Capacitated lot sizing problem with production carryover and setup splitting: Alternate Mathematical Model and extension of the CLSP

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1. Further Work: An Overview

In the main work we have proposed a mathematical model for the Capacitated Lot Sizing Problem (CLSP) with production carryover and setup splitting, which can handle two scenarios namely: (1) situation/scenario where the setup costs and holding costs are product dependent and time independent, and with no backorders or lost sales, and (2) situation where the setup costs and holding costs are product dependent and time dependent, and with no backorders or lost sales. Section 1 of our main paper discusses three ways (numbered as (i), (ii) and (iii)) by which a machine can be setup for producing a product. The figures corresponding to the three ways are shown below in Figure 6. The X-axis in the Gantt chart denotes the time periods represented as 1, 2, 3 and so on. The shaded region indicates the setup and the blank region indicates the production.

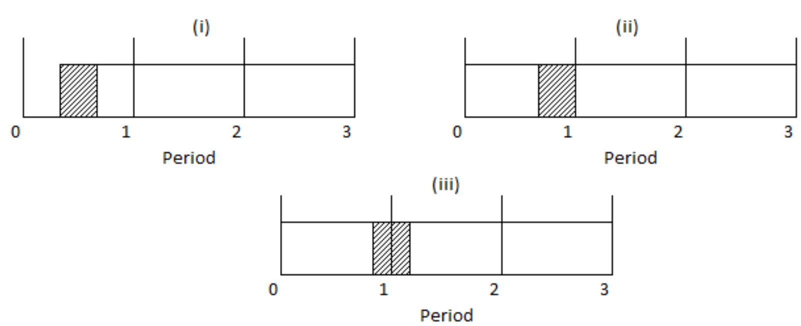

Figure 6. Three ways of a machine being setup in a period for production

We present another mathematical model (Alternate Mathematical Model (AMM)) for the same problem, in Section 2 of this extension work.

Following this, we present another possible extension of the proposed models for the situation where the setup costs and holding costs are product dependent and time dependent with no backorders or lost sales. This extension addresses a situation when there is a splitting of setup between periods t and t+1 for product i, there may arise a possibility (Scenario 2 **-** Variant (3)) in a manufacturing system where the setup cost is split between periods t and t+1 and this cost is proportional to the amount of time used to setup product i in periods t and t+1 respectively. The detail of one of the proposed models (MM) (with respect to Scenario 2 **-** Variant (3)) is presented in Section 3, with the numerical illustration presented in Section 4 of this extension work. We also discuss the results of Scenario 2 (Variants (1) and (2)) presented in the main work when the models are executed using only two binary variables (see Section 5 of this extension work). We use the data given in Tables 1 and 3 of our main work for the discussions given in the following sections.

The solution values (i.e., values of decision variables and Z) obtained by the model of Mohan *et al.* (2012), corresponding to Figure 1 of the main work, are provided in Table 4 of this extension work. The set of initial and terminal (boundary) conditions used to execute Mohan *et al.*'s model are as follows: $i_{i,0} = 0$; $v_{i,0} = 0$; $u_{i,0} = 0$; $z_{i,0} = 0$; $l_{i,0} = 0$ and $u_{i,T+1} = 0$. Also, the solution values obtained for our model for the situation where the setup costs and holding costs are product dependent and time dependent, corresponding to Figures 3 and 4 of the main work, are provided in Tables 5 and 6 of this extension work. The solution values corresponding to the Gantt chart given in Figure 5 of the main work are provided in Table 7 of this extension work. The numbering in this link is in continuation with the numbering in the main work, with respect to constraints, equations, tables and figures.

2. Alternate Mathematical Model (AMM)

Decision Variables

$\delta^{*}_{i,t}$: an indicator (binary) variable that takes value 1 if a complete setup is done for product i in period t with the production starting in period t and completed within period t;
0 otherwise.

$\delta^{**}_{i,t}$: an indicator (binary) variable that takes value 1 if a complete setup is done for product i in period t with the production starting in period t, and the production can also be carried over to the subsequent periods;
0 otherwise.

$\delta^{***}_{i,t}$: an indicator (binary) variable that takes value 1 if a setup for product i is started in period t and completed by the end of the period, followed by its production starting in period t+1; /* this aspect is called end-of-period setup in our paper. */
0 otherwise.

$\delta_{i,t}^{****}$: an indicator (binary) variable that takes value 1 if the setup for a product i is commenced in period t and is completed in period t+1, followed by its production starting in period t+1; /* this aspect is called setup splitting in our paper. */
0 otherwise.

$I_{i,t}$: inventory of product i at the end of period t.

$s_{i,t}^{*}$: setup time for product i in period t that takes the value of ST_i when $\delta_{i,t}^{*}$ = 1;
0 otherwise.

$s_{i,t}^{**}$: setup time for product i in period t that takes the value of ST_i when $\delta_{i,t}^{**}$ = 1;
0 otherwise.

$s_{i,t}^{***}$: setup time for product i in period t that takes the value of ST_i when $\delta_{i,t}^{***}$ = 1;
0 otherwise.

$s_{i,t}^{****}$: setup time for product i in period t when $\delta_{i,t}^{****}$ = 1;
0 otherwise.

$s_{i,t+1}^{*****}$: setup time for product i in period t+1 when $\delta_{i,t}^{****}$ = 1;
0 otherwise.

Note: $s_{i,t}^{****} + s_{i,t+1}^{*****} = ST_i$.

$X^{*}{}_{i,t,t}$: production quantity for product i in period t due to its setup and production started and completed within the same period t.

$X^{**}{}_{i,t,t'}$: production quantity for product i in period t' due to its setup started and completed within the same period t, with the possible production carryover to period t'+1, with $1 \leq t \leq T-1$ and $t \leq t' \leq T$.

$X^{***}{}_{i,t,t'}$: production quantity for product i in period t' due to its setup started and completed by the end of period t, with possible production carryover to period t'+1, with $1 \leq t \leq T-1$ and $t+1 \leq t' \leq T$.

$X^{****}{}_{i,t,t'}$: production quantity for product i in period t' due to its setup starting in period t and ending in period $t+1$, with possible production carryover to period t'+1, with $1 \leq t \leq T-1$ and $t+1 \leq t' \leq T$.

Mathematical model

Objective function:

$$\text{Min Z} = \sum_{i=1}^{N}\sum_{t=1}^{T} SC_i\delta_{i,t}^{*} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC_i\delta_{i,t}^{**} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC_i\delta_{i,t}^{***} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC_i\delta_{i,t}^{****} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{i,t} \qquad (41)$$

subject to the following:

/* Constraints (42)-(46) capture a possible complete setup in period *t* with production starting in period *t* and finished within period *t* */

$$s_{i,t}^{*} \leq ST_i\delta_{i,t}^{*} \qquad \forall i \text{ and } \forall t. \qquad (42)$$

$$s_{i,t}^{*} \geq ST_i\delta_{i,t}^{*} - M(1-\delta_{i,t}^{*}) \qquad \forall i \text{ and } \forall \text{t}. \qquad (43)$$

$$X^{*}{}_{i,t,t} \leq M\ \delta_{i,t}^{*} \qquad \forall i \text{ and} \forall t. \qquad (44)$$

$$X^{*}{}_{i,t,t} \geq \varepsilon - M(1-\delta_{i,t}^{*}) \qquad \forall i \text{ and } \forall t. \qquad (45)$$

$$a_i\ X^{*}{}_{i,t,t} \leq C_t\ \delta_{i,t}^{*} \qquad \forall i \text{ and} \forall t. \qquad (46)$$

/* Constraints (47)-(51) capture a possible complete setup in period *t* with production starting in period *t* and carried over to period *t*+1 and so on */

$$s_{i,t}^{**} \leq ST_i\delta_{i,t}^{**} \qquad \forall i \text{ and } t = 1,..,T-1. \qquad (47)$$

$$s_{i,t}^{**} \geq ST_i\delta_{i,t}^{**} - M(1-\delta_{i,t}^{**}) \qquad \forall i \text{ and } t = 1,..,T-1. \qquad (48)$$

$$X^{**}{}_{i,t,t'} \leq M\ \delta_{i,t}^{**} \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t, t+1,..,T. \qquad (49)$$

$$X^{**}{}_{i,t,t'} \geq \varepsilon - M(1-\delta_{i,t}^{**}) \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t, t+1. \qquad (50)$$

$$a_i\ X^{**}{}_{i,t,t'} \leq C_{t'}\ \delta_{i,t}^{**} \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t, t+1,..,T. \qquad (51)$$

/* Constraints (52)-(56) represent an end-of-period setup in period *t*, followed by the commencement of its production in period *t*+1 */

$$s_{i,t}^{***} \leq ST_i\delta_{i,t}^{***} \qquad \forall i \text{ and } t = 1,..,T-1. \qquad (52)$$

$$s_{i,t}^{***} \geq ST_i\delta_{i,t}^{***} - M(1-\delta_{i,t}^{***}) \qquad \forall i \text{ and } t = 1,..,T-1. \qquad (53)$$

$$X^{***}{}_{i,t,t'} \leq M\ \delta_{i,t}^{***} \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t+1, t+2,..,T. \qquad (54)$$

$$X^{***}{}_{i,t,t'} \geq \varepsilon - M(1-\delta_{i,t}^{***}) \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t+1. \qquad (55)$$

$$a_i\ X^{***}{}_{i,t,t'} \leq C_{t'}\delta_{i,t}^{***} \qquad \forall i, t = 1,..,T-1 \text{ and } t' = t+1, t+2,..,T. \qquad (56)$$

