Capital-labour substitution elasticities in New Zealand: one for all industries?

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Contents

List of tables and figures

Tables

Figures by chapter

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Abstract

This paper tests the assumption of a Cobb-Douglas production function (a unitary elasticity of substitution between capital and labour) for 20 of New Zealand's industries, using Statistics New Zealand's industry-level productivity data. It also assesses how the Leontief production function (zero substitutability) may apply to New Zealand industries.

The econometric estimates of the capital-labour substitution elasticity provide some evidence for the Cobb-Douglas assumption, at former measured sector and industry level in the long run, but show the Leontief function is more appropriate in the short run.

These results facilitate interpretation of the industry-level productivity data, and highlight the variation in substitutability across industries and sectors.

Key words

Capital-labour substitution; productivity; Cobb-Douglas; Leontief; production function; industry; New Zealand

1 Introduction

Purpose

Determining New Zealand's productive capability relies on understanding the role of inputs in producing outputs. Value-added growth can come from growth in labour inputs, capital inputs, or multifactor productivity (MFP). However, calculating MFP (as a residual) depends on specifying the relationship between labour and capital in the production function (which shows how inputs are used to create output).

There is a range of possible forms of the production function, each of which has implications for measuring MFP. In calculating MFP, it is often assumed that the production function is of the Cobb-Douglas form, which has a unitary elasticity of substitution between capital and labour inputs ('the elasticity'). This means a unit increase in the ratio of wages to rental-prices (the price of capital) is matched by a unit increase in the capital-to-labour ratio. The elasticity measures how easy it is for an economy, sector, industry, or firm, to adjust its inputs as the price of labour changes relative to the price of capital.

An alternative to the Cobb-Douglas function is the Leontief function, where it is assumed that labour and capital cannot be substituted. Both functions are specific cases of the general constant elasticity of substitution function, which allows the elasticity to be greater than or equal to zero. As MFP (often viewed as a measure of technological change) is dependent on specifying the inputs to production, assessing the validity of this assumption is key to verifying the estimates for MFP (ie mis-specifying the production function can bias MFP estimates).

The purposes of this paper are to:

- understand the importance of specifying the form of the production function for interpreting and calculating MFP
- examine which form of the production function is most applicable in New Zealand
- gain insight into the nature of the production technology of New Zealand's industries.

Motivation

The value of the elasticity is an important parameter in general equilibrium models. However, some models used in New Zealand currently use estimates based on Australian data,[1] which may not reflect New Zealand's market activity. Zuccollo (2011), for example, demonstrates the impact of using New Zealand-specific elasticities on economic growth following a labour supply shock. While the impact on the total economy is similar to forecasts generated by pre-existing assumptions, industry-level predictions can vary significantly.

The degree of input substitution may also be useful in assessing the likely efficacy of policy that aims to use the price mechanism to increase capital per worker and (ultimately) labour productivity. In addition, the elasticity can be used to explain whether capital accumulation is a driver of growth in real unit labour costs (Lebrun and Perez, 2011).

A lack of appropriate data has previously hampered attempts to estimate the elasticity and therefore the form of the production function at industry level in New Zealand. Hall and Scobie (2005) provide estimates for New Zealand's total economy but do not test the assumption of Cobb-Douglas at industry level (or contrast the findings and implications with the Leontief form of the production function).

However, Statistics NZ's industry-level productivity data, first released in June 2010, provide the required income and volume data, for both capital and labour inputs, that are necessary to test the Cobb-Douglas assumption. These data can be used to assess the relationship between the capital-to-labour ratio and the wage-rental price ratio, and allow econometric tests of the

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[1] The Australian data suggests that an elasticity of 0.5 is appropriate for all industries.

assumption of a unitary elasticity. This framework can also be used to test the assumption underlying the Leontief function. The series is also long enough to provide reliable statistical estimates.

Data and methods

This paper tests the assumption that a Cobb-Douglas production function is applicable to all industries. The data used in this paper are taken from Statistics NZ's industry-level productivity statistics series, which covers 24 industries from 1978 to 2009. To provide reasonable estimates at the industry level, data are for the Australian New Zealand Standard Industrial Classification 1996 (ANZSIC96) industries A to K (see appendix A for a definition of these industries). Property services, business services, cultural and recreational services, and personal and other community services are excluded from the industry-level analysis. Because their series begin in 1996, they have too few observations for time series models. Estimates of the elasticity for the former measured sector (ANZSIC96 industries A–K and P), three core sectors, and 20 of New Zealand's industries are presented in this paper.

The econometric methodology follows that of Balistreri, McDaniel, and Wong (2003) who assessed the assumption of a Cobb-Douglas production function using United States' (US) industry-level data. This approach allows both short- and long-run elasticities to be derived, and the dynamic nature of capital accumulation decisions to be accounted for.

Summary

This paper begins with an overview of capital-labour substitution and an explanation of how the degree of substitution varies, depending on assumptions about the production function. Econometric techniques are then used to estimate the elasticity.

These estimates may be useful in interpreting and understanding the robustness of the industry-level MFP data, and for undertaking sensitivity analysis in broader general equilibrium models.

2 Capital-labour substitution

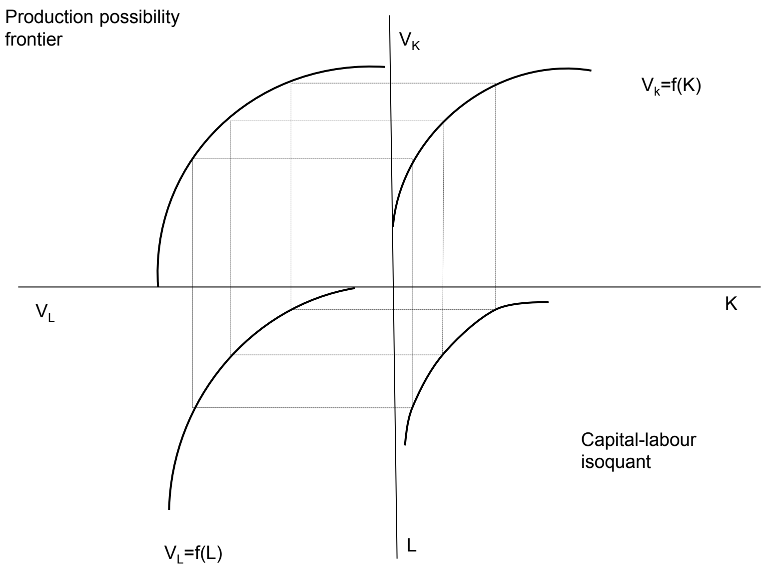
This chapter presents an informal overview of the concept of capital-labour substitution and its relevance to MFP, and discusses the existing literature in this area.

Graphical representation

The concept of capital-labour substitution in a case of two outputs, V_K and V_L, is illustrated in figure 1. Goods V_K and V_L are mainly produced by capital and labour respectively. A neoclassical framework is used to show the (concave) production possibility frontier (north-west quadrant) of an industry using capital *K* and labour *L*. Under perfect competition, industries always operate on the frontier. At any point on this curve, the same amount of total output *V* can be produced from corresponding amounts of labour and capital input. The slope of the frontier reflects the marginal rate of technical substitution (the ratio of relative factor prices). Assuming diminishing marginal returns from each factor of production, the production functions for capital and labour can be plotted (north-east and south-west quadrants).

Figure 1

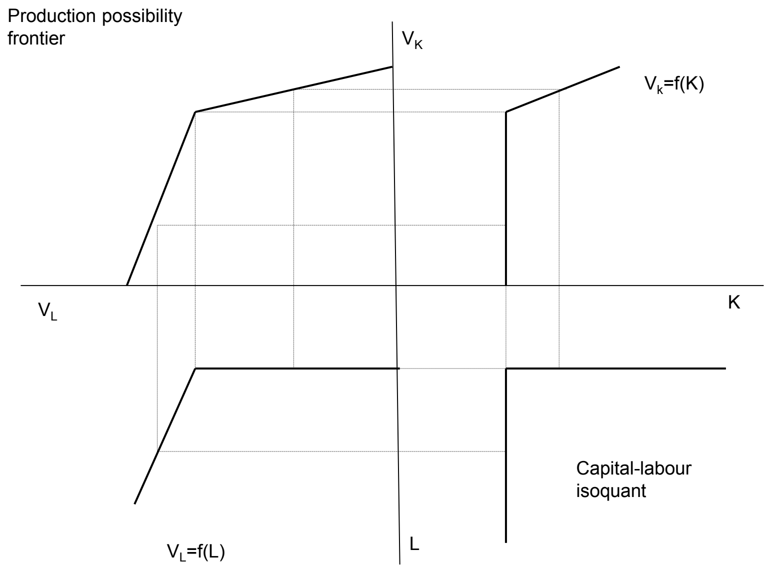
The production possibility frontier, production functions, and capital-labour isoquant in a Cobb-Douglas economy



Suppose that the industry reduces its capital input but maintains the same level of output. This leads to a movement along the production possibility frontier and along the production function for labour. The amount of labour used in production can then be compared against the amount of capital (south-east quadrant). Repeating this exercise reveals a convex relationship between capital and labour inputs (known as the isoquant). This result depends on assuming diminishing marginal returns. As the slope of the production possibility frontier is equal to the ratio of relative factor prices, and the capital-to-labour ratio is derived from the points on the frontier, relative movements along the frontier (ie reflecting a changing slope in the budget constraint), and correspondingly the isoquant, affect the elasticity. Under the Cobb-Douglas framework, the value of the elasticity is equal to unity. It is important to note that shifts in the frontier reflect pure disembodied technological change (ie are independent of an increased use of existing resources or price changes).

Figure 2

The production possibility frontier, production functions, and capital-labour isoquant in a Leontief economy



Sketching the relationship between production possibilities, functions, and the factor input isoquant is problematic when there is zero substitutability between inputs. In this case, the shape of the production possibility frontier is not concave in the short run (Takamasu, 1986). However, it is likely to be concave in the long run, as the short-run frontier is subsumed by the long-run frontier (Landon, 1990). Typically, the frontier and production functions exhibit kinks (see figure 2). Again, moving along the frontier (to reflect changing relative prices) leads to an L-shaped isoquant and shows that the capital-to-labour ratio is independent from such changes. The equilibrium amount of capital and labour used in production is at the corner of the isoquant.

The production function debate

There are strong arguments for using the Cobb-Douglas approach. Economically, this approach satisfies the requirements of Kaldor's stylised facts, which are required for constructing economic growth models (Balistreri et al, 2003). The Cobb-Douglas approach is widely used, so results can easily be compared. The core statistical advantage of the assumption is that labour and capital inputs can be weighted by their current-price value-added shares, which are readily available. The Cobb-Douglas assumption has been tested rigorously, with a number of studies suggesting that it does provide reasonable estimates of the relationship between capital and labour inputs in the production process. Balistreri et al (2003) found that the Cobb-Douglas function could not be rejected in 20 of 28 US industries; they also rejected the Leontief function in seven of those industries. Pendharkar, Rodger, and Subramanian (2008) also found evidence for Cobb-Douglas in a study of software development firms. Fraser (2002) finds some evidence for the Cobb-Douglas function in aggregate New Zealand data from 1920–40.

However, the Cobb-Douglas approach has often been criticised for its inflexibility. Motivated by strong empirical evidence that the substitution between capital and labour is often not equal to unity in US manufacturing firms, Arrow, Chenery, Minhas, and Solow (1961) proposed the constant elasticity of substitution (CES) function, where capital and labour can be substituted at a constant rate but at a value other than unity.

Controversies over the form of aggregate and industry-level production functions remain today. Bhanumurthy (2002) defends the Cobb-Douglas approach by arguing that many of the econometric problems posed in estimation can readily be addressed. In outlining the history of controversies surrounding the form of the aggregate production function, Felipe and McCombie (2005) argue that the lack of microeconomic foundations challenges the assumption that the aggregate production function is of the Cobb-Douglas form. Houthakker (1955), for example, showed that an aggregate Cobb-Douglas function could be derived from linear production functions.

Although often used to represent behaviour for economic aggregates, the Cobb-Douglas function was designed to assess activity at the firm (or microeconomic level). Its use in macroeconomics has not considered the microeconomic foundations on which it is based. Thus, a Cobb-Douglas function for the macroeconomy does not necessarily apply to all industries, nor vice versa. In addition, statistical evidence in support of an aggregate Cobb-Douglas function does not necessarily reveal the true underlying technology of the micro-level components (Felipe & McCombie, 2005).

