Capacity constrained routing algorithms for evacuation planning: a summary of results

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Abstract

Evacuation planning is critical for numerous important applications, e.g. disaster emergency management and homeland defense preparation. Efficient tools are needed to produce evacuation plans that identify routes and schedules to evacuate affected populations to safety in the event of natural disasters or terrorist attacks. The existing linear programming approach uses time-expanded networks to compute the optimal evacuation plan and requires a user-provided upper bound on evacuation time. It suffers from high computational cost and may not scale up to large transportation networks in urban scenarios. In this paper we present a heuristic algorithm, namely Capacity Constrained Route Planner(CCRP), which produces sub-optimal solution for the evacuation planning problem. CCRP models capacity as a time series and uses a capacity constrained routing approach to incorporate route capacity constraints. It addresses the limitations of linear programming approach by using only the original evacuation network and it does not require prior knowledge of evacuation time. Performance evaluation on various network configurations shows that the CCRP algorithm produces high quality solutions, and significantly reduces the computational cost compared to linear programming approach that produces optimal solutions. CCRP is also scalable to the number of evacuees and the size of the network. We also provide a discussion on the formulation of a new optimal algorithm that uses A* search to find the optimal solution for evacuation planning. We prove that the heuristic function used in this A* formulation is monotone and admissible.

Keywords: evacuation planning, routing and scheduling, transportation network

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1 Introduction

Evacuation planning is critical for numerous important applications, e.g. disaster emergency management and homeland defense preparation. Traditional evacuation warning systems simply convey the threat descriptions and the need for evacuation to the affected population via mass media communication. Such systems do not consider capacity constraints of the transportation network and thus may lead to unanticipated effects on the evacuation process. For example, when Hurricane Andrew was approaching Florida in 1992, the lack of effective planning caused tremendous traffic congestions, general confusion and chaos [1]. Therefore, efficient tools are needed to produce evacuation plans that identify routes and schedules to evacuate affected populations to safety in the event of natural disasters or terrorist attacks [12, 14, 7, 8].

The current methods of evacuation planning can be divided into two categories, namely traffic assignment-simulation approach and route-schedule planning approach. The traffic assignment-simulation approach uses traffic simulation tools, such as DYNASMART [27] and DynaMIT [5], to conduct stochastic simulation of traffic movements based on origin-destination traffic demands and uses queuing methods to account for road capacity constraints. However, it may take a long time to complete the simulation process for a large transportation network. The route-schedule planning approaches use network flow and routing algorithms to produce origin-destination routes and schedules of evacuees on each route. Many research works have been done to model the evacuation problem as a network flow problem [15, 4] and to find the optimal solution using linear programming methods. Hamacher and Tjandra [17] gave an extensive literature review of the models and algorithms used in these linear programming methods. Based on the triple-optimization results by Jarvis and Ratliff [21], linear programming method for evacuation route planning works as follows. First, it models the evacuation network into a network graph, as shown by network G in Figure 1, and it requires the user to provide an estimated upper bound T of the evacuation egress time. Second, it converts evacuation network G to a time-expanded network, as shown by G_T in Figure 2, by duplicating the original evacuation network G for each discrete time unit $t = 0, 1, \ldots, T$. Then, it defines the evacuation problem as a minimum cost network flow problem [15, 4] on the time-expanded network G_T. Finally, it feeds the expanded network G_T to minimum cost network flow solvers, such as NETFLO [22], to find the optimal solution. For example, EVACNET [9, 16, 23, 24] is a computer program based on this approach which computes egress time for building evacuations. It uses NETFLO code to obtain the optimal solution. Hoppe and Tardos [19, 20] gave a polynomial time bounded algorithm by using ellipsoid method of linear programming to find the optimal solution for the minimum cost flow problem. Theoretically, ellipsoid method has a polynomial bounded running time. However, it performs poorly in practice and has little value for real application [6].

Linear programming approach can produce optimal solutions for evacuation planning. It is useful for evacuation scenarios with moderate size networks, such as building evacuation. However, this approach has the following limitations. First, it significantly increases the problem size because it requires time-expanded network G_T to produce a solution. As can been seen in Figures 1 and 2, if the original evacuation network G has n nodes and the time upper bound is T, the time-expanded network G_T will have at least $(T+1)n$ nodes. This approach may not be able to scale up to large size transportation networks in urban evacuation scenarios due to high computational run-time caused by the tremendously increased size of the time-expanded network. Second, linear programming approach requires the user to provide an upper bound T of the evacuation time in order to generate the time-expanded network. It is almost impossible to precisely estimate the evacuation time for an urban scenario where the number of evacuees is large and the transportation network is complex. An under-estimated time bound T will result in failure of finding a solution. In this case, the user will have to increase the value of T and re-run the algorithm until a solution can be reached. On the other hand,

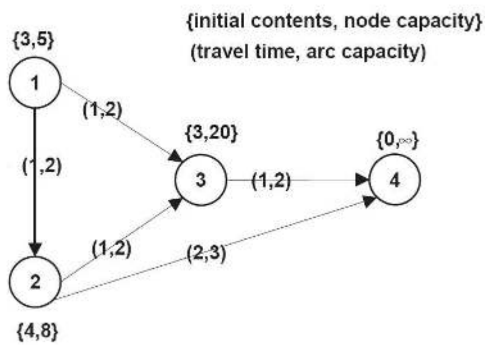


Figure 1: Evacuation Network G, (source: [17])

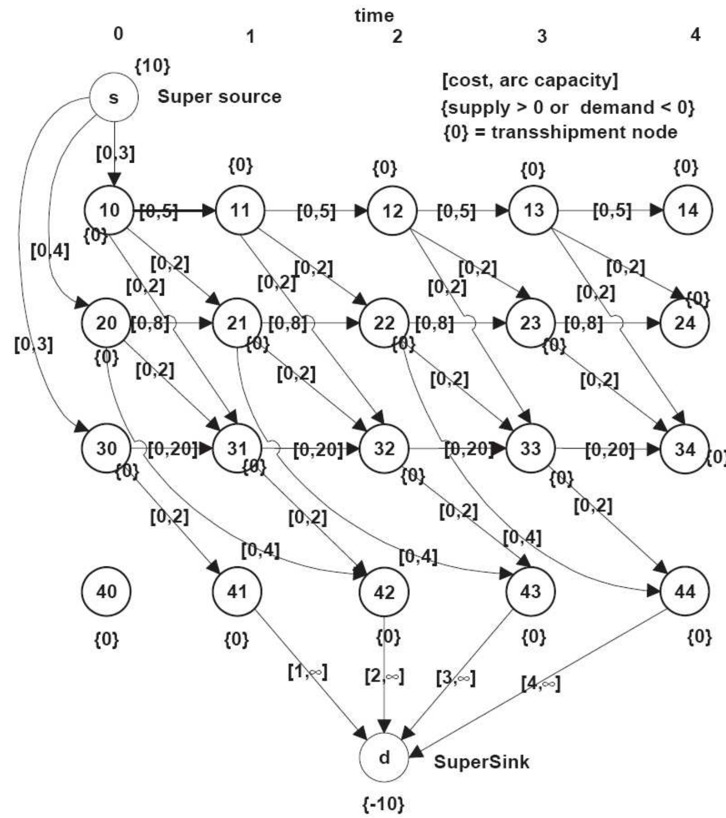


Figure 2: Time-expanded Network G_T, with T=4, (source: [17])

an over-estimated T will result in an over-expanded network G_T and hence lead to unnecessary storage and run-time.

Heuristic routing and scheduling algorithms can be used to find sub-optimal evacuation plan with reduced computational cost. It is useful for evacuation scenarios with large size networks and scenarios that do not require an optimal plan, but need to produce an efficient plan within a limited amount of time. However, old heuristic approaches only compute the shortest distance route from a source to the nearest destination without considering route capacity constraints. It cannot produce efficient plans when the number of evacuees is large and the evacuation network is complex. New heuristic approaches are needed to account for capacity constraints of the evacuation network. Lu, Huang and Shekhar [26] proposed prototypes of two heuristic capacity constrained routing algorithms, namely SRCCP and MRCCP, and tested its performance using small size building networks. SRCCP assigns only one route to each source node. It has very fast run-time but the solution quality is very poor and hence has little value for real application. MRCCP assigns multiple routes to each source node and produces high quality solution with much less run-time compared to that of linear programming approach. However, its scalability to large size networks is unsatisfactory because it has a computational cost of $O(p \cdot n^2 logn)$ (where n the is number of nodes and p is the number of evacuees). In this paper, we present an improved algorithm called Capacity Constrained Route Planner (CCRP). CCRP can reduce the run-time to $O(p \cdot nlogn)$ by conducting only one shortest path search in each iteration instead of the multiple searches used in MRCCP. We also present the analysis of its algebraic cost model and provide the results of performance evaluation using large size transportation networks.

