NBER WORKING PAPER SERIES

CAPITAL CONTROLS AND THE TIMING OF EXCHANGE REGIME COLLAPSE

Daekeun Park

Jeffrey Sachs

Working Paper No. 2250

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1987

The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #2250
May 1987

Capital Controls and the Timing of Exchange Regime Collapse

ABSTRACT

This paper investigates the nature of balance of payments crises in regimes with capital controls. It extends earlier work on capital controls by assuming that households manage their consumption and asset portfolios to maximize intertemporal utility. Our main result is that capital controls are effective in delaying, but not preventing, a breakdown of a fixed exchange rate regime in the presence of money-financed fiscal deficits.

Daekuen Park
Department of Economics
Littauer Center
Harvard University
Cambridge, MA 02138

Jeffrey Sachs
Department of Economics
Littauer Center
Harvard University
Cambridge, MA 02138

I. INTRODUCTION

The vast and growing literature on balance of payments crises focuses on the incompatibility of persistent budgetary deficits and fixed exchange rates. Under the standard assumption of high international capital mobility, a government that finances its deficit through domestic credit expansion and simultaneously pegs the exchange rates, will experience a simultaneous offsetting loss of foreign exchange reserves. Eventually, a minimum level of official reserve holdings will be reached, and the government will be forced to allow the exchange rate to float (assuming that there is no correction of the underlying budget deficit). The transition to floating rates typically involves a speculative attack on the central bank reserves in the "instant" (or period) just before the float begins.

The assumption of internationally mobile capital is critical to the adjustment mechanism just outlined, at two points. First, the domestic credit expansion that finances the budget deficit generally leads to an immediate offset of official reserves because domestic agents are free to convert domestic currency into foreign financial assets at the pegged exchange rate. Second, the possibility of a "speculative attack", in which there is a large, discrete loss of central bank reserves, also depends on the ability of domestic residents instantly to convert part of their money stock into foreign assets on the eve of the exchange rate collapse.

It is often for these reasons that governments resort to capital controls to help to maintain an exchange rate peg. By preventing the instantaneous conversion of domestic money into foreign financial assets, governments try to preserve their central bank reserves for a longer period of time, and thereby forestall the exchange rate collapse. Under tight capital controls, according to which domestic residents are not allowed to hold foreign assets, the central bank loses foreign reserves via current account deficits only. Reserves can decline only gradually, with a current account deficit, rather than in the sudden step of a speculative attack, in which part of the domestic money stock is traded for the central bank's foreign exchange.

As many governments have learned, however, the use of exchange controls does not generally prevent the collapse of the fixed rate (though in some cases it may), but rather it delays the collapse and changes the adjustment mechanism. With exchange controls and fixed rates, domestic credit expansion *does* lead to a rise in domestic real money balances, since the increase in money cannot be converted to foreign assets. The buildup in real money balances leads to a rise in domestic absorption and a current account deficit. The deficit, in turn, leads to reserve losses of the central bank, and the eventual collapse of the fixed rate. However, the collapse comes smoothly in this case, with reserves trickling down to the minimum permissible level, rather than being exhausted in a single stroke by a speculative attack.

Moreover, the exchange rate behavior is different in the cases of free versus restricted capital mobility. With free capital mobility, the transition from the fixed to floating rate occurs without a discrete jump in the exchange rate, while in the capital control case, there is a discrete depreciation of the currency at the moment of breakdown of the exchange rate peg.[1] It should be clear why an *anticipated* discrete depreciation is possible only in the presence of capital controls: if agents are free to swap money for foreign assets, they would always do so the moment before an anticipated depreciation, thus causing it to occur before the "anticipated" date. In equilibrium with capital mobility, the speculative attack is timed precisely so that asset markets clear continuously without any discrete anticipated exchange rate change.

Despite the widespread use of capital controls to forestall exchange rate changes, there has been relatively little modelling of the effects of capital controls on the balance of payments cirses. Wyplosz (1986) provides the pioneering model in this area. This paper extends that work by couching the analysis in a model in which private agents are intertemporal utility maximizers, in contrast to the Wyplosz model in which private sector behavior is not derived from utility maximization. For this purpose we use the elegant model introduced by Calvo (1986, 1987), and follow his notation closely.

Section II of this paper introduces the Calvo model, and section III uses the model to analyze a balance of payments crisis under the standard assumption of complete capital mobility. Section IV then repeats the analysis under the alternative assumption of zero captial mobility, and the adjustment paths in the

---

1. The absence of a discrete jump in the exchange rate holds strictly only in continuous time, deterministic models. In discrete time (Obstfeld, 1984), or with stochastic monetary policy (Penati and Pennachi, 1986), discrete changes in the exchange rate may occur as equilibrium outcomes in models with free capital mobility.

two cases are compared in some detail. Section V offers some conclusions and a discussion of possible extensions.

II. THE MODEL

The basic model is based on Calvo (1987). We have a small open economy with perfect mobility of capital and goods. There is a single representative consumer with infinite time horizon whose utility at time t, U_t, is :

$$U_t = \int_t^{\infty} u(c_s)e^{-r(s-t)}ds \tag{1}$$

where c_s is real consumption at time s.

There is only one good in the world. Since this good is freely tradable, domestic inflation rate is, by purchasing power parity, equal to the rate of exchange depreciation ϵ assuming zero world inflation rate. The world interest rate, denoted by r, is assumed to be constant. Therefore, the domestic interest rate, which is the opportunity cost of holding money, is $r+\epsilon$ by interest arbitrage. Note that for convenience, the subjective rate of time preference in (1) and the world interest rate are assumed to be equal.

Since goods are assumed to be perishable, foreign bonds and money are the only means of storing wealth. Thus, the real wealth, a_t , is :

$$a_t = m_t + b_t \tag{2}$$

where m_t is real money balance held by the consumer and b_t is real bond holdings. a_t evolves according to the following relation :

$$\dot{a} = y + rb + g - \epsilon m - c \tag{3}$$

where y is a constant flow of real output (GDP), g is real lump-sum transfer payment from government, and ϵm is inflation tax paid by the consumer. Using (2), eq. (3) can be rewritten as follows.

$$\dot{a} = y + ra + g - (r+\epsilon)m - c \tag{4}$$

The consumer faces a cash-in-advance constraint, according to which :

$$m_t \geq \alpha c_t \,, \quad \alpha > 0 \tag{5}$$

Money is return-dominated by bond so long as the domestic interest rate is positive. Thus, (5) will be an equality as long as $r + \epsilon > 0$.[2]

Solving the problem of maximizing (1) subject to the constraints (4) and (5), we get the following first order condition.

$$u'(c_t) = \lambda_t \left[1 + \alpha(r+\epsilon_t) \right] \tag{6}$$

$$\dot{\lambda}_t = 0$$

From this condition we know that given a path of g and y , real consumption remains constant so long as the inflation rate is constant, and the higher is the inflation rate the lower is real consumption.

