Capacitated location-routing problem with time windows under uncertainty

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Abstract

This paper puts forward a location-routing problem with time windows (LRPTW) under uncertainty. It has been assumed that demands of customers and travel times are fuzzy variables. A fuzzy chance constrained programming (CCP) model has been designed using credibility theory and a simulation-embedded simulated annealing (SA) algorithm is presented in order to solve the problem. To initialize solutions of SA, a heuristic method based on fuzzy *c*-means (FCM) clustering with Mahalanobis distance and sweep method has been employed. The numerical experiments which were carried out, clearly attest that the proposed solution approach is both effective and robust in solving problems with up to 100 demand nodes in reasonable times.

Keywords. Transportation, Location-routing problem (LRP), Time windows, Uncertainty, Fuzzy variables, Fuzzy clustering.

1. Introduction

Supply chain management (SCM) is the process of planning, implementing and controlling the operations of the supply chain in an efficient way. SCM spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption (Melo, Nickel et al. 2009). Design of a distribution network is a fundamental step in building an efficient supply chain. In design of a distribution network, there are many decisions to be made varying from determination of number of supply chain layers to finding the optimal location(s) of facilities. These decisions are often categorized as strategic, tactical, and operational levels. A strategic or long-term decision does not take place on a regular basis and needs major capital investments. A tactical decision is made more often than a strategic decision. An instance of a tactical decision is vehicle routing problem (VRP). Finally, the operational decisions such as scheduling are those decisions that take place regularly. The location-routing problem (LRP) integrates the strategic (location) and tactical (routing) levels. A location-routing problem (LRP) may be defined as a special case of vehicle routing problem (VRP) in which there is a need to determine the optimal number and location of depots simultaneously with finding distribution routes. LRP is an NP-hard problem, as it encompasses two NP-hard problems (Nagy and Salhi 2007) and has many real-life applications of which some have been addressed in the literature such as management of hazardous wastes (Alumur and Kara 2007). Many 21st century managers believe that the success of their organizations highly depends on the location and routing decisions.

The dynamic and complex nature of supply chain imposes a high degree of uncertainty in supply chain planning decisions and significantly influences the overall performance of the supply chain network (Klibi, Martel et al. 2010). A problem under uncertainty may be modeled using various approaches such as using random variables or fuzzy variables. Whether to use fuzzy or random variables in a model directly depends on the semantic of the problem and also the availability of reliable data. Although many problems can be modeled using random variables, there are some instances in which it becomes almost impossible or irrational to use random variables, such as:

(a) where there are not enough data to be used to model the problem

(b) the available data is not reliable and error-prone

Besides, using scenario-based approaches which are employed in stochastic approaches, a large number of scenarios used in representing the uncertainty can lead to computationally challenging problems (Pishvaee and Torabi 2010) Hence, using fuzzy logic to model many real-world problems seems more reasonable. To put it in simpler terms, fuzzy variables can represent the uncertainty inherent in some problems in a better way. Fuzzy location routing problem (FLRP) arises whenever some elements of the problem are uncertain or ambiguous. For instance, the information about demand of a customer may be imprecise to some extent and may be estimated as “around 10 units” or “between 15 and 20 containers”. Moreover, as is shown in figure 1, the time to travel between the nodes in a graph can be estimated as a fuzzy variable.

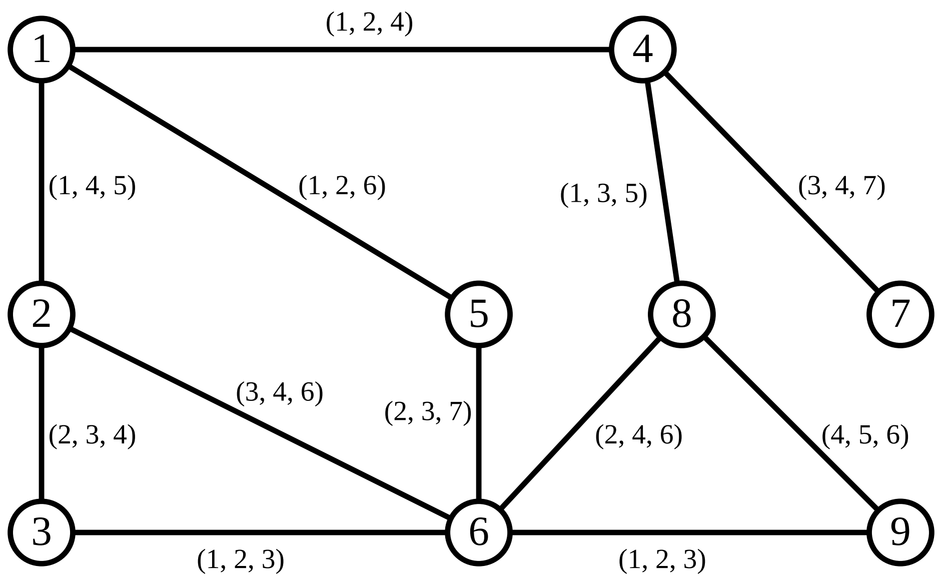

Figure 1. A sample network with fuzzy travel times

To the best of our knowledge, the problem of LRP under fuzziness has not been addressed enough in the literature. Hence, our paper makes the following contributions to the knowledge pool. First, a chance-constrained programming model (CCP) of LRPTW has been proposed with fuzzy demands and fuzzy travel times (FLRPTW). As far as we know, this is the first attempt to model and solve LRPTW with fuzzy demands and fuzzy travel times simultaneously. Secondly, a simulation-embedded simulated annealing has been presented to solve the LRPTW. Moreover, an effective initialization method based on fuzzy clustering has been presented which provides the solution algorithm with good initial solutions.

The rest of the paper is organized as follows. In the next section, the literature review of location routing problems and credibility theory is presented. A concise review of fuzzy variables and basics of credibility theory are presented in section 3. Section 4 is devoted to our problem definition and the mathematical formulation of the problem. The solution methodology is elaborated in section 5 and numerical experiments appear in section 6. Finally, conclusions are made and some possible future research outlooks are proposed.

2. Literature review

A survey on LRP literature shows that the research on LRP has attracted relatively less attention compared to various vehicle routing problems or location variants. There exists some review papers dedicated to the LRP literature such as (Balakrishnan, Ward et al. 1987), (Laporte 1989), (Min, Jayaraman et al. 1998), and (Nagy and Salhi 2007). Interested readers are recommended to refer to (Nagy and Salhi 2007) and references therein for a more detailed review of LRP models, extensions and solution methods.

Different variants of LRP have been targeted in the literature and various solution methods have been employed in order to solve them. These solution methods may be categorized as exact, heuristic, and metaheuristic approaches. For instance, (Cappanera, Gallo et al. 2003) presented an obnoxious facility location-routing (OFLR) problem in which lagrangean relaxation (LR) was used to decompose the problem into two subproblems of location and routing and two lagrangean heuristics were presented. (Marinakis and Marainaki 2008) combined particle swarm optimization (PSO), greedy randomized adaptive search procedure (GRASP), expanding neighborhood search (ENS) and path relinking (PR) to solve LRP. Using a combination of GRASP and evolutionary local search (ELS), (Duhamel, Lacomme et al. 2010) solved a Capacitated LRP. (Ambrosino, Sciomachen et al. 2009) considered a distribution network design problem and a two-phase heuristic is presented to solve the problem. A single depot LRP was studied by (Schwardt and Fischer 2009) and a self-organizing map (SOM) approach was proposed to solve it. (Nguyen, Prins et al.) proposed a multi-start iterated local search and compared the performance of their method with four published metaheuristics. (Barreto, Ferreira et al. 2007) considered integration of several hierarchical and non-hierarchical clustering methods in addition to several proximity measures to solve the LRP. They compared the results of running their procedure on standard LRP datasets and results were analyzed. In another attempt to solve LRPs with metaheuristics, (Nguyen, Prins et al. 2012) presented a hybrid of GRASP, learning process, and also path relinking to solve a two-echelon LRP. (Derbel, Jarboui et al.) considered a LRP with multiple capacitated depots and one uncapacitated vehicle per depot. They presented a combination of genetic algorithm and iterated local search.