In the CCRP algorithm, we model capacity as a time series because available capacity of each node and edge may vary during the evacuation. We use a generalized shortest path search algorithm to account for route capacity constraints. This algorithm can divide evacuees from each source into multiple groups and assign a route and time schedule to each group of evacuees based on an order that is prioritized by each group's destination arrival time. It then reserves route capacities for each group subject to the route capacity constraints. The quickest route available for one group is re-calculated in each iteration based on the available capacity of the network. Performance evaluation on various network configurations shows that the CCRP algorithm produces high quality solutions, and significantly reduces the computational cost compared to linear programming approach. CCRP is also scalable to the number of evacuees and the size of the network. A case study using a nuclear power plant evacuation scenario shows that this algorithm can be used to improve existing evacuation plans by reducing evacuation time.

We also provide a discussion of the formulation of a new optimal algorithm using A* search[28, 29]. This algorithm addresses the limitations of linear programming approach by using only the original evacuation network to find the optimal solution. In addition, it does not require the user to provide an upper bound of the evacuation time. We provide the the proof of monotonicity and admissibility of this A* search algorithm. We also give the design of the experimental evaluation and we expect detailed experimental results within the coming month.

Outline: The rest of the paper is organized as follows. In Section 2, the problem formulation is provided and related concepts are illustrated by an example evacuation network. Section 3 describes the Capacity Constrained Route Planner (CCRP) algorithm and the algebraic cost model. In Section 4, we present the experimental design and performance evaluation. In Section 5, we provide the formulation of new optimal algorithm using A* search. We summarize our work and discuss future directions in Section 6.

2 Problem Formulation

We formulate the evacuation planning problem as follows:

Given: A transportation network with non-negative integer capacity constraints on nodes and edges, non-negative integer travel time on edges, the total number of evacuees and their initial locations, and locations of evacuation destinations.

Output: An evacuation plan consisting of a set of origin-destination routes and a scheduling of evacuees on each route. The scheduling of evacuees on each route should observe the capacity constraints of the nodes and edges on this route.

Objective: (1) Minimize the evacuation egress time, which is the time elapsed from the start of the evacuation until the last evacuee reaches the evacuation destination. (2) Minimize the computational cost of producing the evacuation plan.

Constraint: (1) Edge travel time preserves FIFO (First-In First-Out) property. (2) Edge travel time reflects delays at intersections. (3) Limited amount of computer memory.

We illustrate the problem formulation and a solution with an example evacuation network, as shown in Figure 3. In this evacuation network, each node is shown by an ellipsis. Each node has two attributes: maximum node capacity and initial node occupancy. For example, at node N1, the maximum capacity is

50, which means this node can hold at most 50 evacuees at each time point, while the initial occupancy is 10, which means there are initially 10 evacuees at this node. In Figure 3, each edge, shown as an arrow, represents a link between two nodes. Each edge also has two attributes: maximum edge capacity and travel time. For example, at edge N4-N6, the maximum edge capacity is 5, which means at each time point, at most 5 evacuees can start to travel from node N4 to N6 through this link. The travel time of this edge is 4, which means it takes 4 time units to travel from node N4 to N6. This approach of modelling a evacuation scenario to a capacitated node-edge graph is similar to those presented in Hamacher [17], Kisko [24] and Chalmet [9].

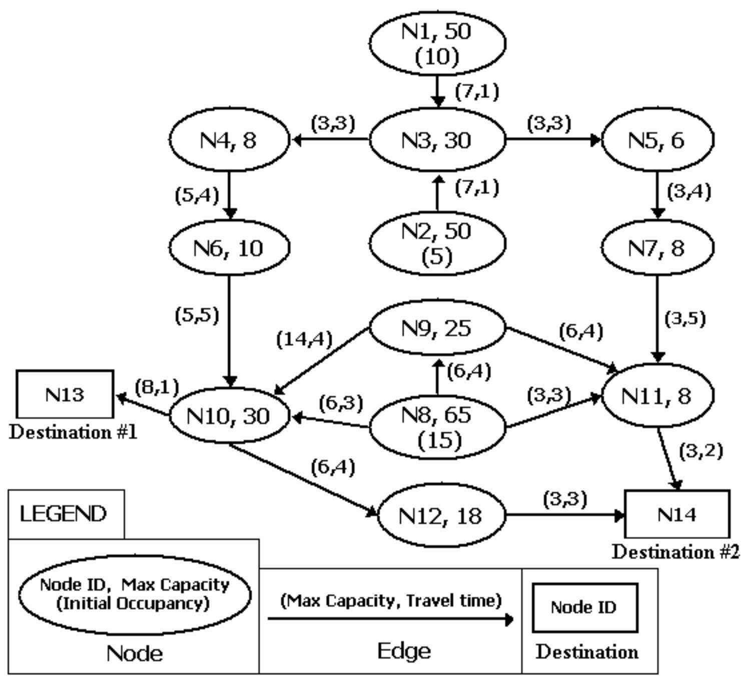


Figure 3: Node-Edge Graph Model of Example Evacuation Network

As shown in Figure 3, suppose we initially have 10 evacuees at node N1, 5 at node N2, and 15 at node N8. The task is to compute an evacuation plan that evacuates the 30 evacuees to the two destinations (node N13 and N14) using the least amount of time.

Example 1 (An Evacuation Plan) Table 1 shows an example evacuation plan for the evacuation network in Figure 3. In this table, each row shows one group of evacuees moving together during the evacuation with a group ID, source node, number of evacuees in this group, the evacuation route with time schedule, and the destination time. The route is shown by a series of node number and the time schedule is shown by a start time associated with each node on the route. Take source node N8 for example; initially there are 15 evacuees at N8. They are divided into 3 groups: Group A with 6 people, Group B with 6 people and Group C with 3 people. Group A starts from node N8 at time 0 to node N10, then starts from node N10 at time 3 to node N13, and reaches destination N13 at time 4. Group B follows the same route of group A, but has a different schedule due to capacity constraints of this route. This group starts from N8 at time 1 to N10, then starts from N10 at time 4 to N13, and reaches destination N13 at time 5. Group C takes a different route. It starts from N8 at time 0 to N11, then starts from N11 at time 3 to N14, and reaches destination N14 at time 5. The procedure is similar for other groups of evacuees from source node N1 and N2. The whole evacuation egress time is 16 time units since the last groups of people (Group H and I) reach destination at time 16. This evacuation plan is an optimal plan for the evacuation scenario shown in Figure 3.

Group of Evacuees			**Route with Schedule**	**Dest. Time**
ID	**Source**	**No. of Evacuees**		
A	N8	6	N8(T0)-N10(T3)-N13	4
B	N8	6	N8(T1)-N10(T4)-N13	5
C	N8	3	N8(T0)-N11(T3)-N14	5
D	N1	3	N1(T0)-N3(T1)-N4(T4)-N6(T8)-N10(T13)-N13	14
E	N1	3	N1(T0)-N3(T2)-N4(T5)-N6(T9)-N10(T14)-N13	15
F	N1	1	N1(T0)-N3(T1)-N5(T4)-N7(T8)-N11(T13)-N14	15
G	N2	2	N2(T0)-N3(T1)-N5(T4)-N7(T8)-N11(T13)-N14	15
H	N2	3	N2(T0)-N3(T3)-N4(T6)-N6(T10)-N10(T15)-N13	16
I	N1	3	N1(T1)-N3(T2)-N5(T5)-N7(T9)-N11(T14)-N14	16

Table 1: Example Evacuation Plan

In our problem formulation, we allow time dependent node capacity and edge capacity, but we assume that edge capacity does not depend on the actual flow amount in the edge. We also allow time dependent edge travel time, but we require that the network preserve the FIFO (First-In First-Out) property.

Alternate problem formulations of the evacuation problem are available by changing the objective of the problem. The main objective of our problem formulation is to minimize the evacuation egress time. Two alternate objectives are: (1) Maximize the number of evacuees that reach destination for each time unit; (2) Minimize the average evacuation time for all evacuees. Jarvis and Ratliff presented and proved the *triple optimization theorem* [21], which illustrated the properties of the solutions that optimize the above objectives of the evacuation problem. A review of linear programming approaches to solve these problem formulations was given by Hamacher and Tjandra [17].

3 Proposed Approach

Linear programming approach can produce optimal solutions for evacuation planning. It is useful for evacuation scenarios with moderate size networks, such as building evacuation. However, it may not be able to scale up to large size transportation networks in urban evacuation scenarios due to high computational cost caused by the tremendously increased size of the time-expanded network. Heuristic routing and scheduling algorithms can be used to find sub-optimal evacuation plan with reduced computational cost. It is useful for evacuation scenarios with large size networks and scenarios that do not require an optimal plan, but need to produce an efficient plan within a limited amount of time.