/* Constraints (57)-(63) capture the possible setup split between periods t and t+1, followed by the commencement of its production in period t+1 */

$$s_{i,t}^{****} \geq \varepsilon - M\left(1-\delta_{i,t}^{****}\right) \qquad \forall i \text{ and } t=1,..,T-1. \qquad (57)$$

$$s_{i,t+1}^{*****} \geq \varepsilon - M(1-\delta_{i,t}^{****}) \qquad \forall i \text{ and } t=1,..,T-1. \qquad (58)$$

$$s_{i,t}^{****} + s_{i,t+1}^{*****} \leq ST_i \delta_{i,t}^{****} \qquad \forall i \text{ and } t=1,..,T-1. \qquad (59)$$

$$s_{i,t}^{****} + s_{i,t+1}^{*****} \geq ST_i \delta_{i,t}^{****} - M(1-\delta_{i,t}^{****}) \qquad \forall i \text{ and } t=1,..,T-1. \qquad (60)$$

$$X^{****}{}_{i,t,t'} \leq M\, \delta_{i,t}^{****} \qquad \forall i, t=1,..T-1 \text{ and } t' = t+1, t+2,..,T. \qquad (61)$$

$$X^{****}{}_{i,t,t'} \geq \varepsilon - M(1-\delta_{i,t}^{****}) \qquad \forall i, t=1,..,T-1 \text{ and } t' = t+1. \qquad (62)$$

$$a_i\, X^{****}{}_{i,t,t'} \leq C_{t'}\, \delta_{i,t}^{****} \qquad \forall i, t=1,..,T-1 \text{ and } t' = t+1, t+2,..,T. \qquad (63)$$

/* Constraints (64)-(65) represent the conditions for a setup */

$$\sum_{i=1}^{N} (\delta_{i,t}^{**} + \delta_{i,t}^{***} + \delta_{i,t}^{****}) \leq 1, \qquad t=1,..,T-1. \qquad (64)$$

$$\left(\delta_{i,t}^{*} + \delta_{i,t}^{**} + \delta_{i.t}^{***} + \delta_{i.t}^{****}\right) \leq 1 \qquad \forall i \text{ and } t=1,..,T-1. \qquad (65)$$

/* Constraints (66)-(68) represent capacity constraints */

$$\sum_{i=1}^{N} (s_{i,1}^{*} + s_{i,1}^{**} + s_{i,1}^{***} + s_{i,1}^{****} + a_i\, X^{*}{}_{i,1,1} + a_i\, X^{**}{}_{i,1,1}) \leq C_1. \qquad (66)$$

$$\sum_{i=1}^{N} (s_{i,t}^{*} + s_{i,t}^{**} + s_{i,t}^{***} + s_{i,t}^{****} + s_{i,t}^{*****}) + \sum_{i=1}^{N} a_i\; X^{*}{}_{i,t,t} + \sum_{i=1}^{N} \sum_{t''=1}^{t} a_i\; X^{**}{}_{i,t'',t}$$
$$+ \sum_{i=1}^{N} \sum_{t''=1}^{t-1} (a_i\, X^{***}{}_{i,t'',t} + a_i\, X^{****}{}_{i,t'',t}) \leq C_t, \quad t=2,..,T-1. \qquad (67)$$

$$\sum_{i=1}^{N} (s_{i,T}^{*} + s_{i,T}^{*****}) + \sum_{i=1}^{N} a_i\, X^{*}{}_{i,t,t} + \sum_{i=1}^{N} \sum_{t''=1}^{T-1} (a_i\, X^{**}{}_{i,t'',T} + a_i\; X^{***}{}_{i,t'',T} + a_i\, X^{****}{}_{i,t'',T}) \leq C_T. \qquad (68)$$

/* Constraints (69)-(71) represent demand constraints */

$$X^{*}{}_{i,1,1} + X^{**}{}_{i,1,1} \geq d_{i,1} \qquad \forall i. \qquad (69)$$

$$\sum_{t''=1}^{t} X^{*}{}_{i,t'',t''} + \sum_{t'''=1}^{t} \sum_{t''=1}^{t'''} X^{**}{}_{i,t'',t'''} + \sum_{t'''=2}^{t} \sum_{t''=1}^{t'''-1} (X^{***}{}_{i,t'',t'''} + X^{****}{}_{i,t'',t'''}) \geq \sum_{t'''=1}^{t} d_{i,t'''}$$

$$\forall i \text{ and } t = 2,..,T-1. \quad (70)$$

$$\sum_{t''=1}^{T} X^{*}{}_{i,t'',t''} + \sum_{t'''=1}^{T} \sum_{t''=1}^{t'''} X^{**}{}_{i,t'',t'''} + \sum_{t'''=2}^{T} \sum_{t''=1}^{t'''-1} (X^{***}{}_{i,t'',t'''} + X^{****}{}_{i,t'',t'''}) = \sum_{t'''=1}^{T} d_{i,t'''}$$

$$\forall i. \quad (71)$$

/* Constraints (72)-(73) represent the inventory balance constraints */

$$X^{*}{}_{i,1,1} + X^{**}{}_{i,1,1} - d_{i,1} = I_{i,1} \qquad \forall i. \quad (72)$$

$$\sum_{t''=1}^{t} X^{*}{}_{i,t'',t''} + \sum_{t'''=1}^{t} \sum_{t''=1}^{t'''} X^{**}{}_{i,t'',t'''} + \sum_{t'''=2}^{t} \sum_{t''=1}^{t'''-1} (X^{***}{}_{i,t'',t'''} + X^{****}{}_{i,t'',t'''})$$

$$- \sum_{t'''=1}^{t} d_{i,t'''} = I_{i,t} \qquad \forall i \text{ and } t = 2,\dots,T. \quad (73)$$

/* Constraints (74)-(85) represent the conditions required for production to occur in period *t* */

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$X^{**}{}_{i',t'',t'''} \leq M(1-\delta^{*}_{i,t'})$ (74)

$X^{**}{}_{i',t'',t'''} \leq M(1-\delta^{**}_{i,t'})$ (75)

$X^{**}{}_{i',t'',t'''} \leq M(1-\delta^{***}_{i,t'})$ (76)

$X^{**}{}_{i',t'',t'''} \leq M(1-\delta^{****}_{i,t'})$ (77)

$X^{***}{}_{i',t'',t'''} \leq M(1-\delta^{*}_{i,t'})$ (78)

$X^{***}{}_{i',t'',t'''} \leq M(1-\delta^{**}_{i,t'})$ (79)

$X^{***}{}_{i',t'',t'''} \leq M(1-\delta^{***}_{i,t'})$ (80)

$X^{***}{}_{i',t'',t'''} \leq M(1-\delta^{****}_{i,t'})$ (81)

$X^{****}{}_{i',t'',t'''} \leq M(1-\delta^{*}_{i,t'})$ (82)

$X^{****}{}_{i',t'',t'''} \leq M(1-\delta^{**}_{i,t'})$ (83)

$X^{****}{}_{i',t'',t'''} \leq M(1-\delta^{***}_{i,t'})$ (84)

$X^{****}{}_{i',t'',t'''} \leq M(1-\delta^{****}_{i,t'})$ (85)

}

$\forall i, \forall i', t = 2,..,T-1,\ t'' = 1,2,..,T-t,$
$t''' = t'' + t$ and $t' = t'' + 1, t'' + 2,.., t''' - 1.$

Note that:
$\delta^{*}_{i,t}$, $\delta^{**}_{i,t}$, $\delta^{***}_{i,t}$ and $\delta^{****}_{i,t}$ are binary variables and all other variables are ≥ 0.

The objective function is the minimization of the setup cost and holding cost of all products across all time periods. Constraints (42), (43), (47), (48), (52), (53), (57)-(60) ensure that when product i is set up in period t (i.e., when $\delta^{*}_{i,t}$, $\delta^{**}_{i,t}$, $\delta^{***}_{i,t}$ and $\delta^{****}_{i,t}$ exist), the total setup time ST_i required by the product i for setup is consumed. Constraints (44), (45), (49), (50), (54), (55), (61) and (62) are the production constraints which indicate that the variables indicating the production quantity, i.e., $X^{*}{}_{i,t,t'}$, $X^{**}{}_{i,t,t'}$, $X^{***}{}_{i,t,t'}$ and $X^{****}{}_{i,t,t'}$ exist only when the variables $\delta^{*}_{i,t}$, $\delta^{**}_{i,t}$, $\delta^{***}_{i,t}$ and $\delta^{****}_{i,t}$ exist. Constraints (46), (51), (56) and (63) denote the capacity constraints with the consideration of the corresponding production time of each product. Constraints (66), (67) and (68) denote the capacity constraints, with the consideration of the corresponding possible setup time and production time of all products. Constraint (64) indicates that there can exist only a complete setup with production carryover or an end-of-period setup or a setup split for at most one product in every period. Constraint (65) indicates that for a product to be produced in a period, it is enough to produce the product using either a complete setup, complete setup allowing production carryover, an end-of-period setup or a split setup given in a period, in order to avoid setting up the machine more than once, for a product demanded in that period. Constraints (69)-(71) represent the demand constraints with no backorders or lost sales. Constraints (72) and (73) indicate the inventory balance constraints. Constraints (74)-(85) allow the production carryover for the product i setup in period t'', from period t'' to period t''' provided that there is no intermittent setup of any other product between the production periods t'' and t''' of product i. For example, production carryover is allowed for product 1 from period $t'' = 1$ to period $t''' = 4$ provided that there is no intermittent setup of any other product in periods $t' = 2$ and $t' = 3$.