The other component of the domestic economy is the government. The government holds foreign bonds as reserves and is committed to two roles. One is to transfer a lump-sum amount g in each period according to a predetermined

2. Since we are going to confine our analysis to the case of a fixed or depreciating exchange rate, this is a reasonable assumption.

schedule and the other is to maintain a fixed exchange peg. These government roles are financed by interest from bond holdings, new money printing and inflation tax.[3] The real government reserves, denoted by k_t , can be changed by two ways. One is by the flow of government surplus or deficit. The other is by the stock adjustment of the consumer to buy or sell foreign bonds. Thus, k_t evolves according to the following equation ,except for the points where m jumps.

$$\dot{k} = rk + \dot{m} + \epsilon m - g \tag{7}$$

At points where m takes a jump of Δm ,

$$\Delta k = \Delta m \tag{8}$$

Following Krugman (1978) we will assume that there is a lower bound to k ,which will be assumed to be zero.[4]

$$k_t \geq 0 , \quad t \in (-\infty,\infty) \tag{9}$$

Since under the fixed exchange regime the government should meet the demand to exchange domestic currency for foreign currency at a predetermined exchange rate, the ability of the government to defend a given level of fixed exchange rate is limited by the availability of foreign reserves. Therefore, when the government runs out of reserves, the fixed exchange regime is no longer sustainable. Unless the government adopts some corrective measures, it has to allow the exchange rate to

3. Of course, inflation tax is not applicable under fixed exchange peg. However, once the fixed exchange regime breaks down and the float begins, inflation tax is the major source of government revenue.

4. A lower bound of zero is the case when the government cannot borrow reserves.

float.

The net claims against the rest of the world of this economy, denoted by w , is the sum of the private and government holdings of bonds, that is, $b + k$. Since the gross national product of this economy is $y + rw$, the current account balance defined as GNP minus domestic absorption can be written as

$$\dot{w} = y + rw - c \tag{10}$$

For this economy to stay in steady state under fixed exchange regime, $\dot{w}$ should be equal to zero. It follows from eq. (10) that real consumption should be constant at $y + rw$. By the cash-in-advance constraint, m is also constant at $m = \alpha (y + rw)$. The steady state economy also requires that $\dot{k}$ be equal to zero. Since $\dot{m} = 0$ in the steady state, and $\epsilon = 0$ under the fixed exchange regime, the level of government expenditure which is compatible with steady state under fixed exchange peg is, from eq.(7), $\bar{g} = r\bar{k}$, which means the government should maintain a balanced budget.

III. BALANCE OF PAYMENTS CRISIS UNDER FREE CAPITAL MOBILITY

We assume that up to time zero the economy has been following a steady state path along which the consumer consumes exactly the amount of his personal income and government maintains a balanced budget. As a result, bond holdings of the government and the consumer are kept constant at the positive levels of $\bar{k}$ and $\bar{b}$ respectively. At time zero, there is an unanticipated permanent increase in the transfer payment from $\bar{g}\ (= r\bar{k})$ to $\bar{g} + \Delta g$,which will be financed by an increase in the money supply. Since the consumer will not hold all of the increased money, the government will eventually lose all of its reserves if it maintains the increased level of transfer.

The consumer knows that the fixed exchange regime will collapse at time T and that starting from time T the exchange rate will depreciate at a constant rate of ϵ . Since consumption and money holdings will be constant from (6), and since k will be equal to zero, the only source of government revenue to support the positive amount of transfer after the collapse is inflation tax of the amount ϵm . Thus, to keep the nonnegative reserve constraint of (9), ϵ should be uniquely set at

$$\epsilon = \frac{g}{m} = \frac{\bar{g} + \Delta g}{m} \tag{11}$$

For the sake of concreteness, we will investigate the particular case of log utility, that is, $u(c) = \ln c$. As was discussed earlier, consumption is constant in the intervals $[0,T)$ and $[T,\infty)$. We'll denote consumption in these intervals as x and z respectively. Then, using (6), we get

$$x\,(1 + \alpha r) = z\,[1 + \alpha(r+\epsilon)] \tag{12}$$

As for k , it jumps at time zero as the consumer adjusts his money holding to his new consumption profile. Thus, k at time zero is

$$k_0 = \bar{k} + \Delta k_0 = \bar{k} + \Delta m_0 \tag{13}$$

$$\Delta m_0 = \alpha\,(x - y - rw_0) \quad , \quad w_0 = \bar{k} + \bar{b}$$

k is also expected to jump at time T. So, we distinguish between the reserve right before the collapse, denoted by k_{T_-} and that right after the collapse, denoted by

k_{T+} . Integrating (7), we can get

$$k_{T-} = (\bar{k} + \Delta k_0) e^{rT} - g (e^{rT} - 1) / r \tag{14}$$

$$= -\alpha (y + rw_0 - x) e^{rT} - \Delta g \frac{e^{rT}}{r} + \frac{g}{r}$$

By the zero-minimum-reserve assumption $k_{T+} = 0$. Thus,

$$\Delta k_T = k_{T+} - k_{T-} = -k_{T-} \tag{15}$$

Since the amount of speculative attack is exactly matched by the size of the change in money holdings,

$$\Delta k_T = -\Delta m_T = m_{T-} - m_{T+} = \alpha (x - z) \tag{16}$$

Integrating eq. (10) we get

$$y + rw_T = (y + rw_0 - x) e^{rT} + x \tag{17}$$

From the fact that the economy is in steady state after T with consumption equal to GNP, we get

$$z = y + rw_T \tag{18}$$

Substituting (18) in (17),

$$x - z = - (y + rw_0 - x) e^{rT} \tag{19}$$

Substituting (19) in (16) and equating (15) and (16), we can get

$$\frac{\Delta g}{r} e^{rT} - \frac{g}{r} = 0 \tag{20}$$

Solving this equation for e^{rT}, we can finally get an expression that implicitly determines the time till collapse, T :

$$e^{rT} = \frac{g}{\Delta g} = \frac{\bar{g} + \Delta g}{\Delta g} \tag{21}$$

The following equations show the behavior of some key variables during the period before the collapse.

