Capacity Scaling in Mobile Wireless Ad Hoc Network with Infrastructure Support

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***Abstract*—We study the throughput capacity of mobile wireless ad hoc networks with infrastructure support. Mobility and infrastructure support independently have been shown to be effective ways to improve capacity, but few work has analyzed the impact of their combination. In our work we consider an ad hoc network with n users and k base stations. All base stations are wired to each other with bandwidth $c(n)$. We adopt a general mobility model where users move with arbitrary patterns within a bounded distance around their home-points, and let the area of the network scales as $f^2(n)$. We show that for different parameters, mobility can be divided into strong, weak and trivial regimes. The per-node capacity is $\Theta(1/f(n)) + \Theta(\min(k^2c/n, k/n))$ under strong mobility , and is $\Theta(\min(k^2c/n, k/n))$ in the two latter cases. We also discuss optimal communication schemes and system parameters in each regime. Our study provides fundamental insight on the understanding and design of wireless ad hoc network.**

***Keywords*-Ad hoc wireless networks; hybrid wireless network; mobility; capacity; scaling law;**

I. Introduction

Wireless ad hoc networks are modeled as a set of n nodes that exchanges messages using radio transmissions. A well-known challenge in these networks is the poor scalability of throughput capacity, i.e., the maximal traffic rate that can be sustained as n grows. In the ground breaking work [1] by Gupta and Kumar, it is shown that in a static network with arbitrary topologies, per-node capacity decreases as $\Theta(1/\sqrt{n})$[1] as n tends to infinity. This is the best performance achievable even allowing for optimal scheduling, routing and relaying of packets.

Various methods have been proposed thereafter to improve the pessimistic result. Among them mobility perhaps is the one that receives most attention. The capacity of mobile ad hoc wireless networks (MANETs) was first explicitly considered in [2], where a two-hop relay algorithm was developed and shown to support constant per-node throughput, if each node uniformly visits the entire network area according to an ergodic mobility process. Garetto et al. [3] further investigate the case that mobility is limited to radius $1/f(n)$ in a network with unit size and show that the corresponding per-node capacity is $\Theta(1/f(n))$.

[1]We use the standard order notations.

Infrastructure is yet another more intuitive and straightforward way to improve capacity. Often referred to as *hybrid networks*, capacity of these ad hoc networks with infrastructure support is studied in [4]–[6], which show similar result that (only with a difference of factor $\log n$, due to difference in models) capacity increases linearly with k, the number of base stations, given that k grows faster than $\sqrt{n}$. As well, Kozat and Tassiulas [7] finds that per-node capacity of $\Theta(1/\log n)$ is achievable if users per base station is bounded above. Agarwal and Kumar [8] extends this result to $\Theta(1)$. Capacity results above are revisited in recent work [9], where a L-maximum-hop resource allocation strategy is proposed, and delay is shown to be costant.

In this paper, we further systematically study the capacity scaling laws of a network featuring both mobility and infrastructure. We model every user to move around a certain home-point within a bounded distance, in a network whose area grows with n as $f^2(n)$. The specific mobility pattern can be arbitrary and is characterized by a general stationary spatial distribution. We also allow the home-points to cluster. For infrastructure, we assume there are k base stations (BSs) wired between each other with bandwidth $c(n)$.

Our major contributions are three folds. First, we find that mobility can be divided into three regimes: the strong, weak and trivial cases. We comment on the role of mobility and infrastructure in different regimes. Secondly, we determine the asymptotic per-node capacity of each regime. Capacity is $\Theta(1/f(n)) + \Theta(\min\{k^2c/n, k/n\})$ in the first regime and $\Theta(\min\{k^2c/n, k/n\})$ in the two latter cases. Thirdly, we propose optimal communication schemes for each regime, such as scheduling and routing protocol, BS bandwidth and BS placement.

Mobility and infrastructure benefit the network in different ways and jointly influence the optimal communication schemes. A main problem is how to handle their interaction and optimally exploit the two features like determining when to prefer one of them to act dominantly, and when their cooperation can better facilitate transmission. An interesting example is, we show that per-node capacity is the same in the weak mobility regime and the trivial mobility regime, but the corresponding optimal communication schemes are different.

Our work is fundamentally different from previous works on hybrid networks [4]–[10], which typically do not take mobility into account and assume a infinite bandwidth between BSs. Besides, we motivate some of the techniques in the analysis of uniformly dense networks (defined in Section III) from [3]. However, because our problem and model are different, substantial efforts are needed to develop the methodology. Moreover, for the ease of representation and a smoother logic, we prove some of the useful results in [3] such as the optimality of scheduling scheme and upper bound of capacity of BS-free networks in simpler ways, which may be of separate interest. Last but not least, our work well unify and generalize prior capacity results on MANETs and hybrid networks.

The paper is organized as follows. In Section II we introduce our assumptions and models. Section III defines the uniformly dense networks and discusses some preliminaries. Capacity analysis of uniformly dense networks is carried out in Section IV, and the non-uniformly dense cases in Section V. Last, we conclude the paper in Section VI.

II. Models and Assumptions

A. Mobility Model

We consider a network with n mobile users (also called mobile stations, MSs) and $k = \Theta(n^K)$ base stations (BSs).

Definition 1. *Network Extension* $\mathcal{O}$ is a Torus, or a square region with wrap-around conditions. Its length of side scales as n according to $f(n) = n^{\alpha}$. However, for convenience, we normalize $\mathcal{O}$ to be a unit Torus, therefore any quantity representing a constant distance independent of n should be scale down by a factor $1/f(n)$ correspondingly.

We note that the normalization of $\mathcal{O}$ is only a technical assumption commonly adopted in previous works [1], [2]. Also, ignoring edge effect is common for avoiding tedious technicalities [3], [11]. Such assumptions will not change the main results of this paper.

Remark 1. In this paper we focus on the case that $\alpha \in [0, 1/2]$. $\alpha = 0$ corresponds to the *dense networks* [1] where network size is constant and $\alpha = 1/2$ the *extended networks* [12] where average node density is constant.

We denote the location of the ith MS at time t as $X_i(t)$, and the position of the jth static BS as Y_j. When MSs and BSs don't need to be distinguished, notation $Z_i(t), 1 \leq i \leq n + k$ is used. Operator $\|\cdot\|$ denotes the distance between two points. $d_{ij} = \|Z_i - Z_j\|$.

Definition 2. *Mobility around Home-points:* We assume $\{X_i(t)\}$ to be independent stationary and ergodic processes with stationary distribution $\phi_i(X)$:

$$\phi_i(X) = \phi(X - X_i^h) = \frac{s(f(n)\|X - X_i^h\|)}{\int_{\mathcal{O}} s(f(n)\|X - X_i^h\|)\mathrm{d}X} \quad (1)$$

where X_i^h is called *home-point* of the ith node and $s(d)$ is an arbitrary, non-decreasing function with finite support.

