tag{6}$$

[9] We assume in the remainder of this paper that r is fixed at 5%.

[10] See Proposition 1 in Deaton and Laroque (1996).

[11] We assume $S = 10$, with nodes $\omega_s = \{\pm 4.8595, \pm 3.5818, \pm 2.4843, \pm 1.4660, \pm 0.4849\}$ and weights $\pi_s = \{4.3107 \times 10^{-6}, 7.5807 \times 10^{-4}, 1.9112 \times 10^{-3}, 0.1355, 0.3446\}$, respectively.

with $\theta \equiv \{a, b, d, k\}$ and where:

$$\hat{J}(p_t) \equiv \begin{cases} \frac{\mathrm{d}F^{-1}}{\mathrm{d}p_t}(p_t), & \text{if } p_t \geq p^*, \\ \\ \frac{\mathrm{d}\hat{f}^{-1}}{\mathrm{d}p_t}(p_t), & \text{if } p^* > p_t. \end{cases} \tag{7}$$

For a given set of parameters θ, we numerically solve for the approximated price function $\hat{f}$ and use it to calculate $\hat{f}^{-1}(p_t), \hat{f}^{-1}(p_{t-1})$ and $\hat{J}(p_t)$. These values, together with the implied consumption levels $F^{-1}(p_t)$, are used to evaluate (6) in the maximization routine.

The approximation in (7) induces discontinuities in (6), as illustrated in figure 1, a problem that we address by choosing an appropriate numerical optimization method. We first use a grid-search routine to locate a candidate maximum, and then use a gradient-based constrained maximization algorithm to search for a maximum in the neighborhood of the candidate.[12] We repeat the maximization of the log-likelihood several times, using different starting values. In our empirical application, the maximum found in this way corresponds to a flat spot of the log-likelihood surface.[13]

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[12]To approximate the solution function f and the derivatives needed to calculate $\hat{J}$ we use the Matlab® Spline Toolbox™. To maximize the function (6) we first use the Matlab® routine `fminsearch`, and then the routine `fmincon`, both included in the Optimization Toolbox™.

[13]We impose the constraints $b < 0$, $k > 0$, and $d > 0$ by programming the likelihood maximization routine in terms of the set of transformed parameters $\eta \equiv \{\eta_1, \eta_2, \eta_3, \eta_4\}$ where: $\eta_1 = a$, $\eta_2 = \ln(-b)$, $\eta_3 = \ln(k)$, and $\eta_4 = \ln(d)$. Having identified a maximum, we form an estimate of the asymptotic variance-covariance matrix of the estimated parameters, $\mathbf{W}$, as the inverse of the outer product of score vectors, evaluated at the estimated values

(Figure 1 here)

4. Small Sample Properties

The small sample properties of some estimators of the storage model have been previously explored. Using Monte Carlo experiments, Michaelides and Ng (2000) find that PML as implemented by Deaton and Laroque (1995, 1996) is more efficient than the Simulated Method of Moments estimator of Duffie and Singleton (1993), the Indirect Inference estimator of Gourieroux, Monfort, and Renault (1993), and the Efficient Method of Moments estimator of Gallant and Tauchen (1996) in estimating a storage model with linear demand, fixed interest rate and storage cost consisting only of proportional decay of the amount stored. In this section we compare the small sample performance of the ML estimator with that of PML. In evaluating estimates of the cutoff price p^* we also consider Generalized Method of Moments (GMM).

(Table 1 here)

$\hat{\eta}$. A consistent estimate of the variance covariance matrix $\mathbf{V}$ of the original parameters is obtained using the delta method, as:

$$\mathbf{V} = \mathbf{D}\mathbf{W}\mathbf{D}'$$

where $\mathbf{D}$ is a diagonal matrix of the derivatives of the transformation functions:

$$\mathbf{D} = \left\{ \begin{array}{llll} 1 & 0 & 0 & 0 \\ 0 & -e^{\hat{\eta}_2} & 0 & 0 \\ 0 & 0 & e^{\hat{\eta}_3} & 0 \\ 0 & 0 & 0 & e^{\hat{\eta}_4} \end{array} \right\}.$$

We conduct two Monte Carlo experiments.[14] The first maintains the same parameterization as Michaelides and Ng (2000) for samples of size 100. Table 1 shows that our ML estimator yields quite precise estimates of the parameters of the model on samples of this size. The Root Mean Square Error (RMSE) for each estimated parameter is substantially lower than the corresponding value obtained using the PML estimator. Note that both use identical information on the structure of the model that generates the data. Table 1 also shows the superior precision of the ML estimates of the stockout price p^*, a key parameter of this model: the RMSE is reduced from 7.53% of the true value with PML to 2.24% with ML. Figure 2 highlights the greater precision of the ML estimates of p^*; indeed PML, though imposing more structure in the estimation procedure, does not perform much better than the parsimonious GMM.

[14]In each experiment, for a given set of parameters we solve for the SREE price function, set initial stocks at zero and generate a series of prices using a series of independent draws from the standard normal distribution of the harvest. We then apply ML, PML and GMM to the simulated series of prices. In applying the ML estimator, one difference from the econometric procedure described in the previous section must be noted. Free disposal and the fact that the support of the harvest distribution is unbounded imply that zero prices have positive probability when $k > 0$, and therefore the simulated series might include price realizations equal to zero. Recognizing this possibility, we form the log-likelihood of a given sample of prices p_t, $t = 0, 1, \ldots, T$ as:

$$\begin{aligned}\ln L(\theta|p_0,\ldots,p_T) \;=\; \sum_{t=1}^{T} \Big\{ & \mathbb{1}_{\{p_t=0|p_{t-1}\}} \ln \mathrm{Prob}(p_t = 0|p_{t-1}) \;+ \\ & \mathbb{1}_{\{p_t>0|p_{t-1}\}} \left[\ln\phi\left(f^{-1}(p_t) - (1-d)\left(f^{-1}(p_{t-1}) - F^{-1}(p_{t-1})\right)\right) + \ln|J_t|\right] \Big\},\end{aligned}$$

where $\mathbb{1}$ is the indicator function.

(Figure 2 here)

This first Monte Carlo experiment uses a parameterization that implies low average storage and frequent stockouts. The mean of the serial correlations measured on all possible consecutive samples of length 108 taken from a series of 300,000 prices generated with this parameterization is 0.2535, much lower for example than 0.6451, the value observed in the sugar price data we introduce in section 5.