Recent international empirical evidence refutes the Cobb-Douglas production function. Chirinko, Fazzari, & Meyer (2004) estimate the elasticity to be 0.4. Barnes, Price, and Barriel (2008) also find evidence that the elasticity is approximately 0.4, using firm-level data from the United Kingdom. Lebrun & Perez (2011) suggest that the elasticity is approximately 0.7. Upender (2009) finds strong supporting evidence for the CES formulation in a study of Indian industries while Raval (2011) finds no supporting evidence for the Cobb-Douglas function in US manufacturing firms. Hall and Scobie (2005) suggest the elasticity for the total economy between 1987 and 2002 in New Zealand is 0.8. Szeto (2001) also rejects the Cobb-Douglas function for New Zealand.[2]

However, the elasticity may not be constant over time. Balk (2010) notes that “the environment in which production units operate is not so stable as the assumption of a fixed production seems to claim” (p S225). The fixed nature of inputs in the short-term, labour and capital market frictions (eg barriers to moving freely between jobs and time required to learn how to use capital), growth in labour- or capital-augmenting technology, and sticky wages and prices (ie those resistant to change) contribute to this instability. Konishi and Nishiyama’s (2002) study of Japanese manufacturing firms suggests the elasticity does vary over time. However, in the long term, many of these issues should not pervade. One factor likely to have a persistent impact is the changing nature of capital inputs used in the production process. The advent of information technology, in particular, may have affected the degree of substitution between capital and labour over time (Jalava, Pohjola, Ripatti, & Vilmunen, 2006). A time-varying elasticity may lead to some industries showing Cobb-Douglas technology in one period and Leontief in another.

Applications across industries

At the industry level, the choice of function may be important because industries differ in the way they mix their labour and capital inputs. For example, how might a highly capital-intensive industry adjust its capital and labour if wages increase sharply relative to payments to capital? A similar question can be posed for labour-intensive industries. If wages increase relative to payments to capital, but capital cannot be substituted for labour and a certain amount of labour is required, the elasticity should be minimal. The relevance of the assumption of uniform production functions for industry-level analysis was summarised by Carlaw and Lipsey (cited in Mawson, Carlaw, & McLellan, 2003) who stated that:

> Given what we know about technological complementarities and the need to adapt technologies for specific uses, identical production functions across industries is not an acceptable assumption. For example, it is difficult to believe that the application of electricity to communications technologies can be considered to be the same production technology as the application of electricity to mining or machining? (p15)

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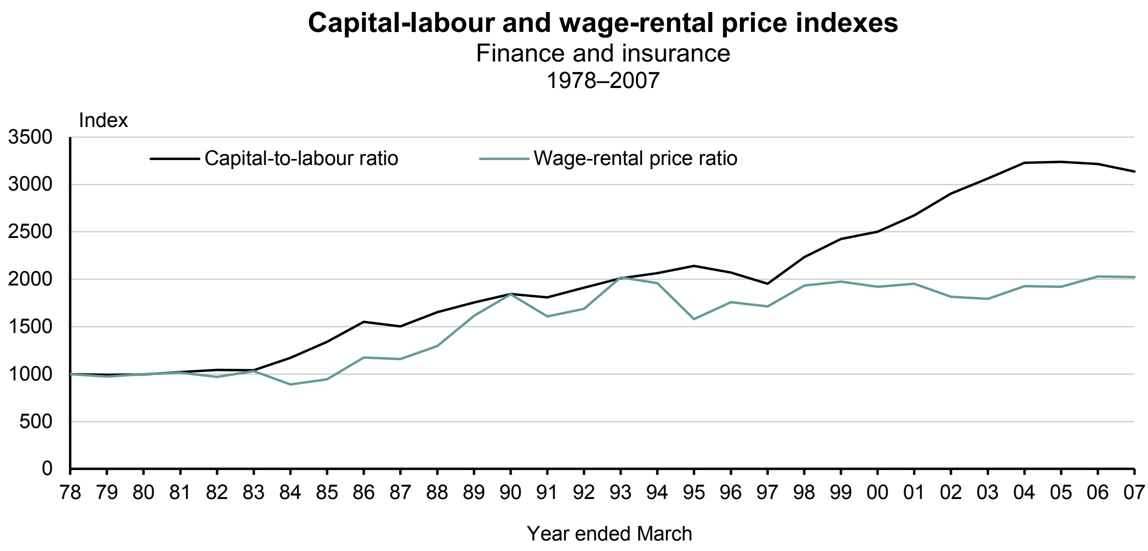
[2] It is worth noting that true elasticities may be much greater than those often observed due to outliers or ‘shocks’. Regression specification error, such as including too few lagged variables or not accounting for mis-specification of technological change, may also bias elasticity estimates.

In other words, Cobb-Douglas might be an appropriate assumption for one industry, but the Leontief function (or another function) might be applicable in another.

These studies suggest that the value of the elasticity of substitution may vary across countries, industries, and time. The elasticity values for New Zealand's industries can be assessed by examining the data underlying Statistics NZ's Industry Productivity Statistics: 1978–2009 (2011). Holding the relative prices of labour and capital constant, there may be relatively low substitutability in agriculture, and transport and storage, as labour and capital inputs have tracked similarly over the last three decades (Statistics New Zealand, 2011). This leads to the labour productivity, capital productivity, and MFP estimates tracking similarly. However, the substitutability depends on the ratio of wages relative to rental-prices. As discussed later, this ratio can be derived as the ratio of labour income, divided by the labour input index to capital income, divided by the capital input index. This can then be compared with the capital-to-labour ratio.

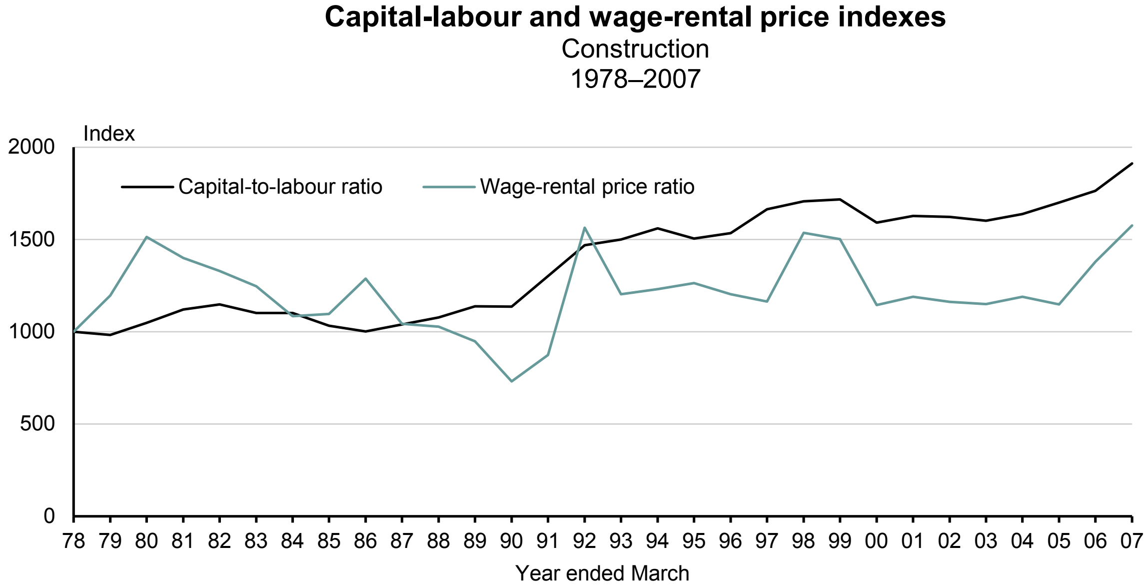
A variety of relationships between these two variables are observed. A close relationship between the capital-to-labour ratio and the ratio of factor prices is seen for finance and insurance until the mid-1990s (see figure 3). Thereafter, the capital-to-labour ratio continues to grow strongly while the factor price ratio remains flat. Such divergences during the series are also present for mining, retail trade, and communication services. They highlight the potential for the elasticity to be non-constant over time and indicate that a transformation occurred in the way industries operate. The two ratios for construction (and for electricity, gas, and water supply) track similarly later in the series, after moving in seemingly opposite directions from 1980–90 (see figure 4).

Figure 3



Source: Author's calculations using Statistics New Zealand data

Figure 4



Source: Author's calculations using Statistics New Zealand data

Figures 3 and 4 suggest the response of capital and labour used in production to changes in relative factor prices may differ across industries. To understand how these differences may be reflected in MFP estimates and growth accounting frameworks, it is necessary to see how the elasticity is defined under different production function formulations.

3 The elasticity under different production functions

This section provides a formal discussion of the relationship between the elasticity of substitution and MFP under different production functions.

The possible forms of the production function each possess different mathematical properties and implications for measuring MFP.[3] In the econometric estimation of MFP, the choice of production function depends on the desired degree of flexibility (relative to the available data), whether the function is linear in the parameters and satisfies the economic assumptions of homogeneity and monotonicity, and the principle of parsimony (Coelli, Rao, O'Donnell, & Battese, 2005, pp211–12). The last three of these criteria also apply to the index number approach to estimating MFP, which is used by Statistics NZ and other international statistical agencies.

Present in production functions are assumptions surrounding the elasticity between capital and labour.[4] The elasticity, denoted by σ_i, for a production process with two inputs is defined as:

$$\sigma_i = \frac{d_i(K_i/L_i)}{d(w_i/r_i)} \frac{w_i/r_i}{K_i/L_i} \quad (1)$$

where L and K are the quantities of labour and capital inputs, w and r denote their respective marginal products (the change in output due to the change in the input), and subscript i denotes industry. Note also that equation 1 assumes that the elasticity is time invariant. Equation 1 can be interpreted as the percentage change in the capital-to-labour ratio over the percentage change in the wage-rental price ratio. As wages and rental-prices are assumed to be equal to the marginal revenue products of labour and capital, respectively, this definition depends on assuming perfect competition. Where this assumption does not hold, estimates of the elasticity may be biased. Theories of imperfect labour markets (eg Manning, 2003) show that if it is costly for workers to change jobs then wages diverge from the marginal revenue product, the difference depending on the elasticity of labour supply.

Cobb-Douglas

To enable MFP to be calculated, it is often assumed that the production function is of the Cobb-Douglas form.[5] This approach is the most frequently used in the productivity literature (Miller, 2008), and has important properties (such as constant returns to scale and constant factor shares) and assumptions that facilitate productivity analysis. For example, the Cobb-Douglas framework permits the rating forward of capital and labour income shares, enabling estimates to be calculated even when current-price national accounts data are unavailable.[6] The production function takes the form:

$$V_i = A_i(t) L_i^{wl_i} K_i^{wk_i} \quad (2)$$

where V_i = industry chain-volume value added

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[3] Further specifications include a quadratic form, normalised quadratic, and the translog function. These forms are not discussed in this section as they do not yield exact assumptions for the capital-labour substitution elasticity and therefore do not present alternative hypotheses that can be tested in this framework.

[4] Different forms of the production function also yield different elasticities of output with respect to inputs and therefore different marginal rates of technical substitution, which is related to the elasticity of substitution. This therefore has implications for computable general equilibrium models.

[5] The production function underlying Statistics NZ's MFP series uses a Tornqvist index to weight capital and labour. The production function therefore is a Cobb-Douglas *type* aggregator, but has greater flexibility as including previous periods weights captures the effect of delayed responses to price changes. However, this only allows for one-year lagged responses, and weights the previous and current income shares equally.

[6] An implication of the Cobb-Douglas approach is that factor shares are constant over time, meaning that these shares can be held constant even when current-price data are not available.

$A_i(t)$ = a parameter that captures disembodied technical shifts over time, for example, outward shifts of the production function allowing output to increase with a given level of inputs (= MFP)
L_i = industry labour inputs
wl_i = industry labour income share
K_i = industry capital inputs
wk_i = industry capital income share

The use of income shares rests on assuming perfect competition, where economic profits are zero, and value added is equal to the cost of labour and capital. Cost shares are thus equal to income shares.

MFP is calculated residually, by dividing the output index by an index of total inputs:

$$A_i(t) = V_i / L_i^{wl_i} K_i^{wk_i} \tag{3}$$

The elasticity under a Cobb-Douglas framework is equal to one, which implies that a unit of capital is perfectly substitutable for a unit of labour. The elasticity is only implicit in the production function. It is worth noting that this result is independent of assuming constant returns to scale.

Leontief

The Leontief production function assumes there is zero substitution between the factors of production. In this case, an increase in the amount of capital used by a firm or industry is not matched by any corresponding change in labour following a change in relative prices. The production function is written as:

$$V_i = A_i(t) \min(wl_i L_i, wk_i K_i) \tag{4}$$

Output is maximised when there are fixed proportions of each input and one (lowest-cost) factor of production dominates total inputs. Therefore, when calculating MFP as a residual, MFP (in a levels sense) will be greater as the elasticity tends to zero (Young, 1998). As with Cobb-Douglas, the elasticity is only implicit in the Leontief function.

