

Can supermassive black holes alter cold dark matter cusps through accretion?

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ABSTRACT

We present some simple models to determine whether or not the accretion of cold dark matter by supermassive black holes is astrophysically important. Contrary to some claims in the literature, we show that supermassive black holes cannot significantly alter a power-law density cusp via accretion, whether during mergers or in the steady state.

Key words: accretion, accretion discs – black hole physics – dark matter.

1 INTRODUCTION

Over the past few decades, the idea of dark matter has become deeply rooted within the astrophysical community. In particular, the Λ CDM (cold dark matter with a cosmological constant) theory has done very well in reproducing experimental results on large scales (see e.g. Bahcall et al. 1999 or Sellwood & Kosowsky 2000).

However, on small scales (galaxies and smaller), the success of cold dark matter has been less striking (Moore 2002). On galactic scales, current experiments which measure the velocity dispersion of a galaxy as a function of radius have found enormous mass-to-light ratios in all galaxies, from dwarfs to giants (Salucci & Borriello 2000). While this is strong evidence for the existence of dark matter, the derived mass profiles do not compare well with those predicted from simulations involving cold dark matter (Moore 2002). In particular, cold dark matter (CDM) simulations predict the presence of a central cusp in the density profile, whereas the observed profiles seem to have a well defined core (Salucci & Borriello 2000). This apparent misalignment of theory and experiment has led to three main types of solution, as follows.

- (i) Dark matter is real but not cold (Moore 2002).
- (ii) Dark matter is not the solution and we should look to alternative theories (Milgrom 1983).
- (iii) CDM is correct but current theories are missing some important physics.

In other words: does the absence of a central dark matter cusp tell us something about the nature of dark matter, or does it tell us something about galaxy formation? Understanding this is essential for facilitating direct search experiments for dark matter, such as DAMA (Bernabei 2001). With any number of particle candidates for dark matter and the difficulty of removing noise from direct detection experiments, particle physicists almost need to know what they are looking for before they can find it. As such, any complimentary method which can rule out some candidates is extremely important.

Unfortunately, before any robust conclusions about the nature of dark matter can be made from the rotation curve measurements of nearby galaxies, we need to be sure that what we are measuring is the dark matter profile as it would have been before the formation of a galaxy. It is this profile which contains information about the detailed nature of dark matter. If the profile has been changed as a result of astrophysical effects such as matter outflow (Gnedin & Zhao 2002) or bars (Athanasoula 2001) then some or maybe all of the information about the nature of dark matter could have been lost. It is important to work out which astrophysical effects we should consider and which are not important.

In this paper, we explore the fate of a CDM cusp in the presence of a central black hole. Recently, Zhao, Haehnelt & Rees (2002) (hereafter ZHR) have suggested that tidal stirring from infalling satellites could lead to significant dark matter accretion, accounting for some 20–40 per cent of the mass of a central supermassive black hole if loss cone refilling can be efficient enough. As such, we consider whether or not supermassive black holes can alter the dark matter distribution in a galaxy in any significant way.

In Section 2, we recall that the steady-state accretion rate of CDM onto a supermassive black hole is negligible. In Section 3, we present a toy model for calculating the mass of CDM accreted during merger events. In Section 4, we discuss the validity of our model and the subsequent effect on the underlying dark matter distribution. Finally, we conclude in Section 5.

2 THE STEADY-STATE ACCRETION RATE

The steady-state accretion rate of matter on to a black hole has been treated by several authors (e.g. Lightman & Shapiro 1977). Any particles which lie on orbits that bring them within the Schwarzschild radius of the hole (see equation 11) will be swallowed. As both low-energy, high angular momentum and high-energy, low angular momentum particles can be swallowed, the problem is necessarily two-dimensional. The fraction of particles in phase space which will be swallowed is described by the ‘loss cone’ (Lightman & Shapiro 1977) and any particles lying within the loss cone will be swept

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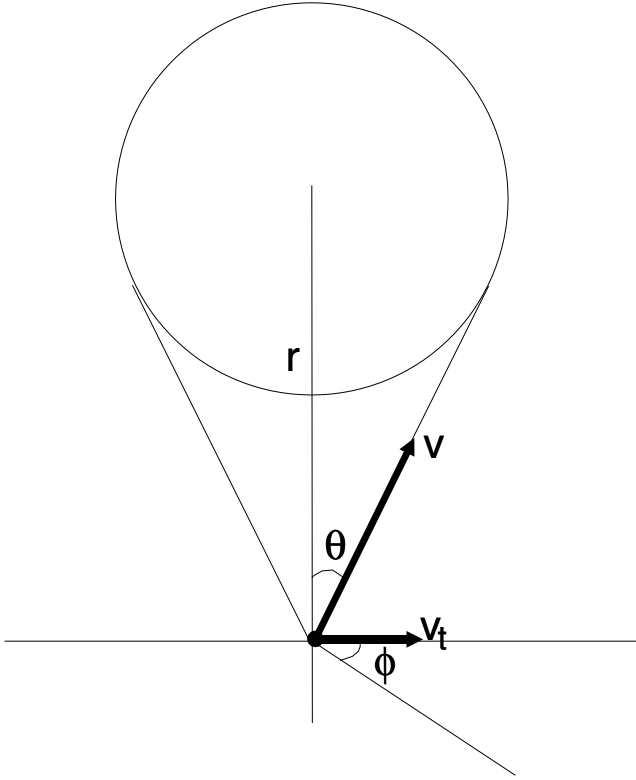


Figure 1. Schematic diagram of a black hole and its loss cone. The solid circle at the origin represents a dark matter particle a distance r away from a black hole (depicted by the large circle). The dark matter particle has velocity v and transverse velocity $v_t = L_{\max}/r$.

away in a dynamic time \sim the time taken for a particle to complete one orbit.

As can be seen in Fig. 1, the loss cone can be thought of as the locus of points swept out from trying to point a particle at the black hole from some distance r away.