Remark 2. Home-point X_i^h is the place visited most often by mobile station i. We let the home-points of BSs to be their static position. $s(d)$ characterizes the actual stationary distribution of a node's presence around its home-point before size normalization. With a little integration we can simplify (1) as:

$$\phi_i(X) \sim f^2(n)s(f(n)\|X - X_i^h\|) \quad (2)$$

Definition 3. *Clustered model* characterizes the distribution of home-points. There are $m(n) = \Theta(n^M)$ clusters with radius $r(n) = \Theta(n^{-R})$, independently and uniformly distributed in $\mathcal{O}$. Each of the n home-points is randomly assigned to a cluster and then uniformly and independently placed inside it.

Remark 3. A smaller m represents a more severe degree of clustering and *vice versa*. Particularly if $m = n$, no clusters are formed, i.e., all home-points are uniformly distributed.

Remark 4. We motivate our mobility model from [3], [13]. Home-points and $s(d)$ are introduced to characterize the restrictive and non-uniform nature which are often observed in real mobility traces [14]. Clustered model on the other hand can capture the preferential attachment phenomena in the formation of real networks [15], [16]. Besides, many classical mobility models like i.i.d. mobility [12], hybrid random walk [11] and Brownian motion [17] preserve the uniform distribution of nodes at all time, thereby are special cases of our model when $m = \Theta(n)$ and $f(n) = \Theta(1)$.

To achieve better utilization, the distribution of base stations should match the distribution of users. With this aim, We let $\{Y_j\}$ be independent random variables. For a particular BS j, we randomly choose a point Q_j according to the clustered model, and let Y_j follow distribution $\phi(Y - Q_j)$. However, under certain condition we can relax this requirement and employ a simpler base station placement scheme, as will be shown in Section IV.

This work focuses on the case that $M - 2R < 0$ so that clusters will not overlap[2] w.h.p.[3]. Clusters should not shrink as n grows so $0 \leq R \leq \alpha$. We assume that every cluster should be equipped with BSs w.h.p., i.e., $k = \omega(m)$.

B. Communication Model

The BSs are wired to each other with bandwidth $c(n)$bps and communication between them won't cause interference. We also assume every node has a constant wireless bandwidth of W bps in a common channel. Since it won't affect our result, we normalize W to 1. We base our analysis on the following interference model which governs direct radio transmissions between nodes.

[2]The case that clusters tend to overlap is similar to the cluster-free case.
[3]with high probability, i.e., probability tends to 1 as n tends to infinity

Definition 4. *Protocol Model [1]* : all nodes use a common transmission range R_T for all their wireless communication. A wireless transmission from node i to j is successful only if : 1) $\|Z_i(t)-Z_j(t)\| \leq R_T$; and 2) For every other node l that is simultaneously transmitting, follows, $\|Z_l(t) - Z_j(t)\| \geq (1 + \Delta)R_T$, where constant Δ defines the area of guard zone.

Definition 5. *Feasible Throughput*: A throughput $g(n)$ is said to be feasible if there is a spatial and temporal scheme for scheduling transmissions, such that by operating the network in a multi-hop fashion, buffering at intermediate nodes when awaiting transmission and possibly utilizing the BSs, every node can send $g(n)$bps to its destination, i.e., there is a $T < \infty$ such that in every time interval $[(i-1)T, iT]$, every node can send $Tg(n)$ bits to its destination.

Definition 6. *Asymptotic per-node capacity* $\lambda(n)$ of the network is said to be $\Theta(g(n))$ if there exist two positive constants c and c' such that:

$$\begin{cases} \lim_{n\to\infty} \Pr\{\lambda(n) = cg(n) \text{ is feasible}\} = 1 \\ \lim_{n\to\infty} \Pr\{\lambda(n) = c'g(n) \text{ is feasible}\} < 1 \end{cases}$$

Similar to previous works we use the uniform permutation model as traffic model. n randomly selected source-destination pairs (s, d) exchange data at rate λ. It's noted that pair selection should ensure that every MS is both source and destination. BSs only act as relays and are not involved. We express the traffic by a $n \times n$ matrix in the form $\lambda\Lambda$, the elements of matrix $\Lambda = [\lambda_{sd}]$ satisfy $\lambda_{sd} \in \{0, 1\}$.

III. Preliminary in Uniformly Dense Networks

A. Uniformly Dense Networks

Definition 7. *Local density* of nodes at X is defined as:

$$\rho(X) = \sum_{i=1}^{n+k} E[\mathbf{1}_{Z_i \in B(X, 1/\sqrt{n})} | \mathcal{F}_{n+k}]$$

where $B(X, 1/\sqrt{n})$ is the disk centered at X with radius $1/\sqrt{n}$, $\mathcal{F}_{n+k}$ is the Borel-field generated by $\{Z_i^h\}_{i=1}^{n+k}$, E is expectation, and $\mathbf{1}_X$ is the indicator function.

Definition 8. A network is said to be *uniformly dense* if for any $X \in \mathcal{O}$, there exist two positive constants h and H, such that

$$h < \rho^n(X) < H \qquad w.h.p. \tag{3}$$

Theorem 1. *If $f(n)\sqrt{\gamma(n)} = o(1)$, where $\gamma(n) = \frac{\log m}{m}$, and $k = O(n)$, then the network is uniformly dense.*

Because of limited space, the proof of this theorem is deferred to the full technical report [18].

Remark 5. Less formally, recall from the connectivity criterium [19] that $\sqrt{\gamma(n)}$ is the critical transmission range for connectivity if all nodes were static. Besides, mobility of MSs is roughly limited to radius of $\Theta(1/f(n))$. Thereby the condition of Theorem 1 implies that mobility is strong

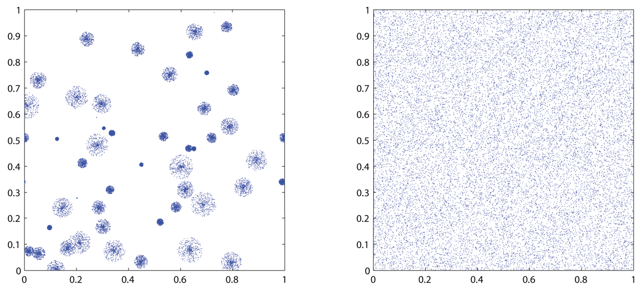

Figure 1: An example for non-uniformly dense network (left) and uniformly dense model (right)

enough to exceed the critical range (in order sense). In such cases, we may expect mobility to help connecting nodes and overcome the clustering nature of home-points. Otherwise, the significance of mobility is weak and clustering may be an obstacle to connectivity. To provide some intuition, we show an example in Fig.1 .We will first focus on the uniformly dense case and lay some foundation for later analysis in the non-uniformly dense case.

