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2017

Li, X., Fang, J., Tang, Y., Wu, X., & Geng, Y. (2018). Capacitor-voltage feedforward with full delay compensation to improve weak grids adaptability of LCL-filtered grid-connected converters for distributed generation systems. *IEEE Transactions on Power Electronics*, 33(1), 749-764. doi:10.1109/TPEL.2017.2665483

<https://hdl.handle.net/10356/80637>

<https://doi.org/10.1109/TPEL.2017.2665483>

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# Capacitor Voltage Feedforward with Full Delay Compensation to Improve Weak Grids Adaptability of *LCL*-filtered Grid-connected Converters for Distributed Generation Systems

Xiaoqiang Li, *Member IEEE*, Jingyang Fang, *Student Member IEEE*, Yi Tang, *Member IEEE*, Xiaojie Wu, *Member IEEE*, and Yiwen Geng

**Abstract**—*LCL*-filtered grid-connected converters are widely used for distributed generation systems. However, the current regulation of such converters is susceptible to weak grid conditions, e.g., grid impedance variation and background harmonics. Paralleling multiple harmonic compensators (HCs) is a commonly used method to suppress the current distortion caused by grid background harmonics, but the control bandwidth should be wide enough to ensure system stability. In order to enhance the adaptability of *LCL*-filtered grid-connected converters under weak grid operation, this paper proposes an improved capacitor voltage feedforward control with full delay compensation. When used with converter-side current feedback, the proposed control can keep system low-frequency characteristic independent of grid impedance and provide a high harmonic rejection capability without using additional HCs. Moreover, it completely avoids the design constraints of *LCL*-filter, i.e.  $\omega_r < \omega_s/6$  is required for single-loop converter-side current control. Therefore, a higher resonant frequency can be designed to achieve a wider control bandwidth and to lower the current distortion caused by the paralleled filter capacitor branch. Experimental results are finally presented to verify the proposed control which are also in good agreement with theoretical analysis.

**Index Terms**—Capacitor voltage feedforward, control bandwidth, delay compensation, grid background harmonics, grid impedance, *LCL*-filter, weak grids.

## I. INTRODUCTION

*LCL*-filtered grid-connected converters have been widely used in power conversion applications, such as distributed generation (DG) systems based on photovoltaic (PV), wind turbine, and energy storage etc. [1], [2]. However, due to their direct interconnection with power grids, *LCL*-filtered grid-connected converters are particularly sensitive to non-

ideal grid conditions, e.g., grid impedance [2], background harmonics [1]-[3], and grid voltage sags or swells, which often occur in weak grids [4].

Compared with *L*-filter, *LCL*-filter features high harmonic attenuation at switching frequencies, but it brings a stability challenge to the control system due to the well-known *LCL* resonance, and additional damping methods are required to ensure system stability. Because of the extra power loss of physical resistor-based passive damping, active damping methods based on additional state-variable feedback, e.g., capacitor current and capacitor voltage [5]-[7], have attracted more research interests. However, one or three more sensors are required for implementing the active damping in single-phase or three-phase systems, which may lead to higher costs and failure rate. Yi Tang *et al.* [8] first explored the inherent damping characteristic of converter-side current feedback control, which results in a more stable closed-loop system than grid-side current feedback. However, it has overlooked the impact of control delay on the damping performance. Xiongfei Wang *et al.* [9]-[11] investigated the active damping with only grid-side current through a negative high-pass filter, which is equivalent to paralleling a virtual impedance with grid-side inductor. Recently, the effectiveness of conventional notch filter-based active damping has been re-investigated with consideration of control delay and variations of filter parameters and grid impedance, and a design guideline of digital notch filters for robust active damping was developed in [12]. In addition, the stable design regions of *LCL*-filter with single-current-loop control are also revealed in [13], i.e.,  $\omega_r < \omega_s/6$  for converter-side current feedback, and  $\omega_r > \omega_s/6$  for grid-side current feedback, where  $\omega_r$  and  $\omega_s$  are the resonant frequency and sampling frequency respectively.

Generally, in order to prevent the large inrush current during startup [14], improve the dynamic response during grid voltage sags or swells [15], and suppress the current distortion caused by grid background harmonics [16], [17], grid voltage at the point of common coupling (PCC) or capacitor voltage is usually measured and fed forward to the control system for disturbances cancellation. However, due to the control delay and high-order *LCL* plants, simple unit voltage feedforward control cannot completely neutralize the impact of grid disturbances on output current regulation, especially those caused by background harmonics. Xuehua

Manuscript received August 13, 2016; revised October 23, 2016 and December 12, 2016; accepted January 22, 2017. This research is supported by the National Research Foundation, Prime Minister's Office, Singapore under the Energy Programme and administrated by the Energy Market Authority (EP Award No. NRF2015EWT-EIRP002-007). (*Corresponding author: Yi Tang*)

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Wang *et al.* [18] thus proposed a full feedforward scheme of grid voltage, which involves a derivative and even a second-order derivative in the feedforward function. It is particularly difficult to implement a precise high-order derivative in the digital control system due to the noise amplification. Paralleling multiple harmonic compensators (HCs) with fundamental proportional-integral (PI) or proportional-resonant (PR) current controller is a commonly used method to attenuate current harmonics [3], [19], [20], but the control bandwidth should be wide enough to cover all the resonant frequencies of HCs [2], [21]. However, the increasing use of distributed generation, low-voltage distribution transformers, and long distribution cables, etc. may cause grid impedance to vary in a wide range, thus leading to a significantly decreased control bandwidth [2]. Instead of compensating harmonics, HCs will then excite low-frequency instability when the control bandwidth is lower than the resonant frequencies of HCs [2]. In addition, the decreased resonant frequency of *LCL*-filter due to inductive grid impedance may cause *LCL*-filter resonance even through the active damping method is employed [5], [6]. Moreover, an additional positive feedback of grid-side current by grid impedance is embedded implicitly in the feedforward path of PCC voltage, which yields a minor current loop. This minor loop changes the low-frequency characteristic of the system, and the stability margin is decreased significantly [22].

Typically, two sets of sensors, one set of current sensors for current feedback control and another set of voltage sensors for grid synchronization, are the basic requirements for proper control of grid-connected converters. Fig. 1 shows the topology of an *LCL*-filter and possible sensed control variables. The control variables can be classified into four groups, i.e.,  $i_1$  &  $v_{pcc}$ ,  $i_1$  &  $v_c$ ,  $i_2$  &  $v_{pcc}$ , and  $i_2$  &  $v_c$ . In practical industrial applications, converter-side current  $i_1$  is usually sensed for current feedback control and overcurrent protection in spite of its poor output power quality, i.e., reactive power and harmonics. Since capacitor voltage  $v_c$  is linear with PCC voltage  $v_{pcc}$ , it is commonly used for grid synchronization. In addition, due to its damping features, capacitor voltage  $v_c$  can also be used as a state-variable for active damping [7], [23]-[25]. Based on the above considerations, the group,  $i_1$  &  $v_c$ , is selected and discussed in this paper due to its low cost, fast overcurrent protection and resonance damping [24], [25]. This group of control variables is also a promising choice for dual-mode applications, i.e., grid-connected mode and stand-alone mode, in DG and micro-grid systems based on renewable energy.

Active damping techniques based on capacitor voltage have been well investigated in literature. One commonly used method is the virtual impedance control [26]-[28]. In [26], harmonic components of the capacitor voltage are extracted to generate the inner current reference. However, it needs a high control bandwidth for harmonic current tracking, thus high dynamic current control such as deadbeat

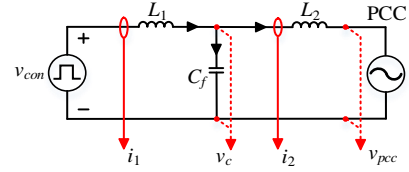


Fig. 1. Topology of an *LCL*-filter and possible sensed control variables.

control, should be employed instead of the conventional PR or PI-based current control. In [27] and [28], an additional state-variable is fed forward with a specific function to form the desirable virtual impedance. However, it demands a noise-sensitive derivative term for the capacitor voltage [29]. In order to avoid the noise amplification at high frequencies, two highly accurate derivatives based on second-order or non-ideal generalized integrator are proposed in [7] recently. Nevertheless, it suffers from damping failure under weak power grids as it is equivalent to that with proportional capacitor-current-feedback [7]. In addition, this derivative-based feedforward form for active damping cannot prevent the large inrush current during startup and suppress the disturbances of grid voltage as mentioned before.

In order to improve the weak grids adaptability of *LCL*-filtered grid-connected converters, this paper proposes a full delay compensation strategy for unit capacitor voltage feedforward, and the digital implementation of the proposed one and a half sampling periods delay compensation does not require derivative operation or additional sensing variables. When incorporated with converter-side current feedback, the proposed method can reshape the low-frequency characteristic of current control and make it independent of grid impedance. Therefore, the control bandwidth keeps unchanged regardless of the variation of grid impedance under weak grid conditions. It is also proved that the system has very low output admittance over a wide range of frequencies, and it is possible to achieve good grid background harmonic rejection even with line frequency variation. The multiple HCs used in parallel with the conventional current control can be eliminated, which greatly simplifies the current controller design. Moreover, the resonance frequency of *LCL*-filter can be designed to be above 1/6 of the system sampling frequency, indicating a wider control bandwidth and lower current distortion caused by the paralleled filter capacitor branch. All these findings have not been reported in literature and could be deemed as the main contributions of this paper. Because of the voltage feedforward mechanism, other grid disturbances, e.g. voltage swells or sags, can be also canceled, ensuring fast dynamic response of current control. Experimental results are finally provided to verify the proposed control method.

## II. SYSTEM DESCRIPTION AND MODELING

Fig. 2 shows a single-phase *LCL*-filtered grid-connected converter and its basic control structure, where converter-side current  $i_1$  is sensed for closed-loop feedback control and overcurrent protection, and capacitor voltage  $v_c$  is

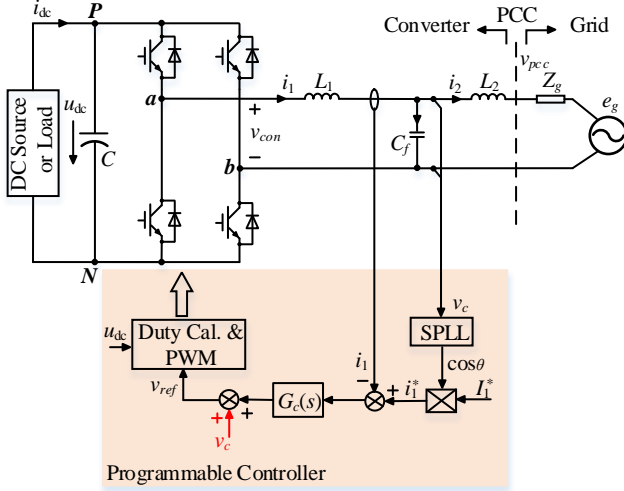


Fig. 2. A single-phase  $LCL$ -filtered grid-connected converter with converter-side current feedback.

sensed for synchronization,  $LCL$  resonance damping, and suppression of grid voltage disturbances with its unit feedforward. The  $LCL$ -filter includes a converter-side inductor  $L_1$ , a grid-side inductor  $L_2$ , and a filter capacitor  $C_f$ . The equivalent grid impedance  $Z_g$  can be modeled by the Thevenin's theorem at PCC, and it is purely inductive with  $L_g$  here to present the worst case without any physical damping.  $e_g$  and  $v_{pcc}$  denote grid voltage and PCC voltage respectively.  $v_{con}$  denotes output voltage of the converter.  $u_{dc}$  and  $i_{dc}$  denote dc-side voltage and current respectively. Unipolar sinusoidal pulse-width-modulation (SPWM) and a second-order generalized integrator-based phase-locked loop (SOGI-PLL) are employed to obtain gating and synchronization signals [30], [31].