3. Scenario 2 - Variant (3) (further to Variants (1) and (2) given in the main paper): Mathematical model when setup costs and holding costs are product dependent and time dependent, and the setup cost is calculated per unit time of setup in a given period

In this variant the setup cost is split between periods t and t+1, and the cost is proportional to the amount of time used to setup product i in periods t and t+1. The objective function (corresponding to MM) is now shown in (86).

Objective function:

$$\text{Min } Z = \sum_{i=1}^{N}\sum_{t=1}^{T} SC'_{i,t}\delta^{*}_{i,t} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC'_{i,t}\delta^{**}_{i,t} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC^{1}_{i,t}s^{***}_{i,t} + \sum_{i=1}^{N}\sum_{t=1}^{T-1} SC^{1}_{i,t+1}s^{****}_{i,t+1} + \sum_{i=1}^{N}\sum_{t=1}^{T} h'_{i,t}I_{i,t} \tag{86}$$

subject to all constraints presented in the main work.

In the above, $SC^{1}_{i,t}$ denotes the rate of cost of setup (cost corresponding to one time unit of setup) of product i in period t, and is given by $SC^{1}_{i,t} = \frac{SC'_{i,t}}{ST_i}$.

4. A numerical illustration and discussion

The illustration of the model given in Section 3 and the corresponding results are presented in this section. For executing the model, the same numerical example presented in the main work

(Table 3) is used. For Scenario 2 - Variant (3), the value of the objective function obtained by our model is equal to 245 mu (monetary units). The resulting solution is shown in Table 8, and its corresponding Gantt chart is shown in Figure 7.

5. Numerical illustration and discussion of the mathematical models when the setup costs and holding costs are product dependent and time dependent (Scenario 2 - Variants (1) and (2)) presented in the main work, while using two binary variables

The objective of including this section is to show that we need the use of three binary variables in the case of time dependent costs. The illustration of the mathematical models presented in Sections 3.2.1 and 3.2.2 of the main work (with the use of only two binary variables) is presented here. For executing the models the numerical example presented in the main work (Table 3) is used. When we set $\delta_{i,t}^{**} = 0 \ \forall i$ and $1 \leq t \leq T-1$ and remove constraints (26) and (27) from the mathematical model presented in the main work (MM), Variants (1) and (2) result in an incorrect value of *Z* on execution. The discussion is presented below.

5.1. Scenario 2 - Variant (1) (see main paper for details): Setup costs and holding costs are product dependent and time dependent, and the setup cost is calculated with respect to period *t* when the setup is initiated

For Scenario 2 - Variant (1), our model results in the value of the objective function being equal to 235 mu, while using two binary variables. The resulting solution is shown in Table 9 with its corresponding Gantt chart shown in Figure 8. While calculating the value of *Z*, $\delta_{3,2}^{***}$ is multiplied with $SC'_{3,2}$ (set up cost is calculated with respect to period *t*) and gives a lower value of *Z* = 235 mu. The resulting value is incorrect because the setup actually takes place at the beginning of period 3 with the setup time in period 2 being zero, when there is a split setup between periods 2 and 3 as indicated by the variable $\delta_{3,2}^{***}$.

5.2. Scenario 2 - Variant (2) (see main paper for details): Setup costs and holding costs are product dependent and time dependent, and the setup cost is calculated with respect to the period when the setup is completed

For Scenario 2 - Variant (2), our model results in the value of the objective function being equal to 245 mu, while using two binary variables. The resulting solution is shown in Table 10 with its corresponding Gantt chart shown in Figure 9. While calculating the value of *Z*, $\delta_{3,1}^{***}$ is multiplied with $SC'_{3,2}$; $\delta_{4,5}^{***}$ is multiplied with $SC'_{4,6}$; and $\delta_{2,9}^{***}$ is multiplied with $SC'_{2,10}$ (set up cost is calculated with respect to the period when the setup is completed), resulting in a lower value of *Z* = 245 mu. The resulting value of *Z* is incorrect because, the setup is started and completed within the same time periods 1, 5 and 9 (here, time periods 1, 5 and 9 indicate the period *t* when the setup is initiated) when there is a split setup indicated between period's *t* and *t*+1, with the setup costs computed corresponding to period *t*+1. This resulted in the incorrect value of *Z*. Hence, we justify the use of three binary variables ($\delta_{i,t}^{*}, \delta_{i,t}^{**}$ and $\delta_{i,t}^{***}$) to handle the generalized situation with respect to Variants (1) and (2) of Scenario 2.

6. Additional Remarks

Remark 1: We wish to state here that for the sample problem instance shown in the main work (Table 1 given in the main paper), our MM model with the consideration of Scenario 1 has generated an alternate optimal solution (Table 11 of this link) resulting in the same value of *Z*. The corresponding Gantt chart is shown in Figure 10. Similarly, for the sample problem instance shown in Table 3 of the main work, our model with the consideration of Scenario 2 (Variants (1), (2) and (3)) has generated alternate optimal solutions with the same value of *Z*. The solutions generated are shown in Tables 12, 13 and 14, with their corresponding Gantt charts shown in Figures 11, 12 and 13. The alternate optimal solutions are generated using CPLEX v12.4.

Remark 2: The AMM allows idle time periods to exist between the course of production of the product.

7. Conclusion

In the preceding sections we have discussed the AMM and a possible extension of the Mathematical Model (MM) presented in our main work. It is to be noted that the AMM can also be extended to address Scenario 2 **-** Variant (3) presented in the preceding section, and well as Variants (1) and (2) of Scenario 2 presented in the main work.

Table 4. Solution generated by the model of Mohan *et al.* (2012) (corresponding terms in that paper are used here) (see Figure 1 of the main work for the corresponding Gantt chart).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$z_{1,1}= 1$ $x_{1,1} = 70$ $l_{3,1} = 20$	$z_{2,2} = 1$ $v_{3,2} = 1$ $x_{2,2}= 40$ $x_{3,2} = 50$	$z_{2,3}= 1$ $u_{3,3} = 1$ $x_{2,3} = 40$ $x_{3,3}= 30$	$u_{2,4} = 1$ $x_{2,4} =100$ $Q_4 = 1$	$z_{1,5}= 1$ $u_{2,5}= 1$ $x_{1,5}= 70$ $l_{3,5}= 20$	$z_{2,6}= 1$ $z_{4,6}= 1$ $v_{3,6}= 1$ $x_{2,6} = 10$ $x_{3,6} = 50$ $x_{4,6}= 10$	$z_{2,7}= 1$ $u_{3,7}= 1$ $x_{2,7} = 20$ $x_{3,7} = 30$	$u_{2,8} = 1$ $x_{2,8}= 95$ $l_{1,8}= 5$ $Q_8 = 1$	$v_{1,9}= 1$ $x_{1,9} = 85$ $f_{1,9} = 5$ $l_{3,9} = 10$	$z_{2,10} = 1$ $v_{3,10}= 1$ $x_{2,10} = 40$ $x_{3,10}= 40$ $f_{3,10} = 10$	$z_{2,11}= 1$ $u_{3,11}= 1$ $x_{2,11} = 40$ $x_{3,11} = 30$	$u_{2,12}= 1$ $x_{2,12} = 100$ Z=320 mu

Table 5. Solution generated by our model with the consideration of Scenario 2 - Variant (1) (see Figure 3 of the main work for the corresponding Gantt chart).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^*_{1,1} = 1$ $\delta^{**}_{3,1} = 1$ $\Delta^*_{1,1,1} = 1$ $s^*_{1,1} = 10$ $s^{**}_{3,1} = 20$ $X^*_{1,1,1} =70$	$\delta^*_{2,2} = 1$ $\Delta^*_{2,2,2} = 1$ $\Delta^{**}_{3,1,2}=1$ $s^*_{2,2} = 10$ $X^*_{2,2,2}=40$ $X^{**}_{3,1,2} =50$	$\delta^*_{3,3} = 1$ $\delta^*_{2,3} = 1$ $\Delta^*_{2,2,3}= 1$ $\Delta^*_{2,3,3}= 1$ $\Delta^*_{3,3,3} = 1$ $s^*_{2,3} = 10$ $s^*_{3,3} = 20$ $X^*_{3,3,3} =30$ $X^*_{2,2,3} =39$ $X^*_{2,3,3} =1$	$\Delta^*_{2,3,4} = 1$ $X^*_{2,3,4}=100$	$\delta^*_{1,5} = 1$ $\delta^{**}_{4,5} = 1$ $\Delta^*_{1,5,5} = 1$ $s^*_{1,5} = 10$ $s^{**}_{4,5} = 20$ $X^*_{1,5,5} =7\ 0$	$\delta^*_{2,6} = 1$ $\delta^*_{3,6} = 1$ $\Delta^*_{2,6,6} = 1$ $\Delta^*_{3,6,6} = 1$ $\Delta^{**}_{4,5,6} = 1$ $s^*_{3,6} = 20$ $s^*_{2,6} = 10$ $X^{**}_{4,5,6}=10$ $X^*_{3,6,6} = 50$ $X^*_{2,6,6} = 10$	$\delta^*_{2,7} = 1$ $\Delta^*_{3,6,7} = 1$ $\Delta^*_{2,7,7} = 1$ $s^*_{2,7} = 10$ $X^*_{3,6,7} = 30$ $X^*_{2,7,7} =20$	$\delta^{***}_{1,8} = 1$ $\Delta^*_{2,7,8} = 1$ $s^{***}_{1,8} = 5$ $X^*_{2,7,8} =95$	$\delta^{**}_{2,9} = 1$ $\Delta^{***}_{1,8,9} = 1$ $s^{****}_{1,9} = 5$ $s^{**}_{2,9} = 10$ $X^{***}_{1,8,9} = 85$	$\delta^*_{3,10} = 1$ $\Delta^{**}_{2,9,10} = 1$ $\Delta^*_{3,10,10} = 1$ $s^*_{3,10} = 20$ $X^{**}_{2,9,10} = 40$ $X^*_{3,10,10} = 40$	$\delta^*_{2,11} = 1$ $\Delta^*_{3,10,11} = 1$ $\Delta^*_{2,11,11} = 1$ $s^*_{2,11} = 10$ $X^*_{2,11,11} = 40$ $X^*_{3,10,11} = 30$	$\Delta^*_{2,11,12} = 1$ $X^*_{2,11,12}=100$ Z=240 mu