The next lemma is a useful result regarding the statistics of home-points. It's not limited to uniformly dense network, but we place the lemma here for it shares a similar implication of "uniform". The proof is available in technical report [18].

Lemma 1. *Suppose that $\{Z_i^h\}_{i=1}^{n+k}$ are placed on $\mathcal{O}$ according to clustered model. Given a regular tessellations $\mathcal{A}$ of $\mathcal{O}$ (or its sub-region), with cell area $|A| \geq (16 + \beta)\gamma(n)$, for some $\beta > 0$, and defined with $N_m(A_l)$ and $N_b(A_l)$ the number of home-points of MSs and BSs inside cell A_l, then uniformly over $\mathcal{A}$, it holds w.h.p. :*

$$\frac{1}{4}n|A_l| < N_m(A_l) < 4n|A_l| \tag{4}$$

$$\frac{1}{4}k|A_l| < N_b(A_l) < 4k|A_l| \tag{5}$$

B. Link Capacity

Definition 9. *Link capacity* between node i and j is the maximal long term data flow between them:

$$\mu^S(i, j) = E[\mathbf{1}_{(i,j)\in\pi^S(t)} | \mathcal{F}_{n+k}]$$

where $\pi^S(t)$ is a set of non-interfering node pairs selected to transmit under a stationary ergodic scheduling policy S.

Definition 10. *Scheduling policy* S^* enables wireless transmission between node i and j when the following conditions are satisfied:

$$d_{ij}(t) < R_T = \frac{c_T}{\sqrt{n}}$$

$$\min(d_{lj}(t), d_{li}(t)) > (1 + \Delta)R_T$$

for every other node l in the network (regardless of node l activity), where c_T is a constant. Moreover, the transmission bandwidth is equally shared in two directions.

We borrow the next lemma from [3] to facilitate the computation of link capacity. It develops the relationship between mobility and link capacity under S^*.

Lemma 2. *In uniformly dense network, under policy S^*, for any pair of nodes (i, j) (at least one of them is MS), and any finite constant $c_T > 0$, the link capacity is:*

$$\mu^{S^*}(i,j) = \mu(Z_i^h, Z_j^h) = \Theta\left(\Pr\left\{d_{ij} \le \frac{c_T}{\sqrt{n}} | \mathcal{F}_{ij}\right\}\right)$$

where $\mathcal{F}_{ij}$ is the Borel-field generated by Z_i^h and Z_j^h.

Corollary 1. *Given the position of home-points of MS i, j and BS k, the link capacity between them is*

$$\mu(X_i^h, X_j^h) = \Theta(f^2(n)\eta(f(n)\|X_i^h - X_j^h\|)/n) \quad (6)$$

$$\mu(X_i^h, Y_l^h) = \Theta(f^2(n)s(f(n)\|Y_l^h - X_i^h\|)/n) \quad (7)$$

where $\eta(\|X_0\|) = \int_{X\in \mathrm{R}} s(\|X - X_0\|)s(\|X\|)\mathrm{d}X$

Proof: The proof of (6) is reported in [3], we now focus on computing $\mu(X_i^h, Y_l^h)$:

$$\begin{aligned}\mu(X_i^h, Y_l^h) &\sim f^2(n)\int_{Y_l\in\mathcal{O}}\int_{X_i\in B(Y_l,R_T)} s(f(n)\|X_i - X_i^h\|)\delta(Y_l - Y_l^h)\mathrm{d}X_i\mathrm{d}Y_l \\ &= f^2(n)\int_{X_i\in B(Y_l^h,R_T)} s(f(n)\|X_i - X_i^h\|)\mathrm{d}X_i \\ &\sim f^2(n)|B(Y_l^h, R_T)|s(f(n)\|Y_l^h - X_i^h\|) \\ &= \frac{\pi c_T^2 f^2(n)}{n} s(f(n)\|Y_l^h - X_i^h\|) \quad (8)\end{aligned}$$

where $\delta(X)$ is the Dirac impulse. ∎

Lemma 3. *In uniformly dense network, under policy S^*, there exists a positive constant p, such that the probability that an arbitrary node i is scheduled to transmit (as source or relay) at an arbitrary time instant is greater than by p.*

Proof: Given an arbitrary node i, to prove the lemma is equivalent to establish the equation:

$$\sum_{j=1,j\neq i}^{n+k} \mu(Z_i^h, Z_j^h) = \Theta(1)$$

It's trivial that $\sum_{j=1,j\neq i}^{n+k} \mu(Z_i^h, Z_j^h) < 1$. On the other hand, according to Lemma 2, it holds:

$$\begin{aligned}\sum_{j=1,j\neq i}^{n+k} \mu(Z_i^h, Z_j^h) &= \sum_{j=1,j\neq i}^{n+k} \Pr\left\{d_{ij} \le \frac{c_T}{\sqrt{n}} | \mathcal{F}_{ij}\right\} \\ &\ge \Pr\left\{\bigcup_j d_{ij} \le \frac{c_T}{\sqrt{n}} | \mathcal{F}_{n+k}\right\} \\ &\ge (1 - \frac{\pi c_T^2}{n})^{n+k} \to e^{-\pi c_T}\end{aligned}$$

Combining the two sides we have the assertion. ∎

Theorem 2. *In uniformly dense networks the scheduling policy S^* is optimal (in order sense).*

Proof: It's proved in [20] that position-based scheduling policy suffice to achieve capacity. Thereby our goal is to show S^* is optimal among this class of scheduling schemes.

First note that S^* only permit transmission when all other nodes are away from a guard zone, whether they are active or not. This is a little stricter than protocol model. However, we argue it will not reduce capacity in order sense. The reasons are: a) it's obvious that further allowing transmission when the number of irrelevant nodes within guard zone is bounded by a constant will not affect the order of link capacity, and b) if there are $\omega(1)$ irrelevant nodes $\{v\}$ inside guard zone and we still wish to transmit, then this decision will suppress all nodes in $\{v\}$ from simultaneously transmitting. This will not be beneficial unless all nodes in $\{v\}$ are not scheduled to transmit. However, Lemma 3 guarantees even under S^*, such event will not happen w.h.p..

Next we should show the transmission range R_T is optimal. It's an immediately observation from Lemma 2 that $R_T = o(\frac{1}{\sqrt{n}})$ is sub-optimal for it results in a smaller link capacity. We now focus on the $R_T = \omega(\frac{1}{\sqrt{n}})$ case.