Now, by considering particle orbits in a Schwarzschild metric (Wald 1984), any particle with an angular momentum per unit mass less than $L_{\max} = \sqrt{12}GM_{\text{BH}}/c$ will fall within the Schwarzschild radius and be swallowed (where M_{BH} is the mass of the black hole, c is the speed of light in vacuo and G is the gravitational constant).

The solid angle subtended by the loss cone, Ω , can be written as:

$$\begin{aligned} \Omega &= \int_0^{2\pi} \int_0^\theta \sin \theta' d\theta' d\phi \\ &\simeq \pi\theta^2 \simeq \pi(v_t/v)^2 = \pi(L_{\max}/rv)^2. \end{aligned} \quad (1)$$

Therefore, given some phase space density of dark matter particles, $f(r, v)$, the total mass inside the loss cone M_{lc} is given by:

$$\begin{aligned} M_{\text{lc}} &= \int_0^\infty \int_{r_s}^{r_{200}} \Omega 4\pi r^2 4\pi v^2 f(r, v) dr dv \\ &= \int_0^\infty \int_{r_s}^{r_{200}} 16\pi^3 L_{\max}^2 f(r, v) dr dv. \end{aligned} \quad (2)$$

Notice that the lower limit for the integral over r is the Schwarzschild radius. This is because the black hole itself must provide a cut-off for the dark matter distribution. The upper limit is given by r_{200} , the virial radius (Navarro, Frenk & White 1996, NFW).

Assuming a Maxwell Boltzmann distribution for the velocity density of the dark matter particles and an NFW profile (Navarro et al. 1996) for the spatial density gives us:

Table 1. Initial conditions. See text for details.

Parameter	Value	Description
Free Parameters		
$M_{\text{bh}} (M_\odot)$	10^6	Black hole mass
r_i (kpc)	1	Black hole start radius
$M_{200} (M_\odot)$	1.9×10^{12}	Halo mass within virial radius
h	1	Hubble parameter
Derived Parameters		
c_n	13.34	NFW halo concentration
r_{200} (kpc)	201.4	Virial radius
$\rho'_0 (M_\odot \text{ kpc}^{-3})$	2.49×10^7	NFW scale density
ρ'_0 (kpc)	15.14	NFW scallength

$$\begin{aligned} f(r, v) &= \rho_{\text{NFW}}(r) g_{\text{MB}}(v) \\ &= \frac{\rho_0}{\left(\frac{r}{r_0}\right) \left(1 + \frac{r}{r_0}\right)^2} \left(\frac{3}{2\langle v^2 \rangle \pi}\right)^{\frac{3}{2}} e^{(-3v^2/2\langle v^2 \rangle)}, \end{aligned} \quad (3)$$

where ρ_0 and r_0 are the scalefactors for a particular galaxy, and $\langle v^2 \rangle^{1/2}$ is the velocity dispersion of the dark matter halo. The velocity dispersion is taken to be a constant, which is a reasonable approximation given the observed rotation curves in many galaxies.

For a $10^6 M_\odot$ black hole at the centre of a Milky Way type galaxy (we take scaling values as in Table 1), the dark mass within the loss cone is $\sim 0.6 M_\odot$, while the total mass of dark matter is some $1.9 \times 10^{12} M_\odot$. Thus a tiny fraction of the dark matter is accreted in a crossing time, but then not much else can happen until the loss cone is refilled. For stars, refilling can be achieved by two-body interactions, continuing satellite mergers and dynamical friction, allowing a continual diffusion of matter into the loss cone (Lightman & Shapiro 1977). However, for CDM, the interaction cross-section is presumed to be so small that the time taken to re-fill the loss cone by diffusion is very much longer than the age of the Universe.

The problem can be avoided by postulating that dark matter can self-interact, thus raising the interaction cross-section and allowing significant steady-state accretion rates (Ostriker 2000). ZHR have also suggested that tidal stirring from infalling satellites could refill the loss cone. We look at this idea in more detail in the following section.

3 ACCRETION DURING MERGER EVENTS

As shown above, the steady-state accretion rate of dark matter is negligible. However, during a galaxy merger event, while a central supermassive black hole is stationary with respect to the dark matter in its host galaxy, it is not stationary with respect to the dark matter in the merger galaxy. In the rest frame of one of the galaxies, the central black hole, belonging to the other galaxy will spiral in, sweeping up dark matter on its way. As the two galaxies merge, their CDM haloes will be tidally stripped out to the Roche limit (Roche 1859) where the density of a halo approximately matches that of its surroundings. We can expect, then, that the CDM cusps from the two galaxies should make it right into the centre of the forming merger remnant.

This idea is borne out by various numerical studies (see e.g. Van Albada & Van Gorkom 1977 and more recently Milosavljevic & Merritt 2002) which show that the initial conditions are statistically preserved in major merger events. As such, it is not clear that a dark matter cusp will survive during galaxy mergers. Two black holes could, in principle, mutually accrete dark matter from each other's

host cusp. The magnitude of the effect will depend on the loss cone refilling rate during such a merger process and it is this that we will discuss in Sections 3.1 and 3.2.

The presence of a significant bulge component in galaxies, apparently correlated with the central black hole mass, can set a limit on the efficiency of this process. If the cusp depletion is found to be very efficient then it must occur in the early stages of bulge growth, or in low surface brightness (LSB) galaxies where the dark matter dominates over the baryonic matter even in the central regions.

Faber et al. (1997) examined the central density profile of 61 elliptical galaxies and spiral bulges. They found that, while all of the galaxies in their sample were well fit by a power law profile, the brightest galaxies contained a central core in the light profile at a break radius of $\sim 10\text{--}100$ pc. They suggest, as have others (e.g. Quinlan 1996; Milosavljevic & Merritt 2001, hereafter MM), that the central cusp depletion could be due to scattering during major merger events as two massive black holes form a hard binary. We consider a complimentary mechanism, operating earlier in the merger process, based on the accretion of scattered material during galaxy mergers. We show that this mechanism is not important for growing black holes or rearranging dark matter cusps.

