Capacity of MIMO Systems in Shallow Water Acoustic Channels

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***Abstract*—Underwater acoustic (UWA) channels are typically characterized by a multipath structure with large delay spread, where only a few propagation paths carry significant energy. Each path exhibits time variability, which, together with the transmitter and receiver motion, induces Doppler spreading and shifting of the signal. In this paper, we analyze the limits on the information rate achievable through multiple-input multiple-output (MIMO) communications over UWA channels. Assuming full channel state information (CSI) at the receiver, we evaluate the ergodic capacity in two scenarios: one with partial CSI at the transmitter, and another with no CSI. Also, we consider the constrained capacity for practical modulations, e.g., BPSK and QPSK, and, exploiting the sparseness of the multipath structure, we provide new lower bounds on the achievable information rate. Statistical characterization and numerical examples are given based on the data collected in a recent experiment, conducted off the coast of Kauai, Hawaii, in June 2008.**

I. Introduction

In recent years, there has been an increased interest in underwater acoustic communications due to a variety of applications (e.g., in marine research, oceanography, and offshore oil industry). Ongoing research investigates the design of systems with improved performance and robustness, which requires the physical nature of the channel to be captured through proper channel modeling. This is a challenging problem because the UWA channel is characterized by frequency-dependent path loss, time-varying multipath propagation, and low propagation speed (i.e., about 1500 m/s) [1]. Time variations of the propagation paths, induced by the system motion as well as by changes in the medium, result in Doppler spreading and shifting of the signal. Also, since acoustic propagation is best supported at low frequencies, the system is inherently wideband. This fact is of particular importance because of the large delay spreads that typically affect the UWA channels, which cause inter-symbol interference (ISI) spanning tens or even hundreds of symbol intervals in the case of wideband single-carrier systems. However, the channel impulse response is typically sparse, i.e., very few propagation paths carry significant energy [1].

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Ray theory has been used to provide a deterministic (average) description of the multipath propagation. However, in order to explore the communication limits of this complex medium, a statistical description of the random time variations of the channel is needed. Recent works [2], [3] provide a short-term statistical characterization of the fading process, showing that a Rician model with a slowly time-varying parameterization satisfactorily matches the experimental data collected at different sites. Hence, we will adopt a time-varying Rician model for the information-theoretical analysis of the UWA channels.

Growing interest in the use of MIMO systems and space-time coding has been sparked by the information-theoretical results in [4], [5], which showed that the ergodic capacity of a fading channel increases linearly with the number of transmit or receive elements, whichever is smaller. The ergodic capacity of UWA MIMO channel is analyzed in [2], where a Rician fading model is assumed, with rank-1 channel matrix and no individual path dispersion. In this paper, we extend these results to more realistic channel models with higher-rank matrices and individual path dispersions. We consider two different scenarios: in one the transmitter has partial CSI and knows the statistics of the channel, while in the other it has no CSI. In both scenarios, perfect CSI at the receiver is assumed.

Besides the capacity analysis for unconstrained inputs, we also investigate the ultimate information rate for practical constrained inputs, chosen from finite-order constellations such as BPSK and QPSK. For this case, analytical results are available that provide upper and lower bounds on the achievable information rates [6] (see [7] for the extension of the bounds in [6] to MIMO systems). Another approach is introduced in [8], according to which we can obtain an unbiased estimate of the information rate by running long simulations of the channel and of the optimal maximum-a-posteriori detector, i.e., the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm. Unfortunately, this second approach is infeasible for an UWA environment since the complexity of the BCJR algorithm exponentially grows with the delay spread of the equivalent discrete-time channel model, which, in UWA channels, can be on the order of hundreds of symbols. In this paper, we present a novel approach that, by exploiting the *sparseness* of the channel impulse response, allows us to compute lower bounds on the achievable information rate that improve upon those in [7].

The paper is organized as follows. In Section II, we introduce the equivalent discrete-time channel model. In Section III, we provide a capacity analysis for UWA MIMO systems with unconstrained inputs. In Section IV, we consider constrained inputs, presenting new bounds on the information rate and comparing them with the existing ones. Finally, in Section V, we give concluding remarks.