Moreover, some location-routing models for realistic scenarios have been published. (Alumur and Kara 2007) studied a multiobjective LRP for collection, transportation, treatment and disposal of hazardous material. They presented a mixed integer programming model for such a problem and solved a real-world sample with 92 generation nodes. (Ambrosino, Sciomachen et al. 2009) is another real-world case of LRP in Italy addressing the problems of location, fleet assignment and routing with one central depot and heterogeneous fleet of vehicles. Moreover, they solved the problem using a large neighborhood search algorithm. Another real-world case study of LRP is (Caballero, González et al. 2007) in which a multiobjective LRP in Andalusia, Spain is presented and a tabu search approach is presented to solve the problem.

As mentioned earlier, FLRP arises whenever some elements of the problem are uncertain, ambiguous, or vague. Generally, fuzzy variables can be emplyed to deal with many uncertain parameters. (Zheng and Liu 2006) and (Zarandi, Hemmati et al. 2011) surveyed routing problems with fuzzy travel times, and presented chance constrained programming (CCP) models using the credibility measure. They integrated fuzzy simulation and metaheuristics to design a hybrid intelligent algorithm to solve their models. The FLRP presented in this paper differs from its deterministic counterpart in several fundamental respects.

Credibility theory has been used in many problems with fuzzy parameters so far, in parallel with some metaheuristics. Table 1 gives a brief review of using credibility theory to solve various problems. Each of the publications cited in table 1 shows the applicability of credibility theory to a real-world problem. In the following sections, the applicability of this theory to solve FLRPTW will be presented.

Table 1. Some problems solved in fuzzy environment using credibility theory

Author	**Problem**	**Solution Procedure**
(Peng and Liu 2004)	Parallel machine scheduling	Genetic algorithm
(Zhao and Liu 2005)	Standby redundancy optimization	Genetic algorithm
(Zheng and Liu 2006)	Vehicle routing problem	Genetic algorithm
(Liu and Li 2006)	Quadratic assignment problem	Genetic algorithm
(Yang and Liu 2007)	Fixed charge solid transportation	Tabu search
(Zhou and Liu 2007)	Location-allocation problem	Genetic algorithm
(Xiaoxia 2008)	Portfolio selection	Genetic algorithm
(Erbao and Mingyong 2009)	Vehicle routing problem	Differential evolution
(Lan, Liu et al. 2009)	Multi-period production planning	Particle swarm optimization
(Liu and Gao 2009)	Multi-job assignment problem	Genetic algorithm
(Li, Zhang et al. 2009)	Portfolio selection	Simulated annealing
(Ke and Liu 2010)	Project scheduling	Genetic algorithm
(Lau, Jiang et al. 2010)	Distribution system design	Genetic algorithm
(Zarandi, Hemmati et al. 2011)	Location-routing problem	Simulated annealing
(Davari, Fazel Zarandi et al.)	Maximal covering location problem	Simulated annealing
(Li, Qin et al. 2011)	Trip distribution problem	Genetic algorithm
(Wang, Fu et al.)	Inventory control	Differential evolution
(Davari and Fazel Zarandi 2011)	Hub location problem	Variable neighborhood search

3. Credibility theory

The term “Fuzzy variable” was coined by (Kaufmann 1975) and then discussed in (Zadeh 1975) and (Nahmias 1978). Later, possibility theory was proposed by (Zadeh 1978) and its extensions and developments were followed by (Dubois and Prade 1988). A modification to possibility theory which is called credibility theory was founded by (Liu 2009) and recently have been studied by many scholars all over the world. Since a fuzzy version of LRP in credibility space will be considered in this paper, a brief introduction to basic concepts and definitions used in this paper are presented as follows:

Definition 1 (Liu 2009). Let Θ be a nonempty set, the power set of Θ, and Cr a credibility measure. Then the triplet (Θ, P, Cr) is called a credibility space.

Definition 2 (Liu 2009)**.** A fuzzy variable is defined as a function from the credibility space (Θ, $P(\Theta)$, Cr) to the set of real numbers.

Definition 3 (Liu 2009)**.** Let ξ be a fuzzy variable on the credibility space (Θ, $P(\Theta)$, Cr). Then its membership function is derived from the credibility measure Cr by:

$$\mu(x)=(2\mathrm{Cr}\{\xi=x\})\wedge 1, \quad x\in\Re \tag{1}$$

Definition 4 (Zheng and Liu 2006). Let ξ be a fuzzy variable on a possibility space (Θ,$P(\Theta)$,Pos). Then the set

$$\xi_\alpha=\{\xi(\theta)\,|\,\theta\in\Theta,\mathrm{Pos}\{\theta\}\geq\alpha\} \tag{2}$$

is called the α-level set of ξ.

Definition 5 (Zheng and Liu 2006)**.** Let (Θ, $P(\Theta)$, Pos) be a possibility space, and A be a set in $P(\Theta)$. Then the credibility measure of A is defined by $\mathrm{Cr}\{A\}=\frac{1}{2}(\mathrm{Pos}\{A\}+\mathrm{Nec}\{A\})$ which is a self-dual measure. (Possibility and necessity measures lack the self-duality property)

If the membership function of ξ is given as μ (u is an event), then the possibility, necessity, credibility of the fuzzy event $\{\xi\geq r\}$ can be represented by:

$$\mathrm{Pos}\{\xi\geq r\}=\sup_{\mathrm{u}\geq \mathrm{r}}\mu(\mathrm{u}) \tag{3}$$

$$\mathrm{Nec}\{\xi\geq r\}=1-\sup_{\mathrm{u}<\mathrm{r}}\mu(\mathrm{u}) \tag{4}$$

$$\mathrm{Cr}\{\xi\geq r\}=\frac{1}{2}(\mathrm{Pos}\{\xi\geq r\}+\mathrm{Nec}\{\xi\geq r\}) \tag{5}$$

Considering equation (5), the credibility of a fuzzy event is defined as the average of its possibility and necessity. The credibility measure is self-dual. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. Now let us consider an example of a triangular fuzzy variable $\xi=(r_1, r_2, r_3)$ as shown in figure 2. From the definitions of possibility, necessity and credibility, it is easy to obtain:

$$\text{Pos}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_2 \\ \dfrac{r_3 - r}{r_3 - r_2} & \text{if } r_2 \leq r \leq r_3 \\ 0 & \text{if } r \geq r_3 \end{cases} \tag{6}$$

$$\text{Nec}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_1 \\ \dfrac{r_2 - r}{r_2 - r_1} & \text{if } r_1 \leq r \leq r_2 \\ 0 & \text{if } r \geq r_2 \end{cases} \tag{7}$$

$$\text{Cr}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_1 \\ \dfrac{2r_2 - r_1 - r}{2(r_2 - r_1)} & \text{if } r_1 \leq r \leq r_2 \\ \dfrac{r_3 - r}{2(r_3 - r_2)} & \text{if } r_2 \leq r \leq r_3 \\ 0 & \text{if } r \geq r_3 \end{cases} \tag{8}$$

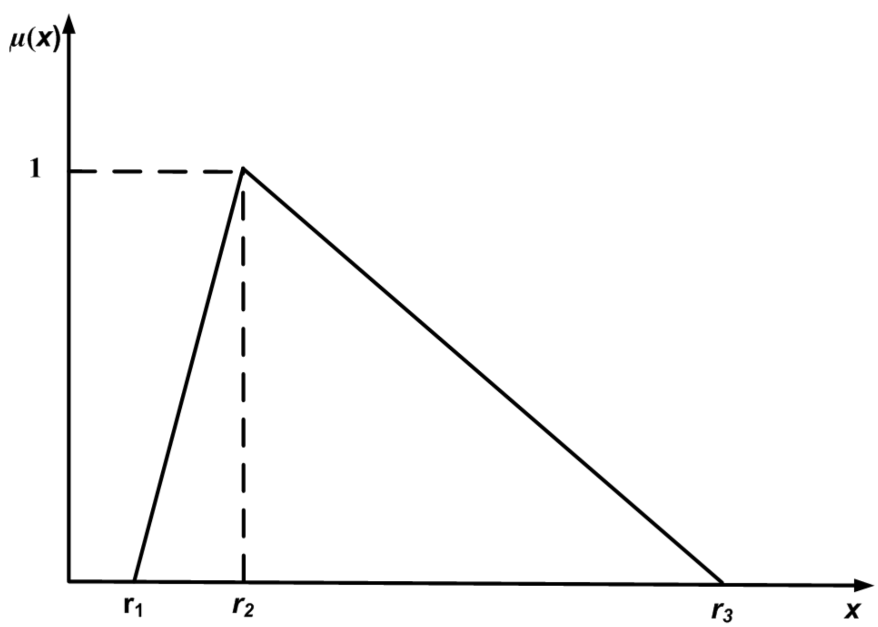

Figure 2. A triangular fuzzy variable

4. FLRPTW model

In this section, the fuzzy chance constrained programming (CCP) model of FLRPTW is presented. It is assumed that:

