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Capacity reservation and utilization for a manufacturer with uncertain capacity and demand

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Abstract

We consider an OEM (Original Equipment Manufacturer) that has outsourced the production activities to a CM (Contract Manufacturer). The CM produces for multiple OEMs on the same capacitated production line. The CM requires that all OEMs reserve capacity slots before ordering and responds to these reservations by acceptance or partial rejection, based on allocation rules that are unknown to the OEM. Therefore, the allocated capacity for the OEM is not known in advance, also because the OEM has no information about the reservations of the other OEMs. Based on a real-life situation, we study this problem from the OEM's perspective who faces stochastic demand and stochastic capacity allocation from the contract manufacturer. We model this problem as a single-item, periodic review inventory system, and we assume linear inventory holding, backorder, and reservation costs. We develop a stochastic dynamic programming model and we characterize the optimal policy. We conduct a numerical study where we also consider the case that the capacity allocation is dependent on the demand distribution. The results show that the optimal reservation policy is little sensitive to the uncertainty of capacity allocation. In that case, the optimal reservation quantities hardly increase, but the optimal policy suggests increasing the utilization of the allocated capacity. Further, in a comparison to a static policy, we show that a dynamic reservation policy is particularly useful when backorder cost and uncertainty are low. Moreover, we show that for the contract manufacturer, to achieve the desired behavior, charging little reservation costs is sufficient.

Keywords: Capacity reservation, stochastic capacity and demand, outsourcing, stochastic dynamic programming

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1. Introduction

Outsourcing has been defined by Chase *et al*. (2004, p.372) as an 'act of moving some of a firm's internal activities and decision responsibilities to outside providers'. In the last two decades, many papers appeared on the rapid growth of outsourcing in many industries (Kremic *et al.*, 2006). A survey in 1997 of more than 600 large companies by the American Management Association finds that substantial numbers of companies are now outsourcing in many areas: information systems, finance, accounting, manufacturing, maintenance, and personnel. Among manufacturing companies, more than half had outsourced at least one component of their production process (Bryce and Useem, 1998).

Due to contractual agreements and limited information transparency, outsourcing complicates the order placing process for the OEM, especially when the contract manufacturer serves a number of OEMs on the same production line (Boulaksil and Fransoo, 2010). It is common in practice to have a contractual agreement that obliges the OEMs to reserve capacity prior to ordering (Zhao *et al.*, 2007). Capacity reservation offers several benefits to supply chain members such as mitigating the bullwhip effect (Lee *et al.*, 1997), providing flexibility to deal with uncertain demand and helps the contract manufacturer with his capacity planning, i.e., to secure capacity prior to receiving orders from the OEMs (Serel *et al.*, 2001).

In this paper, we build on the insight from the real-life case studies of Boulaksil and Fransoo (2010). That paper studied the planning challenges of an OEM that has outsourced the production activities for long-term to a contract manufacturer, who is the only source of supply for a specific product. The contract manufacturer performs the production activities on a non-dedicated capacitated production line on which multiple OEMs are served. Actually, the contract manufacturer does not have his own product portfolio, but only produces by offering outsourcing services to the OEMs.

One of the main insights from that study is that the order process consists of 'different, hierarchical connected decisions in time'. More precisely, the contract manufacturer requires contractually from the OEM to reserve capacity slots in advance. Once the reservations are collected, the contract manufacturer plans his capacity based on allocation rules and priorities that are unknown to the OEM. Therefore, the available capacity for each OEM is not known in advance. Later, the contract manufacturer responds to the OEM with the accepted reservation quantity, which is the upper bound for the order quantity, which is placed by the OEM to meet the uncertain demand.

From the OEM's perspective, it is not obvious what the optimal strategy is to control such a system. Reservation secures capacity for future orders, but also increases costs. A large body of literature deals with production planning models and inventory systems that considers capacitated supply (Federgruen and Zipkin, 1986; Ciarallo *et al.* 1994), but does not consider capacity reservation in their models. Therefore, our main contribution to this line of research is that it includes the capacity reservation problem to the production planning model in case of uncertain capacity and demand.

We explicitly model the case as outlined in Boulaksil and Fransoo (2010). We study a multi-period inventory system from the OEM's perspective that faces uncertain capacity from the contract manufacturer and uncertain customer demand. The OEM has to decide on the reservation and order quantities (to release to the contract manufacturer) in order to minimize the expected total costs. We develop a stochastic dynamic programming model for this problem and characterize the optimal policy. We also conduct a numerical study in which we extend the problem by considering dependency between the demand and capacity distributions.

The numerical results reveal several interesting managerial insights, such as that the utilization of the reservations (the order quantity divided by the accepted reservation quantity) is increased when the capacity uncertainty or the reservation costs increase, while the optimal reservation policy is little sensitive to the level of capacity uncertainty. We also show that the desired reservation and order behavior is achieved when small reservation costs are charged. Moreover, we compare the performance of the dynamic reservation policy with a static reservation policy, in which the reservation level is fixed. The gap in the performance is negligible in case the reservation cost is low or the backorder cost is high, as both policies opt for high reservation levels in those cases. Otherwise, we find that the gap in the performance of the policies increases under higher reservation cost, lower backorder cost, and lower demand uncertainty. The difference in the total expected cost can go as high as 7%.

This paper is organized as follows. In section 2, we discuss the literature review and show our contribution to the literature. In section 3, we present the model and analytical results. In section 4, we present the optimal policies. Then, in section 5, we present and discuss the numerical results and the managerial insights. Finally, in section 6, we draw some conclusions and discuss some managerial insights.

2. Literature review

Federgruen and Zipkin (1986) were one of the first to study a periodic review inventory model with a finite but certain capacity level. They proved the optimality of modified basestock policies. An extension of this work is that of Ciarallo *et al.* (1994) who study the stochastic demand and stochastic capacity setting. They show that in a single-period setting, the optimal policy is not affected by the capacity uncertainty, but in the multi-period setting, order-up-to policies that are dependent on the distribution of the capacity are optimal. Several other papers extended this problem (Güllü, 1998; Hwang and Singh, 1998; Wang and Gerchak, 1996; Iida, 2002;; Atasoy *et al.*, 2012;). Our main contribution to this line of research is that we add the reservation problem to the stochastic demand and stochastic capacity case.

In our model, the OEM makes a reservation by sharing advance demand information (Gallego and Ozer, 2001; Karaesmen *et al.*, 2002). Advance demand information means that future demand information is being shared prior to the moment when it is needed. Gallego and Özer (2001) show the existence of optimal state-dependent policies for periodic inventory systems with advance demand information. The optimal basestock levels depend on a vector of the advance order information. Ozer and Wei (2004) and Wijngaard and Karaesmen (2007) extended this work by assuming a capacitated system. They show how advance demand information can be a substitute for capacity and inventory. Tan *et al.* (2007) contributed to this line of research by assuming that the advance demand information might not be perfect. Gayon *et al.* (2009) also considers imperfect advance demand information, as customers may decide to order prior to or later than the expected due date or even cancel the orders. Also, Tan *et al.* (2009) assume imperfect advance demand information and they characterize the optimal inventory policy for such systems.