To explore the small sample performance of our ML estimator in a model in which storage plays a greater role, the next Monte Carlo experiment uses a specification in which consumption demand is steeper and storage is more frequent. We set $a = 1$, $b = -2$, $k = 0.02$ and $d = 0$. The results are summarized in table 2. With this second parameterization, the overall precision is lower for all considered estimators.[15] The kernel densities of the estimates of p^* reported in figure 3 confirm the fundamental result that ML dominates PML in terms of the RMSE of all estimated parameters.[16] The fatter tails of the density of the estimates of p^* obtained with PML (the dashed line in figure 3), and the skewness of the price distribution, together imply that PML tends to overestimate the number of stockouts, as shown in figure 4. This figure illustrates the histogram of the number of stockouts occurring in the samples of simulated prices used in the Monte Carlo experiment, based on the true value of p^*. The figure also shows the histograms of the number of stockouts in the same samples as implied by the estimates of p^* obtained using ML, PML and GMM respectively.

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[15] This appears to be related to the fact that, with this parameterization, in each sample there are likely to be fewer observations in the stockout region, so estimation of the consumption demand parameters is more difficult.

[16] In evaluating the RMSE for k as a percentage of the true value, note that the latter, 0.02, is only about two percent of the mean price implied by this parameterization.

(Table 2 here)

(Figure 3 here)

The means of the distributions reported on the horizontal axis of figure 4 show that, in these samples, ML predicts the average number of stockouts much more accurately than PML and GMM.

(Figure 4 here)

5. Data

We use the sugar price time series presented in Pfaffenzeller, Newbold, and Rayner (2007) and extend it from 2003 to 2009 using sugar price data provided by the World Bank, Development Prospects Group. We use the monthly figures reported in the World Bank "Pink Sheets" as: "Sugar (world), International Sugar Agreement (ISA) daily price, raw, f.o.b. and stowed at greater Caribbean ports", take their annual calendar average, and then divide them by the 1977-79 average, consistently with the description given in Pfaffenzeller, Newbold, and Rayner (2007). We deflate the nominal values by the corresponding annual average of the United States Consumer Price Index reported by the US Bureau of Labor Statistics.

Previous studies of real price series for many commodities and for aggregate commodity price indexes have identified a structural break between 1920 and 1921.[17] Accordingly, we use data for the period 1921-2009. The real sugar price index we use is plotted in figure 5.

(Figure 5 here)

[17] See Grilli and Yang (1988); Cuddington and Urzúa (1989); Helg (1991); Ardeni and Wright (1992); Cuddington, Ludema, and Jayasuriya (2002); Zanias (2005), among others.

This series exhibits occasional sharp spikes interspersed among longer periods of lower variability. The positive skewness in the series is typical of what one might expect from intertemporal arbitrage via storage, occasionally interrupted by stockouts.

6. Estimation results

Table 3 reports ML, PML and GMM estimates obtained on the real sugar price series. The first panel shows the ML estimates of a, b, k and d. As the estimate of d is not significantly different from zero, we present in the second panel ML estimates with d set at zero. The latter imply a cutoff price p^* that is 82.7% above the sample mean, and stockouts in 1923, 1962, 1974, 1975, and 1980, the same years as implied by the GMM estimate of p^*.

(Table 3 here)

One implication of this model is that, beginning in any period in which the current price is above p^*, the discounted expected price for the following period, net of storage cost, equals p^*. An informal check of this implication is to calculate the discounted mean of prices observed immediately following periods in which price is at or above the estimated p^*, and compare the result, net of storage cost, to p^*. Using the ML estimate of p^*, 0.8914, the discounted mean, net of storage cost, of the small sample of prices in periods following stockouts is $0.8823 - 0.0110 = 0.8713$.

By contrast, the lower PML estimates of p^* imply eleven stockouts over the sample interval. Conducting the above informal check with the PML estimate of p^*, 0.7932, the discounted mean of prices following each of the eleven stockout years, net of storage cost, is $0.8738 - 0.0071 = 0.8667$, substantially above the estimated p^*. The results in table 3 suggest that PML underestimates p^*, thus overstating the number of stockouts, consistent with the Monte Carlo evidence presented in section 4. Note that a lower p^* implies lower autocorrelation.

To assess how well the estimated model reproduces the mean, first and second order autocorrelation, coefficient of variation, skewness and kurtosis of our data, we simulate one long series of 300,000 prices using the estimated parameters, and extract from it all possible sequential samples of 89 prices. Table 4 reports values measured on the observed price sample, and the corresponding percentiles of the simulated distributions depicted in figure 6. It is clear that the estimated model reproduces these features of the sugar price series very well.

(Table 4 here)

(Figure 6 here)

7. Implications for price dynamics

The estimated equilibrium price function depicted in figure 7 has important implications for the dynamics of prices in this model and their relation to the realization of shocks.[18]

(Figure 7 here)

The estimated inverse consumption demand for sugar is much steeper than the market equilibrium function below p^*. This means that price is much more responsive to harvest shortfalls when price is high and there are no stocks. Then available supply equals the harvest and price is given by a point on the consumption demand at or above $p^* = 0.8914$. Next period, if the harvest realization is at its mean, μ (here normalized at zero), availability again equals the current harvest and the price falls to $f(\mu) = 0.7767$ (point C). If the harvest is one standard deviation below the mean, since there are no stocks, the full impact

[18] In figure 7, quantities are measured in units of standard deviation, net of mean harvest.

of the shock falls on consumption forcing the price to jump to $F(\mu - \sigma) = 1.4521$ (point E). The maximum price jump that may be caused from such a harvest realization occurs if the initial price is at p^*. It is equal to $F(\mu - \sigma) - p^* = 1.4521 - 0.8914 = 0.5607$, a jump of 62.9%.

The natural tendency to infer the size of supply shocks from the size of price jumps can be very misleading. Consider an initial situation in which consumption equals mean harvest, μ. This would occur when available supply is at $z^A = 1.5235$ and price is $f(z^A) = F(\mu) = 0.4804$ (point A). In this situation, should the next harvest realization equal the mean, carryout stocks, consumption and price would remain the same. On the other hand, a harvest one standard deviation below the mean results in an available supply of $(z^A - \sigma) = 0.5235$, and a price of $f(z^A - \sigma) = 0.6577$ (point A'). In this case, the rise in price is $f(z^A - \sigma) - f(z^A) = 0.6577 - 0.4804$, an increase of 0.1773, 36.9% of the initial price, and only about one third of the maximum price jump that could occur after a harvest of $(\mu - \sigma)$, as discussed above. A second harvest outcome of $(\mu - \sigma)$ would induce a slightly larger price rise, moving the equilibrium to point A''. Thus, starting at point A where price is at the mean, even two successive harvests of one standard deviation below the mean are not sufficient to induce a stockout. However, a third repetition of a harvest of $(\mu - \sigma)$ would induce a stockout and a price jump of almost 60% to point H on the consumption demand curve F. Starting from the same point A, if the initial harvest is two standard deviations below the mean, the price jumps by 0.4638, from $f(z^A) = 0.4804$ to $f(z^A - 2\sigma) = 0.9442$, (point A'''), above p^*. A repetition of the same very low harvest outcome causes a much larger jump, 1.4796, to $F(\mu - 2\sigma) = 2.4238$. (See point M on the consumption demand curve.)