Constant elasticity of substitution

The production functions in equations 2 and 4 assume that the elasticity is either zero or one. An alternative and more general specification is the CES production function (Arrow et al, 1961). The advantage of the CES function is that it has one less-restrictive assumption by allowing the elasticity to take values other than zero or one. However, constant returns to scale are still assumed. In this case, output is related to inputs as follows:

$$V_i = A_i(t)(\delta_\mathrm{i} L_i^{-\rho_i} + (1-\delta_i)K_i^{-\rho_i})^{-1/\rho_i} \tag{5}$$

$$(0 < \delta_i < 1;\ -1 < \rho_i \neq 0)$$

where $V_i, A_i(t), K_i$, L_i are defined as above, δ_i is a distribution parameter that reflects the relative factor shares, and ρ_i is a parameter that determines the value of the elasticity. The elasticity can be derived as (see Chiang, 1984, p428):

$$\sigma_i = \frac{1}{1+\rho_i} \tag{6}$$

In other words, the elasticity is a constant that can take on a value other than unity and depends on the value ofρ_i:

$$\left.\begin{array}{r} -1 < \rho_i < 0 \\ \rho_i \approx 0 \\ 0 < \rho_i < \infty \end{array}\right\} \to \begin{cases} \sigma_i > 1 \\ \sigma_i \approx 1 \\ \sigma_i < 1 \end{cases}$$

The constraints on ρ_i require the elasticity to be non-negative. Other production functions can be seen as special cases of the CES function, depending on the value of ρ_i. When $\rho_i \approx 0$, the elasticity is approximately equal to unity, and the CES production function approaches the Cobb-Douglas function. This leads to a convex capital-labour relationship. However, the production function is not defined when $\rho_i = 0$ (as there will be division by 0), meaning that the CES and Cobb-Douglas functions are only approximate as ρ_i tends towards 0. When $\rho_i = \infty$, (ie there is no substitutability between the factors of production) the CES isoquant looks like the Leontief isoquant (resulting in the familiar L-shape relationship). When ρ_i = -1, the production function will be linear (ie $V_i = A_i(t)(\delta_i L_i + (1-\delta_i)K_i)$) (see Varian, 1992, pp19–20).

4 Implications for productivity

If MFP was to be calculated using the more general CES approach, then the calculation becomes:

$$MFP_i = A_i(t) = V_i/(\delta_i L_i^{(\sigma_i-1)/\sigma_i} + (1-\delta_i)K_i^{(\sigma_i-1)/\sigma_i})^{-\sigma_i/(1-\sigma_i)} \quad (7)$$

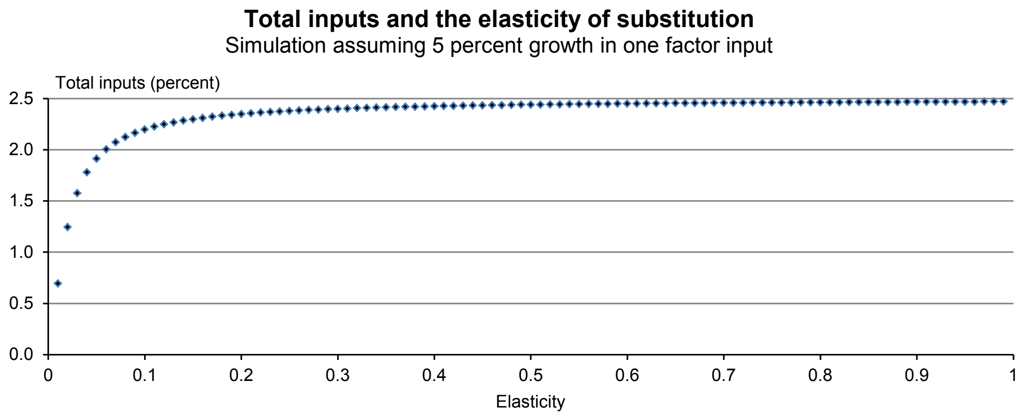
While MFP is still derived as a residual, the calculation differs to the Cobb-Douglas approach. In this instance, MFP explicitly depends on the elasticity between capital and labour. MFP growth under a CES function will be less than that from the Cobb-Douglas function when the elasticity is greater than unity. This is because, holding all else equal, a higher elasticity leads to greater change in the total inputs index and therefore lower change in MFP.

Labour and capital productivity estimates are similarly defined under the CES model and the Cobb-Douglas function (ie the ratio of an index of outputs to respective inputs). It is important to note that MFP estimates will still be between those for labour and capital productivity as MFP is a weighted function of the two factors of production. This implies that the form of the production function will have little empirical relevance for industries showing little difference in their labour and capital productivity growth (eg accommodation, cafes, and restaurants; and transport and storage). Conversely, there may be effects on MFP growth estimates for industries showing diverging labour and capital productivity (eg communication services, finance and insurance, and manufacturing).

The direction of the impact depends on the growth in labour and capital inputs. As the elasticity approaches zero (the Leontief function), total inputs track towards the input showing the slowest growth (see equation 4). Lower elasticities imply slower growth in total inputs, and thus higher MFP growth. However, this assumes the parameter δ_i is constant. If this is non-constant, there may be offsetting effects on the total input and MFP indexes.

Figure 5 illustrates that the potential bias from mis-specifying the production function on MFP estimates is minimal. Assuming there is volume of growth of 5 and 0 percent in the two factors of production, and equal factor income shares, then elasticities between 0.2 and 1 have negligible impact on total inputs. There is a strong non-linear effect on total inputs when the elasticity is between zero and 0.2. This implies that concern about bias in MFP is only warranted when the estimated elasticity is near, or significantly equal to, zero: a linear function, for example, has no effect on MFP growth. Note that, for any value of the elasticity, growth in total inputs is between the growth rates in the factors of production.

Figure 5



Source: Author's calculations

Regardless of the size of the quantitative impact on estimating MFP, the form of the production function has implications for interpreting MFP growth. Under a CES model, MFP reflects technological change as well as the constraints of adjusting production to relative factor prices.

The choice of production function also has implications for growth accounting for both output and labour productivity. As labour productivity is calculated as a ratio of output to labour input, and MFP depends on the elasticity, the contribution of capital deepening must differ when the elasticity is not equal to unity. In other words, the weight used to calculate the contribution of capital deepening is not equal to capital's share of income. Instead, it reflects the responsiveness of capital and labour to relative factor prices. Under a CES approach, the contributions of capital and labour inputs to output growth capture the degree to which inputs are substituted, according to relative prices as well as factor income shares. Using equations 5 and 6, output is decomposed as follows:

$$d\ \ln V_i = d\ \ln A_i(t) - \frac{\sigma_i}{1-\sigma_i} d \ln(\delta_i {L_i}^{(\sigma_i-1)/\sigma_i} + (1-\delta_i)\, K_i^{(\sigma_i-1)/\sigma_i}) \qquad (8)$$

The core contributions are the same as under the Cobb-Douglas function (MFP, capital input, and labour input), but the weights depend on the elasticity. From equation 8, it is observed that the elasticity has an impact on growth rates. If this formulation was extended to allow the elasticity to vary over time, there would be a further effect on growth.

5 Empirical analysis

This section presents the econometric framework used to estimate the elasticity of substitution.

To understand whether the choice of production function may have an impact on the estimate of MFP growth, an estimate for the elasticity (σ) is required. An econometric approach is adopted for two reasons: first, indications of statistical significance are required, and this cannot be obtained by direct computation; and second, the dynamics of capital accumulation need to be taken into account as quantities in one period may depend on their prior values.

The general form of the estimation equation to be applied to the New Zealand data is outlined in Balistreri et al (2003). Maximising the CES production function, subject to the budget constraint, yields the following specification:

$$\ln\frac{K_i}{L_i} = \sigma_i ln\frac{\delta_i}{1-\delta_i} + \sigma_i ln\frac{w_i}{r_i} \quad (9)$$

The left-hand side of the equation is the logarithm of the capital-to-labour ratio, and the right-hand side is equal to a constant plus the logarithm of the wage-rental price ratio multiplied by the elasticity. Equation 9 can be rearranged so it can be estimated by ordinary least squares:

$$\ln y_i = \alpha_i + \beta_i \ln x_i + \varepsilon_i \quad (10)$$

The first term in equation 10, y_i, is the capital-to-labour ratio. The wage-rent ratio is denoted by x_i. The β_i term is the key parameter to be estimated and ε_i is a normally distributed identical and independent error term. The constant term α_i reflects the assumption that the factor-cost ratio is constant.

Balistreri et al (2003) note that a simple linear regression may not provide reliable estimates, as the role of dynamics between capital and labour needs to be considered. They suggest three specifications to account for this:

- a first-order autoregressive model (AR1)
- a first-difference model to account for non-stationarity
- an error-correction model (ECM) to account for non-stationarity and to derive the long-run elasticity.

First-order autoregressive model

The first model employed in this analysis is based on equation 10, but includes a lagged dependent variable as an independent variable (leading to the AR1 model):

$$\ln y_{it} = \alpha_i + \beta_{1i} \ln x_{it} + \beta_{2i} \ln y_{it-1} + \delta_i t + \varepsilon_{it} \quad (11)$$

Lags are important for understanding the evolution of the capital-labour ratio, due to inertia, technological factors, imperfect information, or institutional (contractual) effects (Gujarati, 1995, pp589–90). The choice of lag length is important, as the same structure may not be applicable to all industries. If industry-specific lags are not taken into account, then coefficients may be biased. Using lagged terms means that long-run and short-run elasticity estimates can be derived. The short-run elasticity is β_{1i} .This reflects the percentage change in the capital-to-labour ratio from a percentage change in the wage-rental price ratio **within** one year. The long-run elasticity takes into account the effect of contemporaneous and lagged variables and is calculated as $\beta_{1i}/(1-\beta_{2i})$ where $\beta_{2i} \neq 1$. The elasticity is therefore implicitly defined over a two-year period.

Equation 11 deviates from the approach of Balistreri et al (2003) by including a time trend. This is to account for any factors, other than prices and lagged dependency, that may be affecting the capital-to-labour ratio, such as labour (Harrod-neutral) or capital (Solow-neutral) augmenting technological change. Jalava et al (2006) highlight the importance of including a time trend, to control for possible bias from mis-specifying the nature of technological change. Capital augmenting technological change has probably been occurring since the evolution in information and communication technology.

Consider figure 1, where capital-augmenting technological change leads to a pivoted outward shift of the production possibility frontier and capital production functions. Under perfect competition, the marginal product of capital equals the rental price, thus changing the slope of the budget constraint. The capital-to-labour isoquant flattens as more output can be produced by less capital. These effects are captured by a time trend. Jalava et al (2006) show that the time variable used in the regression model may capture the mis-specification of technological change in the production function. In this case, the capital-to-labour ratio is equal to a constant, plus the elasticity multiplied by the log of the wage-rental price ratio, and a time trend:

$$\ln y_{it} = \alpha_i + \beta_{1i} x_{it} + (\gamma - \theta)(1 - \beta_{1i})t \qquad (12)$$

The coefficient on the time trend reflects the elasticity of substitution and the bias from mis-specifying technological change $\gamma - \theta$, where γ and θ are the rates of labour-augmenting and capital-augmenting technological change, respectively. If $\gamma - \theta > 0$ then labour-augmenting technological change is greater than capital-augmenting change and the coefficient on the time trend will be positive as long as the elasticity is less than one. Econometrically, including a time trend implies that the capital-to-labour ratio is trend stationary.

First-difference model

The second model is based on first differences of the dependent and independent variables:

$$\Delta \ln y_{it} = \alpha_i + \beta_{1i} \Delta \ln x_{it} + \delta_i t + \varepsilon_{it} \qquad (13)$$

where $\Delta \ln x_{it} = \ln x_{it} - \ln x_{it-1}$ denotes the first difference. This specification is preferred if the capital-to-labour and wage-rental price ratios are non-stationary (ie the variances are time dependent). If the ratios are non-stationary, the AR1 regression is 'spurious' and the coefficients and derived elasticities are meaningless. As there are no lagged terms in this specification, only the short-run estimate is derived (and is equal to β_{1i}).

Error-correction model

Balistreri et al (2003) also employ a single equation error-correction model (hereafter ECM) to determine the elasticity. It is based on first differences with lagged dependent and independent variables as regressors:

$$\Delta \ln y_{it} = \alpha_i + \beta_{1i} \Delta \ln x_{it} + \beta_{2i} \ln y_{it-1} + \beta_{3i} \ln x_{it-1} + \delta_i t + \varepsilon_{it} \qquad (14)$$

In this model, the short-run elasticity is again β_{1i} but the long-run elasticity is $-\beta_{3i}/\beta_{2i}$ where $\beta_{2i} \neq 0$. This model is more appropriate when the variables are non-stationary and when an indication of the long-run elasticity is required (as is the case here). It allows the divergence of short-run deviations from long-run equilibrium to be assessed. For some industries, a second-difference model is more appropriate, or the time trend is not required (as determined by the empirical fit of the model).[7] Dickey-Fuller tests were used to guide the degree of differencing and use of trend.