II. Channel model

Let us define a discrete-time equivalent baseband channel model for the MIMO system under consideration. Assuming a spatially correlated channel with N_t transmit and N_r receive elements, the channel matrix of size $N_r \times N_t$ is given by

$$\mathbf{H}[n,m] = \sum_{l=0}^{L} \mathbf{H}_l[m]\delta[n-l], \tag{1}$$

where m is the time index, n denotes the delay, and $\mathbf{H}_l[m]$ is the l-th tap gain matrix at time m. More insights into the channel model are provided in Figure 1. We remark that typically very few channel taps contain energy as a consequence of the sparseness of the UWA channels.

Assuming a Rician model, we can write

$$\mathbf{H}_l[m] = \sqrt{\frac{\Omega_l k_l}{k_l+1}}\bar{\mathbf{H}}_l + \sqrt{\frac{\Omega_l}{k_l+1}}\tilde{\mathbf{H}}_l[m], \tag{2}$$

where k_l and Ω_l are the k-factor and the average power of the l-th tap, respectively, with $\{\Omega_l\}$ representing the MIMO multipath delay profile of the channel, normalized according to the constraint $\sum_l \Omega_l = 1$. Moreover, $\bar{\mathbf{H}}_l$ is a deterministic matrix (i.e., the mean-value matrix of the Rician process) whose Frobenius norm is normalized to $\|\bar{\mathbf{H}}_l\|_F^2 = N_t N_r$ for all values of l, while $\tilde{\mathbf{H}}_l$ contains the random channel components and can be written as

$$\tilde{\mathbf{H}}_l = \mathbf{\Theta}_{R,l}^{1/2}\tilde{\mathbf{H}}_{\omega,l}\mathbf{\Theta}_{T,l}^{1/2}\ , \tag{3}$$

where $\tilde{\mathbf{H}}_{\omega,l}$ is an $N_r \times N_t$ matrix of independent zero-mean complex Gaussian random variables whose Frobenius norm is $\|\tilde{\mathbf{H}}_{\omega,l}\|_F^2 = N_t N_r$ for all values of l, and $\mathbf{\Theta}_{R,l}$ and $\mathbf{\Theta}_{T,l}$ are the receive and the transmit correlation matrices of the l-th tap. We assume that different taps are uncorrelated, i.e., the only correlation in the channel is among the elements of the l-th tap, which is defined by $\mathbf{\Theta}_{R,l}$ and $\mathbf{\Theta}_{T,l}$. This assumption is justified for wideband acoustic systems in which the uniformly-spaced taps of the discrete-time model (1) can be associated with the *physical* propagation paths. We also assume that all the elements of the tap with index l have the same distribution, characterized by Ω_l and k_l. According to (1), we denote by L the total extent of ISI in our channel model. We also denote by L' the number of significant ISI taps, i.e., those taps whose energy is above a pre-specified threshold — in our experimental environment we found $L' \ll L$. In the following analysis we assume that the channel is affected by additive white Gaussian noise (AWGN) modeled as an $N_r \times 1$ circularly symmetric complex Gaussian vector with independent and identically distributed (i.i.d.) elements, with zero mean and variance $1/2$ per complex component.

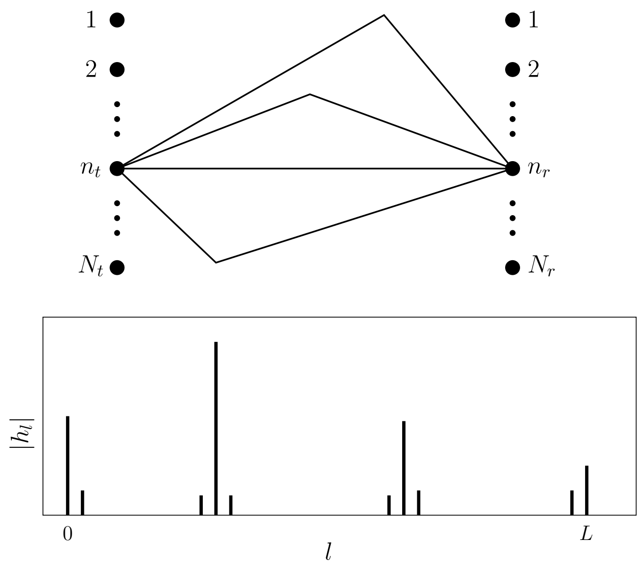

Fig. 1. Pictorial example of a sparse MIMO channel, with focus on the impulse response h_l related to the transmit/receive pair (n_t, n_r).