In this section, we present a heuristic algorithm, namely Capacity Constrained Route Planner (CCRP), that produces sub-optimal solutions for evacuation planning. We model edge capacity and node capacity as a time series instead of fixed numbers. A time series represents the available capacity at each time instant for a given edge or node. We propose a heuristic approach based on an extension of shortest path algorithms [13, 11] to account for capacity constraints of the network.

3.1 Capacity Constrained Route Planner (CCRP)

The Capacity Constrained Route Planner (CCRP) uses an iterative approach. In each iteration, the algorithm first searches for route R with the earliest destination arrival time from any source node to any destination node, taking previous reservations and possible waiting time into consideration. Next, it computes the actual amount of evacuees that will travel through route R. This amount is affected by the available capacity of route R and the remaining number of evacuees. Then, it reserves the node and edge capacity on route R for those evacuees. The algorithm continues to iterate until all evacuees reach destination. The detailed pseudo-code and algorithm description are shown in Algorithm 1.

Algorithm 1 Capacity Constrained Route Planner (CCRP)

Input:
1) $G(N, E)$: a graph G with a set of nodes N and a set of edges E;
Each node $n \in N$ has two properties:
$Maximum\_Node\_Capacity(n)$: non-negative integer
$Initial\_Node\_Occupancy(n)$: non-negative integer
Each edge $e \in E$ has two properties:
$Maximum\_Edge\_Capacity(e)$: non-negative integer
$Travel\_time(e)$: non-negative integer
2) S: set of source nodes, $S \subseteq N$;
3) D: set of destination nodes, $D \subseteq N$;
Output: Evacuation plan : Routes with schedules of evacuees on each route
Method:
Pre-process network: add super source node s_0 to network,
link s_0 to each source nodes with an edge which
$Maximum\_Edge\_Capacity() = \infty$ and $Travel\_time() = 0$; (0)
while any source node $s \in S$ has evacuee do { (1)
find route $R < n_0, n_1, \ldots, n_k >$ with time schedule $< t_0, t_1, \ldots, t_{k-1} >$
using one generalized shortest path search from super source s_0 to all destinations,
(where $s \in S, d \in D, n_0 = s, n_k = d$)
such that R has the earliest destination arrival time among routes between all (s,d) pairs,
and $Available\_Edge\_Capacity(e_{n_i n_{i+1}}, t_i) > 0, \quad \forall i \in \{0, 1, \ldots, k-1\}$,
and $Available\_Node\_Capacity(n_{i+1}, t_i + Travel\_time(e_{n_i n_{i+1}})) > 0, \quad \forall i \in \{0, 1, \ldots, k-1\}$; (2)
$flow = \min($ number of evacuees still at source node s,
$Available\_Edge\_Capacity(e_{n_i n_{i+1}}, t_i), \quad \forall i \in \{0, 1, \ldots, k-1\}$,
$Available\_Node\_Capacity(n_{i+1}, t_i + Travel\_time(e_{n_i n_{i+1}})), \quad \forall i \in \{0, 1, \ldots, k-1\}$
); (3)
for $i = 0$ to $k - 1$ do { (4)
$Available\_Edge\_Capacity(e_{n_i n_{i+1}}, t_i)$ reduced by $flow$; (5)
$Available\_Node\_Capacity(n_{i+1}, t_i + Travel\_time(e_{n_i n_{i+1}}))$ reduced by $flow$; (6)
} (7)
} (8)
Output evacuation plan; (9)

The CCRP algorithm keeps iterating as long as there are still evacuees left at any source node (line 1). Each iteration starts with finding the route R with the earliest destination arrival time from any sources node to any destination node based on the current available capacities (line 2). This is done by generalizing Dijkstra's shortest path algorithm [13, 11] to work with the time series node and edge capacities and edge travel time. Route R is the route that starts from a source node and gets to a destination node in the least amount of time and available capacity of the route allows at least one person to travel through route R to a destination node. Given the evacuation network in Figure 3, the example execution trace of CCRP is as follows

Example 2 (CCRP Execution Trace) At the very first iteration, route R will be N8-N10-N13. Evacuees from source node N8 can take this route to reach destination N13 at time 4 using the time schedule N8(T0)-N10(T3)-N13. At algorithm line 3, the actual number of evacuees that will travel through route R is determined by taking the smallest number among the number of evacuees at the source node and the available capacities of each nodes and edges on route R based on the time schedule that evacuees will travel through each node and edge. Thus, at the first iteration, this flow amount of R will be 6, which is the available edge capacity of edge N8-N10 at time 0.

The next step is to reserve capacities for the evacuees on each node and edge of route R based on the time schedule(lines 4-7). At the first iteration, the algorithm makes a reservation for the 6 evacuees by reducing the available capacity of each node and edge at corresponding time points. This means that available capacities are reduced by 6 for edge N8-N10 at time 0, for node N10 at time 3, and for edge N10-N13 at time 3. The 6 evacuees arrive at destination N13 at time 4. Then, the algorithm goes back to line 1 for the next iteration(line 8). The iteration terminates when the occupancy of all source nodes is reduced to zero, which means all evacuee have been sent to destination nodes. Line 9 outputs the evacuation plan, as shown in Table 1.

Compared with the earlier MRCCP algorithm [26], major improvements in CCRP lie in line 0 and line 2. In MRCCP, finding route R (line 2) is done by running generalized shortest path searches from each source node. Each search is terminated when any destination node is reached. In CCRP, this step is improved by adding a super source node s_0 to the network and connecting s_0 to all source nodes(line 0). This allows us to complete the search for route R by using only one single generalized shortest path search, which takes the super source s_0 as the start node. This search terminates when any destination node is reached. Since the super source s_0 is connected to each source nodes by an edge with infinite capacity and zero travel time, it can be easily proved that the shortest route found by this search is the route R we need in line 2. This improvement significantly reduces the computational cost of the algorithm by one degree of magnitude compared with MRCCP. We give a detailed analysis of the cost model of CCRP algorithm in the next section.

3.2 Algebraic Cost Model of CCRP

We now provide the algebraic cost model for the computational cost of the proposed CCRP algorithm. We assume that n is the number of nodes in the evacuation network, m is the number of edges, and p is the number of evacuees.

The CCRP algorithm is an iterative approach. In each iteration, the route for one group of people is chosen and the capacities along the route are reserved. The total number of iterations equals the number of groups generated. In the worst case, each individual evacuee forms one group. Therefore, the upper bound of the number of groups is p, i.e. the number of iterations is $O(p)$. In each iteration, the computation of the route R with earliest destination arrival time is done by running one generalized Dijkstra's shortest path search. The worst case computational complexity of Dijkstra's algorithm is $O(n^2)$ for dense graphs [11]. Various implementations of Dijkstra's algorithm have been developed and evaluated extensively [4, 10, 32]. Many of these implementations can reduce the computational cost by taking advantage of the sparsity of the graph. Transportation road networks are very sparse graphs with a typical edge/node ratio around 3. In CCRP, we implement Dijkstra's algorithm using heap structures, which runs in $O(m + nlogn)$ time [4, 10]. For sparse graphs, $nlogn$ is the dominant term. The generalization of Dijkstra's algorithm to account for capacity constraints affects only how the shortest distance to each node is defined. It does not affect the computational complexity of the algorithm. Therefore, we can complete the search for route R with $O(nlogn)$ run-time. The reservation step is done by updating the node and edge capacities along route R, which has a cost of $O(n)$. Therefore, each iteration of the CCRP algorithm is done in $O(nlogn)$ time. As we have seen, it takes $O(p)$ iterations to complete the algorithm. The cost model of the CCRP algorithm is $O(p \cdot nlogn)$. CCRP is an improved algorithm based on the same heuristic method of MRCCP [26] which has a run-time of $O(p \cdot n^2 logn)$. CCRP reduces the computational cost of MRCCP by one degree of magnitude.

Algorithm	Computational Cost	Solution Quality
CCRP	$O(p \cdot nlogn)$	Sub-optimal
MRCCP	$O(p \cdot n^2 logn)$	Sub-optimal
Linear Programming Approach	at least $O((T \cdot n)^6)$	Optimal

Table 2: Comparison of Computational Costs (n: number of nodes, p: number of evacuees, T: user-provided upper-bound on evacuation time)

The computational cost of linear programming approach depends on the method used to solve the minimum cost flow problem. Hoppe and Tardos [19] showed that this problem can be solved using ellipsoid method which is theoretically polynomial time bounded. However, the computational complexity of ellipsoid method is at least $O(N^6)$[6](where N is the number of nodes in the network). Since linear programming approach requires a time-expanded network, N equals to $(T+1)n$ (where n is the number of nodes in the original evacuation network, T is the user-provided evacuation time upper bound).

