Capital Structure, Cost of Capital, and Voluntary Disclosures*

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Abstract

This paper develops a model of external financing that jointly determines a firm's capital structure, its voluntary disclosure policy, and its cost of capital. We study a setting in which investors – who provide financing to a firm in exchange for securities issued by the firm – sometimes incur trading losses when they subsequently trade their securities with a superiorly informed trader. Both the firm's disclosure policy and the structure of the firm's securities determine the informational advantage of the superiorly informed trader which in turn determines both the size of investors' trading losses and the firm's cost of capital.

In this setting, among other things, we establish: there is a hierarchy of optimal securities that varies with the volatility of the firm's cash flows, that an increase in the volatility of the firm's cash flows is associated with: an increase in the amount of debt in the firm's capital structure; a reduction in the firm's voluntary disclosures; and an increase in the firm's cost of capital. The model predicts a negative association between firms' cost of capital and the extent of information they disclose voluntarily. This negative association does not imply, however, that more expansive voluntary disclosure *causes* firms' cost of capital to decline. The paper also documents how imposing mandatory disclosure requirements can alter firms' voluntary disclosure decisions, their capital structure choices, and their cost of capital.

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1 Introduction

This paper develops a model that jointly explains a firm's voluntary disclosure policy, its capital structure, and its cost of capital. While links between a firm's disclosure policy and its cost of capital have been established in prior academic accounting research (e.g., Botosan [1997], Botosan and Plumlee [2002]), and links between a firm's capital structure and its cost of capital separately have been identified through research in finance (e.g., in Myers and Majluf's [1984] hierarchical model of financing), we are not aware of any existing literature in accounting or finance that endogenously connects a firm's disclosure policy to its capital structure, and a fortiori, any literature that connects all three of these components of a firm's financial structure.

That developing a theory that jointly explains the links among these three components is important is clear. A firm's owners do not seek to have their firm adopt a particular disclosure policy, or a particular form of capital structure, or even take actions to affect their firm's cost of capital, merely for their own sake. Rather, the owners care about these components of a firm's financial structure only to the extent that the components jointly help the owners maximize the expected value of their residual claims deriving from their ownership of the firm. Even if (contrary to fact) accounting researchers were interested only in understanding, say, the determinants of a firm's voluntary disclosure policy, examining firms' voluntary disclosure policies in isolation of these other components of firms' financial structures would not be very revealing when the firms' owners themselves choose their firms' disclosure policies in conjunction with the actions they take to affect their firms' cost of capital and capital structure.

It is intuitive that there should be a relationship between a firm's capital structure and its disclosure policy: since the owners of a firm will choose the firm's disclosure policy to maximize the market's perceptions of the expected value of the owners' residual claim, the form of their residual claim – equivalently, the form of the securities that the firm has sold to investors – will affect the owners' objective function, and hence also affect what subsequent disclosures will maximize that objective function. A simple example that exhibits this relationship is given by a firm that has issued debt whose face value in some states of the world exceeds the value of the firm's assets when the debt must be repaid. In those states, barring legal or regulatory imperatives, owners of a firm interested in maximizing the market's perceptions of the expected value of their residual claims would *never* disclose that the value of the firm's assets has fallen below the face value of the debt since, by doing so, the owners would reveal that their residual claims are worthless.

In the model we study, there can be a gap between the expected present value of securities a firm issues to investors, and the amount that investors are willing to pay for the securities, because the investors can anticipate at the time they buy the securities that they may subsequently incur trading losses as a consequence of having to liquidate the securities on a market on which there are insiders who have information superior to that of the market maker in those securities. This gap, which is a cost that the firm bears in raising capital, is the source of the "cost of capital" in our model.

A firm has an interest in minimizing the investors' expected trading losses because – as in Jensen and Meckling's [1976] original theory of capital structure – the firm itself ultimately bears the cost of these losses. The firm's manager can reduce the expected trading losses that the investors supplying the firm with capital bear by reducing the information asymmetry between the insiders and the market maker regarding the securities' value in either of two ways. First, since the insiders' private information is likely to overlap with the manager's own private information, the manager can disclose the private information he receives. Second, the manager can have the firm issue securities whose value is not "informationally sensitive" (Sunder [2006]), i.e., whose value does not vary with the insiders' private information. For example, were the owners to issue debt that is genuinely risk-free, then – regardless of the insiders' private information – the insiders have no informational advantage over anyone else in assessing the debt's value. Thus, in the model, a firm's capital structure and its disclosure policy jointly determine the firm's cost of capital because they jointly determine how much information asymmetry remains between the market maker and insiders, and this remaining information asymmetry in turn determines investors' expected trading losses and hence the firm's cost of capital.

The paper develops a formula that shows how a firm's cost of capital varies with the design of the security it offers to investors and its disclosure policy. The formula, which is applicable to all securities, establishes that a firm's cost of capital depends, differentially, on a security's upside potential (the difference between a security's payoff if the firm's realized cash flows assume "high" or "medium" values) and its downside risk (the difference between a security's payoff if the firm's realized cash flows assume "medium" or "low" values). The formula shows that the contribution of a security's downside risk and its upside potential to the firm's cost of capital depends on the firm's disclosure policy. Specifically, the formula shows that the contribution of a security's upside potential to the firm's cost of capital decreases as the firm discloses more information, and, perhaps surprisingly, the contribution of a security's downside risk to the firm's cost of capital *increases* as the firm discloses more information.[1]

The paper also develops a hierarchy involving both the firm's optimal capital structure and the firm's optimal voluntary disclosure policy. This hierarchy is indexed by the amount of volatility in the firm's cash flows. The paper shows that a firm with very low volatility in its cash flows prefers to raise capital by issuing risk-free debt and adopting an "expansive" disclosure policy.[2] Then, as its cash flow volatility increases, the firm prefers to use investment-grade debt (that defaults with low probability) combined with the continued use of an expansive disclosure policy. Then, as its cash flow volatility increases still further, the firm will continue to use investment-grade debt, but it will curtail its disclosures and adopt a "limited" disclosure policy. As its cash flows become even more volatile, the firm will switch to using "junk" debt (which defaults if anything other than the highest cash flows occur), accompanied by limited disclosure. Finally, as its cash flow volatility becomes even greater still, we demonstrate that it is impossible to finance the firm with any form of security accompanied by any form of disclosure.

[1] We defer the explanation for this result to the text below where the result is presented.

[2] The precise meanings of an "expansive" disclosure policy, as well as the related notion of a "limited" disclosure policy, are given formally in the text below.

While reminiscent of Myers and Majluf's [1984] famous financial hierarchy (inside financing is least expensive; outside financing with debt is more expensive; outside financing with equity is most expensive), our hierarchy is distinct from Myers and Majluf's [1984] hierarchy in two fundamental respects: first, our hierarchy combines a firm's capital structure choice with its voluntary disclosure policy, whereas Myers and Majluf make no reference to firms' disclosures. Second, our capital structure hierarchy is indexed to the volatility of a firm's cash flows, whereas Myers and Majluf's hierarchy is indexed to the relative costliness of alternative financing methods while holding cash flow volatility fixed.

The paper also contains an examination of the effects of a firm precommitting to disclose certain information that it receives or, what amounts to the same thing, studying mandatory disclosure requirements. The paper shows how mandatory disclosure requirements interact with firms' (ex post) voluntary disclosure decisions to affect the firms' cost of capital and their securities design choices. The paper shows that, depending on how the mandatory disclosure requirements are structured, mandatory disclosure requirements can either inhibit or encourage firms to make voluntary supplementary disclosures, and also that imposing mandatory disclosure requirements sometimes radically alters what securities firms may wish to offer their investors. In particular, we show that equity securities, which are typically not optimal in our model when all disclosures are voluntary, may become optimal when mandatory disclosure requirements are imposed.

Related literature

Our paper is related to the literature which explores the implications of voluntary and mandatory accounting disclosures. Since this literature is voluminous, we discuss only some illustrative strands of the literature here. Korajczyk, Lucas and McDonald [1991, 1992] started a literature studying the incentives of firms to disclose information in advance of raising capital. The central point of Korajczyk, Lucas and McDonald is that firms will tend to raise new capital in periods where the information asymmetry between insiders of the firm and outside investors is small. Selecting these periods in which to raise capital – or equivalently, making additional disclosures in advance of raising capital – is sensible, because it helps to eliminate the lemons' problem that Myers and Majluf [1984] originally noted: firms will tend to go outside to raise capital only when the insiders think that outsiders overestimate the firm's value. Our paper is distinct from this literature initiated by Korajczyk et al. [1991, 1992] in many respects, but most significantly in that: (a) we are concerned about not when firms raise capital, but rather in what form capital is raised, and (b) we are not concerned about disclosures before the firm raises capital, but rather about disclosures after the firm has raised capital. But, the present paper and Korajczyk et al. [1991, 1992] share the important property that the disclosure of information has real effects, which is also true of a substantial part of the disclosure literature (e.g., Kanodia, Mukherji, Sapra, and Venugopalan [2000], Stocken [2000], Pae [2002]).

Several recent papers discuss the links between the quality of financial statements and the cost of capital. Jorgensen and Kirschenheiter [2003] show that firms have incentives to selectively disclose information that indicates lower cash flow risk, thus leading to low risk premia conditional on a voluntary disclosure. Focusing more specifically on incentive problems, Christensen, Feltham and Wu [2002] develop a theory that explains when and how manager's compensation should be adjusted for the cost of capital. They show that compensating a manager based on residual income (i.e., with a charge for market risk) may make the manager excessively averse to undertaking risky projects. Our analysis differs from these two papers because our notion of cost of capital is not with respect to risk-aversion, but in terms of the endogenous liquidity costs borne by investors when reselling an asset for liquidity reasons.

The present paper is also linked to the large literature on securities design, see, e.g., Harris and Raviv's [1991] survey article. Nachman and Noe [1994] study a problem in which an informed issuer issues a security; they show that there is a unique equilibrium such that all issuers pool and offer the same risky security. When the security is designed prior to the issuer being informed, Demarzo and Duffie [1999] show that issuers optimally offer risky debt but retain a fraction of the issue to signal high future cash flows. Demange and Laroque [1995] and Rahi [1996] examine a security design problem when a risk-averse insider subject to private-information shocks may choose to float the security on the market. Boot and Thakor [1993], Fulghieri and Lukin [2001] and Sunder [2005] analyze the security design problem when some market participants may optimally acquire information as a function of the issued security; these models note that the choice of more informationally sensitive instruments (such as equity) encourage more information acquisition. Finally, several recent papers model the security design problems that emerge in venture finance with moral hazard. Biais et al. [2007] study the evolution of the structure of a firm's financial claims with a wealth-constrained entrepreneur, and they characterize the links between current and past performance, the choice of leverage, the payments of dividends and the scale of operations. Casamatta [2003] and Repullo and Suarez [2004] characterize the optimal mix of debt and equity in the presence of two-sided moral hazard between the entrepreneur and the investor. Curiously, as far as we are aware, none of the present and past research on security design literature has considered the interdependencies between security design and disclosures, which is a core object of focus in the present work.

2 Model setup

We begin by sketching each of the four main steps of the model, the timeline for which is presented in Figure 1, and then we describe each step in detail.

At step $t = 1$, an individual (called the "manager" in what follows) owns a production technology (called the "firm") that converts capital invested in the firm in the first step stochastically into cash flows $\tilde{\theta}$ in the fourth step.[3] The manager, who has no capital of his own, offers a security $\tilde{\psi} = \psi(\tilde{\theta})$ (the payoff of which depends upon the firm's future cash flows) to an investor in return for the investor supplying

[3] In an extension discussed in Section 4.1 below, the technology that produces future cash flows requires two inputs, capital and "effort" (or some other costly productive input) supplied by the manager. To keep the description of the initial model from becoming too complex, we do not incorporate this multi-input technology into the base version of the model.

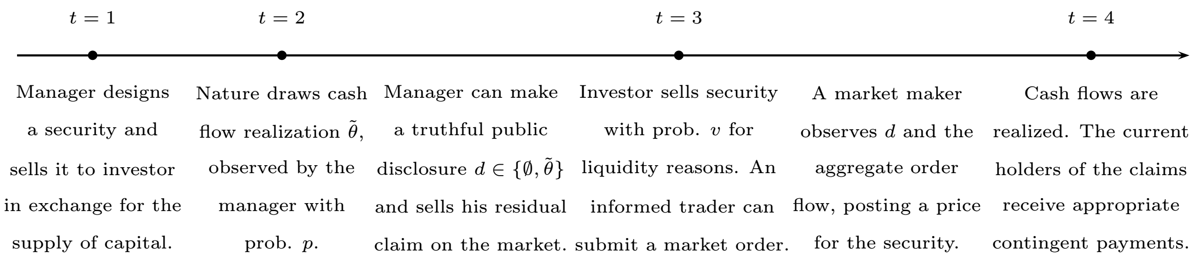

Figure 1: Model Timeline

capital to the firm. Specifying the security $\tilde{\psi}$ simultaneously defines the "residual security" $\tilde{\xi} \equiv \tilde{\theta} - \tilde{\psi}$ whose payoff consists of the difference between the firm's realized cash flows θ and the firm's cash flows $\psi(\theta)$ dedicated to be paid to the holder of security $\tilde{\psi}$ at step 4.

