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Cancellation of Glauber Gluon Exchange in the Double Drell–Yan Process

Abstract For any factorisation proof, a crucial step is a demonstration of the cancellation of so-called Glauber gluons. We summarise a recent paper in which we demonstrated this cancellation for double Drell–Yan production (the double parton scattering process in which a pair of electroweak gauge bosons is produced), both for the integrated cross section and for the cross section differential in the boson transverse momenta.

1 Introduction

In order to make predictions at the LHC one relies on factorisation formulae that separate the short distance/high-scale dynamics of interest from the low-scale nonperturbative physics. The short distance physics is encoded in perturbatively-computable partonic cross sections, whilst the infra-red physics is encoded in parton distribution-type objects (and possibly also soft functions). In single parton scattering, factorisation has been rigorously proven for Drell–Yan production (or, more generally, colour-singlet production), for the total cross section and cross section differential in the p_T of the colour singlet system [8,12–14]. These factorisation formulae are correct at leading power—i.e. up to corrections of order Λ_{QCD}^2/Q^2, with Q the hard scale of the process. For the analogous process in double parton scattering, namely double Drell–Yan production, a factorisation formula was written down long ago for the total cross section [21,22] based on the analysis of the lowest-order Feynman diagrams. A factorisation formula for the cross section differential in the p_Ts of the colour singlet systems has also been written down using the same method [15]. However, serious attempts to rigorously justify these formulae have only begun in recent years [7,15,16,20].

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One approach to proving factorisation formulae at leading power (and potentially beyond) is the so-called Collins–Soper–Sterman (CSS) approach, which is the one that was employed in [8,12–14] (there exists an alternative approach based on effective field theories—see e.g. [1–6,23] —but we will not discuss this here). Let us give a very brief review of this method. The first goal is to identify leading infra-red contributions in Feynman graphs contributing to the process of interest—contributions from small regions around the points at which certain lines go on shell, which despite having a smaller phase space are leading due to propagator denominators blowing up. These infra-red contributions are the ones that will have to be absorbed in the PDF-type objects. To be more precise, one needs to identify regions around pinch singularities—these are points where propagator denominators pinch the contour of the Feynman integral [strictly speaking the singularities only appear in the limit in which parton masses and small transverse momenta are set to zero]. If poles all converge on the Feynman contour from one side, one can perform a contour deformation to avoid them (in that case the contribution originally in the vicinity of the poles is subsumed into some other region). The identification of pinch singular points is facilitated by the Coleman–Norton theorem [11], which states that pinch singularities in Feynman graphs correspond to classically allowed processes, and the pinch singular surfaces for single and double Drell–Yan are known.

The pinch singularity analysis tells us nothing about the strength of the singularities, and so must be complemented with a power counting procedure to identify the leading power regions around the singularities. Doing this, one identifies several types of loop momentum scaling that can give rise to a leading power contribution. These are *collinear* (momentum close to some beam/jet direction), *(central) soft* (all momentum components small and of the same order), *hard* (all momentum components large and of order Q) and *Glauber*. The kinematics of Glauber particles is that of those mediating forward or small-angle scattering—i.e. a Glauber momentum r satisfies $|r^+ r^-| \ll \mathbf{r}^2$, where $r^\pm = (r^0 \pm r^3)/\sqrt{2}$, $\mathbf{r} = (r^1, r^2)$.

Initially, there are many longitudinally-polarised gluon connections between parton lines in the collinear region and the union of the soft and Glauber regions, as well as between lines in the collinear region and the hard region. The next step of the factorisation procedure is to apply approximations appropriate to these momentum regions, followed by Ward identities in the sum over graphs to strip away these multiple attachments, yielding the separate functions that appear in the factorisation formula. Unfortunately, this procedure does not work for the Glauber modes, since the approximations needed to apply the Ward identities may not be made in this case. Thus, to achieve factorisation one must demonstrate that the contribution from the Glauber region cancels for the given observable. Note that by 'cancels' here, we mean that there is no remaining 'distinct' (pinched) Glauber contribution—there can (and will) be remaining contributions associated with the Glauber kinematic region, but these can be absorbed into the collinear or soft functions (this was re-emphasised recently in the effective field theory context in [23]). The cancellation of Glauber exchanges was achieved in single Drell–Yan (total cross section and cross section differential in colour singlet p_T) by CSS, and recently in double Drell–Yan (total cross section and cross section differential in colour singlet p_Ts) by us in [16]—here we summarise this latter work. In this short proceedings contribution we mainly limit ourselves to a discussion of the cancellation of Glauber modes at the one-gluon level in Sect. 2—this is useful to illustrate why the cancellation works for double Drell–Yan as it does for single Drell–Yan. We very briefly mention how the all-order proof works in Sect. 3.