Table 2 provides a comparison of CCRP, MRCCP, and the linear programming approach. As can be seen, linear programming approach produces optimal solutions but suffers from high computational cost. Both CCRP and MRCCP reduce the computation cost by producing sub-optimal solution, while CCRP gives better computational cost than MRCCP.

Lemma 1: CCRP is strictly faster than MRCCP.

The computational costs of CCRP and MRCCP are $O(p \cdot nlogn)$ and $O(p \cdot n^2 logn)$ respectively, as shown in Table 2.

4 Experiment Design and Performance Evaluation

Performance evaluation of the CCRP algorithm was done by conducting experiments using various evacuation network configurations. In this section, we present the experiment design and an analysis of the experiment results.

4.1 Experiment Design

Figure 4 describes the experiment design to evaluate the performance of the CCRP algorithm. The purpose is to compare the algorithm run-time and solution quality of the proposed CCRP algorithms with that of MRCCP [26] and NETFLO [22] which is a popular linear programming package used to solve minimum cost flow problems.

First, we used NETGEN [25] to generate evacuation networks with evacuees. NETGEN is a program that generates transportation networks with capacity constraints and initial supplies based on input parameters. In our experiments, the following three were selected as independent parameters to test their impacts on the the performance of the algorithms: number of evacuees initially in the network, number of source nodes, and network size represented by number of nodes. Number of edges is treated as a dependent parameter as we set the number of edges to be equal to 3 times the number of nodes because 3 is the typical edge/node ratio for real transportation road networks. Next, the same evacuation network generated by NETGEN was fed to the CCRP and MRCCP algorithms. Before feeding the network to NETFLO, we used a network transformation tool to transform the evacuation network into a time-expanded network, which is required by minimum cost flow solvers as NETFLO to solve evacuation problems [17, 9]. This process requires an input parameter T which is the estimated upper-bound on evacuation egress time. If the evacuation cannot be completed by time T, NETFLO will return no solution. In this case, we must increase T to create a new time-expanded network and try to run NETFLO again until a solution can be reached. Finally, after CCRP, MRCCP and NETFLO produced a solution for each test case, the evacuation egress time, which represents the solution quality, and the algorithm run-time were collected and analyzed in the data analysis module.

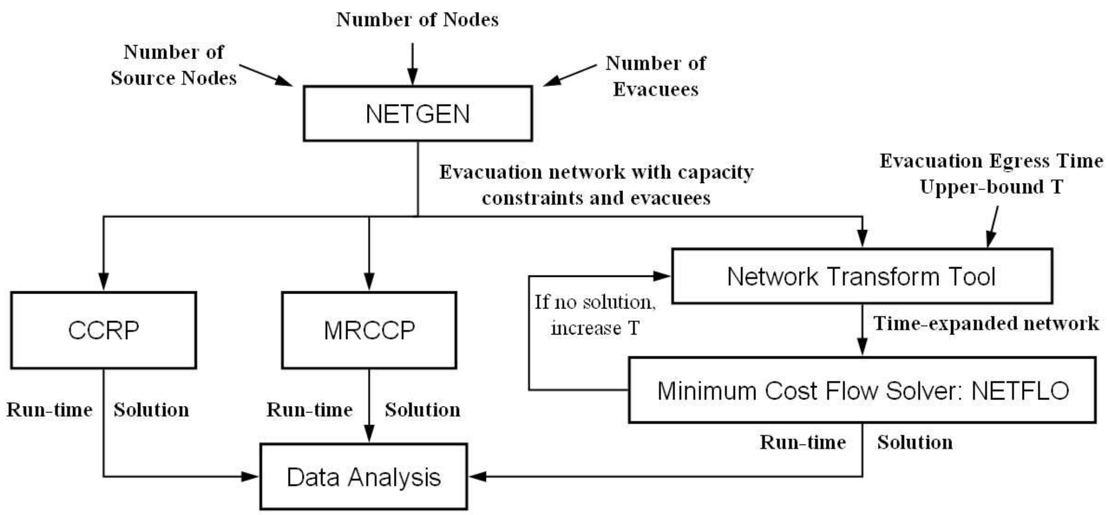


Figure 4: Experiment Design

The experiments were conducted on a workstation with Intel Pentium IV 2GHz CPU, 2GB RAM and Debian Linux operating system.

4.2 Experiment Results and Analysis

We want to answer three questions: (1) How does the number of evacuees affect the performance of the algorithms? (2) How does the number of source nodes affect the performance of the algorithms? (3) Are the algorithms scalable to the size of the network, particularly will they handle large size transportation networks as in urban evacuation scenarios?

Experiment 1: How does the number of evacuees affect the performance of the algorithms?
The purpose of the first experiment is to evaluate how the number of evacuees affects the performance of the algorithms. We fixed the number of nodes and the number of source nodes of the network, and varied the number of evacuees to observe the quality of the solution and the run-time of CCRP, MRCCP and NETFLO algorithms.

The experiment was done with four test groups. Each group had a fixed network size of 5000 nodes and fixed number of source nodes at 1000, 2000, 3000, and 4000 respectively. We varied the number of evacuees from 5000 to 50000. Here we present the experiment results of the test group with number of source nodes fixed at 2000. We omit the results from the other three groups since this group shows a typical result of all test groups. Figure 5 shows the solution quality represented by evacuation egress time and Figure 6 shows the run-times of the three algorithms.

Since CCRP and MRCCP use the same heuristic method to find solution, it is expected that CCRP and MRCCP produced solutions with the same evacuation egress time for each test case. As seen in Figure 5,

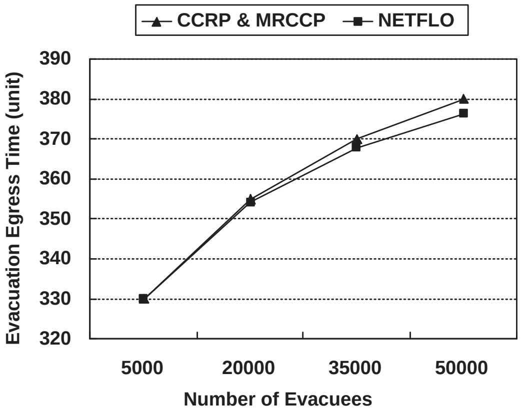


Figure 5: Quality of Solution With Respect to Number of Evacuees

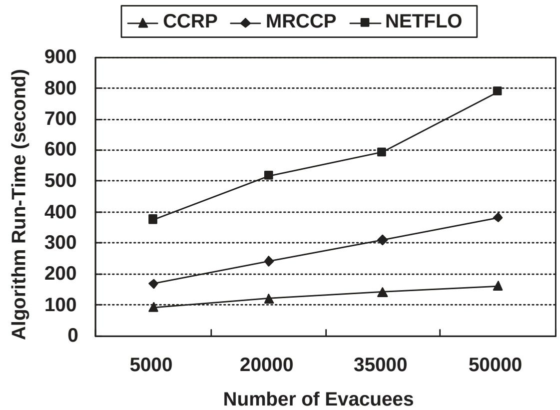


Figure 6: Run-time With Respect to Number of Evacuees

CCRP and MRCCP produced very high quality solution compared with the optimal solution produced by NETFLO. The solution quality of CCRP and MRCCP drops slightly as the the number of evacuees grows. In Figure 6, we can see that, in each case, the run-time of CCRP remains half that of MRCCP and less than 1/3 that of NETFLO. In addition, the CCRP run-time is scalable to the number of evacuees while the run-time of NETFLO grows much faster.

This experiment shows: (1) CCRP produces high quality solutions with much less run-time than that of NETFLO. (2) The run-time of CCRP is scalable to the number of evacuees.

Experiment 2: How does the number of source nodes affect the performance of the algorithms? In the second experiment, we evaluate how the number of source nodes affects the performance of the algorithms. We fixed the number of nodes and the number of evacuees in the network, and varied the number of source nodes to observe the quality of the solution and the run-time. In this experiment, by varying the number of source nodes, we actually create different evacuee distributions in the network. A higher number of source nodes means that the evacuees are more scattered in the network.

Again, the experiment was done with four test groups. Each group had a fixed network size of 5000 nodes and fixed number of evacuees at 5000, 20000, 35000, and 50000 respectively. We varied the number of source nodes from 1000 to 4000. Here we present the experiment results of the test group with number of evacuees fixed at 5000. It shows a typical result of all test groups. Figure 7 shows the solution quality represented by evacuation egress time and Figure 8 shows the run-times of the three algorithms.

As seen in Figure 7, in each test case, CCRP and MRCCP produced high quality solution (within 5 percent longer evacuation time) and the number of source nodes has little effect on the solution quality. It is also noted that the evacuation time is non-monotonic with respect to the number of source nodes and we plan to explore the potential reasons in future works.