Although in our analysis we will talk mainly about the effect on the central dark matter, our analysis could be applied equally to the central baryonic matter cusps which are observed in bright galaxies.

We treat the infalling black hole as a scattering body which spirals into the centre of a galaxy via dynamical friction. Some of the dark matter that it scatters will be scattered back onto itself – *self-scattering* – and some will be scattered into the loss cone of the other hole – *mutual scattering*. The sum of these two effects will give us an estimate of the total accretion rate of dark matter. We now look at these effects quantitatively.

3.1 Self-scattering

Consider two in-spiralling black holes, A and B, from the reference frame stationary relative to hole B. In the self-scattering case, we consider the in-spiralling black hole, A, as an effective cross-sectional area sweeping through phase space.

We need only consider the growth of hole A though, by symmetry, hole B will also grow via this mechanism. We can then parametrize hole B solely by the nature of its host galaxy. This is because of the empirically observed correlation between black hole mass and bulge velocity dispersion (Ferrarese et al. 2001). Small black holes reside in shallow potential wells, larger black holes reside in larger potential wells.

We assume that hole A is brought in via dynamical friction from the dark matter and baryonic matter in the centre of the stationary galaxy (hole B's host galaxy). Following the Chandrasekhar prescription (cf. Binney & Tremaine 1987), we have that the inward velocity of the hole, dr/dt , is given by:

$$\frac{dr}{dt} = \frac{-KM_A}{v_c^3} r \rho(r), \quad (4)$$

where $\rho(r)$ is the density distribution of the central matter (baryons and dark matter), M_A is the mass of hole A, $v_c(r)$ is the local circular velocity and K is approximately a constant. For a Maxwell Boltzmann distribution of particle velocities, K is given by (Binney & Tremaine 1987):

$$K = 0.428G^2 2\pi \ln(1 + \Lambda^2) \quad (5)$$

$$\Lambda = \frac{b_{\max}(v^2)}{GM_A}, \quad (6)$$

where G is the gravitational constant, b_{\max} is the maximum impact parameter for the in-spiralling hole and $\langle v^2 \rangle^{1/2}$ is the mean velocity dispersion of the surrounding baryonic and dark matter.

From equation (4), we can derive the dynamical friction infall time t_{dyn} for the hole starting at radius r_i . This gives us:

$$t_{\text{dyn}} = \int_0^{r_i} \frac{1}{KM_A} \frac{v_c(r)^3}{r\rho(r)} dr. \quad (7)$$

Equation (7) is the Chandrasekhar equation for dynamical friction and is approximately valid provided that the infalling mass does not exceed the mass interior to its orbit (Binney & Tremaine 1987). Notice that the minus sign in equation (4) has been substituted for swapped integration limits.

We can also use equation (4) to derive the amount of mass swept up by the black hole as it moves from a radius a to a radius b . The distance travelled by the hole in a time dt is given by:

$$ds = dt v_c(r) = \frac{v_c(r)^4 dr}{K r \rho(r)}. \quad (8)$$

If the black hole cross-section is given by $\sigma_A(r)$ then the mass of dark matter swept up by the hole by moving through a distance ds is then:

$$dM_A = \frac{v_c(r)^4 \sigma_A(r) \rho(r) dr}{K r \rho(r)}, \quad (9)$$

and so the total mass swept up as the hole moves from a radius a to b is:

$$\Delta M_A = \int_b^a dM_A = \int_b^a \frac{1}{KM_A} \frac{v_c(r)^4 \sigma_A(r)}{r} dr. \quad (10)$$

3.1.1 The black hole cross-section

If the black hole cross section were just πr_s^2 , where r_s is the Schwarzschild radius, then the volume swept up in phase space would be truly tiny. The Schwarzschild radius is given by:

$$r_s = \frac{2GM_A}{c^2}, \quad (11)$$

where c is the speed of light in vacuo. Thus, for a $10^6\text{-}M_\odot$ black hole, $r_s \sim 10^{-7}$ pc.

However, a massive body such as a black hole interacts with the medium through which it moves, preferentially scattering particles on to orbits which will bring them within the Schwarzschild radius, allowing them to be swallowed. Taking this *gravitational focusing* into account gives us a total cross-section of (Binney & Tremaine 1987):

$$\begin{aligned} \sigma_A(r) &= \pi r_s^2 + \frac{2\pi G M_A r_s}{v_c(r)^2} \\ &= \pi r_s^2 \left[1 + \frac{c^2}{v_c(r)^2} \right] \\ &\simeq \frac{4\pi G^2 M_A^2}{c^2 v_c(r)^2}. \end{aligned} \quad (12)$$

Because $v_c(r)$ must always be less than the speed of light, the cross-section due to gravitational focusing will always be much more important than the physical cross-section of the black hole.

Putting this all together we get, for the mass swallowed as the hole moves from radius a to b :

$$\Delta M_A = \int_b^a A M_A \frac{v_c(r)^2}{r} dr. \quad (13)$$

$$A = \frac{2}{0.428 \ln(1 + \Lambda^2)c^2}. \quad (14)$$

Using equation (13) we can now calculate the mass swept up as the black hole spirals in. We cannot, however, naively integrate from some start radius to zero to obtain the mass swallowed. This is because if the black hole manages to complete a whole orbit before moving inwards a distance greater than $\sim \sqrt{a}$ then we have the problem of shell crossing; that is, we would be counting the mass more than once. In order to avoid this problem, we can make the distance a to b some small interval and then numerically sum each mass shell Δm , comparing each Δm with the actual mass of CDM present, and taking the smaller value. In this way, we avoid divergent mass consumption at small radii where the same part of configuration space is swept out many times but the mass within that space can be swept up only once. Quantitatively, this means that we require:

$$\Delta M_A < 4\pi \int_b^a r^2 \rho(r) dr \quad \forall a, b. \quad (15)$$

3.1.2 The density profile

The density profile for the dark and luminous matter in most galaxies can be well approximated, within a few kpc of the centre, by a power law (Blais-Ouellette & Carignan 2002). As such, we adopt the following form for $\rho(r)$:

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\alpha}. \quad (16)$$

For the dark matter, α is observed to be typically $\alpha < 1$, whereas CDM simulations predict dark matter slopes of $\alpha > 1$ (Blais-Ouellette & Carignan 2002).