$$x = y + r\bar{b} + \bar{g} + \frac{1}{1 + \alpha r} \Delta g \tag{22}$$

$$b_t = b_0 = \bar{b} - \frac{\alpha}{1 + \alpha r} \Delta g \tag{23}$$

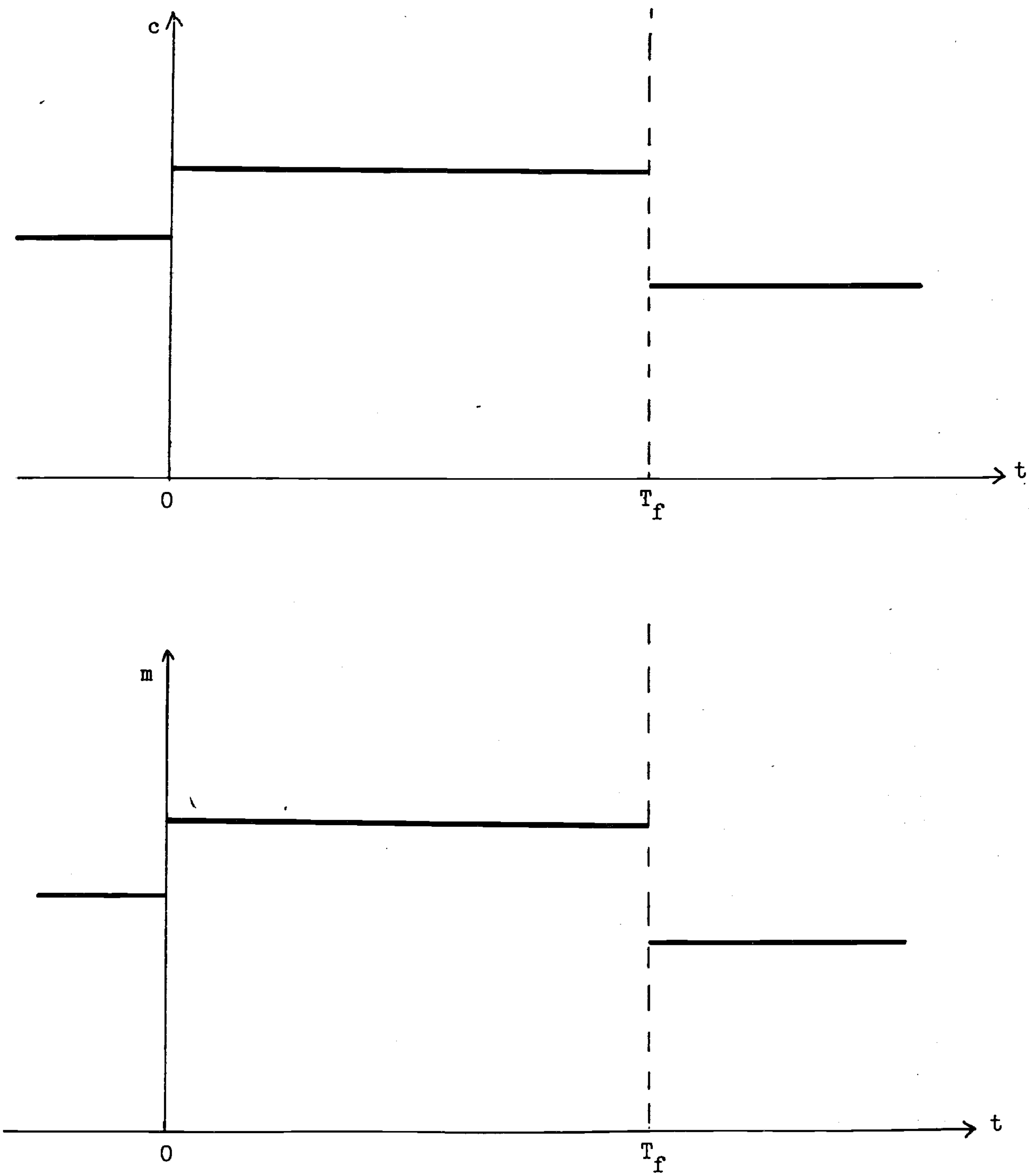
$$k_t = k_0 - \frac{1}{(1 + \alpha r) r} (e^{rt} - 1) \Delta g \tag{24}$$

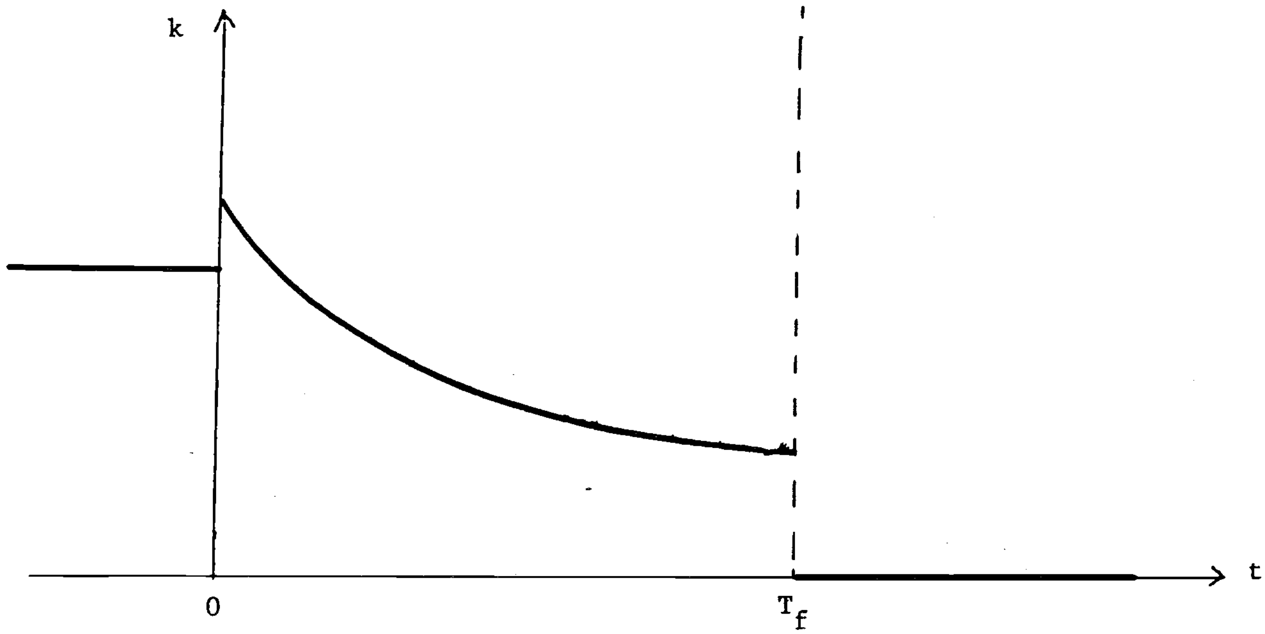
Thus, we can see that consumption does not increase as much as the increase in government transfer. This is because the consumer converts part of his bond holdings into money at time zero to support the increase in consumption and thus his interest income is reduced. The level of consumption after the collapse is lower than that before the collapse, because after the collapse personal income is reduced by the amount of inflation tax.[5] Figure 1. shows the path of consumption together with the paths of other variables of interest. Households adjust their money holdings to the new reduced level of consumption by exchanging money holdings for foreign bonds. This adjustment process serves as the speculative attack, which depletes the remaining central bank reserves in an instant.

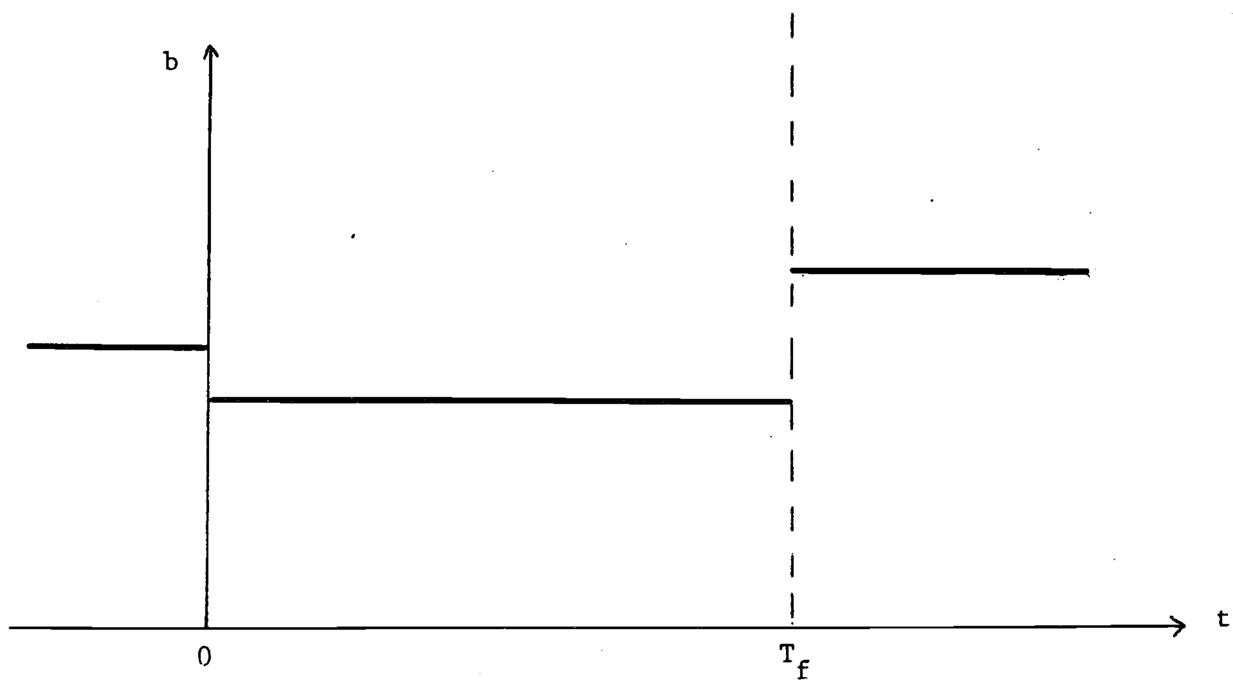
As is shown in eq. (24), the government loses its reserves over time. However, Eq. (23) shows that bond holdings are fixed and thus, except for the speculative attack, capital outflow is not the cause of reserve outflow. Since change in reserves

