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CAPACITY EVALUATION OF VARIOUS MULTIUSER MIMO SCHEMES IN DOWNLINK CELLULAR ENVIRONMENTS

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ABSTRACT

Presented in this paper is a study of the capacity evaluation of various multiuser MIMO schemes in cellular environments. The throughputs per user of the generalized zero-forcing with rank adaptation and vector perturbation schemes are compared with the capacity bound of the Gaussian MIMO broadcast channel, obtained by dirty paper coding under proportional fairness scheduling. The average cell throughputs of these schemes are also compared. From these comparisons, this study provides vital information for applying multiuser MIMO schemes in multicell environments.

I. INTRODUCTION

Recently, the capacity bounds of Gaussian multiple-input multiple-output (MIMO) broadcast channels have been studied and shown to be achieved by dirty paper coding (DPC) [1]–[7], and several practical progresses using source-channel coding in the dirty paper channel have been made in this area [6], [7]. However, there are still open problems to achieve rates closer to capacity with practical transceivers, whereas many suboptimal schemes have been proposed to simplify the transceiver. In [4], the capacity bound obtained from the DPC and the throughput of linear processing, such as zero forcing (ZF)-based orthogonal space-division multiplexing (OSDM) [8]–[10] and time-division multiple-access (TDMA), have been compared for downlink cellular systems.

In this paper, the downlink throughputs of a more general linear OSDM scheme with rank adaptation [11] and nonlinear vector perturbation [12], [13] are evaluated and compared numerically with the capacity bound in the multicell environment. To apply a point-to-point MIMO technique in a point-to-multipoint MIMO system, the proportional fairness (PF) scheduler in [14] is considered. First, the cumulative density functions (CDFs) of the throughput per user are shown, and then the average cell throughputs are compared. The throughput gaps from the capacity bound for suboptimal techniques are shown using computer simulations. All of these results provide useful insights into the design and application of multiuser MIMO techniques in cellular environments.

The organization of this paper is as follows. Section II describes the multiuser MIMO system model in multicell environments. Various multiuser MIMO schemes are then presented in Section III. Section IV shows the computer simulation results to demonstrate the performance evaluation of the optimal and suboptimal methods. Finally, Section V concludes the paper.

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II. MULTIUSER MIMO SYSTEM MODEL

The multiuser downlink MIMO system in cellular environments, where a common base station with N_T transmit antennas transmits different signals to multiple mobile users, is shown in Fig. 1. Let K be the number of users in the cell and $N_{R,k}$ be the number of receive antennas at the k-th user.

To ensure that the users receive their data without coordination, an appropriate preprocessing of the data should be carried out at the transmitter. Let $\mathcal{F}$ denote the preprocessing function and $\mathbf{x}_k$ be the L_k-by-1 data vector for the k-th user, where L_k is the number of spatial modes supported to the k-th user. Then, the transmitted signal is represented by:

$$\mathbf{s} = \mathcal{F}\left(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K\right),$$

where $\mathbf{s}$ is an N_T-by-1 vector.

In multicell environments, the received signal at the k-th user, an $N_{R,k}$-by-1 vector, is represented by

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{s} + \mathbf{n}_k,$$

where $\mathbf{H}_k$ is an $N_{R,k}$-by-N_T matrix that denotes the channel matrix between the base station and the k-th user, and $\mathbf{n}_k$ denotes the additive noise due to both the thermal noise and the interferences from neighboring cells. It is assumed that the entries of $\mathbf{H}_k$ are independent and identically distributed (i.i.d.) random variables whose means are zero and variances are determined by the path loss and shadowing factor of the k-th user. Each user decodes the data vector as follows:

$$\hat{\mathbf{x}}_k = \mathcal{G}_k\left(\mathbf{y}_k\right)\ \forall k,$$

where $\mathcal{G}_k$ is the postprocessing function for the k-th user.