- The capacity of each vehicle is limited and denoted as C.
- A vehicle must be assigned to one and only one route.
- Each demand node must be served by one and only one vehicle.
- Each route must begin from and end at a single depot.
- Each potential depot has its distinct capacity QD_i.
- The numbers of depots to be located and vehicles to be used are variable. Each depot and vehicle has a fixed opening cost, which are denoted as ψ and η respectively. Moreover, there is a unit cost of transportation which is shown as θ.
- The demand of customers can be estimated as triangular fuzzy numbers d_i=(d_{1i}, d_{2i}, d_{3i}).

It is worth to note that after serving the first k customers, the available capacity of the vehicle will equal to $Q_k = C - \sum_{i=1}^{k} d_i$. It can be deduced that Q_k is also a triangular fuzzy number by using the rules of fuzzy arithmetic, where, $Q_k = (C - \sum_{i=1}^{k} d_{3i}, C - \sum_{i=1}^{k} d_{2i}, C - \sum_{i=1}^{k} d_{1i}) = (q_{1,k}, q_{2,k}, q_{3,k})$. The credibility that the next customer's demand does not exceed the remaining capacity of the vehicle can be obtained by (9) as follows:

$$\mathrm{Cr} = \mathrm{Cr}\{d_{k+1} \le Q_k\} = \mathrm{Cr}\{(d_{1,k+1} - q_{3,k}, d_{2,k+1} - q_{2,k}, d_{3,k+1} - q_{1,k}) \le 0\} =$$

$$\begin{cases} 0 & \text{if } d_{1,k+1} \ge q_{3,k} \\ \dfrac{q_{3,k} - d_{1,k+1}}{2 \times (q_{3,k} - d_{1,k+1} + d_{2,k+1} - q_{2,k})} & \text{if } d_{1,k+1} \le q_{3,k}, d_{2,k+1} \ge q_{2,k} \\ \dfrac{d_{3,k+1} - q_{1,k} - 2 \times (d_{2,k+1} - q_{2,k})}{2 \times (q_{2,k} - d_{2,k+1} + d_{3,k+1} - q_{1,k})} & \text{if } d_{2,k+1} \le q_{2,k}, d_{3,k+1} \ge q_{1,k} \\ 1 & \text{if } d_{3,k+1} \le q_{1,k} \end{cases} \tag{9}$$

Obviously, the vehicle's chance of being able to serve the next customer is higher if there is more remaining capacity in the vehicle and the demand at the next customer is less.

The value of $Cr \in [0,1]$ shows the level of certainty about the ability of a vehicle to serve a new customer. When Cr=0, the vehicle is not at all able to serve the next customer and it must return to the depot. On the other side, when Cr=1, the vehicle will be definitely able to serve the next customer. Clearly, the values of Cr between 0 and 1 represent a partial certainty about the ability of a vehicle to serve the next customer.

In this paper, a decision must be made whether to send a vehicle to serve the next customer or return to depot and to dispatch another vehicle to start serving the new demand node. In other words, a vehicle can serve a customer and dispatched to the location of new customer provided that $Cr \geq Cr^*$. Otherwise, the vehicle returns to its depot and starts a new tour. $Cr^* \in [0,1]$ is the dispatcher preference index which can affect the solution considerably. If the dispatcher desires to take a risk, the lower values of Cr^* are preferred which indicates that the dispatcher tries to use vehicle's capacity as much as possible. This can bring about some problems when the vehicle arrives at the next customer and is not able to carry out planned service due to insufficient available capacity. On the other hand, if the dispatcher is risk-averse, they choose the greater Cr^* in order to send the vehicle to the next customer with higher certitude, but they use the capacity of the vehicles less efficiently.

In this paper, an additional cost is considered to calculate the "failure" of the planned route. This idea derives from (Erbao and Mingyong 2009) which considers the additional distance when the vehicle needs to travel due to "failure" arising in some customers along the route when evaluating the planned route. As already stated, Cr^* which is subjectively determined can have a considerable impact on both the total length of the planned routes and on the additional distance covered by vehicles due to "failures" at some customers. Although higher values of Cr* result in shorter planned distances, the lower values increase the number of situations in which vehicles arrive at a customer and are unable to serve them. In addition, higher values of parameter Cr^* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. The problem logically arises of determining the value of parameter CrV^* which results in the least total sum of planned route lengths and additional distance covered by vehicles due to failure.

In this research, we use a simulation-based approach to evaluate the additional distance due to route failure. Before presenting the mathematical model of the problem, notations, parameters and variables are introduced as follows. In this problem, i=1, 2,…, d represent the indices of depots, i=d+1, d+2,…, d+n are allocated to customers, and k=1, 2,…, m are indices of vehicles. The demand of each node i is shown as d_i. The physical capacity of vehicles and depots are represented as C and QD_i respectively. The parameter S_i is the unloading time at customer i=d+1, d+2, …, d+n. Moreover, D_{ij} and T_{ij} are the travel distance and fuzzy travel time between nodes i=1,2,…,d+n and j=1,2,…,d+n. Finally, $[a_i, b_i]$ is the time window of node i where a_i and b_i are the lower and upper limits of the time window and i = 1, 2, ..., d+n.

In this paper, the operational plan is encoded by three decision vectors x, y and z, where $X=(x_1,x_2,...,x_n)$ is an integer decision vector representing n customers as a rearrangement of $\{1,\ldots,n\}$ $1\le x_i \le n$, $x_i \ne x_j$ $(i\ne j)$, i, j=1, 2,…, n. Moreover, $Y=(y_1,y_2,...,y_m)$ is a vector of integer decision variables where $y_0 \equiv 0 \le y_1 \le y_2 \le \ldots \le y_{m-1} \le n \equiv y_m$. In addition, $Z=(z_1,z_2,...,z_m)$ is an integer decision vector concerning depots $1\le z_k \le d$, k=1,2,…, m

Let $f_i(x, y, z)$ show the arrival time of a vehicle at customer i. We assume that a vehicle which arrives at a customer before the start of time window must wait till the time window. On the contrary, if a vehicle arrives within the time window, the service is immediately started. For each used vehicle k, we have:

$$\begin{cases} f_{d+x_{y_{k-1}+1}}(x,y,z) = T_{z_k, d+x_{y_{k-1}}+1} \\ f_{d+x_{y_{k-1}+j}}(x,y,z) = f_{d+x_{y_{k-1}+j-1}}(x,y,z) \vee a_{d+x_{y_{k-1}+j-1}} + S_{x_{y_{k-1}+j-1}} + T_{d+x_{y_{k-1}+j-1}, d+x_{y_{k-1}+j}} \\ f_{z_k}(x,y,z) = f_{d+x_{y_k}}(x,y,z) \vee a_{d+x_{y_k}} + S_{x_{y_k}} + T_{d+x_{y_k}, z_k} \end{cases} \quad (10)$$

where $\vee$ is the maximum operator. It is easy to know that since travel times are considered to be fuzzy variables, $f_i(x, y, z)$ is a fuzzy variable for each i-1, 2, …, n. Let $g(x, y, z)$ be the total travel cost of vehicles. Then, we have:

$$g(x,y,z) = (m \times \eta) + (\theta \times \sum_{k=1}^{m} g_k(x,y,z)) + (\sum_{i=1}^{d} (U_i * \psi_i)) \tag{11}$$

$$g_k(x,y,z) = D_{z_k, d+x_{y_{k-1}+1}} + \sum_{j=y_{k-1}+1}^{y_k - 1} D_{d+x_j, d+x_{j+1}} + D_{d+x_{y_k}, z_k} \tag{12}$$

