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Capital Allocation and International Equilibrium with Pollution Permits*

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ABSTRACT

Since the Kyoto Agreement, the idea of setting up pollution rights as an instrument of environmental policy for the reduction of greenhouse gases has progressed significantly. But the crucial problem of allocating these permits in a manner acceptable to all countries is still unsolved. There is a general consensus that this should be done according to some proportional allocation rule, but opinions vary greatly about what would be the appropriate proportionality parameter. In this paper, we analyze the economic consequences of different allocation rules in a general equilibrium framework. We first show the existence and unicity of an international equilibrium under the assumption of perfect mobility of capital and we characterize this equilibrium according to the dotations of permits. Then, we compare the economic consequences of three types of allocation rules when the permit market is designed to reduce total pollution. We show that a rule which applies some form of grandfathering simply reduces production and emissions proportionally and efficiently. In contrast, an allocation rule proportional to population is beneficial for developing countries. Finally *per capita* allocation rules induce size effect and can reverse these results.

Keywords: Pollution Permits; Capital Allocation; International Equilibrium

1. Introduction

One of the most interesting developments in environmental policy in recent years has been the emergence of global environment as a North-South issue. The close link between global environment and development calls for new insights. In a world of global externalities, national policies have important international repercussions through trade and factor mobility. To be sure that the full impact of environmental policies can be analyzed through to its ultimate effects on factor markets, income and pollution, a general equilibrium approach is needed. This is the way pioneered by Copeland and Taylor [1,2] and Chichinilsky [3] who study the links between trade and environment in a North-South context. Copeland and Taylor [1,4], examine linkages between national income, pollution and international trade in a simple model of North-South trade. By isolating the scale, composition and technical effects of international trade on pollution, they show that free trade increases world pollution. Moreover, an increase in the North's production possibilities increases pollution while similar growth in the South lowers pollution. In their papers, pollution has only a local nature, in the sense that damages are confined to the emitting country, and they analyze the same questions with transboundary pollution in Copeland and Taylor [5] where countries differed only in their endowment of efficient labor which is the one primary factor. Chichinilsky [3], consider two primary factors, physical capital and environmental resource, and focuses mainly on the consequences of differences in property rights on the common-property problem, giving answers to the presumed comparative advantage in "dirty industries" for developing countries or the compatibility of trade policies based on traditional comparative advantages with environmental preservation. In this paper, we adopt the Copeland-Taylor framework with global pollution produced jointly with consumption good, but we introduce international markets for physical capital and pollution permits.[1]

Since tradeable emission permits have been introduced in economic theory by J. H. Dales [6] as a new instrument for environmental policy, they have been the object of many studies (Tietenberg [7]). Many of these studies deal with the comparison between emission permits and emission fees and there is now a growing body of literature

*This paper originates from a research project launched jointly with Philippe Michel.

[1]Copeland and Taylor [4] develop a perfectly competitive general equilibrium model with trade in goods and in emission permits but without capital market.

on their practical application (Noll [8], Hahn [9], Kete [10], Hahn and Stavins [11]). As suggested by Chichinilsky and Heal [12], tradeable emission permits are also a means to secure the biosphere and Chichinilsky *et al.* [13] have analyzed their use as a policy instrument against greenhouse warming. They show that the manner in which emission rights are initially distributed determines the possibility of the market attaining a Pareto efficient outcome (Jouvet *et al.* [14] and Jouvet *et al.* [15]).

Since the Kyoto agreement of 1997, the idea of setting up pollution rights as an instrument of environmental policy for the reduction of greenhouse gases has progressed significantly. Europe, which had been hostile to the creation of such an international market for a long time, seems to have converted to this approach. In spite of the advantages which pollution permits seem to possess in comparison to other systems of environmental regulation (Bohm and Russel [16]), the institutionalizetion of an international market of pollution permits entails several problems (Baumol and Oates [17] Cropper and Oates [18], Pearce and Turner [19]). Among these difficulties, the first one to be aware of is without doubt the definition of an environmental norm necessary for the initial issue of permits.

In fact, seemingly intractable problems emerge as soon as we try to establish what would be the appropriate proportionality parameter in order to implement the initial allocation of permits. Opinions vary greatly in this respect and the list of appropriate parameters, which have been actually been put forward in submissions to the Intergovernmental Panel on Climate Change, is very large (Müller [20]). We have mainly the following:

- *Per capita* emission.
- *Per capita* GDP.
- Relative historical responsibility.
- Land area.
- Size of population.

The main question that remains to be solved concerns the economical consequences of those different rules. This question is particularly relevant in the North-South trade context where developing countries are unlikely to participate in the Kyoto agreement expecting that their costs exceed their benefits. For this reason, Bohm and Larsen [21] do not consider developing countries. They evaluate the distributional implications of the reduction costs brought about by various permit allocations in a tradeable permit regime for carbon emissions reductions, for a region consisting of Europe and the states of the former Soviet Union (FSU). They show that initial permit allocations by population and/or GDP are unlikely to induce the participation of most countries of Eastern Europe and FSU because of the net costs involved. They identify a set of initial allocations that would at least compensate these countries. But their analysis only focuses on the distribution of the economic burden of abatement and misses the general equilibrium implications of the allocation rules. In the same way, Koutstaal [22] focuses on the design, implementation and consesquences of a system of tradeable carbon permits to reduce greenhouse gas emissions within the context of the European Union.

In this paper we study an international equilibrium in a two-country model with capital and permit market. We analyze the effects of allocation rules of permits on capital allocation (and consequently on international equilibrium) by considering permit allocation rules proportional to production, emissions, physical capital (in level or *per capita*) and to population in a general equilibrium framework.

We use the standard technology of production with three factors (capital, labor and emission) in the form proposed by Stokey [23].

We first analyze the international equilibrium. A permit market does not modify the competitive world equilibrium without permits when the total allocation is large enough. When it is not, there exists a unique equilibrium with under-use of the technology, or with full use of the technology in the two countries.

When allocation of permits is not proportional to the emissions in the world without permits, there is a reducetion factor of emissions which results from the equilibrium allocation of capital. The equilibrium level of use of technology is the same in the two countries. It depends both on the total world dotation of permits and its distribution among countries.

The second and main part of the paper is devoted to studying the economic consequences of different permit allocations rules. Three different types of conclusions hold.

A level allocation rule (proportional to outputs, emissions or physical capital) reduces production and emissions in both countries proportionally with a change in the technology used. In this case, each country uses exactly its dotation of permits and the equilibrium allocation of capital is the same as in the economy without permits. In fact, such an allocation is efficient, *i.e.* it allows maximum production for a given total world dotation of permits. The level allocation rules proportionally diminish output in the two countries whatever their relative wealth.

A North-South distinction (Copeland and Taylor [1]) assumes higher level of efficient labor *per capita* in the North. This implies that population allocation rule leads to a North-South ratio of permits smaller than the level allocation. This allocation is beneficial for the developing country, increasing capital and production. Moreover, the South is net seller of permits, which gives him an additional income. However, the *per capita* income remains lower in the South country than in the North country.

Finally, *per capita* allocation rules (proportional to *per capita* output, emissions or physical capital) induce a size effect. If the population in the developing country is lower than the population in the developed country, these rules have the same effects as the population rule. But if it is larger, the developed country benefits from the *per capita* allocation rules.

The remainder of the paper proceeds as follow. Section 2 sets up the model. In Section 3 we study the international equilibrium without permits and in Section 4 we state the conditions under which an international equilibrium with permits exists and is unique. Section 5 deals with the economic consequences of different permit allocations rules and Section 6 presents our conclusions.

2. The Model

We study the international equilibrium for two countries in a simple model with one representative firm in each country. These firms produce the same good with the same technology. We assume perfect mobility of capital but fixed inelastic efficient labor supply $H_i, i=1,2$ in each country and given total capital stock $\bar{K}$. We also assume that emissions of pollution is a joint product and we introduce an international market of emissions permits.