III. Ergodic Capacity

An underwater acoustic channel is characterized by frequency-selective fading and requires dealing with the memory in the channel as described in (1). Therefore, the analysis of the ergodic capacity is to be based on the observation of large input/output signal vectors. Following the treatment in [9], we will study the system under the assumption that the channel can be considered constant for M consecutive time indices, where the choice of the value of M is driven by the coherence time of the channel. Also, we will assume that the only constraint on the signal at the channel input is that the average transmission power cannot exceed P.

For each length-M block, the problem of the capacity evaluation can be conveniently approached in the frequency domain, adopting the input-output relationship [9]

$$\mathbb{Y} = \mathbb{H}\mathbb{X} + \mathbb{W}\ , \tag{4}$$

where $\mathbb{X}$ is the $MN_t \times 1$ space-time vector representing the input block, $\mathbb{Y}$ and $\mathbb{W}$ are $MN_r \times 1$ space-time vectors representing the output block and the noise, respectively, and $\mathbb{H}$ is the $MN_r \times MN_t$ block-diagonal channel matrix induced by (1). In practice, as discussed in [9], a Rician frequency-selective MIMO channel can be represented in the frequency domain as a set of M parallel independent MIMO channels, with different channel gains given by [9]

$$\mathbf{G}_i = \sum_{l=0}^{L} \mathbf{H}_l e^{-j2\pi l(i/M)}, \tag{5}$$

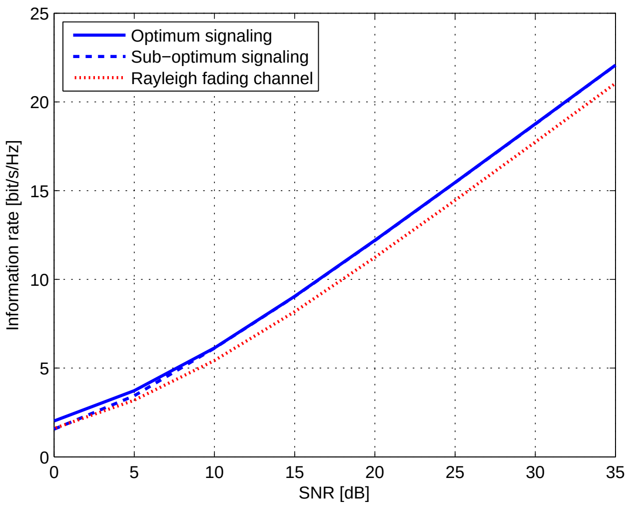

Fig. 2. Ergodic capacity of a 2×2 system for uniform-like eigenvalue distribution of the matrices of the channel mean values.

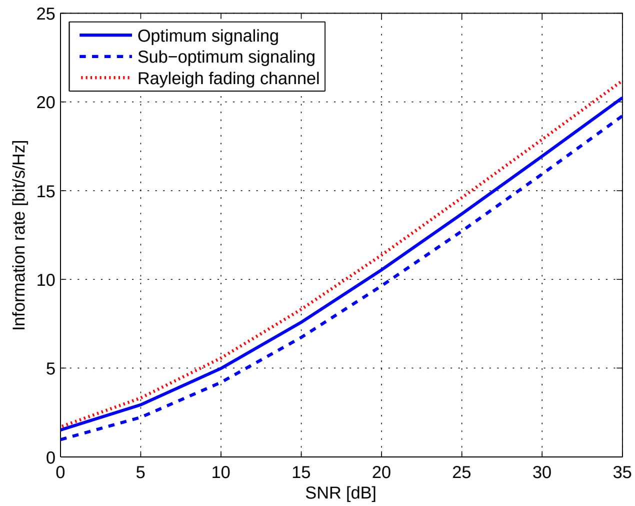

Fig. 3. Ergodic capacity of a 2×2 system for non-uniform-like eigenvalue distribution of the matrices of the channel mean values.