Table 6. Solution generated by our model with the consideration of Scenario 2 - Variant (2) (see Figure 4 of the main work for the corresponding Gantt chart).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^*_{1,1} = 1$ $\delta^{**}_{3,1} = 1$ $\Delta^*_{1,1,1} = 1$ $s^*_{1,1} = 10$ $s^{**}_{3,1} = 20$ $X^*_{1,1,1} = 70$	$\delta^*_{2,2} = 1$ $\Delta^*_{2,2,2} = 1$ $\Delta^{**}_{3,1,2}=1$ $s^*_{2,2} = 10$ $X^*_{2,2,2}=40$ $X^{**}_{3,1,2} =50$	$\delta^*_{3,3} = 1$ $\delta^{**}_{2,3} = 1$ $\Delta^*_{2,2,3} = 1$ $\Delta^*_{3,3,3} = 1$ $s^{**}_{2,3} = 10$ $s^*_{3,3} = 20$ $X^*_{2,2,3} = 40$ $X^*_{3,3,3} =30$	$\Delta^{**}_{2,3,4} = 1$ $X^{**}_{2,3,4} = 100$	$\delta^*_{1,5} = 1$ $\delta^{**}_{4,5} = 1$ $\Delta^*_{1,5,5} = 1$ $s^*_{1,5} = 10$ $s^{**}_{4,5} = 20$ $X^*_{1,5,5} =7\ 0$	$\delta^*_{2,6} = 1$ $\delta^*_{3,6} = 1$ $\Delta^*_{2,6,6} = 1$ $\Delta^*_{3,6,6} = 1$ $\Delta^{**}_{4,5,6} = 1$ $s^*_{3,6} = 20$ $s^*_{2,6} = 10$ $X^{**}_{4,5,6}=10$ $X^*_{3,6,6} = 50$ $X^*_{2,6,6} = 10$	$\delta^*_{2,7} = 1$ $\Delta^*_{3,6,7} = 1$ $\Delta^*_{2,7,7} = 1$ $s^*_{2,7} = 10$ $X^*_{3,6,7} =30$ $X^*_{2,7,7} =20$	$\delta^{***}_{1,8} = 1$ $\Delta^*_{2,7,8} = 1$ $s^{***}_{1,8} = 5$ $X^*_{2,7,8} =95$	$\delta^{**}_{2,9} = 1$ $\Delta^{***}_{1,8,9} = 1$ $s^{****}_{1,9} = 5$ $s^{**}_{2,9} = 10$ $X^{***}_{1,8,9} = 85$	$\delta^*_{3,10} = 1$ $\Delta^{**}_{2,9,10} = 1$ $\Delta^*_{3,10,10} = 1$ $s^*_{3,10} = 20$ $X^{**}_{2,9,10} = 40$ $X^*_{3,10,10}= 40$	$\delta^*_{2,11} = 1$ $\Delta^*_{3,10,11} = 1$ $\Delta^*_{2,11,11} = 1$ $s^*_{2,11} = 10$ $X^*_{2,11,11} = 40$ $X^*_{3,10,11} = 30$	$\Delta^*_{2,11,12} = 1$ $X^*_{2,11,12}=100$ Z=250 mu

Table 7. Solution generated by our model with the consideration of Scenario 2 - Variant (1), and with additional modifications done given in Section 4.3 of our main paper (see Figure 5 of our main work for the corresponding Gantt chart).

t_1	t_2	t_3	t_4	t_5	t_6
$\delta^{*}_{1,1} = 1$	$\Delta^{**}_{2,1,2}=1$	$\delta^{**}_{1,3} = 1$	$\Delta^{**}_{1,3,4}=1$	$\Delta^{**}_{1,3,5}=1$	$\delta^{*}_{2,6} = 1$
$\delta^{**}_{2,1} = 1$		$\Delta^{**}_{2,1,3}=1$		$X^{**}_{1,3,5}=40$	$\Delta^{**}_{1,3,6}=1$
$\Delta^{*}_{1,1,1}=1$		$s^{**}_{1,3}=10$			$\Delta^{*}_{2,6,6}=1$
$s^{*}_{1,1} = 10$		$X^{**}_{2,1,3}=90$			$s^{*}_{2,6} = 20$
$s^{**}_{2,1} = 20$					$X^{*}_{2,6,6}=40$
$X^{*}_{1,1,1}=65$					$X^{**}_{1,3,6}=40$
$I_{1,1} = 10$					$Z=120$

Table 8. Solution generated by our model with the consideration of Scenario 2 - Variant (3).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^{*}_{1,1}=1$	$\delta^{*}_{2,2}=1$	$\delta^{*}_{3,3}=1$	$\Delta^{*}_{2,3,4}=1$	$\delta^{*}_{1,5}=1$	$\delta^{*}_{2,6}=1$	$\delta^{*}_{2,7}=1$	$\delta^{***}_{1,8}=1$	$\delta^{**}_{2,9}=1$	$\delta^{*}_{3,10}=1$	$\delta^{*}_{2,11}=1$	$\Delta^{*}_{2,11,12}=1$
$\delta^{**}_{3,1}=1$	$\Delta^{*}_{2,2,2}=1$	$\delta^{*}_{2,3}=1$	$X^{*}_{2,3,4}=100$	$\delta^{**}_{4,5}=1$	$\delta^{*}_{3,6}=1$	$\Delta^{*}_{3,6,7}=1$	$\Delta^{*}_{2,7,8}=1$	$\Delta^{***}_{1,8,9}=1$	$\Delta^{**}_{2,9,10}=1$	$\Delta^{*}_{3,10,11}=1$	$X^{*}_{2,11,12}=100$
$\Delta^{*}_{1,1,1}=1$	$\Delta^{**}_{3,1,2}=1$	$\Delta^{*}_{2,3,3}=1$		$\Delta^{*}_{1,5,5}=1$	$\Delta^{*}_{2,6,6}=1$	$\Delta^{*}_{2,7,7}=1$	$s^{***}_{1,8}=5$	$s^{****}_{1,9}=5$	$\Delta^{*}_{3,10,10}=1$	$\Delta^{*}_{2,11,11}=1$	Z=245 mu
$s^{*}_{1,1}=10$	$s^{*}_{2,2}=10$	$\Delta^{*}_{3,3,3}=1$		$s^{*}_{1,5}=10$	$\Delta^{*}_{3,6,6}=1$	$s^{*}_{2,7}=10$	$X^{*}_{2,7,8}=95$	$s^{**}_{2,9}=10$	$s^{*}_{3,10}=20$	$s^{*}_{2,11}=10$	
$s^{**}_{3,1}=20$	$X^{*}_{2,2,2}=40$	$s^{*}_{2,3}=10$		$s^{**}_{4,5}=20$	$\Delta^{**}_{4,5,6}=1$	$X^{*}_{3,6,7}=30$		$X^{***}_{1,8,9}=85$	$X^{**}_{2,9,10}=40$	$X^{*}_{2,11,11}=40$	
$X^{*}_{1,1,1}=70$	$X^{**}_{3,1,2}=50$	$s^{*}_{3,3}=20$		$X^{*}_{1,5,5}=7\ 0$	$s^{*}_{3,6}=20$	$X^{*}_{2,7,7}=20$			$X^{*}_{3,10,10}=40$	$X^{*}_{3,10,11}=30$	
		$X^{*}_{2,3,3}=40$			$s^{*}_{2,6}=10$						
		$X^{*}_{3,3,3}=30$			$X^{**}_{4,5,6}=10$						
					$X^{*}_{3,6,6}=50$						
					$X^{*}_{2,6,6}=10$						