8. Conclusions

Our results establish the empirical relevance of storage in determining commodity price behavior. The dynamics of commodity prices can be quite different from those of the shocks that drive them.

One crucial implication of the model is that commodity prices follow two regimes, depending upon whether or not current price is low enough to motivate carrying stocks into the next period. The ability to locate the cutoff price at which discretionary stocks go to zero is thus fundamental for the evaluation of the estimator. On price samples of sizes typically available from commodity markets, our ML estimator identifies the cutoff price far more precisely than estimators previously proposed.

Application of our ML estimator to a series of annual sugar prices from 1921 to 2009 produces estimates consistent with the observed highly skewed long-run distribution of prices. The implied estimate of the cutoff price indicates only five stockouts over the entire period. Stocks are carried most of the time, but when a stockout occurs the steepness of the consumption demand makes the market price extremely sensitive to shocks in net supply. The correlation implied by the model is regime-dependent; we find no indication that, during a stockout, the discounted net price in the next period is correlated with the current price.

Our results indicate that speculative storage has a highly stabilizing effect on sugar prices.

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A. Proof of the Theorem

Consider the function $\gamma : [0, \sup F(Z)) \to \mathbb{R}$, defined by:

$$\gamma(p) \equiv \left(\frac{1-d}{1+r}\right) E[\varphi(p,\omega)],$$

with

$$\varphi(p,\omega) \equiv \begin{cases} F(\omega), & \text{if} \quad \omega \leq F^{-1}(p) \\ p, & \text{if} \quad \omega > F^{-1}(p), \end{cases}$$

where E denotes the expectation taken with respect to the random variable ω.

Claim 1. *γ has a unique fixed point $\hat{p} \in (0, \sup F(Z))$. $p < \gamma(p) \quad \forall\, p < \hat{p}$, and $p > \gamma(p) \quad \forall\, p > \hat{p}$.*

Proof of Claim 1. Since $E(F(\omega)) \in \mathbb{R}$, $\gamma(p) \in \mathbb{R}$, $\forall\, p \in [0, \sup F(Z))$. Note that $\gamma(0) \geq \left(\frac{1-d}{1+r}\right) EF(\omega) > k \geq 0$, and $\lim_{p \uparrow \sup F(Z)} (\gamma(p) - p) < 0$. By continuity of γ, there exists $\hat{p}$ such that $\gamma(\hat{p}) = \hat{p}$.

For $p \in [0, \hat{p})$, $\varphi(\hat{p},\omega) - \varphi(p,\omega) \leq \hat{p} - p$, $\forall\, \omega$, and therefore:

$$\gamma(\hat{p}) - \gamma(p) = \left(\frac{1-d}{1+r}\right) E[\varphi(\hat{p},\omega) - \varphi(p,\omega)] \leq \left(\frac{1-d}{1+r}\right)(\hat{p} - p) < \hat{p} - p,$$

thus, $p < \gamma(p)$.

A similar argument establishes that $p > \gamma(p)$ for $p \in (\hat{p}, \infty)$. □

Define the space:

$\mathcal{G} \equiv \{g : Z \to [0,\infty) :\ g$ is continuous, non-increasing, $g \geq F$, and $g(z) = F(z)$, $\forall\, z \leq F^{-1}(\hat{p})\}$.

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$\mathcal{G}$ is a complete metric space with the metric:

$$||g_1 - g_2||_\infty \;\equiv\; \sup_{z\in Z} |g_1(z) - g_2(z)| \;=\; \sup_{z \geq F^{-1}(\hat{p})} |g_1(z) - g_2(z)| \;\leq\; \hat{p} \;<\; \infty.$$

Given $g \in \mathcal{G}$, define $G : Y \equiv \{(p, z) : z \in Z,\ \max\{F(z), 0\} \leq p < \sup F(Z)\} \to \mathbb{R}$,

$$G(p, z) \equiv \left(\frac{1-d}{1+r}\right) Eg\{\omega + (1-d)x(p, z)\} - k,$$

with

$$x(p, z) \;\equiv\; \begin{cases} z - F^{-1}(p), & \text{if} \quad z < z_g^* \\ z_g^* - F^{-1}(p), & \text{if} \quad z \geq z_g^*, \end{cases}$$

and

$$z_g^* \equiv \inf \left\{ z \geq F^{-1}(0) \;:\; \left(\frac{1-d}{1+r}\right) Eg\{\omega + (1-d)(z - F^{-1}(0))\} - k = 0 \right\}.$$

Given $g \in \mathcal{G}$, define Tg by:

$$Tg(z) = \max\left\{G(Tg(z), z),\ F(z)\right\}, \quad z \in Z. \tag{A1}$$

Claim 2. *Given $g \in \mathcal{G}$, there exists a unique Tg that satisfies (A1), $Tg \in \mathcal{G}$, and:*

$$Tg(z) = \begin{cases} F(z), & \textit{if} \quad z \leq F^{-1}\left(\left(\frac{1-d}{1+r}\right) Eg(\omega) - k\right), \\ G(Tg(z), z), & \textit{if} \quad z > F^{-1}\left(\left(\frac{1-d}{1+r}\right) Eg(\omega) - k\right). \end{cases}$$

Proof of Claim 2. The facts that $Eg(\omega) < \infty$, and that $x = x(p, z)$ and $g = g(z)$ are continuous, imply that $G = G(p, z)$ is continuous. Clearly $G = G(p, z)$ is non-increasing in p and z. Given $z \in Z$, with $F(z) < \infty$, to establish existence and uniqueness of $Tg(z)$, note that $Tg(z)$ is the root of $\psi_z(p) \equiv \max\{G(p, z) - p, F(z) - p\}$. ψ_z is strictly decreasing and continuous, and

$$\psi_z\Big(\big[\max\{F(z), 0\}, \sup F(Z)\big)\Big) = \Big(A_z,\ \psi_z\big(\max\{F(z), 0\}\big)\Big],$$

where $A_z < 0$.

