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CAN THE PRODUCTION SMOOTHING MODEL OF INVENTORY BEHAVIOR BE SAVED?

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ABSTRACT

The production smoothing model of inventory behavior has a long and venerable history, and theoretical foundations which seem very strong. Yet certain overwhelming facts seem not only to defy explanation within the production smoothing framework, but actually to argue that the basic idea of production smoothing is all wrong. Most prominent among these is the fact that the variance of detrended production exceeds the variance of detrended sales.

This paper first documents the stylized facts. Then it derives the production smoothing model rigorously and explains how the model can be amended to make it consistent with the facts. Next, estimates of stock adjustment equations derived from the theory are presented and evaluated. Finally, it reviews the theoretical and empirical evidence and tries to draw some tentative conclusions.

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I. INTRODUCTION

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The production smoothing model of inventory behavior has a long and venerable history. <1> Its theoretical foundations seem very strong. All that is necessary to create a production- smoothing motive for holding inventories is that demand vary through time and that the short-run cost function be convex (i.e., the short-run production function be concave). If, in addition, there is a random element to demand, inventories will also serve as a buffer stock.

These conditions appear to be very weak -- so weak, in fact, that it is hard to imagine how they could fail to hold. In addition, the production smoothing model has been used with some success in empirical work on inventories. <2> Under the special assumption that costs are quadratic, it leads to the "partial adjustment" model that dominates empirical work on the subject. <3>

Yet the production smoothing model is in trouble. Certain overwhelming facts seem not only to defy explanation within the production smoothing framework, but actually to argue that the basic idea of production smoothing is all wrong. To elucidate these facts, consider the following accounting identity:

$$(1.1) \qquad Y_t = X_t + N_{t+1} - N_t ,$$

where Y is production, X is sales, and N_t is the stock of inventories at the beginning of period t. If lower case symbols are used to denote the detrended values of the corresponding upper case symbols, this identity leads to the following decomposition of the variance of Y about trend:

$$(1.2) \quad \mathrm{var}(y) = \mathrm{var}(x) + \mathrm{var}(\Delta n) + 2\mathrm{cov}(x,\Delta n).$$

The first two facts of interest pertain to equation (1.2).

FACT 1: The variance of detrended production exceeds the variance of detrended sales: var(y) > var(x).

If firms use inventories to smooth production in the face of fluctuating sales, it is surprising indeed that production is more variable than sales. This remarkable fact has been known for a long time at the aggregate level, where y stands for real GNP and x stands for real final sales. <4> In Blinder (1981), I showed that var(y) exceeds var(x) for retailing as a whole and for 7 of 8 two-digit retail industries. In Section II, I show that production is more variable than sales in manufacturing as a whole and in 18 of the 20 two- digit manufacturing industries. Thus the finding that var(y) > var(x) seems to hold quite generally. Recently, West (1983a) has used a more elaborate version of this inequality to derive a test of the validity of the production smoothing model in several nondurable manufacturing industries -- with mostly negative results.

FACT 2: The covariance between sales and inventory change is not negative.

If inventories are used to buffer output against shocks to demand, then inventories should fall when sales spurt and rise when sales slump. In fact, the covariance between sales and inventory change is strongly positive for GNP as a whole (that is, inventory investment is strongly procyclical) and weakly positive in the retail sector. <5> I show in Section II that there are only a few manufacturing industries in which cov(x, Δn) is substantially negative. If inventories play a buffer stock role at all, it must be swamped by other considerations. <6>

FACT 3: When a partial-adjustment inventory equation of the form:

$$(1.3) \quad N_{t+1} - N_t = \beta_1 (N^*_{t+1} - N_t) - (1-\beta_2)(X_t - {}_{t-1}X_t) + u_t ,$$

where ${}_{t-1}X_t$ is expected sales, N^*_{t+1} is (some proxy for) desired inventories, and u_t is a stochastic disturbance term, is estimated, the estimated β_1 normally turns out to be quite low while the estimated β_2 turns out to be quite high.

This, of course, is not a "fact" like the others, but depends on estimation techniques, etc. That this finding is troublesome to the production smoothing model can be understood best by using identity (1.1) to write (1.3) as an equation for output:

$$(1.4) \quad Y_t = {}_{t-1}X_t + \beta_1 (N^*_{t+1} - N_t) + \beta_2 (X_t - {}_{t-1}X_t) + u_t .$$

According to equation (1.4), production deviates from expected sales according to how much inventory change is desired -- the term $\beta_1(N^*_{t+1} - N_t)$ -- and how much sales deviate from what was expected -- the term $\beta_2(X_t - {}_{t-1}X_t)$. If cost conditions dictate a great deal of production smoothing, it would seem that both β_1 and β_2 should be low, i.e., output should not react much to either inventory discrepancies or unanticipated sales fluctuations. On the other hand, if cost conditions dictate relatively little smoothing, then both β_1 and β_2 should be high. Empirically, however, we find small β_1 and large β_2, which seems hard to reconcile with the theory. <7>

Taken as a whole, these facts add up to a stunning indictment of the production smoothing/buffer stock model. Yet, as I indicated at the outset, the theory that underlies this model requires little more than a concave short-run production function. There seems to be more than a little tension here between theory and fact.

There are several ways to resolve this dilemma. In my earlier paper on retail inventories (Blinder (1981)), I suggested that the technology of the retail firm is not in fact concave, and nominated the (S,s) model as a replacement for the production smoothing model. This model, which is based on a fixed cost of placing and receiving an order, has little trouble accounting for the stylized facts. Furthermore, I showed that, under certain restrictive assumptions, it leads to an estimating equation that is very similar to the stock adjustment model.

Since the stylized facts of inventory behavior in manufacturing are so similar to those in retailing, it is tempting to adopt the same explanation for manufacturing. And it might even be correct. But I am hesitant to do so for several reasons.

The first is that the basic technological assumption that underlies the (S,s) model is far less appealing on a priori grounds for manufacturers than it is for retailers. The cost function that makes (S,s) inventory behavior optimal is:

$$\begin{aligned} C(Y) &= A + mY \qquad \text{if } Y>0 \\ &= 0 \qquad\qquad\ \ \text{if } Y=0, \end{aligned}$$

where A and m are positive constants. For retailers, A is the fixed cost and m is the (constant) marginal cost at which they can purchase goods from manufacturers. For manufacturers, such a cost structure connotes a substantial set-up cost followed by constant marginal costs thereafter. While this may be an appealing description of costs for some industrial processes (where production in large batches is optimal), it is far from obvious that it typifies manufacturing technology.

Second, the (S,s) model requires that sales be beyond the firm's control, which is quite hard to swallow in many oligopolistic manufacturing industries. (Think, for example, of automobiles, steel, and chemicals.)

Third, if we give up the assumption that the production function is concave, we give up much of neoclassical economic thought in the bargain. Of course, if production functions really are convex, then so much the worse for neoclassical economics. Theory must be bent to fit facts, not the other way around. My point is simply that abandoning concavity is not something that should be taken lightly. <8>

In fact, one way to interpret this paper is as a last-ditch effort to save the assumption that the (short-run) production function is concave from the scrapheap of discarded doctrines. I leave it to the reader to decide whether the rescue mission was successful.

The paper is organized into four sections. Section II is factual: it documents Facts 1 and 2 and several others as well. Section III is theoretical: it derives the production smoothing/ buffer stock model rigorously and explains how the model can be made consistent with the facts. Section IV is econometric: estimates of stock adjustment equations derived from the theory are presented and evaluated. Finally, Section V is impressionistic: it reviews the theoretical and empirical evidence and tries to draw some tentative conclusions. But these are certainly more tentative than conclusive.

II. THE STYLIZED FACTS ABOUT MANUFACTURING INVENTORIES

The empirical parts of this paper study monthly, seasonally adjusted data on sales and inventories in billions of 1972 dollars, provided by the Bureau of Economic Analysis (BEA). Inventories are broken down by stage of processing (materials and supplies, works in progress, and finished goods) and by industry (10 durable sectors and 10 nondurable sectors). The period of study runs from February 1959 (the first month for which opening stocks of inventories are available) through December 1981.

Before looking at the data, it is wise to get some accounting identities straight. Figure 1 will help. In this schema X denotes sales, F denotes the stock of finished goods, and W denotes the stock of works in progress (henceforth "works"). It indicates that:

(1) Items that are started within the period might be counted as works in progress (path a), finished goods (path b), or shipments to customers (path c) by the end of the period.

(2) Items that began the period as works in progress might still be in progress, or might be recorded as finished goods (path d), or as sales (path e) by the end of the period.

(3) Items that started the period as finished goods are either sold within the period (path f) or remain in inventory.

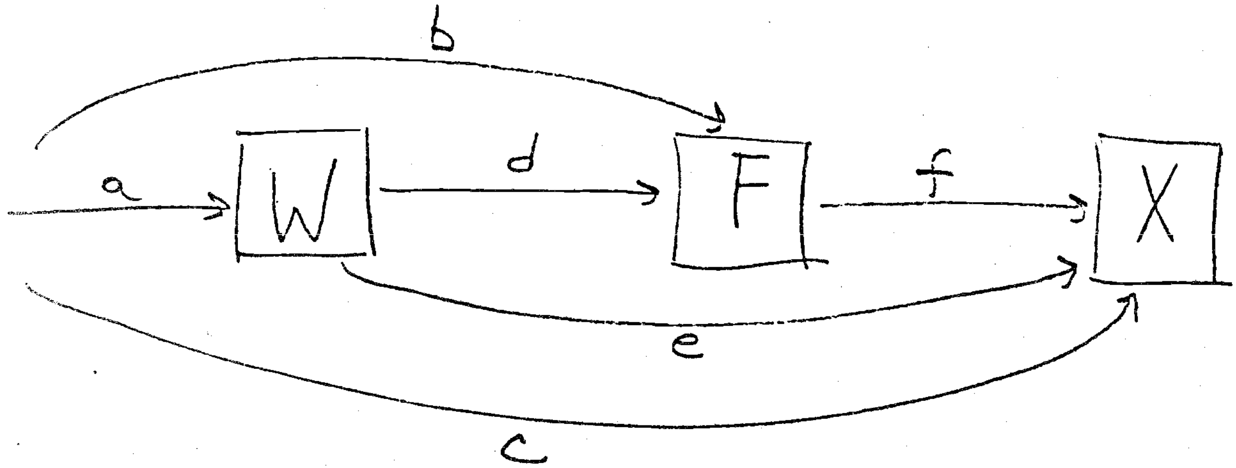

Figure 1
Stages of Processing

Adding up these possibilities (using obvious notation), we see that sales are given by:

$$(2.1)\quad X = c + e + f,$$

while a natural definition of production is:

$$(2.2)\quad Y = a + b + c.$$

Similarly, the change in the stock of finished goods is:

$$(2.3)\quad \Delta F = b + d - f,$$

and the change in the stock of works in progress is:

$$(2.4)\quad \Delta W = a - d - e.$$

Adding up (2.3) and (2.4) gives:

$$\Delta F + \Delta W = a + b - (e + f),$$

which, according to (2.1) and (2.2), is exactly equal to Y-X. So we see that the concept of inventories that satisfies the identity

$$(1.1)\qquad N_{t+1} - N_t = Y_t - X_t,$$

is the sum of finished goods plus works in progress. I will henceforth denote this sum by the symbol N, in accord with (1.1).