We assume that the channel matrices for different users are independent and all the channel matrices are quasi-static, flat fading, and perfectly known at the base station. For simplicity, it is also assumed that every user has the same number of receive antennas, i.e. $N_{R,k} = N_R\ \forall k$.

Throughout this paper, $\mathbf{A}^\dagger$, $\mathrm{tr}(\mathbf{A})$, and $|\mathbf{A}|$ denote the conjugate transpose, trace operation, and determinant of matrix $\mathbf{A}$, respectively.

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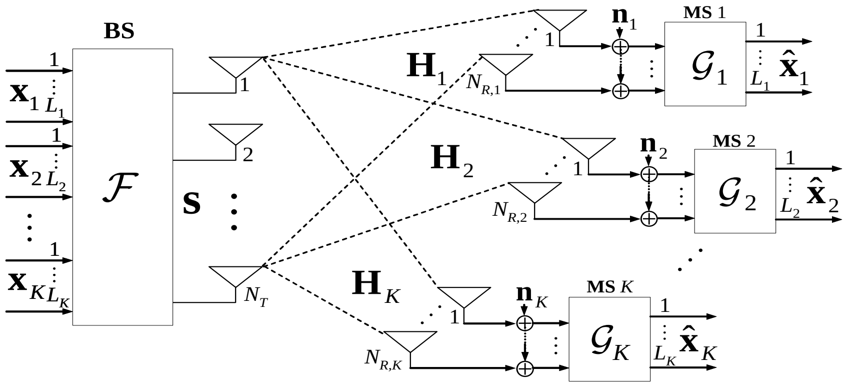

Figure 1: System model of a multiuser MIMO system

III. TECHNIQUES FOR MULTIUSER MIMO DOWNLINK

In this section, we first present the capacity bound of the multiuser MIMO downlink systems, i.e. the DPC achievable rate regions. Next, we present two suboptimal schemes: the generalized zero-forcing (GZF) and the vector perturbation (VP) schemes. Finally, we explain the PF scheduler with rank adaptation that is also under consideration.

A. Capacity Bound (DPC)

With the DPC it is possible to precancel interferences that are known noncausally at the transmitter, resulting the same capacity as if there is no interference. When the DPC is applied to a multiuser MIMO downlink, it can be used to precancel other users' coded signals with an appropriate ordering [3]. The DPC achievable rate region is given by:

$$\mathcal{C}_{DPC} = Co\left(\bigcup_{\pi,\Sigma_i} \mathcal{R}_\pi\right), \tag{1}$$

where $Co(\mathcal{A})$ is the convex hull operation of the set $\mathcal{A}$, $\bigcup$ is the union operation, $\mathcal{R}_\pi = (\mathcal{R}_{\pi(1)}, ..., \mathcal{R}_{\pi(K)})$, π is the user permutation vector, and

$$\mathcal{R}_{\pi(i)} = \log \frac{\left|\mathbf{I}_{N_r} + \mathbf{H}_{\pi(i)}\left(\sum_{j\geq i}\mathbf{\Sigma}_{\pi(j)}\right)\mathbf{H}^\dagger_{\pi(i)}\right|}{\left|\mathbf{I}_{N_r} + \mathbf{H}_{\pi(i)}\left(\sum_{j> i}\mathbf{\Sigma}_{\pi(j)}\right)\mathbf{H}^\dagger_{\pi(i)}\right|}, \quad i = 1, ..., K.$$

Here, $\mathbf{I}_N$ is the N-dimensional indentity matrix, Σ_k is the covariance matrix of the user k, the union operation is performed over all possible permutations $(\pi(1), \pi(2), ..., \pi(K))$, and all possible non-negative covariance matrices that are constrained to $tr(\Sigma_1 + \Sigma_2 + ...\Sigma_K) \leq P$.