Define with $\bar{S}$ a similar policy to S^* only with a different transmission range $R_T = \omega(\frac{1}{\sqrt{n}})$. The following holds:

$$\mu^{\bar{S}}(Z_i^h, Z_j^h) = \Pr\left\{d_{ij} \le R_T, \bigcap_l Z_l \in A_{\Delta(Z_i,Z_j)} | \mathcal{F}_{n+k}\right\}$$

where $A_{\Delta(Z_i,Z_j)}$ is the set $\{X \in \mathcal{O} : \min(\|X - X_j\|, \|X - X_i\|) > (1+\Delta)R_T\}$. Define for short $P_l = E[\mathbf{1}_{x_l\notin A_{\Delta(Z_i,Z_j)}} | \mathcal{F}_{n+k}]$, then:

$$P_l > \int_{X_l\in B(X_i,(1+\Delta)R_T)} \phi(X_l - X_l^h)\mathrm{d}X_l$$

And according to (3), it follows:

$$\sum_{l\neq i,j} P_l > h(1+\Delta)^2 n R_T^2$$

Note that for $x > 0$, it holds $\log(1-x) < -x$, therefore:

$$\sum_{l\neq i,j} \log(1 - P_l) < -\sum_{l\neq i,j} P_l < -h(1+\Delta)^2 n R_T^2$$

So equivalently,

$$\begin{aligned}\Pr\left\{\bigcap Z_l \in A_{\Delta(Z_i,Z_j)} | \mathcal{F}_{n+k}\right\} &= \prod_{l\neq i,j}\int_{X_l\in A_{\Delta(Z_i,Z_j)}} \phi(X_l - X_l^h)\mathrm{d}X_l \\ &= \prod_{l\neq i,j} E[\mathbf{1}_{x_l\in A_{\Delta(Z_i,Z_j)}} | \mathcal{F}_{n+k}] \\ &\le e^{-h(1+\Delta)^2 n R_T^2}\end{aligned}$$

Other other hand, similarly to Corollary 1, it can be shown that:

$$\Pr\{\|X_i - X_j\| < R_T | \mathcal{F}_{ij}\} \sim f^2(n) R_T^2 \eta(f(n)d_{ij})$$

$$\Pr\{\|X_i - Y_j\| < R_T | \mathcal{F}_{ij}\} \sim f^2(n) R_T^2 s(f(n)d_{ij})$$

Due to the independence of node mobility, it follows

$$\mu^{\bar{S}}(X_i^h, X_j^h) = O\left(e^{-nR_T^2} f^2(n) R_T^2 \eta(f(n)d_{ij})\right)$$
$$\mu^{\bar{S}}(X_i^h, Y_j^h) = O\left(e^{-nR_T^2} f^2(n) R_T^2 s(f(n)d_{ij})\right)$$

Comparing with (6),(7), it's obvious that $\mu^{\bar{S}} = O(\mu^{S^*})$. This finishes the proof. ■

Remark 6. $\Theta(\frac{1}{\sqrt{n}})$ is the critical distance that a node can find a neighbor with positive constant probability. A smaller range will interrupt connectivity between nearest neighbors and a larger range will result in too much interference.

Remark 7. Since S^* is optimal, link capacity μ^{S^*} (also denoted as μ for short) can be later used to develop upper bounds on network capacity.

C. Capacity of Uniformly Dense Network without Infrastructure

Lemma 4. *In uniformly dense network with n MSs but no BS, the per-node capacity is upper bounded as*

$$\lambda(n) = O(\frac{1}{f(n)})$$

Informal Proof:: Due to spatial interference, capacity is limited to $\lambda = O(\frac{1}{\sqrt{n}})$ if for a generic session (a series of transmissions from source to relays and finally to destination), wireless transmission is used to cover a distance of $\Theta(1)$. [11] However, there is a constant proportion of source-destination pairs who are $\Theta(1)$ away from each other. Therefore to achieve a better capacity than the static case ($\lambda = \Theta(1/\sqrt{n})$), we have to exploit mobility to cover at least a distance of $\Theta(1)$. However, by denoting constant $D = \sup\{d : s(d) > 0\}$, a single node's movement is limited to radius $D/f(n)$. Therefore we will need at least $\Theta(f(n))$ hops to finish the delivery. By interpreting $\Theta(f(n))$ as redundancy or using a hop-count technique such as that in [21], it's easy to show that capacity is upper bounded by $\Theta(\frac{1}{f(n)})$. ■

Remark 8. The above proof is easy to be made rigorous. A different complete proof of the upper bound is also available in [3]. But our approach is simpler.

Definition 11. *Optimal Routing Scheme A [3]:* The network area $\mathcal{O}$ is partitioned into square tessellation $\mathcal{A}$ with element area $\Theta(1/f^2(n))$. Squarelets $A_{i,j}$ are indexed by (i, j) where i is row index and j is column index. Traffic from A_{i_s,j_s} to A_{i_d,j_d} is first forwarded horizontally to A_{i_s,j_d} along contiguous squarelets and then vertically in the same manner to destination. By saying forwarding to contiguous squarelets, we mean a random node whose home-point is in the adjacent squarelet is chosen as relay.

Lemma 5. [3] *Employing the optimal routing scheme A described above, a per-node throughput $\Theta(\frac{1}{f(n)})$ can be achieved.*

The following theorem is the consequence of Lemma 4 and Lemma 5.

Theorem 3. *In uniformly dense network with n MSs but no BSs, the per-node capacity is $\Theta(\frac{1}{f(n)})$.*

IV. Capacity Analysis in Uniformly Dense Network

A. Upper Bound on Per-node Capacity

Lemma 6. *Given an arbitrary simple, closed ,convex curve $\mathcal{L}$ which divides $\mathcal{O}$ into $I_\mathcal{L}$ and $E_\mathcal{L}$, per-node capacity is upper bounded by:*

$$\lambda \leq \frac{\sum_{i:Z_i^h \in I_\mathcal{L}} \sum_{j:Z_j^h \in E_\mathcal{L}} \mu_{ij}}{\sum_{s:X_s^h \in I_\mathcal{L}} \sum_{d:X_d^h \in E_\mathcal{L}} \lambda_{sd}}$$

Proof: Given the optimality of policy S^* and the definition of μ^S, the inequity

$$\lambda \sum_{s\in I} \sum_{d\in I^c} \lambda_{sd} \leq \sum_{i\in I} \sum_{j\in I^c} \mu_{ij}$$

is evident from the viewpoint of graph cut, for any partition (I, I^c) of the nodes. ■

Lemma 7. *In uniformly dense network with n MSs and k BSs, an upper bound of per-node capacity is*

$$\lambda(n) \leq O(\frac{1}{f(n)}) + O(\frac{k^2 c(n)}{n})$$

Proof: Basically, Lemma 6 can be exploited to develop an upper bound on capacity. However, note that in Lemma 4 an upper bound for network without BSs is already established, therefore we can simplify our analysis to the impact of BSs. That is, for an arbitrary $\mathcal{L}$ with constant length, holds:

$$\lambda(n) \leq O(\frac{1}{f(n)}) + \frac{\sum_{i:Y_i^h \in I_\mathcal{L}} \sum_{j:Y_j^h \in E_\mathcal{L}} \mu(Y_i^h, Y_j^h)}{\sum_{s:X_s^h \in I_\mathcal{L}} \sum_{d:X_d^h \in E_\mathcal{L}} \lambda_{sd}} \quad (9)$$

where by definition, $\mu(Y_i^h, Y_j^h) = c(n)$ is the bandwidth between BSs. Note that we have ignore the term $\sum_i \sum_j \mu(Y_i^h, X_j^h)$ in the numerator, for it's not comparable to $\sum_i \sum_j \mu(X_i^h, X_j^h)$, which is singled out in $O(\frac{1}{f(n)})$.