Fig. 3 shows the mixed-domain current-loop structure that involves the digital current control and continuous  $s$ -domain plant model.  $G_c(s)$  is the current controller consisting of a fundamental PR controller to achieve main power control and multiple HCs to eliminate low-order current harmonics with integral gains  $k_{i1}$ ,  $k_{ih}$  and proportional gain  $k_p$ , expressed as

$$G_c(s) = k_p + \underbrace{\frac{k_{i1}\omega_b s}{s^2 + \omega_b s + \omega_1^2}}_{PR} + \sum_{h=3,5,7,\dots} \underbrace{\frac{k_{ih}\omega_{bh} s}{s^2 + \omega_{bh} s + \omega_h^2}}_{HC}. \quad (1)$$

$T_s$  is the sampling period.  $z^{-1}$  is the computation delay of one sampling period and  $G_{zoh}(s)$  is the zero-order-hold (ZOH) given by

$$G_{zoh}(s) = \frac{1 - e^{-T_s s}}{s}. \quad (2)$$

For the sake of analysis, Fig. 4 shows the simplified current control diagrams in the continuous  $s$ -domain and discrete  $z$ -domain respectively, where  $G_d(s)$  is the equivalent 1.5 sampling periods delay, derived as

$$G_d(s) = \frac{1}{T_s} e^{-T_s s} G_{zoh}(s) \approx e^{-1.5T_s s}. \quad (3)$$

$G_{i1}(s)$  and  $G_{vc}(s)$  are the transfer functions from  $v_{con}(s)$  to  $i_1(s)$  and  $v_c(s)$ , respectively. Define  $L_T = L_2 + L_g$ , it then yields

$$G_{i1}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{L_T C_f s^2 + 1}{s(s^2 + \omega_r^2)}, \quad (4)$$

$$G_{vc}(s) = \frac{1}{L_1 C_f} \cdot \frac{1}{(s^2 + \omega_r^2)}, \quad (5)$$

where  $\omega_r$  is the resonant angular frequency,

$$\omega_r = \sqrt{\frac{L_1 + L_T}{L_1 L_T C_f}}. \quad (6)$$

Applying the ZOH  $z$ -transformation to (4) and (5), it can be derived as

$$\begin{aligned} G_{i1}(z) &= Z[G_{zoh}(s)G_{i1}(s)] \\ &= \frac{T_s}{(L_1 + L_T)(z-1)} + \frac{L_T \sin(\omega_r T_s)}{L_1(L_1 + L_T)\omega_r} \cdot \frac{z-1}{z^2 - 2z \cos(\omega_r T_s) + 1}, \end{aligned} \quad (7)$$

$$\begin{aligned} G_{vc}(z) &= Z[G_{zoh}(s)G_{vc}(s)] \\ &= \frac{L_T(1 - \cos(\omega_r T_s))}{L_1 + L_T} \cdot \frac{z+1}{z^2 - 2z \cos(\omega_r T_s) + 1}. \end{aligned} \quad (8)$$

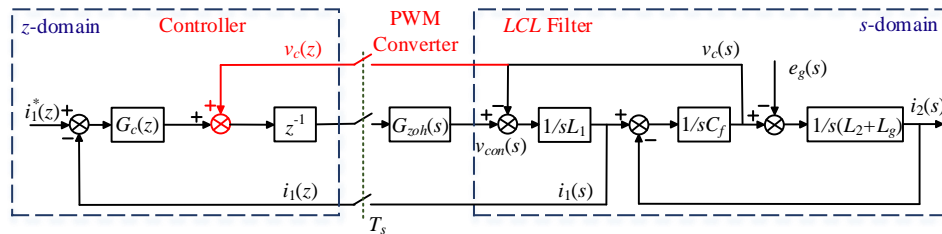


Fig. 3. Current-loop control structure in the mixed-domain.

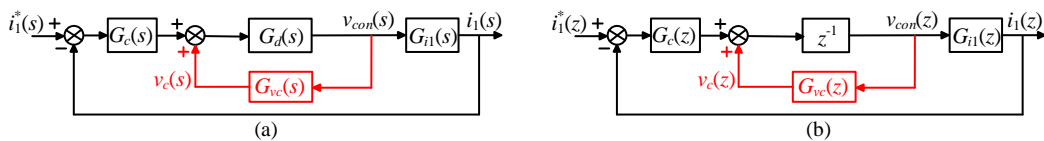


Fig. 4. Simplified current-loop control diagrams. (a) In the continuous  $s$ -domain. (b) In the discrete  $z$ -domain.

### III. SINGLE-CURRENT-LOOP CONTROL WITHOUT CAPACITOR VOLTAGE FEEDFORWARD

As analyzed in literature, single-current-loop control with converter-side current feedback can be stable only when the resonant frequency  $\omega_r$  is less than one-sixth of the sampling frequency  $\omega_s$ , i.e.,  $\omega_r < \omega_s/6$  [13]. The open-loop transfer function  $T_{op1}(z)$  can be derived as (9) from Fig. 4 without considering capacitor voltage feedforward. Fig. 5 shows the bode plots of the open-loop transfer function  $T_{op1}(z)$ , where  $L_g=0$ ,  $C_f=5\ \mu\text{F}$  and  $25\ \mu\text{F}$  respectively, and other parameters are listed in Table I. For the sake of simplicity, only proportional gain  $k_p$  is considered with  $k_p=1$ . As seen, the  $-180^\circ$  crossing takes place at  $\omega_s/6$  when  $\omega_r < \omega_s/6$ , while at  $\omega_r$  and  $\omega_s/6$  when  $\omega_r > \omega_s/6$  (Here, the  $-180^\circ$  crossing at the zero is not considered). According to Nyquist stability criterion, the system is stable only if there are no encirclements of  $(-1, j0)$  due to no unstable open-loop poles. From Fig. 5, it is obvious that the infinite open-loop gain at  $\omega_r$  will lead to two encirclements of  $(-1, j0)$  when  $\omega_r > \omega_s/6$ , then the system is unstable.

$$T_{op1}(z) = z^{-1} \tilde{G}_c(z) G_{i1}(z) = \frac{G_c(z)}{L_1(L_1 + L_T)\omega_r} \cdot \frac{L_1\omega_r T_s [z^2 - 2z \cos(\omega_r T_s) + 1] + L_T \sin(\omega_r T_s)(z-1)^2}{z(z-1)[z^2 - 2z \cos(\omega_r T_s) + 1]} \quad (9)$$

However, it is not sufficient to ensure system stability only with the constraint of  $\omega_r < \omega_s/6$ . The proportional gain  $k_p$  should satisfy the following condition

$$T_{op1}(z = e^{j\frac{\omega}{6}T_s}) < 0 \text{ dB} \Rightarrow k_p < \frac{L_1(L_1 + L_T)\omega_r(1 - 2\cos(\omega_r T_s))}{L_T \sin(\omega_r T_s) - L_1\omega_r T_s(1 - 2\cos(\omega_r T_s))} \quad (10)$$

It is known that the limitation of resonant frequency ( $\omega_r < \omega_s/6$ ) will lead to a low control bandwidth, which then results in slow transient response. Moreover, in order to suppress the current distortion arising from low-order grid background harmonics, multiple HCs are usually paralleled with the fundamental PR controller as (1). This requires a wide enough control bandwidth to cover all the resonant frequencies of HCs, or the system may be unstable. The low control bandwidth will limit the number of harmonics to be compensated by HCs. In addition, the above situations may get worse when large inductive grid impedance  $L_g$  appears which decreases the control bandwidth. Fig. 6 shows the bode plots of the open-loop transfer function  $T_{op1}(z)$  with 5th and 7th HCs, where  $k_p=4$  and  $C_f=25\ \mu\text{F}$  with resonant frequency being  $1.46\ \text{kHz}$  ( $< f_s/6$ ).  $f_c$  is the cut-off frequency. As seen from Fig. 6(b), with the requirements of  $\omega_r < \omega_s/6$  and (10),  $f_{c1}$  is close to the 7th harmonic frequency, thus the highest harmonic to be compensated is 7th. Near the 7th harmonic frequency, there is one negative and one positive  $-180^\circ$  crossing at  $f_1$  and  $f_2$  respectively. Since the magnitude of  $T_{op1}(z)$  is always greater than 0dB at these two frequencies, the total number of encirclements of  $(-1, j0)$  is

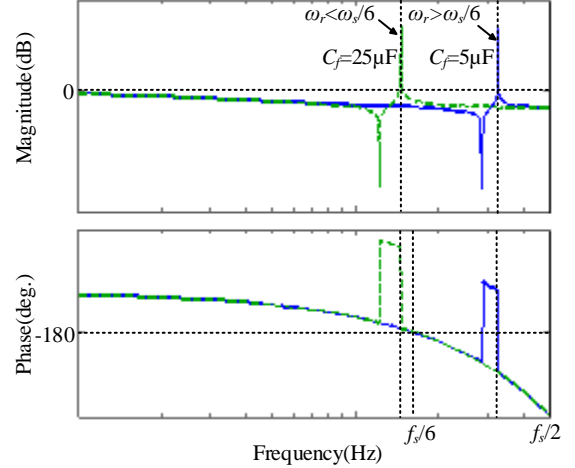


Fig. 5. Bode plots of the open-loop transfer function  $T_{op1}(z)$  with  $\omega_r < \omega_s/6$  and  $\omega_r > \omega_s/6$ .

TABLE I  
PARAMETERS OF LCL-FILTER

Description	Symbol	Value
Filter capacitance	$C_f/\mu\text{F}$	3~25
Converter-side inductance	$L_1/\text{mH}$	1.5
Grid-side inductance	$L_2/\text{mH}$	0.7
Grid impedance	$L_g/\text{mH}$	0 / 2
Sampling frequency	$f_s/\text{kHz}$	10

zero, and the system is stable. However, with the increase of  $L_g$ , the cut-off frequency  $f_{c1}$  decreases to  $f_{c2}$ , and the magnitude of  $T_{op1}(z)$  at  $f_2$  is less than 0dB. Hence, there will be two encirclements of  $(-1, j0)$  (considering the negative frequency), and it causes the system to be unstable.