Figure 8 shows that the run-time of all three algorithms are scalable to the number of source nodes. However, the run-time of CCRP remains less than half that of NETFLO.

This experiment shows: (1)The solution quality of CCRP is not affected by the number of source nodes. (2) The run-time of CCRP is scalable to the number of source nodes.

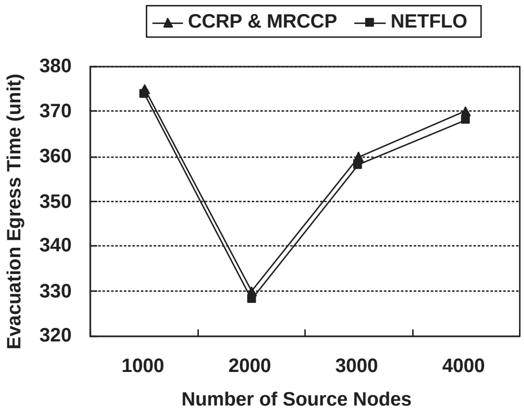


Figure 7: Quality of Solution With Respect to Number of Source Nodes

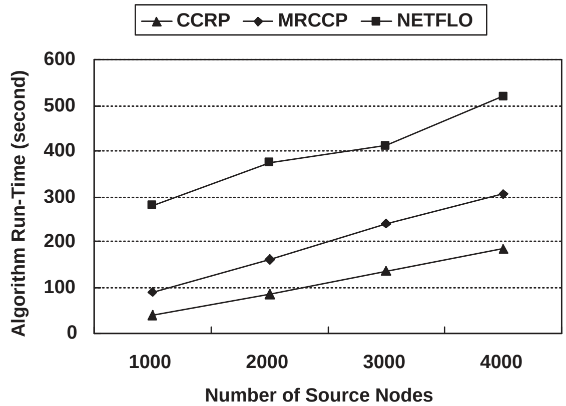


Figure 8: Run-time With Respect to Number of Source Nodes

Experiment 3: Are the algorithms scalable to the size of the network?

In the third experiment, we evaluate how the network size affects the performance of the algorithms. We fixed the number of evacuees and the number of source nodes in the network, and varied the network size to observe the quality of solution and the run-time of the algorithms.

The experiment was done with a fixed number of evacuees at 5000 and the number of source nodes at 10. We varied the number of nodes from 50 to 50000. Figure 9 shows the solution quality represented by evacuation egress time and Figure 10 shows the run-times.

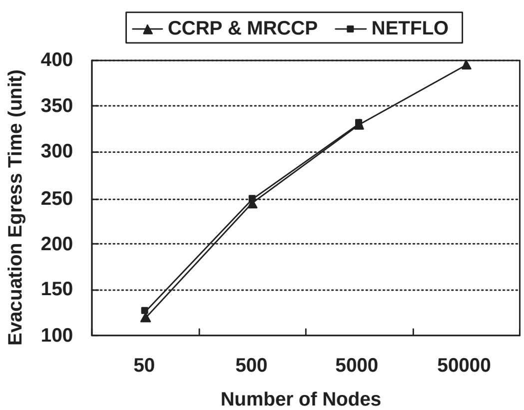
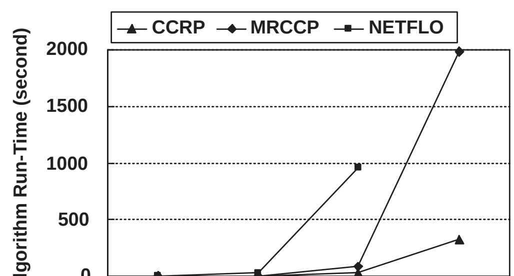


Figure 9: Quality of Solution With Respect to Network Size



Number of Nodes	50	500	5000	50000
CCRP	0.1	1.5	23.1	316.4
MRCCP	0.1	2.8	78.5	1980.1
NETFLO	0.3	25.6	962.1	

Figure 10: Run-time With Respect to Network Size

Note: x-axis(number of nodes) in Figure 9 and 10 is on a logarithmic scale rather than linear. Run-time of CCRP and MRCCP in Figure 10 grow in small polynomial.

There is no data point for NETFLO at network size of 50000 nodes. We were unable to run NETFLO for this setup because the size of the time-expanded network became too large (more than 20 million nodes and 80 million edges)that NETFLO could not produce solution.

As seen in Figure 9, in each of the first three test case, CCRP and MRCCP produced high quality solution

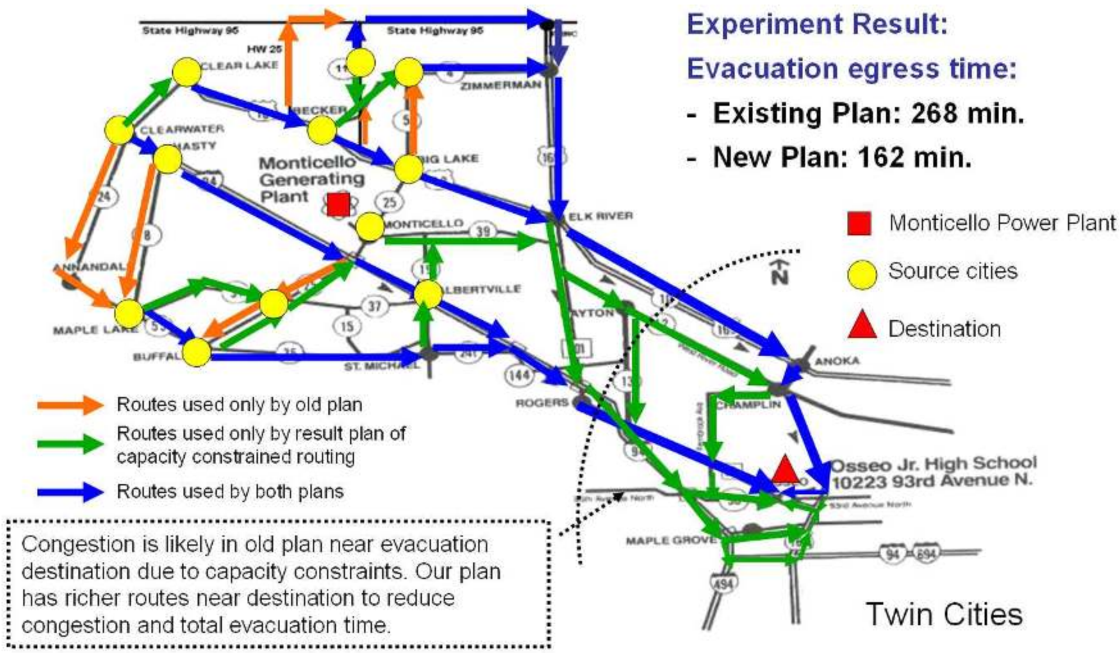


Figure 11: Result Routes Overlay of Monticello Power Plant Evacuation Planning (best viewed in color)

(within 5 percent longer evacuation time) and the solution quality becomes closer to optimal solution as the network size increases. Figure 10 is shown with a data table of each run-time. The x-axis(number of nodes) of Figure 10 is on a logarithmic scale rather than linear and the run-time of CCRP and MRCCP grow in small polynomial. It can be seen that the run-time of CCRP is scalable to the network size while the NETFLO run-time grows exponentially.

This experiment shows: (1) Given a fixed number of evacuees and source nodes, the solution quality of CCRP increases as the network size increases. (2) The run-time of CCRP is scalable to the size of the network.

4.3 A Case Study

We also conducted experiments using a real evacuation scenario. As shown in Figure 11, the Monticello nuclear power plant is about 40 miles to the northwest of the Twin Cities. Evacuation plans need to be in place in case of accidents or terrorist attacks. The evacuation zone is a 10-mile radius around the nuclear power plant as defined by Minnesota Homeland Security and Emergency Management [3]. A hand-drafted evacuation route plan was developed to evacuate the affected population to a high school. However, this plan did not consider the capacity of the road networks and put high loads on two highways.

We conducted an experiment using the CCRP algorithm. The experiment was done using the road network around the evacuation zone provided by the Minnesota Department of Transportation [2], and the Census 2000 population data for each affected city (circles in Figure 11). The total number of evacuees is about 42,000. As can be seen in Figure 11, our algorithm gives a much better evacuation route plan by selecting shorter paths to reduce evacuation time and utilizing richer routes (routes near evacuation destination) to reduce congestions. The old evacuation plan has an evacuation egress time of 268 minutes. CCRP algorithm produced a much better plan with evacuation time of only 162 minutes. This experiment shows that our algorithm is effective in real evacuation scenarios to reduce evacuation time and improve existing plans.

Our approach was presented in the UCGIS Congressional Breakfast Program on homeland security[30], and the Minnesota Homeland Security and Emergency Management newsletter[31]. It was also selected by the Minnesota Department of Transportation to be used in the evacuation planning project for the Twin Cities Metro Area, which evolves a road network of about 250,000 nodes and a population of over 2 million people.