where i specifies the frequency bin. We recall that we are studying the channel over M units of time in which it does not change, so that the dependence of $\mathbf{H}_l$ on the time index can be omitted in (5). The ergodic capacity of the system is equal to the sum of the jointly-maximized information rates for each of the M sub-bands, so that the spectral efficiency yields

$$E\left\{\frac{1}{M}\sum_{i=0}^{M-1} I_i\right\}, \tag{6}$$

where

$$I_i = \log_2\left|\mathbf{I}_{N_r} + \mathbf{G}_i\mathbf{R}_{xx}[i]\mathbf{G}_i^H\right|$$

is the information rate for the i-th sub-channel, and $\mathbf{R}_{xx}[i] = E\{\mathbf{x}_i\mathbf{x}_i^H\}$ is the covariance matrix of the $N_t \times 1$ complex Gaussian vector $\mathbf{x}_i$ at the input of the i-th sub-channel. If we assume, for simplicity, that $\mathbf{\Theta}_{R,l}$ and $\mathbf{\Theta}_{T,l}$ are identity matrices, the columns of the matrix $\mathbf{G}_i$ become uncorrelated with different mean values $\bar{\mathbf{G}}_i$, due to the presence of the full-rank matrix of the mean values $\bar{\mathbf{H}}_l$. This assumption on the channel matrices implies that the distribution of I_i depends on i, unlike for the case of Rayleigh fading [10], where all frequency bins show the same statistical description.

If no CSI is available at the transmitter side, for all sub-channels we have the identity matrix as the optimal solution for the covariance matrix, corresponding to the optimal signaling strategy in Rayleigh fading [4]. In the case of partial CSI (i.e., the channel statistics are known, but not the specific realizations), there is no closed-form solution for the ideal transmission strategy. The optimal solution for all flat-fading sub-bands is provided in [11] using a semi-analytical approach where the eigenvectors of the optimal covariance matrix are identical to the eigenvectors of $\bar{\mathbf{G}}_i\bar{\mathbf{G}}_i^H$ and the optimal eigenvalues are computed based on the iterative power allocation algorithm. Even though this strategy is not the water-filling solution corresponding to the complete CSI at the transmitter, it is similar. In fact, the eigenvector structure indicates that the transmission strategy attempts to decorrelate the channel output, at least on average. This solution, due to the non-orthogonality of the parallel channels after eigenvector decorrelation, cannot avoid part of the power transmitted on each eigenvector spilling as interference onto other channels (which is a consequence of not having full CSI). Beside this semi-analytical optimal solution, we consider a sub-optimal heuristic solution previously discussed in [2] in the UWA context, and originally proposed in [12]. For the full-rank matrix of the channel mean values in (1), the covariance matrix for the i-th sub-band is given by

$$\mathbf{R}_{xx}[i] = \frac{P}{MN_t(1+k[i])}(\mathbf{I}_{N_t} + k[i]\mathbf{\Psi}_{N_t}[i]), \tag{7}$$

where $k[i]$ is the Rician k-factor for the i-th sub-band, and $\mathbf{\Psi}_{N_t}[i]$ is the $N_t \times N_t$ matrix that provides exact water-filling solution in the case of deterministic channel with AWGN when $k[i] \to \infty$. The rationale behind this idea is to exploit the knowledge of the Rician k-factor by linearly combining the optimal solutions for the covariance matrix in two limiting scenarios: Rayleigh fading and deterministic channel [2].

In Fig. 2 and Fig. 3 we illustrate the ergodic capacities for two different 2×2 MIMO systems. The channel parameters have been set according to the data collected in the Kauai Acomms MURI 2008 (KAM08) experiment, which was conducted in shallow water off the western coast of Kauai, Hawaii, in June 2008. In both cases, the channel has $L = 52$ ISI taps, $L' = 3$ of which are significant, with $\Omega_0 = 0.25$, $\Omega_{21} = 0.5$, $\Omega_{34} = 0.15$ and $\Omega_{52} = 0.1$ — we recall that the tap with index 0 is not considered an ISI tap. The values of k_l are all equal to 10, and the correlation between different transmit/receive pairs is neglected ($\mathbf{\Theta}_{R,l} = \mathbf{I}_{N_r}$ and $\mathbf{\Theta}_{T,l} = \mathbf{I}_{N_t}$). The results of Fig. 2 refer to a channel where all $\bar{\mathbf{H}}_l$'s in (2) are different full-rank matrices. In this case the eigenvalue distribution of the matrix of the channel mean values for each sub-band tends to a uniform distribution. We note that the channel capacity is slightly greater than the capacity of the Rayleigh fading channel which is presented as a reference. Despite the sub-optimality of the heuristic solution, we observe a degradation in its performance with respect to the optimal one only at low SNR. Even though it is a sub-optimal solution, due to the low complexity of implementation, it seems to be a convenient choice for system design. The results of Fig. 3 refer to a channel where we assume that all full-rank matrices of the channel mean values are identical, which will result in non-uniform eigenvalue distribution unlike in the previous scenario. Clearly, the channel capacity is degraded in comparison with the results presented in Fig. 2. We also note that the sub-optimal solution causes a more significant degradation than in the former case, which suggests that the heuristic solution is more effective when the eigenvalue distribution of the matrix of the channel mean values tends to a uniform distribution.