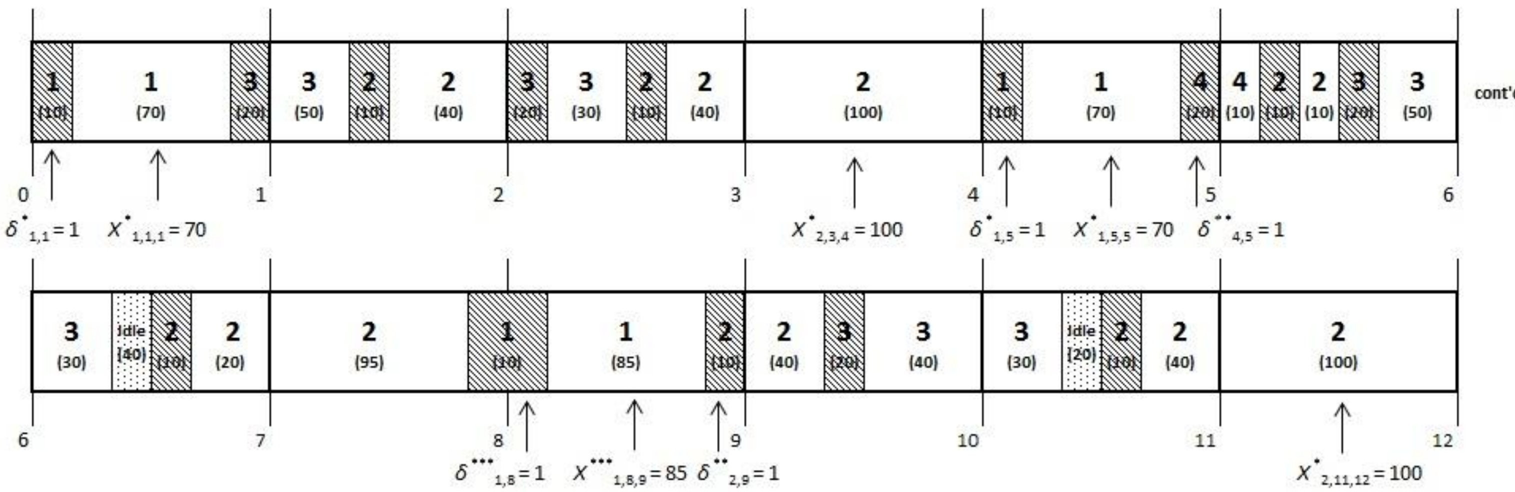

Figure 7. Gantt chart (for the problem instance given in Table 3 of the main work, and the solution given in Table 8) generated by our model with the consideration of Scenario 2 - Variant (3).

Note: Time period is denoted along the X-axis (see Fig. 7, 8, 9, 10, 11, 12 and 13 for the respective GANTT charts). The entries in the chart denote the products. The shaded region denotes the setup of a product. The product which is setup is indicated inside the shaded region with the setup time denoted in brackets. The unshaded region denotes the production of the product setup with the production time denoted in brackets. Idle time of a machine is denoted as 'Idle' with the idle time indicated in brackets. Values of some variables are shown in the figure for the sake of understanding.

Table 9. Solution generated by our model with the consideration of Scenario 2 - Variant (1), and with the use of just two binary variables.

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^{*}_{1,1}=1$ $\delta^{***}_{3,1}=1$ $\Delta^{*}_{1,1,1}=1$ $s^{*}_{1,1}=10$ $s^{***}_{3,1}=20$ $X^{*}_{1,1,1}=70$	$\delta^{*}_{2,2}=1$ $\delta^{***}_{3,2}=1$ $\Delta^{*}_{2,2,2}=1$ $\Delta^{***}_{3,1,2}=1$ $s^{*}_{2,2}=10$ $X^{*}_{2,2,2}=40$ $X^{***}_{3,1,2}=50$	$\delta^{*}_{2,3}=1$ $\Delta^{*}_{2,3,3}=1$ $\Delta^{***}_{3,2,3}=1$ $s^{*}_{2,3}=10$ $s^{****}_{3,3}=20$ $X^{***}_{3,2,3}=30$ $X^{*}_{2,3,3}=40$	$\Delta^{*}_{2,3,4}=1$ $X^{*}_{2,3,4}=100$	$\delta^{*}_{1,5}=1$ $\delta^{***}_{4,5}=1$ $\Delta^{*}_{1,5,5}=1$ $s^{*}_{1,5}=10$ $s^{***}_{4,5}=20$ $X^{*}_{1,5,5}=7\ 0$	$\delta^{*}_{2,6}=1$ $\delta^{*}_{3,6}=1$ $\Delta^{*}_{2,6,6}=1$ $\Delta^{*}_{3,6,6}=1$ $\Delta^{***}_{4,5,6}=1$ $s^{*}_{3,6}=20$ $s^{*}_{2,6}=10$ $X^{***}_{4,5,6}=10$ $X^{*}_{3,6,6}=50$ $X^{*}_{2,6,6}=10$	$\delta^{*}_{2,7}=1$ $\Delta^{*}_{3,6,7}=1$ $\Delta^{*}_{2,7,7}=1$ $s^{*}_{2,7}=10$ $X^{*}_{3,6,7}=30$ $X^{*}_{2,7,7}=20$	$\delta^{***}_{1,8}=1$ $\Delta^{*}_{2,7,8}=1$ $s^{***}_{1,8}=5$ $X^{*}_{2,7,8}=95$	$\delta^{***}_{2,9}=1$ $\Delta^{***}_{1,8,9}=1$ $s^{****}_{1,9}=5$ $s^{***}_{2,9}=10$ $X^{***}_{1,8,9}=85$	$\delta^{*}_{3,10}=1$ $\Delta^{***}_{2,9,10}=1$ $\Delta^{*}_{3,10,10}=1$ $s^{*}_{3,10}=20$ $X^{***}_{2,9,10}=40$ $X^{*}_{3,10,10}=40$	$\delta^{*}_{2,11}=1$ $\Delta^{*}_{3,10,11}=1$ $\Delta^{*}_{2,11,11}=1$ $s^{*}_{2,11}=10$ $X^{*}_{2,11,11}=40$ $X^{*}_{3,10,11}=30$	$\Delta^{*}_{2,11,12}=1$ $X^{*}_{2,11,12}=100$ Z=235 mu

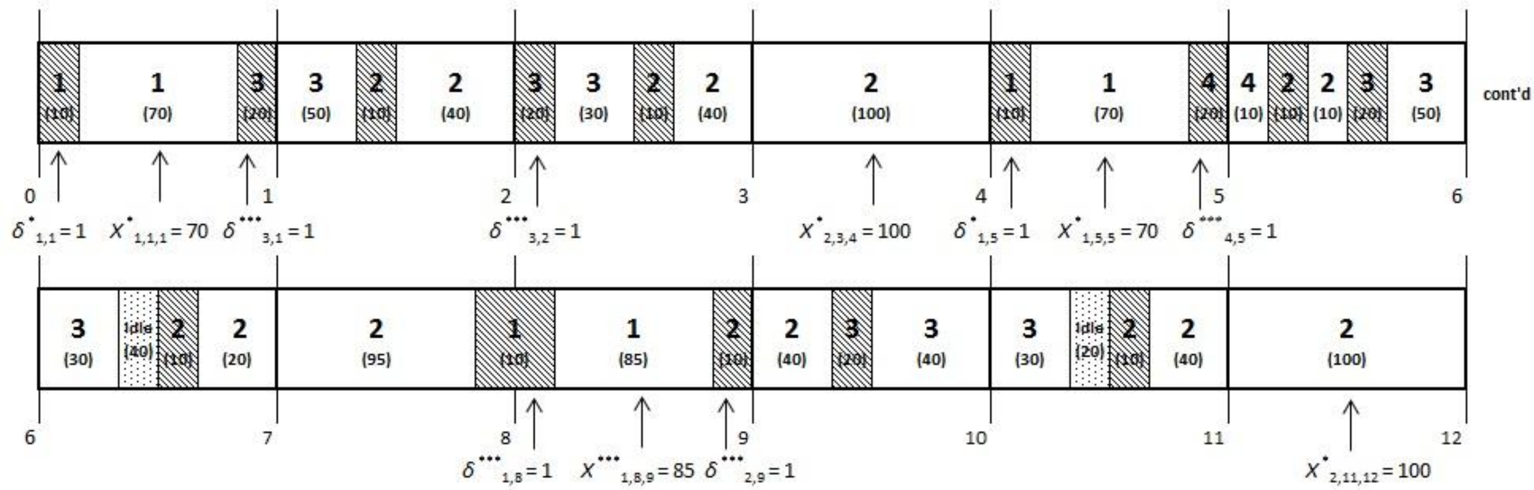

Figure 8. Gantt chart generated (for the problem instance given in Table 3 of the main work, and the solution given in Table 9) by our model with the consideration of Scenario 2 - Variant (1), and with the use of just two binary variables.

Table 10. Solution generated by our model with the consideration of Scenario 2 - Variant (2), and with the use of just two binary variables.