5 Discussion: An Optimal Approach Using A* Search

As discussed in Section 1, the linear programming methods to solve the evacuation planning problem use time expanded networks that require a large amount of memory; these methods also require a prior knowledge of the upper bound of evacuation time. CCRP algorithm presented in Sections 3 and 4 addresses these issues very effectively; but the evacuation times obtained are sub-optimal. There is a need to explore new approaches that would guarantee optimal solutions without using time-expanded networks. In this section, we discuss the possibility of formulating the evacuation planning problem as a search problem implemented as an A* search. We present the heuristic function used in this A* formulation and we prove the properties of the heuristic function which guarantees the optimality of the solution. This approach finds an optimal solution to the evacuation planning problem without using time expanded networks and also eliminates the need for user-provided upper bound of evacuation time. Basic graph search strategies and general outline of an A* algorithm are explained in the Appendix.

5.1 Formulation of the Evacuation Problem as an A* Search

The search space consists of different states of the evacuation network. Each state is the snapshot of the network at each instant of time. The start node of the search tree would be the initial state of the evacuation network at the start of evacuation. The goal node would be the state of the network when there are occupants only at the destination nodes.

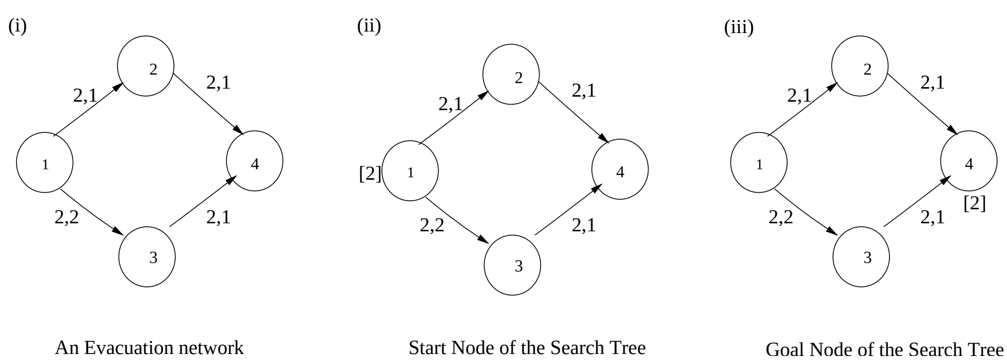


Figure 12: Illustration of Start Node, Goal Node

Figure 12 illustrates the formulation for an evacuation network with four nodes (shown in the first figure) as a search problem. The number in square brackets adjacent to a node indicates the node occupancy. Node 1 is the source node and node 4 is the destination node. The source node has an occupancy of 4. The second figure shows the start node of the search tree which represents the initial state of the network when all evacuees are the source node (node 1). The last figure shows the goal node of the search tree which represents the state where all evacuees are at the destination node (node 4).

5.2 Search-Node Expansion in the Search Tree

Given the occupancy (number of people at the node) of the source node and the capacity constraints of the outgoing edges of the node, all possible feasible combinations are generated. This is formulated as follows.

$$\sum_i x_i \leq min(N, \sum C_i), \text{ subject to the constraints } 0 \leq x_i \leq C_i, i = 1, .., n.$$

where n is the out-degree of the node, N is the node occupancy, C_i is the capacity of the i^{th} outgoing edge.

Each search tree node with nonzero occupancy thus has $\prod_i C_i$, child nodes; each corresponding to one of the possible combinations that are generated. Each step in the expansion would correspond to advancing in time by one unit.

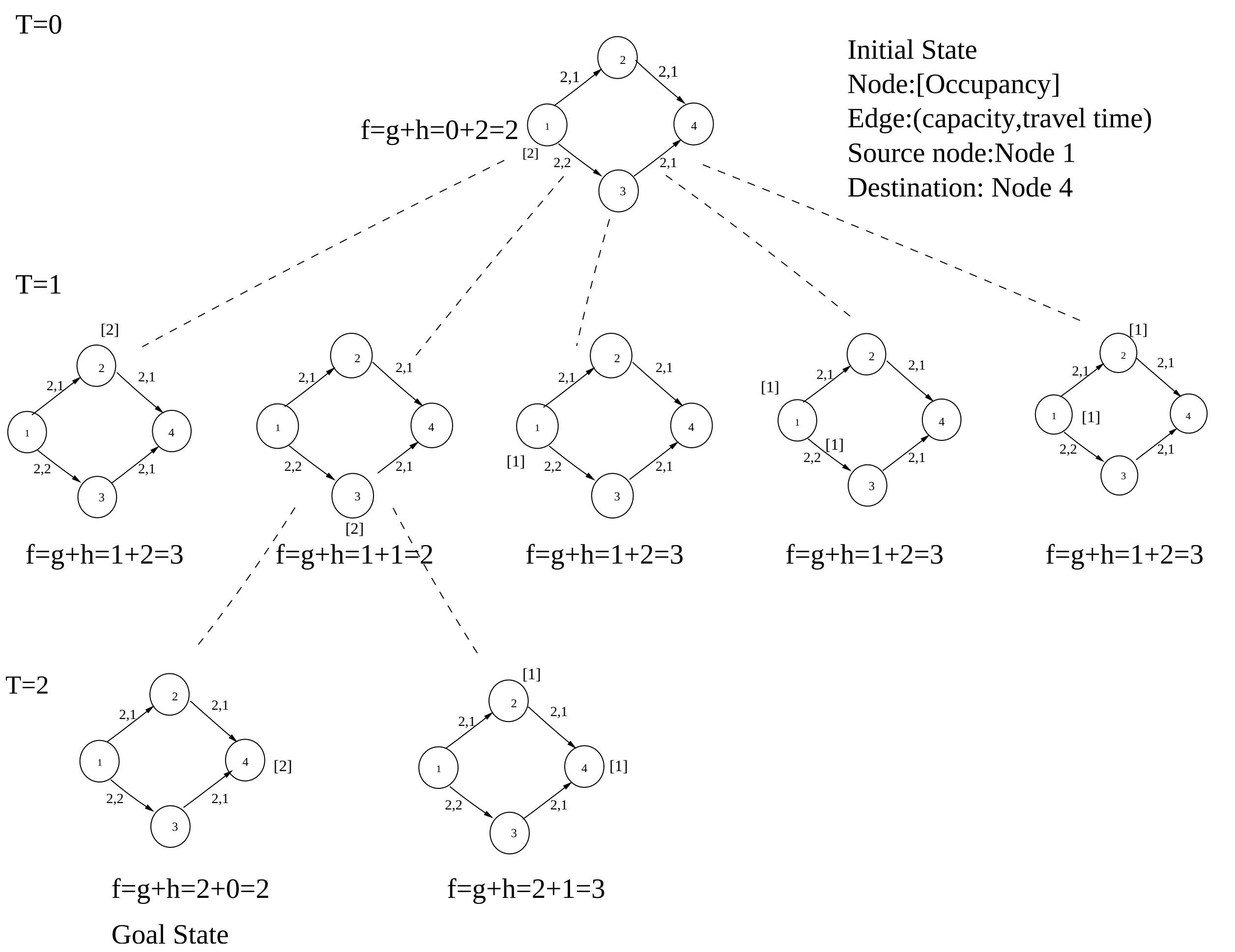


Figure 13: Search Tree Node Expansion

An example has been included to illustrate search node expansion. (See Figure 13). The first level of search tree (T=0) shows the initial state of the evacuation network. The source node is node 1 and destination node is node 4. The number in square brackets indicates the number of people at any node or at an edge. The network in the initial state forms the start node of the search tree. Given the occupancy at each node (except the destination nodes) and capacity constraints of outgoing edges of the nodes, all feasible combinations are generated. For example for node 1 which has two persons, the feasible groups that can travel along the two outgoing edges are $(2,0)$, $(0,2)$, $(1,0)$, $(0,1)$, $(1,1)$. There are five child nodes for the start node of the search tree, each representing the state of the network after one time unit, corresponding to each of these five possible combinations.

5.3 Proposed Heuristic Function

The evaluation function $f(n)$ of a search node n is formulated as $f(n) = g(n) + h(n)$.
$g(n)$ is the actual cost to reach the search node n from the start node which is the time taken to reach the current state from the start state. With the node expansion method used here, $g(n)$ would be the depth of the node n, since every expansion advances the evacuation network by one time unit. $h(n)$ is the estimated cost from the node n to a goal node. We propose $h(n)$ to be the maximum over all groups of the time taken to reach the closest destination node with capacity constraints ignored.
The g and h values for each search node are shown in the figure. The node with the least value of $f = g + h$ is expanded. In this example, the goal state is reached at the third level of the search tree, when all evacuees are at the destination node.
We can trace the evacuation plan by tracing the search tree upwards from the goal node to the start node.