Given the quotas $\bar{E}_i, i=1,2$ for each country, the representative firms can buy or sell it on a permit market, deciding on their emissions as if there was a global world quota. But when the price of permits is positive and there is a reallocation, then the firm's revenues are modified.

Assuming there exists competitive labor market in each country, wage corresponds to the marginal productivity of labor and the firm's revenue net of wages includes the net benefit of the permit market. As a consequence, the rate of return of capital is different from the marginal productivity of capital, as soon as there are transactions on the permit market.

With perfect mobility of capital across countries, only the average returns to capital are equalized to the marginal productivities. Indeed, the permit market modify the net revenue of the firms and thus their value. As a consequence, the equilibrium with perfect mobility of capital will lead to equalizing the values of capital that take into account the net gains on the permit market.

2.1. The Technology

Two countries produce the same good with the same Cobb-Douglas production technology given by

$$Y_i = z_i A K_i^{\alpha} L_i^{1-\alpha}, \quad i=1,2 \tag{1}$$

where K_i and L_i are respectively capital and efficient labor, and z_i an index of the technology used with $z_i \le 1$. With $z_i = 1$, $\tilde{Y}_i = A K_i^{\alpha} L_i^{1-\alpha}$ is the potential output.

The ratio emission E_i on production Y_i is an increasing function of z_i

$$\frac{E_i}{Y_i} = b z_i^{\beta}, \quad \beta > 0 \tag{2}$$

when $z_i = 1$, the use of all productive possibility leads to the largest emissions and pollution.

Remark 1. *This one-good model (see Stokey [23]) can be interpreted as a reduced form of the framework in Copeland and Taylor [1]. In fact, it is equivalent to the following three factor production function*

$$Y_i = Y_i^1 = A^{\frac{\beta}{1+\beta}} b^{-\frac{1}{1+\beta}} K_i^{\frac{\alpha\beta}{1+\beta}} L_i^{\frac{(1-\alpha)\beta}{1+\beta}} E_i^{\frac{1}{1+\beta}} \text{ if } E_i \le bAK_i^{\alpha} L_i^{1-\alpha}$$

$$Y_i = Y_i^2 = AK_i^{\alpha} L_i^{1-\alpha} \text{ if } E_i \ge bAK_i^{\alpha} L_i^{1-\alpha}$$

This function, $Y_i = \min\{Y_i^1, Y_i^2\}$ is homogenous of degree one, continuous and concave with respect to capital, labor and emissions. It is differentiable except at the points at which $Y_i^1 = Y_i^2$.

2.2. Firm's Behavior

In each country $i, i=1,2$, a representative firm maximize profits with respect to the use of technology z_i, efficient labor L_i and capital stock K_i. In addition, firm in country i hold a given stock of permits $\bar{E}_i$. This initial allocation is different from E_i, the firm's demand, which depend of the market price q of the permit on the international market.

Denote by w_i the wage in country *i*. The revenue, including the net gains on the permit market is thus given by

$$Y_i - w_i L_i - q\left(E_i - \bar{E}_i\right) \tag{3}$$

Using relation (2), the problem of firm in country *i*. is

$$\max_{0<z_i\le 1, L_i>0} AK_i^{\alpha} L_i^{1-\alpha}\left(z_i - qbz_i^{1+\beta}\right) - w_i L_i + q\bar{E}_i$$

The first order conditions are

$$w_i = (1-\alpha) m_i A K_i^{\alpha} L_i^{-\alpha} \tag{4}$$

with $m_i = z_i - qbz_i^{1+\beta}$, and

$$1 - qb(1+\beta) z_i^{\beta} \ge 0, (= 0 \text{ if } z_i < 1) \tag{5}$$

This last condition gives

$$z_i = \min\left\{1, \left(\frac{1}{qb(1+\beta)}\right)^{\frac{1}{\beta}}\right\} \equiv z(q) \tag{6}$$

Thus, in (4), $m_i = z(q) - qbz(q)^{1+\beta} = m(q)$

Efficient labor is paid at its marginal productivity according to (4). Decision on the use of technology only depends on the price of permits. Hence, in the two countries the index of the technology used is the same, $z_i = z$. Thus profits satisfy

$$\overline{\Pi}_i = \alpha\left(\frac{1}{bz^{\beta}} - q\right)E_i + q\overline{E}_i \tag{7}$$

As long as the price of permits is low enough, *i.e.* when $q \le \frac{1}{b(1+\beta)}$, in the two countries, the production is equal to its potential output $(z=1)$ which leads to maximum pollution in the two countries. But, as soon as the price of permits exceeds $\frac{1}{b(1+\beta)}$, the index of technology used is less than one which implies a reduction in production and thus in pollution.

Note that pollution is reduced in two ways : emissions decrease both with production and the index of technology used (Equation (2)). Following Hahn and Solow [24] (pages 70-71) "...we take it to be characteristic of capitalist firms that their profits go to the suppliers of capital. We assume, therefore, that savings...are used to buy shares in the gross operating surplus of firms." Therefore the total return per unit of capital, π_i, is defined by

$$\pi_i = \frac{\overline{\Pi}_i}{K_i} \tag{8}$$

This implies that

$$\pi_i K_i = \alpha\left(\frac{1}{bz^{\beta}} - q\right)E_i + q\overline{E}_i \equiv \Pi_i \tag{9}$$

This net revenue Π_i is similar to the gross operating surplus defined by Hahn and Solow. Note that when the price of permits is positive, the permit market modify the firm's income and so the return of capital which is not equal to its marginal productivity.

According to the price q of permits, two cases occur :

$$q \le \frac{1}{b(1+\beta)}, z = 1, \Pi_i = \alpha\left(\frac{1}{b} - q\right)E_i + q\overline{E}_i \tag{10}$$

$$q > \frac{1}{b(1+\beta)}, z = \left(\frac{1}{qb(1+\beta)}\right)^{\frac{1}{\beta}} \equiv \varsigma(q) < 1, \tag{11}$$

$$\Pi_i = \alpha\beta q E_i + q\overline{E}_i$$

3. Equilibrium

In the absence of mobility of labor, in each country, the equilibrium in the labor market implies the equality of the labor demand L_i and the supply H_i.

In the world without permits, the definition of the equilibrium is standard. It is efficient and gives the maximum of the world production.

$$\mathcal{Y}(\overline{K}) = \max_{K_1}\left\{AK_1^{\alpha}H_1^{1-\alpha} + A(\overline{K} - K_1)^{\alpha} H_2^{1-\alpha}\right\}$$

This maximum is obtained when the allocation of total capital $K_1 + K_2 = \overline{K}$ is proportional to efficient labor and this leads to the potential world output,

$$\mathcal{Y}(\overline{K}) = A(H_1 + H_2)^{1-\alpha}\,\overline{K}^{\alpha} \tag{12}$$

The corresponding total emissions is then also maximum: $b(Y_1 + Y_2) = b\mathcal{Y}(\overline{K})$. Emissions are proportional to efficient labor

$$\frac{E_2}{E_1} = \frac{Y_2}{Y_1} = \frac{H_2}{H_1} = \mu$$

with the allocation of permits $\overline{E}_i$, in country i, $i = 1,2$, there is an additional market and we denote q^* the equilibrium price on this market. In addition, this market interact with the capital market. The assumption of *perfect mobility of capital* leads to equality of the two rates of return $\pi_1 = \pi_2 = \pi$, which implies

$$\frac{\Pi_1^*}{K_1^*} = \frac{\Pi_2^*}{K_2^*} \tag{13}$$

Finally, the permit market clears, which means

$$E_1^* + E_2^* \le \overline{E}_1 + \overline{E}_2 \text{ with equality if } q > 0 \tag{14}$$

At equilibrium, emissions are $E_i^* = b(z^*)^{1+\beta} A(K_i^*)^{\alpha} H_i^{1-\alpha}$. Thus, the ratio $e^* = \frac{E_2^*}{E_1^*}$ only depends on the equilibrium ratio $\frac{K_2^*}{K_1^*}$ of capital stocks

$$e^* = \frac{E_2^*}{E_1^*} = \left(\frac{K_2^*}{K_1^*}\right)^{\alpha}\left(\frac{H_2}{H_1}\right)^{1-\alpha} = \left(\frac{K_2^*}{K_1^*}\right)^{\alpha}\mu^{1-\alpha} \tag{15}$$

with $\mu = \frac{H_2}{H_1}$.