The local circular velocity is then given by (Binney & Tremaine 1987):

$$\begin{aligned} v_c(r)^2 &= \frac{4\pi G}{r} \int_0^r r'^2 \rho(r') dr' \\ &= \frac{4\pi G \rho_0 r_0^\alpha}{3 - \alpha} r^{2-\alpha}. \end{aligned} \quad (17)$$

For a power-law profile, it can be shown (see Appendix A) that $\langle v^2 \rangle \simeq v_c^2$,

and if the maximum impact parameter, b_{\max} , is taken to be the starting radius of the hole, r_i , this gives us

$$\Lambda = \frac{4\pi \rho_0 r_0^\alpha r_i}{M_A (3 - \alpha)} r^{2-\alpha}. \quad (19)$$

Fig. 2 plots $\ln(1 + \Lambda^2)$ against r for initial conditions set up as in Table 1 and for varying α . For $r > 0.01$ kpc, or $\alpha > 0.5$, $\ln(1 + \Lambda^2) \sim \text{constant}$. However, in general it is not constant over the full range of r and hence neither is A (see equation 14). We assume that A is constant over the small change in r , $\Delta r = a - b$, but does vary as a function of a .

Thus, putting this all together gives us, for the mass swept up by hole A,

$$\begin{aligned} \Delta M_A &= \frac{4\pi G A(a) M_A \rho_0 r_0^\alpha}{(3 - \alpha)(2 - \alpha)} (a^{2-\alpha} - b^{2-\alpha}) \quad \alpha < 3, \alpha \neq 2 \\ \Delta M_A &= 4\pi G A(a) M_A \rho_0 r_0^2 \ln \left(\frac{a}{b} \right) \quad \alpha = 2. \end{aligned} \quad (20)$$

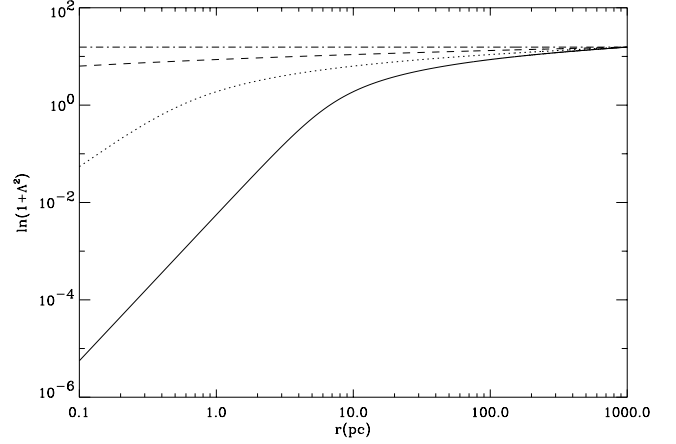


Figure 2. Radial dependence of the Coulomb factor, $\ln(1 + \Lambda^2)$, in power-law density profiles with exponent α . The solid line is for $\alpha = 0.5$, the dotted line is for $\alpha = 1$, the dashed line is for $\alpha = 1.5$ and the dot-dashed line is for $\alpha = 2$.

3.1.3 Initial conditions

From equation (20), we can see that the important factors for determining the amount of mass accreted are M_A , the mass of the hole, and ρ_0 , r_0 and α , the density parameters for the central cusp.

The initial infall radius of the hole can be taken from numerical simulations performed by MM. We use 1 kpc as our starting position for the hole.

The density parameters are scaled such that the galaxy has the same mass as an NFW-type profile within 1 kpc. The NFW profile is given by (Navarro et al. 1996):

$$\rho_{\text{NFW}} = \frac{\rho'_0}{(r/r'_0)(1 + r/r'_0)^2}, \quad (21)$$

$$\lim_{r \ll r'_0} \rho_{\text{NFW}} \simeq \frac{\rho'_0 r'_0}{r}, \quad (22)$$

where ρ'_0 and r'_0 are given by

$$\rho'_0 = \delta_0 \rho_{\text{crit}}, \quad (23)$$

$$\delta_0 = \frac{200}{3} \frac{c_n^3}{\ln(1 + c_n) - (c_n/1 + c_n)}, \quad (24)$$

$$r'_0 = r_{200}/c_n \quad (25)$$

$$r_{200} = \left(\frac{M_{200}}{4/3\pi 200 \rho_{\text{crit}}} \right)^{1/3}, \quad (26)$$

where r_{200} is the virial radius, M_{200} is the mass within the virial radius and is taken to be the mass of the galaxy, c_n is the NFW concentration parameter, and $\rho_{\text{crit}} = 277.3 h^2 M_\odot \text{ kpc}^{-3}$ is the critical density.

The only free parameter in the NFW profile is M_{200} , as the concentration parameter is found, numerically, to be a function of the mass of a galaxy (Navarro et al. 1996).