Knowing that travel times are fuzzy variables and not known precisely, a chance constraint is added to the model estimating the credibility that customers are visited within their time windows. This constraint can be modeled using the credibility measure as follows:

$$Cr\{ f_i(x,y,z) \in [a_i, b_i] \} \geq \alpha \qquad i = 1,2,...,d+n \tag{13}$$

In addition, there are two additional chance constraints in the model. Constraint (14) assures that all customers are visited within the vehicle capacity with a confidence level. Moreover, constraint (15) assures that all routes are visited within their depot capacity with a predetermined confidence level.

$$Cr(\sum_{j=y_{k-1}+1}^{y_k} d_{x_j} \leq C) \geq CrV^*, \qquad k = 1, 2, \ldots, m \tag{14}$$

$$Cr(\sum_{k=1}^{m} \sum_{j=y_{k-1}+1}^{y_k} d_{x_j} . t_{ki} \leq QD_i) \geq CrD^*, \qquad i = 1, 2, \ldots, d \tag{15}$$

Equation (16) states that the assignment of a vehicle to a depot is a binary variable. Constraint (17) holds that each vehicle is assigned to one and only one depot. Constraint (18) guarantees that U variables are binary. Constraint (19) holds that a vehicle is assigned to a depot, if and only if the depot is opened. Because of the strategic nature of determining depots in this paper, we assume that CrD^* is equal to 1. It means that the depot has sufficient capacity to serve a route.

$$t_{ki} = \begin{cases} 1 & \text{If vehicle } k \text{ is assigned to depot } i \\ 0 & \text{Otherwise} \end{cases} \tag{16}$$

$$\sum_{i=1}^{d} t_{ki} = 1 \quad k = 1,...,m \tag{17}$$

$$U_i = \begin{cases} 1 & \text{If depot } i \text{ is used} \\ 0 & \text{Otherwise} \end{cases} \tag{18}$$

$$t_{ki} \le U_i \quad i = 1,2,...,d; k = 1,2,...,m \tag{19}$$

Thus, the objective of our model is to minimize the total travel cost of vehicles which is represented as $g(x,y,z)$. Moreover, the objective function c' seeks to minimize total additional travel distance due to route failures which is found using the proposed simulation mechanism. The model of this paper can be summarized as follows:

$$\begin{aligned}
&\text{Min}\, g(x, y, z) \\
&\text{Min}\, c' \\
&\text{subject to:} \\
&Cr\{ f_i(x, y, z) \in [a_i, b_i]\} \ge \alpha \quad i = 1,2,...,d+n \\
&Cr\{ \sum_{j=y_{k-1}+1}^{y_k} d_{x_j} \le C\} \ge CrV^* \quad k = 1,2,...,m \\
&Cr\{\sum_{k=1}^{m} \sum_{j=y_{k-1}+1}^{y_k} d_{x_j} t_{ki} \le QD_i\} \ge CrD^* \quad i = 1,2,...,d \\
&1 \le x_i \le n \quad i = 1,2,...,n \\
&x_i \ne x_j; i \ne j; i, j = 1,2,...,n \\
&0 = y_0 < y_1 < y_2 < ... < y_m = n \\
&1 \le \text{z}_\text{k} \le d \quad k = 1,2,...,m \\
&\sum_{i=1}^{d} t_{ki} = 1 \quad k = 1,2,...,m \\
&t_{ki} \le U_i \quad i = 1,2,...,d; k = 1,2,...,m \\
&x_i, y_j, z_k \text{ are integers} \quad i = 1,2,...,n; j = 1,2,...,m; k = 1,2,...,m \\
&t_{ki}, U_i \in \{0,1\}
\end{aligned} \tag{20}$$

LRP is easily reducible to VRP considering one located depot. Therefore, knowing that VRP is an NP-Hard problem, LRP is proven to be more combinatorial and is NP-Hard. This means that for larger instances of LRP, exact solution procedures are handicapped to solve the problem efficiently. Hence, one should resort to heuristics and metaheuristics to solve the problem efficiently. In this paper, a simulation-embedded simulated annealing is proposed in order to solve the fuzzy version of the proposed LRP.

5. Solution approach

In this work, due to the fuzzy nature of the travel times and the demands for each customer, the problem is not deterministic. Therefore, a simulation-embedded procedure is applied to solve the problem in the fuzzy environment. Moreover, to increase the efficiency of the proposed SA, an efficient initialization approach based on FCM is presented.

One may pose a question regarding the reasons behind selection of SA as the solution algorithm of this paper. Based on some preliminary experiments and results obtained in a relatively similar paper (Zarandi, Hemmati et al. 2011), it has been found out that SA can perform well in reaching high-quality solutions for LRPTW. Moreover, (Arostegui, Kadipasaoglu et al. 2006) showed that SA provides the best performance under solution limited situations of facility location problems and their variants. Due to the fact that the proposed solution algorithm needs simulation, there is a need to limit the runtime of solution algorithm. Hence, SA is preferred to be used in order to solve FLRPTW. In addition, the fuzzy simulation can be easily embedded within SA. Another justification to use SA is the fact that local search algorithms are easier to be employed in solving LRPTW and their runtimes are relatively less. These issues have led us to prefer SA to solve FLRPTW.

The following sections will elaborate these modules and how they contribute to the solution algorithm.

5.1. The proposed simulation algorithm

As mentioned above, it has been assumed that the travel times and the demand of each customer are triangular fuzzy numbers. In many of the real-world problems, the actual demand of a customer is known when the vehicle reaches the customer. In other words, there is not a pre-defined demand to be considered when routes are defined. The most prominent application of such an assumption is in reverse logistics where the amount of returns from a customer is completely unknown before meeting the customer's site. Hence, a simulation algorithm is useful to determine an approximation of additional distances (c') due to route failures. The proposed simulation algorithm of this paper is summarized as follows. Let $T=\{T_{ij}, i,j=0,1,2,\ldots,n\}$ and T_{ij} as the travel time between nodes i and j. Then, N random numbers T_{ij}^l are generated from the ε-level set of T_{ij} where l is the index of iteration number. In this step, N should be large enough and ε must be a sufficiently small number. Then, we set $T^l=\{T_{ij}^l;\ i,j=0,1,\ldots,n\}$ for $l=1,2,\ldots,N$ and $\mu(T^l)=\mu_{11}(T_{11}^l)\wedge\mu_{12}(T_{12}^l)\wedge\ldots\wedge\mu_{(n-1)n}(T_{(n-1)n}^l)\wedge\mu_{nn}(T_{nn}^l)$ where μ_{ij} is the membership degree of T_{ij}, i, $j=0,1,\ldots,n$. Then, the credibility can be estimated by the equation (21).

$$\mathrm{Cr}([a,b])=\frac{1}{2}\left(\max_{1\le l\le N}\{\mu(T^l)\mid f(x,y,t)(T^l)\in[a,b]\}+\min_{1\le l\le N}\{1-\mu(T^l)\mid f(x,y,t)(T^l)\notin[a,b]\}\right) \qquad (21)$$

Now, the additional distance to travel can be estimated by getting the average extra distance in M iterations.

5.2. Simulated annealing

Simulated annealing (SA) is a local search procedure which is capable of exploring the solution space stochastically and effectively. It tries to escape from local optima by accepting worse solutions during its search with a probability which is monotonically decreasing by temperature. SA was first introduced by (Metropolis, Rosenbluth et al. 1953) and has been applied to various combinatorial optimization problems such as scheduling (Damodaran and Vélez-Gallego 2012), facility layout (Wang, Wu et al. 2001), and network design (Xu, Wei et al. 2009).