To evaluate $\psi_z\big(\max\{F(z), 0\}\big)$ we consider three cases:

Case 1. $z \leq F^{-1}(0)$:

$\psi_z\big(\max\{F(z), 0\}\big) = \max\left\{\left(\frac{1-d}{1+r}\right) Eg(\omega) - k - F(z), 0\right\}$.

If $F(z) \geq \left(\frac{1-d}{1+r}\right) Eg(\omega) - k$, then $\psi_z\big(F(z)\big) = 0$, and $Tg(z) = F(z)$.

If $F(z) < \left(\frac{1-d}{1+r}\right) Eg(\omega) - k$, then $\psi_z\big(F(z)\big) > 0$, and $Tg(z)$ satisfies $Tg(z) = G\big(Tg(z), z\big)$.

Case 2. $F^{-1}(0) < z < z_g^*$:

$\psi_z\big(\max\{F(z), 0\}\big) = \psi_z(0) = \max\{G(0, z), F(z)\} = G(0, z) > 0$, then $Tg(z)$ satisfies $Tg(z) = G\big(Tg(z), z\big)$.

Case 3. $z \geq z_g^*$:

$\psi_z\big(\max\{F(z), 0\}\big) = \psi_z(0) = \max\{G(0, z), F(z)\} = G(0, z) =$
$= \left(\dfrac{1-d}{1+r}\right) Eg\left\{\omega + (1-d)(z_g^* - F^{-1}(0))\right\} - k = 0,$ then $Tg(z) = 0$ and satisfies $Tg(z) = G\big(Tg(z), z\big)$.

To see that $Tg(z)$ is continuous and non-increasing, note that $\max\{G(p, z) - p, F(z) - p\}$ is continuous and non-increasing.

Since $\left(\dfrac{1-d}{1+r}\right) Eg(\omega) - k \;\leq\; \left(\dfrac{1-d}{1+r}\right) Eg(\omega) \;\leq\; \gamma(\hat{p}) \;=\hat{p}$, we conclude that $Tg(z) = F(z), \quad \forall\, z \leq F^{-1}(\hat{p})$. $\square$

A standard argument shows that if $g_1, g_2 \in \mathcal{G}$ and $\alpha \equiv ||g_1 - g_2||_\infty$, then:

$$||Tg_1 - Tg_2||_\infty \leq \left(\frac{1-d}{1+r}\right)\alpha,$$

thus T is a contraction.

The SREE f is the unique fixed point of T. Indeed, take $h : Z \to (-\infty, \infty]$ to be any continuous, non-increasing function that satisfies (1) and (2), that is:

$$h(z) = \max\left\{\left(\frac{1-d}{1+r}\right) Eh\big(\omega + (1-d)x(z)\big) - k, F(z)\right\},$$

and

$$x(z) = \begin{cases} z - F^{-1}\big(h(z)\big), & \text{if} \quad z < z^* \equiv \inf\{z : h(z) = 0\} \\ z^* - F^{-1}(0), & \text{if} \quad z \geq z^*. \end{cases}$$

We now prove that $h(z) = F(z), \quad \forall\ z \leq F^{-1}(\hat{p})$.

Let $p_h^* \equiv \left(\dfrac{1-d}{1+r}\right) Eh(\omega) - k$. Since $h(z) = F(z) \quad \forall\ z \leq F^{-1}(p_h^*)$, it follows that $h(\omega) \leq \varphi(p_h^*, \omega)$, for any ω in its support, and $p_h^* \leq \gamma(p_h^*)$. Therefore, $p_h^* \leq \hat{p}$, concluding that $h \in \mathcal{G}$.

f is strictly decreasing whenever it is strictly positive: If not, since f is non-increasing, there is an interval where f is a positive constant. We have two cases:

Case 1. Suppose there exists a first interval $I \equiv [z', z'']$ where f is constant. Let

$B \equiv f(z'), \forall\ z \in I$,

$$B = f(z) = \left(\frac{1-d}{1+r}\right) Ef\left(\omega + (1-d)\left(z - F^{-1}(B)\right)\right) - k.$$

Since f is non-increasing, $f\left(\omega + (1-d)\left(z - F^{-1}(B)\right)\right)$ is constant ($\leq B$), for $z \in I$, for any ω in its support. Therefore, $B \leq \left(\frac{1-d}{1+r}\right) B - k$, a contradiction.

Case 2. Suppose there is no first interval where f is constant.

Let $\mathcal{I} \equiv \{I \ : \ I \text{ is an interval where } f \text{ is constant}\}$ and let $\bar{f} \equiv \sup\{f(z) : z \in I \text{ and } I \in \mathcal{I}\}$. Since there is no first interval where f is constant, $\bar{f}$ is accumulated by a sequence of values of f in I, $I \in \mathcal{I}$.

Take any $\epsilon > 0$ and consider an interval I such that the value of f in I is $\geq \bar{f} - \epsilon$. Let $B \equiv$ value of f in I. $\forall z \in I$,

$$B = f(z) = \left(\frac{1-d}{1+r}\right) Ef\left(\omega + (1-d)\left(z - F^{-1}(B)\right)\right) - k.$$

Since f is non-increasing, $f\left(\omega + (1-d)\left(z - F^{-1}(B)\right)\right)$ is constant for $z \in I$, for any ω in its support, and

$$f\left(\omega + (1-d)\left(z - F^{-1}(B)\right)\right) \leq \bar{f}.$$

Therefore,

$$B \leq \left(\frac{1-d}{1+r}\right)\bar{f} - k,$$

and then,

$$B \leq \left(\frac{1-d}{1+r}\right)(B+\epsilon) - k.$$

Since $\epsilon > 0$ is arbitrary, we obtain a contradiction.