In a world without permits the equilibrium allocation of capital and emissions are proportional to efficient labor and given by $K_i^0 = \frac{H_i}{H_1 + H_2}\overline{K}$ and $E_i^0 = \frac{bAH_i\overline{K}^{\alpha}}{(H_1 + H_2)^{\alpha}} i = 1,2.$

More generally, when the sum of the allocation of permits is at least equal to the maximum of emissions the equilibrium price of permits is zero, total production is equal to potential world output. This holds if $\overline{E} = \overline{E}_1 + \overline{E}_2 \ge E_1^0 + E_2^0 = b\mathcal{Y}(\overline{K})$.

4. World Equilibrium with Reduction of Emissions

When the total dotation of permits does not allow for the maximum of pollution, *i.e.*

$$\bar{E} = \bar{E}_1 + \bar{E}_2 < b\mathcal{Y}\left(\bar{K}\right)$$

The following study shows the existence of a unique equilibrium, either with under-use of the technology or with full use of the technology in the two countries.

This second possibility occurs when the allocation of permits is not proportional to the emissions in the world without permits. There is then a reduction factor of emissions which results from the equilibrium allocation of capital.

The equilibrium level of use of technology is the same in the two countries. It depends both on the total world dotation of permits and its distribution among countries.

4.1. Equilibrium with Under-Use of Potential Outputs

We begin with some useful concepts in order to study the existence of an equilibrium with under-use of potential outputs.

Equilibrium ratio: At the equilibrium with under-use of potential outputs $\left(z^* < 1\right)$, emissions $E_i^*, i = 1,2$ are proportional to $\left(K_i^*\right)^\alpha$ (relation (15)), capital stocks are proportional to incomes Π_i^* (relation (13)) and incomes are proportional to $\alpha\beta E_i^* + \bar{E}_i$ (relation (11)).

This leads to an equilibrium ratio $e^* = \tilde{e}_\mu\left(\bar{e}\right)$ as a function of $\bar{e} = \dfrac{\bar{E}_2}{\bar{E}_1}$ depending on $\mu = \dfrac{H_2}{H_1}$. The equilibrium ratio of emissions $\tilde{e}_\mu\left(\bar{e}\right)$ increases with $\bar{e}$ and its value is located between $\bar{e}$ and μ. (for details see Appendix A1, Lemma 6).

Proportional allocation: We have a proportional allocation when the allocation of permits is proportional to efficient labor, $\left(\bar{e} = \mu\right)$, then, there are no transactions on the permit market $\left(\tilde{e}_\mu\left(\mu\right) = \mu\right)$. The index of technology used z^* is simply defined by the level of total permits $\bar{E} = \bar{E}_1 + \bar{E}_2$, *i.e.* $\bar{E} = \left(z^*\right)^{1+\beta} b\mathcal{Y}\left(\bar{K}\right)$ which result from the proportionality properties.

Non-proportional allocation: When an allocation is not proportional to efficient labor $\left(\bar{e} \neq \mu\right)$, there are permit's transactions which draw the economy in the direction of the proportional allocation.

Since the allocation of factors are not proportional, then the sum of potential outputs $\tilde{Y}_1 + \tilde{Y}_2$ is smaller than the world potential output and we have $\tilde{Y}_1 + \tilde{Y}_2 = \gamma\mathcal{Y}\left(\bar{K}\right)$ where γ is a reduction factor smaller than 1.

At the equilibrium, this reduction factor is a function of the equilibrium ratio: $\gamma^* = \gamma_\mu\left(e^*\right)$[2] With $e^* = \tilde{e}_\mu\left(\bar{e}\right)$, the reduction factor at equilibrium $\gamma^* = \gamma_\mu\left(e^*\right) = \varphi_\mu\left(\bar{e}\right)$ where $\varphi_\mu\left(\bar{e}\right) = \gamma_\mu\left(\tilde{e}_\mu\left(\bar{e}\right)\right)$. This reduction factor γ^* is smaller than 1 for $\bar{e} \neq \mu$. More precisely, the larger the gap between $\bar{e}$ and μ, the smaller the reduction factor at equilibrium.

Equilibrium: Given $\bar{E}_1$ and $\bar{E}_2$, the equilibrium index of technology used z^* is determined by

$$\bar{E}_1 + \bar{E}_2 = \gamma^*\left(z^*\right)^{1+\beta} b\mathcal{Y}\left(\bar{K}\right) \tag{16}$$

and q^* is determined by $q^* = \dfrac{\left(z^*\right)^{-\beta}}{b\left(1+\beta\right)}$.

Thus $z^* < 1$ is equivalent to $\gamma^* b\mathcal{Y}\left(\bar{K}\right) > \bar{E}_1 + \bar{E}_2$

To summarize, we have shown the following.

Proposition 1. *Given the dotations of permits, and the total capital stock, there exists an equilibrium with under-use of technology if and only if the total dotation of permits is smaller than the product of maximum of emissions with the reduction factor. The equilibrium ratio of emissions is an increasing function of the ratio of dotation and determines the reduction factor.*

4.2. Equilibrium with Full Use of Potential Outputs

With full use of potential outputs and positive price of permits we have $z^* = 1$ and $q^* > 0$.

In the proportional case, $\left(\bar{e} = \mu\right)$, at equilibrium there is no transactions on the permit's market.

In the particular case where $\bar{E} = b\mathcal{Y}\left(\bar{K}\right)$, any value of the permit's price $q^* \in \left]0, \dfrac{1}{b\left(1+\beta\right)}\right]$ leads to the same allocation as in the economy without permits. (Appendix A2, Lemma 10)

In the non proportional case $\left(\bar{e} \neq \mu\right)$, there is a reduction factor γ^* and with $z^* = 1$ we have

$$\bar{E}_1 + \bar{E}_2 = \gamma^* b\mathcal{Y}\left(\bar{K}\right) \tag{17}$$

This implies $\bar{E} < b\mathcal{Y}\left(\bar{K}\right)$ and the corresponding value of γ^* verifies $\gamma^* = \gamma_\mu\left(e^*\right)$ which determines the equilibrium value of $e^* = \dfrac{E_2^*}{E_1^*}$.

Assume $\bar{e} \neq \mu$. When $\bar{E}$ is large enough, ($\bar{E} \geq b\mathcal{Y}\left(\bar{K}\right)$ the equilibrium allocation is proportional to efficient labor $\left(e^* = \mu, z^* = 1, \gamma^* = 1\right)$. When it is small enough, $\bar{E} < \varphi_\mu\left(\bar{e}\right) b\mathcal{Y}\left(\bar{K}\right)$, there is under use of potential outputs, the equilibrium ratio is $e^* = \tilde{e}_\mu\left(\bar{e}\right)$, the reduction factor is $\gamma^* = \varphi_\mu\left(\bar{e}\right)$ and $z^* < 1$.

[2]This function first increases, reaches a maximum equal to 1 in the proportional case $\left(e^* = \mu\right)$ and then decreases (see Appendix A1, Lemmas 7 and 8).

In the intermediate case, $\left(\varphi_\mu(\bar{e})b\mathcal{Y}(\bar{K}) \le \bar{E} < b\mathcal{Y}(\bar{K})\right)$, there is full use of potential outputs but it remains a reduction factor which is smaller than 1 and larger than $\varphi_\mu(\bar{e})$.