The power-law scale-density and radius (ρ_0 , r_0) we use are then scaled to contain the same mass as the NFW profile within 1 kpc. Thus

$$r_0 = r'_0 \quad (27)$$

$$\rho_0 = \rho'_0 \left(\frac{r_0}{1 \text{ kpc}} \right)^{1-\alpha} \frac{(3 - \alpha)}{2}. \quad (28)$$

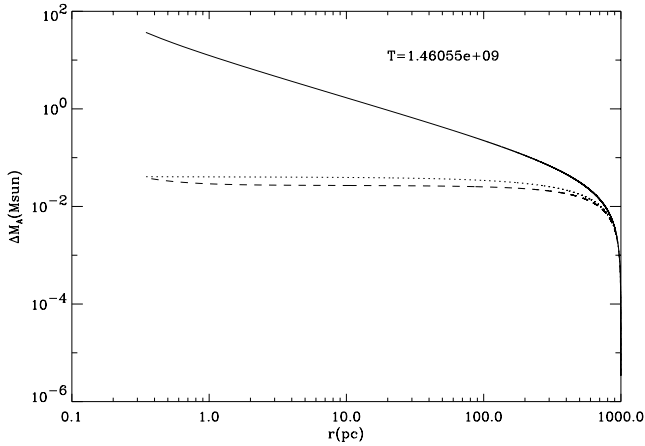


Figure 3. Mass accreted by a $10^6 M_\odot$ black hole spiralling in to a Milky Way type galaxy from 1 kpc away from the centre due to self-scattering. The infall time, T , is marked in yr. The solid curve is for $\alpha = 2.9$, the dotted curve is for $\alpha = 1$ and the dashed curve is for $\alpha = 0.5$.

Recall that this density profile describes the galaxy which is in the stationary frame. The infalling galaxy is parameterised only by the mass of its black hole. Thus, by setting up the profiles for a Milky Way type galaxy, we can model the full range of mergers, from equal mass to minor mergers, by scaling the infalling black hole mass.

A summary of all of the initial conditions used is given in Table 1. The parameters are broken up into those which are put into the model and those which are subsequently derived. The scaling parameters for the NFW profile, for example, are set up consistently for a halo which contains a mass of $1.9 \times 10^{12} M_\odot$ within the virial radius. A halo of this mass has a concentration parameter of about 13.3 (Navarro et al. 1996).

3.1.4 Results

The numerical integration was performed using IDL, with the initial conditions set up as in Table 1 and with a numerical resolution of $\Delta r = a - b = 0.1$ pc. The results are displayed in Fig. 3.

As can be seen, most of the accretion occurs in the outer regions and increases with α . However, even for $\alpha = 2.9$, the effect is small, with less than $100 M_\odot$ accreted in total. This is a tiny fraction of the mass of the black hole, which is some $10^6 M_\odot$. For an NFW profile, with $\alpha = 1$ (dotted line, Fig. 3), the total mass accreted is less than in the steady state case. Finally, from equation (20), we can see that the total mass accreted scales linearly with M_A , so the fractional increase in black hole mass is unaffected by any increase in M_A .

3.2 Mutual scattering

Now we consider the case where the black holes scatter dark matter into each others' loss cones. Consider, as in the self-scattering scenario, a black hole, A, spiralling into a hole B, from a reference frame that is stationary with respect to the hole B. As before, each black hole has its own bound CDM cusp. The dark matter that hole A scatters, for example, is that which resides in hole B's CDM cusp.

Hole A will scatter some fraction, η_A , of particles within its sphere of influence into the loss cone of hole B, and these particles will then be quickly swallowed in a dynamical time. Similarly, hole B will scatter particles into hole A's loss cone and hence both holes will grow via accretion as they merge.

The problem, then, is similar to the self-scattering case, except that now, the effective black hole cross-section is due to the sphere of influence of hole A, and hole A causes the growth of hole B.

Because each hole induces the growth of the other, we should consider both M_A and M_B as functions of r and consider the growth of both holes. However, we will make the assumption that $\Delta M_A/M_A \sim \Delta M_B/M_B \sim$ small, so that M_A and M_B are \sim constant.

The sphere of influence of hole A is given by:

$$r_{\text{sph,a}} = \frac{GM_A}{\langle v^2 \rangle}, \quad (29)$$

where M_A is the mass of hole A and $\langle v^2 \rangle^{1/2}$ is the local spherical velocity dispersion in hole B's host galaxy. For a $10^6 M_\odot$ black hole in a Milky Way type galaxy, $r_{\text{sph,a}} \sim 0.3$ pc. Thus the effective black hole cross-sectional area, $\pi r_{\text{sph,a}}^2$ is ~ 0.1 pc², whereas the effective cross sectional area for the self scattering case was just $\sim 10^{-6}$ pc². As such, if the scattering efficiency, η_A , is large enough, this could be an important mechanism for growing black holes.

From equations (10) and (29), for the mass accreted by hole B as black hole A moves from a radius a to b we get

$$\Delta M_B = \int_b^a \frac{\eta_A \pi G^2 M_A v_c(r)^4}{K \langle v^2 \rangle^2 r} dr. \quad (30)$$

For a power-law density profile (see equation 16), and using equation (18), this gives us

$$\Delta M_B = A'(a) M_A \eta_A \ln \left(\frac{a}{b} \right), \quad (31)$$

$$A' = \frac{1}{0.856 \ln(1 + \Lambda^2)}. \quad (32)$$

3.2.1 The scattering efficiency

Comparing equations (20) and (31), we can see straight away that, if the scattering efficiency η_A was nearly 1, nearly all of the central dark matter would be swept up as the holes spiral together. This is because the constant, A' is larger than A by a factor $\sim c^2$. As such, the value of η_A is key to determining the magnitude of the accretion effect.

We can obtain a crude estimate for η_A if we assume that all dark matter particles are scattered randomly. If this is the case then the probability, P_A , that a particle will be scattered into hole B's loss cone will be simply the fraction of phase space subtended by its loss cone.

Thus we get, from equations (1), (3), (17) and (18),

$$P_A \simeq \frac{1}{4\pi} \int_0^\infty \Omega 4\pi v^2 g(v) dv \quad (33)$$

$$= \frac{9GM_B^2(3-\alpha)}{2\pi c^2 \rho_0 r_0^\alpha r^{(4-\alpha)}}. \quad (34)$$

If N_A is the number of times hole A crosses the same section of configuration space, then the fraction of particles scattered into B's loss cone will be

$$\eta_A = 1 - (1 - P_A)^{N_A}. \quad (35)$$

N_A is essentially the number of attempts hole A can make at scattering the same particles into hole B's loss cone.