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^*_{1,1} = 1$ $\delta^{***}_{3,1} = 1$ $\Delta^*_{1,1,1} = 1$ $s^*_{1,1} = 10$ $s^{***}_{3,1} = 20$ $X^*_{1,1,1} = 70$	$\delta^*_{2,2} = 1$ $\Delta^*_{2,2,2} = 1$ $\Delta^{***}_{3,1,2}=1$ $s^*_{2,2} = 10$ $X^*_{2,2,2}=40$ $X^{***}_{3,1,2}=50$	$\delta^*_{3,3} = 1$ $\delta^{***}_{2,3} = 1$ $\Delta^*_{2,2,3} = 1$ $\Delta^*_{3,3,3} = 1$ $s^{***}_{2,3} = 10$ $s^*_{3,3} = 20$ $X^*_{2,2,3} = 40$ $X^*_{3,3,3}=30$	$\Delta^{***}_{2,3,4} = 1$ $X^{***}_{2,3,4}=1\ 00$	$\delta^*_{1,5} = 1$ $\delta^{***}_{4,5} = 1$ $\Delta^*_{1,5,5} = 1$ $s^*_{1,5} = 10$ $s^{***}_{4,5} = 20$ $X^*_{1,5,5}=7\ 0$	$\delta^*_{2,6} = 1$ $\delta^*_{3,6} = 1$ $\Delta^*_{2,6,6} = 1$ $\Delta^*_{3,6,6} = 1$ $\Delta^{***}_{4,5,6} = 1$ $s^*_{3,6} = 20$ $s^*_{2,6} = 10$ $X^{***}_{4,5,6}=10$ $X^*_{3,6,6} = 50$ $X^*_{2,6,6} = 10$	$\delta^*_{2,7} = 1$ $\Delta^*_{3,6,7} = 1$ $\Delta^*_{2,7,7} = 1$ $s^*_{2,7} = 10$ $X^*_{3,6,7}=30$ $X^*_{2,7,7}=20$	$\delta^{***}_{1,8} = 1$ $\Delta^*_{2,7,8} = 1$ $s^{***}_{1,8} = 5$ $X^*_{2,7,8}=95$	$\delta^{***}_{2,9} = 1$ $\Delta^{***}_{1,8,9} = 1$ $s^{****}_{1,9} = 5$ $s^{***}_{2,9} = 10$ $X^{***}_{1,8,9} = 85$	$\delta^*_{3,10} = 1$ $\Delta^{***}_{2,9,10} = 1$ $\Delta^*_{3,10,10} = 1$ $s^*_{3,10} = 20$ $X^{***}_{2,9,10} = 40$ $X^*_{3,10,10}= 40$	$\delta^*_{2,11} = 1$ $\Delta^*_{3,10,11} = 1$ $\Delta^*_{2,11,11} = 1$ $s^*_{2,11} = 10$ $X^*_{2,11,11} = 40$ $X^*_{3,10,11} = 30$	$\Delta^*_{2,11,12} = 1$ $X^*_{2,11,12}=100$ Z=245 mu

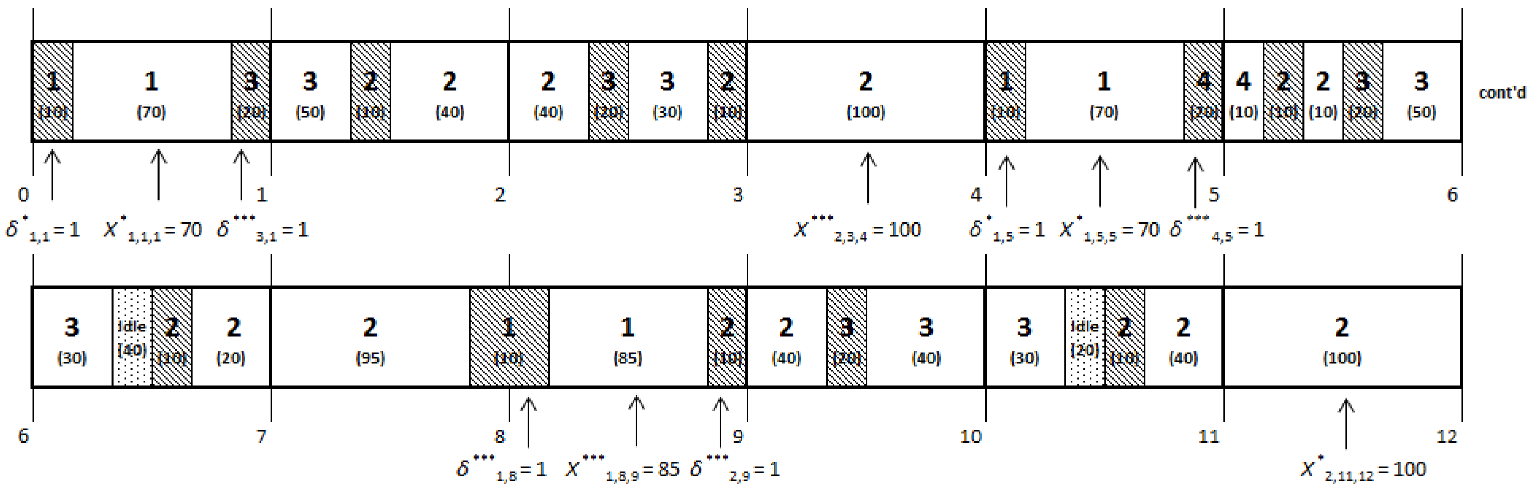

Figure 9. Gantt chart (for the problem instance given in Table 3 of the main work, and the solution given in Table 10) generated by our model with the consideration of Scenario 2 - Variant (2), and with the use of just two binary variables.

Table 11. Solution generated by our model with the consideration of Scenario 1.

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^*_{1,1}=1$ $\delta^{**}_{3,1}=1$ $\Delta^*_{1,1,1}=1$ $s^*_{1,1}=10$ $s^{**}_{3,1}=20$ $X^*_{1,1,1}=70$	$\delta^*_{2.2}=1$ $\Delta^{**}_{3,1,2}=1$ $\Delta^*_{2,2,2}=1$ $s^*_{2,2}=10$ $X^*_{2,2,2}=40$ $X^{**}_{3,1,2}=50$	$\delta^*_{2,3}=1$ $\delta^*_{3,3}=1$ $\Delta^*_{2,2,3}=1$ $\Delta^*_{2,3,3}=1$ $\Delta^*_{3,3,3}=1$ $s^*_{2,3}=10$ $s^*_{3,3}=20$ $X^*_{2,3,3}=1$ $X^*_{2,2,3}=39$ $X^*_{3,3,3}=30$	$\Delta^*_{2,3,4}=1$ $X^*_{2,3,4}=100$	$\delta^*_{1,5}=1$ $\delta^{**}_{4,5}=1$ $\Delta^*_{1,5,5}=1$ $s^*_{1,5}=10$ $s^{**}_{4,5}=20$ $X^*_{1,5,5}=70$	$\delta^*_{2,6}=1$ $\delta^*_{3,6}=1$ $\Delta^*_{2,6,6}=1$ $\Delta^*_{3,6,6}=1$ $\Delta^{**}_{4,5,6}=1$ $s^*_{3,6}=20$ $s^*_{2,6}=10$ $X^{**}_{4,5,6}=10$ $X^*_{3,6,6}=50$ $X^*_{2,6,6}=10$	$\delta^*_{2,7}=1$ $\Delta^*_{3,6,7}=1$ $\Delta^*_{2,7,7}=1$ $s^*_{2,7}=10$ $X^*_{3,6,7}=30$ $X^*_{2,7,7}=20$	$\delta^{***}_{1,8}=1$ $\Delta^*_{2,7,8}=1$ $s^{***}_{1,8}=5$ $X^*_{2,7,8}=95$	$\delta^{**}_{2,9}=1$ $\Delta^{***}_{1,8,9}=1$ $s^{****}_{1,9}=5$ $s^{**}_{2,9}=10$ $X^{***}_{1,8,9}=85$	$\delta^*_{3,10}=1$ $\Delta^{**}_{2,9,10}=1$ $\Delta^*_{3,10,10}=1$ $s^*_{3,10}=20$ $X^{**}_{2,9,10}=40$ $X^*_{3,10,10}=40$	$\delta^*_{2,11}=1$ $\Delta^*_{3,10,11}=1$ $\Delta^*_{2,11,11}=1$ $s^*_{2,11}=10$ $X^*_{2,11,11}=40$ $X^*_{3,10,11}=30$	$\Delta^*_{2,11,12}=1$ $X^*_{2,11,12}=100$ Z=350 mu