The equilibrium level of inventories, $x(z)$, is strictly increasing for z in $[F^{-1}(p^*), z^*)$:

Let $z_1 < z_2$ in $[F^{-1}(p^*), z^*)$. Since f is strictly decreasing in this interval, $f(z_1) > f(z_2)$. Therefore,

$$\left(\frac{1-d}{1+r}\right)Ef\left(\omega + (1-d)x(z_1)\right) - k > \left(\frac{1-d}{1+r}\right)Ef\left(\omega + (1-d)x(z_2)\right) - k,$$

which implies that $x(z_1) < x(z_2)$. □

Table 1. Monte Carlo experiment with the parameterization of Michaelides and Ng (2000)†

Estimator	Parameters			
	a	b	d	p^*
ML				
mean	0.6010	-0.2968	0.1015	0.5820
st. dev.	0.0187	0.0236	0.0155	0.0130
bias	0.0010	0.0032	0.0015	-0.0020
	(0.17%)	(1.08%)	(1.54%)	(-0.33%)
RMSE	0.0187	0.0238	0.0156	0.0131
	(3.11%)	(7.92%)	(15.58%)	(2.24%)
PML				
mean	0.5995	-0.2919	0.1037	0.5782
st. dev.	0.0291	0.0478	0.0364	0.0436
bias	-0.0005	0.0081	0.0037	-0.0058
	(-0.08%)	(2.71%)	(3.70%)	(-1.01%)
RMSE	0.0291	0.0476	0.0371	0.0440
	(4.85%)	(15.87%)	(37.15%)	(7.53%)
GMM‡				
mean	n.a.	n.a.	0.1113	0.5710
st. dev.	n.a.	n.a.	0.0431	0.0471
bias	n.a.	n.a.	0.0113	-0.0130
			(11.28%)	(-2.22%)
RMSE	n.a.	n.a.	0.0446	0.0488

Table 1—Continued

Estimator	Parameters			
	a	b	d	p^*
			(44.56%)	(8.35%)

[†]True values: $a = 0.6$, $b = -0.3$, $d = 0.1$, with r fixed at 0.05 as in Michaelides and Ng (2000). These values imply $p^* = 0.5840$.

[‡]For GMM, the estimated value of d is inferred from the estimate of the parameter $\gamma = (1 + r)/(1 - d)$.

Note. — Monte Carlo experiments selecting 500 valid replications, as in Michaelides and Ng (2000), and sample size 100. The starting values are randomly chosen in the range between 90% and 110% of the true values. When any one of the estimators did not converge, the sample was discarded.

Table 2. Comparison of Monte Carlo experiment results†

Estimator	Parameters			
	a	b	k	p^*
ML				
mean	1.0148	-1.9449	0.0244	1.8278
st. dev.	0.1437	0.3264	0.0138	0.1596
bias	0.0148	0.0551	0.0044	-0.0239
	(1.48%)	(2.75%)	(21.89%)	(-1.29%)
RMSE	0.1444	0.3310	0.0143	0.1614
	(14.44%)	(16.55%)	(71.35%)	(8.71%)
PML				
mean	1.0443	-1.9954	0.0273	1.8776
st. dev.	0.2260	0.7852	0.0200	0.4766
bias	0.0443	0.0046	0.0073	0.0259
	(4.43%)	(0.23%)	(36.73%)	(1.40%)
RMSE	0.2303	0.7852	0.0213	0.4773
	(23.03%)	(39.26%)	(106.51%)	(25.77%)
GMM				
mean	n.a.	n.a.	0.0397	1.7881
st. dev.	n.a.	n.a.	0.0450	0.4298
bias	n.a.	n.a.	0.0197	-0.0636
			(98.44%)	(-3.44%)
RMSE	n.a.	n.a.	0.0491	0.4344

Table 2—Continued

Estimator	Parameters			
	a	b	k	p^*
			(245.73%)	(23.46%)

[†]True values: $a = 1$, $b = -2$, $k = 0.02$ and $d = 0$, which imply $p^* = 1.8517$. Monte Carlo experiments with 1859 replications and sample size 108. The starting values are randomly chosen in the range between minus and plus twenty percent of the true values. When the optimization routine did not converge, the estimation was discarded. In the end, we had a total of 1809 valid estimations for ML, 1605 for PML and 1810 for GMM. The table reports the 1533 estimates obtained on common samples. Including all valid estimations for each method does not change the results noticeably.

Table 3. Estimation results

a	b	k	d	log(L)	p^*	no. of stockouts
ML						
0.4793	-0.9810	0.0108	0.0033	61.1976	0.8913	5
(0.0731)	(0.1320)	(0.0078)	(0.0256)			
ML, setting $d = 0$						
0.4804	-0.9717	0.0110	–	61.1835	0.8914	5
(0.0729)	(0.1085)	(0.0076)				
PML						
0.4182	-0.9049	0.0076	0.0187	38.0746	0.7888	11
(0.0768)	(0.2549)	(0.0062)	(0.0393)			
PML, setting $d = 0$						
0.3867	-0.9183	0.0071	–	37.8381	0.7932	11
(0.1718)	(0.3267)	(0.0089)				
GMM, imposing $(1 + r)/(1 - d) = 1.05$ and $k \geq 0$.[†]						
–	–	0.00	–	–	1.0315	5
		–	–	–	(0.4041)	

[†]The OID statistic is 6.4076, above the 5% critical value for the relevant χ^2 distribution.

Note. — Asymptotic standard errors in parentheses.

Table 4. Comparison of data features and model predictions

	Mean	1st order a.c.	2nd order a.c.	C.V.	Skewness	Kurtosis
Actual values	0.4879	0.6451	0.3464	0.6795	2.7295	11.2492
Percentiles†	53.404	62.677	29.450	66.300	70.640	75.844

†Percentiles of the distributions of mean, first and second order autocorrelation, coefficient of variation, skewness and kurtosis, obtained by calculating those values from all possible consecutive sequences of length 89 taken from a simulated series of 300,000 prices using the parameters' estimates in table 3, panel 2.

Fig. 1.— The log-likelihood function in (a, b) space evaluated at $k = 0.011$, $d = 0$, using sugar prices for the period 1921-2009