Thus, if the time taken for hole A to complete one orbit at a radius, r is given by $T(r) = 2\pi r/v_c(r)$, then N_A is given by

$$\begin{aligned}
 dN_A &\simeq \frac{dt}{T(r)} \\
 &= \frac{v_c(r)^4}{2\pi K M_A r^2 \rho(r)} dr \\
 \Rightarrow \\
 N_A &= \frac{8A'(a)\rho_0 r_0^\alpha}{M_A(3-\alpha)^3} (a^{3-\alpha} - b^{3-\alpha}), \tag{36}
 \end{aligned}$$

so that N_A is now the number of times hole A crosses the same section of phase space as it moves from a radius a to a radius b . Thus, if $a - b = \Delta r \ll a$, such that $P_A(a) \simeq P_A(b)$ and if $\eta_A \ll 1$, then

$$\begin{aligned}
 \eta_A(a) &\simeq 1 - e^{-N_A P_A} \\
 &\simeq 1 - \exp\left(\frac{-36GA'(a)M_B^2(a-b)}{\pi c^2 M_A(3-\alpha)a^2}\right). \tag{37}
 \end{aligned}$$

Notice that the scattering efficiency is maximized for $M_A \ll M_B$. This is because the size of hole B's loss cone is proportional to M_B^2 (see equation 2) and so as M_B increases, the chance of hole A scattering matter into B's loss cone increases as M_B^2 . As M_A increases, the infall time for hole A decreases and so hole A will cross the same section of configuration space fewer times, reducing the scattering efficiency.

The dependence of the scattering efficiency on α is seemingly very weak. However, A' is a strong function of α , particularly for small r (see Fig. 2 and equation 32). As A' increases as α is reduced, we can expect that η_A will be similarly correlated.

Notice also that η_A is independent of ρ_0 and r_0 . This is because increasing the central density normalization $\rho_0 r_0^\alpha$, while keeping the black hole mass constant reduces the fraction of phase space subtended by the loss cone, and so P_A falls. However, the number of times hole A sweeps through the same section of phase space, N_A , is subsequently increased. The two effects cancel out, so that the scattering efficiency depends only on the slope of the density profile and not its normalization.

Finally, notice that the scattering efficiency is a strong function of radius and will be maximized for small a . For most radii, the scattering efficiency is absolutely tiny. Fig. 4 plots $\eta_A(a)$ against a for $M_A = M_B = 10^8 M_\odot$, $\alpha = 1$. As can be seen for almost all radii, as might be expected physically, the scattering efficiency is practically zero. Only when the holes are close does the efficiency become large enough to produce any significant amount of accretion.

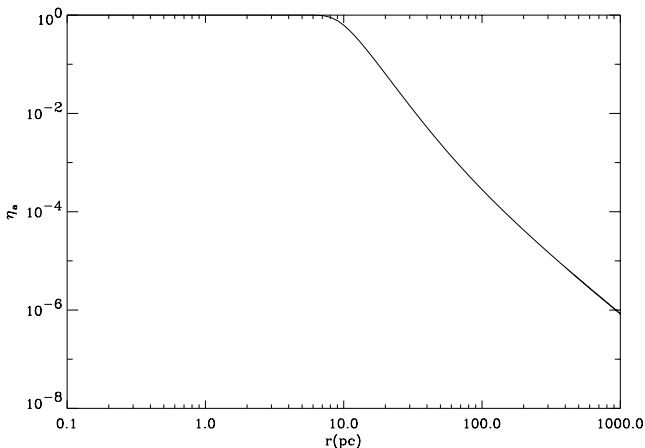


Figure 4. The scattering efficiency η_A , as a function of radius for $M_A = M_B = 10^8 M_\odot$, $\alpha = 1$.

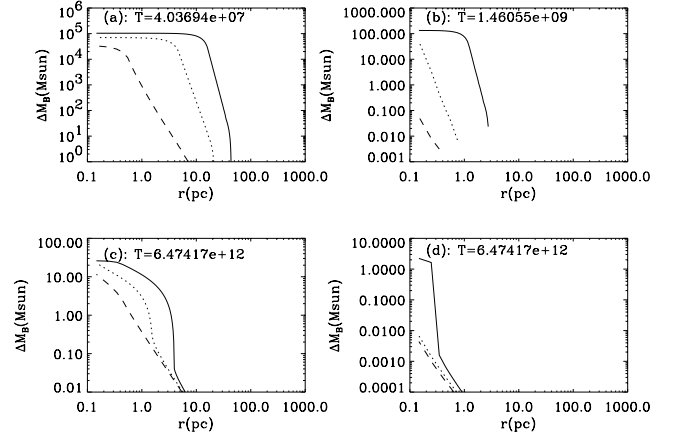


Figure 5. Mass accreted by hole B due to the scattering of dark matter particles by hole A. Graph (a) is for $M_A = M_B = 10^8 M_\odot$. Graph (b) is for $M_A = M_B = 10^6 M_\odot$. Graph (c) is for $M_A = 10^2 M_\odot$, $M_B = 10^8 M_\odot$. Graph (d) is for $M_A = 10^2 M_\odot$, $M_B = 10^6 M_\odot$. The solid lines are for $\alpha = 0.5$, the dotted lines are for $\alpha = 1$ and the dashed lines are for $\alpha = 1.5$. The infall times, T , are marked in yr for each case.

3.2.2 Results

The numerical integration was performed using IDL, with the initial conditions set up as in Table 1 and with a numerical resolution of $\Delta r = a - b = 0.1$ pc. The results are displayed in Fig. 5.