In the following sections, the proposed SA of our paper will be discussed in detail, including solution representation, neighborhood generation, and fitness evaluation. Moreover, the tuning steps of the algorithm are presented.

5.2.1. Solution representation and initialization of solutions

In this paper, each solution is represented using a string of numbers. A solution representation should determine the assigned customers to each vehicle, the depots to be established and the sequence of customers to be served by a specific vehicle starting and ending at a depot. Considering *n* customers, *m* vehicles and *d* candidate locations for depots, a solution representation is comprised of $n+2m$ elements and incorporates three sections. The first section of each solution which has *n* elements is the *X* vector which has been already explained. This section shows the sequence of customers to be served by vehicles. The second and third sections of a solution representation show the *Y* and *Z* vectors respectively. The second section which is comprised of *m* elements is used to determine the customer indices to be served by a vehicle. Finally, the last section which has *m* elements exhibits the vehicles to start from each established depots. While the first two sections of the solution representation (The first $n+m$ elements) should be used together to decode the solution, the final *m* elements which represent the depots to be established may be decoded separately.

To clarify the encoding, a simple example of six customers, four vehicles and three candidate locations is presented. The representation and its counterpart are shown in figure 4. The first section of the string shows that customers should be served according to the order [7 1 6 3 4 2 5 8]. The second section determines which customers are served by each vehicle. Since there are four vehicles in this example, the second section is comprised of four elements. In the proposed representation, the customers with indices lying between the values of i^{th} and $(i+1)^{th}$ elements in the second section are served by a single vehicle. Clearly, the last element of the second section must be equal to *n*, considering *n* customers. In addition, the values in the second section must be ordered from smallest to the largest. In our sample solution representation, the customers with indices 1 and 2 (first and seventh customers) are served by a single facility, the customer with index 3 is served by a different vehicle, demand nodes with indices 4, 5, and 6 (second, third, and fourth customers) in the first section are served separately, and finally demands of customers with indices 7 and 8 in first section (Fifth and eighth customers) are served by the fourth vehicle. Moreover, the third part shows that vehicle 1 starts from the first depot, the second vehicle starts from the second depot, and the last two vehicles start serving customers from the third depot. It is easy to validate that the proposed representation is effective, short and easily decodable. Table 2 summarizes our representation considering *n* customers, *m* vehicles and *d* candidate locations for depots.

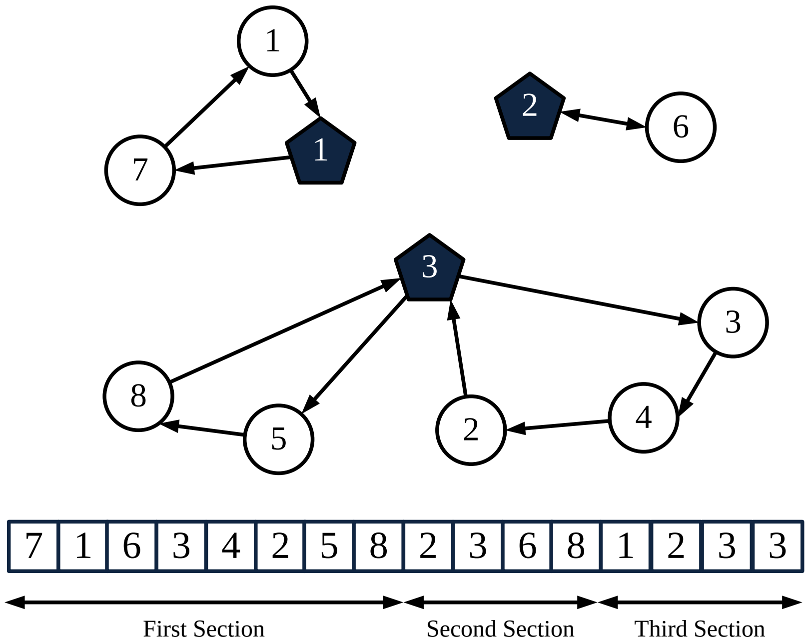

Figure 4. A sample solution representation of 8 customers and 3 available depots

Table 2. General information about the solution representation

	Range of values	Length	Note
First section	1, 2, …, *n*	*n*	
Second section	Some values between 1 and *n*	*m*	The last element must equal to *n*
Third section	Some values between 1 and *d*	*m*	

Initialization of solutions plays a significant role in reaching good solutions using local search methods. Hereby, a heuristic is proposed to generate initial solutions which are feasible and satisfy the relation $Cr \geq Cr^*$ before allocating any other customer to the current route. To do so, first a fuzzy *c*-means (FCM) algorithm is employed to cluster the customers, and then the sweep method is used to generate the customer arrangements. Finally, the proposed approach (which is derived from the idea presented in (Erbao and

Mingyong 2009)) is used to assign customers to routes and vehicles to depots, considering the chance constraints.

5.2.1.1. Fuzzy *c*-means clustering

Since the introduction of fuzzy partitioning by (Ruspini 1969), fuzzy clustering has been a valuable tool in various fields such as data mining, medicine, etc. Contrary to hard clustering algorithms, in a soft clustering algorithm, gradual membership values of data points to clusters are allowed. Later, the proposal of fuzzy *c*-means algorithm (Bezdek 1973) was a great move towards popularity of fuzzy clustering among scientists all over the world.

In a classic FCM, there are *n* data to be allocated to *c* clusters; *m* is a number greater than 1 (often equals 2), x_i is the i^{th} data, c_j is the center of j^{th} cluster and ||*|| is a norm representing the similarity of two vectors (in this paper, the Mahalanobis norm is used). Since its introduction, various extensions have been made to the classical hard clustering approach such as fuzzy *c*-means and possibilistic *c*-means algorithms; the main virtue of these methods is the softness they incorporate in assigning degrees of membership to the data points. The objective function of FCM is as follows (Bezdek 1973) and (Dunn 1973):

$$J_f(X,U_f,C)=\sum_{i=1}^{c}\sum_{j=1}^{n}u_{ij}{}^{m}d_{ij}{}^{2} \tag{22}$$

The procedure of FCM can be summarized as follows (Bezdek 1973) and (Dunn 1973):

I. Initialization of the *U* matrix (the size of matrix is *c***n*) representing the membership value of the i^{th} data to the k^{th} cluster.

II. In each step (step *k*), update the cluster centers using the following equation:

$$C_j=\frac{\sum_{i=1}^{n}x_{ij}u_{ij}{}^{m}}{\sum_{i=1}^{n}u_{ij}{}^{m}} \tag{23}$$

III. Update the *U* matrix using:

$$u_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{\|x_i - c_j\|}{\|x_i - c_k\|}\right)^{\frac{2}{m-1}}} \tag{24}$$

IV. If $\|U^{k+1}-U^{k}\|<\epsilon$, then stop the algorithm, otherwise go to step II.

For instance, figure 5 shows the clustered data (customers and depots) using FCM.

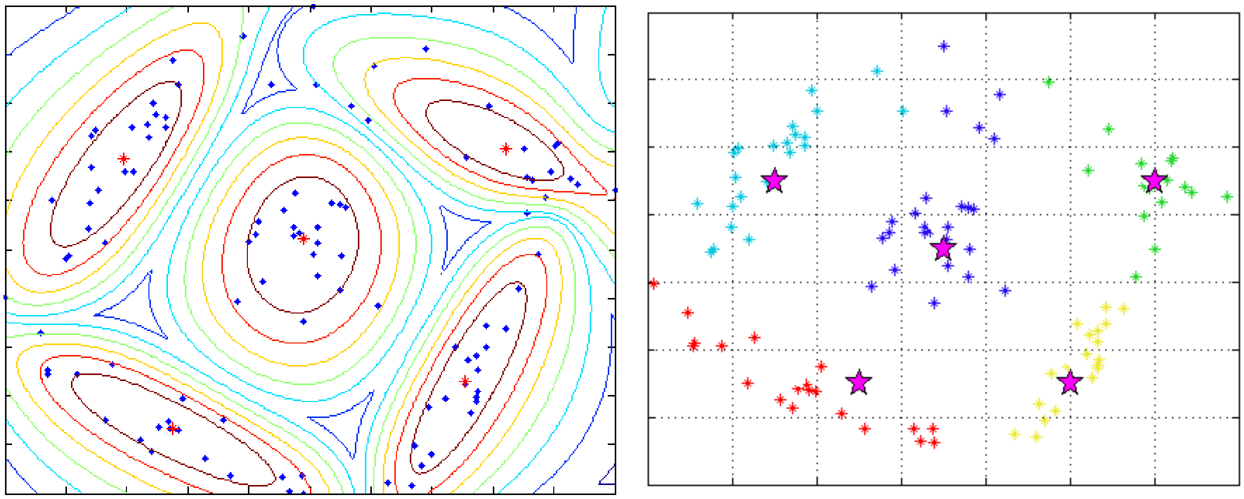

Figure 5. Left: FCM behavior for clustering random customers and depots

Right: Clustered customers (points) and depots (stars)