From the supply side, the OEM's reservations are shared with the CM without knowing exactly what the supply quantity will be, which can be considered as a form of supply uncertainty. A large stream of papers studies the supply uncertainty problem (Bassok and Akella, 1991; Parlar *et al.*, 1995; Güllü *et al.*, 1999; Pac *et al.*, 2009). Most of these papers consider completely uncertain supply quantities, whereas in our case, the supply uncertainty can be partly controlled by the reservation decisions. A few studies have considered the advance supply information case (Jaksic *et al.*, 2011; Altug and Muharremoglu, 2011; Jaksic and Fransoo, 2015). In this stream of papers, inventory systems are studied where the supplier shares forecasts of future supply availabilities, which are referred to as 'advance supply information', which can be useful to the customer for its own planning purposes. Altug and Muharremoglu (2011) present the optimal policy and they show the value of sharing the advance supply information. Jaksic *et al.* (2011) compare inventory models that assume advance supply information with models that assume advance demand information. They show that under some restrictive assumptions these two might be equivalent.

Another related part of the literature is about capacity reservation, which has been studied at both the tactical and the operational level. At the tactical level, the main objective is to study contract types and the conditions under which coordination in the supply chain can be achieved. Erkoc and Wu (2005) study the so-called deductible reservation contract, which means that the buyer pays a fee in advance for each reserved unit of capacity. When the buyer places a firm order, the reservation fee is deducted from the order payment, but the fee is not refundable in case the reserved capacity is not fully utilized within the specified time.

At the operational level, many papers have studied capacity reservation (Bonser and Wu, 2001; Hazra and Mahadevan, 2009; Serel *et al*., 2001; Serel, 2007; Van Norden and Van de Velde, 2005; Mincsovics *et al.*, 2009). The main objective of these studies is to decide on getting materials supplied either at a lower price by reserving capacity in advance with the long-term supplier or at a higher price from the spot market (Hazra and Mahadevan, 2009) or making reservations to *guarantee* the delivery of (a portion of) the reserved quantity, given the existence of the more expensive spot market (Serel *et al.*, 2001) or given the uncertain availability of the item in the spot market (Serel, 2007).

Serel *et al.* (2001) show that the existence of the spot market alternative significantly reduces the capacity reservation quantity from the long-term supplier. A similar case is considered by Hazra and Mahadevan (2009), who derive the supplier's optimal capacity reservation price in such a setting. Another paper that has studied capacity reservation is that of Jain and Silver (1995). They consider a single-period setting with stochastic demand and supplier's capacity, but dedicated capacity can be ensured by paying a premium to the supplier. The paper shows that the cost function is not convex in the dedicated capacity, but an algorithm is developed for finding the best level of dedicated capacity.

The literature on capacity reservation considers a single-period (or two-period) dual sourcing setting. We contribute to this line of research by considering a multi-period setting where the reservation problem is integrated with the inventory control problem. The paper that is the closest to our work is that of Costa and Silver (1996). A multi-period inventory problem is considered where the supplier capacity and the customer demand are uncertain. In that paper, the decision maker has the option to reserve some capacity for one or more periods, but the reservations have to be made prior to the start of the planning horizon, whereas in our model, the reservations can be done in each period of the planning horizon, based on more updated information. Furthermore, we characterize the optimal policy for our setting.

3. Model

Table 1. Notation

T	number of periods in the planning horizon
h	inventory holding cost per unit per period
b	backorder cost per unit per period
s	reservation cost per unit per period
x_t	inventory position in period *t* before ordering
y_t	inventory position in period *t* after ordering
r_t	reservation quantity in period *t* for period *t+1*
z_t	reservation position in period *t* after reserving
a_t	actual accepted reservation quantity in period *t*
A_t	(random) accepted reservation quantity in period *t*
q_t	order quantity in period *t*
D_t	(random) demand in period *t*
$f_t(d_t)$	probability density function of the demand in period *t*
d_t	actual demand in period *t*
C_t	(random) capacity in period *t*
c_t	actual capacity in period *t*
α	discount factor ($0 < \alpha \leq 1$)

As discussed earlier, we consider an OEM that has outsourced the production activities for a long-term to a contract manufacturer who serves a number of OEMs on the same production line. The contract manufacturer is the only source of supply for the product. According to the contractual agreement, the OEM reserves capacity before ordering. The reservations are needed by the contract manufacturer for his capacity planning and the contract manufacturer responds to a reservation r_{t-1} one period later by the accepted reservation quantity a_t.

At the moment of reservation, the OEM does not know what the actual allocated capacity c_t will be, because the contract manufacturer decides on the capacity allocation based on rules and priorities that are unknown to the OEM. Further, the OEM also has no information about the reservations of other OEMs. Therefore, from the OEM's perspective, we model a_t as the minimum of r_{t-1} and the uncertain allocated capacity C_t at the contract manufacturer. We consider the situation where the relationship between the OEM and the CM is a long-term relationship that lasts for many years, as one can find in e.g. the pharmaceutical industry. In such a setting, the OEM would be able to empirically estimate the probability distribution of C_t. Since the relationship is there for many years, sufficient historical information should be available to accurately estimate the distribution. Actually, this is also what we observed in a few real-life situations, as documented in Boulaksil and Fransoo (2010). Note that the CM does only reveal a_t to the OEM and not c_t.

Once a_t is announced, the reservation costs are charged, which are equal to sa_t with s being the unit reservation cost. These costs are introduced by the contract manufacturer to avoid that the OEMs inflate their reservations which may result in underutilization of the CM's production capacity. The OEM is not charged for each unit reserved r_t, as this would be 'unfair' if part of the reservation is rejected. This is similar to what we observed in real-life situation (Boulaksil and Fransoo, 2010). The OEM is required to reserve capacity slots in advance, which can be accepted or (partially) rejected by the contract manufacturer. The mutual contract states that the accepted reservation slots are charged by the contract manufacturer, as they are considered to be an early order commitment. Based on the accepted reservation quantities, the contract manufacturer ensures the availability of expensive materials that are needed for the production process, which justifies charging the reservation costs. In essence, our reservation cost structure ensures that the OEM has the incentive to share realistic reservations and it is similar to the deductible reservation fee of Erkoc and Wu (2005), as in both cases, a premium is also paid for reservations that are not utilized. After knowing a_t, the OEM decides on the order quantity q_t to meet the uncertain demand D_t. The order quantity q_t cannot exceed a_t and is delivered by the contract manufacturer just before the real demand d_t is observed.

To model this finite-horizon planning problem, we use a stochastic dynamic programming approach with two state variables: the inventory position before ordering x_t and a_t, which forms an upper bound on q_t. These state variables are needed to make decisions on r_t and q_t. We assume a periodic review inventory system with stochastic demand and stochastic capacity. As far as the model is concerned, we do not need any assumptions on the probability distributions of D_t and C_t. However, to show some optimality results in section 4, we assume that the distributions of D_t and C_t are independent of each other. In section 5.2, we relax this assumption and investigate the effects of dependency between the distributions.

We consider the following sequence of events. At the start of period t, the decision maker reviews x_t and a_t, where $a_t = min\{r_{t-1}, c_t\}$. Then, the reservation costs are incurred: sa_t. Based on the current state of the system (x_t, a_t), the decision maker decides on $r_t \geq 0$. The decision maker also decides on q_t, which raises the inventory position to $y_t = x_t + q_t$, where $0 \leq q_t \leq a_t$. Then at the end of period t, q_t that was ordered at the beginning of period t arrives and d_t is observed and satisfied as much as possible from inventory; unsatisfied demand is backordered. Then, inventory holding and backorder costs are incurred.