Data on shipments and inventories of finished goods and works in progress were used, in conjunction with identity (1.1) to create a series on production for each industry. <10> Then all the series were detrended by the following model of the trend component:

$$\log(Z_t) = a_0 + a_1\text{TIME} + a_2\text{DTIME} + a_3 \ \ \text{D66} + u_t,$$

where TIME is a time trend beginning at 1 in January 1959, DTIME is a second time trend beginning at 1 at the first OPEC shock (October 1973), and D66 is a dummy variable equal to 1 for all observations in 1966-1982. (D66 is motivated by a data revision that went back only to 1966.) To get more efficient estimates of the trends, estimation was by generalized least squares with u assumed to follow a second-order autoregressive scheme. <11> This is exactly the same procedure I used earlier on the retailing data (Blinder (1981)), which facilitates comparisons.

With these definitions understood, Table 1 shows the decomposition of the variance of detrended production as in equation (1.2). <12> A number of conclusions are apparent.

First, and most important, the variance of production is generally larger than the variance of sales, and sometimes much larger. Primary metals is the only industry in which sales has a bigger variance that production. The ratio of var(y)/var(x) ranges from a high of 2.40 to a low of 0.95, and is 1.14 for manufacturing as a whole. (The corresponding ratio for retailing was 1.15.)

Second, notice from (1.2) that var(y) cannot possibly be less than var(x) unless the covariance between x and Δn is negative enough to overwhelm the variance of Δn. In the durables sector, this covariance is negative in only 2 of 10 industries; and the only nontrivial negative value occurs in primary metals (where $\rho=-.22$). By contrast, large positive covariances are found in electrical machinery ($\rho=.33$), non-electrical machinery ($\rho=.45$), and transportation equipment

Table 1

Summary of Variances and Covariances

Sector	(1) var(y)	(2) var(x)	(3) var(Δn)	(4) 2cov(xΔn)	(5) ρ(x,Δn)	(6) var(y)/var(x)	(7) var(Δn)/var(x)
All Manufacturing	10.22	8.90	.177	.999	.40	1.14	.020
Durable Goods	6.23	5.21	.147	.775	.44	1.20	.028
Primary metals	.244	.257	.011	-.024	-.22	.95	.042
Fabricated metals	.146	.131	.012	.0049	.06	1.11	.092
Electrical machinery	.197	.163	.0097	.026	.33	1.21	.060
Non-elect. machinery	.230	.154	.021	.051	.45	1.49	.138
Transportation equip.	.802	.657	.041	.096	.29	1.22	.063
Lumber & Wood Products	.0146	.0130	.0019	-.00027	-.03	1.12	.148
Furniture & Fixtures	.0046	.0036	.00064	.00032	.10	1.27	.176
Stone, Clay, & Glass Products	.0098	.0086	.00103	.00015	.02	1.14	.120
Instruments & Related Products	.0096	.0057	.0030	.0012	.14	1.69	.537
Miscellaneous Manufacturing Industries	.0037	.0025	.00093	.00017	.06	1.46	.371
Nondurable Goods	0.728	0.694	.032	.0029	.01	1.05	.046
Food & Kindred Prods.	.0449	.0365	.0108	-.0025	-.06	1.23	.296
Tobacco Manufacturing	.00133	.00056	.00078	.000005	.00	2.40	1.405
Textile Mill Products	.0134	.0124	.0012	-.00064	-.08	1.08	.098
Apparel Products	.0208	.0149	.0042	.0015	.09	1.40	.283
Leather & Leather Products	.00130	.00097	.00029	.000056	.05	1.34	.300
Paper & Allied Prods.	.00958	.00917	.00051	-.000078	-.02	1.04	.056
Printing & Publishing	.0162	.0136	.0020	.00014	.01	1.18	.149
Chemicals & Allied Products	.0538	.0522	.0048	-.0030	-.09	1.03	.092
Petroleum & Coal Prods.	.0207	.0207	.0016	-.0012	-.11	1.00	.078
Rubber & Plastic Prods.	.0181	.0162	.0010	.00066	.08	1.12	.064

(ρ=.29). Thus Fact 2 in the introduction holds particularly strongly in the durables sector. Things are more mixed in the nondurables sector: the covariance is positive half the time and negative half the time, but generally of trivial magnitude.

Believers in a buffer stock role for inventories will raise several questions about this finding. First, recall that:

$$\text{cov}(x,\Delta n) = \text{cov}(x,\Delta f) + \text{cov}(x,\Delta w).$$

Could it be that a negative covariance between sales and changes in finished goods inventories (evidence for a buffer stock role for inventories) is hidden by an even stronger positive covariance between sales and changes in works in progress? Regrettably, the answer is no. The correlation between x and Δf is negative in only 7 of 20 industries, and more negative than -.10 in only 3 industries.

Second, would the buffer stock role of finished goods inventories look more important if we replaced the deviation of sales from trend by the change in sales, or by unexpected sales? Only a little. Cov (Δx,Δf) is negative in only 9 of 20 industries. The same is true of cov(x_t^u, Δf), where x_t^u is a proxy for unexpected sales explained later in the paper.

Once the disaggregation of inventories by stage of processing is brought up, several additional questions arise. Tables 2 and 3 address some of these questions.

Which of the three types of inventory is most important quantitatively? As Tables 2 and 3 show, there are some systematic differences between durable and nondurable

Table 2

Inventories by Stage of Processing: Means

Sector	Mean Inventory Investment (1) Finished	(2) Works	(3) Materials	Mean Inventory/Sales Ratio (4) Finished	(5) Works	(6) Materials
All Manufacturing	.084	.108	.096	.60	.60	.65
Durable Goods	.047	.095	.065	.57	.93	.71
Primary metals	.0037	.0098	.0088	.65	.77	.92
Fabricated metals	.0040	.0078	.0068	.57	.80	.97
Electrical machinery	.0077	.0212	.0120	.56	1.02	.64
Non-electrical machinery	.0171	.0256	.0179	.82	1.28	.78
Transportation equipment	.0046	.0216	.0069	.21	1.16	.46
Lumber & Wood Products	.0012	.0009	.0022	.69	.38	.60
Furniture & Fixtures	.0014	.0014	.0016	.58	.48	.92
Stone, Clay, & Glass Products	.0024	.0009	.0031	.82	.22	.56
Instruments & Related Products	.0038	.0048	.0042	.67	1.06	.69
Miscellaneous Manufacturing Industries	.0021	.0010	.0015	.96	.60	.80
Nondurable Goods	.036	.013	.031	.63	.21	.57
Food & Kindred Products	.0084	.0023	.0059	.62	.09	.38
Tobacco Manufacturing	.0002	.0001	.0007	.42	.09	5.27
Textile Mill Products	.0026	.0020	.0015	.71	.52	.59
Apparel Products	.0041	.0013	.0038	.70	.31	.55
Leather & Leather Prods.	.0003	-.0001	-.0002	.73	.40	.60
Paper & Allied Products	.0041	.0009	.0047	.48	.15	.73
Printing & Publishing	.0023	.0012	.0024	.36	.24	.40
Chemicals & Allied Prods.	.0106	.0027	.0081	.74	.22	.55
Petroleum & Coal Prods.	.0013	.0009	.0012	.72	.29	.31
Rubber & Plastic Prods.	.0024	.0013	.0030	.84	.25	.53

Table 3

Inventories by Stage of Processing: Variances and Covariances

Sector	Variances of Inventory Investment (1) Finished	(2) Works	(3) Materials	Correlation Coefficients (4) ρ_{FW}	(5) ρ_{FM}	(6) ρ_{WM}
All Manufacturing	.036	.047	.058	.09	.03	.15
Durable Goods	.016	.044	.040	.17	.03	.14
Primary metals	.0028	.0021	.0023	.43	.04	.03
Fabricated metals	.0014	.0031	.0050	.28	-.29	-.22
Electrical machinery	.0011	.0021	.0017	.25	.35	.35
Non-electrical mach.	.0024	.0049	.0024	.16	.05	.24
Transportation equip.	.0027	.0193	.0110	-.00	.30	-.23
Lumber & Wood Products	.0077	.00031	.00054	.08	-.06	-.08
Furniture & Fixtures	.00012	.00012	.00015	.21	-.01	-.17
Stone, Clay & Glass Products	.00048	.00006	.00017	.13	-.19	.11
Instruments & Related Products	.00036	.00056	.00045	.01	-.03	-.25
Miscellaneous Manufacturing Industries	.00026	.00011	.00017	.21	-.03	.16
Nondurable Goods	.016	.0019	.010	.11	-.07	.02
Food & Kindred Prods.	.0070	.00038	.0042	.06	-.03	.04
Tobacco Manufacturing	.000036	.000030	.00092	-.01	-.35	.01
Textile Mill Products	.00056	.00018	.00045	.09	-.18	.01
Apparel Products	.0016	.00045	.0011	.22	-.21	-.12
Leather & Leather Prods.	.00013	.000029	.000055	.09	-.12	.09
Paper & Allied Prods.	.00029	.000077	.00033	-.22	-.03	-.06
Printing & Publishing	.00037	.00035	.00046	.01	.04	.08
Chemicals & Allied Prods.	.0017	.00031	.00082	.14	-.12	.04
Petroleum & Coal Prods.	.00092	.00016	.00016	.06	.04	-.01
Rubber & Plastic Prods.	.00043	.000069	.00026	.16	-.11	.11

industries in this respect. <13>

Over the period 1959-1981, 6 of the 10 durables industries did more investment in works in progress than in any other type of inventory; 3 of the remaining 4 industries invested most in materials and supplies. (The largest number in each line is underlined.) However, if we look at stocks instead of flows, that is, at inventory/sales ratios, the picture is more mixed. Inventory holdings are mostly stocks of works in progress in 4 industries, mostly finished goods in 3, and mostly materials and supplies in the remaining 3.

In the nondurables industries, works in progress inventories are rather unimportant. Six of the 10 nondurables sectors did most of their inventory investment in the form of finished goods, and the others concentrated on materials and supplies. This picture changes but slightly if we focus on stocks (inventory/sales ratios) rather than on flows.

Variances, rather than means, are more important for business cycle analysis. Table 3 continues to show that inventory investment in works in progress is quite unimportant in nondurable goods industries, but quite important in durables. (In each line of Table 3, the largest of the three variances is underlined.) However, it is noteworthy (given this paper's concentration on finished goods) that inventories of finished goods look more important in the durables industries when we consider variances instead of means.

It is natural to ask whether the different types of inventories display different behavior or tend to march in lock step with one another. For this purpose, Table 3 displays correlation coefficients among the three types of inventory change. The major conclusion, I think, is that these numbers are very small; each type of inventory movement seems to have a life of its own.

Changes in finished goods and changes in works in progress typically are positively correlated; but the correlation is large only in the primary metals industry. Few other generalizations can be made. Changes in finished goods and changes in materials and supplies are negatively correlated in 14 of 20 sectors, but the strongest correlations are positive (in electrical machinery and transportation equipment). Changes in works and materials are nearly orthogonal in the nondurable industries, but show mixed results in the durable industries.