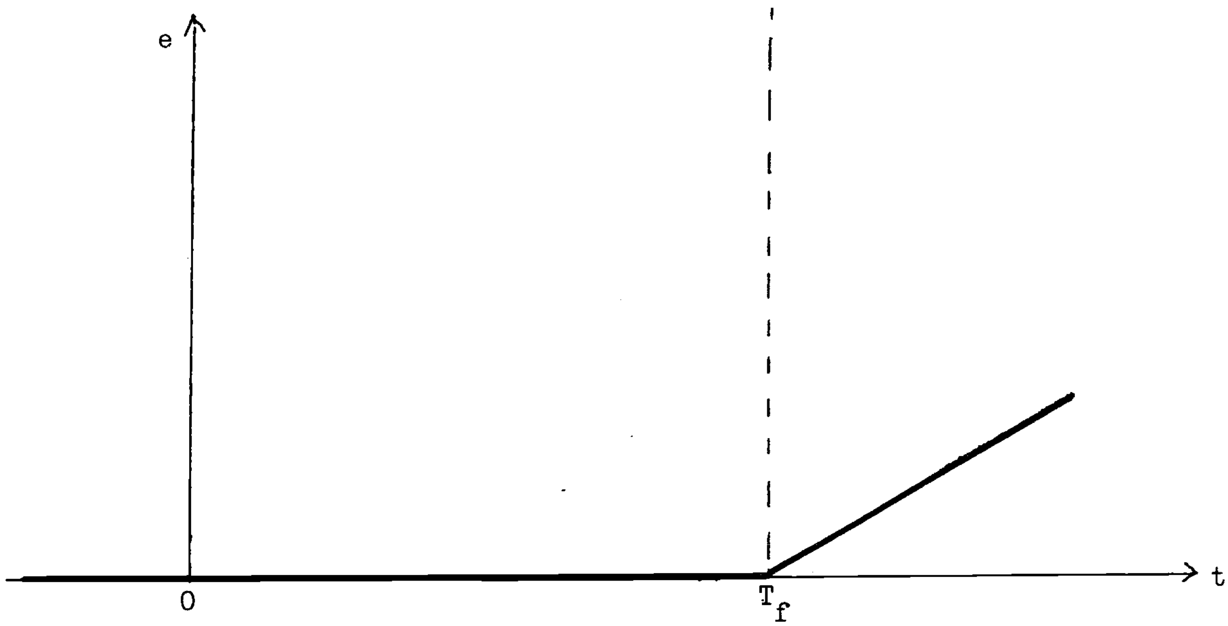
5. In other words, the opportunity cost of consumption is higher after the collapse. When the cash-in-advance constraint is binding, (4) can be rewritten as $\dot{a} = y + ra + g - [1+\alpha(r+\epsilon)]\ c$. Obviously, the cost of consumption, $1 + \alpha(r+\epsilon)$ is higher after the collapse, when $\epsilon > 0$.

FIGURE 1. Paths of key variables under full capital mobility

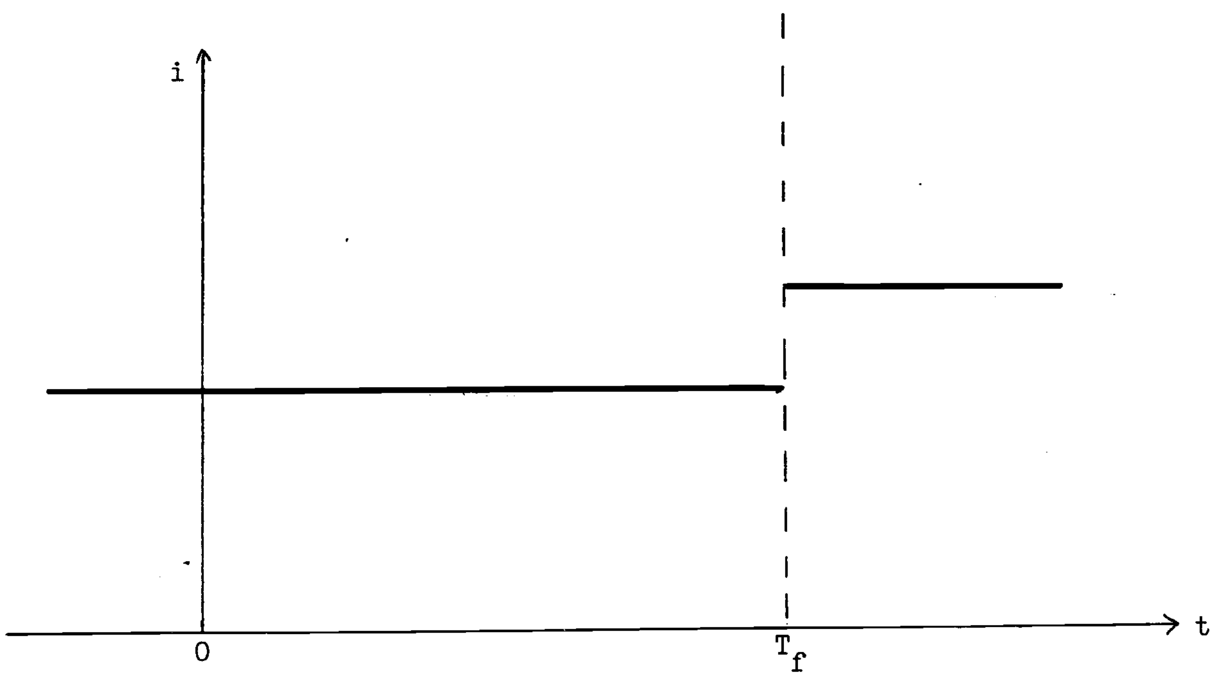



k
0
T_f
t


b
0
T_f
t



e
0
T_f
t



i
0
T_f
t

is the sum of the current and capital account balance, it is obvious that the reserve outflow before the final run on the government reserves is caused by a current account deficit.

After the collapse, the economy will be in steady state with key variables constant at the following levels :

$$b = \bar{b} + \frac{\alpha}{1 + \alpha r}\bar{g} \tag{25}$$

$$z = y + rb = y + r\bar{b} + \frac{\alpha r}{1 + \alpha r}\bar{g}$$

In conclusion, a permanent increase in government expenditure gives the economy a nonneutrality result of reducing steady state consumption and money holdings and raising the rate of exchange depreciation from zero to ϵ.