2 Glauber Cancellation for One-Gluon Exchange

In order to show the cancellation of Glauber modes at the one-gluon level, we adopt a model in which all partons (except the exchanged Glauber gluon) are scalar (represented by a solid line), and the hadrons are also scalars (represented by a dashed line). These partons may be predominantly collinear to the lower hadron travelling in the plus direction (we colour plus-collinear lines in red), or collinear to the upper hadron travelling in the minus direction (we colour minus-collinear lines in blue). Since the Glauber cancellation argument below depends only on the analyticity properties of the Feynman integrands, which in turn is determined only by the propagator denominators, this argument applies beyond the model, also to diagrams in QCD.

The lowest-order diagram contributing to the double Drell–Yan amplitude (which one simply squares to get the lowest order contribution to the cross section) is given in Fig. 1, along with the nonzero virtual one-gluon corrections to this (these are combined with the tree-level amplitude to get the virtual one-gluon corrections to the cross section). The real one-gluon corrections to the tree-level amplitude squared are zero by colour considerations, and anyway can have no contribution associated with the Glauber kinematic region due to the on-shell constraint for the gluon. Let us consider the virtual corrections in turn, starting with the 'double box'
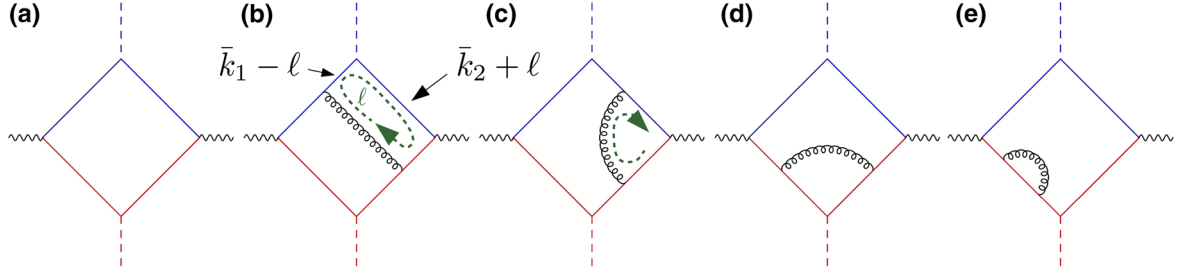
Fig. 1 Lowest order diagram contributing to double Drell–Yan in the model described in the text (**a**), and the nonzero virtual one-gluon corrections to this (**b–e**)
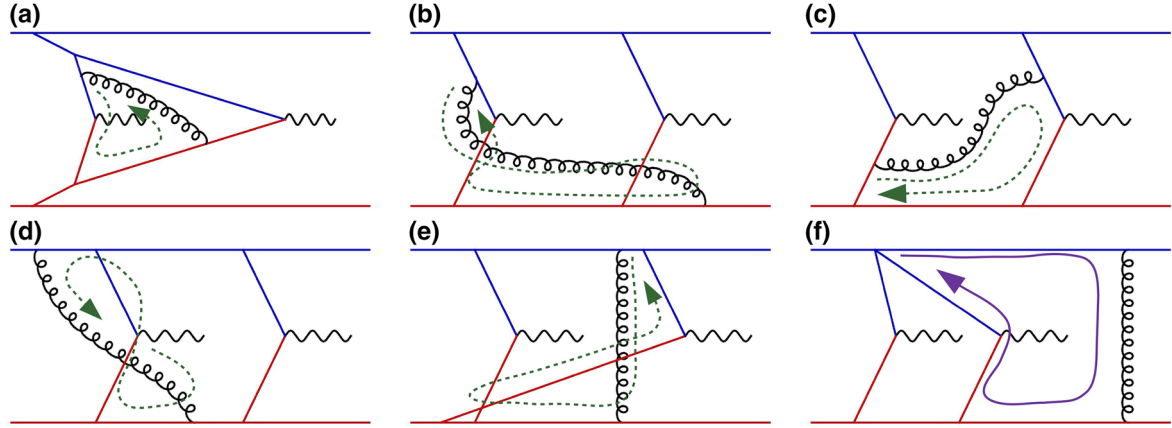

Fig. 2 Example graphs for the double Drell–Yan amplitude within the model described in the text. The incoming lines on the left of each graph could represent the proton, or partons emerging from the proton

graph in Fig. 1b. In this graph, the gluon momentum ℓ runs 'against' and 'with' a large minus momentum—this results in ℓ^+ being trapped at small values, as one can see by looking at the propagator denominators associated with these minus-collinear lines:

$$2\ell^+ \bar{k}_2^- + \cdots + i\varepsilon \qquad -2\ell^+ \bar{k}_1^- + \cdots + i\varepsilon \qquad (1)$$

The terms indicated by '$\cdots$' are products of small components only. This is a general principle - to trap the plus/minus component of a soft momentum at small values, the momentum must run both 'against' and 'with' a collinear line (or multiple lines) with large minus/plus momentum. Now, since ℓ in Fig. 1b only runs against a large plus momentum, ℓ^- is not trapped, and we can deform ℓ^- until ℓ is out of the Glauber region (and is in the collinear region). So there is no distinct Glauber contribution associated with Fig. 1b.