IV. Information Rate for Standard Modulations

The ergodic capacity investigated in the previous section gives the performance limit of the system according to the Shannon's definition, which relies on the concept of infinitely-long codewords and allows the use of arbitrary input alphabets [13]. However, in any practical communication system, the codewords adopted to protect the information are of finite length and the input alphabet is a finite-order constellation such as BPSK, QPSK, or QAM. In this section, we investigate the performance limit of the system under the practical constraints of finite-length codewords and independent and uniformly distributed (i.u.d.) inputs, drawn from finite-order modulation alphabets. Specifically, our analysis is carried out under the assumption of *quasi-static channel*, according to which the codeword length is much lower than the coherence time of the channel, so that each transmitted codeword sees a time-invariant channel. Hence, the problem reduces to characterizing the constrained capacity (also known as *information rate*) of time-invariant frequency-selective MIMO systems with finite-order modulation alphabets. Clearly, because of the time evolution, different codewords may see different channel realizations, each supporting a different information rate. The aim of this work is to characterize the information rate supported by a specific realization of the MIMO channel, and not to give a statistical characterization of the system in terms of *outage capacity* [13], i.e., evaluate the fraction of time that a given information rate is supported by the time-varying channel.

The information rate supported by a time-invariant MIMO channel can be expressed as [13]

$$I(X;Y) = \lim_{N\to\infty} \frac{I(X_1, X_2, \ldots, X_N; Y_1, Y_2, \ldots, Y_N)}{N}, \quad (8)$$

where, for each time index n, X_n is the $N_t \times 1$ channel input vector and Y_n is the $N_r \times 1$ channel output vector. Currently, no single-letter expression of the information rate in (8) is available [7], [8]. In principle, we can obtain an estimate of the information rate as accurately as desired by means of the simulation-based algorithm described in [8]. This approach requires the simulation of a full-complexity BCJR receiver that processes a trellis with $|X|^{N_t L}$ states, where $|X|$ is the cardinality of the modulation alphabet. For example, in a $2 \times N_r$ system adopting QPSK over a channel with memory $L = 3$, the number of states is $4^{2\cdot 3} = 4096$. Unfortunately UWA channels are often characterized by large values of L, which makes the adoption of the simulation-based algorithm infeasible — the data collected in the KAM08 experiment show values of L on the order of 50 symbols. Hence, to the best of our knowledge, the only tools for the characterization of the information rate of UWA channels with large memory are the analytical bounds presented in [7], which are the extension of the bounds proposed in [6] to MIMO channels. Specifically, the author of [7] introduced a provable upper bound (UB) on the achievable information rate, together with a provable lower bound (LB) and a conjectured LB. Such bounds are shown in Fig. 4 for two different realizations of the UWA channel with memory $L = 52$ already considered in Fig. 2 and Fig. 3. Note that, in both cases, the gap between the upper and lower bounds is significant, which does not allow us to satisfactorily characterize the information rate of interest. In the following, we introduce two new lower bounds that are much tighter than the existing ones.

Let us consider an arbitrary MIMO receiver that, processing the received samples $\{Y_n\}$, produces the decisions $\{Z_n\}$. For example, in the case of hard-output detection, Z_n is the estimate of the symbol X_n transmitted at time n, and thus belongs to the signaling constellation. In general, we can consider soft-output detection, in which case Z_n does not necessarily belong to the signaling constellation. In both cases, the data-processing inequality [13] guarantees that the information rate

$$I(X;Z) = \lim_{N\to\infty} \frac{I(X_1, X_2, \ldots, X_N; Z_1, Z_2, \ldots, Z_N)}{N} \quad (9)$$

is lower than $I(X;Y)$. The mutual information in (9) still involves infinite-length sequences, which makes it impractical to compute when the channel and the receiver have memory. On the other hand, the chain rule for the mutual information [13] guarantees that $I(X;Z)$ is lower bounded by

$$I_{\text{LB}} = I(X_n; Z_n) \quad (10)$$

for each value of the time index n at which the system is not affected by border effects due to the channel variations. Note that the evaluation of I_{LB} requires computing the mutual information between elements of the sequences, and not between the whole sequences as in (9). Hence, we can evaluate I_{LB} first by collecting the joint statistics of X_n and Z_n through long simulations of the channel and the receiver, and then by numerically computing the mutual information $I(X_n; Z_n)$.