The equilibrium ratio of emissions e^* is intermediate between μ and $\tilde{e}_\mu(\bar{e})$. Indeed, $\frac{\partial\gamma_\mu}{\partial\bar{e}}$ is positive for $\bar{e} < \mu$ and negative for $\bar{e} > \mu$ (see Appendix A2, Lemma 11).

To summarize, we obtain :

Proposition 2. *Assume that allocation of permits is not proportional to efficient labor and total allocation is below the maximum of pollution. Then, there exists a minimum level of total allocation for which the world equilibrium uses potential outputs and the price of permits is positive.*

Again, the equilibrium ratio of emissions is located between the ratio of efficient labor μ and the ratio of dotations $\bar{e}$. More precisely, it is located between μ and the value $\tilde{e}_\mu(\bar{e})$. As shown in the Appendix A2, we have

$$\text{if } \bar{e} > \mu,\ \mu < e^* < \tilde{e}_\mu(\bar{e}) < \bar{e}$$

$$\text{if } \bar{e} < \mu,\ \bar{e} < \tilde{e}_\mu(\bar{e}) \le e^* < \mu$$

The unicity of equilibrium results from the three preceding propositions.

The three preceding propositions are illustrated in **Figure 1** below.

In the $(\bar{E}_1, \bar{E}_2)$ plane, we have drawn regions corresponding to the different equilibria. In region A, total dotation of permits is at least equal to the maximum of emissions and $q^* = 0$ (Proposition 1), in region B total dotation of permits is smaller than the product of maximum of emissions with the reduction factor and there is under-use of potential output (Proposition 2), and in region C there is full use of potential output and the price of permits is positive (Proposition 3).

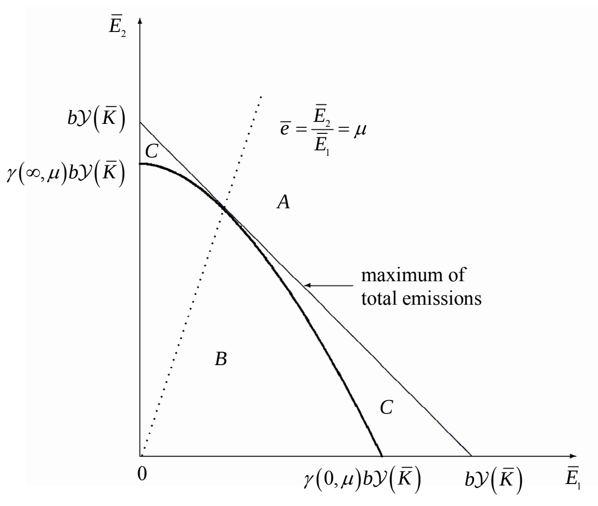

Figure 1. Regions corresponding to different types of equilibrium.

5. The Economic Consequences of Allocation Rules of Permits

In order to study the consequences of different allocation rules of permits, we compare the equilibrium with permits to the equilibrium without permits.

Without permits, the equilibrium values of capital stocks K_i^0, production Y_i^0, emissions E_i^0 are proportional to efficient labor supplies H_i. Profits per unit of capital are equal in the two countries (perfect mobility of capital) and equal to the marginal productivity of capital. As shown in Section 3, the equilibrium with permits coincides with the equilibrium without permits when the total dotation of permits allow for the potential world output, *i.e.* $\bar{E}_1 + \bar{E}_2 \ge b\mathcal{Y}(\bar{K})$ and pollution is maximum in this case: $E_1^0 + E_2^0 = \mathcal{Y}(\bar{K}) = b\mathcal{Y}(\bar{K})$. This is our benchmark case defined by

$$\frac{K_2^0}{K_1^0} = \frac{Y_2^0}{Y_1^0} = \frac{E_2^0}{E_1^0} = \frac{H_2}{H_1} = \mu \tag{18}$$

We assume now that the total dotation of permits does not allow for the maximum of pollution, *i.e.*

$$\bar{E} = \bar{E}_1 + \bar{E}_2 < b\mathcal{Y}(\bar{K}) \tag{19}$$

and we consider three types of allocation rules.

5.1. Level Allocation Rules

The proportionality at the equilibrium without permits of capital, output, emissions and efficient labor (Equation (18)) implies that any allocation of permits proportional to one of these levels, leads to the same allocation which we call the level allocation rules. These rules can be viewed as some form of grandfathering[3]. All these rules are equivalent and they imply that the ratio $\bar{e} = \frac{\bar{E}_2}{\bar{E}_1}$ is equal to $\mu = \frac{H_2}{H_1}$.

This implies that the equilibrium reduction factor $\gamma = \varphi_\mu(\mu) = 1$. Under (19), the equilibrium value of the technology index is (Proposition 1 with $\bar{e} = \mu$)

$$\bar{\bar{z}} = \left(\frac{\bar{E}}{b\mathcal{Y}(\bar{K})}\right)^{\frac{1}{1+\beta}} < 1.^4$$

[3]In the simple grandfathering allocation, all countries receive permits in proportion to their baseline emissions.

[4]For further comparison, we denote $\bar{\bar{z}}$, $\bar{\bar{K}}_i$, $\bar{\bar{E}}_i$, $\bar{\bar{Y}}_i$ the equilibrium values with $\bar{e} = \mu$

The capital stocks remain unchanged, $\overline{\overline{K}}_i = K_i^0$, productions and emissions are reduced,

$$\overline{\overline{Y}}_i = \overline{\overline{z}}A\left(K_i^0\right)^{\alpha} H_i^{1-\alpha} = \overline{\overline{z}}Y_i^0$$

and

$$\overline{\overline{E}}_i = b\overline{\overline{z}}^{\beta}\,\overline{\overline{Y}}_i = b\overline{\overline{z}}^{1+\beta} Y_i^0 = \overline{\overline{z}}^{1+\beta} E_i^0 = \overline{E}_i.$$

The price of permits $\overline{\overline{q}} = \dfrac{1}{b(1+\beta)\overline{\overline{z}}^{\beta}}$ is positive, but there are no transactions on the permit market. A level allocation rule simply reduces proportionally production and emissions by applying the technology index $\overline{\overline{z}}$.

This is a consequence of the assumption that the technology of production and the corresponding emission function are the same in the two countries. Because of the effect of the index of pollution, emissions diminish more than the production: $\dfrac{\overline{\overline{E}}_i}{\overline{\overline{Y}}_i} = b\overline{\overline{z}}^{\beta} < \dfrac{E_i^0}{Y_i^0}$ implies $\dfrac{\overline{\overline{E}}_i}{E_i^0} < \dfrac{\overline{\overline{Y}}_i}{Y_i^0}$.

We have the following result of efficiency of this allocation rule: it leads to the maximum of the world production for given total capital stock $\overline{K}$ and total emission $\overline{E}$ (see Prat [25]).

Proposition 3. *Given the total capital stock, the maximum of the world production subject to a total emissions constraint is reached at the equilibrium obtained by an allocation rule which is proportional to efficient labor.*

Proof. Consider first any allocation $K_1 > 0$ and $K_2 > 0$ of $\overline{K} = K_1 + K_2$. $\tilde{Y}_i = AK_i^{\alpha}H_i^{1-\alpha}$ is the potential production in country i. The maximum of $z_1\tilde{Y}_1 + z_2\tilde{Y}_2$ subject to

$$E_1 + E_2 = bz_1^{1+\beta}\tilde{Y}_1 + bz_2^{1+\beta}\tilde{Y}_2 \le \overline{E}$$

leads to $z_1 = z_2$. This results from the concavity of the problem and the maximization on the Lagrangian

$$\mathcal{L} = z_1\tilde{Y}_1 + z_2\tilde{Y}_2 + \lambda\left(\overline{E} - bz_1^{1+\beta}\tilde{Y}_1 - bz_2^{1+\beta}\tilde{Y}_2\right)$$

As a consequence, the maximum of world production can be formulated as follow: Maximize with respect to z, K_1 and K_2, $Y_1 + Y_2 = z\left(\tilde{Y}_1 + \tilde{Y}_2\right)$, with $\tilde{Y}_i = AK_i^{\alpha}H_i^{1-\alpha}$, $i = 1,2$ subject to $K_1 + K_2 = \overline{K}$ and $bz^{1+\beta}\left(\tilde{Y}_1 + \tilde{Y}_2\right) = \overline{E}$.