5.2.1.2. The proposed sweep method

In this paper, we use the results of the clustering of the customers and also depots for generating a sequence of customers such as the one shown in figure 6. To do so, in each cluster we follow the following steps:

- Set the cluster center as the core of sweeping.
- Set the sweep line in zero degree.
- For each customer in the current cluster, calculate the angle between the zero line (the line from cluster center through the zero degree) and a line from customer to cluster center.
- Sort the customers ascendingly by the angles.
- Sweep the customer with the sweep line from the lowest angle to the highest.

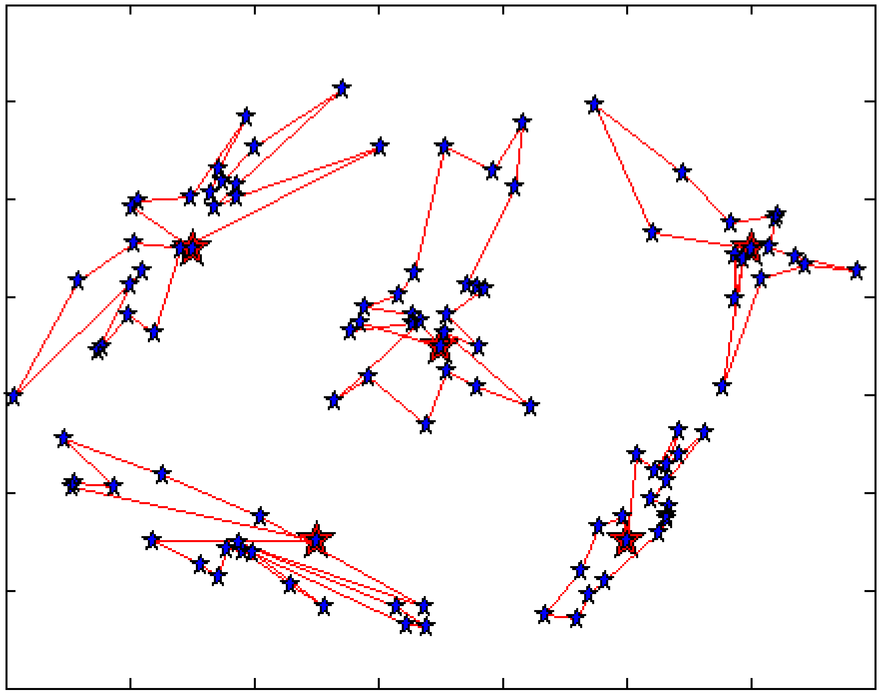

Figure 6. A sample of using the sweep method for clustered data

5.2.1.3. Feasibility of solutions

In the proposed solution procedure, the feasibility of routes are examined against both of the chance constraints. Verification of the chance constraints is composed of the following steps.

I. Select the first customer of a sequence according to the customer demand and the remained vehicle capacity and obtain the credibility of the solution using (9). For a dispatcher preference indices value CrV^* and CrD^*, if $Cr_1 \geq CrV^*$ and $Cr_2 \geq CrD^*$, then the customer is assigned to the current vehicle and depot; Cr_1 is related to the vehicle's capacity and Cr_2 is related to the depot capacity. Otherwise if $Cr_1 < CrV^*$, another vehicle (but the same depot) is used to service the customer. Finally, if $Cr_2 < CrD^*$, another vehicle is used in addition to a new starting depot.

II. Remove the first customer from the sequence.

III. Repeat steps I and II. If all of the customers have been assigned to routes, a feasible chromosome is obtained.

5.2.2. Neighborhood Structure

A neighborhood search structure (NSS) is a mechanism to get new solutions by slightly changing the current solution. In this paper, four NSS types are used called two-opt, shuffle (used for the first section), reorder (used for the second section) and mutate (used for the third section). A sample of these four moves is shown in figure 8. Generally, in an *r*-opt move, the values of *r* randomly chosen elements are substituted. A solution is *r*-optimal when it cannot be improved by any *r*-opt move and is shown as $r\text{-opt}^*$. In this paper, a two-opt move follows the same rule. In a shuffle move, two random indices are selected and the values between these two are shuffled randomly to get a new solution. Such a move has a stochastic character and is used in order to diversify the solutions. Moreover, when a solution is reordered, the second section is modified considering the rule that the last element of this section must be equal to the number of customers. In other words, while the last element does not change at all, the other elements may be changed completely. This move changes the group of customers served by each vehicle. Finally, a mutation is employed to change the allocation of vehicles to depots. To mutate, the value of one element is changed, so a new allocation plan of vehicles to depots is achieved. While the mutation move keeps the number of vehicles in the solution constant, it searches the solution space for a better utilization of vehicles.

In each itearation, one of these moves is used based on a Monte-Carlo approach. Then, the new generated solution is considered as the current solution, provided that the SA rules are not violated.

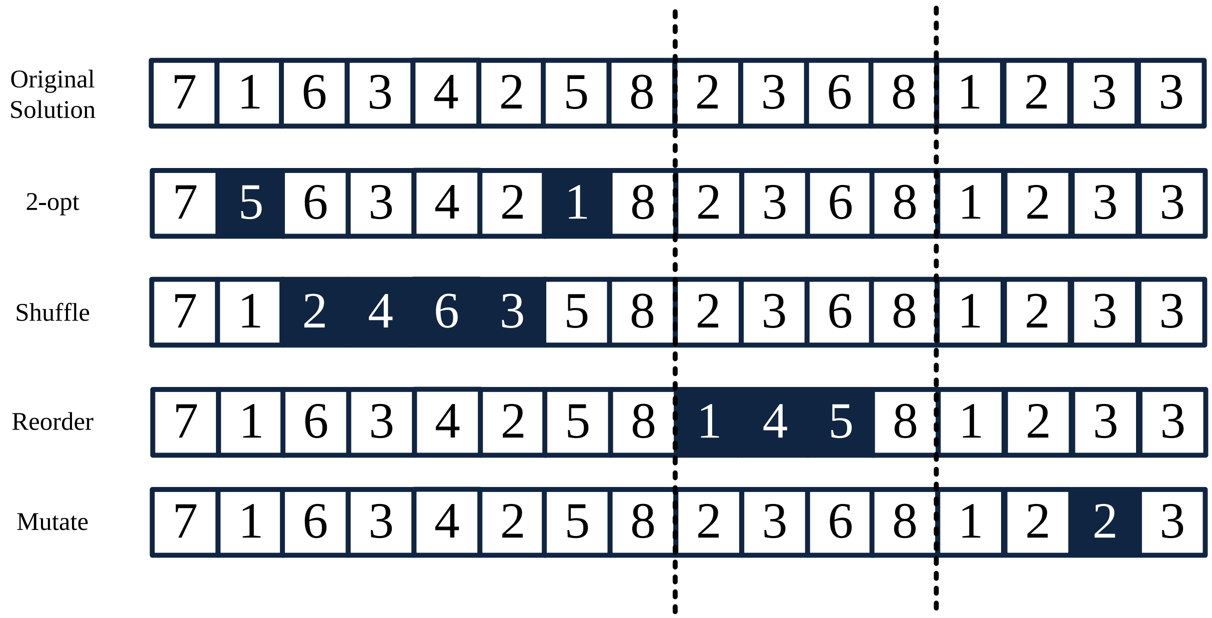

Figure 7. The four moves used in this paper

5.2.3. Initial temperature and cooling schedule

Initial temperature and the procedure to update temperatures are extremely important in success of any SA. As already mentioned, SA avoids getting trapped in local minima by letting worse moves based on a cooling schedule. There exist several types of cooling schedule such as linear or nonlinear methods. Three of these methods are linear, exponential, and hyperbolic cooling schedules as follows. Further details are obtainable in Lundy & Mees (Lundy and Mees 1986):