Before leaving these data, one further fascinating observation should be made. Look at the second column of Table 1. While the variance of sales in the durables sector is 5.21, the sums of the variances in the 10 industries that comprise this sector is only 1.40. The remaining 3.81 -- or three-quarters of the total variance -- is accounted for by the covariances among the sales of the 10 industries. Much the same pattern holds in the nondurables sector, where the variances of sales of the 10 industries sum to only .177 while the variance of nondurable sales as a whole is .694. This domination by the covariances suggests, but does not prove, that there is a dramatic common "business cycle" element in the sales of the various manufacturing industries. <14>

To what conclusions are we led by this quick perusal of the facts? First, the production smoothing/buffer stock model looks dubious at best. Consequently, the next section is devoted to modifying the theory so as to make it more consistent with the facts.

Second, all three types of inventories appear to make independent contributions to the variance of total inventory investment. This suggests seeking separate empirical explanations for each type of inventory. The empirical work reported in Section IV deals only with finished goods; research on the other two types of inventory investment would be worthwhile.

Third, there are enough differences across industries in Tables 1-3 to make aggregation look hazardous; hence I adopt a disaggregated approach to the empirical work in Section IV.

Fourth, most of the variance in manufacturing sales and output is contributed by pervasive positive covariances among the sales and output of component industries, which suggests a very strong common business cycle element in U.S. manufacturing industries.

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III. THE THEORY OF PRODUCTION SMOOTHING

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1. Concepts and Notation

The model used here is a generalization of Blinder (1982), and uses the notation employed there. Specifically, consider a value- maximizing firm with linear demand curve:

$$(3.1) \quad X_t = 2d_0 - 2dP_t + 2d\,\eta_t,$$

where P_t is the price in period t and X_t is the quantity sold. The demand shock, η_t, has a complex structure that will be specified presently.

The firm is assumed to have quadratic costs of production:

$$(3.2) \quad C(Y_t) = c_0 + (c_1 + \Gamma_t)Y_t + (1/2c)Y_t^2 \quad ,$$

where Y_t is output and Γ_t is a cost shock representing stochastic disturbances to either technology or factor prices. The curvature parameter c is critical to the production smoothing issue. A low value of c connotes a steeply increasing marginal cost curve, and hence a strong motive to smooth production.

Similarly, the firm is assumed to have quadratic costs of holding inventories:

$$(3.3) \quad B(N_t) = b_0 + b_1N_t + (b/2)N_t^2 \quad ,$$

where N_t is the stock of inventories at the beginning of period t. The model treats only inventories of finished goods, which is why the empirical work is restricted to this type of inventory. The curvature parameter b is again critical to the production smoothing issue. A large value of b makes it costly to vary inventories, and hence will discourage the firm from using inventory movements to smooth production.

If $D=1/(1+i)$ is the discount rate, the firm seeks to maximize:

$$(3.4) \qquad E_t \sum_{s=0}^{\infty} D^s \{ X_{t+s} P_{t+s} - C(Y_{t+s}) - B(N_{t+s}) \},$$

where X, C(Y), and B(N) are as given above. Before solving the problem, it is useful to define production smoothing precisely, because that is the central issue of the paper. One obvious definition is:

DEFINITION 1: A firm is said to smooth production if the (unconditional) variance of production is less than the (unconditional) variance of sales: $\mathrm{var}(Y) < \mathrm{var}(X)$.

For obvious reasons, I call this "long-run production smoothing." Fact 1 in the introduction can be interpreted as saying that firms do not smooth production by this definition.

For "short-run production smoothing," I offer two definitions. The first is more useful in empirical applications, while the second is easier to work with theoretically.

DEFINITION 2: A firm is said to smooth production if, in response to a positive (negative) demand shock, production rises (falls) less than sales so that inventories decline (rise):

$$\frac{\partial(Y_t - X_t)}{\partial \eta_t} < 0 .$$

DEFINITION 3: A firm is said to smooth production if its production responds less to a sales shock than it would if it could not carry inventories:

$$\frac{\partial Y_t}{\partial \eta_t} < \left(\frac{\partial Y_t}{\partial \eta_t}\right)^*$$

where the * denotes a firm that cannot carry inventories.

If either inequality is reversed, I will say there is production "bunching" instead. It is easy to show that if production is smoothed by Definition 3, then it is also smoothed by Definition 2. But, the converse does not hold.

2. The Informational Structure

Because expectations are assumed to be rational, the information structure is critical to the solution. I assumed that the firm observes its cost shock for period t before making its decisions on production and price for period t. This seems a natural specification if cost shocks represent fluctuations in input prices, but not so natural if they represent stochastic aspects of the technology.

For the demand shock, I employ a general structure that admits of several interpretations. Specifically, the demand shock is assumed to have two independent components:

$$(3.5)\quad \eta_t = \eta_t^1 + \eta_t^2$$

which differ only in that the firm can observe η_t^1 , but not η_t^2, before it makes its decisions on P_t and Y_t . The idea is that what the econometrician, using monthly or quarterly data, labels as "unanticipated sales" is only partly unanticipated by the firm, which actually knows η_t^1.

Two polar cases are evident. If the η_t^2 shock is absent, then the firm knows its demand curve before making its decisions and the sales "surprise" is a surprise only to the econometrician. If the η_t^1 shock is absent, the firm must make its decisions before it knows its demand curve, and sales surprises really are surprises. <15> The distinction between these two versions of "unanticipated" sales turns out to be critical to reconciling the theory with the data. I interpret the empirical evidence as suggesting that η_t^1 shocks are far more important than η_t^2 shocks.

To recapitulate briefly, the firm inherits an opening stock of inventories (N_t) which is the legacy of the past. It then observes its cost shock (Γ_t) and part of its demand shock (η_t^1) before choosing its level of production (Y_t), price (P_t), and expected sales. After these decisions are made, the rest of the demand shock (η_t^2) is observed and actual sales (X_t) are determined. The beginning-of-period inventory for period t+1 then follows from the identity:

(3.6) $N_{t+1} = N_t + Y_t - X_t$,

and the whole process repeats.

3. The Solution: Optimal Inventory Policy

Details of the solution are presented in a lengthy mathematical appendix available on request. Here I confine myself to establishing the notation and stating the results. First, for the variables X_{t+s}, Y_{t+s} , N_{t+s}, and P_{t+s}, let lower case symbols denote the expectations of the corresponding upper case symbols. All expectations are conditional on the information available when the period t decision is made. For example:

$$y_{t+s} = E_t Y_{t+s},$$

where the information set available at time t includes N_t, Γ_t, η^1_t, and all variables dated t-1 or earlier. Similar definitions hold for x_{t+s}, p_{t+s}, and n_{t+s}. Analogously, let the new symbols:

$$\gamma_{t+s} = E_t \Gamma_{t+s}$$

$$i=1,2.$$

$$\varepsilon^i_{t+s} = E_t \eta^i_{t+s}$$

denote the expected values of the period t+s shocks.

If Q_t is defined as the shadow value of inventories, i.e., the costate variable attached to the dynamic constraint (3.6); q_{t+s} is the period t expectation of Q_{t+s}; and λ_{t+s} is the deviation of q_{t+s} from its nonstochastic steady state; then the appendix shows that the first-order conditions for maximizing (3.4) subject to the sequence of constraints (3.6) are as follows for s=0:

$$(3.7)\quad Y_t = \bar{Y} + c(\lambda_t - \Gamma_t)$$

$$(3.8)\quad x_t = \bar{X} + d(\varepsilon_t - \lambda_t).$$

$$(3.9)\quad \lambda_{t+1} - (1+i)\lambda_t = b(n_{t+1} - \bar{N}),$$

where $\bar{Y}$, $\bar{X}$, and $\bar{N}$ are respectively the nonstochastic steady state values of output, sales, and inventories (naturally, $\bar{Y}=\bar{X}$).

These first-order conditions have straightforward interpretations. Equation (3.7) equates marginal cost to the shadow value of inventories; equation (3.8) equates expected marginal revenue to the shadow value of inventories; and equation (3.9) states that the increase in the shadow value must just compensate the firm for both the interest costs and explicit holding costs of carrying inventories.

Notice that (3.7) is a decision rule for actual production, but (3.8) is a rule only for expected sales. Actual sales are related to expected sales by:

$$(3.10)\quad X_t - x_t = 2d(\eta_t - E_t\eta_t),$$

which follows from (3.1).

To make these rules operational, we must solve for the initial value of the shadow value of inventories, λ_t. This is a tedious calculation which is done in the appendix. The result is:

$$(3.11)\quad \lambda_t = \left(\frac{1-z_1}{c+d}\right)[\bar{N} - N_t + d(\varepsilon_t + F_t) + c(\Gamma_t + G_t)],$$

where z_1 is the stable root of the quadratic equation:

$$(3.12)\quad z^2 - [2+i+b(c+d)]z + (1+i) = 0 \text{ , and where:}$$

$$F_t = \sum_{s=1}^{\infty} \theta^s \varepsilon_{t+s}$$

$$G_t = \sum_{s=1}^{\infty} \theta^s \gamma_{t+s} ,$$

where $\theta \equiv 1/z_2$ and z_2 is the unstable root of (3.12). Clearly, the exact solution depends on the specific nature of the shocks.

Shortly, I will deal with a series of more particular cases, but to start I assume that both demand shocks are ARMA(1,1) processes:

$$\eta_t^i = \rho\eta_{t-1}^i + mv_{t-1}^i + v_t^i \qquad i=1,2$$

and the cost shock is AR(1):

$$\Gamma_t = r\Gamma_{t-1} + w_t \qquad .$$

Under these assumptions, the appendix shows that (3.11) becomes:

$$(3.13)\quad \lambda_t = \left(\frac{1-z_1}{c+d}\right) [\bar{N} - N_t + \frac{d}{1-\theta\rho} (\eta_t^1 + m\theta v_t^1 + \rho\eta_{t-1}^2 + mv_{t-1}^2) + \frac{c}{1-\theta r} \Gamma_t] .$$

Using this result in (3.7) and (3.8), the decision rules for optimal output and sales are:

$$(3.14)\quad Y_t - \bar{Y} = \frac{c(1-z_1)}{c+d} (\bar{N} - N_t) + \left(\frac{dc}{c+d}\right)\left(\frac{1-z_1}{1-\theta\rho}\right)[E_t\eta_t + m\theta v_t^1] - \left(1 - \frac{c}{c+d}\frac{1-z_1}{1-\theta r}\right)c\Gamma_t$$

$$(3.15)\quad X_t - \bar{X} = \frac{-d(1-z_1)}{c+d} (\bar{N} - N_t) + d\left(1 - \frac{d}{d+c}\ \frac{1-z_1}{1-\theta\rho}\right)E_t\eta_t - \frac{d}{d+c}\frac{1-z_1}{1-\theta\rho} dm\theta v_t^1 + 2dv_t^2$$
$$- \frac{cd}{c+d}\ \frac{1-z_1}{1-\theta r} \Gamma_t ,$$

where, it will be recalled:

$$E_t\eta_t = \varepsilon_t = \eta_t^1 + \rho\eta_{t-1}^2 + mv_{t-1}^2 \quad .$$

Together with the identity (3.6), these imply the following inventory investment equation:

$$(3.16)\quad N_{t+1} - N_t = (1-z_1)(\bar{N}-N_t) - d(\frac{1+i-\rho}{z_2-\rho})E_t\eta_t + d(\frac{1-z_1}{1-\theta\rho})m\theta v_t^1 - c(\frac{1+i-r}{z_2-r})\Gamma_t - 2dv_t^2 ,$$

where use has been made of the fact $z_1z_2=1+i$.