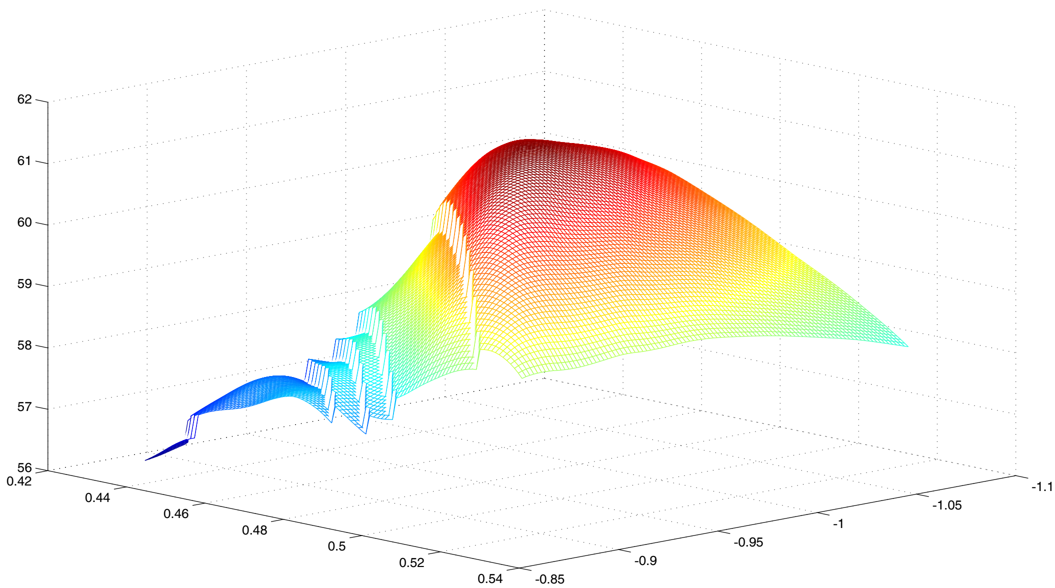
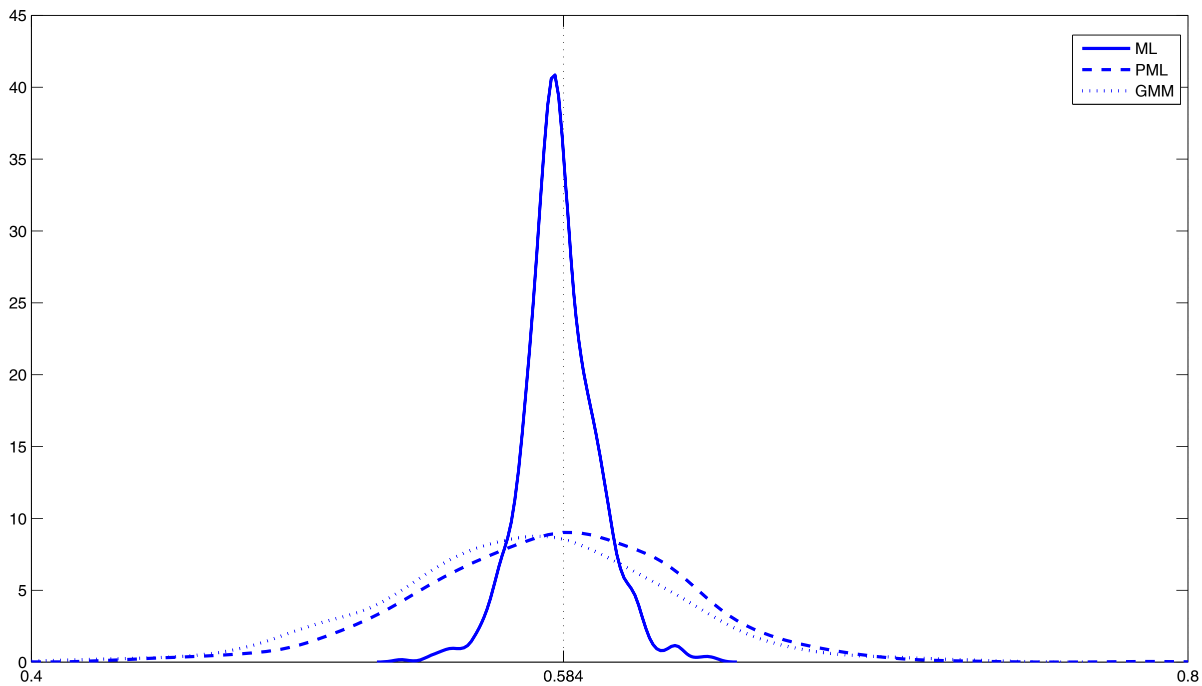
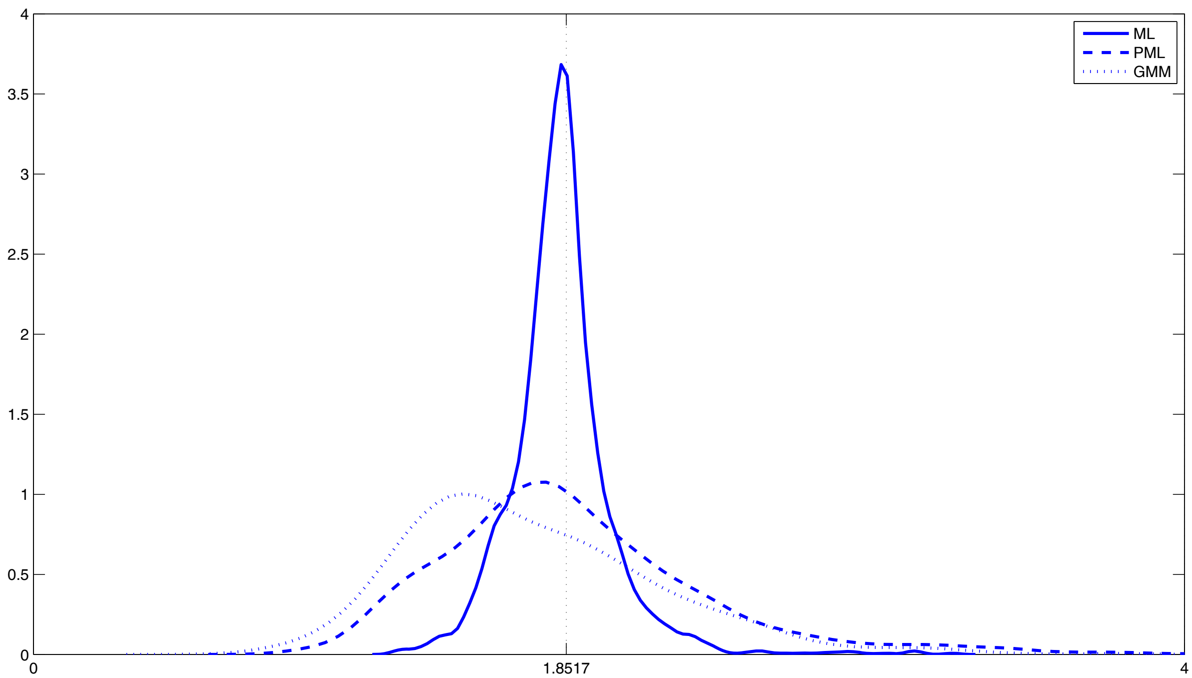


Fig. 2.— Kernel densities of p^* estimates with alternative estimators[†]