IV. THE CASE OF CAPITAL CONTROLS

Now, we consider the case where the government imposes a control on capital transaction at time zero. We assume that neither the capital control nor the increase in g are expected by the consumer. We consider a very restrictive type of capital control which prohibits the consumer from changing his bond holdings. Thus, from time zero private bond holdings will be fixed at the level $\bar{b}$. Since the government continues to supply foreign exchange for trade of goods , it is still possible that government reserves will be eventually depleted by an accumulation of current account deficits. We assume that the government will not remove the capital control after the collapse.[6]

One direct result of the capital control is that the interest parity relation between domestic and world inerest rates, which holds under free capital mobility, will not hold any more because domestic and foreign bonds are no longer freely substitutable. To be able to solve for the domestic nominal interest rate, it is convenient to introduce a market for domestic bonds, with a zero net supply of bonds. Each household then demands domestic bonds in nominal amount D_t, and with nominal interest rate i_t. Real domestic bond holdings are then $d_t = D_t/P_t$. In equilibrium, we will require $d_t \equiv 0$, and will find the equilibrium value of i_t consistent with domestic market clearing. Note that adding such a market under free capital mobility is superfluous. The home interest rate would simply be $i_t = r + \epsilon_t$, and d_t and b_t would be perfect substitutes.

Now, real private wealth, a_t , is defined as :

$$a_t = m_t + d_t \tag{26}$$

a_t evolves according to the following equation,

$$\dot{a}_t = i_t d_t + y + r\bar{b} + g - \epsilon_t (m_t + d_t) - c_t \tag{27}$$

except at times of a discrete jump of the exchange rate, when we have $a_{T+} = a_{T-}/(1 + \gamma)$, where γ is the instantaneous jump in the exchange rate.

Capital controls cannot prevent the eventual collapse of the fixed exchange

6. Actually, removal of capital control after the collapse does not make any difference, for domestic currency is no longer convertible to foreign currency once the government runs out of reserves. However, these alternative assumptions will give different results once we introduce storable goods.

regime. Initially, the economy is in current account balance, with $c_0 = y + r\ (\bar{b} + \bar{k}\ )$, and $m_0 = \alpha c_0$. When transfer payments are raised from $\bar{g}\ (=r\bar{k})$ to $\bar{g} + \Delta g$, the consumer does not instantaneously change consumption, since consumption at time zero is constrained by the real money stock held right before time zero. As a result, consumption at time zero remains equal to the level of consumption the instant before, and the current account remains in balance at $t = 0$. Since private income at time zero is higher than consumption, the consumer ends up with positive saving in the form of money which can be used to increase consumption in the next instant. As consumption increases over time, the current account deficit increases. The accumulation of current account deficits gradually drives the reserves down to zero.

Another important result of the capital control is that, as was mentioned in the introduction, the exchange rate can take a discrete jump in the process of the exchange regime collapse. This is because, unlike the case under free capital mobility, speculative attack is impossible under the capital control[7] and people cannot avoid windfall losses from the jump in exchange rate. Since real stock of wealth, a_t, is defined as nominal stock of wealth divided by exchange rate, the discrete jump in the exchange rate leads to a discrete jump in the real stock of wealth.

7. If goods are storable, speculative attack through current account is possible. In reality, we observe accumulation of consumer durables in expectation of devaluation. However, we assume this possibility away by assuming that goods are perishable.

Understanding that the consumer knows and takes as given the eventual jump in exchange rate, γ, and the timing of exchange regime collapse, T, the problem of the representative consumer can be formulated as follows.

$$\max_{c_t, m_t} \int_0^\infty \ln c_t \; e^{-rt} \, dt \tag{28}$$

$$\text{s.t.} \quad \dot{a}_t = (i_t - \epsilon_t) \, a_t + y + g + r\bar{b} - i_t m_t - c_t$$

$$m_t \geq \alpha c_t$$

$$\epsilon_t = 0 \; , \; t < T \quad \text{and} \quad \epsilon_t = \epsilon \; , \; t < T$$

$$a_{T-} = (1 + \gamma) \, a_{T+}$$

Solving this problem, we can get the following first order condition.

$$(a) \quad \frac{1}{c_t} = \lambda_t \, (1 + \alpha i_t) \tag{29}$$

$$(b) \quad \mu_t = \lambda_t i_t$$

$$(c) \quad \dot{\lambda}_t = [\, r - (\, i_t - \epsilon_t \,) \,] \, \lambda_t$$

$$(d) \quad \mu_t \, (m_t - \alpha c_t) = 0 \quad , \mu_t \geq 0$$

$$(e) \quad (1 + \gamma) \lambda_{T+} = \lambda_{T-}$$

where λ is the costate variable and μ is the Lagrangian multiplier for the cash-in-advance constraint.

To find the optimal path, we make a conjecture on the optimal path and show that this path satisfies the above conditions. Our conjecture is that domestic interest rate remains positive during the period before the collapse. Since the return from holding money is zero under the fixed exchange regime, a positive domestic interest rate requires that the cash-in-advance constraint hold as an equality. This is also obvious from the complementary slackness condition.[8]

Our conjecture for the path after the collapse is that the economy will be in steady state with

$$c = y + r\bar{b} \tag{30}$$

$$m = \alpha\,(y + r\bar{b})$$

$$\epsilon = \frac{g}{\alpha\,(y + r\bar{b})}$$

Appendix A proves that the conjecture we made satisfies the first order condition in (29). In equilibrium, $d_t = 0$, and eq. (25) can be rewritten as follows.

$$\dot{m}_t = y + g + r\bar{b} - c_t \tag{31}$$

Substituting $m_t = \alpha c_t$ in eq. (31), we can get the following differential equation for c_t.

$$\dot{c}_t = -\frac{1}{\alpha} c_t + \frac{1}{\alpha}(y + g + r\bar{b}) \tag{32}$$