At step $t = 2$, sometimes the manager learns the realization of the firm's future cash flows "early," at this step. The manager may either disclose his information or disclose nothing, based on whether the disclosure increases the capital market's perceptions of the expected value of the manager's residual security $\tilde{\xi}$.

At step $t = 3$, two securities' markets open, one for the security $\tilde{\psi}$ and one for the residual security $\tilde{\xi}$.[4] The investor *sometimes* experiences a liquidity shock that forces him to sell his security $\tilde{\psi}$. The manager *always* experiences a liquidity shock (or, what amounts to the same thing, has a sufficiently short time horizon) that forces the manager to sell his residual security $\tilde{\xi}$.[5] The market clearing prices on both securities' markets are established via the same Kyle-type market maker,[6] who sets each security's market price based on knowledge of the aggregate order flows of both securities. The aggregate order flows potentially include orders from an informed trader, or insider, who may have information superior to the market maker's about the securities' payoffs.

At step $t = 4$, the firm's cash flow realizes its value, and the holders of the two securities at this time are paid off according to the terms specified by the securities.

We now elaborate on the preceding description.

Expanding on the first step, in the base model the firm's stochastic production technology requires a fixed amount of capital, which we scale to be \$1, to be supplied by an investor to make the technology run. Less than \$1 of capital generates no cash flows in step 4 at all, and more than \$1 does not increase the (distribution of) step 4's cash flows. For notational convenience only, we normalize the investor's required rate of return on capital at zero.[7] Consequently, the manager can induce the investor to supply

[4] In the following, it will be seen that the role of the $\tilde{\xi}$ securities market is limited to establishing a price at which the manager can liquidate his interest in the security at step 3.

[5] The assumption that the manager always sells his residual stake in the security (in the third step of the model) could be replaced by the assumption that only sometimes does the manager sell his residual stake for life cycle or other reasons unrelated to the security's expected market value, and all of our results will remain valid qualitatively. Since, in practice, managers often adopt 10b(5)-1 plans in which they commit to selling shares in their firm unconditionally, we believe that the assumption that having the manager, at least sometimes, sell his security without regard to a comparison of its current price to its terminal expected value, is a reasonable description of prevailing managers' behavior.

[6] Equivalently, the prices are established by separate market makers who get to see the aggregate order flows associated with both securities. (This is a simple, but explicit, way of representing the information spillovers that naturally occur across securities' markets.)

[7] It should be noted here that how that rate of return is determined – and in particular, whether that rate of return is a competitive return as a consequence of this capital market being competitive, or an above normal rate of return as

sufficient capital to run the technology by ensuring that the security $\tilde{\psi}$ has an expected payoff to the investor of at least \$1.

Conditional on the manager obtaining financing for the firm, the firm's realized cash flows (in step 4) are given by the realization of the random variable $\tilde{\theta}$, which assumes one of three possible values: $\theta_l \equiv \mu - e$, or $\theta_m \equiv \mu$ or $\theta_h \equiv \mu + e$, for some $e > 0$. Each of θ_l and θ_h occurs with probability $\frac{\eta}{2} \in \left(0, \frac{1}{2}\right)$, and θ_m occurs with probability $1 - \eta$. We further assume

$$\mu - e > 0 \text{ and } \mu > 1. \tag{1}$$

That is, the firm's realized cash flows are symmetrically distributed around their mean μ; the cash flows are always positive, and investing in the firm is efficient (in so far as the gross expected cash flows produced by investing in the firm exceed the expected cash flows that would result from investing the same amount of funds at the investor's required rate of return).

The security $\tilde{\psi}$ the manager offers the investor gives the investor (or whoever is in possession of the security at step 4 when $\tilde{\theta}$ realizes its value) a payoff that is potentially contingent on the firm's realized cash flows. In the following, we denote a security interchangeably by $\tilde{\psi}$, $\psi(\tilde{\theta})$, or (ψ_l, ψ_m, ψ_h). The security could be, for example, risk-less debt ($\tilde{\psi} \equiv F$ where $F \leq \mu - e$ is the face value of the debt), risky debt ($\tilde{\psi} \equiv \min\{\tilde{\theta}, F\}$ with $F > \mu - e$), equity ($\tilde{\psi} = \alpha\tilde{\theta}$, for some $\alpha \in (0, 1)$), a mixture of debt and equity (e.g., $\tilde{\psi} \equiv \alpha\tilde{\theta} + \min\{\tilde{\theta}, F\}$), etc. The choice of the security $\tilde{\psi}$ is tantamount to a choice of the firm's capital structure, because it determines whether the manager finances the firm with risky or risk-free debt, with equity, or with some other, potentially more exotic, financial instrument.

Throughout the following, we require that every security $\tilde{\psi}$ we study satisfies three natural restrictions: first, that it subject the manager to limited liability, i.e.,

$$\tilde{\psi} \leq \tilde{\theta}; \tag{2}$$

second, that it not entail further capital injections by the investor – or equivalently, that the security also provides limited liability to the investor – i.e.,

$$0 \leq \tilde{\psi}, \tag{3}$$

and third, that the security be monotone in the sense that:

$$\xi(\theta) = \theta - \psi(\theta) \text{ and } \psi(\theta) \text{ are both weakly increasing in } \theta, \text{ with } \xi(\theta_l) < \xi(\theta_h). \tag{4}$$

We leave further discussion of these natural, standard assumptions – that have also been adopted in other studies of securities' design – to the accompanying footnote.[8]

a consequence of the investor having some market power in this capital market – has no substantive effect on any of our results.

[8]Regarding (2), the assumption of limited liability for the manager is not simply reasonable, but is also necessary for the model to be internally consistent: the only way the manager could offer a security with a payout $\psi(\theta) > \theta$ for some θ is for the manager to have some capital of his own. But, if the manager possessed this capital initially, then why did the manager need capital from the investor to invest in the technology? By positing that (2) holds, such questions do not

To avoid other informational frictions that may complicate the analysis, we assume that the investor has no reason to be concerned that: (1) the manager will fail to invest in the stochastic production technology after receiving the initial capital injection and (2) the manager possesses any private information about the returns to the security at the time the security is issued.

Expanding on the second step, we posit that, after designing a security that makes the investor willing to provide financing, the manager probabilistically receives advanced information about the firm's cash flows. Specifically, we assume that, with probability $p \in (0,1)$, the manager privately learns the exact realization of the firm's cash flows, and with probability $1-p$, the manager learns nothing about those cash flows. We let $\tilde{s}_{knowcash}$ denote the random variable representing the manager's signal about the cash flows, with $s_{knowcash} = \theta$ indicating that the manager learns that the cash flows will be θ, and $s_{knowcash} = \emptyset$ indicates that the manager receives no information about the cash flows.[9]

Subsequent to receiving the realized value of the signal $\tilde{s}_{knowcash}$, the manager decides between disclosing the firm's realized cash flows (when he learns them) or else disclosing nothing. We denote the manager's disclosure or nondisclosure by $d = d(s_{knowcash})$. As is standard in the literature (see, e.g., Dye [1985], Jung and Kwon [1988], or Pae [2005]), we assume: (1) if the manager makes a disclosure, it must be truthful; (2) if the manager receives no information, he necessarily makes no disclosure; and (3) the manager cannot disclose that he has received no information. With $d(s_{knowcash}) = \emptyset$ denoting that the manager makes no disclosure, these restrictions are summarized by the requirements $d(\emptyset) = \emptyset$ and $d(\theta) \in \{\theta, \emptyset\}$.

Because, as was described previously, the manager liquidates his residual stake in the firm in the third step, the manager's decision whether to make a disclosure in the second step (when the manager

arise, and the internal consistency of the model is maintained.

Regarding (3), the investor sometimes is forced to sell the security he receives from the manager before the firm's cash flows realize their value on a capital market. Among other things, trading securities that do occasionally demand capital injections by the security's holder – with $\psi(\theta) < 0$ – would require solvency checks for those seeking to acquire the security in order for the stated payoff of the security to correspond to the security's actual payoff. To be in accordance with most commonly traded securities, and to avoid concerns over the viability of solvency checks, we posit that the investor is also subject to limited liability.

Regarding (4), all this condition states is that the value of the manager's claim $\xi(\theta)$ and the investor's claim $\psi(\theta)$ increase weakly as the firm's cash flows θ increase, and that over some pair of cash flows, the manager's claim strictly increases in value. This is an economically appealing condition on its face, and it can be supported formally in a variety of ways. First, since the manager can be viewed as an agent of the investor, the manager's claim $\xi(\theta)$ can be viewed as the manager's compensation for having the firm's cash flows turn out to be θ. When so viewed, the assumption that $\xi(\theta)$ is monotone increasing has been formally justified by agency theory (see, e.g., Holmstrom [1979], Grossman and Hart [1983]) when the relationship between the manager's action and the distribution of the firm's output satisfies the monotone likelihood ratio property (MLRP). Moreover, Grossman and Hart [1983] have shown in their formulation of the agency problem that the principal's residual security must be increasing over some sets of states; Dye and Sridhar [2008] have shown, in another formulation of the agency problem, that when the distribution of the manager's output satisfies an increasing inverse hazard rate condition, the manager's compensation and the principal's residual stake both must be strictly increasing. So, these monotonicity conditions emerge naturally from agency-theoretic considerations.

Second, monotonicity of the residual security must hold under "free disposal" assumptions commonly invoked in the economic literarture. That is, if the residual security holder can waste, or give away, the firm's cash flows so as to increase the value of the residual security, then the actual, cash flow-contingent payout of the residual security, $\bar{\xi}(\theta) \equiv \max\{\xi(\theta')|\theta' \leq \theta\}$, will be monotone regardless of whether the original residual security $\xi(\theta)$ was monotone. Since the manager never benefits by wasting cash, there is no loss in restricting the residual security to be monotone from the outset.

Finally, we note that various monotonicity assumptions similar to those we have imposed are commonplace in models of security design. See, e.g., Harris and Raviv [1990], Innes [1990], Nachman and Noe [1994], Demarzo and Duffie [1999]).

[9]As is standard in such models, the results are essentially unchanged were the manager to receive a noisy signal about the firm's cash flows; were the latter assumption adopted, $s_{knowcash} = \theta$ is to be interpreted as the firm's expected cash flows are θ based on the manager's private information.

receives information) is guided by whether the disclosure will increase or decrease the expected market value of his residual security $\tilde{\xi}$ when he sells it on the residual securities market.

Expanding on the third step, after the manager has made his disclosure d (which could be "no disclosure," i.e., $d = \emptyset$), two securities markets open, one for the security $\tilde{\psi}$ and one for the residual security $\tilde{\xi}$. While, as was noted above, the manager always sells his residual stake, whether the investor sells his security $\tilde{\psi}$ is assumed to depend on whether the investor is subject to a liquidity shock. In the event he is subject to such a shock, the investor is forced to sell his entire interest in $\tilde{\psi}$. We introduce the random variable $\tilde{q} \in \{0,1\}$ to indicate whether or not the investor is subject to a shock, with $q = 1$ (resp., $q = 0$) indicating that he is (resp., is not) subject to a shock, and we let $v = \Pr(\tilde{q} = 1)$ denote the ex ante probability he is subject to a shock. If not subject to a shock, though the investor may still sell his security, it will be clear in what follows that he will elect not to.

An informed trader or insider (whom we generally refer to simply as "the trader" in the following) and a market maker who sets the market price for each security based on all information available to him are the two other potential participants in each securities market. The trader's private information consists of knowing, when the manager makes no disclosure, whether the reason for the manager's nondisclosure was due to the manager not having received information (which we indicate by $\tilde{s}_{withheld} = 0$) or having deliberately withheld information he did receive (indicated by $\tilde{s}_{withheld} = 1$). In the latter case – where the investor knows that the manager withheld information – we presume that the trader does not know the value of the information the manager received.[10] When the manager makes no disclosure, the market maker in the firm's securities, unlike the informed trader, does not know the reason for the manager's nondisclosure. The market maker sets the price for each security based on public knowledge of the manager's disclosure d and the aggregate order flows for both securities, as in Kyle [1985].

All of the following events happen simultaneously: the manager sells his residual security $\tilde{\xi}$; the investor learns whether he is subject to a liquidity shock and if so, sells his security $\tilde{\psi}$; the trader submits his market order(s) for one or both securities.

We assume that the trader is prohibited from engaging in short-sales on either market.[11] Combined with the assumption that, when the investor experiences a liquidity shock, the investor must sell all of his interest in his security, this leads to the conclusion that the trader – who has an interest in masking the identity of his trades – either does not participate in the $\tilde{\psi}$ securities market, or else he submits a market order for exactly one unit of security $\tilde{\psi}$.[12] That is, with $y(d, s_{withheld})$ denoting the investor's market

[10] It is easy to think of a variety of practical accounting settings in which this informational assumption (that a trader exists who knows whether the manager has received undisclosed information, but not what that information is) is applicable. As just two of myriad examples: a trader may know that if a firm's auditor resigns from an audit, then - notwithstanding the (often uninformative) required 8K filing - there was a dispute between the firm and the auditor over how the firm's financial statements should be presented, but when an auditor does resign from an audit, the trader typically does not know the specific information concerning the firm's financial statements that gave rise to the auditor's resignation. As another example, a trader may know that if a firm's earnings release has been delayed considerably, this indicates problems with the firm's earnings report, but when a firm's earnings release is delayed considerably, the trader typically will not know what specific problems with the earnings report gave rise to the delay.