[7] For the ECM with second differences, the regression specification is
$\Delta^2 \ln y_{it} = \alpha_i + \beta_{1i} \Delta^2 \ln x_{it} + \beta_{2i} \ln y_{it-1} + \beta_{3i} \ln x_{it-1} + \beta_{4i} \ln y_{it-2} + \beta_{5i} \ln x_{it-2} + \varepsilon_{it}$
and the long-run elasticity is $(\beta_{3i} + \beta_{5i})/(-\beta_{2i} - \beta_{4i})$

A further constraint to estimation is that the elasticity must be positive. A negative elasticity has no economic interpretation: “it implies a decline in the availability of one input can be **made up** by a decline in the availability of other factors.” (World Bank, 2006, p117). Negative elasticities are found in some studies (see results of Balistreri et al, 2003, and Raval, 2011) and can arise when the dynamic structure of an industry is not fully considered (eg when too few lags are specified in the regression equation). Therefore, economic theory needs to be considered alongside the results of statistical tests, and further investigation is warranted where this condition is violated.

6 Data

Data were required for labour and capital volumes to construct the dependent capital-to-labour ratio variable, and wage and rental prices were required to calculate the independent wage-rental price ratio.[8] The wage and rental-price variables are expressed in nominal terms, consistent with the approach of Balistreri et al (2003). Labour income includes compensation of employees, net taxes on production attributable to labour income, and the labour income of working proprietors. Capital income is the sum of gross operating surplus (adjusted for the labour income of working proprietors) and net taxes on production attributable to capital.[9] As stated by Coelli et al (2005), substitution elasticities are invariant to the units of measurement because they depend on first-order conditions. This implies that labour and capital input indexes can be used instead of actual values.

The capital-to-labour ratio for a given industry is defined as the ratio of an index of capital input to an index of labour input. The wage rate is calculated as labour income divided by the labour input index. Rental prices are calculated in a similar manner, by dividing capital income by the capital input index. The wage rental-price ratio is then derived as the ratio of these two ratios.

The wage and rental prices can be considered implicit (rather than explicit) prices. Rental price calculations in particular may not match those underlying productivity data because the implicit approach assumes an endogenous rate of return (so user costs completely exhaust capital income). However, Statistics NZ assumes an exogenous rate of return (set at 4 percent).[10] Therefore, there may be differences between explicit rental prices used to calculate MFP and the implicit series used in this analysis.

Labour input for each industry is measured as a sum of industry hours paid, with the data sourced from labour surveys such as the Linked-Employee-Employer Dataset, Household Labour Force Survey, Quarterly Employment Survey, and the Business Demography Database. To measure the flow of capital services, the perpetual inventory method is used to derive the productive capital stock and supplemented with estimates of land and inventories to create capital inputs. Measured sector (and other aggregated) capital and labour series are weighted together using their respective income shares.[11]

The measured sector covers ANZSIC96 industries A–K, LA, LC, PA, and QA (see appendix A). However, property and business services, and personal and other community services are included in the measured sector only from 1996. This means that estimating the elasticity from 1978 to 2007 is problematic as the industry composition changes over time. The former measured sector (A–K and P) has consistent industry coverage from 1978 to 2007 and is therefore preferable.

For the industry-level analysis, property and business services, cultural and recreational services, and personal and other community services were excluded – their productivity time series only begins in 1996, and longer-time series were required to obtain reliable estimates for the elasticity at the industry level. This meant 20 industries were included in the model; nine of these were the manufacturing sub-industries. In 2007, the measured sector covered 80 percent of the economy (in terms of current-price gross domestic product (GDP)). Data were only available until 2007, as this is the last year for which current-price estimates of GDP by industry are were available under the ANZSIC96 industrial classification.

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[8] Unrounded data were used in this analysis.

[9] See sections 3.5.2 and 4.6 in Productivity statistics: Sources and methods for further information on calculating labour and capital income, respectively.

[10] See MacGibbon (2010) for further details.

[11] Further details on the data sources and construction of the series are in Productivity statistics: Sources and methods.

7 Results

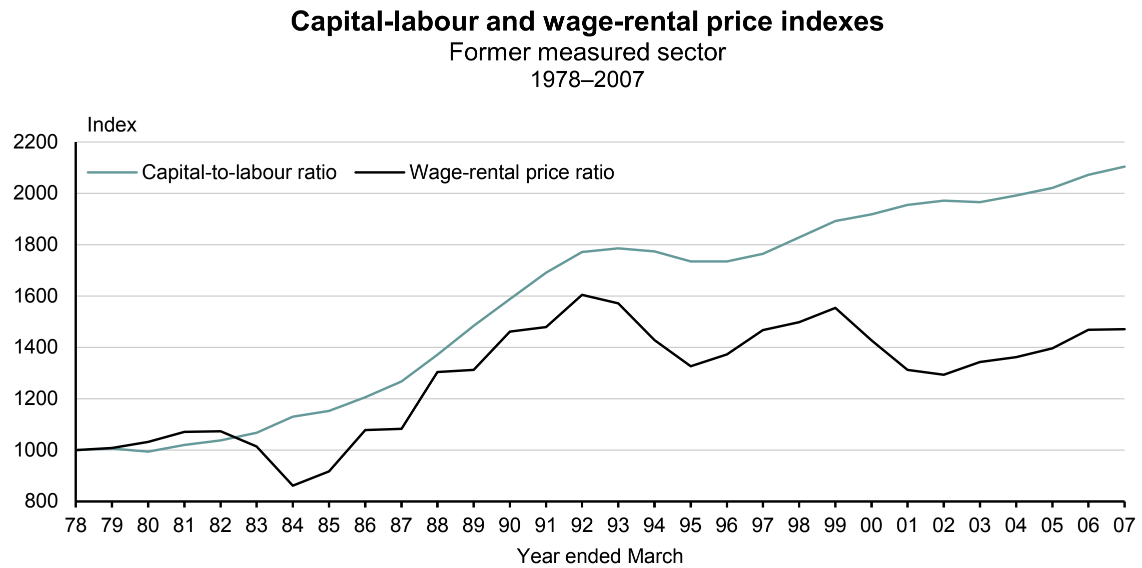
The previous discussion raised three issues that are worth exploring empirically. First, an estimate of the elasticity is required, along with a statistical test to determine whether the data supports a Cobb-Douglas, Leontief, or CES production function. Second, it needs to be assessed whether one common production function is applicable across different levels of aggregation. Third, the applicability of Cobb-Douglas across the same level of aggregation needs to be assessed, as it is expected that not all industries respond to changing factor prices to the same degree, or at the same pace.

Two null hypotheses were tested for both the short and long-run elasticity. The first was that the elasticity was equal to unity. Rejecting this null means the data does not provide evidence for the Cobb-Douglas function. The second hypothesis was that the elasticity was equal to zero. In this case, rejection means that the data does not provide evidence for the Leontief specification. All tests were performed at the 95 percent confidence level. Regression results are presented in appendix C. As the long-run elasticities involve testing a non-linear hypothesis, the delta method was used to calculate the standard errors. This method can be applied where there is a normal distribution; where this is the not the case, the delta method can lead to implausible results (such as suggesting the Cobb-Douglas and Leontief specification hold simultaneously) and tests of the Cobb-Douglas and Leontief functions cannot be made. Standard errors from the delta method are also presented in appendix C.

Former measured sector elasticities

Figure 6 shows the trends in the capital-to-labour ratio and wage-rental price ratio over the series for the former measured sector. Both ratios have trended upwards over time, implying that payments to labour rose at a higher rate than payments to capital, and that there was more capital available per worker (consistent with the trend in the chained capital-to-labour ratio). However, there was stronger growth in the capital-to-labour ratio than in the wage-rental price ratio. The step-pattern in the wage-rental price ratio in the 1980s is of note. The wage-rental price ratio declines in three periods, but the capital-labour ratio only declines in one of these.

Figure 6



Source: Author's calculations using Statistics New Zealand data

Table 1 presents the econometric estimates for the elasticity for the former measured sector. Coefficients and standard errors are in appendix C. The data suggests that the short-run elasticity is close to zero under all models, but is significantly different from zero (ie the null hypothesis that the production function is of the Leontief form is rejected). However, in the long run, the data suggests the production function is of the Cobb-Douglas form, meaning the capital-labour ratio adjusts in line with changing factor prices. While the elasticity is less than one (estimated to be around 0.83), it is not significantly different from one. These estimates are similar but marginally higher than those found by Hall and Scobie (2005) for New Zealand's total economy.[12]

Table 1

Former measured sector elasticities

		AR1	First difference	ECM
Former measured sector	SRE	0.16	0.12(2)	0.17
	LRE	0.82(1)	…	0.83(1)

1. Cobb-Douglas function cannot be rejected.
2. Leontief function cannot be rejected.
Symbol: … not applicable

While the short-run estimate under the AR1 model is low (but still significantly different from zero) the long-run elasticity shows a much stronger relationship between the capital-to-labour ratio and the wage-rental price ratio. This highlights the role of lags in forming the capital-to-labour ratio. Across models, the short-run estimates are broadly consistent, and the AR1 model and ECM produce nearly identical values for the elasticity.

Specification issues

The capital-to-labour ratio and the wage-rental price ratio are both non-stationary but not cointegrated. Dickey-Fuller tests tested whether the capital-to-labour ratio and wage-rental-price ratio for each sector and industry had a unit root. Where evidence for a unit root is found, the first-difference model is more appropriate. If the unit root hypothesis is rejected then the AR1 model is appropriate. The first-difference model is preferred to the AR1 model, because the Dickey-Fuller tests indicate both series are non- stationary at the former measured-sector level. However, the Phillips-Ouliaris test indicates the series are not cointegrated, implying the ECM does not provide additional information on the long-run response.

Jarque-Bera tests indicate that the data are normally distributed, and both the Durbin-Watson statistic (applied to the first-difference model and ECM) and the Durbin *h* test (applied to the AR1 model) indicate that autocorrelation is present. Autocorrelation is also suggested by Godfrey's Lagrange multiplier test. This latter feature of the data may inflate the standard error, leading to incorrect conclusions about significance, and overinflate the R^2 value. The R^2 values of 0.99 under the AR1 model are much greater than those of the first-difference model or ECM (0.17 and 0.27, respectively). However, the presence of autocorrelation does not bias the estimates of the coefficients.

Heteroscedasticity is also present in the AR1 model, but not in either of the differenced models. The Ramsey RESET tests indicate that the functional form of the AR1 model is mis-specified (potentially biasing the estimates) but the first-difference model and ECM are correctly specified. This means that the long-run Cobb-Douglas assumption may not have been rejected in the AR1 models, due to autocorrelation, heteroscedasticity, or mis-specifed functional form, but only autocorrelation may have affected statistical inference in the ECM.

Sector-level elasticities

Elasticities of capital-labour substitution were calculated using productivity estimates for the primary, goods-producing, and service sectors. Differences in the elasticity can be expected

[12] The marginal differences between the findings of Hall and Scobie (2005) and the results presented here may be due to any combination of different data sources, length of time series, and industry composition.

across sectors for many reasons: each sector uses different types of assets, the service sector is more labour-intensive, and capital-deepening is more pronounced in the goods-producing sector.[13]

As shown in table 2, the elasticity varies across sectors. The first-difference model suggests similar elasticities for all sectors, and that the Leontief production function holds in the short run. However, the Leontief function is rejected in the short run under the AR1 model and the ECM (except for services). Some evidence for the Cobb-Douglas hypothesis is found in the long run for the goods-producing sector. The ECM for the goods-producing sector uses second-difference, with two-year lags and no time trend. This means the effect of an increase in the wage-rental price ratio takes longer to affect the capital-to-labour ratio in the goods-producing sector than in the primary or service sectors.

Specification issues

The Phillips-Ouliaris tests again indicate the ECM is preferable to the first-difference model. Note that the service sector includes additional industries from 1996, which may bias the estimates.[14] The aggregate elasticity masks differences across sectors, with goods-producing industries likely to be driving the estimate of the aggregate elasticity up. It is important to note the standard errors for the ECM, for the goods-producing sector and service sector, are large, which affects the reliability of tests for Cobb-Douglas and Leontief functions. However, the Phillips-Ouliaris test indicates that the series are not cointegrated for these sectors. Also, the Cobb-Douglas assumption cannot be rejected under the AR1 model when the hypothesis is tested using a linearised Wald test. The delta method produces more efficient estimates in this case.[15]

Table 2

Sector elasticities

Sector		AR1	First difference	ECM
Primary	SRE	0.26	0.08(1)	0.12
	LRE	0.30	…	0.37
Goods-producing	SRE	0.37	0.10(1)	0.17
	LRE	0.55	…	1.33(2)
Service	SRE	0.38	0.07(1)	0.08(1)
	LRE	0.44	…	0.33(2)

1. Leontief function cannot be rejected.
2. Test is indeterminate due to wide standard errors.
Symbol: … not applicable

Specification issues are pertinent in the sector-level elasticity estimates. Autocorrelation was found in all sectors and models except for the goods-producing ECM. Incorrect functional forms were only present in the AR1 models for goods-producing and service sectors. Heteroscedascity was observed in all AR1 models, the service sector first-difference model, and the goods-producing and service sector ECMs.