Lemma 2: The proposed h function is admissible.

Proof: The h function clearly underestimates the time required by the group to reach the destination because the group that requires the largest time to reach the destination node definitely needs to move to a destination node to reach a goal state. Here, we consider the nearest destination and also ignore the constraints on the arcs. Bringing in the capacity constraints can only add to the evacuation time. Hence, h is clearly admissible.

Lemma 3: The proposed h function is monotone.

Proof: To prove monotonicity of the heuristic $h(n)$, we need to prove that
$h(n) + c(m,n) \geq h(m)$ where node n is a child of node m and $c(m,n)$ is the cost of the arc from m to n.
We call this inequality "triangle inequality" in the rest of the proof.
We define $h(n)$ as the largest travel time taken by a group to reach the closest destination when all the groups travel along the shortest paths, ignoring all capacity constraints.
We prove the triangle inequality by considering the groups g_i and g_j whose shortest travel times to the destination are the largest over all groups in search nodes m and n and hence are the values of h function at these search nodes. We prove the inequality for the following cases which exhaust all possibilities that can arise in an evacuation scenario.

Case 1. g_i is the same as g_j

Case 1a. g_i stays at the same network node in both search nodes m and n.

Case 1b. g_i moves through one time unit in the node expansion.

Case 2. g_i is not the same as g_j

Case 2a. g_i and g_j stay at the same network node.

Case 2b. g_i stays at the same network node, but g_j moves.

Case 2c. g_i moves , but g_j stays at the same network node.

Case 2d. g_i and g_j move by one time unit.

Detailed proof is given in Appendix 3 with Figure 15 which illustrates the evacuation network scenarios for the various cases listed above.

The h values shown in the search tree (Figure 13) clearly illustrate that h is admissible and monotone. It can be observed that h function never overestimates the time required to reach the goal node from any search node. Also, the h value of every node and that of its parent node do satisfy the triangle inequality.

5.4 Experimental Evaluation

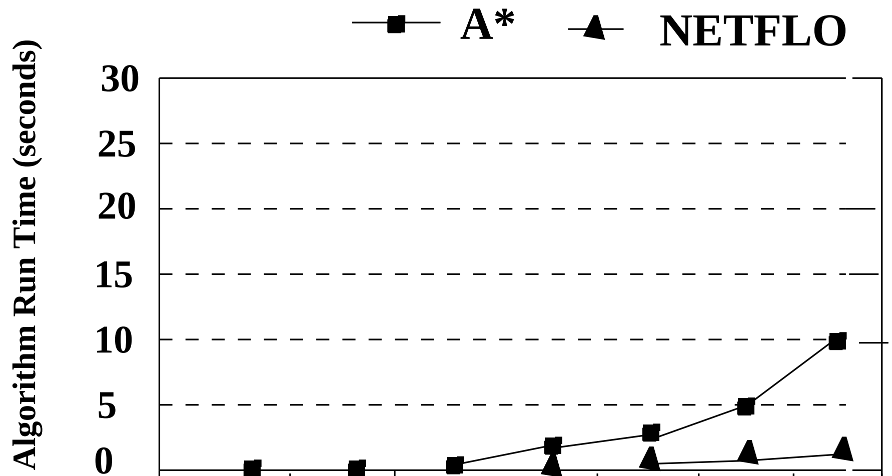
The experimental evaluation for this A* search algorithm needs to answer the following questions: (1) Does the A* algorithm produce optimal evacuation plans? (2) Does the current implementation of A* algorithm need performance tuning? (3) How does the memory usage of A* algorithm compared with other algorithms? Do we need new data structures to reduce memory used by A* algorithm?

We implemented the A* search algorithm and conducted experiments using similar method as described in Section 4. First, we generate evacuation networks using NETGEN and convert it to a time-expanded network. Then, the evacuation network is fed to the A* algorithm and the time-expanded network is fed to NETFLO. Finally, we compare the results from the two algorithms.

For question (1) experiments on all networks configurations show that, in each of the test cases, the A* algorithm produces evacuation plan with the same evacuation time as that of NETFLO. It shows that the A* algorithm can produce optimal evacuation plan as the linear programming approach. For questions (2), we vary the network size from 10 nodes to 40 nodes. Figure 14 shows the algorithm run-time of A* algorithm and NETFLO. It can be seen that the current implementation of the A* algorithm produces higher run-time than that of NETFLO. It shows that further performance tuning are needed to improve the performance of the A* implementation. For question (3), initial results show that the current implementation of A* requires high memory usage. Detailed analysis of the cost model of the A* memory usage remains as our future work as we plan to explore the possibility of introducing new data structures to reduce the memory usage of the A* algorithm.

6 Conclusions and Discussions

In this paper, we proposed a new capacity constrained routing algorithm for evacuation planning problem. Existing linear programming approach uses time-expanded network and requires user provided upper bound on evacuation time. To address these limitations, we presented a heuristic algorithm, namely Capacity Constrained Route Planner(CCRP), which produces sub-optimal solution for evacuation planning problem without using time-expanded networks. We provided the algebraic cost model and the performance evaluations using various network configurations. Experiments show that CCRP algorithm produces high quality solution and significantly reduces the computational cost compared to linear programming approach which produces optimal solution. It is also shown that the CCRP algorithm is scalable to the number of evacuees and the size of the transportation network. A case study using real evacuation scenario shows that CCRP algorithm can be used to improve existing evacuation plans by reducing total evacuation time.



Number of Nodes	10	15	20	25	30	35	40
A*	0.1	0.2	0.4	0.8	2.1	4.9	10.4
NETFLO	0.1	0.1	0.2	0.2	0.2	0.4	0.5

Figure 14: Performance of A* and NETFLO with respect to Netswork size

The limitation of CCRP algorithm remains the follows. First, we assume that maximum capacity of an edge does not depend on traffic flow amount on the edge. We understand that it is a challenging task to accurately model the capacity of each road segment in a real evacuation scenario as the actual traffic flow rate may depend on vehicle speed as well as road occupancy. Second, the generalized shortest path algorithm we used in CCRP requires that the edge travel time reflects traffic delays at intersections. For future work, we plan to incorporate existing research results, such as Ziliaskopoulos and Mahmassani [33], to better address this problem.

To address the sub-optimality issue of the CCRP algorithm, we also explored the possibility of formulating the evacuation problem as a search problem using A* algorithm. Our A* search formulation addresses the limitations of linear programming approach by only using the original evacuation network to find optimal solution. Thus, it does not require prior knowledge of evacuation time. We proved that the heuristic function used in our A* formulation is monotone and admissible thus guaranteeing the optimality of the solution. We plan to evaluate the performance of this approach (as indicated in Section 5.4) in the coming month.

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A Appendix

A.1 Basic Graph Search

A graph consists of a set of nodes which represent the encodings of subproblems. Every graph used in search would have a start node that represents the initial state of the problem being solved. Certain pairs of nodes are connected by directed arcs. If the arc is directed from n to n', n' is a successor of n and n is the parent of n'. Often the arcs are assigned weights that represent the cost of traversing the arc. A sequence of nodes $n_1, ..n_k$, where each n_i is a successor of n_{i-1} is a path from n_1 to n_k. The cost of a path is the sum of the costs of the arcs along the path.

The most basic operation in graph search is node expansion. This involves computing the representations of all successor nodes from that of its parent. A search procedure is a well defined method for determining the order in which nodes are expanded. If the search procedure uses the information collected by the search up to that point, it is called uninformed search. If the procedure uses partial information about the unexplored portion of the graph to guide the node expansion, search is informed. Most search procedures distinguish between nodes that were expanded called closed nodes and nodes that have been generated but not expanded called open nodes. Familiarity with problem domain would sometimes indicate that certain directions of search are more promising than others. This knowledge can be used in deciding which node to expand first. The promise of a node n is estimated numerically by a heuristic evaluation function $f(n)$, which would depend on the description of n, description of the goal, information gathered by the search up to that point and any extra knowledge about the problem domain.

In $A*$ search, the evaluation function $f(n)$ is formulated as $f(n) = g(n) + h(n)$ where $g(n)$ is the cost of the path in the search tree from the start node to n and $h(n)$ is the estimated minimum cost of the path from n to the goal node. The strategy followed is to expand the open node n with the minimum f [29, 28].