- Linear cooling rate: $T_l = T_0 - l\dfrac{T_0 - T_f}{N}; \quad l = 1,2,...,N$
- Exponential cooling rate: $T_l = \dfrac{A}{l+1} + B; A = \dfrac{(T_0 - T_f)(N+1)}{N}; B = T_0 - A; l = 1,2,...,N$
- Hyperbolic: $T_l = \dfrac{1}{2}(T_0 - T_f)(1 - tgh(\dfrac{10l}{N} - 5)) + T_f; l = 1,2,...,N$

In these equations, T_0, T_f, and T_l represent initial temperature, stopping temperature, and temperature of iteration l, respectively. Moreover, N is the number of temperatures between T_0 and T_f, and *tgh* is the tangent hyperbolic function. It should be noted that our proposed SA uses the exponential cooling rate based on outputs from some preliminary experiments.

To set the initial temperature, we have used the method of (Crama and Schyns 2003). The aim of this method is to get roughly equal probabilities of acceptance (χ_0=0.8 in this paper which has been selected among twenty values between 0.7 and 0.9) during the first *L* steps of SA. Therefore, in a preliminary phase, SA is run for *L* steps without rejecting any move at all. Then, the average increase of the objective function over this period is calculated and noted as Δ. Then, the initial temperature is found using (25):

$$T_0 = \frac{\Delta}{\ln \chi_0} \tag{25}$$

6. Numerical experiments and discussion

This section is devoted to the computational evaluation of test problems and some discussions. Hereby, an example is presented to show models that we have discussed before and how the proposed simulation-embedded simulated annealing performs.

It has been assumed that there are 100 customers and 5 depots and the coordinates of all customers and depots are generated randomly. The triangular fuzzy variables of travel times including the travel times between candidate depots are determined randomly based on the distance matrix. It should be noted that the time windows are identical for all the depots and also for all the non-depot nodes.

The fuzzy demands of customers are generated as a triangular fuzzy variable within the vehicle capacity *C*. The additional distances are obtained due to routes failure by the simulation algorithm which has been already explained. The relative parameters of the problem are listed in Table 3. We obtain the planned distances, additional distances and the total distances, and reveal the dispatcher preference index for vehicles CrV^* how influence these traveling distances.

Table 3. The parameters of the test problem used in this paper

n	*D*	*C*	*l*	*M*	CrD*
100	5	10	10000	100	1

Using a Monte-Carlo approach as neighborhood search, needs finding the best combination of move ratios. In other words, the rates by which each move is used should be tuned, in order to achieve good results. In this paper, five instances of (Prodhon) have been targeted using various combinations of moves. First, the chance of shuffle was set to 5% and then excluding the infeasible cases, we increase the value of two-opt and reorder moves from 0 to 1 with steps of 0.05. Among all the 210 possible combinations, the one with the best fitness is selected to solve test problems of this paper. Table 4 represents the optimal ratios of moves and the ratio of moves to be used. Results of this step clearly show that changing the location of depots should be regarded as the most significant factor to reach better solutions. Furthermore, the next effective change of a solution is changing the allocation of customers to vehicles.

Table 4. The ratio of moves used in the proposed model

Two-opt	Shuffle	Reorder	Mutate
0.15	0.05	0.3	0.50

Then, to show the performance of the proposed SA, the "20-5-1a" instance of (Prodhon) was solved using the proposed SA and results were compared with some other methods in the literature as is shown in table 5. The results show that the proposed SA reaches the optimal solution.

Table 5. Comparing results of the proposed approach and the exact solutions of two well-known datasets

	Problem Name	Random initial solution			Heuristic initial solution			Optimal Solution	Gap
		Min	Mean	Max	Min	Mean	Max		
(Prins and Prodh	20-5-1a	55835	57020	60159	54793	55835	57396	54793	0.00%
	20-5-1b	41478	42847	48858	39253	41928	43439	39104	0.38%

	20-5-2a	49199	50597	53272	48908	49457	50362	48908	0.00%
	20-5-2b	37936	40553	43095	37542	39280	41096	37542	0.00%
	50-5-1	99524	103304	115317	94084	98006	100908	90111	4.40%
Barreto (Barreto 2004)	Gaskell67-21x5	443.7	499.4	586.9	430.561	469.8	500.4	424.9	1.33%
	Gaskell67-22x5	599.6	690.5	801.2	586.698	607.6	636.7	585.1	0.27%
	Gaskell67-32x5	623.872	691.4937	758.3425	589.467	647.0904	684.3992	571.7	3.10%
	Gaskell67-32x5b	576.957	638.379	674.628	510.66	579.697	629.08	504.3	1.26%

Table 6. Comparing some solution procedures using the 20-5-1 dataset of (Prins and Prodhon 2004)

GRASP*				MAPM**			
Cost	SC***	RC****	Gap	Cost	SC	RC	Gap
55021	25549	29472	0.42	54793	25549	29244	0

LRGTS*****				Proposed Alg.			
Cost	SC	RC	Gap	Cost	SC	RC	Gap
55131	25549	29582	0.62	54793	25549	29244	0

* "Greedy adaptive search procedure" Presented in (Prins, Prodhon et al. 2006)
** "Memetic algorithm with population management" presented in (Prins, Prodhon et al. 2006)
*** The cost corresponding to the setup costs of the depots
**** The cost corresponding to the routing
**** "Lagrangean relaxation-granular tabu search" presented in (Prins, Prodhon et al. 2007)

The value of dispatcher preference index for vehicles CrV^* varied within [0,1] with a step size of 0.1. The computational results are presented in Table 6. Moreover, figure 9 shows the tendencies about the planned distances, additional distances due to failures at the customers, and total distances of the problem with different dispatcher preference indices for vehicles.

Table 7. The results with different CrV^*

CrV^*	Planned Distance	Additional Distance	Total Distance
0	71,211.51	27,611.72	98,823.23

0.1	80,506.69	18,357.26	98,863.95
0.2	86,754.3	16,763.13	103,517.4
0.3	85,915.81	16,188.48	102,104.3
0.4	90,226.42	10,747.92	100,974.3
0.5	95,553.26	12,78.374	96,831.64
0.6	**93,549.4**	**0**	**93,549.4**
0.7	97,206.98	0	97,206.98
0.8	96,873.37	0	96,873.37
0.9	102,561	0	102,561
1	102,454.6	0	102,454.6