In this paper, we evaluate the achievable rates using the PF scheduler, which needs to find the optimal point on the boundary of the rate region (1) that maximizes the weighted sum rates for a given weight vector. Finding the optimal operating point with (1) is formidable since it is neither convex nor concave. Thus, this problem is transformed into a dual multiple access channel (MAC) optimization problem that is concave of the covariance matrices; then, the following result is obtained [2]:

$$\mathcal{C}_{DPC}(P, \mathbf{H}) = \bigcup_{\mathbf{P}:\sum_{i=1}^K P_i = P} \mathcal{C}_{MAC}(\mathbf{P}, \mathbf{H}^\dagger),$$

which shows that the capacity of the Gaussian broadcast channel is the same as the union of the MAC capacity regions over all individual power constraints $\mathbf{P} = (P_1, ..., P_2)$ that sum to P. Here, the MAC capacity region for a given power constraint and channel instance, $\mathcal{C}_{MAC}(\mathbf{P}, \mathbf{H}^\dagger)$, is given by the union of the rate regions over all possible non-negative covariance matrices, as follows:

$$\mathcal{C}_{MAC}(\mathbf{P}, \mathbf{H}^\dagger) = \bigcup_{\substack{\mathbf{Q}_i \geq 0 \\ \mathrm{tr}(\mathbf{Q}_i) \leq P_i \ \forall i}} \Big\{ \mathbf{R} : \textstyle\sum_{i\in S} R_i \leq \log \left|\mathbf{I}_{N_t} + \sum_{i\in S}\mathbf{H}_i^\dagger \mathbf{Q}_i \mathbf{H}_i\right| \ \forall S \subseteq \{1, ..., K\} \Big\},$$

where $\mathbf{Q}_i$ is the covariance matrix for the i-th user and $\mathbf{R} = (R_1, R_2, ..., R_K)$. We locate the operating point on the boundary of the optimal region for a given weight vector, μ, by maximizing the weighed sum rates using the algorithm in [4], which is calculated using the results of the dual MAC and standard convex optimization procedures shown in (2), where $\bar{\mathcal{R}}$ is a vector of user rates of the set $\mathcal{R}$ and μ is a weight vector given by PF. The numerical methods for the optimization procedure in (2) are shown in detail in [4].

B. Suboptimal Schemes

1) Generalized Zero-Forcing (GZF)

For the downlink of multiuser MIMO systems, one of the suboptimal schemes is the linear OSDM. The OSDM enables the users to receive their own data with zero co-channel interference. Recently, [10] proposed an iterative algorithm that finds the preprocessing and postprocessing linear operators for the OSDM, which maximize the effective channel gains. In this paper, however, to avoid the burden from iterations, we use the conventional ZF block-diagonalization [9], in which the multiuser MIMO channels are decomposed into multiple, single-user MIMO channels. For the case of $N_{R,k} > L_k$, the dominant L_k left singular vectors are considered as the effective left

$$\max_{\mathcal{R}\in\mathcal{R}_{DPC}} \mu\cdot\bar{\mathcal{R}} = \max_{\mathbf{Q}_i:\sum_{i=1}^{K}\mathrm{tr}(\mathbf{Q}_i)=P} \sum_{i=1}^{K-1}\left(\mu_{\pi(i)}-\mu_{\pi(i+1)}\right)\log\left|\mathbf{I}+\sum_{l=1}^{i}H_{\pi(l)}^{\dagger}Q_{\pi(l)}H_{\pi(l)}\right| + \left(\mu_{\pi(K)}\right)\log\left|\mathbf{I}+\sum_{l=1}^{K}H_{\pi(l)}^{\dagger}Q_{\pi(l)}H_{\pi(l)}\right| \quad (2)$$

singular vectors of $\mathbf{H}_k$ for the k-th user in order to preserve the maximum spatial diversity. Then, we apply the singular value decomposition approach [8] to each single-user MIMO channel to achieve the maximum single-user MIMO capacity. This scheme is termed GZF. In the aspect of complexity, GZF has a merit in that it needs only linear processing at both the transmitter and receivers.