As can be seen from equations (37) and (31) and from Fig. 4, accretion becomes significant only for small r . Reducing α , the slope of the cusp, pushes the radius at which accretion becomes significant to larger radii but does not significantly affect the total mass accreted.

Fig. 5(a) shows an extreme case – $M_A = M_B = 10^8 M_\odot$. Here, some $10^5 M_\odot$ is accreted. While this amounts to only ~ 0.1 per cent of the mass of hole B, it is sufficient to completely remove all of the dark matter within $r_c \sim 20$ pc (see Fig. 6). Fig. 5(b) shows a more likely scenario – $M_A = M_B = 10^6 M_\odot$. For an NFW-type inner slope of $\alpha = 1$ or shallower, $\sim 100 M_\odot$ is accreted. However, for steeper slopes this rapidly falls to a mere $0.1 M_\odot$, which is accreted only within the inner few pc. Fig. 5(c) shows the case for $M_A = 100 M_\odot$, $M_B = 10^8 M_\odot$. While only a few tens of solar masses are accreted, in the early stages of black hole growth, many low mass holes could have coalesced to form a supermassive central black hole. Miralda-Escude & Gould (2000) estimate that some 25 000 low mass black holes would have fallen into the central few parsecs of the Milky Way by the present day. If each of these scatters $\sim 10 M_\odot$ of dark matter onto the central black hole then all of the dark matter within $r_c \sim 10$ pc ($\sim 2.4 \times 10^5 M_\odot$) would be swept up. As with the case (a), this amounts to only ~ 0.1 per cent of the mass of hole B. Finally, Fig. 5(d) shows the case for $M_A = 100 M_\odot$, $M_B = 10^6 M_\odot$. As with the case (c), many infalling black holes could lead to all of the dark matter within $r_c \sim 1$ parsec or so being swept up. However, this amounts to only $\sim 2000 M_\odot$, again only ~ 0.1 per cent of the mass of hole B.

4 DISCUSSION

We have looked at some simple models which treat black holes in spherical systems as scattering bodies. A fraction of the scattered material is removed by accretion onto the scattering bodies, either through self-scattering or mutual scattering as discussed above.

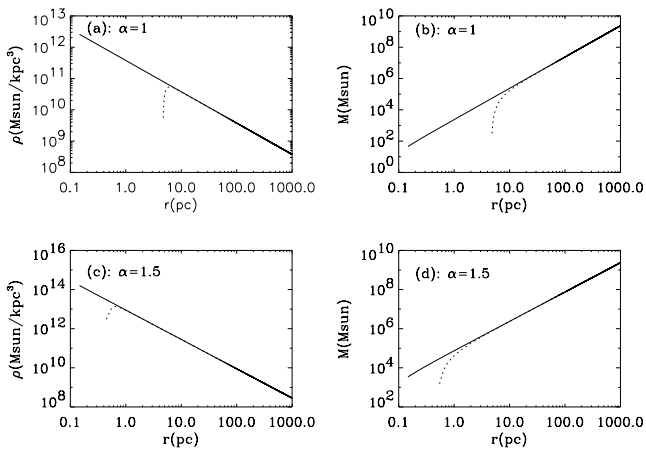


Figure 6. The change in the central dark matter density due to accretion onto two merging $10^8 M_\odot$ black holes. Graphs (a) and (c) show the central density before (solid line) and after (dotted line) the merger, for an initial density cusp of slope, $\alpha = 1$ and $\alpha = 1.5$, respectively. Graphs (b) and (d) show the central mass interior to r before (solid line) and after (dotted line) the merger, also for an initial density cusp of slope, $\alpha = 1$ and $\alpha = 1.5$, respectively.

Self-scattering is most effective when the black holes are at their maximum separation, and for steep central density slopes. However, even in the most extreme situations, the overall mass accreted is negligible, and usually less than that accreted in the steady-state scenario.

Mutual scattering is more effective, but only at very small radii. The main problem with this is that at radii of a few to a few tens of pc, the black holes are likely to form a stable hard binary (Milosavljevic & Merritt 2001). Dynamical friction then no longer applies because, as the binary hardens, much of the central matter is scattered away leaving little to disrupt the binary system. This mechanism, proposed by Quinlan (1996) and others, is effective on a scale of hundreds of parsecs. However, dense central clusters are found around massive black holes, so this process cannot be universal. Lauer et al. (2002) have recently identified several galaxies where central luminosity profiles are seen with cores of ~ 20 pc. It seems likely, then, that well before any significant accretion could take place, much of the central matter would have already been removed by the formation of a hard supermassive black hole binary. This binary system would then decay, in the usual way, via gravitational radiation.

The mutual scattering mechanism can, however, also be applied to infalling massive satellites. A small bound system would survive right into the centre but be disrupted well before the formation of a binary. It seems unlikely, however, that the satellite would remain bound as close as 10 pc to a supermassive black hole. The main point though is that, *even if this were the case*, the total mass accreted would still be a tiny fraction of the central black hole's starting mass and would affect a CDM cusp only on the scale of a few parsecs. It seems that supermassive black holes cannot alter a central dark matter distribution on the kpc scale required to align CDM theory predictions with data from rotation curves.

Finally, removing the constraint of spherical symmetry is unlikely to drastically alter our results. Although the model we present is rather crude, the magnitude of the accretion affect is found to be so small, that it is hard to imagine how a change in initial assumptions could lead to significant accretion. While it is true that particles lying on highly eccentric orbits are more likely to be accreted, the effect of strong dynamical friction as required in our model is likely

to circularize the orbits of the stars and dark matter, reinforcing our initial assumptions (Polnarev & Rees 1994).

This seems to contradict the value cited by ZHR of up to 20–40 per cent of the central black hole's mass being comprised of dark matter, however the reason for this discrepancy is clear. ZHR look at the effect of the diffusion of dark matter and stars into the loss cone via relaxation, whereas we look at refilling the loss cone by scattering from massive infalling bodies. As mentioned above, if the loss cone refilling rate is significant, then all of the dark matter within 1 kpc could be swept up leading to a significant fraction of a black hole's mass comprising of dark matter. In previous literature (see e.g. Lightman & Shapiro 1977) the diffusion rate of matter into the loss cone was calculated by making three main assumptions as follows.