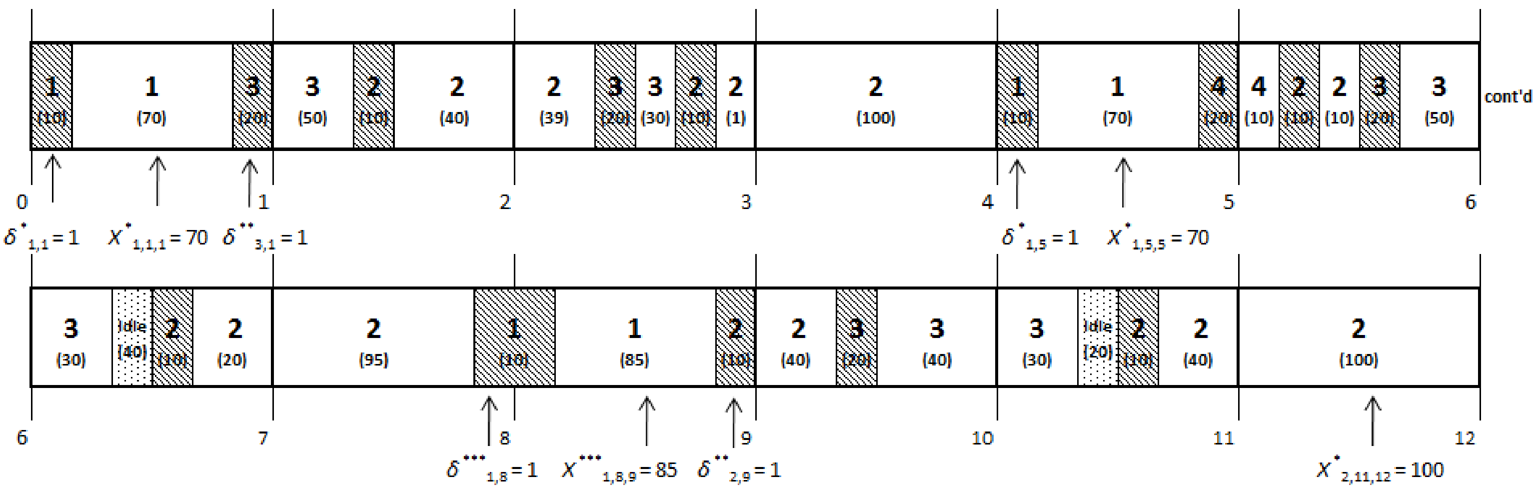

Figure 10. Gantt chart (for the problem instance given shown in Table 1 of the main work, and the solution given in Table 11) generated by our model with the consideration of Scenario 1.

Table 12. Solution generated by our model with the consideration of Scenario 2 - Variant (1).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^{*}_{1,1}=1$ $\delta^{**}_{3,1}=1$ $\Delta^{*}_{1,1,1}=1$ $s^{*}_{1,1}=10$ $s^{**}_{3,1}=20$ $X^{*}_{1,1,1}=70$	$\delta^{*}_{2,2}=1$ $\Delta^{*}_{2,2,2}=1$ $\Delta^{**}_{3,1,2}=1$ $s^{*}_{2,2}=10$ $X^{*}_{2,2,2}=40$ $X^{**}_{3,1,2}=50$	$\delta^{*}_{3,3}=1$ $\delta^{*}_{2,3}=1$ $\Delta^{*}_{2,3,3}=1$ $\Delta^{*}_{3,3,3}=1$ $s^{*}_{2,3}=10$ $s^{*}_{3,3}=20$ $X^{*}_{3,3,3}=30$ $X^{*}_{2,3,3}=40$	$\Delta^{*}_{2,3,4}=1$ $X^{*}_{2,3,4}=100$	$\delta^{*}_{1,5}=1$ $\delta^{**}_{4,5}=1$ $\Delta^{*}_{1,5,5}=1$ $s^{*}_{1,5}=10$ $s^{**}_{4,5}=20$ $X^{*}_{1,5,5}=7\ 0$	$\delta^{*}_{2,6}=1$ $\delta^{*}_{3,6}=1$ $\Delta^{*}_{2,6,6}=1$ $\Delta^{*}_{3,6,6}=1$ $\Delta^{**}_{4,5,6}=1$ $s^{*}_{3,6}=20$ $s^{*}_{2,6}=10$ $X^{*}_{3,6,6}=50$ $X^{*}_{2,6,6}=10$ $X^{**}_{4,5,6}=10$	$\delta^{*}_{2,7}=1$ $\Delta^{*}_{3,6,7}=1$ $\Delta^{*}_{2,7,7}=1$ $s^{*}_{2,7}=10$ $X^{*}_{3,6,7}=30$ $X^{*}_{2,7,7}=20$	$\delta^{***}_{1,8}=1$ $\Delta^{*}_{2,7,8}=1$ $s^{***}_{1,8}=5$ $X^{*}_{2,7,8}=95$	$\delta^{**}_{2,9}=1$ $\Delta^{***}_{1,8,9}=1$ $s^{****}_{1,9}=5$ $s^{**}_{2,9}=10$ $X^{***}_{1,8,9}=85$	$\delta^{*}_{3,10}=1$ $\Delta^{**}_{2,9,10}=1$ $\Delta^{*}_{3,10,10}=1$ $s^{*}_{3,10}=20$ $X^{**}_{2,9,10}=40$ $X^{*}_{3,10,10}=40$	$\delta^{*}_{2,11}=1$ $\Delta^{*}_{3,10,11}=1$ $\Delta^{*}_{2,11,11}=1$ $s^{*}_{2,11}=10$ $X^{*}_{2,11,11}=40$ $X^{*}_{3,10,11}=30$	$\Delta^{*}_{2,11,12}=1$ $X^{*}_{2,11,12}=100$ Z=240 mu

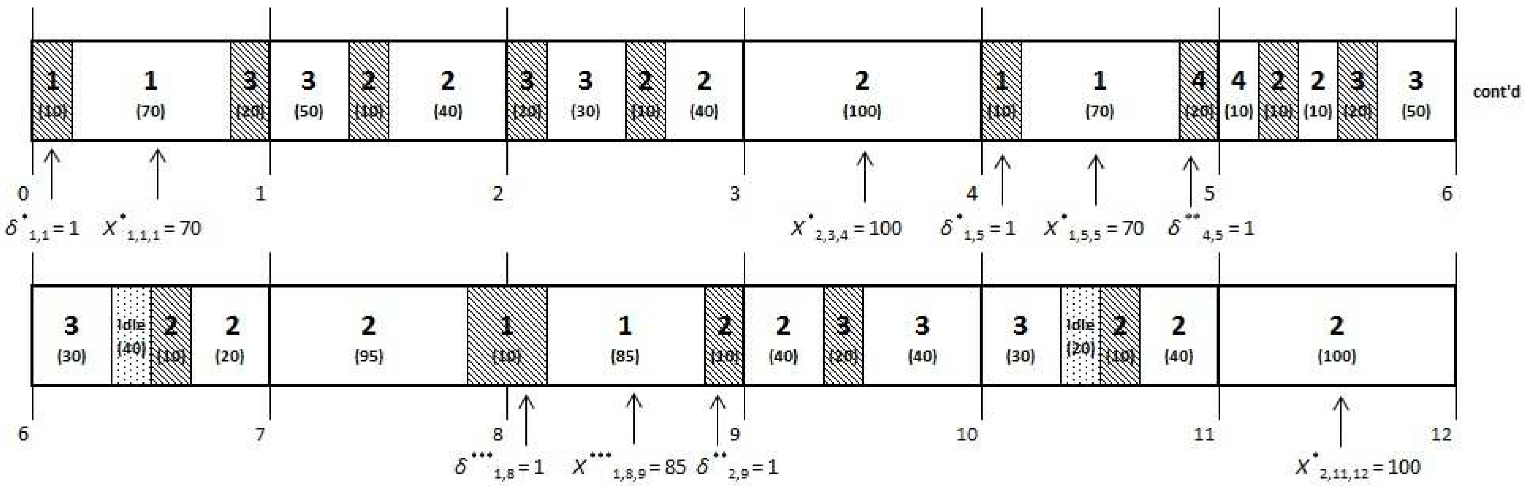

Figure 11. Gantt chart generated (for the problem instance given in Table 3 of the main work, and the solution given in Table 12) by our model with the consideration of Scenario 2 - Variant (1).

Table 13. Solution generated by our model with the consideration of Scenario 2 - Variant (2).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^{*}_{1,1}=1$	$\delta^{*}_{2,2}=1$	$\delta^{*}_{3,3}=1$	$\Delta^{*}_{2,3,4}=1$	$\delta^{*}_{1,5}=1$	$\delta^{*}_{2,6}=1$	$\delta^{*}_{2,7}=1$	$\delta^{***}_{1,8}=1$	$\delta^{**}_{2,9}=1$	$\delta^{*}_{3,10}=1$	$\delta^{*}_{2,11}=1$	$\Delta^{*}_{2,11,12}=1$
$\delta^{**}_{3,1}=1$	$\Delta^{*}_{2,2,2}=1$	$\delta^{*}_{2,3}=1$	$X^{*}_{2,3,4}=100$	$\delta^{**}_{4,5}=1$	$\delta^{*}_{3,6}=1$	$\Delta^{*}_{3,6,7}=1$	$\Delta^{*}_{2,7,8}=1$	$\Delta^{***}_{1,8,9}=1$	$\Delta^{**}_{2,9,10}=1$	$\Delta^{*}_{3,10,11}=1$	$X^{*}_{2,11,12}=100$
$\Delta^{*}_{1,1,1}=1$	$\Delta^{**}_{3,1,2}=1$	$\Delta^{*}_{2,3,3}=1$		$\Delta^{*}_{1,5,5}=1$	$\Delta^{*}_{2,6,6}=1$	$\Delta^{*}_{2,7,7}=1$	$s^{***}_{1,8}=5$	$s^{****}_{1,9}=5$	$\Delta^{*}_{3,10,10}=1$	$\Delta^{*}_{2,11,11}=1$	Z=250 mu
$s^{*}_{1,1}=10$	$s^{*}_{2,2}=10$	$\Delta^{*}_{3,3,3}=1$		$s^{*}_{1,5}=10$	$\Delta^{*}_{3,6,6}=1$	$s^{*}_{2,7}=10$	$X^{*}_{2,7,8}=95$	$s^{**}_{2,9}=10$	$s^{*}_{3,10}=20$	$s^{*}_{2,11}=10$	
$s^{**}_{3,1}=20$	$X^{*}_{2,2,2}=40$	$s^{*}_{2,3}=10$		$s^{**}_{4,5}=20$	$\Delta^{**}_{4,5,6}=1$	$X^{*}_{3,6,7}=30$		$X^{***}_{1,8,9}=85$	$X^{**}_{2,9,10}=40$	$X^{*}_{2,11,11}=40$	
$X^{*}_{1,1,1}=70$	$X^{**}_{3,1,2}=50$	$s^{*}_{3,3}=20$		$X^{*}_{1,5,5}=7\ 0$	$s^{*}_{3,6}=20$	$X^{*}_{2,7,7}=20$			$X^{*}_{3,10,10}=40$	$X^{*}_{3,10,11}=30$	
		$X^{*}_{2,3,3}=40$			$s^{*}_{2,6}=10$						
		$X^{*}_{3,3,3}=30$			$X^{**}_{4,5,6}=10$						
					$X^{*}_{3,6,6}=50$						
					$X^{*}_{2,6,6}=10$						