Since the grid-side current feedback control is the direct regulation of output current, the current distortion can be well suppressed by HCs. While for the converter-side current feedback control, the output current inevitably contains certain harmonics due to the paralleled filter capacitor branch. However, to satisfy the stability requirement,  $\omega_r < \omega_s/6$ , a large capacitance is usually employed to obtain a lower resonant frequency. Large capacitance will then aggravate the output current distortion. In order to explain this, Fig. 7 shows the grid-side current diagram with converter-side current feedback control, where grid voltage  $e_g(s)$  as a harmonic source needs to be considered. The transfer function  $G_{i2}(s)$ ,  $G_{gi2}(s)$  and  $G_{gi1}(s)$  are given as follows

$$\begin{cases} G_{i2}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{1}{s(s^2 + \omega_r^2)} \\ G_{gi2}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{L_1 C_f s^2 + 1}{s(s^2 + \omega_r^2)} \\ G_{gi1}(s) = \frac{1}{L_1 L_T C_f} \cdot \frac{1}{s(s^2 + \omega_r^2)} \end{cases} \quad (11)$$

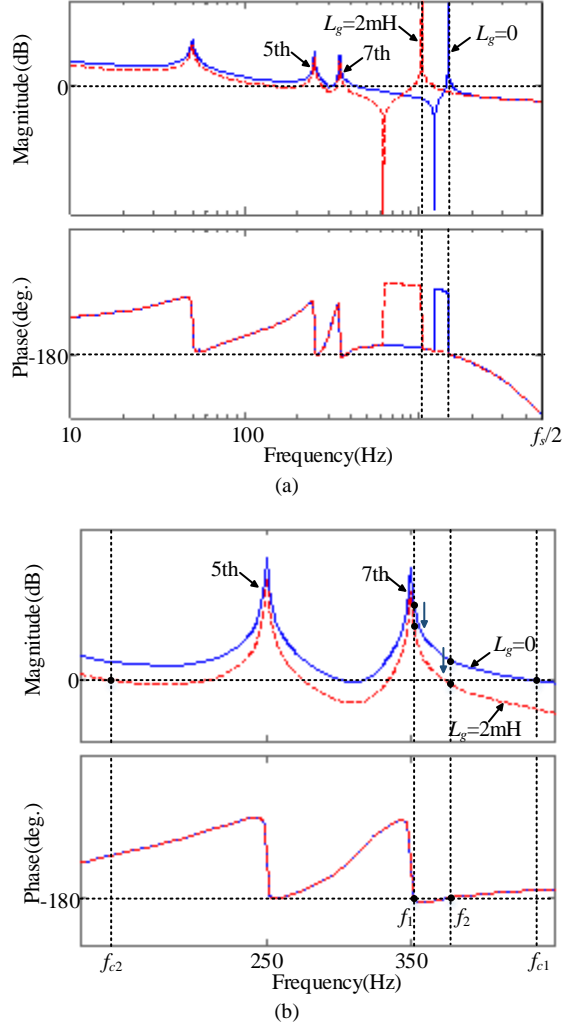


Fig. 6. Bode plots of the open-loop transfer function  $T_{op1}(z)$  with 5th and 7th HCs. (a) Full view. (b) Zoomed-in view in 5th and 7th harmonic frequencies.

Then, the output admittance  $Y_{o1}(s)$  can be derived as (12) from Fig. 7.

$$Y_{o1}(s) = -\frac{i_2(s)}{e_g(s)} = G_{gi2}(s) - \frac{G_d(s)G_c(s)G_{i2}(s)G_{gi1}(s)}{1 + G_d(s)G_c(s)G_{i1}(s)} \quad (12)$$

Fig. 8 shows the magnitude-frequency curves of the output admittance  $Y_{o1}(s)$  with three different filter capacitances  $C_f=5 \mu\text{F}$ ,  $10 \mu\text{F}$  and  $25 \mu\text{F}$ , where  $L_g=0$  and  $k_p=4$ . As seen, with the increase of  $C_f$ , the magnitude of the output admittance  $Y_{o1}(s)$  at the compensated harmonic frequencies by HCs increases, which indicates that the capability of suppressing grid background harmonics decreases.

From the above analysis, it can be known that the single-current-loop control with converter-side current feedback has a poor adaptability under weak grids, which is due to its limited design region of LCL-filter and also its sensitivity to grid impedance variation.

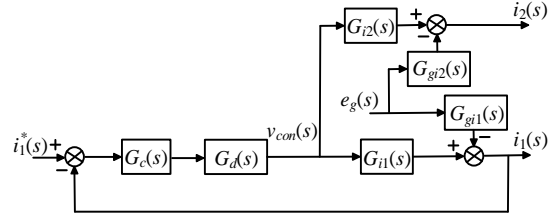


Fig. 7. Grid-side current diagram with converter-side current feedback control.

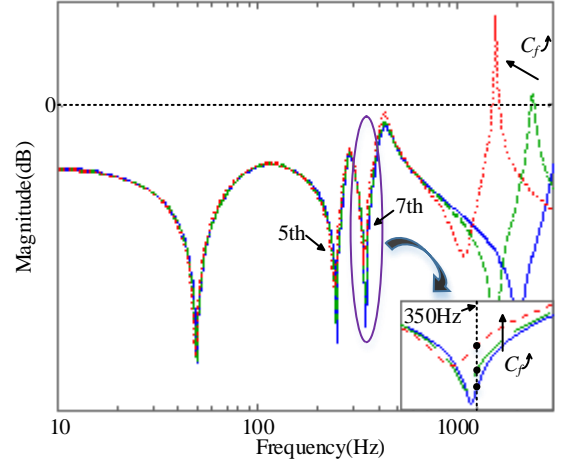


Fig. 8. Magnitude-frequency curves of the output admittance  $Y_{o1}(s)$  with different filter capacitance  $C_f$ .

#### IV. CONVERTER-SIDE CURRENT CONTROL WITH UNIT CAPACITOR VOLTAGE FEEDFORWARD

Generally, in order to prevent the large inrush current during startup and suppress the disturbances of grid voltage, unit PCC voltage or capacitor voltage can be used as a feedforward term to decouple the actual grid voltage. In this paper, considering the resonance damping, unit capacitor voltage feedforward is employed as discussed in Introduction, shown in Fig. 2.

From Fig. 4(b), the open-loop transfer function  $T_{op2}(z)$  is derived as follows

$$T_{op2}(z) = G_c(z)G_{in}(z)G_{i1}(z) = \frac{z^{-1}G_c(z)G_{i1}(z)}{1 - z^{-1}G_{vc}(z)}, \quad (13)$$

where  $G_{in}(z)$  is the minor loop.

According to Nyquist stability criterion, the pole locations of  $T_{op2}(z)$  need to be investigated. From (13), the unstable poles of  $T_{op2}(z)$  can be identified by applying Nyquist stability criterion to  $T_{in}(z) = z^{-1}G_{vc}(z)$ . Since  $G_{vc}(z)$  has no unstable poles, the unstable poles of  $T_{op2}(z)$  are then determined by the encirclements of  $(-1, j0)$  by  $T_{in}(z)$ . Fig. 9 shows the bode plots of the minor loop gain  $T_{in}(z)$ , where  $L_g=0$ ,  $L_1$ ,  $L_2$  and  $f_s$  are listed in Table I, and  $C_f$  is purposely set as  $3 \mu\text{F}$ ,  $5.5 \mu\text{F}$  and  $15 \mu\text{F}$  for analysis with resonant frequency  $4.21 \text{ kHz}$ ,  $3.12 \text{ kHz}$  and  $1.88 \text{ kHz}$  respectively. As seen, the  $-180^\circ$  crossing takes place at  $\omega_r/3$  when  $\omega_r < \omega_s/3$ , while at  $\omega_r$  when  $\omega_r > \omega_s/3$ . In order to explain this situation, the value of  $T_{in}(z)$  at  $\omega_s/3$  can be calculated as



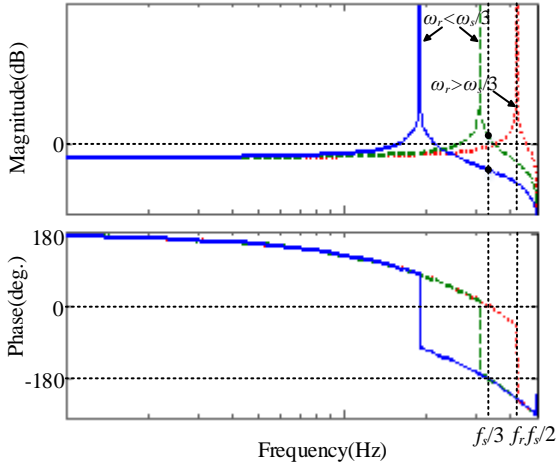


Fig. 9. Bode plots of the minor loop gain  $T_{in}(z)$ .

$$T_{in}(z = e^{j\frac{\omega_s T_s}{3}}) = -\frac{L_T(1 - \cos(\omega_r T_s))}{(L_1 + L_T)(1 + 2\cos(\omega_r T_s))}. \quad (14)$$

From (14), it can be seen  $T_{in}(z)$  at  $\omega_s/3$  is negative when  $\omega_r < \omega_s/3$  and positive when  $\omega_r > \omega_s/3$ . Thus,  $-180^\circ$  crossing always takes place at  $\omega_s/3$  when  $\omega_r < \omega_s/3$ . In order to avoid the encirclements of  $(-1, j0)$ , the resonant frequency  $\omega_r$  should be less than  $\omega_s/3$ . However, it is not sufficient to ensure that  $T_{op2}(z)$  has no unstable poles. As the dashed bode plot shown in Fig. 9, there are two encirclements of  $(-1, j0)$  due to the magnitude of  $T_{in}(z)$  at  $\omega_s/3$  being greater than 0 dB even though  $\omega_r < \omega_s/3$ . Hence, in addition to the design of resonant frequency  $\omega_r$ , the magnitude of  $T_{in}(z)$  at  $\omega_s/3$  should be less than 0 dB. Based on (14), it can be derived as follows

$$20\lg \left| T_{in}(z = e^{j\frac{\omega_s T_s}{3}}) \right| < 0\text{dB} \Rightarrow \cos(\omega_r T_s) > -\frac{L_1}{2L_1 + 3L_T}. \quad (15)$$

The constraint conditions,  $\omega_r < \omega_s/3$  and (15), are then given to ensure that the open-loop transfer function  $T_{op2}(z)$  has no unstable poles. As seen, (15) is always valid when  $\omega_r < \omega_s/4$  due to  $\cos(\omega_r T_s) > 0$ . While for  $\omega_s/4 < \omega_r < \omega_s/3$ , it is difficult to determine which side of (15) is greater due to the potential grid impedance  $L_g$  and its complicated functions, e.g., trigonometric function and square-root operation. It may decrease the upper limit of resonant frequency and make it lower than  $\omega_s/3$ . Given that  $L_g=0$ , and  $L_1=1.5$  mH,  $L_2=0.7$  mH, and  $f_s=10$  kHz listed in Table I, the available range of filter capacitance is found to be  $C_f > 6$   $\mu\text{F}$  by (15), and thus the upper limit of  $\omega_r$  is 2.97 kHz which is less than  $\omega_s/3=3.33$  kHz. Taking  $C_f=7\mu\text{F}$ , Fig. 10 shows the curves of two functions in (15) with the increase of  $L_g$ . As seen, the inequality is always valid which indicates that  $T_{op2}(z)$  has no open-loop unstable poles with this set of  $LCL$  parameters regardless of grid impedance variation.