The specific receiver adopted for the computation of the lower bound in (10) does not affect its validity, but does

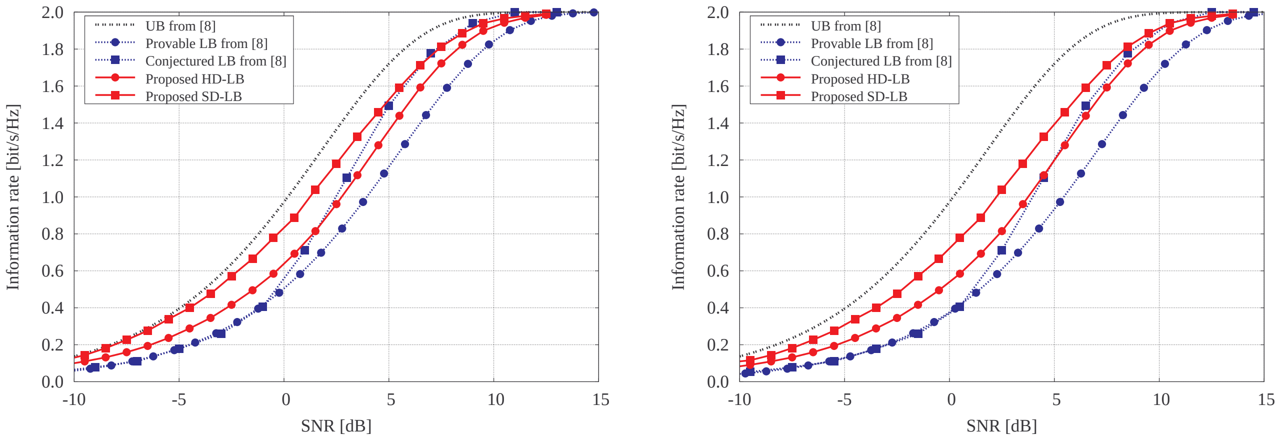

Fig. 4. Comparison of various bounds on the information rate of a 2×2 system with BPSK inputs. The two plots refer to two different realizations of the channel matrix, both randomly generated according to the same fading distribution.

affect its tightness. In fact, the value of I_{LB} gives the ultimate information rate supported by a system adopting that specific receiver, when concatenated with a fully-interleaved outer code [14]. Hence, the better the receiver, the tighter the lower bound. We have considered several receivers in the literature and found that the best performance/complexity tradeoff is provided by that proposed in [15]. The most attractive feature of this receiver is its complexity, which is proportional to $|X|^{N_t L'}$ and thus increases exponentially not with the channel memory L (as in the BCJR algorithm), but with the number of non-zero taps L'. The advantage is very clear in the case of the UWA channel considered in the examples given throughout the paper, where we have $L' = 3$ and $L = 52$. The value of I_{LB} provided by the adopted receiver is shown in Fig. 4 for the soft-decision (SD) version originally proposed in [15] as well as for the hard-decision (HD) version induced by the original one. Note that the SD-LB significantly improves the existing bounds, and even the HD-LB is satisfactory at low SNR values. Interestingly, Fig. 4 also shows that the gap between the upper and lower bounds depends on the specific channel realization.

V. Conclusions

We have investigated the ultimate information rate achievable by MIMO communications over time-varying UWA channels. Considering a slowly-varying Rician model for the fading process, we have first evaluated the channel capacity with average transmission-power constraint. Under the assumption of full CSI at the receiver, we have analyzed two different scenarios: one with partial CSI at the transmitter, and another with no CSI. The transmission rate limits have been studied under the practical constraint of independent and uniformly distributed inputs drawn from finite-order constellations. Particularly, we have presented new lower bounds on the information rates, making use of a sparse channel representation. These bounds improve upon the existing ones, and indicate the utility of receivers that exploit the sparseness of the multipath structure.

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