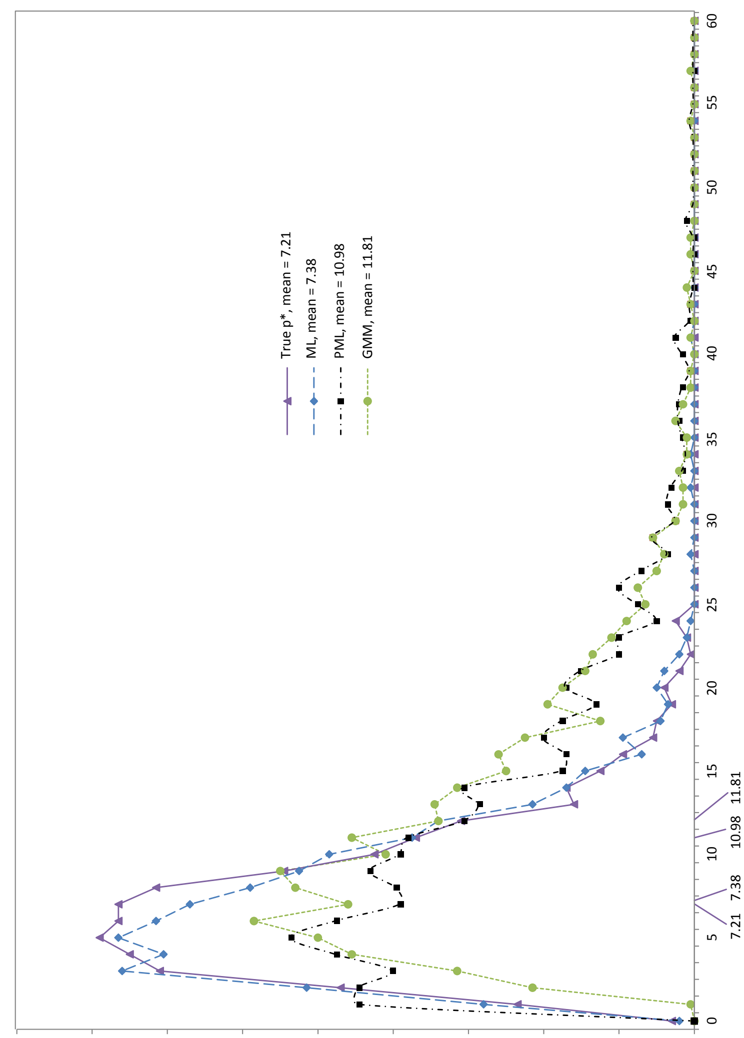
[†] Monte Carlo experiment with sample size 100 and parameters $a = 0.6$, $b = -0.3$, $k = 0$ and $d = 0.1$, as in Michaelides and Ng (2000).

Fig. 3.— Kernel densities of p^* obtained with alternative estimators†



† Monte Carlo experiments with sample sizes of 108 and parameters $a = 1$, $b = -2$, $k = 0.02$ and $d = 0$.

Fig. 4.— Frequencies of occurrence of stockouts in the simulated price samples†



† The figure shows the mean number of stockouts and the histograms of the actual number of stockouts occurring in the simulated price samples using the true value of p^*, and of the predicted numbers of stockouts using the p^* obtained with alternative estimators on the same set of simulated price samples in the Monte Carlo experiment with sample size of 108 and parameters $a = 1$, $b = -2$, $k = 0.02$ and $d = 0$. Each line in the figure connects the middle points of the tops of the histogram bars.