Equation (3.16) is the basis for the effort to reconcile the theory with the stylized facts that follows. It also provides the theoretical underpinning for the econometric work in Section IV.

4. Short-Run Production Smoothing

More apparatus is necessary before we can deal with long-run production smoothing, but we already know enough to study short-run production smoothing. First note that a firm with no inventories would solve a static profit- maximization problem and equate marginal cost:

$$MC_t = c_1 + (1/c)Y_t + \Gamma_t ,$$

to expected marginal revenue. Since the inverse demand curve is:

$$P_t = (d_0/d) - (1/2d)X_t + \eta_t,$$

expected marginal revenue is:

$$MR_t = (d_0/d) - (1/d)X_t + \eta_t .$$

Equating the two and noting that $Y_t = E(X_t)$ in the absence of inventories yields:

$$Y_t - \overline{Y} = (dc/(d+c))(\eta_t - \Gamma_t).$$

From Definition 3, a natural quantitative measure of the degree of production smoothing is:

$$S_y = 1 - \frac{\frac{\partial Y}{\partial \eta}}{(\frac{\partial Y}{\partial \eta})_*}$$

Using this formula and the above expression for Y, we find:

$$S_y = 1 - ((c+d)/cd)\frac{\partial Y_t}{\partial v_t} \quad .$$

Using (3.14), this is readily seen to be:

$$S_y = 1 - \frac{c+d}{cd}\left(\frac{dc}{c+d}\right)\frac{1-z_1}{1-\theta\rho}(1+m\theta) = \frac{z_1 - \theta\rho - m\theta(1-z_1)}{1-\theta\rho} \text{ for } v_t^1 \text{ type shocks;}$$

$$S_y = 1 \qquad \text{for } v_t^2 \text{ type shocks.}$$

Thus production smoothing is complete for truly unexpected sales, but production bunching is actually possible for the econometrician's version of "unexpected sales." Remembering that $\theta = 1/z_2$, we see that some smoothing will take place if and only if:

$$(3.17) \quad 1+i - \rho > m(1-z_1),$$

which must be true for AR(1) demand shocks (m=0), but can be false if m is large enough.

The inuition behind this result is as follows. Consider the implied moving average coefficients when the stochastic process is ARMA(1,1):

$$\eta_t = \rho\eta_{t-1} + mv_{t-1} + v_t .$$

We have:

$$\frac{\partial \eta_t}{\partial v_t} = 1$$

$$\frac{\partial \eta_{t+1}}{\partial v_t} = \rho + m$$

$$\frac{\partial \eta_{t+2}}{\partial v_t} = \rho(\rho + m)$$

$$\vdots$$

So if $\rho+m>1$, the response pattern "builds" at first before decaying.

Suppose a firm sees a positive value of v_t, connoting a good period for sales. If the shock is AR(1), the firm will expect the ensuing periods to also be good, but not quite as good as period t. Hence it has an incentive to sell out of inventory, i.e., to smooth production. However, if the shock is ARMA(1,1), the firm will expect next period to be even better than this period (if $\rho+m>1$). If $\rho+m$ exceeds 1 by enough (as defined by (3.17)), the firm will actually want to build inventories for future sale. So it will bunch, rather than smooth, production.

Naturally, a firm that smooths production will "bunch" sales, and conversely. It is easy to show that a smoothing measure for sales, S_x, that is defined analogously to S_y, is related to S_y by the simple formula:

$$S_x = -(d/c)S_y.$$

Equally naturally, a firm will smooth its price behavior only if it bunches its sales, that is, only if it also smooths its production. <16> In the model, a measure of price smoothing defined analogously to S_y is related to S_y by the simple formula:

$$S_p = (d/(2d+c))S_y \quad .$$

Thus condition (3.17) is pivotal to the firm's behavior. If it holds, as it must unless demand shocks have a strong moving- average component, the firm smooths both production and sales and plans to draw down inventories when demand is high. These reactions are just what we expect. But if (3.17) fails to hold, the firm's optimal behavior is counterintuitive. It smooths sales, not production (nor price), and plans to build inventories in periods when demand is unusually high.

With this analysis complete, we are now ready to address the four stylized facts mentioned in the introduction.

5. Fact 3: Puzzling Regression Estimates

I begin with Fact 3, which was that econometric inventory investment equations like:

$$(1.3) \quad N_{t+1} - N_t = \beta_1(N^*_{t+1} - N)_t) - (1-\beta_2)(X_t - {}_{t-1}X_t) + u_t$$

tend to produce low estimates of the parameter β_1 and high estimates of the parameter β_2 . It is useful to compare the theoretical equation (3.16) to the empirical specification (1.3).

Notice first that (3.16) does have the partial adjustment form assumed in (1.3). In the absence of shocks, inventory change is a fixed fraction of the gap between $\bar{N}$ and N_t. This fraction -- "the speed of adjustment" -- depends on the curvatures of the revenue and cost functions, and on the rate of interest (see (3.12)). Low estimated adjustment speeds, therefore, suggest a high value of z_1.

However, $\bar{N}$ is not the firm's desired inventory stock. A natural definition of the desired inventory stock, call it N^*_{t+1}, is the value of N_t that makes desired inventory change equal to zero, conditional on the information available at time t. Equating the expectation of the righthand side of (3.16) to zero, we can express N^*_{t+1} as a function of $\bar{N}$ and the stochastic shocks that are known at time t. If this definition of N^*_{t+1} is then substituted back into (3.16), a little algebra shows that (3.16) can be written as:

$$N_{t+1} - N_t = (1-z_1)\ (N^*_{t+1} - N_t) \quad - \quad (\eta_t - E_t\eta_t),$$

where (3.10) has been used to replace $2dv_t$ by unexpected sales.

Thus we have two distinct concepts of "target" or "desired" inventories. $\bar{N}$ is the steady state level; its value depends on cost parameters, the rate of interest, and the mean position of the demand curve. We may assume that it moves rather slowly through time and is quite insensitive to fluctuations in sales. By contrast, N^*_{t+1} is the current target. Because it responds to new information, it may well exhibit rapid swings from one period to the next. The distinction between $\bar{N}$ and N^*_{t+1} is reminiscent of Feldstein and Auerbach's (1976) notion that actual inventories adjust rapidly to their target, but that the target itself adjusts only slowly. The model therefore provides a rigorous justification for inventory behavior that is consistent with the spirit, but not the letter, of Feldstein and Auerbach's analysis.

Next consider the parameter β_2 in (1.3). The model offers two possible theoretical interpretations of "unexpected sales." The most natural definition is $2dv_t^2$ -- the difference between what the firm sells and what it expected to sell when it made its production and price decisions. By this definition, unexpected sales enters the inventory equation with a coefficient of exactly -1. However, the econometrician's version of "unexpected sales" is likely to include both v_t^1 and v_t^2. As noted earlier, the theoretical coefficient of a v_t^1 shock is negative if and only if (3.17) holds.

Putting all this together, we have the following potential explanation for Fact 3:

(a) Technological conditions produce a high value of z_1, that is, a slow speed of adjustment. This will occur if $b(c+d)$ is fairly small, which means either that inventory storage costs are nearly linear or that the marginal cost and/or marginal revenue schedules are quite steep.

(b) Much of what looks like unexpected sales to the econometrician is not actually unexpected by the firm, and condition (3.17) fails to hold. The empirical coefficient $1-\beta_2$ will then be a weighted average of -1 and the (positive) coefficient of a v_t^1 demand shock. Therefore, with v_t^1 shocks quantitatively more important than v_t^2 shocks, it would not be surprising to find a very small estimated value of $1-\beta_2$.

On this view, the coefficients of initial inventories and unexpected sales in an econometric inventory equation depend upon fundamentally different characteristics of the firm. The former depends on technology and demand parameters, while the latter depends principally on how quickly the firm learns about demand shocks. Accordingly, no relationship between the two coefficients can possibly refute the production smoothing model, in contradiction to the claim made by Feldstein and Auerbach (1976) and others.

6. Fact 1: Var(Y) Exceeds Var(X)

The remaining stylized facts mentioned in the introduction involve the unconditional variances and covariances of X, Y, and N. Some fairly tedious manipulations detailed in the appendix lead to the following expression for the shadow value of inventories:

$$(3.18)\quad (1-z_1L)\lambda_t = \left(\frac{1-z_1}{1-\theta r}\right)\left(\frac{c}{c+d}\right)\left(\frac{1-\theta rL}{1-rL}\right)w_t + \left(\frac{1-z_1}{1-\theta\rho}\right)\left(\frac{d}{d+c}\right)\left[\frac{1+m\theta-\{\theta\rho(1+m)-m(1-\theta)\}L}{1-\rho L}\right]v_t^1$$

$$+ \left(\frac{1-z_1}{1-\theta\rho}\right)\left(\frac{d}{d+c}\right)\left[\frac{2(1-\theta\rho)+\rho+m-\{2\rho-\theta\rho(\rho-m)\}L}{1-\rho L}\right]v_{t-1}^2 .$$

Given this expression for the shadow value, and equations (3.7) and (3.8) for Y_t and X_t, can the model ever rationalize the fact that var(Y) > var(X)? Clearly it can, if cost shocks are big enough. Intuitively, it seems clear that demand shocks tend to produce high variance in X while cost shocks tend to produce high variance in Y. More specifically, the appendix establishes the following:

PROPOSITION 1: In a simplified version of the model with no demand shocks and r=0 (serial correlation of the cost shock is an unnecessary complication for this purpose), var(Y) > var(X) for any admissible values of the parameters.

Hence cost shocks are always a potential explanation of Fact 1 if the variance of cost shocks is large enough relative to the variance of demand shocks. However, this explanation comes perilously close to assuming the conclusion, and for this reason is not very satisfying.

The remaining propositions, all proven in the appendix, pertain to a model in which cost shocks are absent.

PROPOSITION 2: In a simplified version of the model with no cost shocks and a serially independent demand shock which the firm sees before deciding on production, var(X) > var (Y) for any admissible values of the parameters. However, as z_1 approaches zero, the ratio var(X)/var(Y) approaches 1. <17>

Together, Propositions 1 and 2 provide a potentially more satisfying explanation of the fact that var(Y) > var(X). Suppose the technology makes z_1 very small, so that in the absence of cost shocks var(X) would be only slightly larger than var(Y). Then even relatively minor cost shocks could tip the balance and turn var(Y) > var(X).

The problem with this explanation is that the empirical evidence suggests rather slow adjustment speeds, that is, rather high values of z_1. According to the logic of Proposition 2, a high value of z_1 will leave var(X) substantially larger than var(Y). An alternative explanation of Fact 1 would be desirable. One is provided by the next proposition.