[11] We justify this assumption by noting that there are obvious and well-known restrictions and frictions that make short-selling difficult, and these restrictions have become even more onerous with the SEC's September 2008 actions (SEC's 'Financial Firm Emergency Order' No. 34-58592; Sept 18, 2008).

[12] Were the informed trader to submit market orders in quantities different from that of the investor, he would reveal his

order for security $\tilde{\psi}$, which depends on the manager's disclosure (d) and the investor's private information ($s_{withheld}$), we have $y(d, s_{withheld}) \in \{0, 1\}$. Without loss of generality, we assume the manager does not trade when the manager makes a disclosure $d \neq \emptyset$ given that, in those cases, the informed trader has no opportunity to earn positive expected trading profits because he has no informational advantage over the market maker.

It is also clear, since the manager trades deterministically in the market for the residual security $\tilde{\xi}$ (because he always sells his interest in the security), that were the informed trader ever to trade on the residual securities market, the market maker – who sees the aggregate order flow for each security – would be fully aware of that trade. As a consequence, the informed trader won't make any trades based on his private information in the residual securities market because – were he to engage in any such trading – not only will those trades not earn him any trading profits (because the market maker will see those trades, deduce the trader's private information from those trades, and modify the residual security's equilibrium price to reflect that deduced information[13]), but those trades would also jeopardize the profits he might get from trading on the $\tilde{\psi}$ securities market (because the market maker will also modify the price of security $\tilde{\psi}$ to reflect the deduced information). Since the informed trader will not make informationally based trades on the residual securities market, he is just as well off by making no trades in that market.

Relying on this last observation, we can dispense with further discussion of the informed trader's trading behavior on the residual securities market.

X denotes the aggregate order flow in the $\tilde{\psi}$ securities market. It follows that, given the previous assumptions and observations, X can take on three possible values: -1 (if the investor experiences a liquidity shock and the trader stays out of the market), 0 (if either (a) the investor experiences a liquidity shock and the trader trades or (b) neither the investor experiences a liquidity shock nor does the trader trade), or $+1$ (if the investor does not experience a liquidity shock and the trader trades).

Upon observing the manager's disclosure and the aggregate order flow X for security $\tilde{\psi}$, the market maker posts a price $P_\psi(d, X)$ for the security such that $P_\psi(d, X) = E(\tilde{\psi}|d, X)$.[14] This price $P_\psi(d, X)$ is the price the investor receives for selling his security. On average, an investor who faces a liquidity

trades and thereby eliminate his informational advantage over the market maker.

[13]To state this somewhat more precisely: there is no equilibrium in which the trader can make positive expected trading profits by trading in the residual securities market. To see this, write the trader's order in the residual securities market as $y = y(s_{withheld})$. Since the market maker sees y, the market maker sets the price of the residual security at $P_\xi(y) = E[\tilde{\xi}|y]$. Then, conditional on y, the trader's expected trading profits are $y \times (E[\tilde{\xi}|y] - P_\xi(y))$. So, his ex ante (pre - $\tilde{s}_{withheld}$ realization) expected trading profits are:

$$\begin{aligned} & E[y(\tilde{s}_{withheld}) \times (E[\tilde{\xi}|\tilde{s}_{withheld}] - P_\xi(y(\tilde{s}_{withheld})))] \\ = \; & E[y(\tilde{s}_{withheld}) \times (E[\tilde{\xi}|\tilde{s}_{withheld}] - E[\tilde{\xi}|y(\tilde{s}_{withheld})])] \\ = \; & E[E[y(\tilde{s}_{withheld})\tilde{\xi}]|\tilde{s}_{withheld}] - E[y(\tilde{s}_{withheld})\tilde{\xi}|y(\tilde{s}_{withheld})]] \\ = \; & E[y(\tilde{s}_{withheld})\tilde{\xi}] - E[y(\tilde{s}_{withheld})\tilde{\xi}] \\ = \; & 0. \end{aligned}$$

[14]In view of the remarks made above, it is unnecessary to also condition the equilibrium price of security $\tilde{\psi}$ on the aggregate order flow for the residual security $\tilde{\xi}$, since that order flow will be constant and equal to -1 and consist entirely of the manager's sale of his residual stake.

shock trades should anticipate having to sell the security for less than its expected value because of the presence of the informed trader in the securities' market .[15] Consequently, the ex ante value of the security to the investor,

$$I(\tilde{\psi}, d) \equiv vE(P_{\psi}(d, X)|\tilde{q} = 1) + (1 - v)E(\tilde{\psi}), \tag{5}$$

typically will be below the unconditional expected value $E(\tilde{\psi})$ of the security. The difference between $E(\tilde{\psi})$ and $I(\tilde{\psi}, d)$ is the firm's "cost of capital" (CC) in this model:

$$CC(\tilde{\psi}, d) \equiv E(\tilde{\psi}) - I(\tilde{\psi}, d). \tag{6}$$

$CC(\tilde{\psi}, d)$ is the cost the manager must pay for raising capital from the investor. Exactly what the firm's cost of capital is, and how that cost of capital varies with both the security the manager offers the investor and the manager's disclosure policy will be key to the analysis that follows.

Expanding on the fourth and final step is not necessary: all that happens in this step is that the firm's cash flows are realized, and the parties in possession of the securities at that time are paid the cash flow-contingent amounts specified by the securities.

2.1 The downside risk and upside potential of securities

So far, we have characterized a security $\tilde{\psi}$ exclusively in terms of the triple (ψ_l, ψ_m, ψ_h), or equivalently, in terms of the relationship between the *level* of the firm's cash flows and the *level* of the security's payoff. But, for several of our later results, it turns out to be both more convenient and more descriptive to characterize a security in terms of how the value of the security *changes* with *changes* in the firm's realized cash flows. To present this alternative description of a security, we introduce the following pair of definitions.

Definition 1

(i) The "downside risk" of security (ψ_l, ψ_m, ψ_h) is given by $\Delta_m = \psi_m - \psi_l$;

(ii) the "upside potential" of security (ψ_l, ψ_m, ψ_h) is given by $\Delta_h = \psi_h - \psi_m$.

The downside risk of a security is the amount by which the payoff of the security drops as a consequence of having the low state θ_l, rather than the medium state θ_m, occur, and the upside potential of a security is the amount by which the payoff of the security increases as a consequence of having the high state θ_h, rather than the medium state θ_m, occur. Because of the obvious relationships: $\psi_m = \psi_l + \Delta_m$ and $\psi_h = \psi_l + \Delta_m + \Delta_h$, any security (ψ_l, ψ_m, ψ_h) can be described equivalently as the triple $(\psi_l, \Delta_m, \Delta_h)$.

We now provide several uses of this alternative characterization of a security: in Lemma 1, we give an alternative description of the monotonicity and limited liability conditions (2)-(4); in Definition 2,

[15] This fact establishes, as was asserted earlier, that were the investor not subject to a liquidity shock, the investor would not participate on this market. Instead, he would wait until the random variable describing the firm's cash flows realizes its value and collect the cash flow-contingent payment due him as specified by the security.

we define debt securities; in Definition 3, we partition debt securities into three subclasses: risk-free debt, investment-grade debt, and speculative debt; in Definition 4, we define equity securities; and in Definition 5, we define hybrid securities.

Lemma 1 *The limited liability and monotonicity conditions (2)-(4) are equivalent to the satisfaction of (1) and (2) below:*

(1) $0 \le \Delta_i \le e$ *for* $i = m, h$ *and* $\Delta_i \ne e$ *at least one of* $i = m, h$ *and*

(2) $0 \le \psi_l \le \theta_l = \mu - e$.[16]

This lemma will be useful when we develop a program to characterize optimal securities later in the paper.

Definition 2 *The security* $(\psi_l, \Delta_m, \Delta_h)$ *is a "debt security" provided the following three conditions all hold: (a)* $\Delta_m = 0$ *implies* $\Delta_h = 0$; *(b)* $\Delta_m > 0$ *implies* $\psi_l = \mu - e$; *and (c)* $\Delta_h > 0$ *implies* $\psi_l = \mu - e$ *and* $\Delta_m = e$.

This definition is consistent with calling a security "debt" when written as the triple (ψ_l, ψ_m, ψ_h) and $\tilde{\psi} = \min\{F, \tilde{\theta}\}$ for some face value F.

There are three natural subclassifications of debt securities.

Definition 3 *A debt security is "risk-free debt" when* $\Delta_m = 0$; *it is "investment-grade debt" when* $\Delta_h = 0$ *and* $\Delta_m > 0$; *and it is "speculative (or junk) debt" when* $\Delta_h > 0$.

This classification of debt securities makes clear that the basic difference among different types of debt securities is determined by the states in which default occurs (or, alternatively, the probability of default): a firm never defaults on risk-less debt; it defaults on investment-grade debt firm only if the low state occurs; and it defaults on speculative debt if anything other than the high state occurs.

Definition 4 *A security is an "equity security" provided it satisfies* $(\psi_l, \Delta_m, \Delta_h) = (\alpha\theta_l, \alpha(\theta_m - \theta_l), \alpha(\theta_h - \theta_m))$ *for some* $\alpha \in (0, 1]$.

As will become apparent in the following, equity securities are generally not optimal securities when all the disclosures the manager makes are voluntary. But, perhaps surprisingly, we show (in section 4.2 below) that equity securities can become optimal in the presence of some mandatory disclosure requirements. Equity securities are also important because, in conjunction with debt securities, they define a third class of securities that is sometimes optimal.

[16] The proof is easy. For example, $\tilde{\theta} - \tilde{\psi}$: monotone is equivalent to $\Delta_m \le e$ and $\Delta_h \le e$, since $\theta_l - \psi_l \le \theta_m - \psi_m$ if and only if $\Delta_m \le \theta_m - \theta_l = e$, and similarly, $\theta_m - \psi_m \le \theta_h - \psi_h$ if and only if $\Delta_h \le \theta_h - \theta_m = e$. (The preceding argument also shows that as long as at least one of $\Delta_m \ne e$ and $\Delta_h \ne e$ holds, then $\theta_l - \psi_l < \theta_h - \psi_h$ will be satisfied.)

Also $\tilde{\psi}$: monotone is equivalent to $\Delta_m \ge 0$, $\Delta_h \ge 0$, since, for example, $\psi_l \le \psi_m$ is identical to $\Delta_m \ge 0$.

Also observe that when $\tilde{\psi}$ is monotone, then $\psi_l \ge 0$ implies $\psi_i \ge 0$ for $i = m$ and h; and when $\tilde{\theta} - \tilde{\psi}$ is monotone, then $\theta_l - \psi_l \ge 0$ implies $\theta_i - \psi_i \ge 0$ for $i = m$ and h.

Also, we make the technical note here that when using this lemma to state the optimal security design programs below, we will not impose the condition that at least one of $\Delta_m \ne e$, $\Delta_h \ne e$ must hold. Rather, we will simply observe that the solution to the programs always satisfy this additional requirement (so the additional requirement will turn out not to be binding).

Definition 5 *A security is a "hybrid security" provided it is neither a debt security nor an equity security.*

2.2 Capital structure-dependent disclosure policies

In this section, we characterize how the manager's preferred disclosure policy varies with the firm's capital structure/security design choice $\tilde{\psi}$. Proposition 1 provides this characterization.

Proposition 1 *Suppose the manager offers the investor a security $(\psi_l, \Delta_m, \Delta_h)$ which satisfies the limited liability and monotonicity conditions described in (2)-(4) above. Then, the possible disclosure policies $d(\cdot)$ the security can induce the manager to adopt are the following: $d(\theta_h) = \theta_h$, $d(\theta_l) = \emptyset$, and:*

$$\textit{if } pe < \Delta_m - (1-p)\,\Delta_h, \ \textit{then } d(\theta_m) = \emptyset; \tag{7}$$

$$\textit{if } pe > \Delta_m - (1-p)\,\Delta_h, \ \textit{then } d(\theta_m) = \theta_m; \tag{8}$$

and if each strict inequality in (7) and (8) is replaced by an equality, then the security can induce the manager either to disclose, or to not disclose, $\tilde{\theta} = \theta_m$.

Proposition 1 shows that any security that satisfies the monotonicity and limited liability conditions always induces the manager to disclose θ_h and never induces him to disclose θ_l, but whether the security induces the manager to disclose θ_m depends upon how large the security's upside potential is as compared to its downside risk. The proposition thus formalizes one of the claims made in the Introduction: a firm's capital structure – that is, the securities it uses to raise capital from investors – affects the firm's disclosure policy.