Let $\mathbf{F}$ denote the preprocessing matrix at the base station and $\mathbf{G}_k$ denote the postprocessing matrix at the k-th user. We note that $\mathbf{F}$ can be separated as $[\mathbf{F}_1\ \mathbf{F}_2\ \cdots\ \mathbf{F}_K]$, where $\mathbf{F}_k$ is the preprocessing matrix for the k-th user, and the transmitted signal can be represented by:

$$\mathbf{s} = \sum_{k=1}^{K}\mathbf{F}_k\mathbf{x}_k.$$

To obtain the optimal $\mathbf{F}_k$ and $\mathbf{G}_k$, let $\mathbf{W}_k$ be a matrix with orthogonal columns such that:

$$\mathbf{W}_k \in \mathrm{null}\left(\left[\mathbf{H}_1^{\dagger}\mathbf{U}_1\cdots\mathbf{H}_{k-1}^{\dagger}\mathbf{U}_{k-1}\ \mathbf{H}_{k+1}^{\dagger}\mathbf{U}_{k+1}\cdots\mathbf{H}_K^{\dagger}\mathbf{U}_K\right]^{\dagger}\right),$$

where $\mathrm{null}(\mathbf{A})$ denotes the null space of $\mathbf{A}$ and $\mathbf{U}_k$ consists of the dominant L_k left singular vectors of $\mathbf{H}_k$. If $[\mathbf{W}_1\ \cdots\ \mathbf{W}_K]$ is multiplied to the transmitted signal as the preprocessing matrix, the effective block-diagonal channel matrix becomes:

$$\mathbf{H}_{\text{eff}} = \left[\mathbf{H}_1^{\dagger}\mathbf{U}_1\ \cdots\ \mathbf{H}_K^{\dagger}\mathbf{U}_K\right]^{\dagger}\left[\mathbf{W}_1\ \cdots\ \mathbf{W}_K\right],$$

where the effective channel of the k-th user is given by:

$$\mathbf{H}_{\text{eff},k} = \mathbf{U}_k^{\dagger}\mathbf{H}_k\mathbf{W}_k.$$

Let the columns of $\mathbf{U}_{\text{eff},k}$ ($\mathbf{V}_{\text{eff},k}$) denote the left (right) singular vectors of $\mathbf{H}_{\text{eff},k}$, such as:

$$\mathbf{H}_{\text{eff},k} = \mathbf{U}_{\text{eff},k}\mathbf{D}_{\text{eff},k}\mathbf{V}_{\text{eff},k}^{\dagger},$$

where $\mathbf{D}_{\text{eff},k}$ denotes a diagonal matrix that consists of singular values of $\mathbf{H}_{\text{eff},k}$. To achieve the maximum capacity, $\mathbf{F}_k$ and $\mathbf{G}_k$ are

$$\mathbf{F}_k \triangleq \mathbf{W}_k\mathbf{V}_{\text{eff},k}\mathbf{E}_k,\ \text{and}\ \mathbf{G}_k \triangleq \mathbf{U}_{\text{eff},k}^{\dagger}\mathbf{U}_k^{\dagger}, \quad (3)$$

where the power-loading matrix $\mathbf{E}_k$ is an L_k-by-L_k diagonal matrix, whose diagonal elements are determined by the well-known waterfilling algorithm [8]. Then, the postprocessed data vector is represented by:

$$\begin{aligned}\hat{\mathbf{x}}_k &= \mathbf{G}_k\mathbf{y}_k \\ &= \mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\mathbf{x}_k + \mathbf{G}_k\mathbf{n}_k. \quad (4)\end{aligned}$$

2) Vector Perturbation (VP)

In our comparison, VP, a one-dimensional dirty paper approach, is considered as a simple nonlinear technique. By perturbing the data vector at the transmitter and taking modulo operation at the receivers, VP achieves an excellent performance gain over the linear processing schemes, especially at high signal-to-noise ratios (SNRs) [12]. Recently, VP has been extended to systems with multiple receive antennas and multiple spatial modes, and an optimum VP for minimizing the mean square error has been proposed [13].