Fig. 5.— Deflated Sugar Price Series

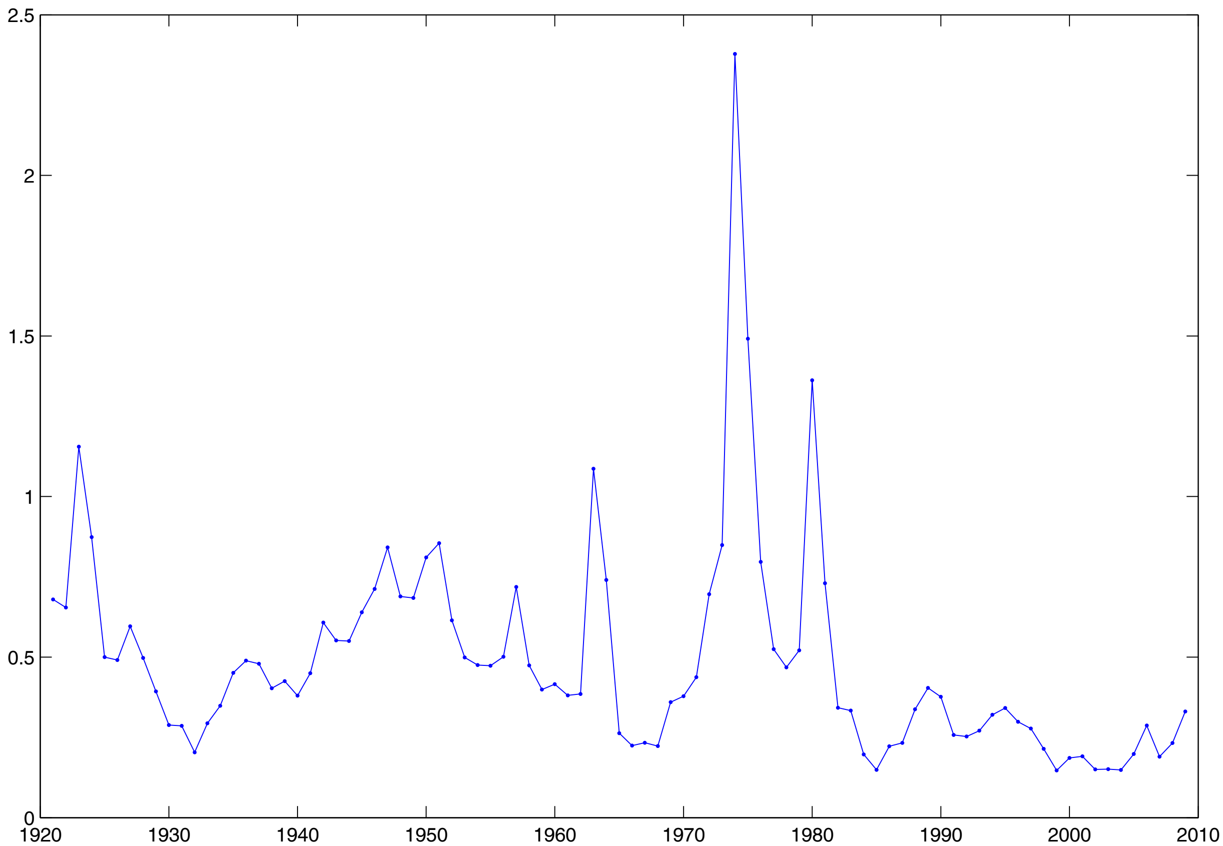
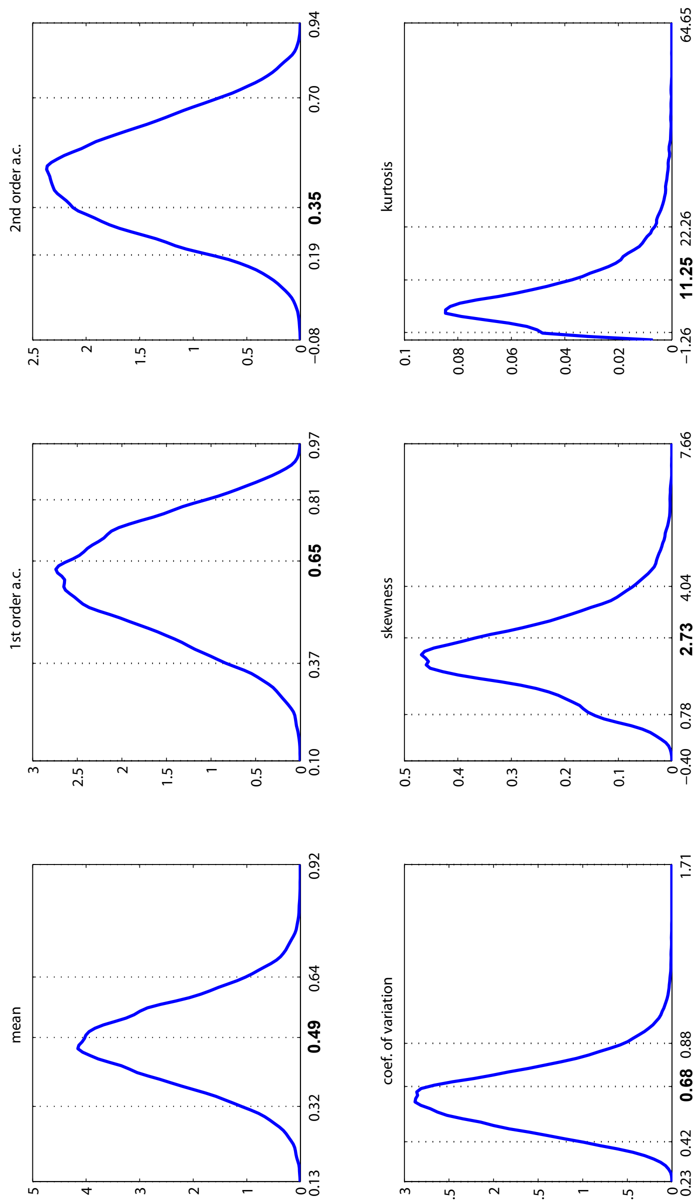
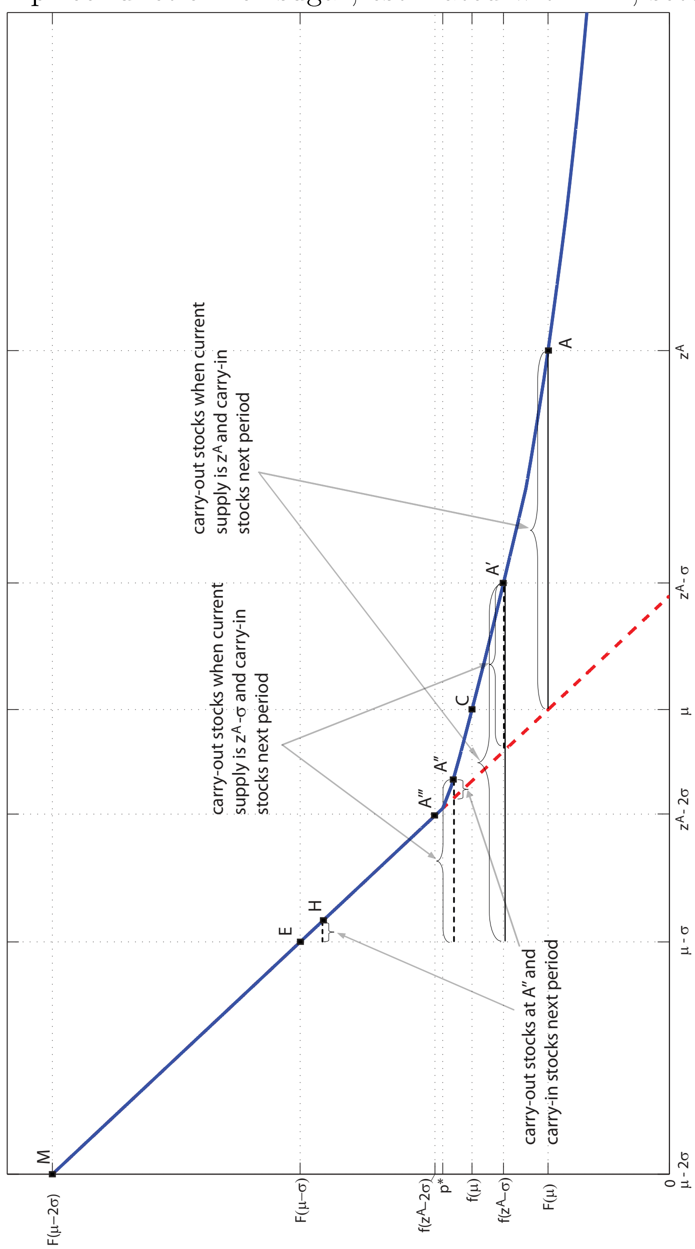


Fig. 6.— Predicted distributions of small sample price characteristics, using the ML estimates†



† The figure is based on the estimates reported in panel 2 of table 3. In each figure, the grid on the horizontal axis reports the minimum, the 5th percentile, the 95th percentile and the maximum of the values obtained on all possible subsamples of 89 consecutive prices extracted from a series of 300,000 prices simulated using a model parameterized with the estimated values. The value observed on the actual series of sugar prices is reported in boldface.

Fig. 7.— Equilibrium price function for sugar, estimated with ML, setting $d = 0^\dagger$



† The horizontal axis measures quantities in units of standard deviation, net of mean harvest.