PROPOSITION 3: In a simplified version of the model with no cost shock and an AR(1) demand shock which is known before output is set, var(X) > var(Y) for any admissible parameter values. However, the ratio var(X)/var(Y) approaches 1 as the autoregressive parameter ρ approaches 1, that is, as demand shocks become permanent. This holds for any value of z_1. <18>

The intuition behind Proposition 3 is clear. As the stochastic structure of the demand shock gets closer to a random walk, demand disturbances become more permanent. Hence the firm is more likely to adjust its production fully.

Propositions 1 and 3 together provide a potentially better explanation of the fact that var(Y)>var(X). The speed of adjustment can be as slow as we please. But as long as ρ is close to unity, as it is empirically, var(Y) will be nearly as large as var(X). In that case, only minor cost shocks are necessary to make var(Y) greater than var(X).

For example, in a numerical example with z_1=.85, no cost shocks, d=c, b=.013c, and a zero rate of interest, var(Y)/var(X) is only 0.55 when ρ=.9. But if ρ gets as high as .98, var(Y)/var(X) rises to .95.

To summarize, a combination of a rapid speed of adjustment (i.e., low z_1) and high serial correlation in demand disturbances (i.e., high ρ) can leave var(Y) so close to var(X) that it takes only very minor cost shocks to make var(Y) > var(X).

7. Fact 2: Sales and Inventory Change Do Not Covary Negatively

The fact that cov(X,ΔN) is typically zero or positive is the hardest to deal with, because conflicting factors produce theoretically ambiguous results. The reader is spared the detailed analysis. Suffice it to say that a positive cov(X,ΔN) can be produced by cost shocks or by demand shocks that are seen before sales decisions are made and that "build" before decaying. Other types of demand shocks produce a negative cov(X,ΔN). Thus, cov(X,ΔN) can have either sign, depending on which types of shocks dominate.

8. The Stylized Facts: Summary

In summary, then, the production smoothing/buffer stock model seems compatible with with all three facts mentioned in the introduction under the following circumstances:

(a) Cost shocks are present, though they need not be large.

(b) Most demand shocks are seen by firms before they must make their production and pricing decisions.

(c) Demand shocks build before they decay. (ARMA(1,1) and AR(2) processes are simple examples.)

(d) Either the technology parameters dictate a rapid speed of adjustment or demand disturbances have strong positive serial correlation.

None of these requirements seem outlandish and, most importantly, none forces us to jettison the basic idea that the production function is concave. In this respect, then, the production smoothing/buffer stock model is "saved," though with rather little emphasis on the buffer stock aspects.

IV. ECONOMETRIC INVENTORY EQUATIONS

This section presents econometric estimates of inventory investment equations for finished goods based on the theoretical specification (3.16). I concentrate on finished goods because that is the only type of inventory for which we have a coherent and operational theory. However, the stylized facts show that works in progress are just as important.

The data are monthly, real, and seasonally adjusted, and (after allowing for lags) span the period December 1960 - March 1981. <19> In accord with the findings in Section II, each two-digit industry is treated separately. However, as a kind of convenient summary, I also present results for all manufacturing and for the durable and nondurable sectors. The theoretical equation (3.16) was made operational as follows.

Demand disturbances were proxied by two variables: expected sales, X_t^e, is the one-period-ahead forecast from a 12-th order autoregressive fit to each industry's actual data on shipments; and unexpected sales, X_t^u, is the residual from this autoregression. Thus expectations are assumed to be "rational," albeit in a limited sense. Experimentation with other expectational proxies led to substantially identical results. In 13 of the 20 industries, data on new orders were available. For these industries, the collinearity between the two sales measures was almost always too great to include both, so two versions of the regressions were run. Normally, a better fit was obtained using shipments.

Cost disturbances were treated by including both the real product wage and the real cost of raw materials in each regression. The nominal wage series was the average hourly earnings series specific to that industry or sector. The nominal materials cost series was the PPI for Crude Materials for Further Processing (and is the same for every industry). Each nominal factor price was deflated by an industry- specific price index.

In addition, the interest rate was included as a potentially important determinant of the nonstochastic steady state level of desired inventories. For reasons described in Blinder (1981), the nominal interest rate (bank prime rate) and the expected rate of inflation (generated by an autoregression) were entered as separate variables rather than combined into a real interest rate.

Before presenting the estimates, a word on autocorrelation is in order. It has been well known for years that econometric procedures have a hard time distinguishing between partial adjustment and autocorrelation (Griliches (1967)). Write the basic stock adjustment model of inventory change as:

$$(4.1) \quad N_{t+1} - N_t = \beta(\bar{N} - N_t) + u_t ,$$

Variables other than N_t are irrelevant for present purposes, and hence ignored. If the error term follows an AR(1) scheme:

$$(4.2) \qquad u_t = \rho\, u_{t-1} + e_t ,$$

the natural procedure is to quasi-difference (4.1) before estimating to get:

$$(4.3) \qquad N_{t+1} = (\rho - \beta + 1)N_t - \rho(1-\beta)N_{t-1} + e_t.$$

This is an AR(2) model for the stock of inventories. But notice the fundamental identification problem. Suppose ρ and β are approximately equal, then the two coefficients in (4.3) are approximately 1 and $\beta(\beta-1)$. Hence, we cannot tell β from $1-\beta$. For example, if either $\rho = \beta = .9$ or $\rho = \beta = .1$, then the coefficients in (4.3) are respectively 1.0 and -.09. Thus any estimation technique will have trouble distinguishing between a model with strong serial correlation and fast adjustment and one with little serial correlation but slow adjustment. <20>

All the equations reported in this section were fit by nonlinear least squares under the assumption that the error term was AR(1). <21> In several cases, two local minima of the sum of squared residuals function were found. In such cases, one of the minima always had high ρ and rapid adjustment while the other had low ρ and slow adjustment, precisely as suggested by this simple argument. This point is important because the extremely high adjustment speeds recently found by Maccini and Rosanna (1984) result from an estimation technique that, I believe, settles on the local minimum with high ρ. The estimation method used here typically shows that the low ρ solution is the global minimum.

The model in Section III recognized the existence of only one type of inventory. But, in fact, there are three types and, in many industries, also backlogs of unfilled orders. Preliminary regressions showed clearly that investment in finished goods inventories reacts differently to the initial stock of each kind of inventories, so Table 4 presents estimates of the following extended stock adjustment model of finished goods inventories:

$$(4.4)\quad \Delta F_t = \beta_1 F_t + \beta_2 W_t + \beta_3 M_t + \beta_4 U_t + \alpha_1 X_t^e + \alpha_2 X_t^u + \gamma_1 R_t + \gamma_2 \pi_t + $$
$$+ \delta_1 w_t + \delta_2 c_t + u_t ,$$

where U_t is the stock of unfilled orders and the error term, u_t, is assumed to be generated by (4.2). (In the table, t-ratios are in parentheses. Asterisks indicate variables for which distributed lags were found to be significant, as will be explained later.)

First, note that the initial stock of finished goods always enters with a significant negative coefficient, indicative of partial adjustment. However, in accord with much previous work (see Fact 3), most of the estimated speeds of adjustment are rather slow. Among the 17 industries for which the "low ρ" solution was the global minimum, the speeds of adjustment range from 5% to 38% per month. These speeds are slightly faster than those typically found in work at a more aggregative level, but are not out of line with earlier estimates. <22> In this context it is interesting to observe

Table 4

Simple Stock Adjustment Regressions, 1960:12 - 1981:3

Coefficient (absolute t ratio) of:

Sector	Initial Stock of: Finished Goods	Initial Stock of: Work in Progress	Initial Stock of: Materials & Supplies	Unfilled Orders	Sales: Expected	Sales: Unexpected	Interest Rate	Expected Capital Gains	Wages	Materials Costs	$\hat{\rho}$	R^2
All Manufacturing	-.080 (3.3)	.006 (0.4)	.029 (2.8)	.002 (0.7)	.003 (0.4)	-.052 (3.2)	-128* (0.7)	-9.9 (0.1)	15.9* (4.5)	-9.4* (3.4)	.13 (1.8)	.19
Durable goods	-.061 (2.6)	-.002 (0.2)	.018 (2.3)	.001 (0.4)	.005* (0.6)	-.039 (2.7)	8.8* (0.1)	-20 (0.3)	10.3 (1.3)	-3.7 (2.2)	.12 (1.7)	.12
Primary metals	-.271 (4.7)	.151 (3.5)	-.051 (2.6)	.008 (1.3)	-.057* (2.8)	-.133 (7.6)	-82* (2.4)	-31 (3.4)	6.0 (2.6)	-4.0 (4.4)	.42 (5.0)	.37
Fabricated metals	-.257 (5.7)	.056 (2.9)	.034 (4.5)	.001 (0.4)	-.001 (0.1)	.025 (0.9)	-10 (0.4)	-9.2 (1.0)	1.7 (1.5)	-.69* (2.1)	-.10 (1.3)	.21
Electrical mach.	-.112 (2.8)	.013 (1.2)	.000 (0.0)	.010 (1.8)	.037 (2.3)	.004 (0.1)	-28* (0.8)	83 (4.1)	5.0 (2.8)	-.52 (1.3)	.03 (0.4)	.12
Nonelectrical[a] machinery	-.106 (3.9)	.026 (1.2)	.029 (1.8)	-.005 (1.0)	.010 (0.9)	-.033 (2.3)	1.2* (0.0)	7.7 (0.4)	4.9 (1.8)	-2.0 (3.4)	.01 (0.1)	.17
Transportation equipment	-.049 (1.7)	.001 (0.2)	-.027 (2.0)	.002 (1.3)	.027 (3.4)	.017 (1.4)	-56 (1.5)	104 (3.1)	-1.5 (0.9)	.17 (0.3)	-.14 (2.0)	.12
Lumber & Wood Products	-.106 (3.0)	.036 (0.9)	.040 (1.8)	--	-.044* (2.2)	.061 (1.5)	14 (1.0)	1.3 (0.5)	-.05 (0.1)	-.55 (2.0)	-.03 (0.4)	.08
Furniture and fixtures	-.115 (3.9)	.040 (1.2)	.024 (2.4)	.009 (1.3)	-.020* (0.7)	.070 (1.8)	-1.6 (0.3)	11.3 (2.6)	-.18 (0.3)	-.16 (1.6)	-.23 (3.4)	.18

Table 4 (continued)