Once the precoding and decoding matrices for a ZF block diagonalization are determined, VP chooses the best perturbation vector to minimize the power in the transmitted signal when added to the data vector. In this paper, the data vector is perturbed based on the GZF. The perturbation vector is constrained to an integer vector so as not to affect the zero co-channel interference condition [12]. The transmitted signal can be represented by:

$$\mathbf{s} = \sum_{k=1}^{K}\mathbf{F}_k\left(\mathbf{x}_k + \tau\mathbf{l}_k\right),$$

and

$$\mathbf{l} = \arg\min_{\mathbf{l}'}\left\|\sum_{k=1}^{K}\mathbf{F}_k\left(\mathbf{x}_k + \tau\mathbf{l}'_k\right)\right\|^2,$$

where τ is a scalar of a positive real value, $\mathbf{l}_k$ is the integer vector, and $\|\cdot\|$ denotes the vector 2-norm.

Using the same pre- and postprocessing matrices with GZF as in (3), the postprocessed data vector for VP is given by:

$$\hat{\mathbf{x}}_k = \mathbf{G}_k\mathbf{H}_k\mathbf{F}_k\left(\mathbf{x}_k + \tau\mathbf{l}_k\right) + \mathbf{G}_k\mathbf{n}_k. \quad (5)$$

Taking the modulo-τ operations for (5), the estimate of $\mathbf{x}$ with VP is obtained and is the same as (4).

3) Scheduling with Rank Adaptation

When the total number of receive antennas is more than N_T, both GZF and VP need an extra scheduling algorithm, while the DPC provides the optimal user selection implicitly. Moreover, if the number of receive antennas for a user can be more than one, the rank adaptation technique [11] should be applied to improve the sum rates.

In our comparison, the PF scheduler is employed under the consideration of fairness among the users. The set of spatial modes for users, $(L_1, L_2, \ldots L_K)$, are determined by brute-force searching.

IV. Comparison and Discussion

The CDFs of throughput per user and the average cell throughputs for the DPC, GZF, VP, and TDMA are evaluated. MIMO channels are obtained by generating independent Gaussian random variables with a zero mean, and the results shown below are the averages over 350 independent trials under the PF

Table 1: Simulation Parameters

Parameter	Values	Parameter	Values
# of antennas $\{N_T, N_{R,k}\}$	$\{2,1\},\{4,2\}$	Inter-cell	18 (2nd-tier)
# of users (K)	$1, 2, 4, 8$	Tx antenna pattern	Omni-direction
Scheduling	PF	Path loss model	3GPP2/TSG-C.R1002
Sync. protocol	Synchronous	Path loss exponent	3.5
Tx power	$47\ dBm$	Shadowing STD	$8.9\ dB$
Rx noise level	$-94\ dBm$	BS correlation	0.5
Radius of cell	$1000\ m$	Min. separation (d_0)	$100\ m$
Channel model	Rayleigh	Max. achievable SINR	$17.8\ dB$

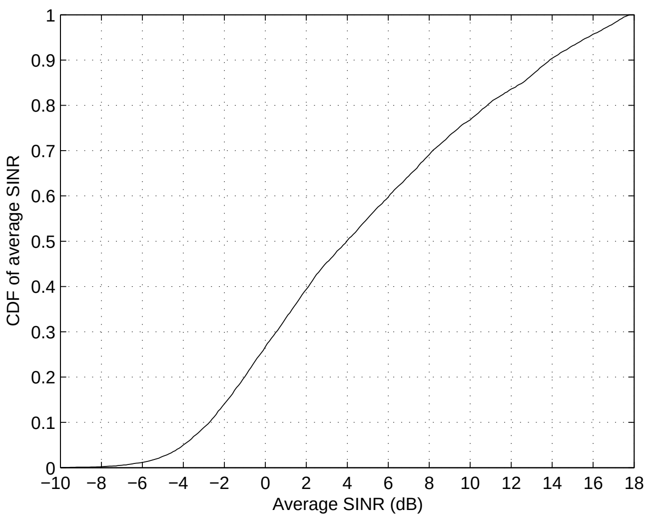

Figure 2: SINR CDF of multicell environments in Table 1.