The intuition for the relationship between the inequalities in the proposition and the associated induced disclosure policy is the following. Consider downside risk first. In view of the monotonicity restrictions (4), a security's downside risk ranges from 0 (in case $\psi_m = \psi_l$) to $\theta_m - \theta_l = e$ (in case $\theta_m - \psi_m = \theta_l - \psi_l$). If the security's downside risk is minimal, i.e., $\Delta_m = 0$, and cash flows θ_m, rather than θ_l, occur, then the value of the manager's residual stake $\xi(\theta) = \theta - \psi(\theta)$ increases by 100% of the increase in the firm's cash flows $\theta_m - \theta_l$ as a consequence of θ_m rather than θ_l occurring. In this case, inequality (8) clearly holds, and so, according to the proposition, the manager prefers disclosing θ_m. Recalling that the manager's disclosure decision is based on whether the disclosure increases the market's perceptions of the value of his residual claim, it follows that when the value of his residual claim rises substantially due to having the medium state rather than the low state occur (which certainly occurs when the manager's residual stake increases one-for-one in the firm's cash flows across these two states), the manager wants the capital market to know it. In contrast, if the security's downside risk is maximal, i.e., $\Delta_m = e$, the manager does not benefit from having it known that the medium state rather than the low state occurred, because with such a security, all of the "fruits" from having the medium state rather than the low state occur go to the investor holding the security $\tilde{\psi}$, and not to the manager.[17] More

[17] This is also easy to see formally. Recall $\theta_m - \theta_l = e$. When $\Delta_m = e$, the inequality in (7) can be written as

$$pe < e - (1-p)\,\Delta_h.$$

generally, it is clear that, as the security's downside risk decreases, the manager is more likely to prefer disclosing θ_m. A similar analysis shows that the manager's preference for disclosing θ_m also depends on the upside potential of the security.[18] When the inequality in (7) (resp., 8) holds, we say that security $\tilde{\psi}$ induces the manager to adopt a *limited* (resp., *expansive) disclosure policy* and also that $d(\cdot)$ is the *disclosure policy induced by* $\tilde{\psi}$.

Proposition 1 also reinforces and extends results in the extant literature on voluntary disclosures. For example, recall that, in the original voluntary disclosure models of Dye [1985] and Jung and Kwon [1988], the value-maximizing manager of an all equity firm sometimes privately gets advanced information about the expected value of his firm's (continuously distributed) cash flows, which he could withhold or disclose. In equilibrium, there is a threshold level determining whether the manager discloses his advanced information. Jung and Kwon [1988] develop the following comparative static concerning this disclosure threshold: the disclosure threshold when the distribution of the firm's cash flows is **f** is higher than the disclosure threshold when the distribution of the firm's cash flows is **g**, if the distribution **f** second order stochastically dominates the distribution **g** (Jung and Kwon [1988], Proposition 3). This is consistent with the following implication of Proposition 1: as the volatility of the firm's cash flows – as measured by e [19] – declines, the manager is less likely to adopt the expansive disclosure policy. (The consistency follows immediately from the fact that decreases in e correspond to the distribution of the firm's cash flows second-order stochastically increasing.)

As a second example, recall that another result of Dye [1985] and Jung and Kwon [1988] is that as the probability p the manager receives information increases, the threshold level determining the manager's disclosure declines, and so the manager is more likely to disclose his information. This is consistent with the following implication of Proposition 1: (7) and (8) implicitly define a cutoff level of p, call it $p^* = \frac{\Delta_m - \Delta_h}{e - \Delta_h}$, such that if the probability that the manager receives information is $p < p^*$ (resp., $p > p^*$), then the manager will engage in limited (resp., expansive) disclosure.[20]

But, Proposition 1 goes beyond Dye [1985] and Jung and Kwon [1988] by revealing how a firm's disclosure threshold varies in a "not all equity" firm. In particular, since the threshold p^* is decreasing in Δ_h and increasing in Δ_m, the proposition shows that a firm is likely to disclose more often (i.e., for a wider range of $p's$) as either the upside potential of the security increases or as the downside risk

Since (4) requires that $\Delta_h < e$ by Lemma 1, inequality (7) necessarily holds, so no disclosure of θ_m occurs.

[18] Given the monotonicity restriction (4), the upside potential ranges between 0 (in case $\psi_h = \psi_m$) and $\theta_h - \theta_m = e$ (in case $\theta_h - \psi_h = \theta_m - \psi_m$). If $\Delta_h = e$, then it is easy to see that the manager prefers to disclose θ_m, i.e., (8) holds, as in that case (8) can be rewritten as:

$$e = (1 - p)\,\Delta_h + pe > \Delta_m,$$

which always holds since $\Delta_m < e$ by Lemma 1. This is also intuitive: if the security exhibits maximal upside potential, then the manager is as well off when the firm's cash flows are θ_m as he is when the highest possible cash flows occur, and so the manager is better off disclosing θ_m rather than staying silent. Finally, it is more likely that the manager prefers to withhold θ_m (when he learns that information) as the investor's upside potential declines, as can be seen by rewriting (7) as

$$(1 - p)\,\Delta_h + pe < \Delta_m.$$

[19] We could alternatively measure the volatility of the firm's cash flows by the variance of the cash flows, $e^2 \times \eta$, without substantively affecting any of our results.

[20] We wish to thank one of the anonymous referees for making this observation.

of the security decreases. In particular, if the manager issues debt with a face value below θ_m, then further reductions in the debt's face value increase the likelihood the manager will engage in expansive disclosure, ceteris paribus.

In addition to demonstrating the consistency of our results with those in the extant literature, the preceding observations demonstrate that the results from our three state model often have analogues in models where firms' cash flows are continuously distributed.

2.3 Informed trader's optimal trading strategy

Next, we derive the informed trader's optimal trading strategy for a given disclosure policy. This is straightforward. Regardless of whether a security $\tilde{\psi}$ induces a limited or expansive disclosure policy, the trader wants to buy the security when the trader believes – based on his (the trader's) information – that the market maker is going to under-price the security, and to stay out of the market for the security when the trader believes, based on his information, that the market maker is going to overprice the security.

When the manager discloses information he received, the market maker is at informational parity with the trader; hence, the trader does not trade, since he knows there is no opportunity to earn positive expected trading profits. But, when the manager discloses nothing, the trader potentially has an informational advantage over the market maker, because the trader then knows the reason for the manager's nondisclosure. We confirm in the appendix that in equilibrium the trader believes the market maker is likely to overprice (resp., underprice) the security when (the trader knows) the reason the manager did not disclose information is that the manager was withholding information (resp., did not receive information), in which case it is optimal for the trader to adopt the following trading strategy.

Proposition 2 *The trader's optimal trading strategy is* $y(\emptyset, s_{withheld}) = 1 - s_{withheld}$*, and* $y(d, s_{withheld}) = 0$ *when* $d \neq \emptyset$.

That is, the trader submits a market order to buy security $\tilde{\psi}$ when the manager makes no disclosure and the trader knows that the reason the manager made no disclosure was because the manager did not receive information ($s_{withheld} = 0$). In all other circumstances, the trader does not trade.

2.4 How a firm's cost of capital depends on the design of its securities and the firm's disclosure policy

As was observed in (5) above, the firm's cost of capital in this model is the difference between a security's expected payout and the security's value to the investor. It is the purpose of this section to explore, in detail, the determinants of a firm's cost of capital.

First, consider the case of a risk-free security $\psi(\tilde{\theta})$, i.e., a security whose payout $\psi(\theta)$ is constant across all realizations of $\tilde{\theta}$. Notice that the market maker always correctly prices risk-free securities, because the informed trader never has an informational advantage over the market maker in assessing a risk-free security's value. Hence, the cost of capital for a risk-free security is zero.

Next, consider a risky security $\psi(\tilde{\theta})$, i.e., a security whose payout $\psi(\theta)$ varies with θ. The cost of capital would always be zero for a risky security, too, if the investor never experienced a liquidity shock, and hence never had to trade in a market populated with a superiorly informed trader. The cost of capital would also be zero even when the investor experiences a liquidity shock and trades in a market where there is an informed trader, as long as this trader has no informational advantage over the market maker. The market maker and trader share the same information set in each of three situations: (a) the manager discloses the firm's realized cash flows; (b) the manager makes no disclosure and the aggregate market order for security $\tilde{\psi}$ is either (bi) $X = 1$ (because $X = 1$ occurs only when the trader places a market order and the investor did not experience a liquidity shock, which reveals that the trader knew the manager's nondisclosure was due to the manager not receiving information) or (bii) $X = -1$ (because $X = -1$ occurs only when the investor, and not the trader, places a market order, which reveals that the trader knew the manager's nondisclosure was due to the manager withholding information).

What creates a positive cost of capital for an investor who experiences a liquidity shock is the situation in which the manager makes no disclosure and $X = 0$, since - from the market maker's perspective - this event is consistent with either: (c) neither the investor nor the informed trader place a market order for security $\tilde{\psi}$ or (d) both the investor (because he faced a liquidity shock) and the market maker (because he knew that the manager's nondisclosure was due to the manager not having received information) place a market order for security $\tilde{\psi}$. By determining how probable this situation is, and the size of the investor's expected trading losses when it occurs, we can calculate the firm's cost of capital. The next proposition displays the results of this calculation, in a form that can be applied to any security the manager constructs and to any disclosure policy the manager adopts. (In stating Proposition 3, it is convenient to write $d = 1$ for an expansive disclosure policy (in place of $d(\theta_m) = \theta_m$) and $d = 0$ for a limited disclosure policy (in place of $d(\theta_m) = \emptyset$).)

Proposition 3 *The firm's cost of capital when it issues security $\tilde{\psi} = (\psi_l, \Delta_m, \Delta_h)$ and adopts disclosure policy d is given by:*

$$CC(\tilde{\psi}, d) = A_h^d \times \Delta_h + A_l^d \times \Delta_m, \tag{9}$$

where:

$$\begin{aligned} A_h^d &= \upsilon(1-p)\frac{\eta}{2}\frac{p(1-\upsilon)\left(\frac{\eta}{2} + (1-d)(1-\eta)\right)}{(1-p)\upsilon + p(1-\upsilon)\left(\frac{\eta}{2} + (1-d)(1-\eta)\right)}; \\ A_l^d &= \upsilon(1-p)\frac{\eta}{2}\frac{p(1-\upsilon)\left(\frac{\eta}{2} + d(1-\eta)\right)}{(1-p)\upsilon + p(1-\upsilon)\left(\frac{\eta}{2} + (1-d)(1-\eta)\right)}. \end{aligned}$$

Before discussing Proposition 3, we introduce Corollary 1, which helps to illustrate some of the economic consequences of the proposition.

Corollary 1

(a) $0 < A_l^0 < A_h^1 < A_h^0 < A_l^1 < \frac{\eta}{2}$*;*

(b) $A_l^0 + A_h^0 < A_l^1 + A_h^1 < \frac{\eta}{2}$.

Proposition 3 shows that the firm's cost of capital varies with each of: (1) the security's downside risk Δ_m and its upside potential Δ_h; (2) the specifics of the information and trading environments in which the securities are traded (i.e., the probability that the manager receives information p and the probability that the investor is subject to a liquidity shock v); (3) the technology that produces the firm's cash flows (i.e., the probability distribution of $\tilde{\theta}$, as summarized by the parameter η); and (4) the manager's disclosure policy d (and, in particular, whether or not the manager discloses the intermediate cash flows θ_m when he knows it).

The ordering of the coefficients $A_l^1 > A_l^0$ in Corollary 1 shows that the downside risk of a security contributes more to the firm's cost of capital when the firm's disclosure policy is expansive ($d = 1$) than when the firm's disclosure policy is limited ($d = 0$). Stated alternatively, *less disclosure favors securities with higher downside risk.*

As this is perhaps one of the more unexpected findings in the paper, we expand on our intuition for it at some length.[21] As we have noted previously, the firm's cost of capital in this model is the same as the investor's expected trading losses. The investor bears trading losses only when both (a) and (b) occur: (a) he sells his security following a liquidity shock and (b) the informed trader buys the security. Note that the probability these two events both occur is independent of whether the manager adopts an expansive or limited disclosure policy, since this probability is, in all cases, $v \times (1 - p)$.[22] Hence, the only reason the investor's expected trading losses would be higher under one disclosure policy than another is if the equilibrium price at which the investor sells his security when the aggregate order flow is zero is lower under one disclosure policy than the other.

The firm's equilibrium price when the manager makes no disclosure and the aggregate order flow is zero is always a weighted average of the form

$$\begin{aligned} & wE[\tilde{\psi}|\text{manager had no info}] + (1 - w)E[\tilde{\psi}|\text{manager had info he withheld}] \\ = \quad & wE[\tilde{\psi}] + (1 - w)E[\tilde{\psi}|\text{manager had "bad" info}], \end{aligned} \tag{10}$$

for some weights w and $1 - w$, i.e., a weighted average of the security's unconditional expected value (applicable when the manager makes no disclosure because he did not receive information) and the security's expected value conditional on the manager receiving, and withholding, "bad" information (applicable in the expansive disclosure policy when he learned that the firm's cash flows were low, and applicable in the limited disclosure policy when he learned that the firm's cash flows were low or medium). Now, to isolate the effects of expanded disclosure on a security's downside risk, consider a security that has only downside risk (so $\Delta_h = 0$). Such a security provides two distinct possible payoffs to its holder, a "low" payoff when $\tilde{\psi} = \psi_l$ or a "high" payoff (which is the same whether $\tilde{\psi} = \psi_m$ or $\tilde{\psi} = \psi_h$). Under the

[21] We are grateful to one of the referee's for helping us develop this intuition.