$$c_0 = \frac{1}{\alpha} m_0 = y + \bar{g} + r\bar{b}$$

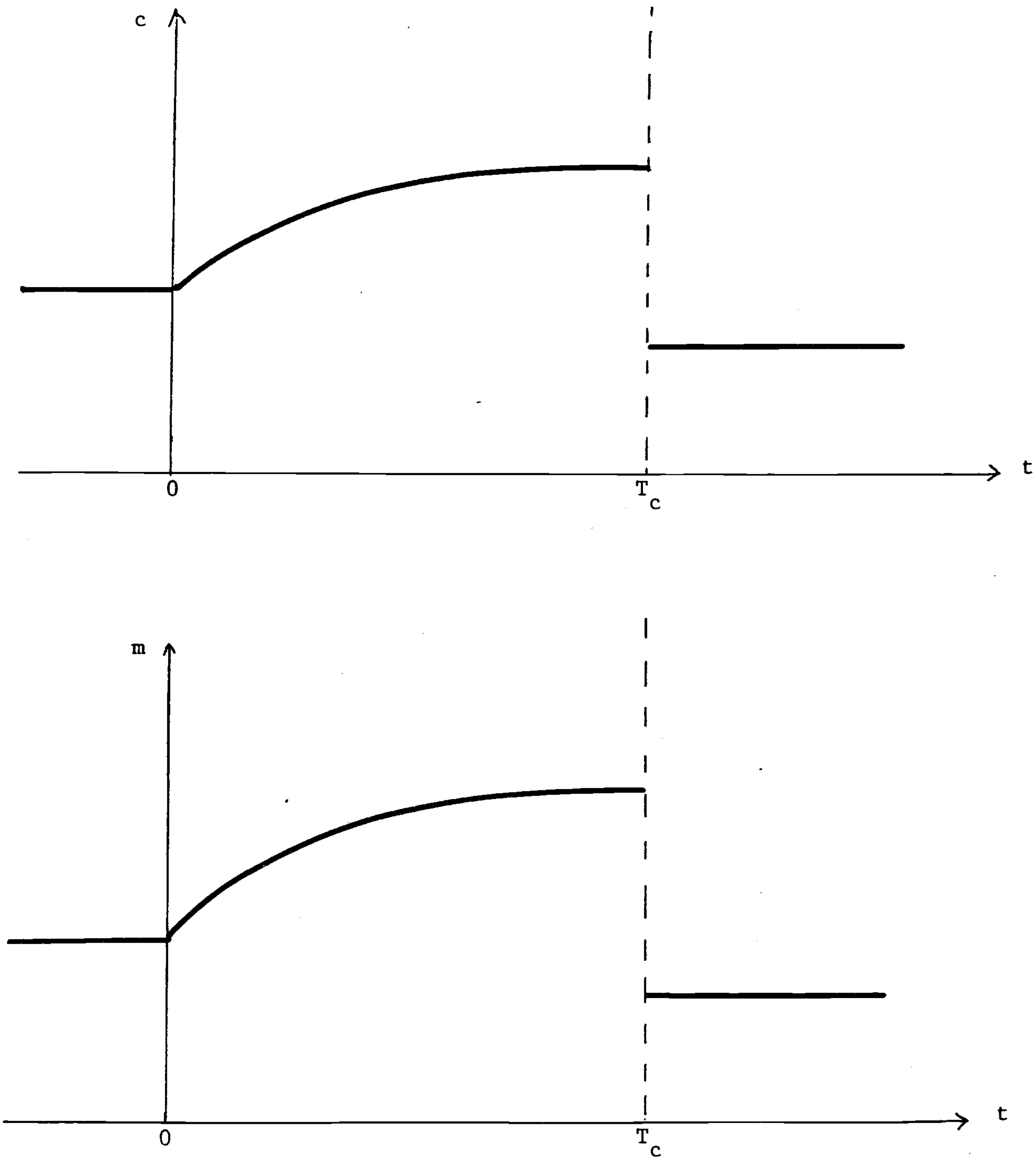
Solving (32), we can get

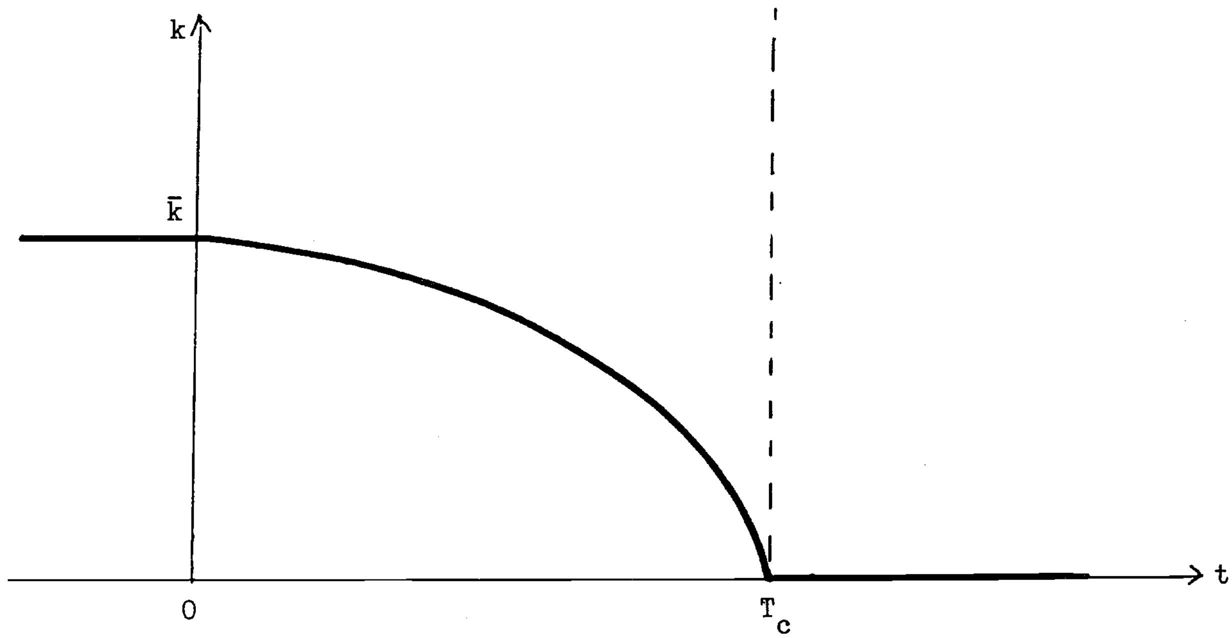
$$c_t = y + \bar{g} + r\bar{b} + \Delta g\,(1 - e^{-t/\alpha}) \tag{33}$$

Thus, real consumption is increasing between time zero and T, but it is always less than personal income, which equals $y + r\bar{b} + \bar{g} + \Delta g$. Figure 2. shows the optimal consumption path under the capital control together with the optimal

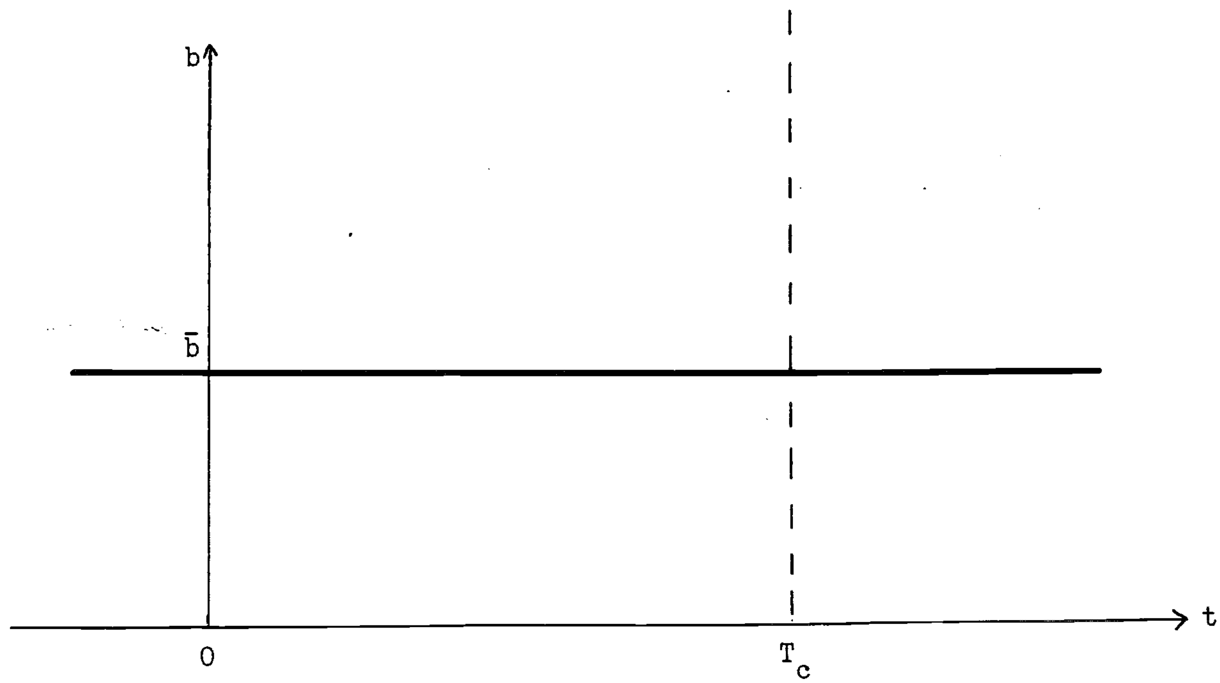
8. From (29b), $\dot{i}_t > 0$ implies $\mu_t > 0$. From (29d), $\mu_t > 0$ implies $m_t = \alpha c_t$.