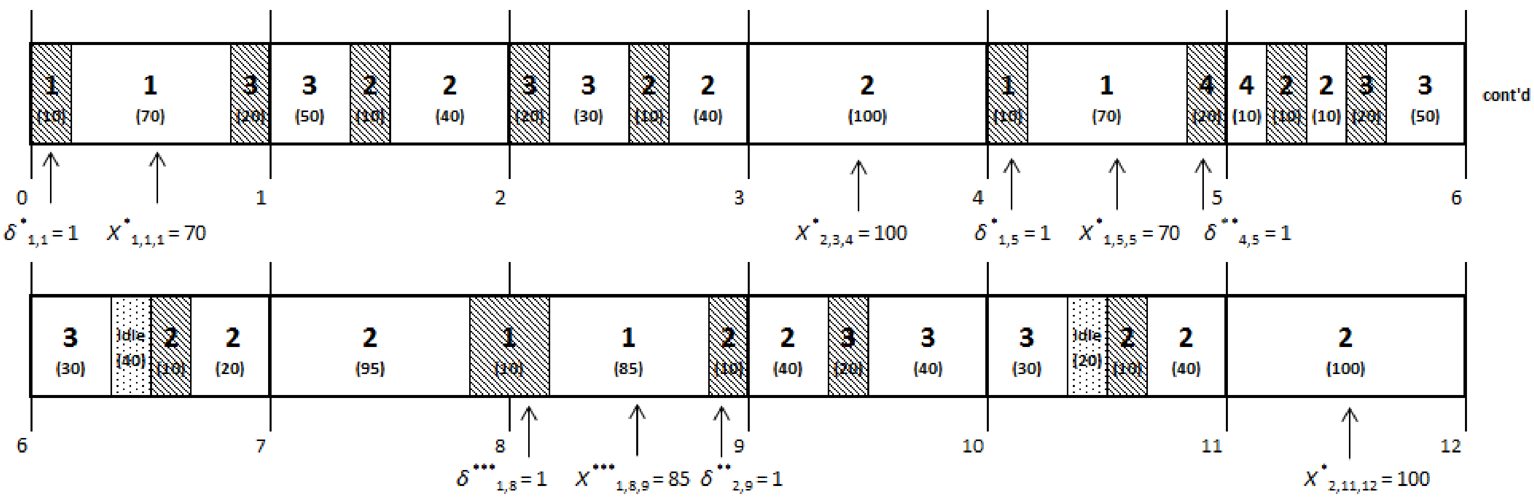

Figure 12. Gantt chart (for the problem instance given in Table 3 of the main work, and the solution given in Table 13) generated by our model with the consideration of Scenario 2 - Variant (2).

Table 14. Solution generated by our model with the consideration of Scenario 2 - Variant (3).

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
$\delta^*_{1,1}=1$	$\delta^*_{2,2}=1$	$\delta^*_{3,3}=1$	$\Delta^*_{2,3,4}=1$	$\delta^*_{1,5}=1$	$\delta^*_{2,6}=1$	$\delta^*_{2,7}=1$	$\delta^{***}_{1,8}=1$	$\delta^{**}_{2,9}=1$	$\delta^*_{3,10}=1$	$\delta^*_{2,11}=1$	$\Delta^*_{2,11,12}=1$
$\delta^{**}_{3,1}=1$	$\Delta^*_{2,2,2}=1$	$\delta^*_{2,3}=1$	$X^*_{2,3,4}=100$	$\delta^{**}_{4,5}=1$	$\delta^*_{3,6}=1$	$\Delta^*_{3,6,7}=1$	$\Delta^*_{2,7,8}=1$	$\Delta^{***}_{1,8,9}=1$	$\Delta^{**}_{2,9,10}=1$	$\Delta^*_{3,10,11}=1$	$X^*_{2,11,12}=100$
$\Delta^*_{1,1,1}=1$	$\Delta^{**}_{3,1,2}=1$	$\Delta^*_{2,2,3}=1$		$\Delta^*_{1,5,5}=1$	$\Delta^*_{2,6,6}=1$	$\Delta^*_{2,7,7}=1$	$s^{***}_{1,8}=5$	$s^{****}_{1,9}=5$	$\Delta^*_{3,10,10}=1$	$\Delta^*_{2,11,11}=1$	Z=245 mu
$s^*_{1,1}=10$	$s^*_{2,2}=10$	$\Delta^*_{2,3,3}=1$		$s^*_{1,5}=10$	$\Delta^*_{3,6,6}=1$	$s^*_{2,7}=10$	$X^*_{2,7,8}=95$	$s^{**}_{2,9}=10$	$s^*_{3,10}=20$	$s^*_{2,11}=10$	
$s^{**}_{3,1}=20$	$X^*_{2,2,2}=40$	$\Delta^*_{3,3,3}=1$		$s^{**}_{4,5}=20$	$\Delta^{**}_{4,5,6}=1$	$X^*_{3,6,7}=30$		$X^{***}_{1,8,9}=85$	$X^{**}_{2,9,10}=40$	$X^*_{2,11,11}=40$	
$X^*_{1,1,1}=70$	$X^{**}_{3,1,2}=50$	$s^*_{2,3}=10$		$X^*_{1,5,5}=7\ 0$	$s^*_{3,6}=20$	$X^*_{2,7,7}=20$			$X^*_{3,10,10}=40$	$X^*_{3,10,11}=30$	
		$s^*_{3,3}=20$			$s^*_{2,6}=10$						
		$X^*_{2,2,3}=39$			$X^{**}_{4,5,6}=10$						
		$X^*_{2,3,3}=1$			$X^*_{3,6,6}=50$						
		$X^*_{3,3,3}=30$			$X^*_{2,6,6}=10$						

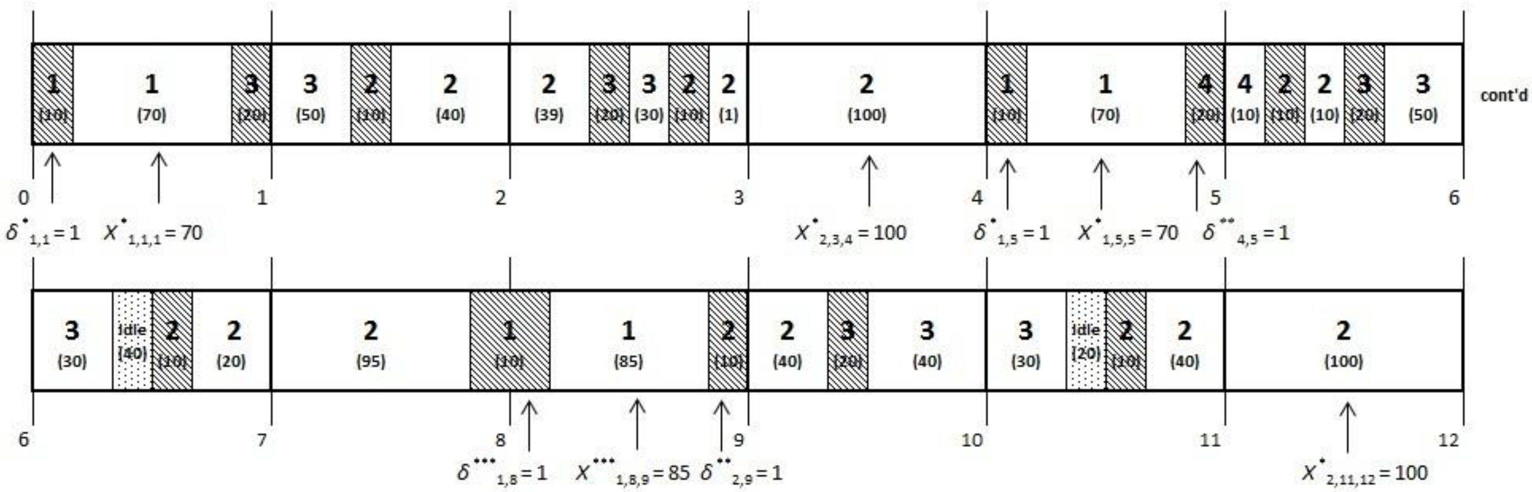

Figure 13. Gantt chart (for the problem instance given in Table 3 of the main work, and the solution given in Table 14) generated by our model with the consideration of Scenario 2 - Variant (3).