[22] Since v is the probability of a liquidity shock and the informed trader only buys when he knows that the reason for the manager not making a disclosure is that the manager did not receive any information, and the latter event occurs with probability $1 - p$.

expansive disclosure policy, the manager only withholds information when the security's payoff is "low," while under the limited disclosure policy, the manager withholds information both when the security's payoff is "low" and also sometimes when the security's payoff is "high" (specifically, when $\tilde{\psi} = \psi_m$). Consequently, when the market maker calculates the equilibrium price when $d = \emptyset$ and $X = 0$ for a security with no upside potential as the weighted average (10), the calculation will result in lower price when the manager adopts the expansive disclosure policy than when he adopts the limited disclosure policy, because $E[\tilde{\psi}|\text{manager had "bad" info}] = E[\tilde{\psi}|\tilde{\psi} = \psi_l] = \psi_l$ for the expansive disclosure policy, whereas $E[\tilde{\psi}|\text{manager had "bad" info}] = E[\tilde{\psi}|\tilde{\psi} = \psi_l \text{ or } \tilde{\psi} = \psi_m] > \psi_l$ for the limited disclosure policy. This provides intuition for the ordering $A_l^1 > A_l^0$.

The result $A_h^1 < A_h^0$ in Corollary 1 shows that the upside potential of a security contributes less to the firm's cost of capital for an expansive disclosure policy than for a limited disclosure policy. Stated differently, *more disclosure favors securities with higher upside potential.* The explanation proceeds along lines similar to that in the preceding paragraph: to isolate the effects of expanded disclosure on a security's upside potential, consider a security that has only upside potential (so $\Delta_m = 0$). Such a security provides two distinct possible payoffs to its holder, a "low" payoff (which is the same whether $\tilde{\psi} = \psi_l$ or $\tilde{\psi} = \psi_m$) or a "high" payoff when $\tilde{\psi} = \psi_h$. Under the limited disclosure policy, the manager always withholds information when the security's payoff is "low" (i.e., regardless of whether $\tilde{\psi} = \psi_l$ or $\tilde{\psi} = \psi_m$), while under the expansive disclosure policy, the manager sometimes discloses information even though the security's payoff is "low" (specifically, when $\tilde{\psi} = \psi_m$). Consequently, when the market maker calculates the equilibrium price when $d = \emptyset$ and $X = 0$ for a security with no downside risk as the weighted average (10), the calculation will result in a lower price when the manager adopts the limited disclosure policy than when he adopts the expansive disclosure policy (even though $E[\tilde{\psi}|\text{manager had "bad" info}]$ is the same for both policies (and equal to $\psi_m = \psi_l$)), because the weight $1 - w$ multiplying $E[\tilde{\psi}|\text{manager had "bad" info}]$ will be higher for the limited disclosure policy (since the manager withholds information under more states for that policy) and $E[\tilde{\psi}|\text{manager had "bad" info}] < E[\tilde{\psi}]$. This provides intuition for the ordering $A_h^1 < A_h^0$.

The results reported in Corollary 1 also show that if the upside potential and downside risk of a security (ψ_l, ψ_m, ψ_h) are the same, then the firm's cost of capital is higher under expansive disclosure than under limited disclosure; that is, with $\Delta \equiv \psi_m - \psi_l = \psi_h - \psi_m$, $CC(\tilde{\psi}, 0) = (A_l^0 + A_h^0) \times \Delta < (A_l^1 + A_h^1) \times \Delta = CC(\tilde{\psi}, 1)$. The result follows by reasoning analogous to that in the preceding two paragraphs: when the aggregate order flow in $\tilde{\psi}$ is zero and the manager makes no disclosure, under expansive disclosure the security's equilibrium price depends on the manager withholding information only when the security's payoff is ψ_l, whereas under the limited disclosure policy the manager withholds information both when the security's payoff is ψ_l and when its payoff is ψ_m. Since, for a security with equal downside risk and upside potential, $\psi_m = E[\tilde{\psi}] > \psi_l$,[23] it follows that the firm's equilibrium price

[23] $E[\tilde{\psi}] = \psi_m$ for a security with equal downside risk and upside potential, since $E[\tilde{\psi}] = (1-m)\psi_m + \frac{m}{2}(\psi_l + \psi_h) = (1-m)\psi_m + \frac{m}{2}(\psi_m - \Delta + \psi_m + \Delta) = \psi_m$.

will be lower when $d = \emptyset$ and $X = 0$ under the expansive disclosure policy than under the limited disclosure policy, which explains why $CC(\tilde{\psi}, 0) < CC(\tilde{\psi}, 1)$ in that case. This provides intuition for the ordering $A_l^0 + A_h^0 < A_l^1 + A_h^1$.

The orderings depicted in the corollary are important to studying both the design of optimal securities while holding a firm's disclosure policy fixed, and the optimal choice among disclosure policies, as will seen by review of the proofs of Propositions 5, 6, and 7 below.

From the formulas for A_h^d and A_l^d in Proposition 3, a variety of additional results can be obtained.[24] As examples, we have shown that for any risky security, the firm's cost of capital: (i) increases in the probability $\frac{\eta}{2}$ that either of the "tail" cash flows (θ_l, θ_h) occurs; (ii) increases as the probability that the manager receives information, or the probability the investor is subject to liquidity shocks approach certain "intermediate" values,[25] and (iii) diminishes to zero as uncertainty about either whether the manager receives information or whether the investor is subject to a liquidity shock gets eliminated. All these results are intuitive, because: (1) the firm's cost of capital grows/diminishes as the investor sustains greater/smaller expected trading losses to the trader with private information; (2) the investor's expected trading losses are the same (apart from sign) as the informed trader's expected trading profits; (3) the informed trader generates more/less expected trading profits as his informational advantage over the market maker improves/worsens; and (4) the informational advantage of the informed trader improves/worsens as the environment in which the trader trades becomes more/less uncertain.

3 Optimal security design

3.1 Preliminaries

In this section, holding the manager's disclosure policy – expansive or limited – fixed, we identify the manager's most preferred security among all securities that satisfy the limited liability and monotonicity conditions (2), (3), (4).

We start the study of this security design problem by specifying the manager's objective function. If the manager offers the investor the security $\tilde{\psi} = (\psi_l, \Delta_m, \Delta_h)$, and the investor anticipates that security $\tilde{\psi}$ will induce the manager to adopt disclosure policy d, then – recalling the normalization that the investor's required rate of return is zero – it follows that the amount of capital the investor is willing to supply to the manager at step 1 is $\$I(\tilde{\psi}, d) \equiv E[\tilde{\psi}] - A_l^d \Delta_m - A_h^d \Delta_h$. From that $\$I(\tilde{\psi}, d)$, the manager then takes \$1 and invests it into the stochastic production technology. The manager retains the excess financing of $I(\tilde{\psi}, d) - 1$.[26] In addition, the manager owns the residual security $\tilde{\xi}$, which he subsequently

[24] To save space, we omit the (straightforward) proofs underlying these results.

[25] The exact values of these "intermediate" values are available from the authors.

[26] Note that we allow the manager to raise more than \$1 from the investor. The reason for doing so is this. If the manager were only allowed to collect \$1 from the investor, then the manager's objective function in the Optimal Security Design d Program below would be $\mu - E[\tilde{\psi}] - 1$. But the solution to the resulting optimization problem could entail that constraint (13) not be binding. That is, the amount that the investor is willing to give the manager for the security, $\$I$, would exceed the amount, \$1, that the manager would receive from issuing the security. The manager would literally be leaving money on the table. To avoid this economically unimportant possibility, we allow the manager to raise more than \$1 from the investor, which – as the discussion in the text in this paragraph indicates – results in the manager's objective function correctly being specified as $\mu - CC(\tilde{\psi}, d) - 1$.

sells on the residual securities market. Since the market maker for any security in a Kyle-type market sets the price of the security at its expected value conditional on the information he has, the ex ante (i.e., before any random variables realize their values) expected value of the residual security's selling price is $E[\tilde{\xi}] = E[\tilde{\theta} - \tilde{\psi}] = \mu - E[\tilde{\psi}]$.[27] It follows that the manager's expected profits from offering security $\tilde{\psi}$ are given by: $\mu - E[\tilde{\psi}] + I(\tilde{\psi}, d) - 1 = \mu - A_l^d \Delta_m - A_h^d \Delta_h - 1$.

Given this observation, it follows that a security that maximizes the manager's expected profits while inducing the manager to implement disclosure policy $d \in \{0, 1\}$ solves the following program.

The Optimal Security Design d (OSDd) Program

$$\max_{\psi_l, \Delta_m, \Delta_h} \mu - A_l^d \Delta_m - A_h^d \Delta_h - 1 \tag{11}$$

subject to:

$$(1-2d)pe \leq (1-2d)(\Delta_m - (1-p)\Delta_h); \tag{12}$$

$$1 \leq I = E\tilde{\psi} - A_l^d \Delta_m - A_h^d \Delta_h; \tag{13}$$

$$0 \leq \psi_l \leq \mu - e; \tag{14}$$

$$0 \leq \Delta_i \leq e, \; i \in \{\mu, h\}. \tag{15}$$

The OSDd Program incorporates three results derived earlier in the paper. Constraint (12) reflects Proposition 1's observation about how a firm's choice among securities influences its disclosure policy. Constraint (13) reflects Proposition 3's observation that a firm's choice among securities and disclosure policies influences its cost of capital. Constraints (14) and (15) reflect Lemma 1's observation that the monotonicity and limited liability constraints can be compressed into the conditions (14) and (15). We say that a security is *feasible* if it satisfies all the constraints of this program.

Our first observation concerning the OSDd Program is that it imposes an upper bound on the manager's objective function of $\mu - 1$, since Δ_m and Δ_h are both nonnegative, and that this upper bound can be attained if and only if $\Delta_m = \Delta_h = 0$ is part of the specification of some feasible security $(\psi_l, \Delta_m, \Delta_h)$ for the program. We call the situation in which the upper bound $\mu - 1$ can be obtained "first-best;" when a first-best situation obtains, the firm's cost of capital is zero.

It is immediate that a first-best situation can never arise for securities that implement the limited disclosure policy ($d = 0$), since constraint (12) is never satisfied by $\Delta_m = \Delta_h = 0$ when $d = 0$. In fact, Proposition 3 makes clear that any security that succeeds in getting the manager to adopt the limited disclosure policy must have downside risk at least equal to $pe > 0$.

We can find out exactly when the first-best situation arises for securities that implement the expansive

[27] Spelling this out in detail, let $P_\xi(d, X)$ be the price the market maker sets, and the manager receives, for the residual security $\tilde{\xi}$ were the manager to make the specific disclosure d and were the aggregate order flow in the $\tilde{\psi}$ (not $\tilde{\xi}$) securities market to turn out to be X. (We omit reference to the aggregate order flow of the residual security $\tilde{\xi}$ because, as was discussed above, the aggregate order flow in the residual securities market is always deterministic (and equal to -1).) The price $P_\xi(d, X)$ is the market maker's best unbiased assessment of the residual security's expected value conditional on the information available to him: $P_\xi(d, X) = E[\tilde{\theta} - \tilde{\psi} | \tilde{d} = d, X]$. Thus, ex ante, $E[P_{\theta - \psi}(\tilde{d}, \tilde{X})] = E[\tilde{\theta} - \tilde{\psi}]$, as asserted in the text.

disclosure policy ($d = 1$) by writing $E\tilde{\psi}$ as:

$$E\tilde{\psi} = \frac{\eta}{2}\psi_l + (1-\eta)(\psi_l + \Delta_m) + \frac{\eta}{2}(\psi_l + \Delta_m + \Delta_h) = \psi_l + (1 - \frac{\eta}{2})\Delta_m + \frac{\eta}{2}\Delta_h, \tag{16}$$

and restating constraint (13) using RHS(16). Since the objective function (11) does not depend on ψ_l, and constraints (13) and (15) are made more slack by replacing security $(\psi_l, \Delta_m, \Delta_h)$ with security $(\psi_l', \Delta_m, \Delta_h)$ for $\psi_l' > \psi_l$, it follows that $(\psi_l', \Delta_m, \Delta_h)$ is feasible whenever $(\psi_l, \Delta_m, \Delta_h)$ is feasible and $\psi_l' \leq \mu - e$. This shows that the manager is never worse off by setting ψ_l at its upper bound: $\psi_l = \mu - e$.