- (i) The supermassive black holes are fed from shallow cores.
- (ii) The stellar distribution is spherical or axisymmetric and angular momentum is conserved.
- (iii) Star–star relaxation is the dominant process for repopulating the loss cone.

ZHR question these three assumptions in their paper and show that, the loss cone refilling rate could be significantly higher than previously thought. They then go on to add that if there were no depletion of orbits for the dark matter, that is, if the loss cone were kept continually filled then some 20–40 per cent of the mass of a supermassive black hole could be composed of dark matter. While this is true, it is somewhat misleading.

The relaxation time for N particles in a collisionless system, assuming that each particle moves in the *mean* potential of all the others, is given by (Binney & Tremaine 1987):

$$t_{\text{relax}}(r) = \frac{N}{8 \ln N} \left[\frac{r^3}{GM(r)} \right]^{1/2}, \quad (38)$$

where $M(r)$ is the mass interior to radius r , and N is the number of particles.

Now, given that the number of dark matter particles must be very, very large, and because $t_{\text{relax}} \propto N$, the dark matter–dark matter relaxation time will be very much longer than the age of the Universe. However, a dark matter particle moving in the granular field of many stars will behave in the same way a particle of any mass moving through the same field. As such, the dark matter, even if cold and with very low self-interaction cross-section, can be relaxed in the same way as the stars *by the stars*.

The question is, at what radius does the relaxation time for stars and dark matter fall below a Hubble time? If we simply model a galaxy as an exponential disc of stars, a central massive black hole and a power-law dark matter profile, then the relaxation time is given by

$$t_{\text{relax}}(r) = \frac{N_*}{8 \ln N_*} \left[\frac{r^3}{G[M_{\text{bh}} + M_{\text{b}}(r) + M_{\text{dm}}(r)]} \right]^{1/2}, \quad (39)$$

where M_{bh} is the black hole mass, $M_{\text{b}}(r)$ is the mass distribution of the baryons and M_{dm} is the mass distribution of the dark matter. We can approximate $N_* \sim M_{\text{b}}(r)/M_\odot$ and so, for given mass profiles and a given central black hole mass, the relaxation time can be found as a function of radius. For a Milky Way type galaxy, the relaxation time is only less than a Hubble time within 2 pc of the central black hole. The total matter out to this radius is not a significant fraction of the start mass of the central black hole. ZHR argue that the relaxation time will be less than given in equation (39) as a result of the relaxation of assumptions (i)–(iii) above. They present some

plausible arguments that could reduce the relaxation time and then state that, should it be sufficiently low that the loss cone can be kept refilled out to the sphere of influence of the black hole, then 20–40 per cent of the mass of a central supermassive black hole could comprise dark matter. It is not clear, however, whether the relaxation time really could be low enough to produce significant loss cone refilling. Even if it were, the radius to which the effect can be considered important must be less than the sphere of influence of the black hole, and so cannot be important for producing dark matter cores on the scale of a kpc or so – the kind of core which would be required to reconcile measurements from rotation curves with CDM theory. Finally, they do not consider the effect of mergers on the central matter profile. As stated above, a merging binary black hole can eject much of the central matter from a galaxy on the scale of 10–100 pc or so. Even if the relaxation time at 100 pc were quite low, it is likely that the matter in this region would be removed by merging black holes before it would have the chance to be accreted – either by diffusion due to the coarse grained stellar potential, as suggested by ZHR, or by scattering from merging supermassive black holes as suggested here.

5 CONCLUSIONS

We have shown that supermassive black holes cannot alter a power-law density cusp via accretion, whether during mergers or in the steady state. This represents another vital piece in the debate surrounding the nature of dark matter. If winds and bars also prove to be ineffective at altering a primordial dark matter cusp, we will be drawn inexorably towards the conclusion that dark matter is more complex than the simple cold particle envisaged so far.

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APPENDIX A: THE VELOCITY DISPERSION IN A POWER-LAW DENSITY PROFILE

For spherically symmetric, isotropic, systems, we have (Binney & Tremaine 1987)

$$\frac{1}{\rho} \frac{d(\rho \langle v^2 \rangle)}{dr} = -\frac{v_c^2}{r}. \quad (\text{A1})$$

If the density, $\rho(r)$, is given by equation (16), then v_c^2 is given by equation (17), and (A1) becomes

$$-\alpha r^{-1} \langle v^2 \rangle + \frac{d\langle v^2 \rangle}{dr} = \frac{-4\pi G \rho_0 r_0^\alpha}{3 - \alpha} r^{1-\alpha} \quad (\text{A2})$$

⇒

$$\begin{aligned} \langle v^2 \rangle &= \frac{2\pi G \rho_0 r_0^\alpha}{(3 - \alpha)(\alpha - 1)} r^{2-\alpha} \quad 1 < \alpha < 3 \\ &= -2\pi G \rho_0 r_0^2 x \ln x \quad \alpha = 1 \\ x &= \frac{r}{r_0}. \end{aligned} \quad (\text{A3})$$

As the power-law density is describing the *inner* parts of the halo, we require $r < r_0$ always, and so $x < 1$. Thus $\ln x$ will always be negative such that $\langle v^2 \rangle$ is still positive for $\alpha = 1$. Thus, substituting everywhere for v_c^2 (see equation 17), we get

$$\begin{aligned} \langle v^2 \rangle &= \frac{v_c^2}{2(\alpha - 1)} \\ &\simeq v_c^2 \quad 1 < \alpha < 3. \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \langle v^2 \rangle &= -v_c^2 \ln x \\ &\simeq v_c^2 \quad \alpha = 1. \end{aligned} \quad (\text{A5})$$

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