Before handling (9), we first introduce some notations. Define $\underline{\mathcal{A}}$ as the inner tessellation of $I_\mathcal{L}$, whose elements are squarelets strictly within $I_\mathcal{L}$, and $\bar{\mathcal{A}}$ the outer tessellation which are the union of $\underline{\mathcal{A}}$ and the set of boundary squarelets crossed by $\mathcal{L}$. Define $\underline{\mathcal{B}}, \bar{\mathcal{B}}$ for $E_\mathcal{L}$ similarly.

Define for short $\mu_\mathcal{L}^B = \sum_{i:Y_i^h \in I_\mathcal{L}} \sum_{j:Y_j^h \in E_\mathcal{L}} \mu(Y_i, Y_j)$, which is a random variable over $\mathcal{F}_{n+k}$, then it is upper bounded by:

$$\sum_{A_i \in \bar{\mathcal{A}}} \sum_{B_j \in \bar{\mathcal{B}}} c(n) N_b(A_i) N_b(B_j)$$

and lower bounded by:

$$\sum_{A_l \in \underline{\mathcal{A}}} \sum_{B_h \in \underline{\mathcal{B}}} c(n) N_b(A_l) N_b(B_h)$$

To apply Lemma 1 for a bound on $N_b(\cdot)$, we select the tessellations $\bar{\mathcal{A}}, \underline{\mathcal{A}}, \bar{\mathcal{B}}, \underline{\mathcal{B}}$ in such a way that their elements have area $(16+\beta)\gamma(n)$. Then the following holds with high probability:

$$\mu_{\mathcal{L}}^B \leq 16k^2(16+\beta)^2 \sum_{A_i \in \bar{\mathcal{A}}} \sum_{B_j \in \bar{\mathcal{B}}} c(n)\gamma^2(n) \tag{10}$$

$$\mu_{\mathcal{L}}^B \geq \frac{1}{16}k^2(16+\beta)^2 \sum_{A_i \in \underline{\mathcal{A}}} \sum_{B_j \in \underline{\mathcal{B}}} c(n)\gamma^2(n) \tag{11}$$

Because $\gamma^2(n) = o(1)$ is proportional to squarelet size, (10) and (11) can be interpreted as upper and lower Riemann sum with mesh size vanishing to 0 as $n \to \infty$:

$$\mu_{\mathcal{L}}^B \sim k^2 c(n) \int_{X \in I_{\mathcal{L}}} \int_{Y \in E_{\mathcal{L}}} \mathrm{d}X \mathrm{d}Y \sim k^2 c(n)$$

Since the denominator in (9) actually counts the number of source-destination pairs crossing boundary $\mathcal{L}$ and equals $\Theta(n)$ w.h.p, it's now evident that:

$$\lambda(n) \leq O\left(\frac{1}{f(n)}\right) + O\left(\frac{k^2 c(n)}{n}\right) \qquad \blacksquare$$

The next lemma provides another upper bound on the influence of BSs.

Lemma 8. *In uniformly dense network with n MSs and k BSs, an upper bound of per-node capacity is*

$$\lambda(n) \leq O(\frac{1}{f(n)}) + O(\frac{k}{n})$$

Proof: Due to protocol model, a BS can at most exchange $\Theta(1)$ traffic to MSs in unit time. Therefore the traffic rate between MSs and global infrastructure (k BSs in aggregation) is limited to $\Theta(k)$. However, such resource is shared by n MSs, and its impact on per-node capacity cannot exceed $\Theta(\frac{k}{n})$. As a result, the per-node capacity is bounded by $O(\frac{1}{f(n)}) + O(\frac{k}{n})$. $\blacksquare$

Remark 9. Lemma 7 mainly considers the global infrastructure capacity while Lemma 8 is an obvious result of limited accessibility of resource due to competition of infrastructure among MSs under interference model. Combining the two we have the following:

Theorem 4. *An upper bound for per-node capacity in uniformly dense network with n MSs and k BSs is*

$$\lambda(n) \leq O\left(\frac{1}{f(n)}\right) + O\left(\min\left\{\frac{k^2 c(n)}{n}, \frac{k}{n}\right\}\right)$$

B. Lower Bound on Per-node Capacity

Lemma 9. *A traffic rate of $\Theta(\frac{k}{n})$ can be sustained between an arbitrary MS and global infrastructure.*

To prove the lemma we need the following proposition, whose proof is reported in [18].

Proposition 1. *If $s(x)$ is a non-increasing function with finite support and $X \in \mathcal{O}$, then*

$$\int_{Y \in \mathcal{O}} s(f(n)\|Y - X\|)\mathrm{d}Y \sim \frac{1}{f^2(n)}$$

Proof (Lemma 9): Given a generic MS i, define for short μ_i^A as the maximal throughput at which it can communicate with global infrastructure, then:

$$\mu_i^A = \sum_{j=1}^{k} \mu(X_i^h, Y_j^h)$$

which is again a random variable defined over $\mathcal{F}_{n+k}$.

To determine μ_i^A, we define $\mathcal{A}$ as tessellation with element area of $(16+\beta)\gamma(n)$ over $\mathcal{O}$. For $A_l \in \mathcal{A}$, define $\bar{d}_{A_l}$ and $\underline{d}_{A_l}$ as the maximal and minimal distance from X_i^h to A_l, respectively. Recall from Corollary 1 that μ is a function of distance, we have the following bound:

$$\inf_{Y_j \in A_h} \{\mu(X_i^h, Y_j^h)\} \geq \Theta(\frac{f^2(n)}{n} s(f(n)\bar{d}_{A_h}))$$

$$\sup_{Y_j \in A_l} \{\mu(X_i^h, Y_j^h)\} \leq \Theta(\frac{f^2(n)}{n} s(f(n)\underline{d}_{A_l}))$$

Therefore,

$$\sum_{A_h \in \mathcal{A}} \frac{f^2(n)}{n} s(f(n)\bar{d}_{A_h}) N_b(A_h) \leq \mu_i^A$$

$$\leq \sum_{A_l \in \mathcal{A}} \frac{f^2(n)}{n} s(f(n)\underline{d}_{A_l}) N_b(A_l)$$

Applying Lemma 1, it holds w.h.p.:

$$\frac{(16+\beta)kf^2(n)}{4n} \sum_{A_h \in \mathcal{A}} s(f(n)\bar{d}_{A_h})\gamma(n) \leq \mu_i^A$$

$$\leq \frac{4(16+\beta)kf^2(n)}{n} \sum_{A_l \in \mathcal{A}} s(f(n)\underline{d}_{A_l})\gamma(n)$$