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[13] The primary sector includes the agriculture, forestry, and fishing; and mining industries. The goods-producing sector includes manufacturing; electricity, gas, and water supply; and construction. The service sector includes these industries from 1978: wholesale trade; retail trade; accommodation, cafes, and restaurants; transport and storage; communication services; finance and insurance; and cultural and recreational services. Business services, property services, and personal and other community services are included in the service sector from 1996.

[14] Ideally, the service sector would exclude business services, property services, and personal and other community services to obtain consistent industry coverage. Preliminary analysis using measured sector data showed that the long-run elasticity differed from the former measured sector elasticity by only 0.01. This suggests minimal bias in the service sector estimate.

[15] Greene (2003) notes there is no assurance that the delta method and Wald test will lead to identical results. While the Wald test is not invariant to the formulation of the hypothesis, it does not rely on the assumption of normality.

Industry-level elasticities

Table 3 presents the short and long-run elasticities for each industry as calculated under each method. Across industries, a range of values for the elasticity is found, suggesting that a uniform production function may not be applicable to all industries.

Table 3

Industry-level elasticities

Industry	Model				
	AR1		First difference	ECM	
	SRE	LRE	SRE	SRE	LRE
Agriculture, forestry, and fishing	0.14	0.16	0.07	0.08	0.67(1)
Agriculture	0.05	0.08	0.06	0.06	0.08(3)
Forestry and fishing	-0.05(2)	-0.18(2)	0.11(2)	0.21	0.26(3)
Mining	0.10(2)	0.27(2)	0.09(2)	0.17(2)	0.50(3)
Manufacturing	0.25(2)	0.45	0.05(2)	0.09(2)	0.90(3)
Food, beverage, and tobacco manufacturing	-0.02(2)	-0.03(2)	0.04(2)	0.03(2)	-0.27(2)
Textile and apparel manufacturing	0.32	0.38	0.10(2)	0.18	0.95(1)
Wood and paper product manufacturing	0.02(2)	0.03(2)	0.10(2)	0.11(2)	1.28(3)
Printing, publishing, and recorded media	0.19	0.23	0.05(2)	0.11	0.36
Petroleum, chemical, plastic, and rubber product manufacturing	0.13(2)	0.67(3)	0.08(2)	0.12(2)	0.67(3)
Non-metallic mineral product manufacturing	0.30	0.35	0.09(2)	0.12	0.54(1)
Metal product manufacturing	0.23	0.42	0.05(2)	0.08(2)	0.48
Machinery and equipment manufacturing	0.38	0.50	0.14	0.23	0.75(1)
Furniture and other manufacturing	0.14	0.18	0.08	0.11	0.62(3)
Electricity, gas, and water supply	0.50	0.77(1)	0.13(2)	0.27(2)	1.71(1)
Construction	0.25	0.29	0.12	0.13	0.07(2)
Wholesale trade	0.18	0.21	-0.02(2)	-0.02(2)	0.03(2)
Retail trade	0.08(2)	0.10	0.02(2)	0.00(2)	0.89(1)
Accommodation, cafes, and restaurants	0.13	0.16	0.11	0.12	0.15(2)
Transport and storage	0.11	0.16	0.04(2)	0.03(2)	0.36
Communication services	-0.13(2)	3.29(3)	0.14(2)	0.04(2)	1.47(3)
Finance and insurance	0.22	0.23	0.15(2)	0.13(2)	0.56(3)

1. Cobb-Douglas function cannot be rejected.
2. Leontief function cannot be rejected.
3. Test is indeterminate due to wide standard errors.

Evidence for the Cobb-Douglas production function was found in some of New Zealand's industries in the long run. The only industry where Cobb-Douglas held in the AR1 model in the long run was electricity, gas, and water supply. However, this finding needs to be interpreted with caution as the error terms were found to be non-normally distributed.

Under the ECM, there was evidence to support Cobb-Douglas in the long run for the: agriculture, forestry, and fishing, textile and apparel manufacturing, non-metallic mineral product manufacturing, machinery and equipment manufacturing, electricity, gas, and water supply, and retail trade industries. The communication services industry, which has shown the strongest MFP growth of all industries in New Zealand, also recorded the highest long-run elasticity under the AR1 model and the second-highest under the ECM. The high elasticity estimates for this industry are unlikely to be reflecting technological change, as any influence of technological change on the capital-to-labour ratio is captured in the time trend.

The elasticity for the goods-producing sector is likely to be driven up by manufacturing, and electricity, gas, and water supply but offset by construction. The range of elasticities for any sector is greatest across the service industries. This highlights the argument that the production function for the aggregate sector or economy may not reflect the production functions of its underlying components.

The low estimates for agriculture suggest fixed proportions technology; although land is a non-productive asset (it enables production) it is essential to the production process and no amount of labour can be substituted for it. While the Leontief function could not be rejected in the long run for accommodation, cafes, and restaurants, the impact on MFP would be minimal as that industry's labour and capital inputs have tracked similarly over time.

Time effects

The Leontief specification held for many industries under the first-difference model and ECM in the short run. The short-run elasticities under the AR1 model and ECM were generally greater than those from the first-difference model. Long-run elasticities are greater than short-run elasticities for virtually all industries. Exceptions are when the ECM is used to measure the elasticity in the construction and the wholesale trade industries. The Dickey Fuller tests show the ECM is not appropriate for these industries, and the results of the AR1 model are preferable. Noticeable differences between short and long-run elasticities are observed for electricity, gas, and water supply; communication services; finance and insurance; manufacturing; petroleum, chemical, plastic, and rubber product manufacturing; metal product manufacturing; and machinery and equipment manufacturing. This is due to strong lagged effects from the capital-labour ratio. For these industries, accounting for the dynamics of capital accumulation is especially important. It is worth noting that the rate of capital-deepening has been strongest in these industries.

One-period lags are not appropriate for all industries under the ECM. Modifications to the generic ECM were required for industries in all sectors, to provide economically plausible estimates. An ECM with second differences, two-year lags, and no time trend was used for agriculture, forestry, and fishing; forestry and fishing; wood and paper product manufacturing; furniture and other manufacturing; and finance and insurance. The effect of differencing is highlighted in agriculture, forestry, and fishing. An ECM with first differences produces a long-run elasticity of 0.22; applying second differences leads to an elasticity of 0.67. The time trend was omitted for textile and apparel manufacturing, wholesale trade, and retail trade.

The AR1 and first difference was applied uniformly to all industries: variants of these models were examined but had no major impact on the results. Mining is a peculiar case. Applying a standard ECM to this industry, using any lag length and differences of up to three years, results in large negative elasticities (which have no economic meaning). Diminishing marginal labour productivity is pronounced in the mining industry. The greatest change in output occurs once extraction begins and, holding labour input constant, there is little scope to increase extraction in subsequent years. This implies an adaptive expectations model, where current investment depends on the observed realisations of prior investments. The capital-to-labour ratio and wage-rental price ratio are constructed by comparing the current capital input in production and associated income with a two-year lag of labour input and income. This results in a positive elasticity for mining.

Attributing significance under the ECM (especially for industries where second differences are appropriate) is problematic due to large standard errors. Although the ECM may reflect the ability of industries to return to equilibrium, and also be less prone to specification error, it may not necessarily provide the most efficient estimates. This is because the delta method, used to calculate the standard errors, depends on the covariance matrix.

The value of the elasticity and effect on MFP

In most industries and models, the elasticity is less than unity. This matches expectations as the estimates are between the values proposed by the Leontief and Cobb-Douglas functions. Assuming rental prices and capital are constant, an elasticity less than unity implies a wage increase has a less-than-proportionate effect on labour demand. In other words, it is not as easy for most industries to shift between capital and labour as the Cobb-Douglas assumption implies.

Recalling the discussion on the implications of different elasticities for MFP estimation, these results suggest that MFP estimates, derived from a Cobb-Douglas production function, may be biased downwards. Therefore MFP may be contributing more to output growth than expected, and labour and capital less than estimated. In only a few cases were the elasticities above unity. Some negative elasticities were also found. However, the Leontief function cannot be rejected in any of these cases and a value of zero can be assumed for the food, beverage, and tobacco manufacturing industry. The negative elasticity for this industry persists under any specification.

The time trend is positive and significant for all industries except petroleum, chemical, plastic, and rubber products manufacturing, and communication services, in the AR1 model. It is negative but insignificant for communication services, while it is positive but insignificant for petroleum, chemical, plastic, and rubber products manufacturing. This potentially indicates the production functions at the industry level are of the Harrod-neutral form. However, the time trend may capture factors other than the bias from mis-specifying technological change, especially if the assumption of perfect competition does not hold or the industry was subject to random shocks. It could also reflect the mis-specification of lagged variables. For these reasons, the time trend should be interpreted with caution. The time trend was less significant in the ECM (where it was used), although was positive in most cases.

Specification issues

The AR1 models have strong explanatory power, with R^2 values of approximately 90 percent observed for most industries. The importance of accounting for deviations from equilibrium is highlighted by the weak explanatory power of the first-difference models and the higher R^2 values from the ECM. The high AR1 R^2 values reflect the presence of autocorrelation. Durbin-Watson *h* tests for the AR1 model suggest autocorrelation is present at the 95 percent confidence level in all industries except printing, publishing, and recorded media, and furniture and other manufacturing. Durbin-Watson statistics for the first-difference models and the ECM show less evidence of autocorrelation.[16] Lagrange multiplier tests for autocorrelation (the Godfrey test) also show autocorrelation to be strongly present in the AR1 models but far less so under the ECM. Multicollinearity is present for most industries and sectors. This has the effect of inflating standard errors, leading to potentially incorrect inferences for significance.

A Lagrange multiplier test was used to test for heteroscedasticity. It was significant at the 95 percent confidence level in seven industries under the AR1 model, and in nine under the ECM.[17] Where present, heteroscedasticity is likely to inflate the standard errors.

Combined, these three specification issues may lead to rejecting the null hypothesis of a Leontief or Cobb-Douglas function. The Ramsey RESET test for functional form shows the ECM performs much better. Nine industries had incorrect functional form under the AR1 model using a squared power transform, but this increased to 17 for a cubic transform. There were only three instances where the RESET test indicated mis-specification of functional form under the ECM: food, beverage, and tobacco manufacturing; printing, publishing, and recorded media manufacturing; and aggregate manufacturing. The standard errors of the AR1 model may therefore be inflated due to mis-specification error as well as autocorrelation, heteroscedasticity, and multicollinearity (although the relative importance of each of these issues varies across industries). The ECM may be performing better than the AR1 model in terms of functional form because more consideration went into choosing the lagged variables and specification.

[16] For the AR1 model, Jarque-Bera tests indicate the error terms are normally distributed for all industries except forestry and fishing; mining; manufacturing; petroleum, chemical, plastic, and rubber product manufacturing; and furniture and other manufacturing. For the ECM, the test suggests the error terms are normally distributed in all industries except forestry and fishing; food, beverage, and tobacco manufacturing; petroleum, chemical, plastic, and rubber product manufacturing; electricity, gas, and water supply; and transport and storage.

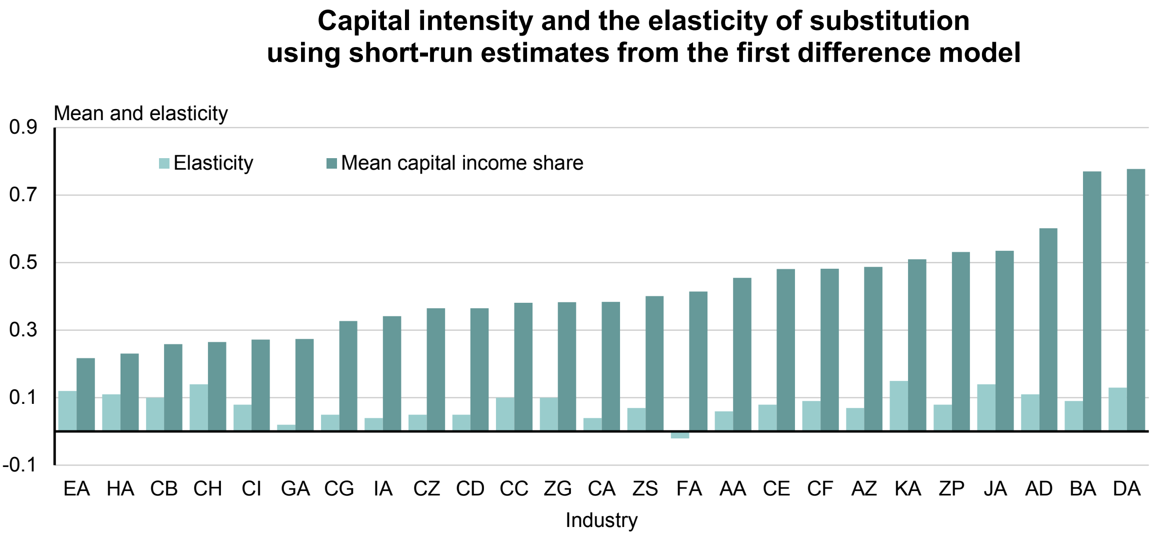
[17] Heteroscedasticity was present in both the AR1 model and ECM for: petroleum, chemical, plastic, and rubber products manufacturing; metal products manufacturing; machinery and equipment manufacturing; and aggregate manufacturing. It was present in only the AR1 models for: agriculture, forestry, and fishing; textile and apparel manufacturing; and furniture and other manufacturing. It was present in only the ECM model for: forestry and fishing; food, beverage, and tobacco manufacturing; printing, publishing, and recorded media; wholesale trade; and transport and storage.