	N_t	W_t	M_t	U_t	X^e_t	X^u_t	R_t	π_t	w_t	c_t	$\hat{\rho}$	$\bar{R}^2$
Stone, Clay, & Glass Products	-.179 (4.0)	.107 (1.1)	.005 (0.3)	-.008 (0.9)	.071* (2.9)	-.103 (2.8)	25 (1.7)	8.8 (0.8)	0.22* (0.4)	-.48 (2.1)	.09 (1.2)	.16
Instruments and related prods.	-1.04 (16.4)	.167 (4.5)	.068 (2.4)	.074 (1.7)	.030 (0.3)	.002 (0.0)	29 (0.7)	-13.2 (1.7)	-4.6 (1.4)	1.32 (2.3)	.98 (80.5)	.15
Miscellaneous Manufacturing Indust.	-.221 (4.9)	.103 (2.0)	.045 (2.5)	.021 (0.9)	.137* (3.2)	-.016 (0.3)	-13 (0.9)	-1.4 (0.3)	.44 (1.3)	.23* (1.0)	.29 (3.7)	.19
Nondurable Goods	-.142 (3.8)	.184 (1.9)	.054 (2.3)	.017 (0.3)	-.026 (1.1)	-.067 (2.0)	98* (1.1)	14.5 (0.3)	12.3* (2.2)	-10.5 (4.0)	.15 (2.0)	.15
Food and Kindred Products	-.785 (10.1)	.293 (1.3)	.115 (1.8)	--	.285* (3.7)	.237 (0.6)	-99* (0.7)	-21.8 (1.9)	10.8 (2.1)	-1.85 (0.4)	.91 (25.0)	.12
Tobacco Manufacturing	-.382 (4.5)	.076 (0.8)	.020 (2.8)	--	.209 (1.6)	-.047 (0.6)	1.7 (0.2)	1.6 (0.4)	.58 (2.4)	-.24 (1.1)	.13 (1.2)	.18
Textile Mill Prods.	-.999 (15.2)	.319 (3.4)	.109 (2.3)	-.057 (2.6)	.064 (0.9)	.013 (0.4)	6.9 (0.2)	-20.6 (2.7)	3.1 (2.7)	-.68 (1.2)	.98 (68.0)	.17
Apparel Products	-.248 (4.3)	.027 (0.5)	.134 (3.9)	--	.033 (0.7)	.111 (2.4)	-12 (0.4)	-53 (1.6)	1.8 (1.0)	-.51* (1.4)	.26 (2.8)	.17
Leather & Leather Products	-.177 (4.4)	.092 (1.2)	.052 (1.9)	-.048 (2.8)	-.042* (1.1)	.032 (0.6)	8.4* (1.3)	-52 (2.1)	.31 (1.1)	.05* (0.6)	-.14 (1.8)	.17

Table 4 (continued)

	$\bar{N}_t$	W_t	M_t	U_t	X^e_t	X^u_t	R_t	π_t	w_t	c_t	$\hat{\rho}$	R^2
Paper & Allied Products	-.237	.160	.056	.001	-.001	-.074	27*	-3.4	2.4	-.54	-.11	.21
	(5.0)	(2.1)	(3.8)	(0.1)	(0.1)	(2.2)	(2.8)	(0.8)	(3.3)	(2.9)	(1.3)	
Printing and Publishing	-.151	-.014	.031	-.081	.068	.041	-28	-1.0	.08	.03	-.02	.11
	(4.6)	(0.4)	(3.3)	(3.4)	(3.0)	(1.1)	(1.9)	(0.3)	(0.1)	(0.1)	(0.2)	
Chemicals & Allied Products	-.081	.093	.009	--	.021	-.251	43	3.0	2.2	-2.6	.10	.27
	(2.9)	(1.3)	(0.4)		(1.1)	(6.5)	(1.5)	(0.0)	(2.0)	(4.0)	(1.5)	
Petroleum & Coal Products	-.123	-.248	.133	--	.008*	-.159	26*	-9.2	.84	-1.3	.02	.16
	(3.3)	(2.7)	(1.5)		(0.4)	(3.2)	(1.5)	(3.4)	(1.8)	(2.5)	(0.3)	
Rubber & Plastic Products	-.076	-.041	.048	--	.015	-.006	-34	.32	1.66*	-.76	.09	.14
	(3.3)	(0.5)	(2.9)		(0.5)	(0.2)	(2.4)	(0.1)	(3.4)	(3.2)	(1.3)	

Note: Estimation was by nonlinear least squares, with allowance for first-order autocorrelation. All regressions also included a constant, not shown here.

[a] For this industry only, new orders are used to measure sales.

that aggregation seems to bias the speed of adjustment downward. The estimated adjustment speeds for durables and nondurables as a whole are lower than those of most of the constituent industries.

In the remaining three industries -- instruments, food, and textiles -- the global minimum turned out to be the "high ρ" solution, and estimated adjustment speeds were very rapid (104%, 79%, and 100% per month, respectively).

The cross-adjustment coefficients, β_2 and β_3, are more novel and display a rather consistent pattern across industries. High opening stocks of either works in progress (W_t) or raw materials (M_t) usually are associated with higher investment in finished goods inventories, that is, with higher production. Whether or not this empirical regularity implies causation, of course, is another matter entirely. For example, higher planned production could induce stockpiling of works in progress and materials.

Studies that merge all three types of inventory into a single stock necessarily produce an estimated "adjustment speed" that is an amalgam of the three adjustment coefficients, β_i. Since one of these is negative and the other two are positive, we should expect this procedure to understate the speed of adjustment if the three types of inventories covary positively. To test this idea, a version of (4.4) was run in which all three types of inventory were lumped together into a single aggregate. The results were as expected: estimated adjustment speeds generally declined, sometimes dramatically.

Turning to specifics, the coefficient of works in progress is positive in 17 of 20 industries, though it is significantly positive in only 4 of these. The petroleum refining industry is the only important exception to this rule; here, high stocks of work in progress apparently lead to lower levels of output.

The coefficient of the opening stock of materials and supplies inventory is positive in 18 of 20 industries, and is significantly positive in 10 of these. The only exceptions are the primary metals and transportation equipment industries, where high levels of raw materials apparently lead to cutbacks in production.

In contrast to these rather good results, the stock of unfilled orders performed poorly. Among the 13 industries reporting data on unfilled orders, the estimated coefficient was positive 7 times (the "correct" sign, it seems to me) and negative 6 times. Only three coefficients were significant; and they were all negative.

As noted already, sales were measured alternatively by shipments and, in those industries offering such data, unfilled orders. Fortunately, the estimated equations proved quite insensitive to the choice of a sales measure. Since shipments performed slightly better than new orders, and are available for all industries, Table 4 reports only the results with shipments. <23>

In general, results for the sales variables were somewhat disappointing and not always in line with a priori expectations. For example, many of the coefficients were insignificantly different from zero, suggesting either that production reacts virtually one-for-one to sales (whether expected or unexpected) or that the difference between production and sales shows up mostly in works in progress rather than in finished goods. <24>

Specifically, the coefficient of expected sales, X_t^e , is normally quite small (values of .05 or less are typical) and insignificantly different from zero. Its sign is positive in 14 cases and negative in 6, and only 8 of the 20 industries (all in durables) display significant coefficients.

The unexpected sales variable is significant in only 7 industries. A positive coefficient for this variable is impossible to interpret in the context of the model; taken literally, it implies that inventories of finished goods rise when there is an unexpected surge in sales. Presumably, a positive coefficient means that the sales fluctuations which we label "unexpected" are really expected by firms, in accord with the discussion in Section III. Yet the point estimate is positive in 11 of 20 industries. There is evidence of a strong negative effect of X_t^u on ΔF_t in only 6 industries.

Interest rates, represented here by the (monthly) nominal interest rate (R_t) and the (monthly) industry-specific expected rate of inflation (π_t) do not perform as the theory suggests. The expected signs are negative for R_t and positive for π_t; but only 4 of 20 industries display this pattern. Taking the two variables individually, we see that R_t gets the expected negative coefficent in only 10 of 20 cases and π_t gets the expected positive coefficient in only 9 of 20 cases. Only 5 of the 19 correctly-signed coefficients are significant; as are 5 of the 21 incorrectly-signed coefficients. This is not much better than what you would expect if the coefficients were randomly distributed around zero, so the overall conclusion seems to be that interest rates do not matter. This finding is consistent with older empirical work on inventory investment, but contradictory to some newer work in which significant inventory effects have been found. <25>

The wage rate is probably the least successful variable of all. Of the 20 industries, only 4 estimates get the expected negative sign. Of the 16 positive coefficients, 9 are significantly different from zero. The results here strongly suggest reverse causation running from higher production to higher wages, perhaps due to overtime premia. Thus, I conclude that wage rates are not good representations of cost shocks.

Raw materials costs are far more successful in this role. The estimated coefficient of c_t is negative in 15 of 20 cases, and is significant in about half the industries. And many of the coefficients are of an economically meaningful size. For example, the coefficient for all manufacturing indicates that a 10% rise in raw materials prices (the variable c_t is an index number with January 1972=100) will lower the desired stock of finished goods inventories by \$2 billion (in 1972 dollars), or about 5% of the mean inventory stock.

Finally, I note in passing that the fits of the regressions -- as measured by R^2 -- are modest at best. Time series analysis of noisy, virtually trendless series like ΔF_t encourages humility.

One objection to the standard stock adjustment model is that it assumes that all the righthand variables enter only contemporaneously. But if there are lags in adjustment, noncontemporaneous values of variables like interest rates and raw materials costs may also matter. <26> In fact, Irvine (1981c) has argued that omission of such variables may bias estimated adjustment speeds downward. There are so many possible combinations of distributed lags that might be added to (4.4) that I adopted a sequential search procedure to economize on computing costs. The results are summarized in Table 5.

First, I tested for whether expectations of sales more than one month ahead contribute anything to the explanation of inventory change by adding X^e_{t+1}, X^e_{t+2}, and X^e_{t+3} to the regression. <27> Column (1) of Table 5 reports the appropriate likelihood ratio tests, showing that these additional variables were significant in 8 of the 20 industries. <28> However, adding these variables to the regression led to a meaningful change in the estimated adjustment speed in only two industries. One of these (miscellaneous manufacturing) jumped

Table 5

Likelihood Ratio Tests for Distributed Lags

	(1) Expected Sales	(2) Interest Rates	(3) Materials Costs	(4) Wages
All Manufacturing	1.364	15.110**	9.915*	9.175*
Durable Goods	6.833*	17.204**	4.963	0.001
Primary metals	17.177**	13.757**	1.191	3.150
Fabricated metals	3.701	5.920	10.439*	0.946
Electrical machinery	2.794	17.485**	6.293	1.962
Nonelectrical machinery	0.040	27.529**	6.453	4.056
Transportation Equipment	3.588	8.377	1.531	2.778
Lumber & Wood Products	6.554*	3.656	6.544	3.402
Furniture & Fixtures	14.975**	5.382	0.148	1.734
Stone, Clay & Glass Products	7.309*	4.047	1.081	9.057*
Instruments & Related Prods.	4.454	6.230	2.312	2.176
Miscellaneous Manufacturing Industries	6.127*	6.029	11.741**	4.110
Nondurable Goods	0.712	25.663**	4.340	8.020*
Food & Kindred Products	6.032*	10.494*	7.004	0.998
Tobacco Manufacturing	0.215	0.708	0.401	0.420
Textile Mill Products	0.721	6.165	4.125	1.890
Apparel Products	0.305	2.108	26.431**	0.579
Leather & Leather Products	9.162*	29.221**	11.186*	5.639
Paper & Allied Products	4.670	10.100*	6.406	0.580
Printing & Publishing	2.995	3.676	0.832	3.523
Chemicals & Allied Products	3.569	3.145	0.989	0.599
Petroleum & Coal Products	9.442**	10.580*	3.934	0.101
Rubber & Plastic Products	0.883	4.640	4.873	7.403*

*Denotes significant at 5% level.