Because a) $\bar{d}_{A_h} - \underline{d}_{A_h} < \sqrt{(16+\beta)\gamma(n)}$, $f(n)\sqrt{\gamma(n)} = o(1)$; and b) $\gamma(n) = o(1)$ is proportional to squarelet size, the summation above can be interpreted as lower and upper Riemann sums with mesh size vanishing to 0 as $n \to \infty$:

$$\mu_i^A \sim \frac{kf^2(n)}{n} \int_{Y \in \mathcal{O}} s(f(n)d)\mathrm{d}Y$$

where $d = \|X_i^h - Y\|$. Last by applying Proposition 1,

$$\mu_i^A = \Theta\left(\frac{k}{n}\right) \qquad \blacksquare$$

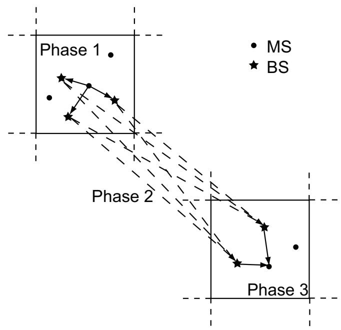

Figure 2: An example for optimal routing scheme B.

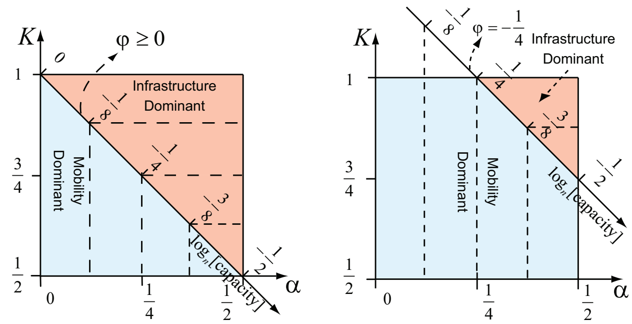

Figure 3: Graphical representation of capacity result in uniformly dense network. We plot per-node capacity as a function of $f(n) = n^{\alpha}$ and $k = \Theta(n^K)$, with $\mu^c = \Theta(n^{\varphi})$ as parameter. Left plot shows the bottleneck at accessing phase and right plot shows the case that infrastructure network itself is the limitation.

Definition 12. *Optimal Routing Scheme B:* We partition $\mathcal{O}$ into regular square tessellation $\mathcal{A}$ with constant element area A. A MS i whose home-point is in squarelet A_l will relay its traffic to all the BSs which also lie in A_l. Then BSs in the source squarelet will communicate with BSs in destination squarelet (the squarelet that contains the home-point of the destination) and exchange data for i. Last BSs in destnation squarelet will finish the delivery to destination. (See Fig. 2 for an example.)

Theorem 5. *In uniformly dense network with n MSs and k BSs, the lower bound of per-node capacity is*

$$\lambda(n) \geq \Theta\left(\frac{1}{f(n)}\right) + \Theta\left(\min\left\{\frac{k^2 c(n)}{n}, \frac{k}{n}\right\}\right)$$

Proof: Note that optimal routing scheme A can guarantee the throughput of $\Theta(\frac{1}{f(n)})$, our main goal is to show optimal routing scheme B can achieve throughput $\Theta(\min\{\frac{k^2 c(n)}{n}, \frac{k}{n}\})$.

We now divide scheme B into three phases (source-BSs, BSs-BSs, BSs-destination) and discuss the sustainable throughput of each phase. Lemma 9 shows that traffic rate $\frac{1}{4A}\mu_i^A \sim \frac{k}{n}$ is feasible in phase I & III. For phase II, the maximal traffic flowing between two squarelets via infrastructure is bounded by $n\lambda$. It can be sustained if no infrastructure edge connecting BS from the two squarelets is overloaded. This is true w.h.p. if:

$$\frac{\lambda n}{N_b(S) N_b(D)} \sim \lambda \frac{n}{k^2} \leq c(n)$$

where S and D is the source and destination squarelet, respectively. Therefore $\lambda \leq \Theta(\frac{k^2 c(n)}{n})$ is feasible in this phase. This completes the proof. ∎

In combination with Theorem 4, the per-node capacity in uniformly dense network is obtained:

Corollary 2. *The lower bound in Theorem 5 is tight.*

Remark 10. It's now evident that the network can be divided into two state. It's in *mobility dominant state* if $\lambda = \Theta(1/f(n))$ or *infrastructure dominant state* if $\lambda = \Theta(\min\{k^2 c/n, k/n\})$. Let's further look into the infrastructure dominant state. Define $\mu^c = kc(n) \sim n^{\varphi}$, then it is the aggregate bandwidth between a BS and global infrastructure. Note φ characterizes the bottleneck of capacity. If $\varphi < 1$, the bottleneck lies within the infrastructure network, if $\varphi \geq 1$, the bottleneck is the accessing phase between BSs and MSs. Therefore $\varphi = 1$ is optimal, for a larger value means a waste of resource and a smaller value will decrease capacity. Equivalently, it's optimal to keep $c(n)$ a constant. See Fig.3 for a graphical representation of the result.

The next theorem reveals a good property of uniformly dense network.

Theorem 6. *In uniformly dense network, changing the deployment model of BSs from the original clustered model to uniform model or regular placement will not affect per-node capacity of the network (in order sense).*

Proof: Obviously the upper bound in Theorem 4 still holds. We only need to focus on the performance of optimal routing scheme B. It's evident that changing the placement of BSs to uniform model or regular model will not affect the sustainable throughput of phase II. For phase I & III, one may follow the main body of the proof of Lemma 9 and finds that the only difference is a constant coefficient in bounding $N_b(A_i)$. Thus the lower bound is also the same as Theorem 5. We skip the details due to space limit. ∎

Remark 11. This theorem guarantees the optimality of simpler BS deployment schemes such as uniform placement or deterministic regular placement. It may also imply a decrease in cost and increase in feasibility to deploy BSs in complicated environment.

Remark 12. It's interesting that the mismatch of distribution between MSs and BSs is not deleterious. The fundamental reason is that in uniformly dense network, Corollary 1 will hold, ensuring $\mu(X_i^h, Y_j^h)$ independent of BS placement. However, it's easy to show this is not true for the non-uniformly dense case.

V. Capacity Analysis in Non-uniformly Dense Network

We have previously mentioned (Remark 5) that mobility is less significant in this regime. This is addressed by the following two results, which are quite intuitive if we regard clusters as nodes.