Exploiting this observation, i.e., substituting $\mu - e$ for ψ_l in (16), and once again rewriting constraint (13) using (16), we conclude that the security $(\psi_l, \Delta_m, \Delta_h) = (\mu - e, 0, 0)$ is feasible, i.e., a first-best situation can arise, if and only if $1 \leq \mu - e$. Restated, a first-best situation can arise if and only if the volatility e in the firm's cash flows does not exceed $\mu - 1$.

Since the security $(\psi_l, \Delta_m, \Delta_h) = (\mu - e, 0, 0)$ is risk-free debt with face value $\mu - e$, the preceding observations establish:

Proposition 4 *(a) There is no security which implements the limited disclosure policy that yields the first-best outcome for any value of e.*

(b) There is a security which implements the expansive disclosure policy that yields the first-best outcome if and only if $e \leq \mu - 1$. When $e \leq \mu - 1$, a security that yields the first-best outcome is risk-free debt with face value $\mu - e$.

Four additional points concerning the proposition are worth making. First, it might seem that when the manager issues risk-free debt, the manager would be willing to implement either disclosure policy, and hence that the first-best outcome could be obtained for both disclosure policies. The proposition shows that this contention is not true: if the security $\tilde{\psi}$ the manager issues is risk-free debt, he is unwilling to adopt the limited disclosure policy because in that case the value of the manager's residual security $\tilde{\xi}$ increases dollar-for-dollar when the medium state, rather than the low state, occurs, and so, a manager interested in maximizing the price of his residual claim is unwilling to be silent when θ_m, rather than θ_l, occurs.

Second, while – as has already been noted – there is no loss in the manager's expected profits by assuming that for any security, $\psi_l = \mu - e$, such a security need not be uniquely optimal. This nonuniqueness is not economically substantive though, and merely amounts to whether the manager chooses to receive "up front" in step 1 (and then return in step 4 through payment under $\tilde{\psi}$) more capital than is required to finance the project. It is clear that the complete set of first-best securities is described by those risk-free securities with face value $F \in [1, \mu - e]$.

Third, the proposition provides the first illustration of the sensitivity of the structure of optimal securities to the size of the firm's cash flow volatility, as measured by the parameter e. This sensitivity will be shown to be a central feature of the analysis that follows, since it will be the basis for our demonstration of the existence of a hierarchy of optimal securities and optimal disclosure policies that will vary depending on how large this volatility parameter is.

Finally, fourth, in view of the complete characterization of optimal securities just given when $e \leq \mu - 1$, in the following we will concentrate primarily on those cash flow volatilities e for which $e > \mu - 1$.

3.2 Optimal securities that implement the limited disclosure policy $(d = 0)$

This section studies optimal securities that implement the limited disclosure policy. The main result is to establish the existence of a hierarchy of optimal securities, where what constitutes an optimal security varies with how large the volatility parameter e is.

To proceed, we start by using the result established in the preceding section that we can set $\psi_l = \mu - e$ with no loss in profits to the manager, and we can rewrite constraint (13) using RHS(16) in place of $E\tilde{\psi}$. With these substitutions, and with the observation that optimal securities are securities that minimize the firm's cost of capital, we conclude that any optimal security that implements the limited disclosure policy must solve the following program.

The Optimal Security Design $d = 0$ (OSD0) Program

$$\min_{\psi_l, \Delta_m, \Delta_h} \quad A_l^0 \Delta_m + A_h^0 \Delta_h \tag{17}$$

$$\begin{aligned} \text{subject to:} \quad & pe \leq \Delta_m - (1-p)\,\Delta_h; & (18)\\ 1 + e - \mu \quad \leq \quad & \left(1 - \frac{\eta}{2} - A_l^0\right)\Delta_m + \left(\frac{\eta}{2} - A_h^0\right)\Delta_h; & (19)\\ 0 \quad \leq \quad & \Delta_i \leq e;\ i \in \{\mu, h\}. & (20) \end{aligned}$$

Since this is a linear program in the two variables Δ_m and Δ_h, it can be solved geometrically. While we leave most of the details of the geometric proof to the appendix, we note here that the set of values of Δ_m and Δ_h that satisfy (18) and (20) consists of the pie-shaped subset of the square $[0, e] \times [0, e]$ in (Δ_m, Δ_h)-space lying below the line

$$\Delta_h = \frac{\Delta_m - pe}{1 - p}. \tag{21}$$

The feasible set of Δ_m and Δ_h is then determined by (19) to be that subset of this pie-shaped set lying in the half-space above the investor's "supply-financing" line

$$\Delta_h = \frac{1 + e - \mu - \left(1 - \frac{\eta}{2} - A_l^0\right)\Delta_m}{\frac{\eta}{2} - A_h^0}. \tag{22}$$

See Figure 2. To obtain the optimal security, all that has to be done is to search for the lowest iso-cost line for the objective function, i.e., the lowest value k among the lines $A_l^0 \Delta_m + A_h^0 \Delta_h = k$ that lies within this constraint set.

The following proposition summarizes the findings from the geometric investigation of the OSD0 Program in the appendix.

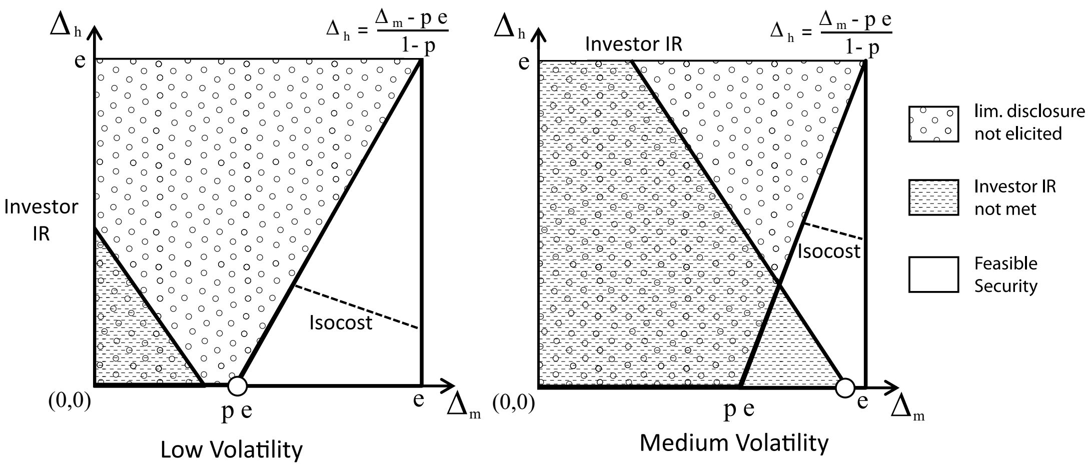

Figure 2: Security Design under Limited Disclosure

Proposition 5 *The optimal security that implements the limited disclosure policy (and solves the OSD0 program) is described as follows:*

(1) it is never first-best for any e*;*

(2) if $\mu - 1 < e \leq \frac{\mu-1}{1-\left(1-\frac{\eta}{2}-A_l^0\right)p} \equiv \underline{e}_{ID}^0$*, the optimal security is given by investment-grade debt with face value* $\mu-(1-p)e$*, and the manager's expected profits are* $\mu - 1 - A_l^0 pe$*;*

(3) If $\underline{e}_{ID}^0 < e \leq \frac{\mu-1}{\frac{\eta}{2}+A_l^0} \equiv \underline{e}_{SD}^0$*, the optimal security is given by investment-grade debt with face value* $\mu - e + \frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0}$ *and the manager's expected profits are* $\mu - 1 - A_l^0 \frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0}$*;*

(4) if $\underline{e}_{SD}^0 < e < \frac{\mu-1}{A_l^0+A_h^0} \equiv \overline{e}_{SD}^0$*, the optimal security is given by speculative debt with face value* $\frac{\mu(\frac{\eta}{2}-A_h^0-1)+1+(\frac{\eta}{2}+A_l^0)e}{\frac{\eta}{2}-A_h^0}$ *and the manager's expected profits are* $\mu - 1 - A_l^0 e - A_h^0 \frac{1-\mu+\left(\frac{\eta}{2}+A_l^0\right)e}{\frac{\eta}{2}-A_h^0}$*;*

(5) if $e \geq \overline{e}_{SD}^0$*, the limited disclosure policy cannot be implemented by any feasible security.*

The proposition establishes that there is a simple two-level hierarchy of optimal securities that induce the manager to implement the limited disclosure policy: investment-grade debt is optimal when the volatility of the firm's cash flows is sufficiently low, and speculative debt is optimal when the volatility of the firm's cash flows is high. When the firm's cash flow volatility is extremely high, no feasible security of any kind implements the limited disclosure policy.

The intuition for the results in the proposition is clear: as was noted in the discussion of Proposition 4 above, risk-less debt is never a feasible security for inducing the manager to implement the limited disclosure policy. Since the only securities that achieve first-best are risk-less debt, this explains part (1) of the proposition.

Among the risky securities that induce the manager to implement the limited disclosure policy, the manager prefers those securities whose payoffs do not vary much with the firm's cash flows, because the absence of much volatility in a security's payoff reduces the informational advantage of the informed trader over the market maker, thereby reducing the trader's expected trading profits, and thus driving down the firm's cost of capital. More than this can be said, however. When choosing among those risky securities that induce the manager to adopt the limited disclosure policy, the manager prefers securities that have only downside risk. Such securities have a lower cost of capital than do securities that also have upside potential, for the reasons described in the discussion following Corollary 1. So, if risky securities with only downside risk provide the investor with an acceptable level of expected return, they will be optimal. When $\psi_l = \mu - e$, securities with only downside risk are of the form $(\psi_l, \Delta_m, \Delta_h) = (\mu - e, \Delta_m, 0)$; such securities are investment-grade debt. This explains part (2) of the proposition.

As the volatility in the *firm's* cash flows increases, the volatility of the payoffs the *security* the firm issues the investor must also increase. The reason is that, as the firm's cash flows become more volatile, the lowest value of the firm's cash flows declines, so a security with relatively uniform payoffs is possible only if those payoffs are uniformly low, in view of the limited liability constraint (2). Because a security that has uniformly low payoffs is a security that will fail to entice the investor to exchange his capital in return for the security, it follows that as a firm's cash flow volatility increases, the volatility of its securities must increase too. There is, of course, a cost to issuing a security whose payoffs are more volatile: the informational advantage of the trader over the market maker goes up, which increases the trader's expected trading profits and hence the firm's cost of capital. While the firm must issue securities with more volatility as the firm's cash flow volatility increases, the manager continues to prefer to issue a security where the volatility manifests itself only in the security's downside risk when such a security is feasible – i.e., acceptable to the investor – for the same reasons discussed above. So, the optimal security continues to be investment-grade debt, but now the investment grade debt has a higher face value than when the firm's cash flow volatility was lower. This explains part (3) of the proposition.

Eventually, even securities with maximal downside risk are unable to induce the investor to supply financing, so to construct feasible securities, the manager must resort to securities that have both downside risk and upside potential. Such securities divert all of the firm's cash flows to the security holder, unless the most favorable (highest cash flow) state occurs. These securities are speculative debt. This explains part (4) of the proposition.

Financing the project eventually becomes infeasible at the highest levels of the firm's cash flow volatility: while, by assumption, (1) always holds, i.e., the expected value μ of the firm's cash flows always exceeds 1 (the amount of capital that must be injected into the firm to make the firm's production technology run), the manager can design a security that makes the investor willing to finance the project in extremely high volatility cases only by offering the investor a security with exceptionally high volatility. When the manager does that, the resulting informational advantage of the investor over the market maker is so large – and the firm's cost of capital is so great – that the manager prefers not to finance the project. This explains part (5) of the proposition.

Besides documenting how the optimal security varies with the volatility of the firm's cash flows, the proposition reports results about the manager's maximum expected profits from issuing the optimal security. It is easy to check, by combining the various cases in the proposition, that the manager's maximum expected profits always decline as the firm's cash flow volatility increases.[28]

3.3 Optimal securities that implement the expansive disclosure policy $(d = 1)$

The following program, which is the counterpart to the OSD0 Program defined by (17)–(20) above, determines the optimal security that implements the expansive disclosure policy.