**Denotes significant at 1% level.

from the "low ρ" solution to the "high ρ" solution, with a correspondingly large increase in the estimated speed of adjustment. <29>

In the second step, expectations of future sales were retained in these 8 industries, but dropped in the other 12, and I tested for the effects of lagged interest rates by appropriate likelihood ratios. <30> Column (2) reports the results. Lagged interest rates proved to be significant in 7 industries. Inspection of the results shows that it was usually the lagged values of R_t, not of π_t, that obtained significant coefficients. While several estimated adjustment speeds changed in this step, there was no clear pattern -- some increased while others decreased.

In the next step, the lagged values of interest rates were retained in the 7 industries in which they proved to be significant, but dropped in the remaining 13, and I tested for the inclusion of lagged raw materials prices. The results of the likelihood ratio tests are reported in column (3) of Table 5. The χ^2 value is significant in 4 industries but, with one exception, inclusion of lagged c had only minor effects on the estimates of ρ and β_1. <31> The exception was miscellaneous manufacturing, which returned to the "low ρ" solution.

The last step was to retain lagged materials costs in these 4 sectors, drop them from the remaining 16, and go on to look for significant effects of lagged wages. <32> Lagged wages proved to be significant in only two industries, and did not change the estimates of ρ and β_1 substantially in either case.

Table 6 reports the end results of this search procedure for each industry and aggregate in which at least one significant distributed lag effect was found. Once a final specification was selected, the computing algorithm was started from different initial points to see if it would converge to a different local minimum in the sum of squared residuals function. This never happened. Among the 20 industries, only food jumped from one local minimum to another between Tables 4 and 6. Consequently, in the end only two industries are estimated to have rapid adjustment speeds (instruments and textiles) and severely autocorrelated disturbances. The remaining 18 have adjustment speeds ranging from 5% to 38% per month and autocorrelation parameters ranging from -.21 to +.32. However, it is worth reemphasizing that our ability to pin down the speed of adjustment is not nearly so good as the t-statistic indicates. <33>

Comparing Tables 4 and 6 shows that -- except in the one case in which the global minimum shifts from one local minimum to another -- the inclusion of distributed lags does not have any notable effects on the estimated speed of adjustment. Sometimes it goes up, sometimes it goes down, but it never changes dramatically. The same is more or less true of the coefficients of the other three stock variables: no dramatic, or even terribly systematic, changes in coefficients were

Table 6

Stock Adjustment Regressions with Distributed Lags

Sector	Finished Goods	Work in Progress	Materials & Supplies	Unfilled Orders	Expected Sales Current	Expected Sales Future[a]	Unexpected Sales	Interest Rate Current	Interest Rate Lagged[b]	Expected Capital Gains Current	Expected Capital Gains Lagged[b]	Wages Current	Wages Lagged[b]	Materials Current	Costs Lagged[c]	$\hat{\rho}$	R^2
All Manufacturing	-.051 (1.7)	.000 (0.0)	.016 (0.8)	.003 (0.8)	.001 (0.1)	--	-.042 (2.4)	-487 (2.1)	-3069 (7.9)	-280 (2.0)	42 (0.1)	-58.3 (1.7)	+75.6 (1.9)	-10.9 (2.1)	14.3 (2.0)	-.003 (0.0)	.30
Durables	-.039 (1.9)	.015 (1.6)	.000 (0.0)	-.002 (1.0)	-.087[f] (1.5)	-.160 (0.1)	-.038 (2.3)	-208 (1.4)	206 (0.9)	-154 (2.0)	325 (3.4)	-7.5 (0.8)	--	-3.3 (2.0)	-- (0.1)	.00 (0.1)	.20
Primary metals	-.216 (4.2)	.128 (3.1)	-.045 (2.4)	.000 (0.0)	-.090[g] (0.7)	-1.71 (0.3)	-.150[g] (7.8)	30 (0.5)	-119 (1.6)	-25 (2.6)	26 (1.3)	3.1 (1.4)	--	-3.2 (3.3)	--	.32 (3.8)	.45
Fabricated metals	-.295 (5.8)	.057 (2.9)	.041 (4.7)	.001 (0.5)	.001 (0.1)	--	.025 (0.9)	10.7 (0.4)	--	-3.2 (0.3)	--	1.8 (1.6)	--	1.7 (2.0)	-2.8 (2.5)	-.07 (0.9)	.24
Electrical machinery	-.113 (2.9)	.091 (1.9)	.014 (1.2)	-.007 (1.3)	.008 (0.5)	--	-.000 (0.0)	37 (0.9)	-146 (3.1)	46 (1.7)	-9.4 (0.3)	4.2 (2.4)	--	-.31 (0.8)	--	-.04 (0.5)	.18
Nonelectrical mach.	-.045 (1.8)	-.001 (0.0)	.017 (1.2)	-.003 (0.5)	.023 (1.8)	--	-.024 (1.7)	-103 (1.8)	159 (1.7)	.14 (0.0)	76 (2.0)	1.3 (0.5)	--	-1.8 (3.4)	--	-.15 (2.1)	.26
Lumber & wood prods.	-.099 (3.0)	.017 (0.4)	.025 (1.1)	--	-.228 (1.2)	6.08 (0.7)	.065 (1.6)	11.6 (0.9)	--	-.61 (0.2)	--	-.49 (1.2)	--	-.48 (1.8)	--	-.06 (0.8)	.11
Furniture & fixtures	-.219 (5.0)	.062 (1.9)	.041 (3.6)	.003 (0.4)	-.279 (2.5)	14.57 (2.9)	.086 (1.9)	4.8 (0.8)	--	10.2 (2.3)	--	-.89 (1.5)	--	-.11 (1.0)	--	-.21 (2.9)	.23
Stone, Clay & Glass products	-.206 (4.0)	.122 (1.3)	.012 (0.7)	-.014 (1.4)	-.150 (1.2)	-1.68 (0.3)	-.157[d] (3.0)	24 (1.5)	--	-11 (0.9)	--	-4.1 (2.5)	4.9 (2.9)	-.56 (2.3)	--	.12 (1.4)	.22
Miscellaneous Manufacturing Ind.	-.188 (4.9)	.107 (2.3)	.050 (2.4)	.042 (1.8)	.085 (0.6)	4.80 (0.6)	-.008 (0.2)	10 (0.7)	--	.25 (0.1)	--	.16 (0.5)	--	1.23 (2.9)	-1.30 (2.8)	.16 (2.0)	.25
Nondurables	-.262 (4.9)	.292 (2.8)	.040 (1.5)	.025 (0.5)	.009 (0.3)	--	-.064 (2.0)	-564 (3.4)	1013 (3.7)	-135 (2.0)	460 (2.6)	-4.9 (0.2)	23.1 (1.0)	-16.3 (5.0)	--	.17 (2.1)	.26
Food	-.122 (2.2)	-.282 (1.7)	.038 (0.9)	--	-.143 (0.6)	13.46 (1.4)	-.050 (1.1)	-220 (2.2)	+210 (1.7)	-30 (2.0)	-85 (1.5)	3.7 (1.4)	--	0.59 (0.2)	--	.21 (2.4)	.16
Apparel Products	-.206 (4.5)	.087 (1.6)	.117 (4.1)	--	.045 (1.2)	--	.118 (2.7)	59 (1.9)	--	-33 (1.1)	--	-1.4 (0.9)	--	0.63 (0.7)	-1.35 (1.6)	.11 (1.4)	.26

Table 6 (continued)

Stock Adjustment Regressions with Distributed Lags

	Finished Goods	Work in Progress	Materials & Supplies	Unfilled Orders	Expected Sales: Current	Expected Sales: Future[a]	Unexpected Sales	Interest Rate: Current	Interest Rate: Lagged[b]	Expected Capital Gains: Current	Expected Capital Gains: Lagged[b]	Wages: Current	Wages: Lagged[b]	Materials Costs: Current	Materials Costs: Lagged[c]	$\hat{\rho}$	$\bar{R}^2$
Leather & Leather Products	-.261	.150	.004	.003	-.365[e]	-10.22	-.090[e]	-7.9	36	-7.1	16	1.4	--	.35	-.45	-.18	.33
	(5.4)	(1.9)	(0.1)	(0.2)	(1.9)	(1.5)	(1.3)	(0.7)	(2.3)	(2.1)	(3.3)	(4.1)		(1.4)	(2.0)	(2.3)	
Paper and Allied Products	-.295	.207	.048	.036	-.008	--	-.087	-4.2	56	-2.0	7.1	3.2	--	-.68	--	-.07	.24
	(5.1)	(2.3)	(3.1)	(1.9)	(0.3)		(2.6)	(0.2)	(2.3)	(0.4)	(1.1)	(3.8)		(3.2)		(0.7)	
Petroleum & Coal Products	-.148	-.242	.163	--	-.605	.942	-.142	3.8	3.9	-6.4	-0.7	1.3	--	-2.0	--	.01	.22
	(4.6)	(2.6)	(1.8)		(2.4)	(0.1)	(0.7)	(0.1)	(0.1)	(2.1)	(0.1)	(2.5)		(3.6)		(0.1)	
Rubber & Plastic Products	-.065	-.057	.034	--	.034	--	-.001	-30	--	6.7	--	4.6	-33	-.73	--	.05	.16
	(3.0)	(0.6)	(1.8)		(1.3)		(0.0)	(2.1)		(1.1)		(3.4)	(2.3)	(3.2)		(0.7)	

Note: Estimation was by nonlinear least squares, with allowance for first-order autocorrelation. All regressions also included a constant, not shown here.

[a] Sum of three lag coefficients, constrained to fall on a straight line.

[b] Sum of 11 lag coefficients, constrained to fall on a straight line.

[c] Sum of 11 lag coefficients constrained to fall on a quadratic.

[d] This equation also included unexpected new orders, with a coefficient of .056 (t=1.4).

[e] This equation also included expected new orders, with a coefficient of .074 (t=0.7) and unexpected new orders with a coefficient of .131 (t=2.7).

[f] This equation also included expected new orders, with a coefficient of .030 (t=2.2).

[g] This equation also included expected new orders, with a coefficient of 0.62 (t=4.7) and unexpected new orders with a coefficient of 0.26 (t=2.1).

observed.

For each of the other variables, Table 6 reports the coefficient of the contemporaneous value and the sum of the remaining coefficients. The few systematic changes that can be observed in going from Table 4 to Table 6 are easily summarized:

EXPECTED SALES: In the 8 industries in which future expected sales were significant, there was a clear tendency for the contemporaneous coefficient to change in the negative direction when future expected values were added.

INTEREST RATES: The distributed lag specification typically yielded larger (in absolute value) coeffients for both nominal rates and expected inflation than the simpler specifications in Table 4. In the case of nominal rates, the coefficients of current and lagged interest rates typically were of opposite sign. While this suggestion of intertemporal substitution (temporarily high rates lead to temporary inventory liquidation) is tantalizing, it is easy to resist given the imprecision of the estimates.

MATERIALS COSTS: In those few cases in which lagged materials prices were significant, the coefficient of current price was usually positive while the sum of the coefficients of past prices was negative.

