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CANONICAL DUAL APPROACH TO SOLVING 0-1 QUADRATIC PROGRAMMING PROBLEMS

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Abstract. By using the *canonical dual transformation* developed recently, we derive a pair of canonical dual problems for 0-1 quadratic programming problems in both minimization and maximization form. Regardless convexity, when the canonical duals are solvable, no duality gap exists between the primal and corresponding dual problems. Both global and local optimality conditions are given. An algorithm is presented for finding global minimizers, even when the primal objective function is not convex. Examples are included to illustrate this new approach.

1. **Introduction.**

In this paper, we consider a simple 0-1 quadratic programming problem in the following form:

$$(\mathcal{P}): \qquad \min/\max\left\{P(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T Q\mathbf{x} - \mathbf{f}^T\mathbf{x} \ \mid \ \mathbf{x} \in \mathcal{X}_a\right\}, \tag{1}$$

where $\mathbf{x}$ and $\mathbf{f}$ are real n-vectors, $Q \in \mathbb{R}^{n\times n}$ is a symmetrical matrix of order n and

$$\mathcal{X}_a = \{\mathbf{x} \in \mathbb{R}^n \mid 0 \le x_i \le 1, \ i = 1, 2, ..., n\} \cap \mathcal{I}^n. \tag{2}$$

with $\mathcal{I}^n = \{\mathbf{x} \in \mathbb{R}^n | \ \ x_i \text{ is an integer}, \ \ i = 1, 2, ..., n\}$. Since the Q matrix in the objective function $P(\mathbf{x})$ can be indefinite, we use the notation "min/max" to indicate that we are interested in finding both minimizers and maximizers.

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