The state variables of the stochastic dynamic programming model (x_t, a_t) are updated at the start of period $t + 1$ in the following way:

$$x_{t+1} = x_t + q_t - d_t \tag{1}$$

$$a_{t+1} = min\{r_t, c_{t+1}\} \tag{2}$$

We assume linear inventory holding, backorder and reservation costs. Let $g_t(x_t, a_t)$ denote the minimum expected cost function, optimizing the cost over the finite planning horizon T from t onward and starting in the initial state (x_t, a_t). Then, we have the following DP recursion:

$$g_t(x_t, a_t) = sa_t + \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t + a_t}} \{\mathcal{L}(y_t) + \alpha E_{D_t, C_{t+1}}[g_{t+1}(y_t - D_t, A_{t+1})]\}, 1 \leq t \leq T \tag{3}$$

where

$$\mathcal{L}(y_t) = h \int_0^{y_t} (y_t - d_t) f_t(d_t) dd_t + b \int_{y_t}^{\infty} (d_t - y_t) f_t(d_t) dd_t \tag{4}$$

and

$$A_{t+1} = min\{r_t, C_{t+1}\} \tag{2'}$$

The last part of (3) is the expected future cost, which is derived by taking the expectation over D_t and C_{t+1}. Further, the stopping condition is $g_{T+1}(\cdot) = 0$.

4. The optimal order and reservation policy

In this section, we characterize the optimal solution of (3) by assuming that the demand (D_t) and capacity (C_t) are independently distributed across periods and are independent of each other. We prove the optimality of a state-dependent reservation policy and a modified basestock policy.

Let $h_t(y_t, a_t) = \mathcal{L}(y_t) + \alpha E_{D_t, C_{t+1}}[g_{t+1}(y_t - D_t, A_{t+1})]$ denote the cost-to-go function in period t. Accordingly, the minimum expected cost function $g_t(.)$ can be rewritten as

$$g_t(x_t, a_t) = \; sa_t + min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_{t+a_t}}} h_t(y_t, a_t), \qquad 1 \leq t \leq T \tag{3'}$$

In order to describe the optimal reservation policy, we introduce the "reservation position": $z_t = x_t + r_t$. Let $(\hat{y}_t, \hat{z}_t)$ be the unconstrained minimizers of $h_t(y_t, a_t)$ for given state variables (x_t, a_t). We first show the convexity results that allow us to find the structure of the optimal policy. Note that the loss function $\mathcal{L}(y_t)$ is convex in y_t (Porteus, 2002). The optimal decisions at any period t (y_t, z_t) are made by minimizing $h_t(\cdot)$ over the feasible region.

Theorem 1:

a. For any period $1 \leq t \leq T$, $g_t(x_t, a_t)$ and $h_t(y_t, a_t)$ are (jointly) convex functions.
b. For any period $1 \leq t \leq T$, the optimal order policy is given by:

$$y_t^* = \begin{cases} x_t + a_t & if \quad x_t < \hat{y}_t - a_t \\ \hat{y}_t & if \quad \hat{y}_t - a_t \leq x_t \leq \hat{y}_t \\ x_t & if \quad x_t > \hat{y}_t \end{cases} \tag{5}$$

c. For any period $1 \leq t \leq T$, the optimal reservation policy is given by:

$$z_t^*(a_t) = \begin{cases} \hat{z}_t(a_t) & if \quad x_t \leq \hat{z}_t(a_t) \\ x_t & if \quad x_t > \hat{z}_t(a_t) \end{cases} \tag{6}$$

See appendix A for the proof.

The optimal order policy (5) is a modified basestock policy, as the order quantity is bounded by a_t if $x_t < \hat{y}_t - a_t$. The optimal reservation policy (6) is a state-dependent reservation-up-to policy. This policy implies that at a given a_t, the reservation quantity should bring the reservation position $z_t = x_t + r_t$ to the optimal reservation position $\hat{z}_t(a_t)$ if $x_t \leq \hat{z}_t(a_t)$. Otherwise, $z_t^*(a_t) = x_t$, which means to reserve nothing.

To summarize, the inventory system can be optimally controlled by two critical parameters: the optimal order-up-to level y_t^* and the optimal reservation-up-to level $z_t^*(a_t)$ according to policies (5) and (6).

5. Numerical study

In this section, we present and discuss a numerical study that we conducted by solving the stochastic dynamic programming formulation given in (3). We construct a number of experiments and we are mainly interested in:

- the effects of different levels of demand and capacity uncertainty and different reservation costs in case of stationary distributions (section 5.1),
- the difference between a dynamic and static reservation policy (section 5.2),and
- the effects of dependency between the demand and capacity distributions (section 5.3)

on the optimal decisions and the system performance.

The following parameters are set at fixed values: T=12*,* α=0.99, and h=1. Furthermore, we assume a Gamma distribution for the demand and a Uniform distribution for the capacity (Burgin, 1975).

5.1. Stationary demand and capacity availability

In this section, we consider different levels of demand and capacity uncertainty and we vary the unit reservation cost. In experiments 1-24 (see Table 2), $E[D_t] = E[C_t]$ = 5, b=10, but we vary:

- the coefficient of variation of the demand $CV(D_t)$ between 0.5, 1, 1.5, 2, and 3;
- the coefficient of variation of the capacity $CV(C_t)$ between 0.28 and 0.52;
- the unit reservation cost between 0, 2, 5, and 10.

Table 2 shows the results of these experiments, where $\hat{z}_1(a_1)$ is shown as a vector in $\overrightarrow{a_1}$. Moreover, the expected costs are shown for $(x_t, a_t) = (0,6)$, as this is a feasible state for all experiments.

Table 2. Results with varying demand uncertainty, capacity uncertainty, and reservation cost when $E[D_t]$= $E[C_t]$=5

Exp	$CV(D_t)$	$CV(C_t)$	*s*	$\hat{y}_1$	$\hat{z}_1(a_1)$	*E[Cost]*
1	0.5	0.28	0	19	36	306.32
2	0.5	0.28	2	22	{20,…,16}	425.00
3	0.5	0.28	5	25	{19,…,15}	598.52
4	0.5	0.28	10	28	{18,…,14}	882.65
5	1	0.28	0	30	59	573.06
6	1	0.28	2	33	{30,..,26}	690.27
7	1	0.28	5	35	{27,…,23}	860.26
8	1	0.28	10	38	{24,…,20}	1136.68
9	1.5	0.28	0	37	77	704.83
10	1.5	0.28	2	39	{36,…,32}	820.01
11	1.5	0.28	5	41	{31,…,27}	985.60
12	1.5	0.28	10	43	{27,…,23}	1250.49
13	0.5	0.52	0	22	39	379.69
14	0.5	0.52	2	28	{25,…,17}	495.67
15	0.5	0.52	5	36	{25,…,17}	663.92
16	0.5	0.52	10	45	{24,…,16}	937.70
17	1	0.52	0	32	60	614.62
18	1	0.52	2	38	{34,…,26}	729.24
19	1	0.52	5	43	{33,…,25}	892.21
20	1	0.52	10	49	{32,…,24}	1154.19

21	1.5	0.52	0	38	78	735.05
22	1.5	0.52	2	43	{40,…,32}	847.08
23	1.5	0.52	5	47	{37,…,29}	1003.41
24	1.5	0.52	10	51	{34,…,26}	1251.89

The results from Table 2 show that higher demand uncertainty increases $\hat{y}_1$, $\hat{z}_1(a_1)$ and leads to higher costs. However, the higher the unit reservation cost s, the lower the effect of an increase of the demand uncertainty, because the incremental increase of the optimal reservation quantities decreases. Figure 1 shows the optimal order-up-to and reservation-up-to levels for different unit reservation cost and different levels of demand uncertainty. From the results, we see that when s increases, it is optimal to increase the order quantity (much more than the reservation quantity) such that a larger part of the accepted reservation is utilized, instead of increasing the reservation quantities. Figure 2 confirms this insight by showing the optimal ratio $\frac{z_t}{y_t}$.