FIGURE 2. Paths of key variables under capital control.

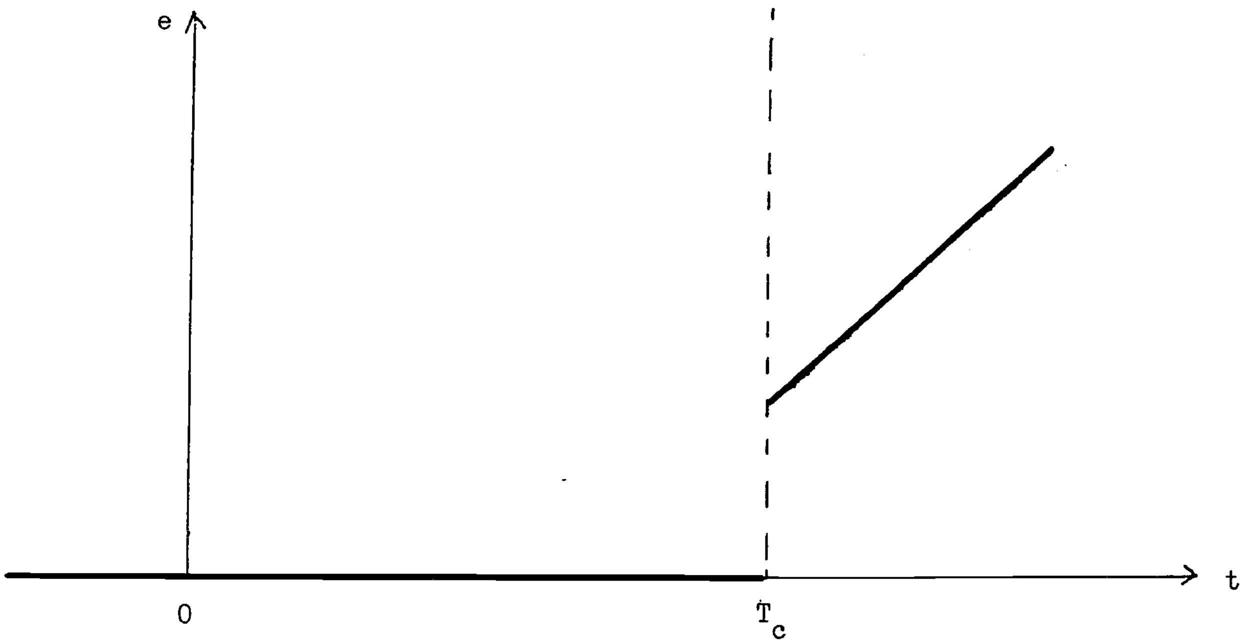




k
k̄
0
T_c
t



b
b̄
0
T_c
t


e
t
0
T_c

paths of the other variables of interest.

Since the cash-in-advance constraint is binding, m_t increases over time. The amount of money held by the consumer right before the collapse is

$$m_{T-} = \alpha (y + r\bar{b}) + \alpha\bar{g} + \alpha\Delta g (1 - e^{-t/\alpha}) \tag{34}$$

Since the second term in eq. (32) is positive, $m_{T-} > m_{T+}$. Thus, we have a discrete jump down of the real balance at the time of collapse. Unlike the case of free capital mobility, this jump in m_t is not caused by speculative attack against central bank reserves, for the consumer is not allowed to exchange money for foreign bond. Instead, it is caused by a discrete jump in exchange rate. Thus,

$$\mathrm{e}_{T+} = (1 + \gamma) \mathrm{e}_{T-} = \frac{m_{T-}}{m_{T+}} \mathrm{e}_{T-} \tag{35}$$

The reason why the exchange rate jumps is that when reserves run out, the flow supply of goods to the economy drops from $y + r\bar{b} - k_{T-} (= c_{T-})$ to $y + r\bar{b} (= c_{T+})$. The consumer tries in vain to buy goods with a now excess level of money, and ends up with depreciating the currency and raising the price level. After the collapse, the exchange rate will depreciate at the constant rate of $g/(y + r\bar{b})$. Note that since money holdings after the collapse under the capital control are lower than those under free capital mobility, the steady state rate of depreciation is higher in the case of the capital controls.

The movement of the shadow domestic interest rate under the capital control can be tracked by the following differential equation.

$$\frac{di_t}{dt} = \left(i_t + \frac{1}{\alpha} \right) \left(i_t - r - \frac{\dot{c}_t}{c_t} \right) \quad , \; t \leq T \tag{36}$$

As is discussed in appendix A, it can be easily shown that the domestic interest remains higher than r until the time of collapse and equals the steady-state level $r + \epsilon$ at the moment of the collapse.

As was discussed before, there is no speculative attack under the capital control. Hence, the exchange regime will collapse when k_t hits zero. Combining (7) and (31), we can get the following path of k_t .

$$k_t = \bar{k} - \Delta g \left(\frac{e^{rt} - 1}{r} + \frac{e^{rt} - e^{-t/\alpha}}{r + 1/\alpha} \right) \tag{37}$$

Setting $k_T = 0$, we can get the following equation for the timing of collapse.

$$e^{rT} + \alpha r e^{-T/\alpha} = (1 + \alpha r) \frac{g}{\Delta g} \tag{38}$$

Appendix B shows that the solution to the eq. (38) is bigger than the T that satisfies (20). Thus, the capital control specified in this section *does* delay the timing of collapse, and it does so by eliminating the chance of speculative attack and by restricting consumption and thus reducing the average current account deficit in the period before the collapse.

V. EXTENSIONS

This paper showed how controls on capital flows can be imposed to delay the timing of the balance of payments crisis when the crisis is inevitable. An interesting extension may be to introduce storable goods with a positive rate of economic depreciation. Although the existence of storable goods would not change the results in the case of free capital mobility, it would make a difference under capital controls. There are two reasons why storable goods can make difference.

First, a speculative attack would be possible even under the capital control. This is because households, facing a loss of financial wealth via devaluation, would import storable goods as an inflation hedge. Note that unlike the case with free capital mobility, the speculative attack would be channeled through the current account. Second, households would have the incentive to save in the form of money before the collapse. This is because households can convert their excess money to storable goods right before the collapse and consume these goods afterwards.

These two factors would present offsetting forces in determining the timing of collapse. A speculative attack shortens the period before the collapse, whereas additional money holdings would delay the collapse. A formal analysis will be necessary to show which force is stronger.