Lemma 10. [3] *If $f(n)\sqrt{\gamma(n)} = \omega(1)$, without infrastructure support, a transmission range $R_T = \Omega(\sqrt{\gamma(n)})$ is necessary to guarantee connectivity.*

Corollary 3. *Under the condition of Lemma 10, per-node capacity is*

$$\lambda = \Theta\left(\frac{1}{nR_T(n)}\right) = \Theta\left(\sqrt{\frac{m(n)}{n^2 \log m(n)}}\right)$$

Remark 13. $\lambda = (1/nR_T(n))$ is strictly smaller than the per-node capacity of uniformly dense case. The smaller m is, the larger the gap becomes. The reason is we need to adopt a larger transmission range to ensure connectivity, which will cause more interference.

Recall that a smaller m represents a more sever degree of clustering, the implication of non-uniformly dense regime is that mobility is not strong enough to overcome clustering effect, leading to inefficient use of spatial concurrency (see Fig.1 for an example). However, it's intuitive that in such cases infrastructure can play a more important role, for BSs can be used to link clusters together.

Now we introduce a simple lemma which is a direct application of Chernoff bound:

Lemma 11. *Define n_i and k_i as the number of MSs and BSs belonging to cluster i, $1 \le i \le m$. If $m = o(n)$ and $m = o(k)$, then $\forall \epsilon > 0$, uniformly over i, it holds w.h.p.:*

$$\frac{(1-\epsilon)n}{m} < n_i < \frac{(1+\epsilon)n}{m}$$
$$\frac{(1-\epsilon)k}{m} < k_i < \frac{(1+\epsilon)k}{m}$$

In the non-uniformly dense networks, MSs belonging to different clusters are separated. This motivate us to regard clusters as subnets, so that we can extend our previous techniques. Therefore, given a generic cluster i, we are interested in whether the nodes inside it are "uniformly dense" or not, according to a criterion similar to Theorem 1: recall that the radius of a cluster is $r(n) \sim n^{-R}$, we further divide mobility in non-uniformly dense regime into two cases. Define $\sqrt{\tilde{\gamma}(n)} = r\sqrt{\frac{\log(n/m)}{n/m}}$, then we say nodes are with *weak mobility* if $f(n)\sqrt{\tilde{\gamma}(n)} = o(1)$ and $f(n)\sqrt{\gamma(n)} = \omega(1)$, while *trivial mobility* corresponds to the case $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(\log \frac{n}{m})$. For consistency, the uniformly dense case is called *strong mobility*.

Remark 14. It may be a little misleading that we say a node's mobility is strong or weak. In fact the movement of a particular node is characterized only by $s(d)$, which is independent of n, and has nothing to do with the definition of strong/weak/trivial mobility. The mobility regime is an attribute of the network, but not the nodes. Specifically, it's determined by the expanding speed of network size and the clustering level of home-points.

A. Weak Mobility

We first introduce a lemma. The proof is available in [18].

Lemma 12. *With a transmission range $R_T = r(n)\sqrt{\frac{m(n)}{n}}$, nodes belonging to different clusters do not interfere with each other, with high probability.*

Theorem 7. *In a network of n MSs with weak mobility and k BSs, the per-node capacity is*

$$\lambda = \Theta\left(\min\left(\frac{k^2 c}{n}, \frac{k}{n}\right)\right)$$

Proof: We now regard a single cluster as a subnet and try to map our previous methods of analysis on it. First the key parameters $(\tilde{f}(n), \tilde{n}, \tilde{k}, \tilde{m}, \tilde{r})$ should be determined. Note that the subnet area is a disk of radius $r(n) + D/f(n)$ where $D = \sup\{d : s(d) > 0\}$ is a constant, and by assumption $f^{-1}(n) = O(r(n))$. So we renormalize the network correspondingly and $\tilde{f}(n) = n^R$. Besides, $\tilde{n} = n/m$ and $\tilde{k} = k/m$ (Lemma 11). It's obvious that $\tilde{m} = \tilde{n}$, and therefore we don't need to care about $\tilde{r}$.

Recall the definition of weak mobility, the subnet is uniformly dense. Then Theorem 2 asserts that transmission range $R_T = r\sqrt{1/\tilde{n}} = r\sqrt{m/n}$ is optimal if there is no inter-cluster interference, which is ensured by Lemma 12. Therefore by Corollary 2 the per-node capacity that can be supported within the subnet is:

$$\tilde{\lambda} = \Theta\left\{\max\left[\frac{1}{\tilde{f}(n)}, \min\left(\frac{\tilde{k}^2}{\tilde{n}}c(n), \frac{\tilde{k}}{\tilde{n}}\right)\right]\right\}$$
$$= \Theta\left\{\max\left[\frac{1}{n^R}, \min\left(\frac{k^2}{nm}c(n), \frac{k}{n}\right)\right]\right\} \quad (12)$$

Now we return our focus to the global network and discuss the inter-cluster traffic. Lemma 12 implies that link capacity between clusters are zero, together with Lemma 6, λ can be upper bounded as:

$$\lambda \le \frac{k\tilde{k}c(n)}{\tilde{n}} = \frac{k^2 c(n)}{n}$$

Combining with (12) yields:

$$\lambda = O\left(\min\left(\frac{k^2 c(n)}{n}, \frac{k}{n}\right)\right)$$

For lower bound, we slightly modify optimal routing scheme B such that a squarelet is replaced be a subnet. Traffic in phase I & III, as (12) shows, is sustainable if $\lambda = O(k/n)$. Also note that w.h.p., the number of source-destination pairs between two clusters is n^2/m, and traffic in phase II is feasible if no edge is overloaded. This happens with high probability if:

$$\frac{\lambda \frac{n}{m^2}}{\frac{k}{m}\frac{k}{m}} \leq c(n) \Rightarrow \lambda \leq \frac{k^2 c(n)}{n}$$

This finishes the proof. ■

B. Trivial Mobility

Under trivial mobility, $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(\log \frac{n}{m})$. We first show the triviality of mobility in this case, then determine capacity with an equivalent static model.

Theorem 8. *If $f(n)\sqrt{\tilde{\gamma}(n)} = \omega(\log \frac{n}{m})$, and if strong connectivity within clusters is required, then per-node capacity of the network and the corresponding scheduling scheme is the same as the case that all nodes are static. By strong connectivity, we mean MSs of a cluster should be connected without the help of infrastructure.*

Proof: The connectivity criterion [19] shows that $R_T \geq r\sqrt{\log(\tilde{n})/\pi\tilde{n}}$ is necessary[4], where $\tilde{n} = n/m$.

Now we take a snapshot of the network at an arbitrary time instant t_0, and observe a successful wireless transmission between nodes i and j. According to the protocol model, it holds: i)$\|Z_i(t_0) - Z_j(t_0)\| \leq R_T$; and ii) $\forall l \in \mathcal{T}, l \neq i, j, \|Z_l(t_0) - Z_j(t_0)\| \geq (1+\Delta)R_T$, where $\mathcal{T}$ is the set of active nodes. Our goal is to show because these two conditions hold at t_0, they hold for all t w.h.p..