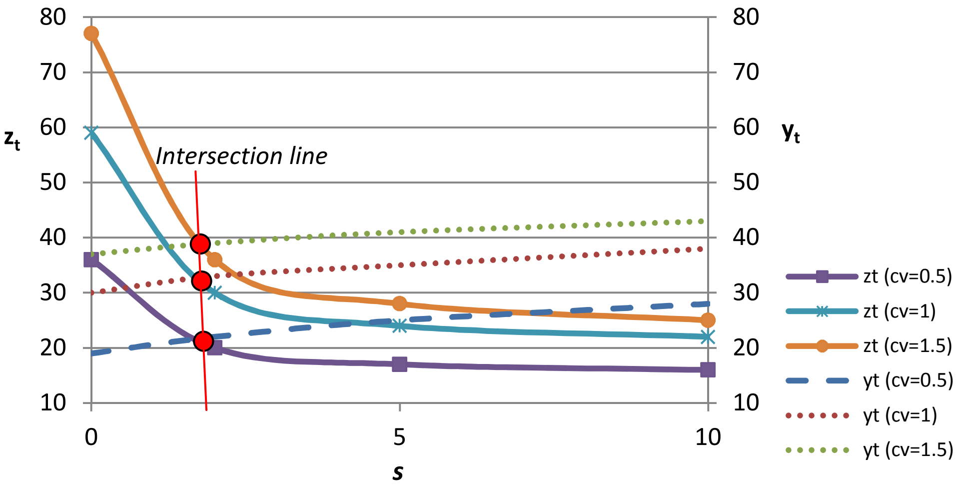

Figure 1. Order-up-to (y_t) and reservation-up-to (z_t) levels for different levels of demand uncertainty and reservation cost and the intersection line.

On the other hand, the contract manufacturer would prefer a situation where the difference between the reservation and order quantities is minimal, ideally zero. In Figure 1, we see that these ideal situations ($y_t = z_t$) are reached at a relatively small unit reservation cost. The intersection line that connects the intersection points is almost vertical, which means that the optimal s is not very sensitive to the level of demand uncertainty. In Appendix C, we show that the same result is obtained when varying other model parameters. That is, although z_t^* and y_t^* are quite sensitive to several model parameters, the value of s that yields $y_t = z_t$ appears to be quite robust. We note that it is critical to set the best reservation cost s, as too low s results in very high reservation levels that are not materialized as orders, and too high s results in a significant decrease of the reservation levels, which may result in a lower service level. Therefore, it is important to optimize the reservation cost by making use of a model, as we propose in this article.

Figure 2. The optimal ratio z_t/y_t at different unit reservation cost and different levels of demand uncertainty.

Another insight from Table 2 is that the higher the demand uncertainty, the lower the effect of an increase of capacity uncertainty on the optimal cost (see Figure 3, where *ΔCost* is given by (7)).

$$\Delta Cost = \frac{E[Cost|CV(C_t) = 1.5] - E[Cost|CV(C_t) = 0.5]}{E[Cost|CV(C_t) = 0.5]} \quad (7)$$

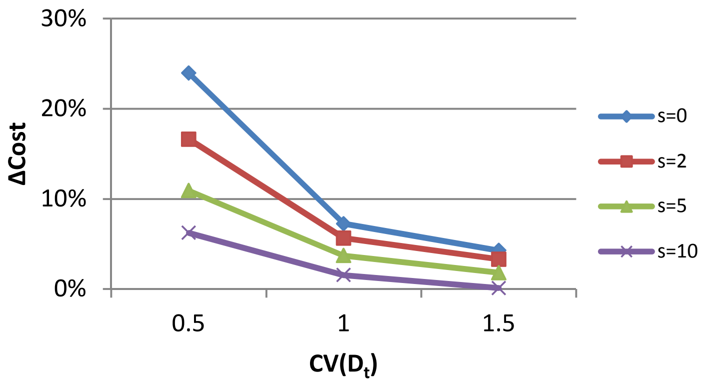

Figure 3. Relative cost increase due to increased capacity uncertainty.

The explanation for this effect is that when the capacity uncertainty increases, $\hat{z}_1(a_1)$ increases little compared to $\hat{y}_1$, i.e., the order quantity increases much more than the reservation quantity (see Figure 5.4). Therefore, when the capacity uncertainty increases, it is optimal to increase the order-up-to level much more than the reservation-up-to level. Therefore, when the capacity uncertainty increases, the optimal ratio $\frac{z_t}{y_t}$ decreases.

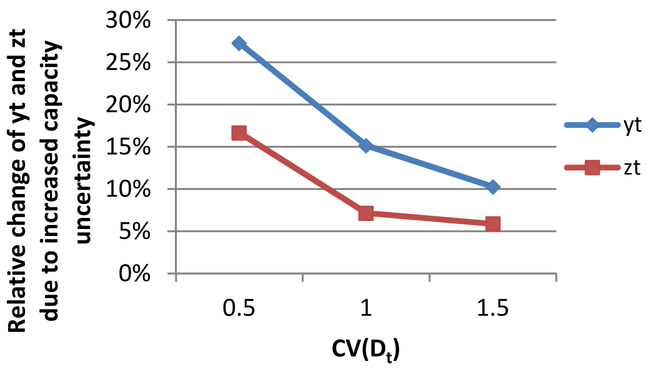

Figure 4. Relative change in y_t and z_t due to increased capacity uncertainty (when s=2).

In the last part of this section, we examine the effect of varying b and s on the optimal strategies. Figure 5 and Table 3 present the results. We notice that increasing b has two effects: both z_t and y_t increase but at different levels. We notice that as b increases, it is optimal to target a much higher z_t (especially when s is low) to increase the chances of getting a big portion of the reservations accepted. Then, y_t^* also increases to minimize the chance of facing a shortage. However, the first effect is stronger than the second one, especially when s is low. Hence, when s/b is low, it is optimal to secure the largest possible acceptance of the reservations to avoid shortages in the future.

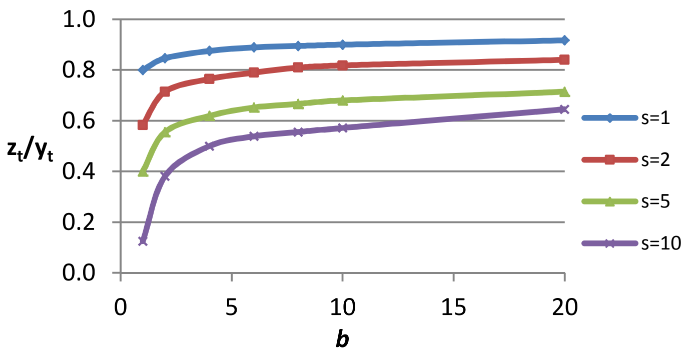

Figure 5. The effect of the unit backorder cost on y_t and z_t.