Replacing $z = \left(\dfrac{\overline{E}}{b}\right)^{\frac{1}{1+\beta}}\left(\tilde{Y}_1 + \tilde{Y}_2\right)^{-\frac{1}{1+\beta}}$, this leads to maximize $\left(\tilde{Y}_1 + \tilde{Y}_2\right)^{\frac{\beta}{1+\beta}}$ and to the solution $K_i = \overline{\overline{K}}_i = K_i^0$, $i = 1,2.$

We have shown that for any allocation of capital $K_1 + K_2 = \overline{K}$, the maximum of the world production $Y_1 + Y_2$ subject to $E_1 + E_2 \le \overline{E}$ is obtained with the same index of technology used z for the two countries and that the reduction factor is equal to one.

5.2. Population Allocation Rule

A population allocation rule leads to an allocation of permits proportional to population.

Independently of the size of population in the two countries, $N_i, i = 1,2$ a reasonable measure of standard of living *per capita* is efficient labor *per capita*. Thus, as in Copeland and Taylor [1], the North-South distinction arises from an assumed higher level of efficient labor in the North, *i.e.* a larger efficient labor *per capita*.

We assume that country 2 is a developing country because it has a lower efficient labor *per capita* than in country 1, say a developed country.

$$h_2 = \frac{H_2}{N_2} < \frac{H_1}{N_1} = h_1$$

Then an allocation rule proportional to population implies

$$\overline{e} = \frac{\overline{E}_2}{\overline{E}_1} = \frac{N_2}{N_1} > \frac{H_2}{H_1} = \mu$$

We compare the effects of this rule of allocation to the preceding rule proportional to μ, with the same dotation of permits $\overline{E} = \overline{E}_1 + \overline{E}_2$ verifying (19).

When $\overline{e} \ne \mu$, the equilibrium reduction factor γ is smaller than 1 and there are two possibilities for the equilibrium according to if $\overline{E}$ is larger or smaller than $\varphi_{\mu} b\mathcal{Y}\left(\overline{K}\right)$. If the equilibrium reduction factor is not too low $\left(\varphi_{\mu} b\mathcal{Y}\left(\overline{K}\right) > \overline{E}\right)$, the equilibrium holds with non-use of potential output $\left(z^* < 1\right)$. If not $\left(\varphi_{\mu} b\mathcal{Y}\left(\overline{K}\right) \le \overline{E}\right)$ the equilibrium holds with use of potential output $\left(z^* = 1\right)$. More precisely, as a function of $\overline{e}$, $\varphi_{\mu}\left(\overline{e}\right) = \gamma_{\mu}\left(\tilde{e}_{\mu}\left(\overline{e}\right)\right)$ is decreasing with respect to $\overline{e}$, for $\overline{e} > \mu$ and admits a finite limit φ_{∞} when $\overline{e}$ tends to $+\infty$ (Appendix A1, Lemma 11). Thus

- If $\overline{E} \le \varphi_{\infty} b\mathcal{Y}\left(\overline{K}\right)$, then for all $\overline{e} > 0$, $\overline{E}$ is smaller than $\varphi_{\mu}\left(\overline{e}\right) b\mathcal{Y}\left(\overline{K}\right)$ and the international equilibrium holds with $z^* < 1$.
- If $\overline{E} > \varphi_{\infty} b\mathcal{Y}\left(\overline{K}\right)$ and $\overline{E}$ verifies (19), there exists a threshold $e^{\#}$ solution of $\overline{E} = \varphi_{\mu}\left(e^{\#}\right) b\mathcal{Y}\left(\overline{K}\right)$ such that the international equilibrium holds with $z^* < 1$ if $\overline{e} < e^{\#}$ and with $z^* = 1$ if $\overline{e} \ge e^{\#}$.

Let us define the threshold $\hat{e}$ such that at equilibrium $z^* < 1$ if and only if $\overline{e} < \hat{e}$. This threshold is $\hat{e} = e^{\#}$ if $\overline{E} > \varphi_{\infty} b\mathcal{Y}\left(\overline{K}\right)$, if not, $\hat{e} = +\infty$.

Proposition 4. *With the population allocation rule, the world production is reduced; the developing country is* *net seller of permits, receives more capital, produces more and thus emits more pollution. The developed country is net buyer of permits, receives less capital, produces less and emits less pollution.*

Proof. Consider first the case $\overline{e} < \hat{e}$, then $z^* < 1$ and the international equilibrium verifies (Proposition 1, Appendix A1, Lemmas 9 and 10)

$$e^* = \tilde{e}_\mu\left(\overline{e}\right), \mu < e^* < \overline{e}$$

and

$$\left(z^*\right)^{1+\beta} = \frac{\overline{E}}{\varphi_\mu\left(\overline{e}\right) b\mathcal{Y}\left(\overline{K}\right)} > \overline{\overline{z}}^{1+\beta}$$

since $\varphi_\mu\left(\overline{e}\right) < 1$.

World production is reduced because its maximum for given $\overline{K}$ and $\overline{E}$ is reached at the equilibrium with allocation $\overline{e} = \mu$.

The capital ratio $\frac{K_2^*}{K_1^*}$ is larger than $\frac{\overline{\overline{K}}_2}{\overline{\overline{K}}_1}$ because we have from relation (15)

$$\frac{K_2^*}{K_1^*} = \left(e^*\right)^{\frac{1}{\alpha}} \mu^{1-\frac{1}{\alpha}} > \mu = \frac{\overline{\overline{K}}_2}{\overline{\overline{K}}_1}$$

But the sum is the same: $K_1^* + K_2^* = \overline{K} = \overline{\overline{K}}_1 + \overline{\overline{K}}_2$. As a consequence, $K_2^* > \overline{\overline{K}}_2$ and $\overline{K}$. The increase in z and K_2 implies an increase in production for country 2.

$$Y_2^* = z^* A\left(K_2^*\right)^\alpha H_2^{1-\alpha} > \overline{\overline{z}} A\left(\overline{\overline{K}}_2\right)^\alpha H_2^{1-\alpha} = \overline{\overline{Y}}_2$$

This also implies an increase in emissions $E_2^* > \overline{\overline{E}}_2$. Since the world production decreases Y_1 decreases (more than Y_2 increases)

$$Y_1^* < \overline{\overline{Y}}_1 + \overline{\overline{Y}}_2 - Y_2^* < \overline{\overline{Y}}_1$$

Emissions also decrease: $E_1^* < \overline{\overline{E}}_1$ (the sum is constant)

Moreover, $\frac{E_2^*}{E_1^*} = e^* < \overline{e} = \frac{\overline{E}_2}{\overline{E}_1}$ implies that the developing country is a net seller and the developed country a net buyer on the permit's market.

Consider now the case $\overline{e} \geq \hat{e}$. Then, $z^* = 1$. At this equilibrium $\frac{E_2^*}{E_1^*} = e^*$ is the solution of

$$\overline{E} = \gamma_\mu\left(e^*\right) b\mathcal{Y}\left(\overline{K}\right)$$

and it verifies $\mu < e^* < \overline{e}$ (Proposition 2). The preceding arguments then applies without modification. □

Clearly, the allocation rule proportional to population is in favor of the developing country increasing capital and production. An additional advantage is the income from selling permits.

The situation of the developed country is the complete opposite: it looses in all respect: capital and production are reduced and it must buy more permits.