Following the above design constraints,  $T_{op2}(z)$  should have no encirclements of  $(-1, j0)$  to ensure system stability by Nyquist stability criterion. With the same  $L_1$ ,  $L_2$  and  $f_s$  as above and  $k_p=1$ , Fig. 11 shows the bode plots of the open-

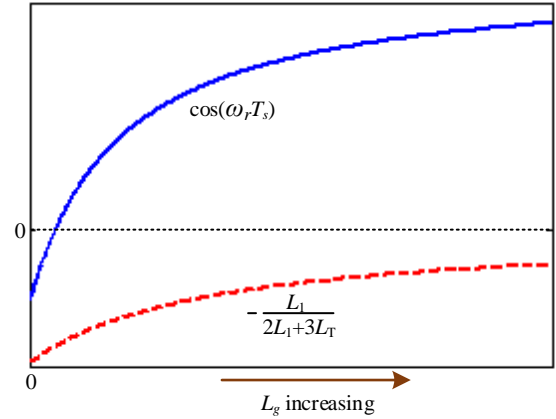


Fig. 10. Curves of two functions in (15) with the increase of  $L_g$ .

loop transfer function  $T_{op2}(z)$ , where  $C_f$  is set as  $7\mu\text{F}$  and  $10\mu\text{F}$  with resonant frequency being 2.75 kHz and 2.30 kHz respectively. As seen,  $LCL$  resonance damping is enhanced with the increase of filter capacitance  $C_f$ . It indicates that a large proportional gain  $k_p$  can be allowed by employing a large filter capacitance, thus leading to a high control bandwidth. Even though large filter capacitance can result in higher control bandwidth to parallel more HCs, it causes increased current distortion due to paralleled filter capacitor branch.

Further, in order to investigate the robustness against grid impedance variation, Fig. 12 shows the bode plots of  $T_{op2}(z)$  with grid impedance  $L_g$ , where  $C_f$  is 10  $\mu\text{F}$  to damp the resonance peak. As seen, the phase-frequency curve falls off rapidly when  $L_g=2$  mH, thus leading to a small phase margin. In addition, the decreased phase margin and decreased magnitude response of  $T_{op2}(z)$  will limit its harmonic rejection capability since only a few harmonics can be compensated as analyzed in Fig. 6.

## V. PROPOSED CAPACITOR VOLTAGE FEEDFORWARD WITH FULL DELAY COMPENSATION

Section IV has derived the stable design region of  $LCL$ -filter when unit capacitor voltage feedforward is employed to suppress grid voltage disturbances, which is obviously wider than single-current-loop control ( $\omega_r < \omega_s/6$ ). However, it still has a poor rejection capability of grid background harmonics and is quite sensitive to grid impedance variation.

From Fig. 4(a), the open-loop transfer function  $T_{op2}(s)$  in the continuous  $s$ -domain is derived as

$$\begin{aligned} T_{op2}(s) &= \frac{G_c(s)G_d(s)G_{il}(s)}{1 - G_d(s)G_{vc}(s)} \\ &= \frac{G_c(s)G_d(s)(L_T C_f s^2 + 1)}{L_1 L_T C_f s^3 + (L_1 + L_T)s - \underbrace{G_d(s)L_T s}_{\text{Damping Term}}} \end{aligned} \quad (16)$$

From (16), it can be seen a damping term is introduced by unit capacitor voltage feedforward, which contributes to the attenuation of  $LCL$  resonance peak shown in Fig. 11 and Fig. 12. Further, the low-frequency characteristic of  $T_{op2}(s)$

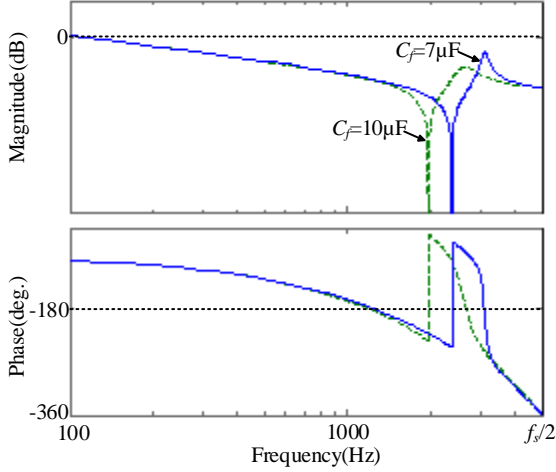


Fig. 11. Bode plots of the open-loop transfer function  $T_{op2}(z)$  with different filter capacitance  $C_f$ .

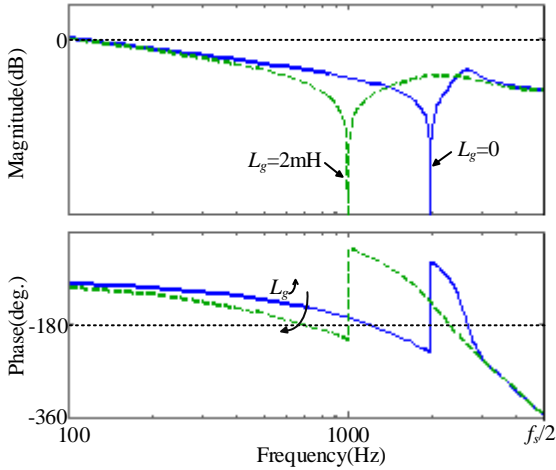


Fig. 12. Bode plots of the open-loop transfer function  $T_{op2}(z)$  with grid impedance  $L_g$ .

can be approximated by neglecting the high-order terms in (16) as

$$T_{op2}(s) \approx \frac{G_c(s)G_d(s)}{(L_1 + L_r)s - G_d(s)L_r s}. \quad (17)$$

As seen from (17), grid impedance  $L_g$  (in  $L_r$ ) changes the magnitude and phase of the denominator. The low-frequency characteristic of  $T_{op2}(s)$  is thus affected by  $L_g$  as shown in Fig. 12, which leads to a poor grid-adaptability, e.g., reduced stability margin and harmonic rejection capability. If a compensation link  $G_{ic}(s)$  is inserted in the feedforward path, as shown in Fig. 13, the open-loop transfer function  $T'_{op2}(s)$  then can be derived as

$$T'_{op2}(s) = \frac{G_c(s)G_d(s)(L_r C_f s^2 + 1)}{L_1 L_r C_f s^3 + (L_1 + L_r)s - G_d(s)G_{ic}(s)L_r s}. \quad (18)$$

From (18), if  $G_{ic}(s) = 1/G_d(s)$ , it can be yielded as  $T'_{op2}(s) = G_c(s)G_d(s)/L_1 s$ . The current control loop becomes a first-order  $L$ -filtered system with only converter-side inductance  $L_1$  and independent of grid impedance  $L_g$ , which

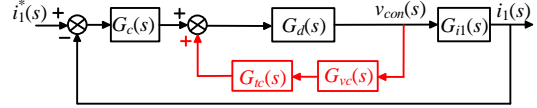


Fig. 13. Current-loop control diagram with compensation link  $G_{ic}(s)$ .

will lead to a strong adaptability under weak grids. From (3),  $G_d(s)$  is the time delay link with one and a half sampling periods. Consequently,  $G_{ic}(s)$  should be a time advance link to compensate the delay. Actually, same thought can also be found in [24], where only one sampling period delay is considered to compensate, and its realization takes up too much storage space and is not accurate enough.

#### A. One Sampling Period Delay Compensation

Several prediction techniques for compensation of one sampling period delay have been proposed earlier, e.g., linear prediction, smith prediction, and repetitive prediction, etc. [32], [33]. However, these techniques are either sensitive to plant parameters or not accurate enough especially in the high-frequency range. [6], [25] and [34] proposed a reduced delay method by shifting the sampling instant of state-variables. However, the sampling number in one sampling period is doubled in [6]. In [25] and [34], the sampling instant needs to be set carefully to implement the control algorithm, and meanwhile, it is difficult to achieve one sampling period delay compensation. More importantly, the sampling-induced aliasing and switching noise may be introduced to the control system. In view of these, an improved reduced delay method is proposed, which shifts the update instant of reference voltage instead of the sampling instant of state-variables.

For comparison, Fig. 14(a) shows the conventional current control structure with unit capacitor voltage feedforward, where  $v_{co}(k)$  is the output of current controller,  $v_{ref}(k)$  is the reference voltage, and  $k$  denotes the  $k_{th}$  sampling period. From Fig. 14(a), it can be yielded as

$$v_{ref}(k) = z^{-1}v_r(k) = v_r(k-1) = v_{co}(k-1) + v_c(k-1). \quad (19)$$

From (19), it can be seen the reference voltage  $v_{ref}$  at the  $k_{th}$  sampling period is the sum of  $v_{co}$  and  $v_c$  at the  $(k-1)_{th}$  sampling period due to one sampling period delay for control algorithm implementation.

Fig. 14(b) shows the sampling and update instants of the digital PWM, where,

**Event 1:** Sampling instant of converter-side current  $i_1$  and capacitor voltage  $v_c$ .

**Event 2:** Calculation finished instant of  $v_r$  which is subsequently used for updating reference voltage  $v_{ref}$ .