Table 3. Results with varying backorder cost and reservation cost in case $E[D_t] = E[C_t] = 5, CV(D_t) = 0.5, and\ CV(C_t) = 0.28$.

Exp	***s***	***b***	$\hat{y}_1$	$\hat{z}_1(a_1)$	***E[Cost]***	***Exp***	***s***	***b***	$\hat{y}_1$	$\hat{z}_1(a_1)$	***E[Cost]***
53	1	1	10	{10,…,6}	106.30	68	5	1	15	{8,…,4}	306.61
54	1	2	13	{13,…,9}	139.96	69	5	2	18	{12,…,8}	355.58
55	1	4	16	{16,…,12}	199.35	70	5	4	21	{15,…,11}	424.64
56	1	6	18	{18,…,14}	256.12	71	5	6	23	{17,…,13}	485.49
57	1	8	19	{19,…,15}	311.99	72	5	8	24	{18,…,14}	543.36
58	1	10	20	{20,…,16}	367.46	73	5	10	25	{19,…,15}	600.17
59	1	20	24	{24,…,20}	642.78	74	5	20	28	{22,…,18}	878.79
61	2	1	12	{9,…,5}	159.34	75	10	1	16	{4,…,0}	517.58
62	2	2	14	{12,…,8}	195.79	76	10	2	21	{10,…,6}	603.09
63	2	4	17	{16,…,12}	256.75	77	10	4	24	{14,…,10}	693.04
64	2	6	19	{17,…,13}	314.17	78	10	6	26	{16,…,12}	761.90
65	2	8	21	{19,…,15}	370.45	79	10	8	27	{17,…,13}	824.53
66	2	10	22	{20,…,16}	426.19	80	10	10	28	{18,…,14}	885.05
67	2	20	25	{23,…,19}	702.17	81	10	20	31	{22,…,18}	1169.74

5.2. Static versus dynamic reservation policy

In this section, we compare the performance of the dynamic reservation policy to a static reservation policy, within which a single optimal reservation level is applied across all periods, i.e., $r_1^* = r_i\ \forall i = \{2, \dots, T\}$. Figure 6 shows the cost increase of a static reservation policy compared to the dynamic reservation policy as a function of the unit reservation cost for different levels of demand uncertainty. The performance of the two policies is almost similar when the unit reservation cost is low, as both policies opt for high reservation levels in those cases, but the gap increases when the unit reservation cost increases.

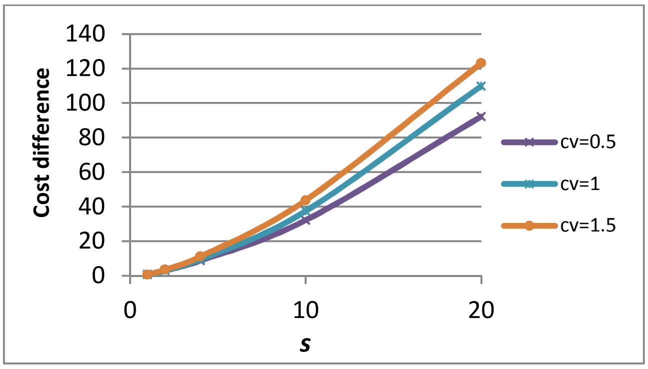

Figure 6. Cost difference between the two policies as a function of the unit reservation cost

Figure 7 shows the cost increase of the static reservation policy compared to the dynamic reservation policy as a function of the unit backorder cost for different levels of demand uncertainty. We notice that the gap between the performance of the two reservation policies gets closer when the unit backorder cost increases, as both policies opt for high reservation levels in those cases. When the backorder cost is low, the static reservation policy performs worse than the dynamic reservation policy, especially in case of low demand uncertainty.

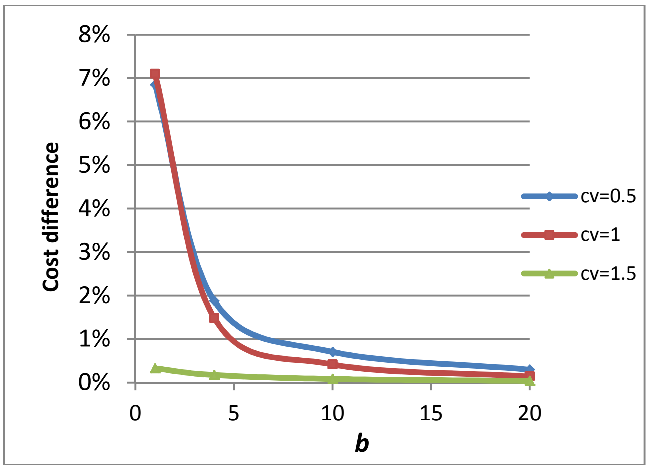

Figure 7. Cost difference between the two policies as a function of the unit backorder cost

5.3. Dependency between the distributions

In this section, we show the results of a numerical study in which we consider dependency between the demand and capacity distributions. In particular, we assume that the capacity allocation C_t of the contract manufacturer is dependent on the OEM's demand D_t and therefore, the results of section 4 do not hold anymore. We consider both the situations where the dependency is positive (section 5.2.1.) and negative (section 5.2.2.). In appendix B, we show how the conditional probability distributions are determined.

5.3.1. Positive dependency

In this section, we consider the case where the contract manufacturer is allocating more capacity when the OEM's demand is higher. The idea is that when the OEM's demand is higher, the OEM will request more (in terms of reservations and orders) and the contract manufacturer is then willing to allocate more capacity to the OEM to avoid the OEM searching for another source of supply. This situation is also possible when the OEMs' demand quantities are negatively correlated, which means that the more the OEM reserves and orders the less the other OEMs reserve and order, the more capacity is available for the OEM. This could happen when the total demand of all OEMs is relatively stable due to a fixed market size and an increase in the OEM's demand represents an increased market share.

We discuss the numerical results and compare them with non-correlated case. Based on the numerical studies, we observed that the optimal order policy remains a modified basestock policy with the same structure as (5). However, the reservation policy does not remain the same. The policy can be characterized by two optimal reservation-up-to levels, where the second level is lower than the first one. When the (starting) inventory position exceeds some point, it is optimal to target for a lower reservation-up-to level, i.e., to reserve much less. Due to the positive dependency, less has to be reserved (which also limits the reservation costs) to get the same amount of capacity allocated.

Table 4 shows the numerical results for 9 experiments that we conducted with positive dependency between the demand and capacity distributions. The results show that for all experiments, the expected cost is lower than in case with no dependency (on average 20.6 %). Due to the positive dependency, less has to be reserved with lower risk of getting too high accepted reservations, which results in lower (reservation) costs.

This result suggests that it is worthwhile to collect market information of the competitors (that produce at the same contract manufacturer) and to assess the dependency between the own demand and that of the competitors. In case of a negative dependency between OEMs' demand (which means there is a positive dependency between the own demand and the available capacity at the contract manufacturer), which is for example the case when the competitors operate in different market sectors, it is wise to adapt the reservation policy towards the contract manufacturer.