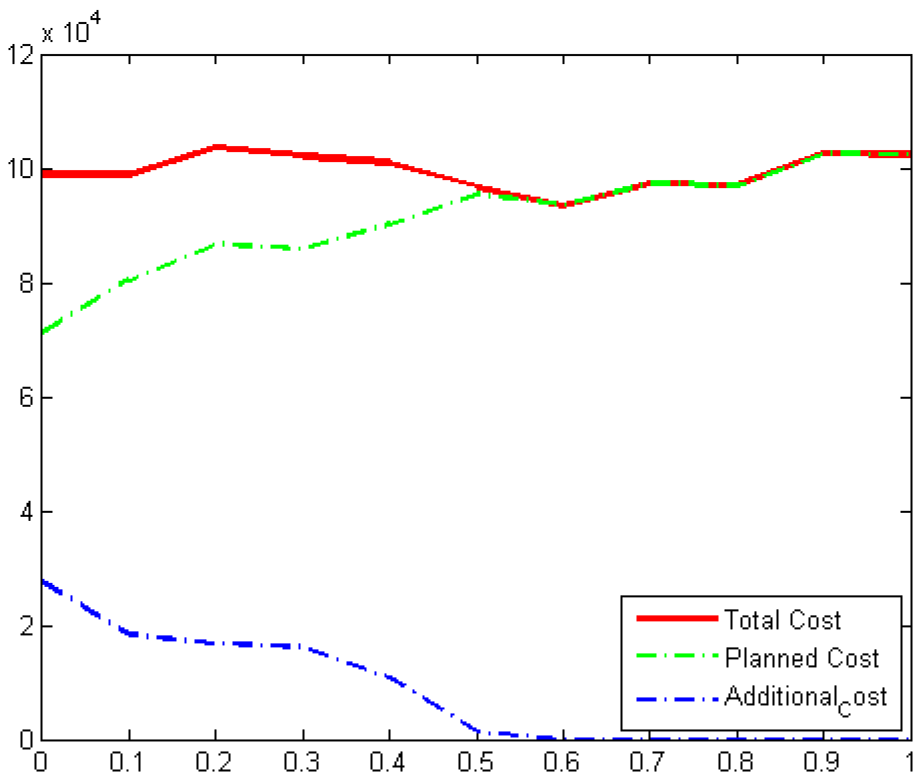

Figure 8. The costs of the problem with different CrV*

Table 6 shows that when dispatcher preference index for vehicles CrV* is higher, a strictly rising tendency in the planned routes and a strictly decrease in the additional distance that vehicles had to make due to failures at the customers is observed. When the dispatcher's preference index CrV^* is equal to 0.6, the value of total distance is minimized. Consequently, lower values of CrV^* bring about maximum use of the vehicle capacity. These values correspond to routes with shorter planned distances. On the other hand, lower values of parameter CrV^* increase the number of cases in which vehicles arrive at a customer and are unable to service that specific customer. This leads to increasing the total additional distance they cover due to the "failure". Higher values of parameter CrV^* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. Therefore, the dispatcher preference index for vehicles should be around 0.6 to get the optimal performance of the system.

Then, to examine the sensitivity of the best solution with respect to the values of α, another complementary test was carried out and results are reported in figure 10. The fuzzy version of the problem was solved using various levels of α between 0.9 and 1 with step sizes of 0.01. Figure 10 indicates that larger values of confidence level leads to larger values for optimal costs. Moreover, such an increase in the cost level is almost linear. This clearly shows that restricting the model to meet customer time windows strictly adds to the total cost. This finding seems reasonable and can be regarded as both a validation criterion for the proposed algorithm and also a sensitivity index.

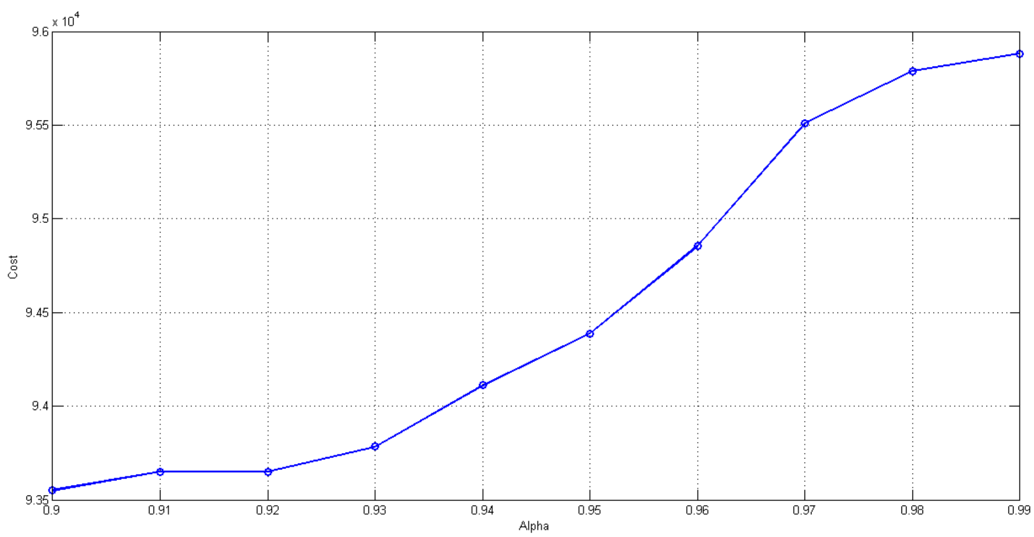

Figure 9. The sensitivity of solution with respect to the value of α

Figure 11 represents the performance of the proposed simulation-embedded simulated annealing and the final output of the problem.

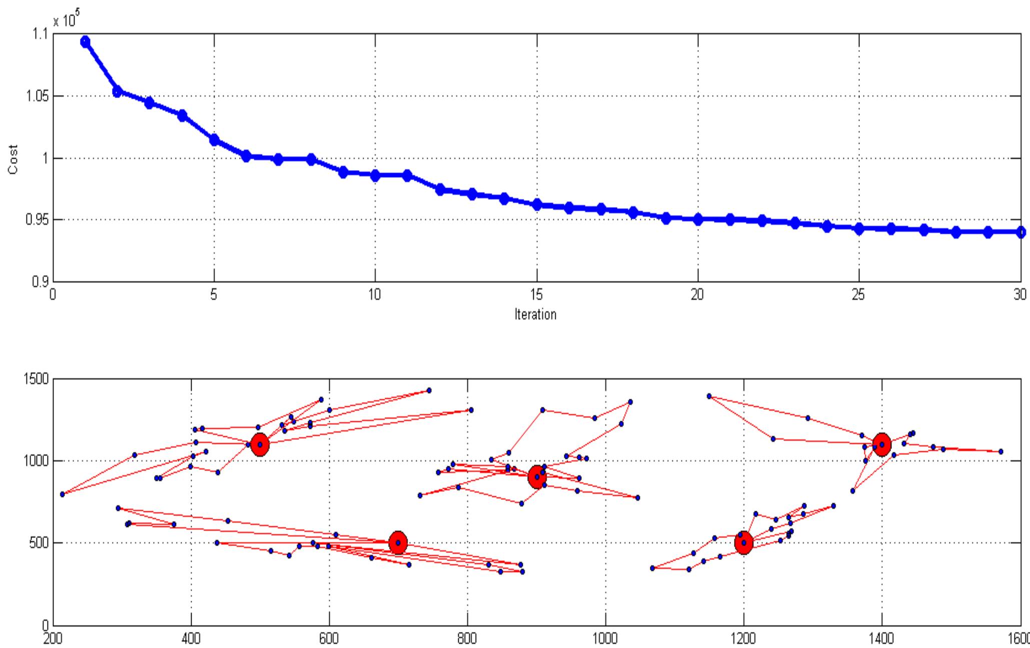

Figure 10. The performance of the proposed algorithm for a LRPTW under uncertainty

7. Conclusions and outlooks for future research

In this paper, an LRPTW is presented under uncertainty and a fuzzy chance-constrained programming formulation is given for it. To solve the problem, a simulation-embedded simulated annealing algorithm is proposed and its effectiveness was shown by a numerical example.

The paper contributes to the knowledge pool of LRP in the following respects. First, a chance-constrained mathematical formulation has been proposed for FLRPTW assuming fuzzy demands and travel times. Besides, a combination of FCM with Mahalanobis norm and a sweeping method has been employed to generate high-quality initial solutions which satisfy chance constraints. Another clear contribution of this paper is the proposal of a simulation-embedded SA which shows promising performance in dealing with various test problems.

However, there is still much opportunity to extend our work in some respects. One may approach other variants of the LRP such as LRPs where backups and deliveries are needed. Another valuable avenue for future research is to consider some other parameters of the problem as fuzzy variables, such as time windows. We believe that this can add to the ability of our paper to model real-world problems and will be a valuable extension. Although SA has shown an outstanding ability to solve the problem at hand, there is a possibility to use other heuristics or metaheuristics to solve the same problem or to conduct an empirical study to compare the strength of various approaches in solving the problem of this paper. This can be carried out on larger datasets available in the literature to show the strengths and weaknesses associated with each solution method. Last but not least, there is the possibility to extend our work using heterogeneous vehicles.

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