Note that by mobility nodes can at most cover a disk of $D/f(n)$. So two nodes can move $4D/f(n)$ closer or farther. Therefore, condition i) will hold for all t if the two nodes are at most $R_T - 4D/f(n)$ away at t_0. This is indeed true w.h.p.:

$$\begin{aligned}
&\Pr\{\|Z_i(t) - Z_j(t)\| \leq R_T | \mathcal{T}_{ij}\} \\
&\quad \geq \Pr\left\{\|Z_i(t_0) - Z_j(t_0)\| \leq R_T - \frac{4D}{f(n)} | \mathcal{T}_{ij}\right\} \\
&\quad = \frac{\pi(\sqrt{\tilde{\gamma}(n)} - \frac{4D}{f(n)})^2}{\pi\tilde{\gamma}(n)} \rightarrow \frac{\pi\tilde{\gamma}(n)}{\pi\tilde{\gamma}(n)} = 1
\end{aligned}$$

where $\mathcal{T}_{ij}$ denotes the event of $\|Z_i(t_0) - Z_j(t_0)\| \leq R_T$.

For condition ii), it is guaranteed for all t if any other active nodes are at least $(1+\Delta)R_T + 4D/f(n)$ away from node j at t_0. This is also true w.h.p.:

$$\begin{aligned}
&\Pr\{\cap_{l\in\mathcal{T}}\|Z_l(t) - Z_j(t)\| \geq (1+\Delta)R_T | \mathcal{E}_j\} \\
&\geq \Pr\left\{\cap_{l\in\mathcal{T}}\|Z_l(t_0) - Z_j(t_0)\| \geq (1+\Delta)R_T + \frac{4D}{f(n)} | \mathcal{E}_j\right\} \\
&\geq \left(\frac{\pi r^2(n) - \pi\left((1+\Delta)\sqrt{\tilde{\gamma}(n)} + \frac{4D}{f(n)}\right)^2}{\pi r^2(n) - \pi\left((1+\Delta)\sqrt{\tilde{\gamma}(n)}\right)^2}\right)^{\frac{n}{m}} \\
&\sim \left(1 - \frac{8D(1+\Delta)\frac{\sqrt{\tilde{\gamma}(n)}}{f(n)}}{r^2(n)}\right)^{\frac{n}{m}} = \left(1 - \frac{8D(1+\Delta)m}{nF(n)}\right)^{\frac{n}{m}} \\
&\rightarrow e^{-\frac{8D(1+\Delta)}{F(n)}} \rightarrow 1
\end{aligned}$$

where $\mathcal{E}_j$ is the event that $\forall l \in \mathcal{T}, l \neq i, j, \|Z_l(t_0) - Z_j(t_0) \leq (1+\Delta)R_T\|$ and $F(n) = f(n)\sqrt{\frac{\tilde{\gamma}(n)}{\log\frac{n}{m}}}$. Note that by the definition of trivial mobility, $F(n) = \omega(1)$.

Then whether a wireless transmission is successful is independent of t, i.e., mobility, and the theorem holds. ■

Definition 13. *Optimal Routing & Scheduling Scheme C:* In every cluster we regularly place the BSs to divide the subnet area into hexagon[5] tessellation. Each hexagon is called a cell with a BS at the center. We arrange cells into non-interfering groups and schedule groups to be active sequentially. When a cell is set active, MSs within the cell access the BS in a TDMA manner, employing a transmission range same as the side length of the cell. Besides, the bandwidth is divided into two symmetric channels for uplink(MS-BS) and downlink(BS-MS) respectively.

Theorem 9. *The per-node capacity of network of n MSs with trivial mobility and k BSs is*

$$\lambda = \Theta\left(\min\left(\frac{k^2 c(n)}{n}, \frac{k}{n}\right)\right)$$

and is achieved by optimal routing & scheduling scheme C.

Proof: According to Theorem 8, we can regard all nodes to be static. It is clear that the only challenge is to validate traffic rate $\Theta(k/n)$ between BSs and a MS is still feasible. Under Scheme C, this is to ensure the existence of a grouping scheme that each cell can be scheduled to be active for a constant fraction of time. However, this is a simple consequence of the well-known fact about vertex coloring of graphs of bounded degree [1]. ■

VI. Conclusion

This paper introduce mobility and infrastructure, two effective ways to increase throughput capacity, to wireless ad hoc network and study the impact of their combination. Our methods feature the concept of link capacity, which is

[4]Even if we relax the strong connectivity requirement, [7] shows that $R_T > c_4 r/\sqrt{\pi\tilde{n}}$ is necessary for any finite c_4.

[5]The shape of a cell is in fact not important. We choose hexagons to resemble realistic cases.

Table I: Capacity and Optimal Transmission Range in Different Regime

Network Regime	Condition[1]	R_T[2]	Per-node Capacity
Strong Mobility without BSs	$f(n)\sqrt{\gamma(n)} = o(1)$	$\frac{1}{\sqrt{n}}$	$\Theta\left(\frac{1}{f(n)}\right)$
Strong Mobility with BSs	$f(n)\sqrt{\gamma(n)} = o(1)$	$\frac{1}{\sqrt{n}}$	$\Theta\left(\frac{1}{f(n)}\right) + \Theta\left(\min\left(\frac{k^2c(n)}{n}, \frac{k}{n}\right)\right)$
Weak/Trivial Mobility without BSs	$f(n)\sqrt{\gamma(n)} = \omega(1)$	$\sqrt{\frac{\log m}{m}}$	$\Theta\left(\sqrt{\frac{m}{n^2\log m}}\right)$
Weak Mobility with BSs	$f(n)\sqrt{\gamma(n)} = \omega(1)$ and $f(n)\sqrt{\tilde{\gamma}(n)} = o(1)$	$r\sqrt{\frac{m}{n}}$	$\Theta\left(\min\left(\frac{k^2c(n)}{n}, \frac{k}{n}\right)\right)$
Trivial Mobility with BSs	$f(n)\sqrt{\tilde{\gamma}(n)} = \omega(\log\frac{n}{m})$	$r\sqrt{\frac{m}{k}}$	$\Theta\left(\min\left(\frac{k^2c(n)}{n}, \frac{k}{n}\right)\right)$

[1] $\gamma(n) = \frac{\log m}{m}, \tilde{\gamma}(n) = r^2\frac{\log(n/m)}{n/m}$ [2] R_T is the optimal transmission range.

convenient and efficient in forming a unified view of the effect of mobility and infrastructure.

Mobility are classified into strong, weak and trivial cases. We determine the per-node capacity as well as optimal communication schemes and system parameters in each regime. A summary of our results is presented in Table I and compared with the cases without infrastructure support. Our work generalizes results from previous related works on MANETs or hybrid networks as special cases, and provides fundamental insight on the understanding and design of wireless ad hoc network.

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