Table 4. Results with positive dependency when $E[D_t]$=$E[C_t]$=5

Exp	$CV(D_t)$	$CV(C_t)$	s	$\hat{y}_1$	***E[Cost]***
25	0.5	0.28	0	17	234.74
26	0.5	0.28	2	20	358.88
27	0.5	0.28	5	24	538.13
28	0.5	0.28	10	28	830.40
29	1	0.28	2	31	598.66
30	1.5	0.28	2	38	717.91
31	0.5	0.52	2	25	370.53
32	1	0.52	2	35	554.45
33	1.5	0.52	2	40	684.23
34	2	0.28	2	41	750.22
35	2	0.52	2	45	769.50
36	3	0.28	2	42	787.79
37	3	0.52	2	54	864.16

5.3.2. Negative dependency

In this numerical study, we consider the negative dependency case. Such a situation is likely when the different OEMs who all reserve and order at the same contract manufacturer operate in the same market, which results in a positive correlation between the demand D_t of the different OEMs. That means that all OEMs will increase their reservations and orders in case of a demand increase and vice versa. In such a situation, the contract manufacturer faces increased demand from all OEMs simultaneously, which results in a smaller capacity allocation for each OEM. Based on the results of the numerical studies, we observe that the optimal order policy remains the same as (5), but the structure of the reservation policy changes.

Like in the positive dependency case, the optimal policy can be characterized by two optimal reservation-up-to levels, but the second one is now higher than the first one. When the starting inventory position exceeds some point, it is optimal to target for a higher reservation-up-to level to get less capacity allocated and consequently not facing too high reservation and inventory holding costs.

Table 5 shows the numerical results of the experiments that we conducted with negative dependency between the demand and capacity distributions. The results show that for all experiments, the costs are higher than in case with no dependency. Due to the negative dependency, the probability of not getting enough supplied to meet the demand increases, which increases the backorder costs.

Like in the positive dependency case, it is worthwhile to assess whether there is dependency between the own demand and that of the competitors, by which the dependency between the own demand and the contract manufacturer's available capacity level can be estimated. If the latter dependency appears to be negative, it is recommended to take measures to eliminate the negative dependency, as this leads to higher costs. The elimination can be done by keeping some safety stock to avoid backorders or by agreeing (contractually) on paying slightly more to make an appeal to (in case needed) a fixed amount of the contract manufacturer's production capacity.

Table 5. Results with negative dependency when $E[D_t]$=$E[C_t]$=5

Exp	$CV(D_t)$	$CV(C_t)$	s	$\hat{y}_1$	***E[Cost]***
38	0.5	0.28	0	20	395.51
39	0.5	0.28	2	22	507.63
40	0.5	0.28	5	25	673.77
41	0.5	0.28	10	28	947.42
42	1	0.28	0	32	719.15
43	1	0.28	2	35	825.08
44	1.5	0.28	2	41	953.94
45	0.5	0.52	0	24	575.52
46	0.5	0.52	2	29	681.81
47	1	0.52	2	40	983.23
48	1.5	0.52	2	46	1116.38
49	2	0.28	2	44	999.53
50	2	0.52	2	50	1163.27
51	3	0.28	2	48	1040.79
52	3	0.52	2	53	1207.47

6. Conclusions

In this paper, we consider the case where a manufacturing company has outsourced the production activities to a contract manufacturer. The contract manufacturer produces on a non-dedicated production line on which multiple OEMs are served. For capacity planning purposes, the contract manufacturer requires that the OEM reserves capacity before ordering and responds to the reservations by acceptance or partial rejection based on rules that are unknown to the OEM. Therefore, the allocated capacity to the OEM is not known in advance.

We study this problem from the OEM's perspective who faces stochastic customer demand and stochastic capacity allocation from the contract manufacturer and who has to decide on the reservation and order quantities under uncertainty. We develop a stochastic dynamic programming model for this problem and we characterize the optimal reservation and order policies. The optimal reservation policy is a state-dependent policy, as the optimal target reservation-up-to level is dependent on the accepted reservation quantity. The optimal order policy is a modified basestock policy; the order quantity is bounded by the accepted reservation quantity.

We conduct a numerical study which reveals several interesting (managerial) insights. First, in case the unit reservation cost or the capacity uncertainty increases, it is optimal to increase the order quantity (much more than the reservation quantity) and so, the utilization of the accepted reservation quantity. This might be counterintuitive, as one would expect to mainly increase the reservation quantities in case the capacity uncertainty increases to hedge against the uncertainty faced from the contract manufacturer. Another insight is that the effect of an increase of the capacity uncertainty decreases substantially when the demand uncertainty increases. When the demand uncertainty increases, the optimal order quantities increase, by which the order will be (closely) equal to the accepted reservation quantity. The action of increasing the order quantity is also required when the capacity uncertainty increases, and therefore, we see that the effect of an increase of the capacity uncertainty is very little when the demand uncertainty increases.

Another managerial insight follows from the fact that from the contract manufacturer's perspective, it is desired to have the reservation equal to the order quantity. We have seen that this can be achieved when little reservation costs are charged. This optimal unit reservation cost is independent of the level of demand uncertainty. Charging no reservation costs leads to over reservation and charging higher reservation costs leads to under reservation. This managerial insight is helpful when having contract negotiations with the OEMs on setting the reservation cost, which is a contract parameter. We also compare the performance of the dynamic reservation policy with a static reservation policies, within which a single reservation level is applied across all periods. The dynamic reservation policy performs substantially better when the reservation cost is high, the backorder cost is low, or when the demand uncertainty is low.

Finally, we studied the case where the capacity allocation of the contract manufacturer depends on the OEM's demand distribution. When the distributions are dependent, the structure of the optimal order policy is the same as in the independent case, but the optimal reservation policy changes to a policy with two optimal target reservation-up-to levels. Dependent on whether the dependency is positive (or negative), the second optimal reservation-up-to level is lower (or higher) than the first one by which the model adapts its reservation quantities to the higher (or lower) capacity allocation. We have seen that the expected cost decreases when the dependency is positive and increases when the dependency is negative. These results suggest that it is worthwhile to collect market information of the competitors (that produce at the same contract manufacturer) to assess the dependency between the own demand and the available capacity at the contract manufacturer. In case the dependency is positive, it is wise to adapt the reservation policy towards the contract manufacturer to save costs. In case of negative dependency, one can think of measures like keeping safety stocks to hedge against the little capacity allocation of the contract manufacturer or agreeing on paying an additional premium to ensure (in case needed) a fixed amount of capacity from the contract manufacturer. Of course, these measures should be cheaper than the extra cost due to the negative dependency.