**Event 3:** Update instant of reference voltage  $v_{ref}$  which is also the sampling instant of next period.

Fig. 15(a) shows the current control structure with the proposed one sampling period delay compensation, and it can be yielded as



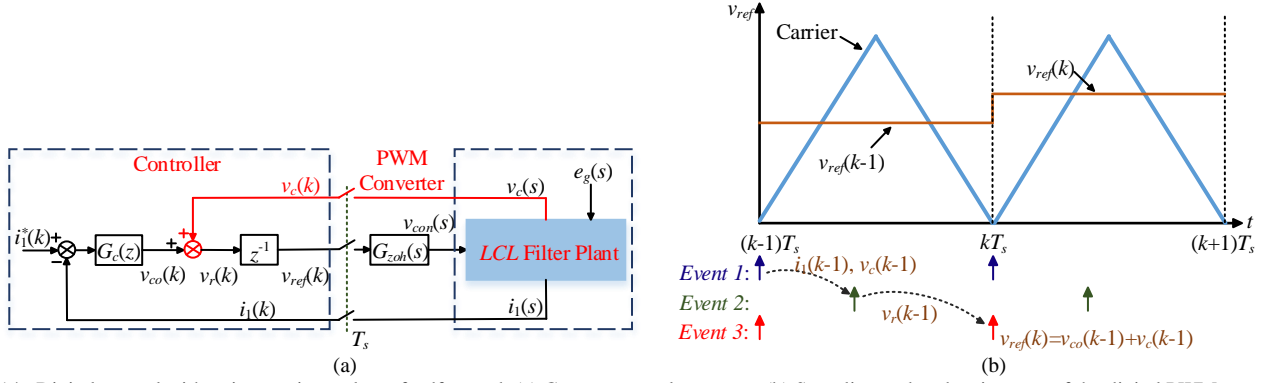


Fig. 14. Digital control with unit capacitor voltage feedforward. (a) Current control structure. (b) Sampling and update instants of the digital PWM.

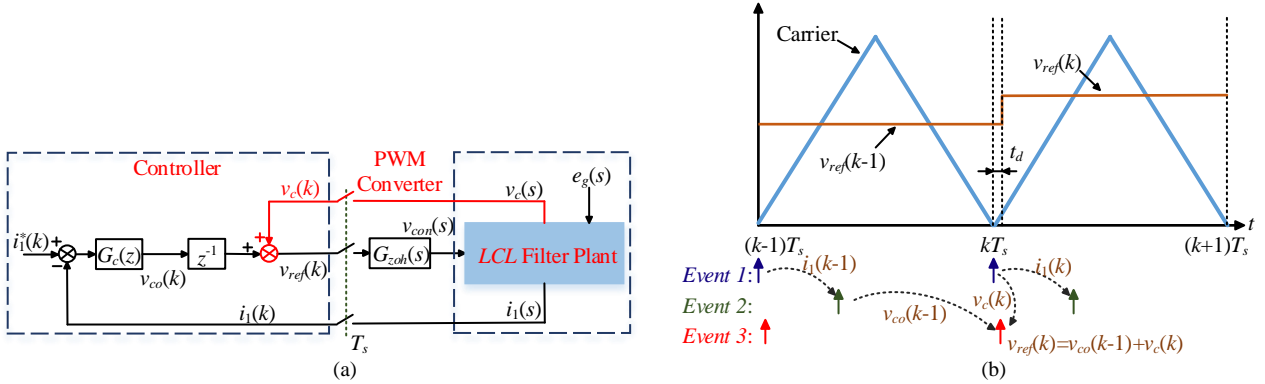


Fig. 15. Digital control with the proposed one sampling period delay compensation. (a) Current control structure. (b) Sampling and update instants of the digital PWM.

$$v_{ref}(k) = z^{-1}v_{co}(k) + v_c(k) = v_{co}(k-1) + v_c(k). \quad (20)$$

In comparison to (19), the reference voltage  $v_{ref}$  in (20) at the  $k_{th}$  sampling period is the sum of  $v_{co}$  at the  $(k-1)_{th}$  sampling period and  $v_c$  at the  $k_{th}$  sampling period. Obviously, the one sampling period delay compensation is realized by changing the feedforward instant of capacitor voltage. With this idea, Fig. 15(b) shows the sampling and update instants which indicates the digital implementation of one sampling period delay compensation, where,

**Event 1:** Sampling instant of converter-side current  $i_1$  and capacitor voltage  $v_c$ .

**Event 2:** Calculation finished instant of  $v_{co}$  which is subsequently used for calculating reference voltage  $v_{ref}$ .

**Event 3:** Update instant of reference voltage  $v_{ref}$  with negligible delay  $t_d$  to Event 1, where  $t_d$  consists of the sampling time  $t_{ad}$  of analog-to-digital converter (ADC) and the calculation time  $t_{cd}$  of half sampling period delay compensation which will be discussed in the following subsection.

As seen from Fig. 15(b),  $v_c(k)$  is used for obtaining the reference voltage  $v_{ref}(k)$  instead of  $v_c(k-1)$  in order to eliminate the one sampling period delay of voltage feedforward. The update instant of reference voltage  $v_{ref}(k)$  is slightly shifted by  $t_d$  for implementing this delay compensation. It should be noted that the delay time  $t_d$  is much smaller as compared to the switching period, and its

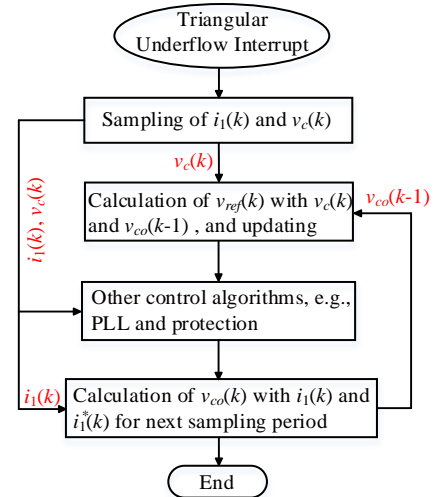


Fig. 16. Digital implementation flowchart of the main control algorithm with one sampling period delay compensation.

impact on the control system could be neglected. If  $t_d$  is too large and cannot be ignored due to some slow conversion rate ADCs or very high sampling frequency, some additional control techniques should be employed to improve the control performance of the system, such as the two-polarity PWM method and asymmetric PWM method

proposed in [35], [36]. These modulation methods can tolerate the maximum time of  $0.25T_s$  for  $t_d$ .

For the sake of easy understanding, Fig. 16 shows the digital implementation flowchart of the main control algorithm with the proposed one sampling period delay compensation.

### B. Half Sampling Period Delay Compensation

From (3), the half sampling period delay is caused by the ZOH characteristic of PWM. Unlike one sampling period delay compensation which is realized by changing the feedforward instant of capacitor voltage, an additional compensation link is inserted in the feedforward path to handle the half sampling period delay. Fig. 17 shows the digital current control structure with full delay compensation, where  $G_{hc}(z)$  is to compensate the half sampling period delay, expressed as (21). Actually, the similar link can also be found in [5] and [37] to improve the performance of active damping.

$$G_{hc}(z) = \frac{2}{1+z^{-1}}. \quad (21)$$

Fig. 18 shows the bode plots of  $G_{hc}(z)$  and an ideal half sampling period in advance  $z^{0.5}$  with sampling frequency  $f_s=10$  kHz. As seen, the phase of  $G_{hc}(z)$  has a perfect match with  $z^{0.5}$  within Nyquist frequency  $f_s/2$ . However, its magnitude response increases with the increase of frequency and becomes infinite at  $f_s/2$ , which is unacceptable because of the sampling noise amplification. In order to explain this, (21) can be expanded as

$$G_{hc}(z = e^{j\omega T_s}) = \frac{2}{1+e^{-j\omega T_s}} = \frac{e^{j0.5\omega T_s}}{\cos(0.5\omega T_s)}. \quad (22)$$

As seen from (22), the phase of  $G_{hc}(z)$  is  $0.5\omega T_s$  same as  $z^{0.5}$ , while the magnitude is  $1/\cos(0.5\omega T_s)$  which is infinite at  $f_s/2$  due to  $\cos(0.5\omega T_s/2)=0$ . Based on the control structure of repetitive controller, a modified version of (21) is thus proposed shown as Fig. 19, where  $m$  is a proportional coefficient with  $0 < m \leq 1$ , and  $G_{zf}(z)$  is a first-order zero-phase-shift low-pass filter (LPF) having  $G_{zf}(z) = d_1 z + d_0 + d_1 z^{-1}$ , where  $d_0 = 1 - 2d_1$  [38], [39]. The modified link is expressed as

$$G'_{hc}(z) = \frac{1+m}{1+mG_{zf}(z)z^{-1}}. \quad (23)$$

It is noted that there is a 'z' in the  $G_{zf}(z)$  which can be hardly realized in the digital control system. Fortunately, another term 'z<sup>-1</sup>' in the feedback path can be integrated into  $G_{zf}(z)$  to cancel 'z'. The value of  $G_{zf}(z)$  is calculated as  $d_0 + 2d_1 \cos(\omega T_s)$  which features zero phase in the whole frequency range as shown in Fig. 20. By calculation,  $G_{zf}(z)$  is equal to  $d_0 - 2d_1$  at  $f_s/2$ , and thus, if  $d_1 = 0.25$ ,  $20 \lg |G_{zf}(z = e^{j\omega T_s})|_{\omega=\omega_s/2} = -\infty$ , as shown in Fig. 20. In order to attenuate the magnitude peak of  $G'_{hc}(z)$  effectively,  $d_1 = 0.25$  is selected in this paper, and then the magnitude of  $G'_{hc}(z)$  at  $f_s/2$  is  $1+m$  with the maximum value of 2 due to  $0 < m \leq 1$ .

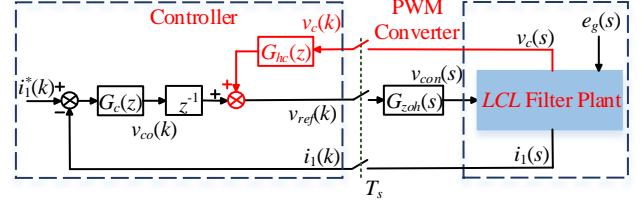


Fig. 17. Digital current control structure with full delay compensation.

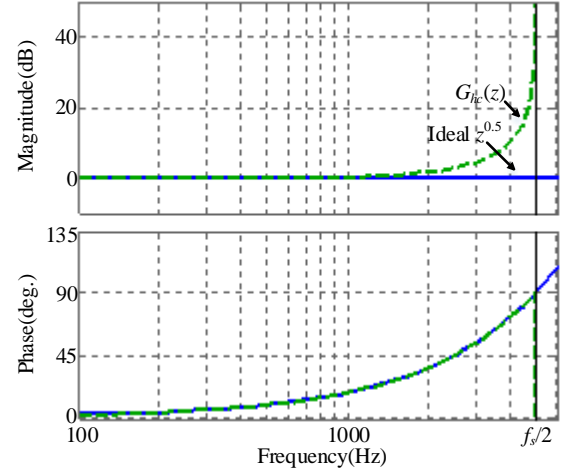


Fig. 18. Bode plots of  $G_{hc}(z)$  and ideal  $z^{0.5}$ .

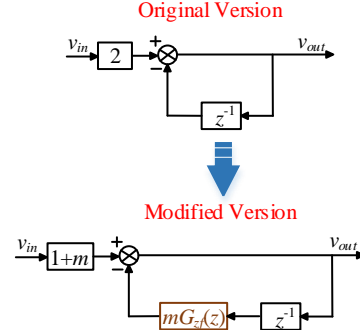


Fig. 19. Modified delay compensation link of half sampling period.