We should also remark that production *per capita* remains larger in the developed country when $\overline{e} = \frac{N_2}{N_1}$, since

$$\frac{Y_2^*}{Y_1^*} = \frac{E_2^*}{E_1^*} = e^* < \overline{e} = \frac{N_2}{N_1}$$

Moreover we have

Proposition 5. *The per capita income remains lower in the developing country than in the developed country*

Proof. When $\frac{H_2}{H_1} < \frac{N_2}{N_1} = \frac{\overline{E}_2}{\overline{E}_1}$ we have $\mu < e^* < \overline{e}$

The ratio of total income is

$$\chi = \frac{Y_2 + q^*\left(\overline{E}_2 - E_2^*\right)}{Y_1 + q^*\left(\overline{E}_1 - E_1^*\right)} = \frac{\beta E_2^* + \overline{E}_2}{\beta E_1^* + \overline{E}_1}$$

Because $Y_i = \frac{E_i^*}{b\left(z^*\right)^\beta}$ and $q^* = \frac{1}{b(1+\beta)\left(z^*\right)^\beta}$ we have $e^* < \chi < \overline{e} = \frac{N_2}{N_1}$

which implies that *per capita* income in the developing country is smaller than in the developed country. □

5.3. *Per Capita* Allocation Rules

Per capita allocation rules lead to an allocation of permits proportional to *per capita* outputs, emissions or physical capital.

We note ν the ratio of population $\frac{N_2}{N_1}$. The three *per capita* allocation rules are equivalent and lead to a ratio of permits $\overline{e} = \frac{\mu}{\nu}$. Indeed, from Equation (18) we have

$$\overline{e} = \frac{Y_2^0/N_2}{Y_1^0/N_1} = \frac{E_2^0/N_2}{E_1^0/N_1} = \frac{K_2^0/N_2}{K_1^0/N_1} = \frac{\mu}{\nu}$$

Per capita allocation rules induce a size effect relative to the level allocation rules except when $\nu = 1$. In this case, the two kind of allocation rules lead to $\overline{e} = \mu$ and we have the same results as in Subsection 4.1.

When $\nu \neq 1$, size effect exists.

If population in country 2 (the developing country) is lower than population in country 1, we have $\nu < 1$ and *per capita* allocation rules imply $\overline{e} > \mu$.

Thus, all the conclusions of the subsection 5.2 hold and a developing country will prefer *per capita* allocation rules to level allocation rules.

On the contrary, if population in country 2 is larger than population in country 1, we have $\nu > 1$ and *per capita* allocation rules imply $\overline{e} < \mu$.

This is equivalent to $\frac{1}{\bar{e}} > \frac{1}{\mu}$. Relabelling countries 1 as the developing country and 2 as the developed country, the analysis of subsection 5.2 hold without other modifications.

This is to say that now, the developing country is a net buyer of permits, receives less capital, produces less and emits less pollution.

In this case, *per capita* allocation rules are in unfavor of the developing country.

6. Conclusions

The Second Assessment Report of the IPCC (Bruce *et al*. [26]), contains the results of a study appraising the economic effects of two allocation rules, the grandfathering rule and the population rule. Developed countries would be net beneficiaries if should quotas grandfathering be adopted and under the population allocation rule the net beneficiaries would be the developing countries.

Our analysis allows us to be more specific on the economic consequences of these different allocation rules. The level allocation rules (which include the grandfathering rule) are efficient and lead to maximum world output once total emissions are given. They imply proportional reduction of pollution in all countries and have no effect on international capital allocation, under the assumption of the same technology in all countries.

The population allocation rule confirms the benefits for developing countries in every respects: production, movement of capital and income from the permit market. Nevertheless, *per capita* income remains lower in the developing country.

Per capita allocation rules have different size effects, depending on the ratio of population in the two countries. With the same level of population, the *per capita* rules lead to the efficiency allocation, and thus performs exactly like the level allocations rules. With a different level of population, the developing country benefits if and only if it has a lower level of population than in the developed country which benefits in the opposite case.

Our results shed some light on the recurrent discussion between countries about the initial distribution of permits in a tradeable market. Regarding efficiency, the level allocation rules seems to be the best. But it does not allow for any evolution of the relative income between countries. This shows that this allocation should be linked to redistribution policies. Further research will analyze the welfare effect of the abatement of pollution and the allocation rule.

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Appendixes

Characterization of an Interior Equilibrium (z* < 1)

Define $\varsigma(q) = (qb(1+\beta))^{-\frac{1}{\beta}}$

Dotation of permits $\bar{E}_1$ and $\bar{E}_2$, and the total capital stock $\bar{K}$ are given.

- Assume $q > 0$ and $z = \varsigma(q) < 1$

With capital stock K_1 and K_2, emissions and profits in country i are

$$\tilde{E}_i = b\varsigma(q)^{1+\beta} AK_i^{\alpha} H_i^{1-\alpha}$$

and

$$\tilde{\Pi}_i = q\left(\alpha\beta\tilde{E}_i + \bar{E}_i\right)$$

- The equilibrium condition (13) on the capital market implies

$$\begin{aligned}\frac{\tilde{E}_2}{\tilde{E}_1} &= \mu^{1-\alpha}\left(\frac{\tilde{K}_2}{\tilde{K}_1}\right)^{\alpha} = \mu^{1-\alpha}\left(\frac{\tilde{\Pi}_2}{\tilde{\Pi}_1}\right)^{\alpha} \\ &= \mu^{1-\alpha}\left(\frac{\alpha\beta\tilde{E}_2 + \bar{E}_2}{\alpha\beta\tilde{E}_1 + \bar{E}_1}\right)^{\alpha}\end{aligned} \tag{A1}$$

- The equilibrium condition (14) on the permit market with $q > 0$ implies

$$\tilde{E}_1 + \tilde{E}_2 = \bar{E}_1 + \bar{E}_2 \equiv \bar{E} \tag{A2}$$

Lemma 6. *Equations* (*A*1) *and* (*A*2) *imply that* $\tilde{e} = \frac{\tilde{E}_2}{\tilde{E}_1}$ *verifies* $\Delta(\tilde{e},\bar{e},\mu) = 0$ *where* $\bar{e} = \frac{\bar{E}_2}{\bar{E}_1}$ *and*

$$\begin{aligned}\Delta(e,\bar{e},\mu) &= \alpha\beta + \frac{1+e}{1+\bar{e}} \\ &- \left(\frac{\mu}{e}\right)^{\frac{1-\alpha}{\alpha}}\left(\alpha\beta + \frac{\bar{e}(1+e)}{(1+\bar{e})e}\right)\end{aligned} \tag{A3}$$

The equation $\Delta(e,\bar{e},\mu) = 0$ admits a unique solution $\tilde{e} > 0$ and $\tilde{e} = \tilde{e}_{\mu}(\bar{e})$ is increasing with respect to $\bar{e}$ and μ. If $\bar{e} = \mu$, then $\tilde{e} = \tilde{e}_{\mu}(\mu) = \mu$. If $\bar{e} > \mu$ (resp. $\bar{e} < \mu$), then $\tilde{e} = \tilde{e}_{\mu}(\bar{e})$ verifies $\mu < \tilde{e} < \bar{e}$ (resp. $\bar{e} < \tilde{e} < \mu$)

Proof. With $\bar{e} = \frac{\bar{E}_2}{\bar{E}_1}$ and $\tilde{e} = \frac{\tilde{E}_2}{\tilde{E}_1}$, the equilibrium condition on the permits market (A2) implies:

$\bar{E}_1 = \frac{\bar{E}}{(1+\bar{e})}$, $\bar{E}_2 = \frac{\bar{e}\bar{E}}{(1+\bar{e})}$, $\tilde{E}_1 = \frac{\bar{E}}{(1+\tilde{e})}$ and $\tilde{E}_2 = \frac{\tilde{e}\bar{E}}{(1+\tilde{e})}$. Thus (A1) implies

$$\tilde{e}^{\frac{1}{\alpha}} = \mu^{\frac{1-\alpha}{\alpha}}\left(\frac{\alpha\beta\tilde{e} + \frac{(1+\tilde{e})\bar{e}}{(1+\bar{e})}}{\alpha\beta + \frac{(1+\tilde{e})}{(1+\bar{e})}}\right)$$

and this condition is equivalent to $\Delta(\tilde{e},\bar{e},\mu) = 0$. $\Delta(e,\bar{e},\mu)$ is decreasing with respect to $\bar{e}$ and μ

$$\frac{\partial\Delta}{\partial e} > 0,\ \frac{\partial\Delta}{\partial\bar{e}} < 0,\ \frac{\partial\Delta}{\partial\mu} < 0$$

For fixed positive values of $\bar{e}$ and μ, $\Delta(e,\bar{e},\mu)$ increases from $-\infty$ to $+\infty$ when e increases from 0 to $+\infty$. Thus, there exists a unique solution $\tilde{e}$ of $\Delta(e,\bar{e},\mu) = 0$ and $\tilde{e} = \tilde{e}_{\mu}(\bar{e})$ is increasing with respect to $\bar{e}$ and μ.