The Optimal Security Design $d = 1$ (OSD1) Program

$$\min_{\psi_l, \Delta_m, \Delta_h} \; A_l^1 \Delta_m + A_h^1 \Delta_h \tag{23}$$

$$\text{subject to:} \qquad pe \geq \Delta_m - (1-p)\,\Delta_h; \tag{24}$$

$$1 + e - \mu \;\leq\; \left(1 - \frac{\eta}{2} - A_l^1\right)\Delta_m + \left(\frac{\eta}{2} - A_h^1\right)\Delta_h; \tag{25}$$

$$0 \;\leq\; \Delta_i \leq e;\; i \in \{\mu, h\}. \tag{26}$$

This is also a linear program in the two variables Δ_m and Δ_h that can be solved geometrically. In this case, (24) and (26) consists of a trapezoid, so the feasible set of Δ_m and Δ_h is that subset of the trapezoid lying in the half-space above the line

$$\Delta_h = \frac{1 + e - \mu - \left(1 - \frac{\eta}{2} - A_l^1\right)\Delta_m}{\frac{\eta}{2} - A_h^1}. \tag{27}$$

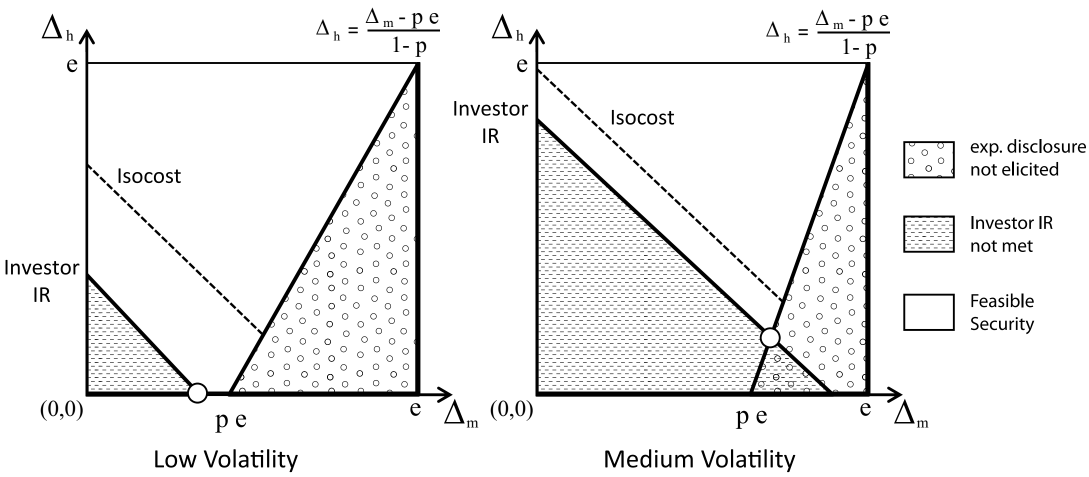

Figure 3: Security Design under Expansive Disclosure

See Figure 3. As was the case for the OSD0 Program in the preceding section, to obtain the optimal security, all that has to be done is to search for the lowest iso-cost line for the objective function, i.e., the lowest value k among the lines $A_l^1 \Delta_m + A_h^1 \Delta_h = k$ that lies within this constraint set.

[28] When, later in the paper, we evaluate the manager's preferences over limited and expansive disclosure policies, the additional details provided in the proposition about how the manager's maximum expected profits change with changes in the firm's cash flow volatility will also become important.

The following proposition summarizes the findings from the geometric investigation of the OSD1 Program in the appendix.

Proposition 6 *Among the securities that can implement the expansive disclosure policy:*

(1) if $e \leq \mu - 1$, an optimal security is given by risk-free debt with face value $F \in [1, \mu - e]$; the manager's expected profits are $\mu - 1$;

(2) if $\mu - 1 < e \leq \frac{\mu-1}{1-\left(1-\frac{\eta}{2}-A_l^1\right)p} \equiv \underline{e}_{ID}^1$, an optimal security is given by investment-grade debt with face value $e + \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1}$; the manager's expected profits are $\mu - 1 - A_h^1 \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1}$;

(3) if $\underline{e}_{ID}^1 < e < \frac{\mu-1}{A_l^1+A_h^1} \equiv \bar{e}_{ID}^1$, an optimal security is given by a hybrid security; the manager's expected profits are $\mu - 1 - A_h^1 \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1}$;

(4) if $e \geq \bar{e}_{ID}^1$, then no feasible security implements the expansive disclosure policy.

The proposition establishes that there is a simple three-level hierarchy of optimal securities that induce the manager to implement the expansive disclosure policy: risk-free debt is optimal when the volatility of the firm's cash flows is extremely low; investment-grade debt is optimal when the volatility of the firm's cash flows is too high to make risk-free debt feasible, but is still sufficiently low; and hybrid securities are optimal for higher levels of volatility. As was also true for the limited disclosure policy discussed in the previous section, when the firm's cash flow volatility is extremely high, no feasible security of any kind implements the expansive disclosure policy

The intuition for part (1) of the proposition was explained in the discussion surrounding Proposition 4 above. The intuition for part (2) is somewhat similar to that for Proposition 5 above: as the volatility in the firm's cash flows increases, the volatility of the feasible securities that induce the investor to supply financing also increases, which raises the firm's cost of capital. However, for volatility levels close to, but larger than $e = \mu - 1$, the manager can issue investment-grade debt with face value close to $\mu - 1$ that allows the manager to obtain nearly first-best payoffs. This explains part (2) of the proposition. As the firm's cash flow volatility increases further, the manager must increase the face value of the debt to satisfy the supply financing constraint. However, doing so also reduces his incentives to implement the expansive disclosure policy; when the cash flow volatility extends beyond the critical threshold $\underline{e}_{ID}^1$, no feasible investment-grade debt will continue to induce the manager to adopt the expansive disclosure policy. For higher volatilities, the manager needs to issue hybrid securities with not only downside risk but also upside potential in order to induce the manager to disclose the intermediate outcome. Thus, in contrast to the case of the limited disclosure policy, debt securities alone do not have sufficient flexibility to be optimal securities for all volatility levels for which it is feasible to implement the expansive disclosure policy. This explains part (3) of the proposition. Part (4) of the proposition follows for reasons similar to part (5) of Proposition 5: at very high levels of cash flow volatility, the manager prefers not to finance the project.[29]

[29] Before leaving the discussion of optimal securities that implement the expansive disclosure policy, it should be noted that such securities may not be uniquely optimal: there may be an interval of securities that are optimal for some volatility levels e. In stating Proposition 6, we resolved this potential indeterminacy by selecting the optimal security that has the

Further discussion of Proposition 6, and in particular, its relationship to Proposition 5, is reserved to the next section, which contains results for the problem of optimizing across both securities and disclosure policies.

3.4 Global hierarchy of securities and disclosure policies

So far, we have established the existence of two hierarchies of securities, one for securities that optimally implement the limited disclosure policy and one for securities that optimally implement the expansive disclosure policy. Now, we develop a combined hierarchy that involves optimizing across both securities and disclosure policies. We call the optimal security and disclosure policy that solves this joint optimization problem the "globally optimal" security and disclosure policy.

By relying on the results already established, we can get an immediate sense of what constitutes the globally optimal security and disclosure policy in the "corner" case where the firm's cash flow volatility is sufficiently low. For that case, Propositions 5 and 6 clearly establish that the expansive disclosure policy necessarily dominates the limited disclosure policy because, as was also noted in the statement and discussion of Proposition 4 above, risk-free securities induce the expansive disclosure policy but not the limited disclosure policy. Combining this result with the well-known result that the maximum value of an objective function of a program is continuous in a parameter of the program, we can further conclude that the expansive disclosure policy is also the globally optimal disclosure policy for all cash flow volatilities in some (right-) neighborhood of those (low) volatilities for which first-best results are attainable. The associated globally optimal security is then defined by the optimal security that implements the expansive disclosure policy for these low volatility levels, namely (by Proposition 6) either risk-free debt (when these volatility levels are very low) or investment-grade debt (when these volatility levels are somewhat higher).

We can also get an immediate sense of what constitutes the globally optimal security and disclosure policy in the other "corner" case where the firm's cash flow volatility is sufficiently high. When the firm's cash flow volatility is sufficiently high, the only potentially feasible security is one which transfers nearly all of the firm's cash flows to the investor, i.e., one for which both the downside risk and upside potential of the security are nearly maximal: $\Delta_m \approx e \approx \Delta_h$. Given this observation, then the result in Corollary 1 that $A_l^0 + A_h^0 < A_l^1 + A_h^1$ leads immediately to the conclusion that the globally optimal disclosure policy must be the limited disclosure policy, since that result implies that a firm's cost of capital under the limited disclosure policy, $A_l^0 \Delta_m + A_h^0 \Delta_h e \approx (A_l^0 + A_h^0)e$, will be less that its cost of capital under the expansive disclosure policy, $A_l^1 \Delta_m + A_h^1 \Delta_h \approx (A_l^1 + A_h^1)e$. Appealing to continuity a second time, we can further conclude that the limited disclosure policy is the globally optimal disclosure policy for all cash flow volatilities in some (left-)neighborhood of those sufficiently high volatilities as well. The associated globally optimal security is then defined by the optimal security that implements the expansive disclosure policy for these high volatility levels, namely (by Proposition 5) speculative

smallest level of downside risk. Whether this choice, or some other choice, of optimal security is made has no influence on the firm's cost of capital, and it will be clear when we discuss optimal disclosure policies in Proposition 7 below, that such choices have no influence on the manager's preferences among disclosure policies either.

debt.

Having settled the question of what constitutes the globally optimal security and disclosure policy when the firm's cash flow volatility fall in the "corners," it remains to determine the globally optimal security and disclosure policy when the firm's cash flow volatility assumes "intermediate" values. The next proposition provides a complete answer to this question.

Proposition 7 *There exists a unique value, e^*, for the volatility of the firm's cash flows, with $e^* \in (\mu - 1, \underline{e}^1_{ID})$, such that:*

1. *if $0 < e \leq e^*$, then expansive disclosure combined with either risk-less or investment-grade debt, as described in Proposition 6, is optimal;*
2. *if $e^* < e < \bar{e}^0_{SD}$, then limited disclosure combined with either investment-grade debt (when $e \leq \underline{e}^0_{SD}$) or speculative debt (when $e > \underline{e}^0_{SD}$), as described in Proposition 5, is optimal;*[30]
3. *if $e \geq \bar{e}^0_{SD}$, then no feasible security implements either the expansive or the limited disclosure policy.*

The proposition establishes that the manager prefers the expansive disclosure policy in all environments characterized by a sufficiently low level of cash flow volatility, whereas he prefers the limited disclosure policy in all environments characterized by higher levels of cash flow volatility, up to some maximum cash flow volatility beyond which no disclosure policy (either limited or expansive) can be implemented by any feasible security. In addition, the proposition establishes that, for all cash flow volatilities for which there exists a feasible security that implements the globally optimal disclosure policy, the optimal security is always a debt security. The debt security that is globally optimal is risk-less debt when the cash flow volatility is sufficiently low, then it becomes investment-grade debt as the cash flow volatility increases, then it becomes speculative debt as the cash flow volatility increases still further.

While the formal proof of Proposition 7 is deferred to the appendix, the heart of the proof is easily explained: it follows from the rankings of the limited and expansive disclosure policies established for the two corner cases (as was discussed above preceding the statement of Proposition 7), and the fact (established in the proof) that over the interval of volatilities $e \in (\mu - 1, \underline{e}^1_{ID})$, the manager's maximum expected profit function obtained from implementing the limited disclosure policy and the manager's maximum expected profit function obtained from implementing the expansive disclosure policy are both linear in e. Hence, these two functions must cross exactly once, at the volatility level e^* specified in the proposition.

Another feature of the proof of Proposition 7 is worth mentioning. The proof explains why hybrid securities, which we know from Proposition 6 are sometimes optimal within the class of feasible securities that implement the expansive disclosure policy, are never globally optimal. The reason is that the region of cash flow volatility levels in which hybrid securities optimally implement the expansive disclosure

[30] The proof of the proposition establishes that that $\underline{e}^1_{ID} < \underline{e}^0_{ID}$. So, since $\underline{e}^0_{ID} < \underline{e}^0_{SD}$ by Proposition 5, each of the intervals $(e^*, \underline{e}^0_{SD}]$ and $(\underline{e}^0_{SD}, \bar{e}^0_{SD})$ is nonempty.

policy is a region for which the volatility of the firm's cash flows, e, exceeds $e > \underline{e}^1_{ID}$ (see the statement of Proposition 6), and the proof of Proposition 7 shows that, in this region (i.e., for $e > \underline{e}^1_{ID}$), whenever there exists a feasible hybrid security – call it "security 1" – that implements the expansive disclosure policy, there exists a feasible debt security that implements the limited disclosure policy that increases the manager's expected profits relative to that obtained from security 1.

Perhaps the most important prediction that emerges from Proposition 7 is the following corollary.

Corollary 2 *In equilibrium, there is a negative association between firms' cost of capital and the amount of information voluntarily disclosed by firms.*

This prediction follows from two previously established facts: first, that as a firm's cash flow volatility e increases, its maximum expected profits decline solely due to the firm's cost of capital increasing (this follows by combining Propositions 5, 6, and 7), and second, that as a firm's cash flow volatility increases, it optimally switches from the expansive disclosure policy to the limited disclosure policy (by Proposition 7).