The first-difference model results are generally preferred over the AR1 model, due to the results of the Dickey-Fuller tests. However, the Phillips-Ouliaris test indicates the ECM is preferable for a number of industries. Evidence for cointegration could not be found for: petroleum, chemical, plastic, and rubber product manufacturing; non-metallic mineral products manufacturing; metal products manufacturing; manufacturing; electricity, gas, and water supply; or communication services. The preference for the ECM is also evident as it is less susceptible to autocorrelation. Appendix C provides further information on the Durbin-Watson, normality, and cointegration tests applied to the industry-level data.[18]

Further considerations

The different values for the elasticity across industries may reflect genuine differences in production technology, which may result from differing degrees of capital intensity. Average capital income shares (used to show capital intensity) show a weak correlation (approximately 0.2) with the estimates of the short-run elasticity. However, the relationship may be non-linear. The first-difference model, in particular, suggests that industries with the lowest and highest capital intensities had the highest elasticities, while those with mid-range capital intensities had some of the lowest (see figure 7).[19] In using one factor of production intensely, a shock to the price of labour or capital will have more effect on the budget constraint. Adjusting the quantity of that input becomes necessary to satisfy the budget constraint. While this pattern is present in the AR1 model and ECM for both the short and long-run elasticities, the relationship is insignificant.

Figure 7



Source: Author's calculations using Statistics New Zealand data
Note: refer to appendix A for industry abbreviations

The divergence from a unitary elasticity may reflect other factors that were not accounted for. Theoretically, the elasticity depends on assuming perfect competition in capital and labour markets. The competitive model implies the elasticity of wages to productivity is equal to unity. However, Rosenberg (2010) finds these elasticities are substantially less than unity, implying a degree of imperfection in labour markets. Given the definition of the elasticity, these imperfections may be feeding into the estimates for the elasticity of capital-labour substitution. The effect may be present for industries showing negative elasticities, such as forestry and fishing (in the AR1 and first-difference models), and food, beverage, and tobacco manufacturing (in the AR1 model

[18] Further details on the heteroscedasticity, RESET, or Lagrange multiplier tests for autocorrelation are available from the author.
[19] A simple regression model confirms this with all polynomials being significant at the 95 percent confidence level and switching in sign.

and ECM). A negative elasticity implies (holding capital input and rental prices constant) that higher wages lead to more labour input. This is also a result from imperfect labour market theory, which shows that more labour can be employed at higher wages due to the distortions between wages and productivity (Manning, 2003).

Another potential explanation for the divergence from a unitary substitution lies in input utilisation (Felipe and McCombie, 2005). Calculating capital inputs assumes the rate of capacity utilisation is constant. As capacity utilisation adjustment involves adjusting capital inputs, but capital income is held constant, the estimated elasticity should change. Without adjustment, a strong increase in the wage-rental price ratio may have little effect on the capital-to-labour ratio – the effect of firms opting to increase utilisation of their existing inputs, rather than invest in additional capital or labour, will not be captured. This also implies that industries may not be operating on their production possibility frontier, an assumption required for estimating the elasticity.

The industry-level elasticities may be biased by asset leasing. Estimates of the volume of productive capital stock (used to derive the flow of capital services) are based on ownership of assets rather than use. The rental price (calculated as capital income over capital inputs) for an industry that rents its assets from other industries includes the effect of rent paid for assets as part of intermediate consumption. However, it may also reflect the income generated from rented assets. Therefore, the wage-rental price ratio may not be directly comparable with the capital-labour ratio and the elasticity may not reflect the actual substitution that occurs.

If rent could be removed from intermediate consumption, and thus from capital income, then consistency between capital income and input could be attained, but the elasticity would only relate to assets owned. Alternatively, if volumes of rented assets could be included in the perpetual inventory model, capital inputs would reflect both assets owned and leased and the elasticity would reflect total substitution.

To fully account for this problem, an econometric measurement-error model needs to be explored. But this still requires some prior belief about the distribution of rented assets over time; imposing a distribution without prior beliefs can lead to additional bias.

Data on rented assets are unfortunately not available to provide context for the owned/leased assets scenario. A snapshot of expenditure on rented assets from supply/use tables by industry is available, but a time series back to 1978 is not.

8 Conclusion

Under an economic framework, estimation of MFP growth requires a production function with associated assumptions or estimates for the elasticity between capital and labour. This paper has sought to estimate these elasticities, using data from Statistics NZ's productivity series in an econometric framework. Three econometric methods were employed to determine the elasticity: a first order AR1 model, a first-difference model, and an ECM. Each model is advantageous to a varying degree, depending on the time-series nature of the capital-to-labour and wage-rental price ratios. In specifying the dynamics of capital accumulation, estimates of both short- and long-run elasticities for the former measured sector and each industry in the analysis can be derived.

The data suggest a Cobb-Douglas form of the constant elasticity production function is appropriate at the aggregate level in New Zealand in the long run. At the industry-level, the evidence suggests a constant elasticity production function with varying elasticities across industries is appropriate. These findings align with those of Balistreri et al (2003) – a range of estimates for the elasticity is found across industries.

The hypothesis that there is one production function for all is not supported. The findings presented in this paper show that the Leontief function is more applicable in the short run but not the long run. This concurs with Sneessens and Dreze (1986) who found the impact that changing factor costs has on optimal technical coefficients occurs predominantly after one year.

This paper has also shed some light on the dynamics of capital deepening in the New Zealand economy. Growth in the capital-to-labour ratio depends strongly on previous growth and, to a lesser extent, on changes in relative factor prices. However, the effects depend on specifying the econometric model, as different industries are susceptible to feedback effects to varying degrees.

In interpreting the implications of this analysis, a number of caveats need to be kept in mind. For the econometric methods employed:

- the choice of econometric specification is important, and may lead to widely different results
- the sample sizes for the regressions are relatively small (due to the limited time series that are available) meaning that the estimates may be sensitive to revisions or additional years of data[20]
- the large standard errors for the ECM (likely influenced by autocorrelation and multicollinearity) make it difficult to attach statistical significance to the estimates.

While attempts were made to examine the dynamic structure for each industry, the results are often sensitive to the lag length or degree of differencing chosen. The length of the time series also restricts the definition of long run. In computable general equilibrium models, where the elasticity is an important parameter, the long run is considered to be more than three to five years. In this sense, the long-run elasticities presented in this analysis may be considered short-run elasticities in general equilibrium models.

The most important limitation is assuming the elasticity is time invariant. Figures 3, 4, and 6 (which plot the capital-to-labour ratio against the wage-rental price ratio) suggest this assumption does hold. Modifying this assumption with the data used in this analysis is not possible; only a micro-based study could provide enough observations each year to show movements in the elasticity over time. A micro-study would also be complementary to this analysis – to highlight the micro-foundation problem underlying the Cobb-Douglas function.

Productivity measurement uses a variety of data sources, each of which may be subject to a degree of sampling and non-sampling error. However, the deviation from expectation of

[20] The time series is long enough for determining statistical significance.

additional assumptions required for productivity measurement may also be a source of bias. Any mis-specification of these assumptions may be captured in the MFP residual.

This analysis suggests the form of the production function may be generating some bias in the MFP residual, and is likely to bias current estimates downwards. However, as shown in figure 5 the size of the bias is only of concern if the elasticity is (significantly) below 0.2. While the quantitative impact may be small (as MFP growth will still be between labour and capital productivity) there are implications for what MFP estimates mean; MFP does not solely reflect technological change, it also reflects the way inputs are employed and their flexibility to market prices.

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Appendix A: Industry coverage

Productivity industry coverage[1]

Measured sector industries	**Omitted industries**
AA Agriculture	LB Ownership of owner-occupied dwellings
AD Forestry and fishing	MA Government administration and defence
BA Mining	NA Education
CA Food, beverage, and tobacco manufacturing	OA Health and community services
CB Textile and apparel manufacturing	
CC Wood and paper products manufacturing	
CD Printing, publishing, and recorded media	
CE Petroleum, chemical, plastic, and rubber products manufacturing	
CF Non-metallic mineral products manufacturing	
CG Metal products manufacturing	
CH Machinery and equipment manufacturing	
CI Furniture and other manufacturing	
DA Electricity, gas, and water supply	
EA Construction	
FA Wholesale trade	
GA Retail trade	
HA Accommodation, cafes, and restaurants	
IA Transport and storage	
JA Communication services	
KA Finance and insurance	
LA Property services[2][3]	
LC Business services[2][3]	
PA Cultural and recreational services[3]	
QA Personal and other community services[2][3]	

1. Based on the Australian and New Zealand Standard Industrial Classification 1996 (ANZSIC96).
2. Included from March 1996 onwards in the measured sector.
3. Not included in this study.

Appendix B: The delta method

The hypothesis that the elasticity is equal to unity or zero involves testing a non-linear relationship between the coefficients from the regression model. While the long-run elasticity estimates can be readily calculated via algebraic manipulation, the standard errors cannot. The delta method can be used to calculate the standard errors for a non-linear hypothesis test.

The general form of the equation is:

$$var(\sigma) = \mathrm{D}\{\mathrm{Est.Asy.Var}[\beta]\}\mathrm{D}^{\mathrm{T}}$$

where

$$\mathrm{D} = \left(\frac{\partial\sigma}{\partial\beta_1} \cdots \frac{\partial\sigma}{\partial\beta_k} \right)$$

is a 1xk Jacobian matrix, D^{T} is the transpose of D, and

$$Est.Asy.Var[\beta] = \begin{bmatrix} var(\beta_1) & \cdots & cov(\beta_1, \beta_k) \\ \vdots & \ddots & \vdots \\ cov(\beta_k, \beta_1) & \cdots & var(\beta_k) \end{bmatrix}$$

is the k x k variance-covariance matrix. In other words, the variance depends on the derivatives of the elasticity with respect to its components, and the variances and covariances of the variables used in the model. The standard error is then just the square root of the estimated variance. The delta method is a linear Taylor series approximation of the estimated elasticity around the true parameter value.

For the AR1 model, where there are only two variables in the long-run elasticity, the variance is calculated as:

$$var(\sigma) = \left(\frac{\partial\sigma}{\partial\beta_1}\right)^2 .\mathrm{var}(\beta_1) + \left(\frac{\partial\sigma}{\partial\beta_2}\right)^2 .\mathrm{var}(\beta_2) + 2\frac{\partial\sigma}{\partial\beta_1}\frac{\partial\sigma}{\partial\beta_2}\mathrm{cov}(\beta_1, \beta_2)$$

where

$$\frac{\partial\sigma}{\partial\beta_1} = \frac{1}{\beta_2}$$

and

$$\frac{\partial\sigma}{\partial\beta_2} = -\frac{\beta_1}{(1-\beta_2)^2}$$

The standard error for the ECM with second differences is more complex as the long-run elasticity depends on four estimated values:

$$\sigma = \frac{\beta_3 + \beta_5}{-\beta_2 - \beta_4}$$

The associated derivatives for calculating the standard errors are:

$$\frac{\partial\sigma}{\partial\beta_3} = \frac{1}{-\beta_2 - \beta_4}$$

$$\frac{\partial\sigma}{\partial\beta_5} = \frac{1}{-\beta_2 - \beta_4}$$

$$\frac{\partial\sigma}{\partial\beta_2} = \frac{\beta_3 + \beta_5}{(\beta_2 + \beta_4)^2}$$

$$\frac{\partial\sigma}{\partial\beta_4} = \frac{\beta_3 + \beta_5}{(\beta_2 + \beta_4)^2}$$

On calculating the standard error, confidence intervals can be constructed as:

$$Conf.Int = \sigma \pm std.err.critical\ value$$

Critical values were taken from t tables (1.697 for the AR1 model, 1.699 for the ECM with first differences, and 1.701 for the ECM with second differences). It is important to note that the delta method and the derived confidence interval are only valid when the data are normally distributed. Table 6 in appendix C presents the standard errors for the long-run elasticity for both the AR1 model and the ECM.