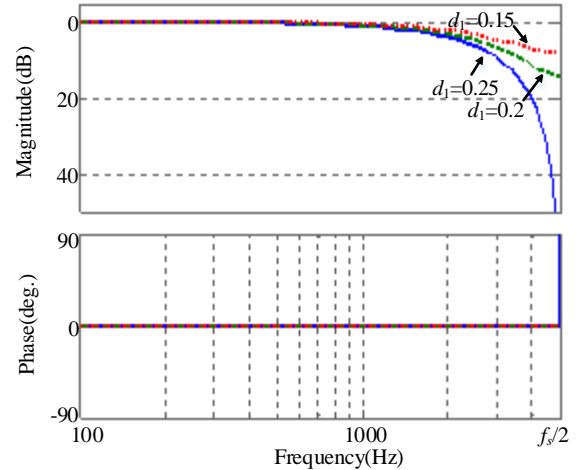


Fig. 20. Bode plots of  $G_{zf}(z)$  with different  $d_1$ .

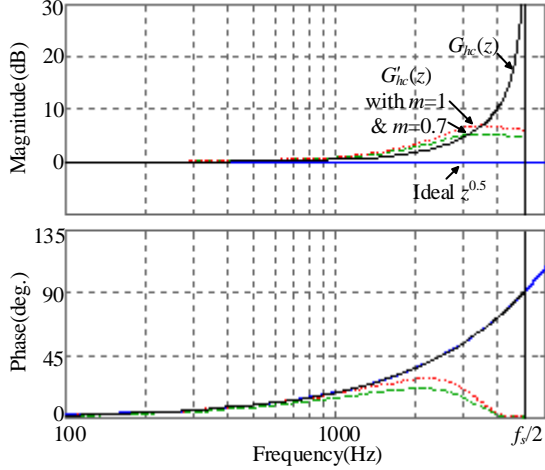


Fig. 21. Bode plots of  $G_{hc}(z)$ ,  $G'_{hc}(z)$  and ideal  $z^{0.5}$ .

Fig. 21 shows the bode plots of  $G_{hc}(z)$ ,  $G'_{hc}(z)$  and ideal  $z^{0.5}$ . As seen, the magnitude peak of  $G'_{hc}(z)$  is attenuated effectively unlike  $G_{hc}(z)$ , but the phase in the high-frequency range decreases gradually until zero at  $f_s/2$ . As discussed in Section IV and the beginning of Section V, grid impedance  $L_g$  has a great impact on the low-frequency characteristic of  $T_{op2}(z)$ , which leads to a poor grid-adaptability. Thus, only the low-frequency range of half sampling period delay needs to be compensated by  $G'_{hc}(z)$ . From Fig. 21, it can be seen the modified  $G'_{hc}(z)$  can simulate ideal  $z^{0.5}$  well in the low-frequency range with a proper  $m$ . Moreover, it can also be noted that a smaller  $m$  gives a better match in magnitude response but poorer phase compensation. In addition to the frequency characteristic of  $G'_{hc}(z)$ , the stability of current control loop should also be considered for the selection of  $m$ , which will be discussed in the following subsection.

### C. Stability Analysis and Its Robustness against Grid Impedance

Fig. 22 shows the current-loop control diagram with full delay compensation in the discrete  $z$ -domain, and its continuous  $s$ -domain form is shown in Fig. 13, where,

$$\begin{cases} G_{ic}(z) = zG'_{hc}(z) \\ G_{ic}(s) = e^{T_s s}G'_{hc}(s) \end{cases} \quad (24)$$

From Fig. 22, the open-loop transfer function  $T'_{op2}(z)$  is derived as

$$T'_{op2}(z) = G_c(z)G_{inc}(z)G_{i1}(z) = \frac{z^{-1}G_c(z)G_{i1}(z)}{1 - G'_{hc}(z)G_{vc}(z)} \quad (25)$$

where  $G_{inc}(z)$  is the minor loop.

As analyzed in Section IV, the poles of  $T'_{op2}(z)$  are first investigated. From (25), its unstable poles are related to the minor loop  $G_{inc}(z)$ . Thus, the unstable poles of  $T'_{op2}(z)$  can be identified by applying Nyquist stability criterion to  $T_{inc}(z) = -G'_{hc}(z)G_{vc}(z)$ . As  $T_{inc}(z)$  has no unstable poles, only its encirclements of  $(-1, j0)$  need to be investigated. Fig. 23 shows the bode plots of  $T_{inc}(z)$ , where  $m=0.7$ ,  $L_g=0$  and  $C_f=5 \mu\text{F}$  with resonant frequency being 3.26 kHz. For the sake of

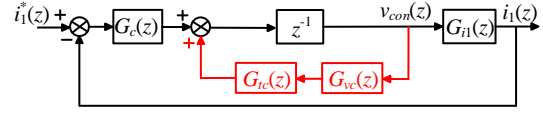


Fig. 22. Current-loop control diagram with full delay compensation in the discrete  $z$ -domain.

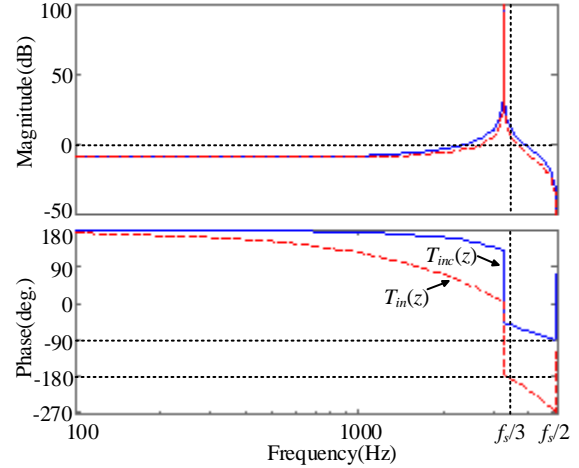


Fig. 23. Bode plots of  $T_{inc}(z)$  and  $T_{in}(z)$ .

comparison,  $T_{in}(z) = -z^{-1}G_{vc}(z)$  is also plotted which indicates unit capacitor voltage feedforward without delay compensation. As seen, there is a  $-180^\circ$  crossing at  $\omega_s/3$  for  $T_{in}(z)$ , and since the magnitude of  $T_{in}(z)$  at  $\omega_s/3$  is greater than 0 dB,  $T_{op2}(z)$  has two unstable poles. Its detail analysis can be found in Section IV. While for  $T_{inc}(z)$ , there is always no  $-180^\circ$  crossing since the phase lag caused by the delay is compensated by  $G_{ic}(z)$ . It indicates that  $G_{inc}(z)$  or  $T'_{op2}(z)$  has no unstable poles, and no additional design constraints for  $LCL$ -filter are required unlike  $T_{op2}(z)$  with  $\omega_r < \omega_s/3$  and (15). Furthermore, a higher resonant frequency can be allowed for converter-side current feedback control, indicating a better rejection capability of grid background harmonics.

Following the analysis of unstable poles of  $T'_{op2}(z)$ , its encirclements of  $(-1, j0)$  are next investigated. Fig. 24 shows the bode plots of  $T'_{op2}(z)$  with different  $m$ , where the used parameters are the same as above and  $k_p=1$ . For comparison, the open-loop transfer function with an  $L$ -filter is also plotted, where the inductance  $L$  is equal to the converter-side inductance  $L_1$  of  $LCL$ -filter. As seen, the low-frequency characteristic of  $T'_{op2}(z)$  has a perfect match with the one in  $L$ -filter, but the high-frequency does not. It is because 1.5 sampling periods delay  $G_d(s)$  is compensated well only in the low-frequency range due to  $G'_{hc}(z)$ . In Fig. 24, there is only one valid  $-180^\circ$  crossing for  $T'_{op2}(z)$  as the one in  $L$ -filter. According to Nyquist stability criterion, the control system can be stable when a proper proportional gain  $k_p$  is selected.

Moreover, it can also be seen that there is a small peak in the magnitude-frequency curves of  $T'_{op2}(z)$ , near the magnitude trap produced by the zeros of  $T'_{op2}(z)$ . Ideally, if the delay  $G_d(s)$  is compensated completely in the whole

frequency range, the zeros can cancel the resonant poles as discussed in the beginning of Section V. Then, the current control loop is an  $L$ -filtered first-order system. However, due to the non-ideal high-frequency characteristic of  $G'_{hc}(z)$  to compensate the half sampling period delay, the poles mismatch the zeros which leads to a small peak, as shown in Fig. 24. The peak caused by the poles is attenuated gradually with the decrease of  $m$ . Thus, a larger proportional gain  $k_p$  can be allowed by selecting a smaller  $m$ .

In order to show the superior robustness of the proposed method under grid impedance variation, Fig. 25 shows the bode plots of  $T'_{op2}(z)$  and  $T_{op2}(z)$ , where  $m=0.7$  for  $T'_{op2}(z)$ ,  $C_f=5 \mu\text{F}$  and other parameters are listed in Table I. As seen, with the proposed method, the low-frequency characteristic of  $T'_{op2}(z)$  keeps unchanged regardless of grid impedance  $L_g$ . Thus, the control system has an invariant phase margin and control bandwidth independent of grid impedance  $L_g$ . While for  $T_{op2}(z)$  with unit capacitor voltage feedforward, its significant phase variation leads to a small stability margin and even instability.

#### D. Suppression of Grid Background Harmonics

In addition to grid impedance variation, grid background harmonics may also exist in weak grids. The harmonic rejection capability will be investigated from the output admittance perspective. Fig. 26 shows the grid-side current diagram with converter-side current feedback control and the proposed capacitor voltage feedforward method, where  $G_{gvc}$  is the transfer function from  $e_g$  to capacitor voltage  $v_c$ , expressed as

$$G_{gvc}(s) = \frac{1}{L_f C_f} \cdot \frac{1}{s^2 + \omega_r^2}. \quad (26)$$

From Fig. 26, it can be derived as

$$Y_{oc}(s) = -\frac{i_2(s)}{e_g(s)} = G_{gi2}(s) - \frac{G_d(s)G_{i2}(s)[G_c(s)G_{gi1}(s) + G_{ic}(s)G_{gvc}(s)]}{1 + G_d(s)G_c(s)G_{i1}(s) - G_{ic}(s)G_d(s)G_{vc}(s)}. \quad (27)$$

Substituting (3)-(5), (11), (26) into (27) yields (28), shown at the bottom of next page.

By neglecting the high-order terms, (28) can be approximated as (29) with  $G_{ic}(s) \approx 1/G_d(s)$  in the low-frequency range, also shown at the bottom of next page.