APPENDIX A

We will prove that the conjecture on the optimal path we made in section IV actually solves the problem (28). For this purpose, we'll define two functions H and Φ as follows.

$$H(a,c,m,\lambda,\mu) = e^{-rt}\,[\,\ln c + \lambda\,(\,(i-\epsilon)\,a + y + g + r\bar{b} - im - c\,) + \mu\,(m - \alpha c)\,]$$

$$\Phi(a_{T-},a_{T+}) = v\,[\,(1+\gamma)a_{T+} - a_{T-}\,]$$

The Euler-Lagrange necessary condition of this problem is :

$$\frac{\partial H}{\partial c} = \frac{\partial H}{\partial m} = 0 \qquad \text{(A1)}$$

$$\frac{\partial}{\partial t}(e^{-rt}\lambda) = -\frac{\partial H}{\partial a} \quad , t \neq T$$

$$\lambda_{T-} = -\frac{\partial \Phi}{\partial a_{T-}} \quad , \lambda_{T+} = \frac{\partial \Phi}{\partial a_{T+}}$$

$$\mu \geq 0 \quad , \frac{\partial H}{\partial \mu} \geq 0 \quad , \mu\frac{\partial H}{\partial \mu} = 0$$

Using the definition of Hamiltonian and Φ , (A1) can be written as follows.

$$\frac{1}{c} = \lambda\,(\,1 + \alpha i\,) \qquad \text{(A2)}$$

$$\mu = \lambda i \qquad \text{(A3)}$$

$$\mu\,(\,m - \alpha c\,) = 0 \quad , \mu \geq 0 \quad , m \geq \alpha c \qquad \text{(A4)}$$

$$\dot{\lambda} = [\,r - (i-\epsilon)\,]\,\lambda \qquad \text{(A5)}$$

$$\lambda_{T-} = v \quad , \lambda_{T+} = (\,1 + \gamma\,)\,v \qquad \text{(A6)}$$

Now, we take the proposed solution in section IV. Our objective is to show that this path satisfies the group of conditions (A2) - (A6). The most crucial part is to show the positivity of i and the nonnegativity of μ. For $t > T$, we can easily get the following values for λ , μ and i by putting $\dot{\lambda} = 0$ and substituting the suggested solution in (30).

$$\lambda = \frac{1}{[1 + \alpha(r+\epsilon)](y + r\bar{b})} \tag{A7}$$

$$i = r + \epsilon > 0$$

$$\mu = \lambda i > 0$$

Thus, it is obvious that our conjecture after the collapse satisfies the F.O.C.

Since the cash-in-advance constraint holds as an equality both right before and after the collapse, we know that :

$$c_{T-} = \alpha m_{T-} \quad , \quad c_{T+} = \alpha m_{T+} \tag{A7}$$

Combining (A7) and the relation $(1 + \gamma)m_{T+} = m_{T-}$,

$$(1 + \gamma)\, c_{T+} = c_{T-} \tag{A8}$$

From (A2),

$$\frac{1}{c_{T-}} = \lambda_{T-} [1 + \alpha i_{T-}] \tag{A9}$$

$$\frac{1}{c_{T+}} = \lambda_{T+} [1 + \alpha i_{T+}]$$

$$\frac{c_{T-}}{c_{T+}} = \frac{\lambda_{T+}}{\lambda_{T-}} \frac{1 + \alpha i_{T+}}{1 + \alpha i_{T-}}$$

Since $\dfrac{c_{T-}}{c_{T+}} = \dfrac{\lambda_{T+}}{\lambda_{T-}} = 1 + \gamma$, it is obvious that

$$i_{T-} = i_{T+} = r + \epsilon \tag{A10}$$

(A10) means that there is no jump in domestic interest rate at time T. Differentiating (A3) with respect to time, we get

$$-\frac{\dot{c}}{c^2} = \dot{\lambda}(1 + \alpha i) + \alpha\lambda\frac{di}{dt} \tag{A11}$$

Rearranging (A11) using (A4) and (A5), we can get the following differential equation for i_t with terminal condition.

$$\frac{di}{dt} = (i + \frac{1}{\alpha})(i - r - \frac{\dot{c}}{c}) \quad , \; i_T = r + \epsilon \tag{A12}$$

Now, we will show that i_t is positive for $t < T$. Suppose $i_t < r$ for some $t < T$. Then, since $\dot{c}/c > 0$ for the proposed consumption path, by (A12), di/dt should be negative. It means that i_t will decrease and remains less than r. We can see that i_t will keep decreasing and remain under r until time T. However, this contradicts the terminal condition. Thus, $i_t > r \;, t < T$. Thus, the domestic interest rate will remain higher than the world interest rate r and settle down to $r + \epsilon$ after the collapse. Since i is positive, it is obvious from (A2) and (A3) that λ and μ are also positive for $t < T$. Thus, the F.O.C. is also satisfied for $t < T$.

APPENDIX B

Suppose T^f is the time of collapse under free capital mobility. Then,

$$e^{rT^f} = \frac{g}{\Delta g}$$

The solution to (38), denoted by T^c, is the zero of the following function.

$$f(t) = e^{rt} + \alpha r e^{-t/\alpha} - (1 + \alpha r)\frac{g}{\Delta g}$$

Since for $t \geq 0$,

$$f'(t) = r\,(e^{rt} - e^{-t/\alpha}) > 0$$

we know that f is monotonically increasing. Moreover,

$$f(0) = -(1+\alpha r)\frac{g}{\Delta g} < 0$$

$$\lim_{t \to \infty} f(t) = +\infty$$

Thus we know that f crosses the x-axis only once and from under. Since $e^{-t/\alpha} < 1$, $t > 0$ and $g/\Delta g > 1$,

$$f(T^f) = \alpha r(e^{-t/\alpha} - \frac{g}{\Delta g}) < 0$$

Therefore, T^c should be greater than T^f.

REFERENCES

Aizenman, J. "On the Complementarity of Commercial Policy, Capital Controls, and Inflation Tax," *Canadian Journal of Economics*, vol.19, no.1, (February 1986), 114-133.

Calvo, G.A. "Temporary Stabilization : Predetermined Exchange Rate," *Journal of Political Economy*, vol.94, no.6, (December 1986), 1319-29.

Calvo, G.A. "Balance of Payments Crises in a Cash-in-Advance Economy," *Journal of Money, Credit and Banking* , vol.19, no.1 (February 1987), 19-32.

Clower, R.W. "A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal*, vol.6, (1967), 1-9.

Connolly, M.B. and Taylor D. "The Exact Timing of the Collapse of an Exchange Rate Regime and Its Impact on the Relative Price of Traded Goods," *Journal of Money, Credit and Banking*, vol.16, (May 1984), 194-207.

Grilli, V.U. "Buying and Selling Attacks on Fixed Exchange Rate Systems ," *Journal of International Economics*, vol.20, no.1/2, (February 1986), 143-156.

Helpman, E. "An Exploration in the Theory of Exchange-Rate Regimes," *Journal of Political Economy*, vol.89, no.5, (October 1981), 865-890.

Krugman, P.R. "A Model of Balance-of-Payments Crises." *Journal of Money, Credit and Banking*, vol.11, (August 1979), 311-325.

McKinnon, R.I. "The Order of Economic Liberalization: Lessons from Chile and Argentina," in K. Brunner and A.H. Metzler(eds.), *Economic Policy in a Changing World*, Carnegie-Rochester Conference Series, vol.16, (1982).

Obstfeld, M. "Balance-of-Payments Crises and Devaluation," *Journal of Money, Credit and Banking* vol.16, no.2, (1984), 208-217

Obstfeld, M. "Capital Controls, the Dual Exchange Rate and Devaluation," *Journal of International Economics* , vol.20, no.1/2, (February 1986), 1-20.

Penati, A. and Pennacchi, G. "Optimal Portfolio Choice and the Collapse of a Fixed-Exchange Rate Regime," mimeo, University of Pennsylvania (1986).

Stockman, A.C. "Anticipated Inflation and the Capital Stock in a Cash-in -Advance Economy," *Journal of Monetary Economics*, vol.8, (November 1981), 387-393

Wyplosz, C. "Capital Controls and Balance of Payments Crises," *Journal of International Money and Finance* , vol.5, (1986), 167-179.