In addition, we have $\Delta(\mu,\mu,\mu) = 0$, thus $\tilde{e}_{\mu}(\mu) = \mu$ is the unique solution of $\Delta(e,\mu,\mu) = 0$.

Assume $\bar{e} > \mu$, then $\tilde{e}_{\mu}(\bar{e}) > \tilde{e}_{\mu}(\mu) = \mu$ and we have

$$\Delta(\bar{e},\bar{e},\mu) = (\alpha\beta+1)\left(1-\left(\frac{\mu}{\bar{e}}\right)^{\frac{1-\alpha}{\alpha}}\right) > 0 = \Delta(\tilde{e},\bar{e},\mu)$$

Thus $\tilde{e}$ verifies $\mu < \tilde{e} < \bar{e}$

Similarly, if $\bar{e} < \mu$, $\tilde{e}$ verifies $\bar{e} < \tilde{e} < \mu$

Lemma 7. *If there exists an equilibrium with* $q > 0$ *and* $\varsigma(q) < 1$, *this equilibrium is unique and it verifies*: $e^* = \tilde{e} = \tilde{e}_{\mu}(\bar{e})$, $z^* = \varsigma(q^*)$

$$\begin{aligned}&\frac{K_2^*}{K_1^*} = (\tilde{e})^{\frac{1}{\alpha}}\mu^{1-\frac{1}{\alpha}} \equiv \tilde{\rho} = \rho(\tilde{e},\mu) \\ &\textit{and}\quad b\varsigma(q^*)^{1+\beta} = \frac{\bar{E}}{\gamma_{\mu}(\tilde{e})\mathcal{Y}(\bar{K})}\end{aligned} \tag{A4}$$

where

$$\gamma_{\mu}(\tilde{e}) = \frac{1+\tilde{e}}{(1+\mu)^{1-\alpha}(1+\rho(\tilde{e},\mu))^{\alpha}} \tag{A5}$$

Proof. The equilibrium verifies (A1) and (A2). The value of the ratio $\frac{K_2^*}{K_1^*}$ results from (A1). The equilibrium condition $K_1^* + K_2^* = \bar{K}$ implies $K_1^* = \frac{\bar{K}}{1+\tilde{\rho}}$ and

$$\begin{aligned}\frac{\bar{E}}{(1+e)} &= E_1^* = b(z^*)^{1+\beta} A(K_1^*)^{\alpha} H_1^{1-\alpha} \\ &= b(z^*)^{1+\beta}\frac{A(H_1+H_2)^{1-\alpha}\bar{K}^{\alpha}}{(1+\mu)^{1-\alpha}(1+\tilde{\rho})^{\alpha}}\end{aligned}$$

Defining $\gamma_{\mu}(\tilde{e})$ according to (A5), we obtain the value of $b\varsigma(q^*)^{1+\beta}$ given by (A4).

Lemma 8. *The function* $e \to \gamma_\mu(e)$ *defined by* (*A*5) *is increasing for* $e < \mu$ *and decreasing for* $e > \mu$; *its ma- ximum* $\gamma_\mu(\mu)$ *is equal to* 1. *The function* $e \to \varphi_\mu(\bar{e}) = \gamma_\mu(\tilde{e}_\mu(\bar{e}))$ *is also increasing for* $\bar{e} < \mu$ *and decreasing for* $\bar{e} > \mu$. *The limits of* $\tilde{e}_\mu(\bar{e})$ *and* $\varphi_\mu(\bar{e})$ *when* $\bar{e}$ *tends to* 0 (*resp.* $+\infty$) *are finite and correspond to dotation of all permits to country* 1 (*resp. country* 2).

Proof. Computing the derivative of $\ln \gamma_\mu(e)$ leads to

$$\frac{\gamma'_e}{\gamma} = \frac{1}{1+e} - \frac{\alpha\rho'_e}{1+\rho}$$

Thus, γ'_e has the same sign as

$1+\rho-\alpha\rho'_e(1+e) = 1-\left(\frac{e}{\mu}\right)^{\frac{1-\alpha}{\alpha}}$ which is positive for $e < \mu$ and negative for $e > \mu$.

Since $\tilde{e}_\mu(\bar{e})$ is increasing with respect to $\bar{e}$, $\varphi_\mu(\bar{e}) = \gamma(\tilde{e}_\mu(\bar{e}))$ is increasing for $\bar{e} < \mu$ and decreasing for $\bar{e} > \mu$.

The limit of $\tilde{e}_\mu(\bar{e})$ when $\bar{e}$ goes to 0 (resp. $+\infty$) is the solution of

$$\Delta(e,0,\mu) = \alpha\beta + 1 + e - \left(\frac{\mu}{e}\right)^{\frac{1-\alpha}{\alpha}} \alpha\beta = 0$$

(resp. $\Delta(e,+\infty,\mu) = \alpha\beta - \left(\frac{\mu}{e}\right)^{\frac{1-\alpha}{\alpha}}\left(\alpha\beta + \frac{1+e}{e}\right) = 0$)

These limits are finite and the corresponding limits of $\varphi_\mu(\bar{e}), \varphi_0 = \varphi_\mu(0)$ and $\varphi_\infty = \varphi_\mu(+\infty)$ are positive and smaller than 1.

The limit values 0 and $+\infty$ of $\bar{e}$ correspond to dotations of all permits to one of the two countries ($\bar{E}_2 = 0$ if $\bar{e} = 0$, $\bar{E}_1 = 0$ if $\bar{e} = +\infty$). These dotations lead to an equilibrium with $e^* = \tilde{e}_\mu(0)$, (resp. $e^* = \tilde{e}_\mu(+\infty)$) and with $z^* < 1$ if and only if $\bar{E}_1 < \varphi_0 b\mathcal{Y}(\bar{K})$ (resp. $\bar{E}_2 < \varphi_\infty b\mathcal{Y}(\bar{K})$.

Characterization of an Equilibrium with $z^* = 1$ and $q^* > 0$

Dotation of permits $\bar{E}_1$ and $\bar{E}_2$, and the total capital stock $\bar{K}$ are given.