Appendix C: Regression results

Appendix table 1

R^2, autocorrelation, and normality tests

Industry	R^2			Autocorrelation tests			Normality test	
	AR1	First difference	ECM	Durbin H (AR1)	Durbin Watson (First difference)	Durbin Watson (ECM)	AR1	ECM
Agriculture	0.97	0.25	0.47	2.42*	1.70	1.61	0.06	0.06
Forestry and fishing	0.94	0.11	0.85	3.35*	0.75*	2.16	7.84*	7.93*
Agriculture, forestry, and fishing	0.98	0.22	0.77	2.52*	1.39^{nc}	1.89	1.20	0.01
Mining	0.90	0.12	0.19	2.92*	1.88*	1.99	10.36*	0.10
Food, beverage, and tobacco manufacturing	0.96	0.11	0.15	4.35*	1.92	1.81	1.25	6.91*
Textile and apparel manufacturing	0.92	0.14	0.27	3.45*	1.20*	1.30*	1.25	0.80
Wood and paper products manufacturing	0.95	0.10	0.67	3.15*	1.53	1.53	2.08	2.32
Printing, publishing, and recorded media	0.96	0.08	0.38	1.03	2.11	1.56	0.53	1.33
Petroleum, chemical, plastic, and rubber products manufacturing	0.86	0.20	0.26	2.88*	0.86*	0.85*	90.18*	35.61*
Non-metallic mineral products manufacturing	0.87	0.10	0.34	3.54*	1.19*	1.32*	1.74	1.48
Metal products manufacturing	0.91	0.20	0.27	2.98*	0.99*	0.91*	3.41	0.37
Machinery and equipment manufacturing	0.95	0.26	0.67	2.87*	1.39^{nc}	1.54	0.39	2.73
Furniture and other manufacturing	0.93	0.23	0.77	0.79	1.69	1.92	14.45*	1.11
Manufacturing	0.95	0.09	0.13	4.05*	1.07*	0.97*	10.57*	0.60
Electricity, gas, and water supply	0.97	0.04	0.25	3.09*	0.98*	1.12*	0.49	17.13*
Construction	0.92	0.21	0.29	3.93*	1.55	1.46^{nc}	0.12	0.22
Wholesale trade	0.64	0.10	0.14	3.23*	1.42^{nc}	1.34^{nc}	0.28	0.12
Retail trade	0.97	0.03	0.11	4.08*	1.71	1.93	1.09	1.30
Accommodation, cafes, and restaurants	0.83	0.31	0.40	2.64*	2.28	1.95	1.29	1.17
Transport and storage	0.40	0.04	0.34	1.89*	1.69	1.66	0.82	24.64*
Communication services	0.99	0.11	0.20	3.74*	1.07*	1.31*	1.65	0.39
Finance and insurance	0.97	0.09	0.64	3.76*	1.58	1.97	0.42	0.11
Former measured sector	0.99	0.17	0.27	4.14*	0.82*	0.73*	0.45	0.59
Goods-producing industries	0.95	0.16	0.86	4.28*	1.00*	2.02	5.19	0.00
Primary industries	0.97	0.12	0.32	2.58*	1.18*	1.20*	1.33	4.31
Service industries	0.96	0.13	0.14	2.83*	0.93*	0.90*	0.94	0.73

Appendix table 2

Stationarity and cointegration tests				
	Dickey-Fuller tests: p-values		Phillips-Ouliaris tests	
Industry	Log capital-to-labour ratio	Log wage-rental price ratio	Rho	Tau
Agriculture	0.91	0.43	-24.78*	-4.54*
Forestry and fishing	0.77	0.62	-11.36	-2.72*
Agriculture, forestry, and fishing	0.92	0.57	-20.66*	-4.01*
Mining	0.45	0.36	-26.38*	-4.93*
Food, beverage, and tobacco manufacturing	0.58	0.25	-26.84*	-5.00*
Textile and apparel manufacturing	0.83	0.71	-17.36	-3.43*
Wood and paper products manufacturing	0.85	0.20	-21.29*	-4.11*
Printing, publishing, and recorded media	0.98	0.39	-29.40*	-5.48*
Petroleum, chemical, plastic, and rubber products manufacturing	0.05	0.12	-14.52	-2.94
Non-metallic mineral products manufacturing	0.76	0.27	-17.68	-3.35
Metal products manufacturing	0.16	0.36	-15.67	-3.28
Machinery and equipment manufacturing	0.82	0.37	-20.56*	-4.07*
Furniture and other manufacturing	0.96	0.54	-24.18*	-4.50*
Manufacturing	0.60	0.44	-15.19	-3.26
Electricity, gas, and water supply	0.73	0.66	-14.40	-3.05
Construction	0.91	0.04*	-23.24*	-4.38*
Wholesale trade	0.27	0.03*	-20.60*	-4.00*
Retail trade	0.99	0.58	-23.74*	-4.48*
Accommodation, cafes, and restaurants	0.81	0.12	-33.32*	-6.13*
Transport and storage	0.11	0.14	-24.70*	-4.50*
Communication services	0.52	0.73	-13.06	-3.14
Finance and insurance	0.80	0.65	-22.96*	-4.30*
Former measured sector	0.71	0.47	-11.88	-2.83
Primary industries	0.64	0.48	-17.49	-3.53*
Goods-producing industries	0.85	0.62	-15.19	-3.14
Service industries	0.56	0.31	-11.69	-2.90

Notes on the R^2, autocorrelation, and normality tests table: For the autocorrelation tests, * indicates evidence for autocorrelation at the 95 percent level of significance, and nc denotes that the test is inconclusive. For the Jarque-Bera normality tests, * denotes significance at the 95 percent confidence level and the null hypothesis of a normal distribution is rejected.

Notes on the stationarity and cointegration test table: For the unit root tests, * denotes the series does not contain a unit root. Tests do not include a time trend. Tests with a time trend also assessed, with the majority of industries exhibiting a unit root. Tests for second differences also performed. Results are available on request. Test statistics for the Phillips-Ouliaris cointegration tests are derived from the first-difference model. Critical values from tables Ib and IIb from Phillips-Ouliaris (1990) are: -20.4935 (rho) and -3.3654 (tau). * denotes the test statistic is less than the critical value, and therefore provides evidence for cointegration.

Appendix table 3

Former measured sector estimates			
Model	**Variable**	**Est**	**SE**
AR1	Intercept	-5.29	4.23
	Log wage-rental price ratio	0.16*	0.06
	Lag capital-to-labour ratio	0.81*	0.09
	Year	2.64	2.13
First difference	Intercept	1.40	1.10
	Wage-rental price ratio (first difference)	0.12	0.07
	Year	-0.69	0.55
ECM	Intercept	-4.45	4.73
	Lag capital-to-labour ratio	-0.17	0.10
	Wage-rental price ratio (first difference)	0.17*	0.07
	Lag wage-rental price ratio	0.14	0.08
	Year	2.23	2.37

Symbol: * denotes the variable is significantly different from zero at the 95 percent confidence level.

Appendix table 4

Sector-level estimates				
Sector	**Model**	**Variable**	**Est**	**SE**
Primary	AR1	Intercept	-30.82	3.57
		Log wage-rental price ratio	0.26	0.05
		Lag capital-to-labour ratio	0.13*	0.05
		Year	15.58	1.80
	First difference	Intercept	-0.62	1.57
		Wage-rental price ratio (first difference)	0.08	0.04
		Year	0.32	0.79
	ECM	Intercept	-10.17*	4.74
		Lag capital-to-labour ratio	-0.33*	0.13
		Wage-rental price ratio (first difference)	0.12*	0.04
		Lag wage-rental price ratio	0.12*	0.05
		Year	5.15*	2.39
Goods-producing	AR1	Intercept	-34.89*	6.85
		Log wage-rental price ratio	0.37*	0.17
		Lag capital-to-labour ratio	0.34*	0.11
		Year	17.50*	3.48
	First difference	Intercept	2.38*	1.38
		Wage-rental price ratio (first difference)	0.10	0.07
		Year	-1.18*	0.69
	ECM	Intercept	-0.03	0.07
		Lag capital-to-labour ratio	1.42*	0.17
		Lag capital-to-labour ratio (two period)	-1.48	0.15
		Wage-rental price ratio (second difference)	0.17*	0.07
		Lag wage-rental price ratio	-0.02	0.08
		Lag wage-rental price ratio (two period)	0.10	0.11
Services	AR1	Intercept	-41.69*	3.36
		Log wage-rental price ratio	0.38*	0.07
		Lag capital-to-labour ratio	0.13*	0.06
		Year	20.94*	1.70
	First difference	Intercept	1.30	1.22
		Wage-rental price ratio (first difference)	0.07	0.05
		Year	-0.64	0.61
	ECM	Intercept	-1.51	6.06
		Lag capital-to-labour ratio	-0.06	0.12
		Wage-rental price ratio (first difference)	0.08	0.06
		Lag wage-rental price ratio	0.02	0.07
		Year	0.77	3.04

Symbol: * denotes the variable is significantly different from zero at the 95 percent confidence level.

Appendix table 5

Industry-level estimates							
		AR1		**First difference**		**ECM**	
	Variable	**Est**	**SE**	**Est**	**SE**	**Est**	**SE**
Agriculture	Intercept	-13.32*	3.33	-1.00	1.07	-11.62*	3.56
	Log wage-rental price ratio	0.05*	0.01	…	…	…	..
	Lag capital-to-labour ratio	0.40*	0.14	…	…	-0.50*	0.16
	Wage-rental price ratio (first difference)	…	…	0.06*	0.02	0.06*	0.02
	Lag wage-rental price ratio	…	…	…	…	0.04*	0.02
	Year	6.72*	1.68	1.00	1.06	5.87*	1.79
Forestry and fishing	Intercept	-30.65*	8.27	-3.01	4.80	0.04	0.02
	Log wage-rental price ratio	-0.05	0.05	…	…	…	…
	Lag capital-to-labour ratio	0.72*	0.09	…	…	1.63*	0.15
	Lag capital-to-labour ratio (two period)	…	…	0.11	0.08	…	…
	Wage-rental price ratio (first difference)	…	…	…	…	0.21*	0.06
	Wage-rental price ratio (second difference)	…	…	…	…	-0.24*	0.09
	Lag wage-rental price ratio	…	…	…	…	-1.70*	0.16
	Lag wage-rental price ratio (two period)	…	…	…	…	0.25*	0.10
	Year	15.44*	4.16	1.53	2.41	…	…
Agriculture, forestry, and fishing	Intercept	-33.43*	2.45	-1.53	1.31	0.02	0.01
	Log wage-rental price ratio	0.14*	0.03	…	…	…	…
	Lag capital-to-labour ratio	0.11*	0.03	…	…	1.18*	0.23
	Lag capital-to-labour ratio (two period)	…	…	…	…	-1.26*	0.21
	Wage-rental price ratio (first difference)	…	…	0.07*	0.03	…	…
	Wage-rental price ratio (second difference)	…	…	…	…	0.08*	0.04
	Lag wage-rental price ratio	…	…	…	…	-0.05	0.04
	Lag wage-rental price ratio (two period)	…	…	…	…	0.10	0.05
	Year	16.86*	1.23	0.78	0.66	…	…
Mining	Intercept	-28.31*	11.70	6.20	3.83	0.16	0.12

Appendix table 5 continued next page

Appendix table 5 continued

Industry-level estimates							
		AR1		First difference		ECM	
	Variable	Est	SE	Est	SE	Est	SE
	Log wage-rental price ratio	0.10	0.15	…	…	...	...
	Lag capital-to-labour ratio	0.63*	0.13	…	…	-0.11	0.08
	Wage-rental price ratio (first difference)	…	…	0.09	0.09	...	...
	Wage-rental price ratio (lagged wages, second difference)	…	…	…	…	0.17	0.11
	Lag wage-rental price ratio (two period)	…	…	…	…	0.06	0.10
	Year	14.37*	5.90	-3.10	1.92	.	.
Food, beverage, and tobacco manufacturing	Intercept	-71.64*	6.66	2.44	1.91	-5.69	9.06
	Log wage-rental price ratio	-0.02	0.06	…	…	...	...
	Lag capital-to-labour ratio	0.13	0.08	…	…	-0.10	0.11
	Wage-rental price ratio (first difference)	…	…	0.04	0.03	0.03	0.04
	Lag wage-rental price ratio	…	…	.	.	-0.03	0.04
	Year	36.23*	3.36	-1.20	0.96	2.92	4.58
Textile and apparel manufacturing	Intercept	-36.83*	7.46	1.41	2.54	-0.17	0.10
	Log wage-rental price ratio	0.32*	0.10	…	…	...	...
	Lag capital-to-labour ratio	0.17*	0.07	…	…	-0.19*	0.09
	Wage-rental price ratio (first difference)	…	…	0.10	0.05	0.18*	0.06
	Lag wage-rental price ratio	…	…	…	…	0.18	0.09
	Year	18.43*	3.80	-0.70	1.28	…	...
Wood and paper products manufacturing	Intercept	-58.04*	6.28	-0.10	2.89	-0.02	0.09
	Log wage-rental price ratio	0.02	0.09	…	…	…	…
	Lag capital-to-labour ratio	0.28*	0.08	…	…	1.23*	0.20
	Lag capital-to-labour ratio (two period)	...	…	0.10	0.06	…	…
	Wage-rental price ratio (first difference)					0.11	0.08
	Wage-rental price ratio (second difference)	…	...	…	...	-0.16	0.08
	Lag wage-rental price ratio					-1.29*	0.21
	Lag wage-rental price ratio (two period)					0.24	0.14