scheduler for each average SINR point. Every average SINR was assigned independently $8,000$ times to all users according to the distribution in Fig. 2, which was generated under the multicell environment outlined in Table 1. The mobile station locations are constrained to be distanced from base station farther than minimum distance (d_0) 100 m and the maximum achievable SINR in the receiver is limited to 20 dB, as shown in Table 1. BS correlation is defined as the correlation factor among the inter-cells' BSs and its value is 0.5.

Fig. 3(a) and 3(b) show the CDFs of the throughput per user for $\{N_T, N_R\} = \{2,1\}$ and $\{4,2\}$, respectively. Here, when $N_T = 4$ and $K = 4$, 10% of the high end users for each scheme in an inter-cell achieve 4.65, 4.19, 3.92, and 2.86 *bit/sec/Hz* using the DPC, VP, GZF, and TDMA, respectively. For the low average SINR users, the gains of the GZF, VP, and DPC over TDMA seem to be negligible. This is because the DPC, GZF, and VP all use the solution of allocating all of the power to the proportionally highest rate user in this regime, which is the same as TDMA with PF. In contrast, in the high SINR regime, the DPC, GZF, and VP have a significant gain over TDMA since the multiplexing gain of TDMA is bounded by $\min\{N_T, N_R\}$, whereas those of the multiuser MIMO schemes are bounded by $\min\{N_T, KN_R\}$. There is an additional gain for the DPC over GZF and VP due to the optimal cancelling of inter-user interferences.

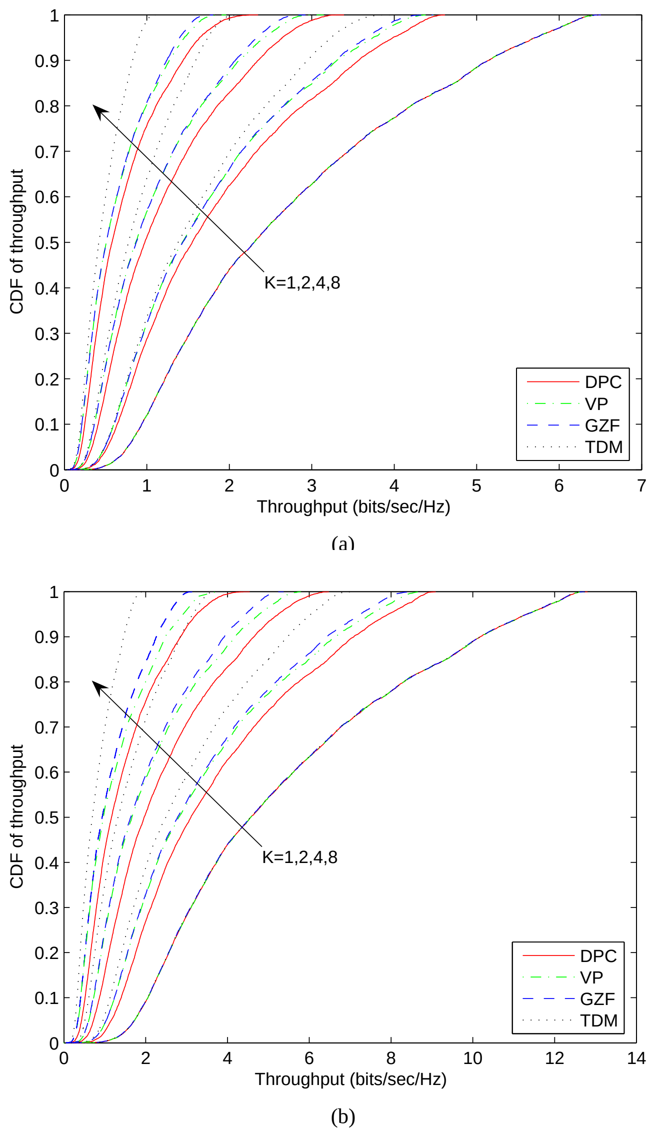

Figure 3: CDF of throughput of various multiuser MIMO schemes when (a) $\{N_T, N_R\} = \{2,1\}$, (b) $\{N_T, N_R\} = \{4,2\}$.