Appendix A. Proof of theorem 1

Let

$$g_t(x_t, a_t) = \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{sa_t + \mathcal{L}(y_t) + \alpha E_{D_t,C_t}[g_{t+1}(y_t - D_t, A_t)]\}$$
$$= \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{sa_t + h_t(y_t, a_t)\} = sa_t + i_t(x_t, a_t)$$

- The functions $g_t(x_t, a_t)$ and $h_t(y_t, a_t)$ are jointly convex functions for any $t \in [1, T]$. We prove this by induction. $g_{T+1}(\cdot) = 0$ and is convex. Assume that $h_{t+1}(\cdot)$ is also convex. Then, the function $h_t(y_t, a_t) = \mathcal{L}(y_t) + \alpha E_{D_t,C_t}[g_{t+1}(y_t - D_t, A_t)]$ is also convex, because:
    - $\mathcal{L}(y_t)$ is a convex funtion;
    - $E[g_{t+1}(y_t - D_t, A_t)]$ is convex due to the convexity of the expected value operator. Rule: If $f: \mathbb{R}^n \to \mathbb{R}$ is convex, then the function $g(x) = E_w\{f(x + w)\}$ is also a convex function, where w is a random vector in $\mathbb{R}^n$, provided that the expected value is finite for every $x \in \mathbb{R}^n$ (Bertsekas, 2005);
    - the linear combination of two convex functions remains convex. Rule: Let K be a non-empty index set, X a convex set, and for each $k \in K$ let $f_k(\cdot)$ be a convex function on X and let $p_k \geq 0$. Then $\sum_{k \in K} p_k f_k(x)$, $x \in X$ is a convex function on any convex subset of X, where the sum takes finite values (Heyman and Sobel, 2004).

- $i_t(x_t, a_t) = \min_{\substack{r_t \geq 0 \\ x_t \leq y_t \leq x_t a_t}} \{h_t(y_t, a_t)\}$ is also convex when $h_t(y_t, a_t)$ is convex. Rule: Let X be a non-empty set with A_x a non-empty set for each $x \in X$. Let $C = \{(x, y): y \in A_x, x \in X\}$, let J be a real-valued function on C, and define $f(x) = inf\{J(x, y): y \in A_x\}, x \in X$. If C is a convex set and J is a convex function on C, then f is a convex function on any convex subset of $X^* = \{x: x \in X, f(x) > -\infty\}$ (Heyman and Sobel, 2004).
- The function $g_t(x_t, a_t) = sa_t + i_t(x_t, a_t)$ is then also convex.

Appendix B. Conditional probability distribution

In part of the numerical studies, we consider the case where the capacity allocation C_t by the contract manufacturer is positively or negatively dependent on OEM's demand D_t. Therefore, we adapt the probability mass function of C_t to a conditional probability mass function in which C_t is conditioned on D_t: $P\{C_t = c | D_t = d\}$. This function is basically the matrix G of size $m \times n$, where $m = d_{max}$ (the maximum demand) and $n = c_{max} - c_{min} + 1$ (where c_{min} and c_{max} are the bounds of the capacity distribution).

In case of positive dependency, elements $G_{m,1} = G_{1,n}$ = 0 and $G_{1,1} = G_{m,n} = \begin{cases} \frac{2}{n} \; if \; n \geq 2 \\ 1 \; if \; n = 1 \end{cases}$.

Then, the rows and columns are filled by a *linear* decrease/increase from 0 to $G_{1,1}$ or $G_{m,n}$. Finally, the probabilities are rescaled, such that distribution sums up to 1.

$$G = \begin{bmatrix} G_{1,1} & \frac{2(n-2)}{n(n-1)} & \frac{2(n-3)}{n(n-1)} & \cdots & \frac{2}{n(n-1)} & 0 \\ & \vdots & & \ddots & \vdots & \\ & 0 & \dots \quad \dots & \cdots & G_{m,n} & \end{bmatrix}$$

In case of negative dependency, the same procedure is applied, but now $G_{1,1} = G_{m,n} = 0$ and $G_{m,1} = G_{1,n} = \begin{cases} \frac{2}{n} \; if \; n \geq 2 \\ 1 \; if \; n = 1 \end{cases}$.

$$G = \begin{bmatrix} 0 & \cdots & G_{1,n} \\ \vdots & \ddots & \vdots \\ G_{m,1} & \cdots & 0 \end{bmatrix}$$

Appendix C. Optimal policy parameters (z_t^*, y_t^*) for different values of h, b, and s.

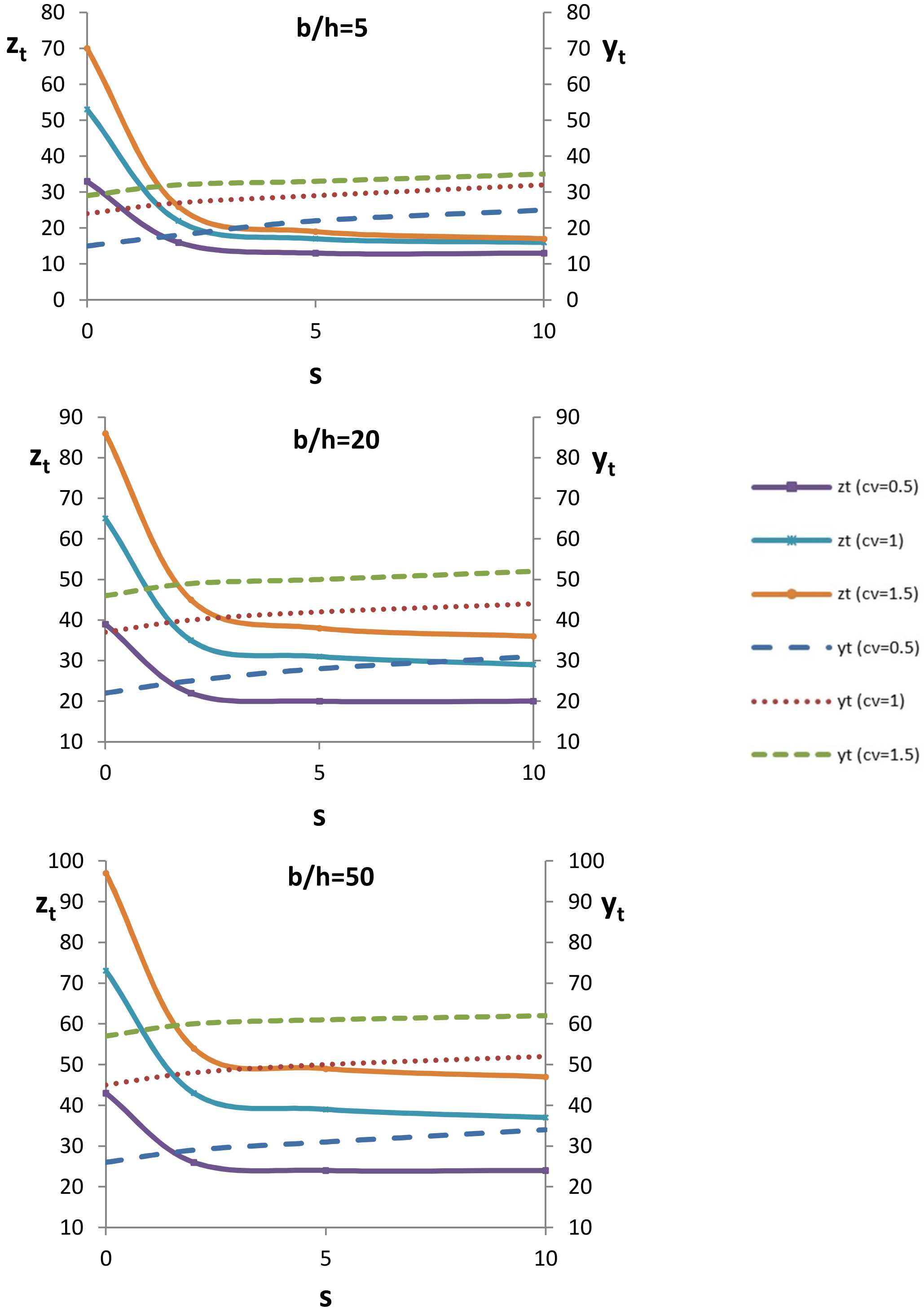

Figure 8. Order-up-to (y_t) and reservation-up-to (z_t) levels for different levels of demand uncertainty, backorder cost, and reservation cost.