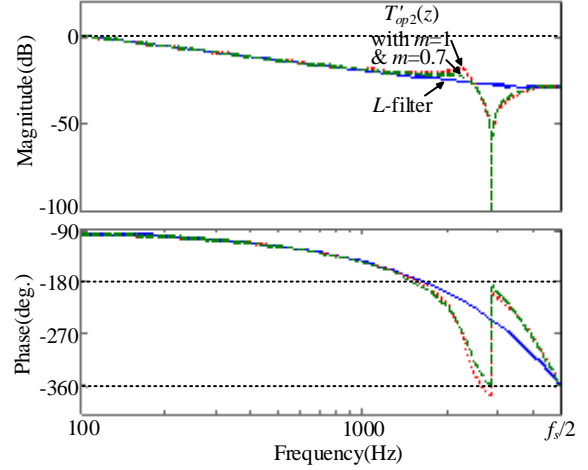


Fig. 24. Bode plots of the open-loop transfer function  $T'_{op2}(z)$  with different  $m$ .

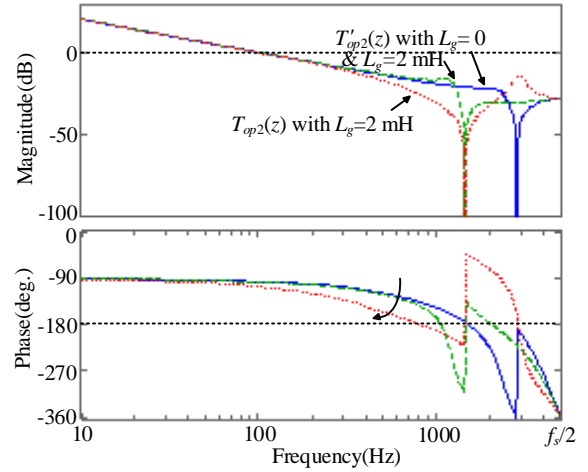


Fig. 25. Bode plots of the open-loop transfer function  $T'_{op2}(z)$  and  $T_{op2}(z)$  with grid impedance  $L_g$ .

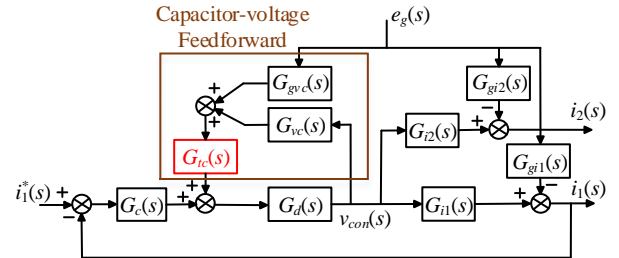


Fig. 26. Grid-side current diagram with converter-side current feedback control and proposed capacitor voltage feedforward method.

For comparison, the low-frequency output admittance without capacitor voltage feedforward or with unit capacitor voltage feedforward are also derived as (30).

When multiple HCs with large enough integral gain  $k_{ih}$  are paralleled with the fundamental PR as (1), (30) can be approximated as (31) at the compensated frequencies by HCs.

$$Y_{o1}(s)\Big|_{s=j\omega_h} = Y_{o2}(s)\Big|_{s=j\omega_h} \approx \frac{1}{(L_1 + L_T)s} \left[ L_1 C_f s^2 + 1 - \frac{1}{(L_T C_f s^2 + 1)} \right] \approx \frac{L_1 C_f s}{L_1 + L_T} \quad (31)$$

From (29) and (31), it can be seen the proposed method has the same harmonic rejection capability as the ones with HCs. Fig. 27 shows the magnitude-frequency curves of  $Y_{oc}(s)$ ,  $Y_{o1}(s)$  and  $Y_{o2}(s)$ , where only 5th and 7th HCs are employed for  $Y_{o1}(s)$  and  $Y_{o2}(s)$ . Here,  $k_p=4$ ,  $L_g=0$  and  $C_f=5 \mu\text{F}$ . As seen, compared with  $Y_{o1}(s)$ , the magnitude of  $Y_{o2}(s)$  has a great dip in the low-frequency range due to the capacitor voltage feedforward, but is the same with  $Y_{o1}(s)$  at the compensated frequencies by HCs. While for  $Y_{oc}(s)$ , its magnitude has a further decrease, and is equal to  $Y_{o1}(s)$  and  $Y_{o2}(s)$  at the compensated frequencies by HCs as analyzed in (29) and (31). It means that the proposed method is equivalent to paralleling a cluster of HCs with the fundamental PR, and meanwhile, due to the smooth magnitude curve of  $Y_{oc}(s)$  in the low-frequency range, the proposed method shows a strong robustness against the variation of harmonic frequencies. For  $Y_{o1}(s)$  and  $Y_{o2}(s)$  with HCs, a small variation of harmonic frequencies  $\Delta f$  will lead to much reduced harmonic rejection capability shown in Fig. 27. Moreover, since HCs are not required for the proposed method, the structure of current control is greatly simplified, and the control parameters can be flexibly designed without having an extremely high bandwidth.

In conclusion, the proposed capacitor-voltage feedforward with full delay compensation can greatly improve the weak grids adaptability of  $LCL$ -filtered grid-connected converters. When incorporated with converter-side current feedback, the overall system features a strong robustness against grid impedance variation and grid background harmonics, and the structure of current control and its design complexity can also be simplified.

$$Y_{oc}(s) = \frac{1}{L_1 L_T C_f s^3 + (L_1 + L_T)s} \left[ L_1 C_f s^2 + 1 - \frac{G_d(s)G_c(s) + G_{tc}(s)G_d(s)L_1 s}{L_1 L_T C_f s^3 + (L_1 + L_T)s + G_d(s)G_c(s)(L_T C_f s^2 + 1) - G_{tc}(s)G_d(s)L_T s} \right] \quad (28)$$

$$Y_{oc}(s) \approx \frac{1}{(L_1 + L_T)s} \left[ L_1 C_f s^2 + 1 - \frac{G_d(s)G_c(s) + L_1 s}{G_d(s)G_c(s)(L_T C_f s^2 + 1) + L_1 s} \right] \approx \frac{L_1 C_f s}{L_1 + L_T} \quad (29)$$

$$\begin{cases} Y_{o1}(s) \approx \frac{1}{(L_1 + L_T)s} \left[ L_1 C_f s^2 + 1 - \frac{G_d(s)G_c(s)}{(L_1 + L_T)s + G_d(s)G_c(s)(L_T C_f s^2 + 1)} \right] \\ Y_{o2}(s) \approx \frac{1}{(L_1 + L_T)s} \left[ L_1 C_f s^2 + 1 - \frac{G_d(s)G_c(s) + G_d(s)L_1 s}{(L_1 + L_T)s + G_d(s)G_c(s)(L_T C_f s^2 + 1) - G_d(s)L_T s} \right] \end{cases} \quad (30)$$

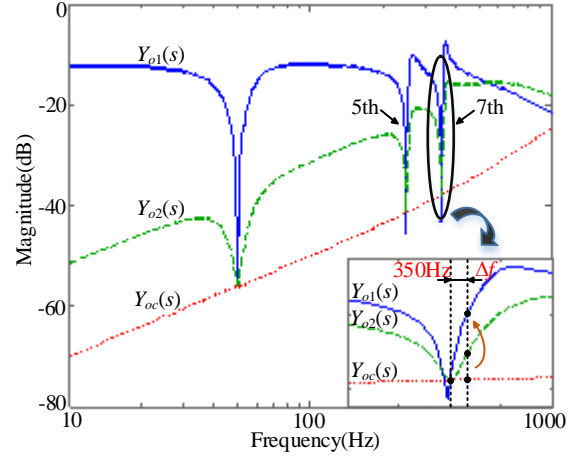


Fig. 27. Magnitude-frequency curves of the output admittance  $Y_{oc}(s)$ ,  $Y_{o1}(s)$  and  $Y_{o2}(s)$ .

TABLE II  
SYSTEM PARAMETERS FOR EXPERIMENTS

Description	Symbol	Value
Grid voltage	$e_g/\text{V}$	90 (RMS)
Dc voltage	$u_{dc}/\text{V}$	200
Fundamental frequency	$f_1/\text{Hz}$	50
Switching frequency	$f_{sw}/\text{kHz}$	10
Sampling frequency	$f_s/\text{kHz}$	10
Proportional gain	$k_p$	4
Compensation coefficient	$m$	0.7

## VI. EXPERIMENTAL RESULTS

A single-phase experimental prototype has been constructed, as shown in Fig.2, to validate the effectiveness of the proposed method. A digital microcontroller is used to perform the control of experimental prototype, which consists of a floating point DSP (Texas Instruments TMS320F28335) as the main algorithm controller, and a Xilinx FPGA XC3S400 as the assistant controller for implementing PWM signals output, ADC control, and fault protection, etc. The key experimental parameters are shown in Table II.

By setting a flag variable in the DSP, the time  $t_d$  used for



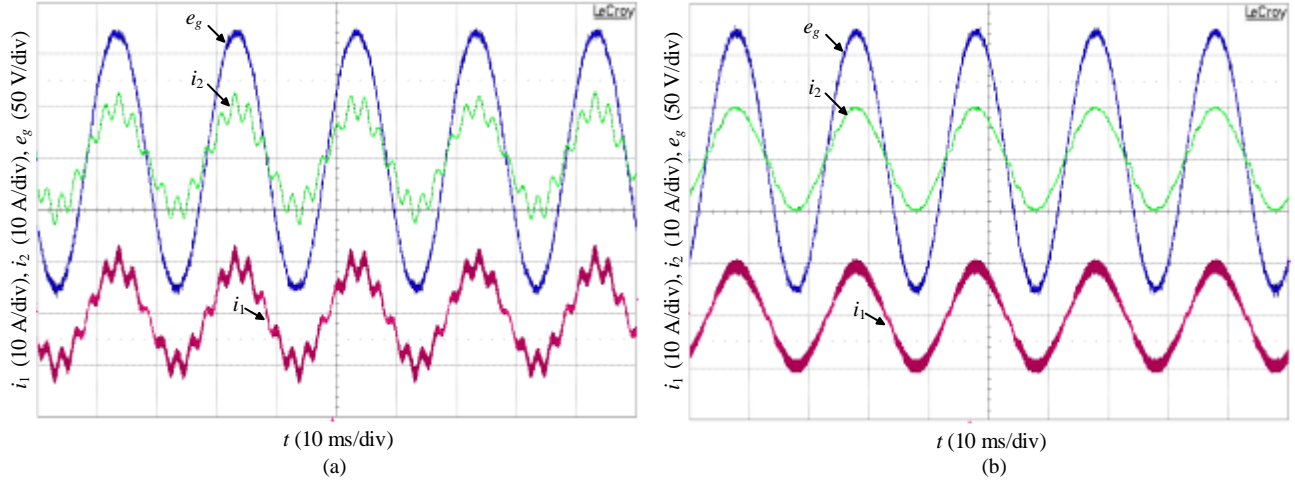


Fig. 28. Steady-state experimental results with 5th and 7th HCs under  $L_g=2$  mH. (a) Unit capacitor voltage feedforward. (b) Proposed method.