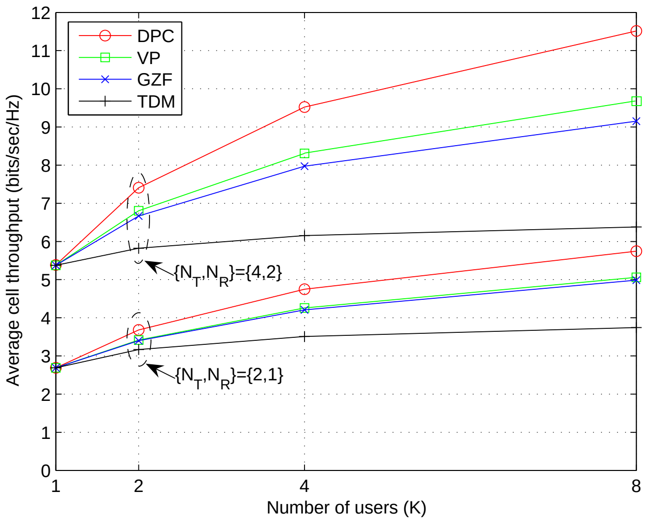

Figure 4: Average cell throughput versus the number of users when $\{N_T, N_R\}$ is $\{2,1\}$ and $\{4,2\}$.

Fig. 4 shows the average cell throughputs for $\{N_T, N_R\} = \{2,1\}$ and $\{4,2\}$ versus the number of users. Obviously, we can see that the average cell throughputs are the same for each scheme when there is only one user in the cell since it is a point-to-point optimization problem without interference cancellation for all schemes. By increasing the number of users, the average throughputs for both $\{N_T, N_R\} = \{2,1\}$ and $\{4,2\}$ increase, but the rate of increase is much larger for the $\{4,2\}$ case with two users, because for more than two users, it will fully utilize the multiplexing gain of 2. For the region where multiplexing gain is fully utilized for both antenna configurations (e.g. users ≥ 2), the rate of increase is slightly larger for $\{N_T, N_R\} = \{4,2\}$ compared with $\{N_T, N_R\} = \{2,1\}$, which results from the array gain of having more receiver antennas. In general, for $KN_R \leq N_T$, the multiplexing gain will be the dominating gain, and this gain is proportional to KN_R; whereas for the $KN_R > N_T$ case, the dominating gain is a relatively smaller multiuser diversity gain. Following this tendency, the cell average throughput for $K = 8$ and $N_T = 2(4)$ shows a gap of $12(16)\%$, $14(20)\%$, and $35(45)\%$ from that of the DPC under the PF scheduler for VP, GZF, and TDMA, respectively. Compared to orthogonal schemes such as TDMA and FDMA, the GZF and VP have a significant throughput improvement of approximately $32(45)\%$ and $34(53)\%$, respectively, with some additional complexity. Despite the gains that the GZF and VP provide over TDMA, there is still a noticeable gap from the optimal bound.

V. Conclusion

The CDFs of the throughput per user and the average cell throughputs for various multiuser MIMO schemes are evaluated. The effects of multiplexing, multiuser diversity, and array gain are observed. Also, the performance gaps between the optimal and various suboptimal schemes are shown. Here, it is surmised that the suboptimal schemes such as the GZF and VP, which also have potential to be improved further, are good candidates for next generation communications. However, all of the previous schemes need perfect channel state information at the transmitter for implementation. Examining the robustness of these schemes against channel uncertainty and the exact complexity comparison of each system remains as work to be undertaken.

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