References

Altug, M.S. and Muharremoglu, A. (2011) Inventory management with advance supply information, *International Journal of Production Economics,* 129, 302-313.

Atasoy, B., Gullu, R., and Tan, T. (2012) Optimal inventory policies with non-stationary supply disruptions and advance supply information, *Decision Support Systems,* 53, 2, 269-281.

Bassok, Y. and Akella, R. (1991) Ordering and production decisions with supply quality and demand uncertainty, *Management Science,* 37, 12, 1556-1574.

Bertsekas, D.P. (2005) Dynamic programming and optimal control, Athena Scientific, Belmont.

Bonser, J.S. and Wy, S.D. (2001) Procurement planning to maintain both short-term adaptiveness and long-term perspective, *Management Science,* Vol. 47, No. 6, 769-786.

Boulaksil, Y. and Fransoo, J.C. (2010) Implications of outsourcing on operations planning: findings from the pharmaceutical industry, *International Journal of Operations & Production Management,* 30, 10, 1059-1079.

Boulaksil, Y. and Fransoo, J.C. (2009) Order release strategies to control outsourced operations in a supply chain, *International Journal of Production Economics,* 119, 149-160.

Bryce, D.J. and Useem, M. (1998) The impact of corporate outsourcing on company value. *European Management Journal*, 16, 635-643.

Burgin, T.A. (1975) The Gamma distribution and inventory control, *Operational Research Quarterly,* 26, 3, 507-525.

Chase, R.B., Jacobs, F.R., and Aquilano, N.J. (2004) Operations Management for Competitive Advantage, 10th ed., Irwin/McGraw-Hill, Bosten, MA.

Ciarallo, F.W., Akella, R., and Morton, T.E. (1994) A periodic review, production planning model with uncertain capacity and uncertain demand – optimality of extended myopic policies, *Management Science,* Vol. 40, No. 3, 320-332.

Costa, D. and Silver, E.A. (1996) Exact and approximate algorithms for multi-period procurement problem where dedicated supplier capacity can be reserved, *OR Spektrum,* 18; 197-207.

Erkoc, M. and Wu, S.D. (2005) Managing high-tech capacity expansion via reservation contracts *Production and Operations Management,* 14, No. 2

Federgruen, A. and Zipkin, P. (1986) An inventory model with limited production capacity and uncertain demand, *Mathematics of Operations Research*, 11, 193-207.

Gallego, G. and Özer, Ö. (2001) Integrating replenishment decisions with advance demand information, *Management Science,* 47, 10, 1344-1360.

Gayon, J.P., Benjaafar, S., and De Véricourt, F. (2009) Using imperfect advance demand information in production-inventory systems with multiple customer classes, *Manufacturing & Service Operations Management*, 11, 1, 128-143.

Güllü, R. (1998) Base stock policies for production/inventory problems with uncertain capacity levels, *European Journal of Operational Research,* 105, 43-51.

Güllü, R., Önol, E., and Erkip, N. (1999) Analysis of an inventory system under supply uncertainty, *International Journal of Production Economics,* 59, 377-385.

Hazra, J. and Mahadevan, B. (2009) A procurement model using capacity reservation, *European Journal of Operational Research,* 193, 303-316.

Heyman, D.P. and Sobel, M.J. (2004) Stochastic models in operations research, Vol. II, Dover Publications, NY.

Hwang, J. and Singh, M.R. (1998) Optimal production policies for multi-stage systems with setup costs and uncertain capacities, *Management Science,* 44, No. 9, 1279-49.

Iida, T. (2002) A non-stationary periodic review production-inventory model with uncertain production capacity and uncertain demand, *European Journal of Operational Research,* 140, 670-683.

Jain, K. and Silver, E.A. (1995) The single period procurement problem where dedicated supplier capacity can be reserved, *Naval Research Logistics,* Vol. 42, 915-934.

Jaksic, M., Fransoo, J.C., Tan, T., De Kok, A.G., and Rusjan, B. (2011) Inventory management with advance capacity information, *Naval Research Logistics,* 58, 4, 355-369.

Jaksic, M. and Fransoo, J.C. (2015) Optimal inventory management with supply backordering, *International Journal of Production Economics,* 159, 254-264.

Karaesmen, F. Buzacott, J.A., and Dallery, Y. (2002) Integrating advance order information in make-to-stock production systems, *IIE Transactions,* 34, 649-662.

Kremic, T., Tukel, O.I., and Rom, W.O. (2006) Outsourcing decision support: a survey of benefits, risks, and decision factors, *Supply Chain Management: An International Journal,* Vol. 11, No. 6, 467-482.

Lee, H.L., Padmanabhan, V., and Whang, S. (1997) Information distortion in a supply chain: the bullwhip effect, *Management Science,* 43, 4, 546-558.

Mincsovics, G., Tan, T., and Alp, O. (2009) Integrated capacity and inventory management with capacity acquisition lead times*, European Journal of Operational Research,* 196, 949-958.

Özer, Ö. and Wei, W. (2004) Inventory control with limited capacity and advance demand information, *Operations Research,* 52, 988-1000.

Pac, M.F., Alp, O., and Tan, T. (2009) Integrated workforce capacity and inventory management under labour supply uncertainty, *International Journal of Production Research,* Vol. 47, No. 15, 4281-4304.

Parlar, M., Wang, Y., and Gerchak, Y. (1995) A periodic review inventory model with Markovian supply availability, *International Journal of Production Economics,* 42, 131-136.

Porteus, E.L. (2002) Foundations of stochastic inventory theory, Stanford University Press.

Serel, D. (2007) Capacity reservation under supply uncertainty, *Computers & Operations Research,* 34, 1192-1220.

Serel, D., Dada, M., and Moskowitz, H. (2001) Sourcing decisions with capacity reservation contracts, *European Journal of Operational Research* 131, 635-648.

Tan, T., Gullu, R., and Erkip, N. (2007) Modelling imperfect advance demand information and analysis of optimal inventory policies, *European Journal of Operational Research,* 177, 897-923.

Tan, T., Gullu, R., and Erkip, N. (2009) Using imperfect advance demand information in ordering and rationing decisions, *International Journal of Production Economics,* 121, 2, 665-677.

Van Norden, L. and Van de Velde, S. (2005) Multi-product lot-sizing with a transportation capacity reservation contract, *European Journal of Operational Research,* 165, 127-138.

Wang, Y. and Gerchak (1996) Periodic review production models with variable capacity, random yield, and uncertain demand, *Management Science,* Vol. 42, No. 1, 130-137.

Wijngaard, J. and Karaesmen, F. (2007) Advance demand information and a restricted production capacity: on the optimality of order base-stock policies, *OR Spectrum,* 29, 4, 643-660.

Zhao, X., Xie, J., and Wei, J.C. (2007) The value of early order commitment in a two-level supply chain, *European Journal of Operational Research,* 180, 194-214.