Appendix table 5 continued next page

Appendix table 5 continued

Industry-level estimates							
		AR1		First difference		ECM	
	Variable	Est	SE	Est	SE	Est	SE
	Year	29.27*	3.18	0.06	1.45	…	...
Printing, publishing, and recorded media	Intercept	-33.20*	2.95	-2.38	1.91	-17.45*	5.89
	Log wage-rental price ratio	0.19*	0.06	…	…	...	...
	Lag capital-to-labour ratio	0.15*	0.05	…	…	-0.48*	0.15
	Wage-rental price ratio (first difference)	…	…	0.05	0.05	0.11*	0.05
	Lag wage-rental price ratio	…	…	…	…	0.18*	0.07
	Year	16.69*	1.50	1.20	0.96	8.75*	2.96
Petroleum, chemical, plastic, and rubber products manufacturing	Intercept	-7.24	10.32	11.52*	4.84	6.70	6.69
	Log wage-rental price ratio	0.13	0.17	…	…	...	...
	Lag capital-to-labour ratio	0.80*	0.13	…	…	-0.11	0.08
	Wage-rental price ratio (first difference)	…	…	0.08	0.09	0.12	0.11
	Lag wage-rental price ratio	…	…	.	.	0.07	0.12
	Year	3.66	5.22	-5.76*	2.43	-3.33	3.38
Non-metallic mineral products manufacturing	Intercept	-32.36*	4.50	1.61	2.67	-6.48	4.61
	Log wage-rental price ratio	0.30*	0.06	…	…	...	...
	Lag capital-to-labour ratio	0.16*	0.07	…	…	-0.26*	0.10
	Wage-rental price ratio (first difference)	…	…	0.09	0.06	0.12*	0.06
	Lag wage-rental price ratio	…	…	…	…	0.14*	0.05
	Year	16.32*	2.27	-0.80	2.64	3.27	2.32
Metal products manufacturing	Intercept	-24.32*	6.96	6.54	3.31	-0.13	6.09
	Log wage-rental price ratio	0.23*	0.05	…	…	...	...
	Lag capital-to-labour ratio	0.46*	0.11	…	…	-0.15	0.11
	Wage-rental price ratio (first difference)	…	…	0.05	0.04	0.08	0.04
	Lag wage-rental price ratio	…	…	…	…	0.07	0.05
	Year	12.21*	3.51	-3.27	1.66	0.08	3.06
Machinery and equipment	Intercept	-36.53*	3.75	-0.25	2.04	-16.82*	4.33
	Log wage-rental price ratio	0.38*	0.06	…	…	...	...

Appendix table 5 continued next page

Appendix table 5 continued

Industry-level estimates							
		AR1		First difference		ECM	
	Variable	Est	SE	Est	SE	Est	SE
manufacturing	Lag capital-to-labour ratio	0.24*	0.07	…	…	-0.39*	0.09
	Wage-rental price ratio (first difference)	…	…	0.14*	0.05	0.23*	0.04
	Lag wage-rental price ratio	…	…	…	…	0.29*	0.05
	Year	18.22*	1.90	0.14	1.02	8.32*	2.17
Furniture and other manufacturing	Intercept	-27.65*	2.55	-3.55	2.08	0.06	0.06
	Log wage-rental price ratio	0.14*	0.04	…	…	…	…
	Lag capital-to-labour ratio	0.23*	0.05	…	…	1.25*	0.19
	Lag capital-to-labour ratio (two period)	…	…	…	…	-1.17*	0.19
	Wage-rental price ratio (first difference)	…	…	0.08*	0.03	…	…
	Wage-rental price ratio (second difference)	…	…	…	…	0.11*	0.04
	Lag wage-rental price ratio	…	…	…	…	-0.05	0.05
	Lag wage-rental price ratio (two period)	…	…	…	…	0.00	0.05
	Year	13.82*	1.29	1.79	1.04	…	…
Manufacturing	Intercept	-33.01*	7.72	2.35	1.69	-1.45	5.39
	Log wage-rental price ratio	0.25	0.13	…	…	...	...
	Lag capital-to-labour ratio	0.44*	0.12	…	…	-0.09	0.09
	Wage-rental price ratio (first difference)	…	…	0.05	0.07	0.09	0.08
	Lag wage-rental price ratio	…	…	…	…	0.08	0.08
	Year	16.58*	3.90	-1.16	0.85	0.73	0.80
Electricity, gas, and water supply	Intercept	-65.15*	8.77	-1.03	2.96	-14.99	9.39
	Log wage-rental price ratio	0.50*	0.22	…	…	...	...
	Lag capital-to-labour ratio	0.35*	0.07	…	…	-0.21*	0.09
	Wage-rental price ratio (first difference)	…	…	0.13	0.13	0.27	0.14
	Lag wage-rental price ratio	…	…	…	…	0.36*	0.17
	Year	33.06*	4.39	0.54	1.49	7.73	4.74
Construction	Intercept	-40.58*	3.09	-0.56	2.01	-9.12	6.29
	Log wage-rental price ratio	0.25*	0.07	…	…	...	...

Appendix table 5 continued next page

Appendix table 5 continued

Industry-level estimates							
		AR1		First difference		ECM	
	Variable	**Est**	**SE**	**Est**	**SE**	**Est**	**SE**
	Lag capital-to-labour ratio	0.12*	0.05	…	…	-0.19	0.14
	Wage-rental price ratio (first difference)	…	…	0.12*	0.05	0.13*	0.06
	Lag wage-rental price ratio	…	…	…	…	0.01	0.08
	Year	20.29*	1.56	0.29	1.01	4.60	3.14
Retail trade	Intercept	-13.89*	3.22	2.37	1.51	0.03	0.02
	Log wage-rental price ratio	0.18*	0.06	…	…	...	...
	Lag capital-to-labour ratio	0.17	0.09	…	…	-0.11	0.07
	Wage-rental price ratio (first difference)	…	…	-0.02	0.02	-0.02	0.03
	Lag wage-rental price ratio	…	…	…	…	0.00	0.05
	Year	7.01*	1.62	-1.19	0.76	…	…
Wholesale trade	Intercept	-37.09*	4.10	-0.63	1.17	0.06*	0.03
	Log wage-rental price ratio	0.08	0.04	…	…	...	...
	Lag capital-to-labour ratio	0.19	0.11	…	…	0.05	0.04
	Wage-rental price ratio (first difference)	…	…	0.02	0.03	0.00	0.04
	Lag wage-rental price ratio	…	…	…	…	-0.04	0.03
	Year	18.69*	2.06	0.33	0.59	…	…
Accommodation, cafes, and restaurants	Intercept	-15.88*	2.25	-1.88	2.00	-6.48*	3.10
	Log wage-rental price ratio	0.13*	0.02	…	…	...	...
	Lag capital-to-labour ratio	0.13*	0.06	…	…	-0.30	0.16
	Wage-rental price ratio (first difference)	…	…	0.11*	0.03	0.12*	0.03
	Lag wage-rental price ratio	…	…	…	…	0.04	0.03
	Year	7.92*	1.13	0.95	1.00	3.24*	1.55
Transport and storage	Intercept	-7.52*	2.63	-1.27	1.96	-6.09*	2.41
	Log wage-rental price ratio	0.11*	0.05	…	…	...	...
	Lag capital-to-labour ratio	0.35*	0.09	…	…	-0.33*	0.14
	Wage-rental price ratio (first difference)	…	…	0.04	0.05	0.07	0.05
	Lag wage-rental price ratio	…	…	.	.	0.12*	0.02

Appendix table 5 continued next page

Appendix table 5 continued

Industry-level estimates							
		AR1		First difference		ECM	
	Variable	**Est**	**SE**	**Est**	**SE**	**Est**	**SE**
	Year	3.73*	1.31	0.64	0.98	3.02*	1.20
Communication services	Intercept	1.53	16.37	4.85	3.62	16.76	15.95
	Log wage-rental price ratio	-0.13	0.10	…	…	...	...
	Lag capital-to-labour ratio	1.04*	0.10	…	…	0.11	0.09
	Wage-rental price ratio (first difference)	…	…	0.14	0.10	0.04	0.12
	Lag wage-rental price ratio	…	…	…	…	-0.16	0.10
	Year	-0.69	8.25	-2.40	1.82	-8.35	8.04
Finance and insurance	Intercept	-73.18*	6.33	0.99	2.37	0.04	0.03
	Log wage-rental price ratio	0.22*	0.09	.	.	...	...
	Lag capital-to-labour ratio	0.03	0.03	.	.	1.18*	0.21
	Lag capital-to-labour ratio (two period)	…	…	0.15	0.10	…	…
	Wage-rental price ratio (first difference)	…	…	…	…	0.13	0.11
	Wage-rental price ratio (second difference)	…	…	…	…	-0.22	0.14
	Lag wage-rental price ratio	…	…	…	…	-1.23*	0.22
	Lag wage-rental price ratio (two period)	…	…	…	…	0.24	0.17
	Year	36.96*	3.20	-0.48	1.19	…	…

Symbols:
… not applicable
* denotes the variable is significant at the 95 percent confidence level

Appendix table 6

Standard errors and confidence interval limits for the long-run elasticity

	AR1				ECM			
Industry	**LRE**	**SE**	**Upper limit**	**Lower limit**	**LRE**	**SE**	**Upper limit**	**Lower limit**
Agriculture	0.08	0.02	0.12	0.05	0.08	0.02	0.12	0.04
Forestry and fishing	-0.18	0.17	0.10	-0.47	0.26	0.68	1.42	-0.89
Agriculture, forestry, and fishing	0.16	0.03	0.21	0.10	0.67	0.27	1.13	0.21
Mining	0.27	0.38	0.91	-0.37	0.50	0.61	1.54	-0.53
Food, beverage, and tobacco manufacturing	-0.03	0.07	0.09	-0.15	-0.27	0.49	0.56	-1.11
Textile and apparel manufacturing	0.38	0.12	0.59	0.18	0.95	0.18	1.25	0.65
Wood and paper products manufacturing	0.03	0.13	0.24	-0.19	1.28	1.00	2.98	-0.42
Printing, publishing, and recorded media	0.23	0.07	0.35	0.12	0.36	0.12	0.58	0.15
Petroleum, chemical, plastic, and rubber products manufacturing	0.67	0.65	1.78	-0.44	0.67	0.83	2.09	-0.74
Non-metallic mineral products manufacturing	0.35	0.07	0.47	0.23	0.54	0.16	0.81	0.26
Metal products manufacturing	0.42	0.08	0.55	0.29	0.48	0.19	0.80	0.16
Machinery and equipment manufacturing	0.50	0.09	0.65	0.35	0.75	0.13	0.97	0.54
Furniture and other manufacturing	0.18	0.05	0.26	0.10	0.62	0.44	1.37	-0.13
Manufacturing	0.45	0.22	0.82	0.08	0.90	0.90	2.42	-0.63
Electricity, gas, and water supply	0.77	0.31	1.29	0.24	1.71	0.75	2.99	0.44
Construction	0.29	0.08	0.42	0.15	0.07	0.37	0.70	-0.56
Wholesale trade	0.21	0.08	0.34	0.08	0.03	0.42	0.73	-0.68
Retail trade	0.10	0.04	0.17	0.03	0.89	0.44	1.63	0.14
Accommodation, cafes, and restaurants	0.16	0.02	0.19	0.12	0.15	0.05	0.24	0.06
Transport and storage	0.16	0.08	0.30	0.02	0.36	0.20	0.71	0.02
Communication services	3.29	7.03	15.21	-8.64	1.47	0.96	3.11	-0.17
Finance and insurance	0.23	0.10	0.39	0.07	0.56	1.42	2.97	-1.85
Former measured sector	0.82	0.26	1.27	0.38	0.83	0.31	1.35	0.30
Primary industries	0.30	0.05	0.39	0.21	0.37	0.10	0.55	0.20
Goods-producing industries	0.55	0.24	0.96	0.15	1.33	0.86	2.79	-0.14
Service industries	0.44	0.07	0.56	0.32	0.33	0.76	1.62	-0.95

Upper limit and lower limit refers to the upper and lower bounds of the 95 percent confidence interval, respectively. Standard errors calculated via the delta method. Where the interval contains zero, the Leontief function cannot be rejected. Where the interval contains unity, the Cobb-Douglas function cannot be rejected. In some cases, the interval contains zero and unity and no economic interpretation can be made. The wide intervals may result from autocorrelation, multicollinearity, or heteroscedasticity. The intervals are not valid where the data are non-normally distributed. Therefore, these results should be compared with the normality tests in the R^2, autocorrelation, and normality tests table above.