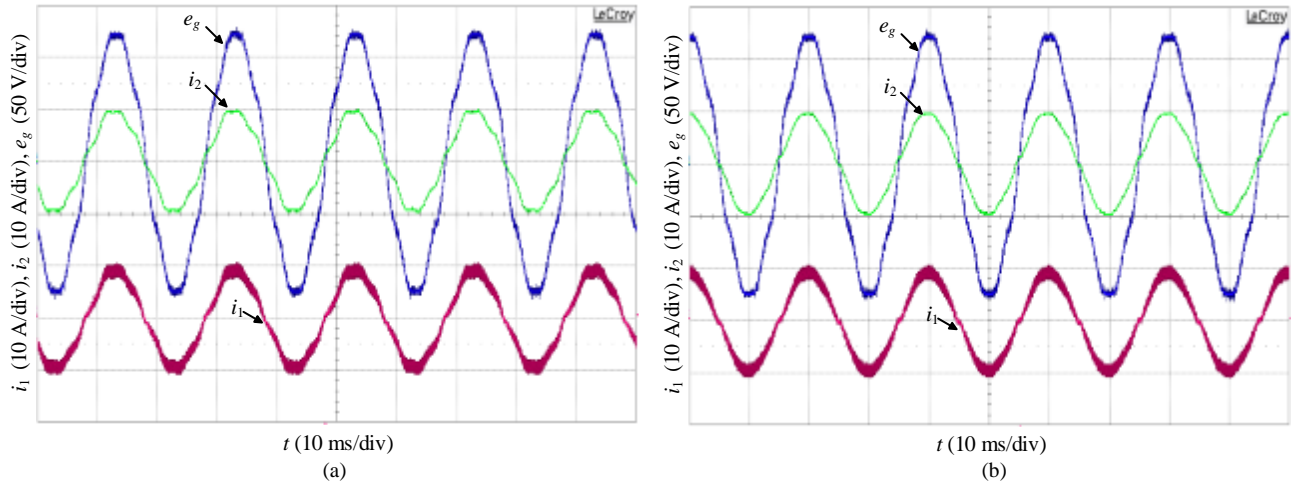


Fig. 29. Experimental results with 5th and 7th grid background harmonics. (a) Unit capacitor voltage feedforward. (b) Proposed method.

execution of capacitor voltage feedforward control is found to be  $4.72 \mu\text{s}$ , which includes  $4 \mu\text{s}$  sampling time  $t_{ad}$  of ADC and  $0.72 \mu\text{s}$  calculation time  $t_{cd}$  for the half sampling period delay compensation link  $G'_{hc}$ . This delay time is much smaller than the sampling period  $T_s=100 \mu\text{s}$ , and its impact on the control system can be neglected. It should be noted that for high power applications where system switching frequency and sampling frequency are significantly reduced, e.g.  $f_s = f_{sw} = 3$  kHz, the performance of the proposed control may be degraded due to the non-ideal half sampling period delay compensation  $G'_{hc}(z)$ . One possible solution is to adopt the multi-sampling technique proposed in [40].

The effect of grid impedance  $L_g$  on the low-frequency control bandwidth is investigated with HCs as discussed in Fig. 6. Here, unit capacitor voltage feedforward is also tested for comparison. From Section IV, the design constraints of LCL-filter,  $\omega_r < \omega_s/3$  and (15), should be satisfied for unit capacitor voltage feedforward.  $C_f=10 \mu\text{F}$  is thus employed to ensure system stability. A 2mH inductor is in series with the grid-side inductor  $L_2$  to simulate the grid impedance  $L_g$ , and in this scenario, (15) is still valid. Fig. 28

shows the steady-state experimental results with only 5th and 7th HCs under  $L_g=2\text{mH}$ . Since the system is always stable when  $L_g=0$  with either unit capacitor voltage feedforward or the proposed method, the experimental results are not presented here. It can be seen from Fig. 28(a) the output current oscillates at about 7th~8th frequency. From Fig. 6(b), it is because the magnitude of the open-loop transfer function at  $f_2$  (the frequency of  $-180^\circ$  crossing caused by 7th HC) is less than or close to 0 dB due to grid impedance. While for the proposed method shown in Fig. 28(b), the output current is stable, because the low frequency characteristic of the system is independent of grid impedance as analyzed in Fig. 25.

Then the rejection capability of grid background harmonics is investigated. Here, 4% of 5th and 4% of 7th harmonics with respect to the fundamental component are injected by the programmable ac power supply. Fig. 29 shows the experimental results with unit capacitor voltage feedforward and the proposed method respectively, where HCs are not employed. It can be seen from Fig. 29(a) both the converter-side and grid-side current are seriously



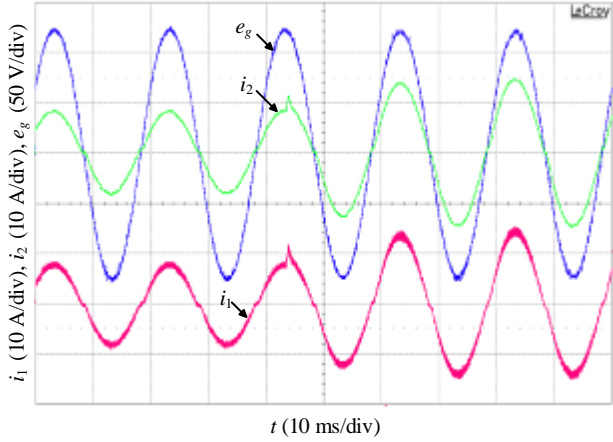


Fig. 30. Transient experimental results with the proposed method under ideal power grids.

distorted. While for the proposed method shown in Fig. 29(b), the converter-side current keeps a good sinusoidal waveform, and the slight distortion in the grid-side current is caused by the paralleled filter capacitor  $C_f$  branch.

From Section V Part C, no additional design constraints are required for the proposed method. To verify this,  $C_f=5 \mu\text{F}$  is selected, which will cause system instability for unit capacitor voltage feedforward. Fig. 30 shows the transient experimental results under ideal power grids. It can be seen the system is stable even though a step change of current reference is performed. The proposed method thus provides a more flexible design of *LCL*-filter without additional constraints.

Furthermore, 5th and 7th harmonics are also injected to test its control performance when  $C_f=5 \mu\text{F}$ . Fig. 31 shows the experimental results, where it is clear that even without using additional HCs, both the grid-side and converter-side currents are kept sinusoidal as those shown in Fig. 29(b). Comparing Fig. 31(a) with Fig. 31(b), the output current without HCs is almost the same as the one with HCs, which verifies the theoretical analysis in Section V Part D. Thus, the proposed method is equivalent to paralleling HCs with the fundamental PR, which greatly reduces the design complexity of current control. Fig. 31(c) shows the transient experimental results without HCs. As seen, the system still keeps good harmonic rejection capability during the transient.

## VII. CONCLUSION

This paper has investigated the adaptability of *LCL*-filtered grid-connected converters with converter-side current feedback under weak grid operation, e.g., grid impedance variation and grid background harmonics. An improved capacitor voltage feedforward with full delay compensation has been proposed, which can keep the low-frequency characteristic of the system independent of grid impedance variation. It is also identified that the system has very low output admittance over a wide range of frequencies. Therefore, it is possible to achieve good grid background

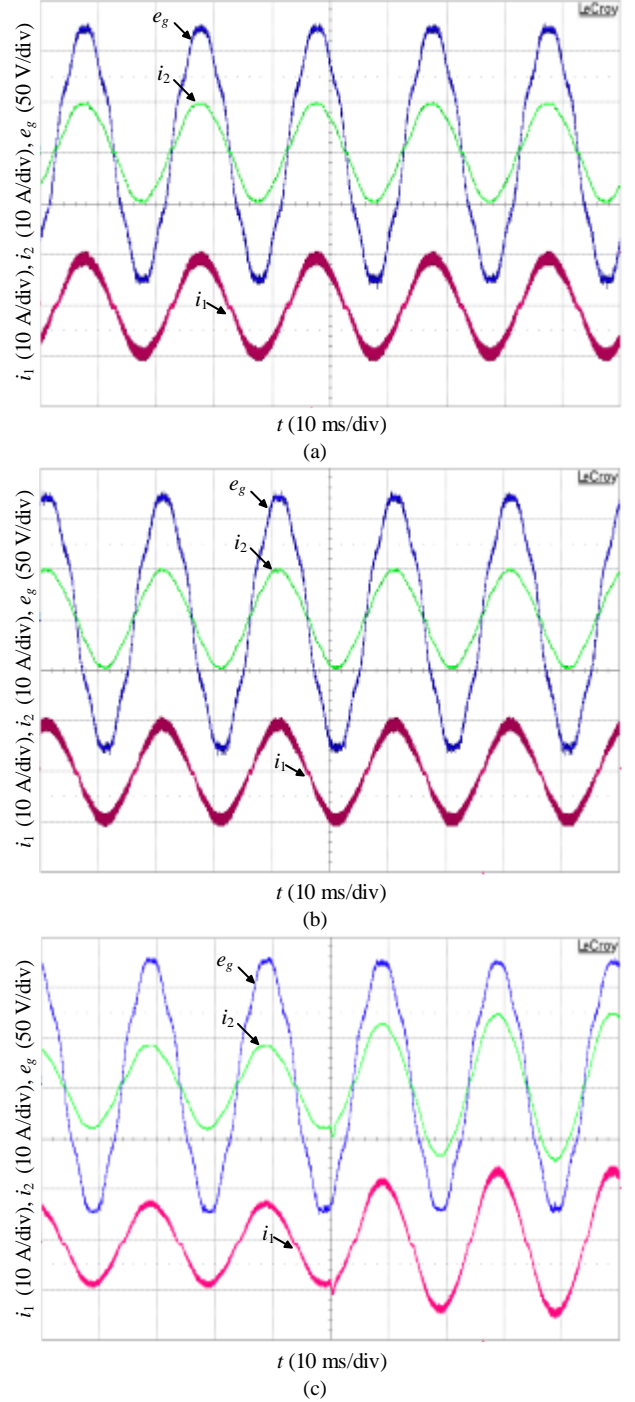


Fig. 31. Experimental results with the proposed method under 5th and 7th grid background harmonics. (a) Steady-state without 5th and 7th HCs. (b) Steady-state with 5th and 7th HCs. (c) Transient without 5th and 7th HCs.

harmonic rejection even with frequency variation. More importantly, the proposed control greatly simplifies the structure of current control and its design, and the multiple HCs used in parallel with the conventional current control can be eliminated. In addition, the resonance frequency of *LCL*-filter can be designed to be above 1/6 of system

sampling frequency, indicating a wider control bandwidth and lower current distortion caused by the paralleled filter capacitor branch. All of these show that the proposed control can greatly enhance the adaptability of *LCL*-filtered grid-connected converters under weak grids. Experimental results are finally presented to verify the proposed control.

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