In the gauge boson vertex correction graph, Fig. 1c, neither ℓ^+ nor ℓ^- is trapped, so we can deform the contour out of the Glauber region there too. This leaves us only with the hadron vertex correction and parton self-energy graphs in Figs. 1d, e, which are topologically factorised already, and thus can present no problem. Thus, for the one-gluon corrections to the tree-level graph, there is no problematic pinched Glauber contribution in the first place.

This kind of consideration can be extended to arbitrarily complex one-gluon diagrams in the model, some of which are sketched in Fig. 2. For many types of diagram, a routing can be found for the gluon momentum ℓ such that ℓ^+ and/or ℓ^- is not trapped at small values. This is the case for diagrams (a–e) in Fig. 2—for each of these diagrams a routing for ℓ that leaves one light-cone component untrapped is given by the green dashed arrow. The only graphs for which both ℓ^+ and ℓ^- are always pinched are the ones in which the gluon attaches to spectator partons that either go directly into the final state or only split into partons that go into the final state, both in the upper and lower parts of the graph. An example of this type of graph is given in Fig 2f (with a routing that gives ℓ^+, ℓ^- trapped denoted by the solid purple arrow). However, these graphs are of essentially the same nature as the Glauber-pinched graphs that appear for single Drell–Yan (except that the latter have the double Drell–Yan production subgraph, as on the left of Fi. 2f, replaced by a simpler single Drell–Yan production). For these graphs one can use the same unitarity-based argument as was used by CSS for single
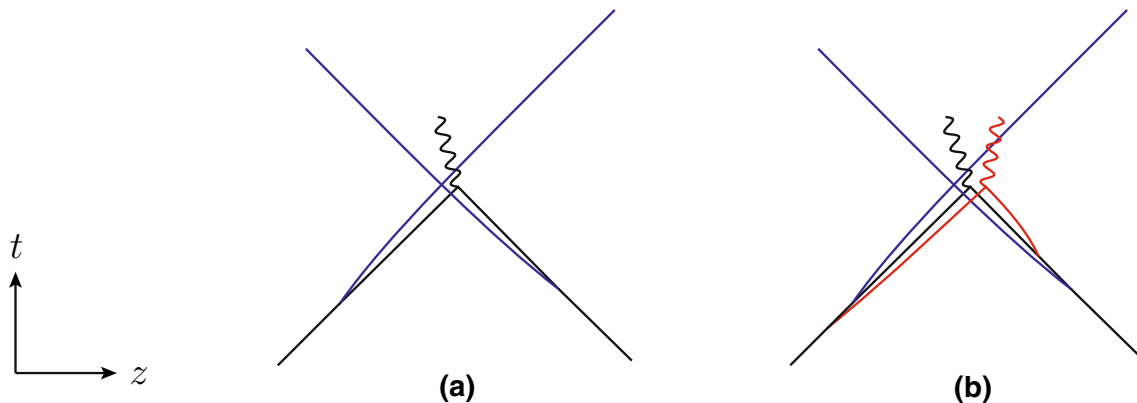
Fig. 3 Spacetime structure of the pinch surfaces for single Drell–Yan (*left*) and double Drell–Yan (*right*). The parton lines move along light-like paths and have been drawn with a slight curvature only for clarity. Figure taken from [15]

Drell–Yan (reviewed in e.g. [17]), to cancel the Glauber contribution after the sum over possible final-state cuts. For this argument to work it is essential that the observable be insensitive to the position of the cut in the 'final state' spectator-spectator system.

3 Glauber Gluon Cancellation at All orders

In order to demonstrate the cancellation of Glauber exchanges in double Drell–Yan production to all orders in QCD perturbation theory, we made use of the same techniques as were used in [12,14] to show the all-order cancellation in single Drell–Yan—in particular, we used the light-front ordered version of QCD perturbation theory (LCPT) (see e.g. [9,10,18,19,24] for an introduction). We do not discuss the details of this proof here, referring the interested reader to [16]. The general idea is the same as the one-loop proof discussed above, however. Also in the all-order LCPT picture, one sees that from the point of view of the Glauber gluons, single and double scattering look rather similar, and that the troublesome 'final state' poles obstructing the deformation out of the Glauber region cancel after the sum over cuts. The argument is based on unitarity, and it can be cast into a form in which it is essentially the same for single and double Drell–Yan.

One can get some insight into why the Glauber cancellation proceeds for double Drell–Yan as it does for single Drell–Yan by looking at the pinch surfaces for the two processes in space–time, given in figure 11 of [15] and reproduced here as Fig. 3. In the diagrams are drawn the collinear lines (black and red solid lines) as well as the hard vertices producing the colour-singlet particles. One observes that the hard vertices occur at the same point in spacetime for double Drell–Yan, and that the locations of the collinear lines and hard vertices are the same for single and double Drell–Yan. Thus, from the perspective of soft long-range gluons the two processes look essentially the same, and the Glauber cancellation that works for single Drell–Yan should also work for double Drell–Yan, as we indeed found.

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