- Assume $q > 0$ and $z = 1$. With capital stocks $\hat{K}_1$ and $\hat{K}_2 = \bar{K} - \hat{K}_1$, emissions are $\hat{E}_i = bA(\hat{K}_i)^\alpha H_i^{1-\alpha}, i = 1,2$ and their ratio $\hat{e} = \frac{\hat{E}_2}{\hat{E}_1}$ verifies (see Equation (15))

$$\left(\frac{\hat{K}_2}{\hat{K}_1}\right)^\alpha = \hat{e}\mu^{\alpha-1} = \hat{\rho}^\alpha$$

with $\hat{\rho} = \rho_\mu(\hat{e}) = \hat{e}^{\frac{1}{\alpha}}\mu^{1-\frac{1}{\alpha}}$

Thus, with $\hat{K}_2 = \hat{\rho}\hat{K}_1$, $\bar{K} = (1+\hat{\rho})\hat{K}_1$ and $H_1 + H_2 = (1+\mu)H_1$ we have

$$\begin{aligned}\mathcal{Y}(\bar{K}) &= A(H_1+H_2)^{1-\alpha}\bar{K}^\alpha \\ &= (1+\hat{\rho})^\alpha(1+\mu)^{1-\alpha}A(\hat{K}_1)^\alpha H_1^{1-\alpha}\end{aligned} \quad \text{(A6)}$$

- At the equilibrium on the permits market, $E_i = \hat{E}_i$ verify $\hat{E}_1 + \hat{E}_2 = \bar{E}_1 + \bar{E}_2 = \bar{E}$, $\bar{E} = (1+\hat{e})\hat{E}_1$ and

$$\bar{E} = (1+\hat{e})bA(\hat{K}_1)^\alpha H_1^{1-\alpha} = \gamma(\hat{e},\mu)b\mathcal{Y}(\bar{K}) \quad \text{(A7)}$$

where $\gamma_\mu(e) = \frac{1+e}{(1+\mu)^{1-\alpha}(1+\rho(e,\mu))^\alpha}$ is the same function γ as defined in Appendix 1 (see Equation (A5))

- The equilibrium condition 13 on the capital market implies (see Equation (10))

$$\hat{e}^{\frac{1}{\alpha}}\mu^{1-\frac{1}{\alpha}} = \hat{\rho} = \frac{\hat{K}_2}{\hat{K}_1} = \frac{\hat{\Pi}_2}{\hat{\Pi}_1} = \frac{\alpha\hat{x}\hat{E}_2 + \bar{E}_2}{\alpha\hat{x}\hat{E}_1 + \bar{E}_1} \quad \text{(A8)}$$

where $\hat{x} = \frac{1}{bq} - 1$ verifies from Equation (10) $\hat{x} \geq \beta$

Lemma 9 *There exists an equilibrium with* $z^* = 1$ *and* $q^* > 0$ *if and only if there exists a solution* $\hat{e}$ *of* (*A*7) *and a solution* $\hat{x} \geq \beta$ *of* $\Gamma(\hat{e},x) = 0$, *where*

$$\Gamma(e,x) = \alpha x + \frac{1+e}{1+\bar{e}} - \left(\frac{\mu}{e}\right)^{\frac{1-\alpha}{\alpha}}\left(\alpha x + \frac{\bar{e}(1+e)}{(1+\bar{e})e}\right)^{5}$$

Proof. The existence of an equilibrium with $z^* = 1$ and $q^* > 0$ implies that $e^* = \frac{E_2^*}{E_1^*}$ verifies (A7) and that $x^* = \frac{1}{bq^*} - 1$ verifies (A8) which is equivalent to $\Gamma(e^*, x^*) = 0$.

Conversely, consider $\hat{e} > 0$ verifying (A7) and $\hat{x} \geq 0$ verifying $\Gamma(\hat{e},\hat{x}) = 0$. Define $K_1^* = \hat{K}_1$ with (A7), $E_1^* = \frac{\bar{E}}{1+\hat{e}}$, $K_2^* = \bar{K} - K_1^*$, $E_2^* = \bar{E} - E_1^*$, $q^* = \frac{1}{b(1+\hat{x})} > 0$. These values verify the equilibrium conditions on both markets of permits and capital with $z^* = 1$. Thus an equilibrium with $z^* = 1$ and $q^* > 0$ exists.

Lemma 10. *There exists an equilibrium with* $z^* = 1$, $q^* > 0$ *and* $e^* = \mu$ *if and only if* $\bar{e} = \mu$ *and* $\bar{E} = b\mathcal{Y}(\bar{K})$. *Then,* $\frac{E_2^*}{E_1^*} = \mu = \frac{K_2^*}{K_1^*}$ *defines an equilibrium with* $z^* = 1$,

[5]This function is similar of the function Δ in Appendix A1 and for $x = \beta$, it coincides with : Thus, $\Gamma(\tilde{e}(\bar{e},\mu),\beta) = 0$

$e^* = \mu$ and any $q^* > 0$, $q^* \le \dfrac{1}{b(1+\beta)}$

Proof. $\Gamma(\mu, x) = \dfrac{1+\mu}{\mu(1+\overline{e})}(\mu - \overline{e})$ does not depend on x. Thus, if $e^* = \mu$ is an equilibrium with $z^* = 1$ and $q^* > 0$, $\Gamma(\mu, x^*) = 0$ implies $\overline{e} = \mu$ and (A7) implies $\overline{E} = b\mathcal{Y}(\overline{K})$ since $\gamma_\mu(\mu) = 1$.

Conversely, under these conditions, $\hat{e} = \mu$ and any $\hat{x} \ge \beta$ verify the existence conditions of Lemma 9.

Lemma 11. *If $\overline{e} \ne \mu$ there exists an equilibrium with $z^* = 1$ and $q^* > 0$ if and only if $\varphi_\mu(\overline{e}) b\mathcal{Y}(\overline{K}) \le \overline{E} < b\mathcal{Y}(\overline{K})$ where $\varphi_\mu(\overline{e}) = \gamma_\mu(\tilde{e}_\mu(\overline{e}))$. this equilibrium is unique and verifies: if $\overline{e} > \mu$, $\mu < e^* \le \tilde{e}_\mu(\overline{e}) < \overline{e}$ and if $\overline{e} < \mu$, $\overline{e} < \tilde{e}_\mu(\overline{e}) \le e^* < \mu$*

Proof. The derivatives of Γ verify : $\Gamma'_e > 0$ and $\Gamma'_x = \alpha\left(1 - \left(\dfrac{\mu}{e}\right)^{\frac{1-\alpha}{\alpha}}\right)$.

- Assume there exists an equilibrium with $z^* = 1$, $q^* > 0$ and $e^* > \mu$.

$\Gamma(e^*, x)$ increases from $\Gamma(e^*, \beta)$ to $+\infty$ when x increases from 0 to $+\infty$. The existence of $x^* \ge \beta$ solution of $\Gamma(e^*, x^*) = 0$ is equivalent to

$$\Gamma(e^*, \beta) \le 0 = \Gamma(\tilde{e}_\mu(\overline{e}), \beta) \Leftrightarrow e^* \le \tilde{e}_\mu(\overline{e})$$

And $\mu < e^* \le \tilde{e}$ implies $\mu < e^* \le \tilde{e} < \overline{e}$ (Lemma 6) and $\gamma_\mu(\mu) > \gamma_\mu(e^*) \ge \gamma_\mu(\tilde{e})$ since $\gamma'_e < 0$ for $e > \mu$. (Lemma 8)

With (A7), for $\hat{e} = e^*$ we obtain the necessary conditions of Lemma 11 and the unicity of $e^* > \mu$ solution of (A5) and of $x^* \ge \beta$ solution of $\Gamma(e^*, x^*) = 0$.

Existence results from Lemma 9.

- Assume there exists an equilibrium with $z^* = 1$, $q^* > 0$ and $e^* < \mu$.

$\Gamma(e^*, x)$ decreases from $\Gamma(e^*, \beta)$ to $-\infty$ when x increases from 0 to $+\infty$. The existence of $x^* \ge \beta$ solution of $\Gamma(e^*, x^*) = 0$ is then equivalent to

$$\Gamma(e^*, \beta) \ge 0 \Leftrightarrow e^* \ge \tilde{e}_\mu(\overline{e})$$

With $e^* < \mu$ it implies $\overline{e} < \tilde{e} \le e^* < \mu$ and $\gamma_\mu(\mu) > \gamma_\mu(e^*) \ge \gamma_\mu(\tilde{e})$ since $\gamma'_e > 0$ for $e < \mu$. Thus the same conclusions as in the case $e^* > \mu$ apply.

The proof is complete since $e^* = \mu$ is excluded when $\overline{e} \ne \mu$ (Lemma 10).