In general, however, the results in Table 6 do not change the overall impression left by Table 4, although the regressions with distributed lags generally fit better.

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V. CONCLUSION

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It is easy enough to see why the production smoothing model looks so bad at first blush. Consider a trivially simple fixed- price macro model of aggregate supply and demand based on the production smoothing idea:

$$\text{Supply:} \quad Y = \alpha + \beta X$$

$$\text{Demand:} \quad X = \bar{X} + e,$$

where Y is production and X is sales. Here e is the random demand shock that drives the model and $\beta<1$ captures the idea that production is smoothed relative to sales.

In this model, $\mathrm{var}(Y)/\mathrm{var}(X) = \beta^2$, which is certainly less than unity. Further, since inventory change is:

$$\Delta N = Y - X = \alpha + (\beta-1)\bar{X} + (\beta-1)e,$$

it is clear that X and ΔN are perfectly negatively correlated in the model. In the data, as we know, var(Y) exceeds var(X) and X and ΔN are nearly orthogonal. The contradiction between the model and reality could hardly be more complete.

This paper has shown, however, that it is possible to amend the production smoothing model in ways that make it consistent with the facts. Section III.8 summarized how this can be done. The two critical ingredients are serially persistent demand disturbances of a particular type and the addition of a cost shock.

It is easy to see how these amendments help. Adding a cost shock changes the model to:

Supply: $Y = \alpha + \beta X + u$

Demand: $X = \bar{X} + e$,

and Section III showed that, for any given cost structure, serial correlation in demand disturbances has the effect of pushing β towards unity. Assuming that u is independent of e at all lags, trivial calculations establish that:

$$\mathrm{var}(Y)/\mathrm{var}(X) = \beta^2 + \tau^2$$

$$\mathrm{corr}(X, N) = \frac{\beta-1}{[\tau^2+(1-\beta)^2]^{\frac{1}{2}}} \quad ,$$

where τ^2 is the ratio var(u)/var(e). Now, if τ^2 is big enough, the variance ratio can exceed unity; and the correlation between X and ΔN, while still negative, can at least be small.

This, in essence, is now Section III attempts to reconcile the production smoothing model with the data. Furthermore, the econometric estimates of inventory equations in Section IV are broadly consistent with the theoretical reconciliation in that (a) the stochastic processes describing demand are highly autocorrelated, (b) the estimated adjustment speeds are quite low, and (c) econometric proxies for unexpected sales appear not to be unexpected by firms. At some level, therefore, the exercise must be judged a success. The production smoothing model, or at least the concavity of the production function, has been saved.

Yet there are some lingering doubts. A skeptic may recall that Ptolemaic astronomy was "saved" many times by the addition of epicycles specifically designed to accommodate each new fact. In addition, many features of the econometric estimates are less than satisfactory, including the tenuous basis for pinning down the adjustment speed parameters, the previously-noted fact that slow adjustment is hard to explain, and the poor results obtained with key variables like unexpected sales, wages, and interest rates. One suspects that Copernicus may be waiting in the wings.

Certainly there are other models of inventory behavior that might be used to explain the stylized facts. For example, the (S,s) model was mentioned in Section I, but judged implausible on a priori grounds. <34> But perhaps the explanation for the puzzling behavior of inventories does not lie in inventory behavior at all. To see what I mean, consider the following trivial "Keynesian cross" model in which demand creates its own supply and inventories never change:

Supply: $Y = X$

Demand: $X = a + bY + e.$

Obviously, in this model $var(Y)/var(X)=1$. And since there are no changes in inventories, $var(\Delta N)=0$ and $cov(X,\Delta N)=0$ in a trivial sense. Clearly, this model cannot be quite right because it ignores some empirically important movements in inventories. Nonetheless, it makes a promising start at "explaining" the first three stylized facts enumerated in Section I.

It does not take much imagination to integrate the Keynesian specification of aggregate demand with the production smoothing model of aggregate supply to get:

Supply: $Y = \alpha + \beta X + u$,

Demand: $X = a + bY + e$.

In this hybrid model, the critical variance ratio is:

$$\frac{var(Y)}{var(X)} = \frac{var(u) + \beta^2 var(e)}{b^2 var(u) + var(e)} .$$

This expression clearly shows that demand shocks lead to a variance ratio smaller than 1 (depending on the degree of production smoothing) while supply shocks lead to a variance ratio bigger than 1 (depending on the MPC). The variance ratio will exceed 1 if and only if:

$$\tau^2 > \frac{1-\beta^2}{1-b^2} .$$

If β is larger than b, this may not require large cost shocks.

The covariance between sales and inventory change is:

$$cov(X,\Delta N) = \frac{b(1-b)var(u) - (1-\beta)var(e)}{(1-b\beta)^2}$$

which can have either sign -- an empirically pleasing prediction, given the mixed results in the data.

Thus it would appear that attaching a Keynesian demand side to our production- smoothing supply side may help the latter account for the stylized facts. Metzler probably knew this forty years ago.

FOOTNOTES

1. See, for example, Holt, Modigliani, Muth, and Simon (1960).

2. Lovell (1961) began this tradition, and many have followed.

3. For a precise derivation, see Blinder (1982).

4. For the period 1947:2-1981:1 (quarterly data), Blinder (1981, p. 446) reports that the variance of real GNP around trend is 32% larger than the variance of real final sales around trend.

5. See Blinder (1981).

6. The time period is obviously crucial here. If the period is a day, for example, it is clear that inventories will serve primarily as buffer stocks whether or not they are held for this purpose. The data I use are monthly.

7. This problem was noticed by Orr (1967), and received prominent attention from Carlson and Wehrs (1974) and from Feldstein and Auerbach (1976).

8. In intermediate position was suggested to me by Geoffrey Heal. It is possible that the production function is convex at low output levels and then becomes concave. This is consistent, for example, with the U-shaped cost curves of elementary textbooks.

9. Alternatively, we could count a portion of a and a portion of d as production. This leads to more cumbersome accounting identities.

10. In doing this, data on inventory stocks were adjusted to reflect the fact that one dollar of inventory stock represents more physical units than one dollar of shipments because inventories are valued at cost rather than market. Part of the appropriate adjustment to convert the data from real values into physical units is presented and explained by West (1983b). The rest is described in Blinder and Holtz-Eakin (1983). The adjustment has the effect of making the variances of y and Δn larger than they appear in the raw data. However, var(y) exceeded var(x) in 18 of the 20 industries (plus all three aggregates) even before the adjustments were made.

11. However, detrending by ordinary least squares led to very similar results, as did entirely different detrending procedures.

12. Since each series was detrended independently, and in logs,

the identity (1.2) does not hold exactly even though (1.1) does.

13. For these comparisons, it was thought that data on (real) dollar values were more meaningful than physical quantities, so the adjustment mentioned in footnote 10 was not made.

14. By contrast, a breakdown of real final sales into consumption, fixed investment, government purchases, and net exports reported in Blinder (1981, p. 448) shows that the sum of the variances of the components is 80% larger than the variance of final sales (the covariances are pervasively negative). For retail sales, the picture is more similar to manufacturing: the individual variances account for 39% of the overall variance.

15. The latter is the case dealt with in Blinder (1982).

16. This statement summarizes succinctly the main point of Blinder (1982).

17. In the case of a v_t^1 demand shock which is unknown to the firm when it makes its production decision, the ratio $var(X)/var(Y) \rightarrow (1+(d/c)) + (d/c) > 1$ as $z_1 \rightarrow 0$. If d is much smaller than c, this will not exceed 1 by much.

18. This proposition does not apply to a v_t^2 demand shock which is unknown to the firm when it makes its production decision. If demand shocks are of this type, var(X)/var(Y) exceeds 1 even as ρ approaches 1.

19. Had they been available, I would have preferred to use data that were not seasonally adjusted, since the production smoothing model presumably applies to seasonal fluctuations in sales. However, such data are not available.

20. Since there are other regressors in (4.1), it is not impossible to distinguish between the two models. But it is difficult.

21. Experiments with more complicated error structures bore little fruit.

22. Feldstein and Auerbach (1976), for example, reported adjustment speeds between 5% and 7% per quarter for finished goods inventories in durable manufacturing. This was fairly typical of work up to that time. Auerbach and Green (1980) got much faster adjustment speeds (from 12% to 85% per quarter) using data on four two-digit industries and a model that treated finished goods and works in progress separately. Blanchard's (1983) study of the divisions of U.S. auto firms found adjustment speeds ranging from 0% to 35% per month. Finally, Maccini and Rossana (1984) found very fast adjustment

speeds (62% to 96% per month).

23. A few cases in which new orders proved to be significant are given in Table 6 below.

24. Recall that $Y_t - X_t = \Delta F_t + \Delta W_t$, so if F_{t+1} does not change when X_t rises either Y_t must rise or W_{t+1} must fall.

25. The earlier literature, summarized, e.g., by Irvine (1981a) found little evidence for a significant effect of interest costs on inventory holdings. However, recent work by Irvine (1981a, 1981b) has detected such effects for retailers and merchant wholesalers, while Rubin (1980) and Akhtar (1983) have found aggregate inventories to be interest sensitive. Only Lieberman (1980), using micro data on a small sample of firms and a specially- constructed cost of capital variable, has found any evidence for interest sensitivity in manufacturing.

26. For example, in the discussion following Blinder (1981) Benjamin Friedman justifiably criticized my work on retail inventories for this reason, and suggested that adjustment speeds might be faster if distributed lags were allowed for.

27. All expectations were based on the information set available for period t (that is, data from period t-1 and earlier). Since expectations are generated by a pure autoregression, each future expectation, like X^e_t itself, is simply a linear combination of lagged X's. The effect of adding future expectations is thus simply to loosen the constraints on how past X's affect current inventory investment.

28. The three distributed lead coefficients were constrained to fall along a straight line, reducing the number of parameters to be estimated from three to two. Thus the relevant χ^2 statistic has two degrees of freedom. The critical values are 5.99 at the 5% level and 9.21 at the 1% level.

29. In these regressions, I also tried using current and expected future new orders in the 13 industries for which such data were available. These variables rarely were significant.

30. In dealing with interest rates, R_t and π_t were always treated symmetrically. For each variable, lags ranging from 1 to 11 months were allowed in the regression, with the lag coefficients constrained to fall along a straight line. Hence, the hypothesis that lagged interest rates do not enter imposes four zero restrictions. The critical points of the χ^2 distribution are 9.49 at the 5% level and 13.3 at the 1% level.

31. In dealing with materials costs, 11 months was again assumed to be the longest lag and lag coefficients were constrained to fall along a quadratic. Hence the relevant χ^2

statistic has three degrees of freedom. The 5% critical point is 7.81 and the 1% critical point is 11.3.

32. For wages a linear lag shape was assumed, so the null hypothesis that lagged wages have no effect imposes two zero restrictions. As before, the longest lag was assumed to be 11 months. The critical points for the χ^2 are 5.99 at 5% and 9.21 at 1%.

33. For example, if we constrain $\rho = 1$ (by estimating the equation in first- difference form), estimated adjustment speeds are extremely high; indeed, many are above 100%.

34. However, the technological assumption mentioned in footnote 8 is not implausible, and is worth exploring.

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