A.2 Outline of A* algorithm [29]

1. Put the start node s, on OPEN.
2. If OPEN is empty, exit with failure.
3. Find the node n in OPEN with minimum f . Remove n from OPEN and place it on CLOSED.
4. If n is goal node, exit with success. The solution can be obtained by tracing back the pointers back from n to s.
5. Otherwise, expand n, generating all its successors. Link the successors to n. For every successor n' of n,
   - If n' is already not on OPEN or CLOSED, estimate $h(n')$ and calculate $f(n') = g(n') + h(n')$ where $g(n') = g(n) + c(n, n'); g(s) = 0$ and $c(n, n')$ i s the cost of the arc from n to n'.
   - If n' is already on OPEN or CLOSED, reassign $g(n')$ to the current minimum.
   - If n' required a reassignment of $g(n')$ and was on CLOSED, reopen it.
6. Go to step 2.

A heuristic function h is said to be admissible if $\forall n h(n) \leq h^*(n)$, where $h^*(n)$ is the cheapest cost of path going from n to the goal node [29]. A search algorithm is called admissible if it is guaranteed to find an optimal path from the start node to a goal node. A^* is admissible if h is admissible [18].

A heuristic function $h(n)$ is monotone if $h(m) \leq h(n) + c(m, n)$, $\forall (m, n) | n$is a child of m in the search tree and $c(m, n)$ is the cost of the arc from m to n. If A* search uses a monotone heuristic, it finds optimal paths to all expanded nodes [29].

A.3 Detailed Proof for Lemma 3 (Monotonicity of h function)

Lemma 3: The proposed h function is monotone.

Proof: To prove monotonicity of the heuristic $h(n)$, we need to prove that
$h(n) + c(m, n) \geq h(m)$ where node n is a child of node m and $c(m, n)$ is the cost of the arc from m to n.
We call this inequality "triangle inequality" in the rest of the proof.
We define $h(n)$ as the largest travel time taken by a group to reach the closest destination when all the groups travel along the shortest paths, ignoring all capacity constraints.

Let $t(g_i, k)$ denote the smallest time taken by a group g_i to reach the closest destination at the search node k.
Now let us consider search nodes m and n where n is a child of m. At node m, let the smallest time taken by the group g_i be the largest among all groups. In other words, $h(m) = t(g_i, m)$.
At node n, let the smallest time taken by the group g_j be the largest amon g all groups. In other words, $h(n) = t(g_j, n)$ We prove the monotonicity by proving the triangle inequality for the following cases.

1. ***Case 1.*** g_i is the same as g_j

   Case 1a. g_i stays at the same network node in both search nodes m and n.
   Here, $h(n) = h(m)$ and the triangle inequality is true.

   Case 1b. g_i moves through one time unit in the node expansion.
   $c(m, n) = 1$ here. We need to show $h(n) + 1 \geq h(m)$. If g_i moved along t he shortest path that was detected at the search node m, $h(n) + 1 = h(m)$ satisfying the inequality. If g_i moved along another path (since we enumerate all possible paths in node expansion , this is possible) towards another destination node,$h(n) + 1 \geq h(m)$. Otherwise this current path would be detected as the shortest path in parent node m.

2. ***Case 2.*** g_i is the not the same as g_j

   Case 2a. Groups g_i and g_j stay at the same network node.
   This is not possible under case 2.

   Case 2b. g_i stays at the same network node, but g_j moves.
   Here, we need to show that $t(g_j, n) + c(m, n) \geq t(g_i, m)$.
   Since $t(g_i, m) = t(g_i, n)$ (group g_i did not move), inequality becomes $t(g_j, n) + c(m, n) \geq t(g_i, n)$.
   This is true since $t(g_j, n) \geq t(g_i, n)$ by definition of $h(n)$

   Case 2c. g_i moves , but g_j stays at the same network node.
   $t(g_i, m) \geq t(g_j, m)$ by the definition of $h(m)$ and $t(g_j, n) \geq t(g_i, n)$ by the definition of $h(n)$.
   Combining these two inequalities and using $t(g_j, m) = t(g_j, n)$, we get $t(g_i, m) \geq t(g_j, n) \geq t(g_i, n)$
   $t(g_i, m) - t(g_i, n) \not> c(m, n)$ since if this were not the case, it contradicts the optimality of the shortest path in search node m. Therefore, $t(g_i, m) - t(g_j, n) \leq c(m, n)$ or $t(g_j, n) + c(m, n) \leq t(g_i, m)$

   Case 2d. g_i and g_j move by one time unit.
   $t(g_i, m) \geq t(g_j, m)$
   $t(g_j, n) \geq t(g_i, n)$ by definition of $h(m)$ and $h(n)$ respectively.
   We need to prove the triangle inequality, $t(g_j, n) + c(m, n) \geq t(g_i, m)$.
   Adding $c(m, n)$ to the left-hand side of the second inequality,
   $t(g_j, n) + c(m, n) \geq t(g_i, n)$
   If $t(g_i, m) \leq t(g_i, n)$, the triangle inequality is satisfied trivially because $t(g_j, n) \geq t(g_i, n)$.
   If $t(g_i, m) > t(g_i, n)$, $t(g_i, m) - t(g_i, n) \leq 1$; if this is not true, this path would be the shortest path in search node m.
   Therefore,$t(g_i, n) + 1 \geq t(g_i, n)$.
   To prove the triangle inequality,
   a)*if* $t(g_j, n) \leq t(g_j, m)$,
   $t(g_j, m) - t(g_j, n) \leq 1$ and $t(g_j, m) \leq t(g_i, m)$
   Therefore, $t(g_i, m) \leq t(g_j, m) \leq t(g_j, n)$
   $t(g_i, m) \leq t(g_j, n) + 1 \Leftrightarrow t(g_j, n) + c(m, n) \geq t(g_i, m)$
   b)*if* $t(g_j, n) > t(g_j, m)$,
   $t(g_j, n) - t(g_j, m) \geq 1$ because the time is always computed in integer units.
   Since $t(g_j, n) \geq t(g_i, n)$ and $t(g_j, m) \leq t(g_i, m)$, we get $t(g_j, n) - t(g_i, m) \geq 1 \Leftrightarrow t(g_j, n) + c(m, n) \geq t(g_i, m)$

The proposed heuristic $h(n)$ is monotone. Figure 15 illustrates the the evacuation network scenarios for the various cases listed above.

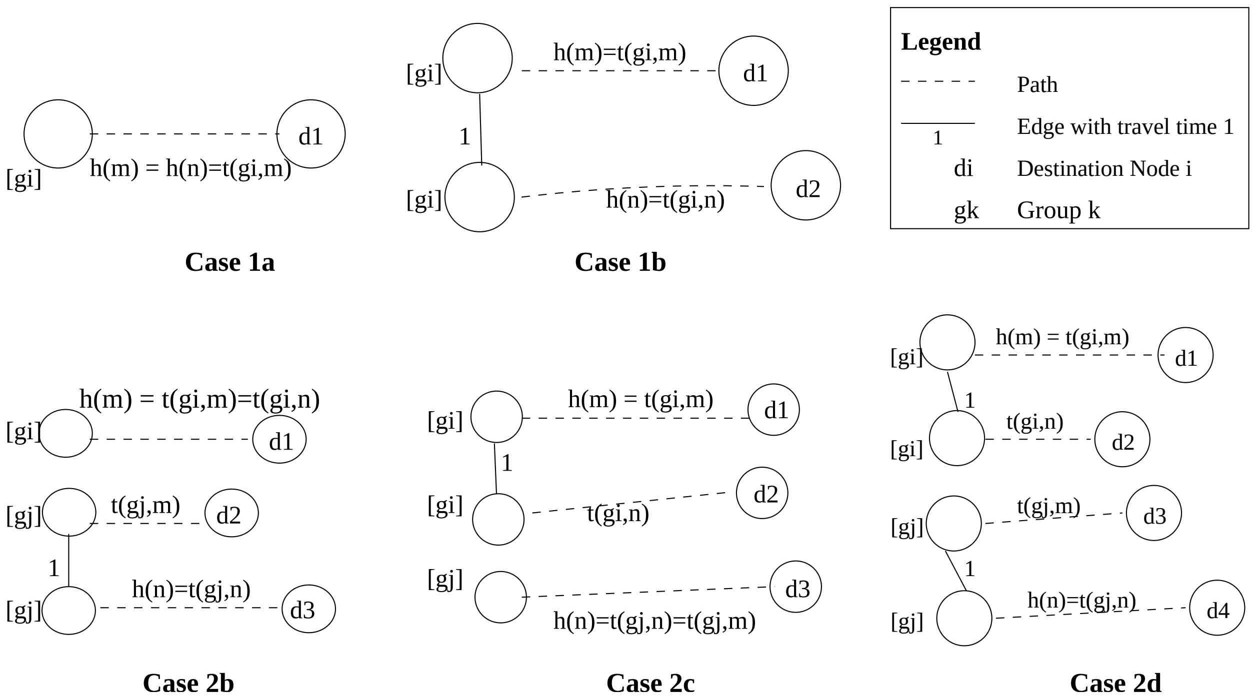

Figure 15: Illustration of Network Scenarios