This negative correlation does not imply, however, that more expansive voluntary disclosure *causes* firms' cost of capital to decline. On the contrary, the model predicts that, whenever the firm issues risky debt, increases in disclosure lead to a strict increase in the firm's cost of capital. To see this, first consider investment-grade debt. The cost of capital associated with issuing investment-grade debt is $A^d_l \Delta_m$. Expansive disclosure yields a greater cost of capital for investment-grade debt than does limited disclosure because, by Corollary 1, $A^0_l < A^1_l$. Next consider speculative debt. The cost of capital associated with issuing speculative debt and adopting the limited disclosure policy is $A^0_l e + A^0_h \Delta_h$. We observe that this is always less than the cost of capital associated with issuing speculative debt and adopting the expansive disclosure policy, $A^1_l e + A^1_h \Delta_h$. To see this, note that $A^0_l e + A^0_h \Delta_h < A^1_l e + A^1_h \Delta_h$ is equivalent to

$$\left(A^0_h - A^1_h\right) \Delta_h < \left(A^1_l - A^0_l\right) e, \tag{28}$$

and that the latter inequality always holds: by Corollary 1 (a), LHS(28) is increasing in Δ_h, and so (28) always holds as long as it holds at $\Delta_h = e$. (28) does hold at $\Delta_h = e$, since $A^0_l + A^0_h < A^1_l + A^1_h$, by Corollary 1(b). Thus, even though for any fixed risky debt security, there is a positive association between a firm's cost of capital and how much it discloses, when the firm adjusts optimally its choice among securities to its environment e, the equilibrium (hence, predicted observed) association between a firm's cost of capital and how much the firm discloses is negative.

4 Extensions

We discuss two extensions here, one involving the introduction of an agency problem into the analysis, and the second involving the effects of having the manager precommit to making certain disclosures. A primary reason for introducing the moral hazard extension is that, with it, it cannot be optimal for the manager to sell, or rent, the firm to the investor – so that the manager's residual stake in the firm does not vary with the realization of the firm's cash flows.[31] A primary reason for introducing the extension involving the manager precommitting to make certain disclosures is that the manager's choice among securities "up front" (before any random variables have realized their values) is itself a form of precommitment, and it is worthwhile to understand whether precommitment to a disclosure policy serves as a substitute for precommitment to a particular capital structure.[32]

4.1 Optimal security design in the presence of managerial moral hazard

In the first extension, we suppose that the firm's stochastic production process requires two inputs, capital supplied by the investor, and labor supplied by the manager. Specifically, we assume that, after obtaining the capital injection I from the investor, the manager can choose which of two binary effort levels to exert. If he exerts the higher effort, he bears a personal cost $c > 0$ and the probability distribution for the firm's cash flows $\tilde{\theta}$ is as was described previously, i.e., θ_l and θ_h each occur with probability $\frac{\eta}{2}$, and θ_m occurs with probability $1 - \eta$. If the manager exerts the lower effort – the cost of which we normalize to zero – then the firm's cash flows are θ_l with certainty.

We suppose that c is sufficiently small so that having the manager supply the higher effort level is efficient. To simplify the presentation, and to make the results developed previously extend most readily to the present situation, we adopt the standard assumption that the manager's effort choice is observable but not contractible. This assumption is common in the incomplete contracting literature, see, e.g., Aghion and Bolton [1992], Hart and Moore [1998], Hermalin and Katz [1991], Dewatripont et al. [2003].[33]

With these assumptions, the specification of the optimal security design program that induces the manager to adopt a particular disclosure policy is almost the same as the ones we have previously specified and analyzed. All that has to be done is to add an appropriate "incentive compatibility" constraint that ensures that the manager prefers to exert the higher effort level. To specify precisely what that incentive compatibility constraint is, we first note that if the manager chooses the zero cost effort, the assumption that the manager's effort is observable (in particular, to the market maker) implies that the market maker will know that the firm's cash flows will be θ_l, and he will price the security $\tilde{\psi}$ at ψ_l, and he will price the residual security $\tilde{\xi}$ at $\theta_l - \psi_l$.[34] Next, with $E[\cdot]$ denoting the expectations operator for the distribution of the firm's cash flows when the manager adopts the higher effort level, we notice that the "ex ante" expected price the market maker will set (and the manager will receive) for the residual security when the manager selects the higher effort level is $E[\tilde{\theta} - \tilde{\psi}] = \mu - E[\tilde{\psi}]$.[35] So the incentive

[31] Consequently, in the moral hazard extension it is unnecessary to impose the restriction (previously embedded in the monotonicity requirement (4)) that the manager's residual stake $\theta_i - \psi_i$ be strictly increasing over some pair of states.

[32] We thank an anonymous referee for this observation.

[33] To be more precise, we require that the market maker gets to set the prices for the two securities after observing the manager's actual effort choice.

[34] Of course, when the manager exerts this lower effort level, it follows that whether the manager discloses (his knowledge of) the firm's cash flows is moot.

[35] This conclusion follows from an argument similar to that discussed in footnote 27 above.

compatibility constraint is given by:

$$\theta_l - \psi_l \leq \mu - E[\tilde{\psi}] - c. \tag{29}$$

Using the representation of $E[\tilde{\psi}]$ in (16) above along with $\theta_l = \mu - e$, this incentive compatibility constraint (29) can be rewritten as:

$$\left(1 - \frac{\eta}{2}\right)\Delta_m + \frac{\eta}{2}\Delta_h \leq e - c. \tag{30}$$

It follows that the optimal security that maximizes the manager's expected profits while inducing the manager to implement disclosure policy $d \in \{0, 1\}$ and to exert the higher effort level solves the following program.

The Optimal Security Design d (OSDd) Program with moral hazard

$$\max_{\psi_l, \Delta_m, \Delta_h} \mu - A_l^d \Delta_m - A_h^d \Delta_h - 1$$

subject to:

$$\begin{aligned}
(1-2d)pe &\leq (1-2d)(\Delta_m - (1-p)\Delta_h); \\
1 &\leq I = E\tilde{\psi} - A_l^d\Delta_m - A_h^d\Delta_h; \\
(1-\frac{\eta}{2})\Delta_m + \frac{\eta}{2}\Delta_h &\leq e - c; \\
0 &\leq \psi_l \leq \mu - e; \\
0 &\leq \Delta_i \leq e, \ i \in \{\mu, h\}.
\end{aligned}$$

Proposition 8 is the moral hazard counterpart to Proposition 7.[36]

Proposition 8 *Take $c \leq \frac{\eta}{2}(\mu - 1)$ and let e^* be the threshold specified in Proposition 7. Then, there is a value of the volatility of the firm's cash flows, $\hat{e}^0_{SD}$, with $\hat{e}^0_{SD} \in (\underline{e}^0_{SD}, \bar{e}^0_{SD})$, such that:*

1. *if $c \leq e \leq e^*$, then expansive disclosure combined with either risk-free (when $e \leq \mu - 1$) or investment-grade debt (when $e > \mu - 1$), as described in Proposition 6, is optimal;*

2. *if $e \in (e^*, \hat{e}^0_{SD}]$, then limited disclosure combined with either investment-grade debt (when $e \leq \underline{e}^0_{SD}$) or speculative debt (when $e > \underline{e}^0_{SD}$) as described in Proposition 5, is optimal.*

3. *If $e > \hat{e}^0_{SD}$, then no feasible security implements either the expansive or the limited disclosure policy.*

The most important observation to be drawn from the proposition is that the introduction of managerial moral hazard need not alter in any significant way the optimality of securities or disclosure policies that were deemed globally optimal in the non-moral hazard case previously studied. While the moral hazard problem shrinks the set of volatility levels for which a feasible security exists, it otherwise has minor effects on the hierarchy of optimal securities developed previously.

[36] The hypotheses of the proposition include a requirement that the volatility e of the firm's cash flows exceeds c. The reason is that the manager's effort increases expected cash flows from $\mu - e$ to μ by e. If the cost of effort c exceeds the increase in expected cash flows e inducing effort is never optimal.

4.2 Optimal security design in the presence of precommitments to disclosure

In this second extension, we preserve the basic structure of our model while extending the model to incorporate consideration of how the manager precommitting to disclosure, i.e., adhering to particular mandatory disclosure requirements, affects the manager's choice among securities, the firm's cost of capital, and the manager's voluntary disclosure decisions. In undertaking this study, we presume that adhering to mandatory disclosure requirements does not preclude the manager from making voluntary supplementary disclosures.

We consider two forms of mandatory disclosure. One form of mandatory disclosure, which we label "loss recognition," requires that the manager disclose the low outcome (θ_l) when he knows it. The other form of mandatory disclosure, which we label "gains recognition," requires the manager to disclose the medium outcome (θ_m) when he knows it. While such disclosure requirements could be implemented in various ways (e.g., by developing internal controls that detect the firm's receipt of news, by allowing for shareholder lawsuits that penalize firms for withholding information or reporting negative earnings surprises, etc.), in the following, we ignore how the disclosure requirements are implemented and instead focus purely on whether the informational effects of mandatory disclosure requirements increase the manager's expected profits.

4.2.1 Optimal security design under mandatory loss recognition disclosures

Mandatory disclosure of some information can affect a manager's incentives to voluntarily disclose other information. Accordingly, when a mandatory disclosure policy is imposed, everything can change, e.g., Proposition 3 may no longer describe the relation between the manager's preferred voluntary disclosure policy and the firm's capital structure; the informed trader's trading strategy may change from that described in Proposition 2; the relation between the firm's cost of capital and its disclosure policies and capital structure may depart from that described in Proposition 3; and the manager's preferred choice among securities may no longer be characterized by Propositions 5 and 6.

We start the formal study of mandatory loss recognition disclosures by considering how loss recognition disclosures change the manager's preferred voluntary disclosure policy.

Proposition 9 *Under mandatory loss recognition disclosures, suppose the manager offers the investor a security* $(\psi_l, \Delta_m, \Delta_h)$ *which satisfies the limited liability and monotonicity conditions described in* (2)−(4) *above. Then, the possible voluntary disclosure policies* $d(\cdot)$ *the security can induce the manager to adopt are the following:* $d(\theta_l) = \theta_l$, $d(\theta_h) = \theta_h$, *and:*

(i)

$$\text{if } \Delta_m > \Delta_h, \text{ then } d(\theta_m) = \emptyset; \tag{31}$$

(ii)

$$\text{if } \Delta_m < \Delta_h, \text{ then } d(\theta_m) = \theta_m; \tag{32}$$

and if each strict inequality in (31) and (32) is replaced by an equality, then the security can induce the manager either to disclosure, or to not disclose, $\tilde{\theta} = \theta_m$.

To provide some intuition for this proposition, begin by noting that $\Delta_m > \Delta_h$ is equivalent to

$$\psi_m > \frac{\psi_l + \psi_h}{2} = E[\tilde{\psi}|\tilde{\psi} \neq \psi_m]. \tag{33}$$

Inequality (33) in turn implies

$$\psi_m > E[\tilde{\psi}] \tag{34}$$

(the expectation $E[\tilde{\psi}]$ here being the security's unconditional expected value) and so, since $\theta_m = E[\tilde{\theta}]$,

$$\theta_m - \psi_m < E[\tilde{\theta} - \tilde{\psi}]. \tag{35}$$

Since the manager does not make a disclosure only if the medium outcome occurs, $E[\tilde{\theta} - \tilde{\psi}|\text{manager does not disclose information}] = f \times (\theta_m - \psi_m) + (1 - f) \times E[\tilde{\theta} - \tilde{\psi}]$, where f is the probability that the manager is withholding information given that he makes no disclosure. So, (35) implies:

$$\theta_m - \psi_m < E[\tilde{\theta} - \tilde{\psi}|\text{manager does not disclose information}]. \tag{36}$$

Thus, since (36) follows from $\Delta_m > \Delta_h$, it follows that the explanation for why the manager prefers not to disclose θ_m when he knows it and $\Delta_m > \Delta_h$ is that the value of his residual security were he to disclose his information is below outsiders' perceptions of its expected value were he to make no disclosure.

It is instructive to compare the condition for nondisclosure in (31) to the corresponding requirement for nondisclosure in Proposition 1. The inequality (7) characterizing nondisclosure in Proposition 1 can be written alternatively as $\Delta_h < \frac{\Delta_m - pe}{1-p}$. Since, for all $\Delta_m \in [0, e)$, it is clear that $\frac{\Delta_m - pe}{1-p} < \Delta_m$, it follows that $\Delta_h < \frac{\Delta_m - pe}{1-p} \implies \Delta_h < \Delta_m$, i.e., any security (Δ_m, Δ_h) that induces the manager not to disclose θ_m when there are no disclosure requirements will also induce the manager not to disclose θ_m when the mandatory loss recognition disclosure requirements are imposed. Thus, in the case of mandatory loss recognition disclosures, *holding the security the firm issues fixed, mandatory disclosure inhibits voluntary disclosure.* We shall show in Proposition 11 below, however, that *if the manager is allowed to optimize his choice among securities* (when the disclosure regime changes from all disclosures being voluntary to the mandatory loss recognition regime)*, then mandatory disclosure leads to expanded voluntary disclosure.* This conclusion thus provides a second illustration of the importance of endogenizing a firm's capital structure when making predictions about disclosures: the predictions about disclosures that emerge from holding capital structure fixed are the opposite of the predictions that emerge when the firm's capital structure is endogenized.[37]

When the manager does not disclose the intermediate outcome (case *i* in Proposition 9), no disclosure might be due to either the manager not receiving information or the manager withholding the intermediate outcome. The trader can distinguish between these two events and hence can earn an informational

[37] The first example was presented and discussed following Corollary 2, where we showed that, holding a firm's debt securities fixed, expanded disclosure is associated with an increase in the firm's cost of capital, whereas when the manager is allowed to optimize across securities, expanded disclosure is associated with a lower cost of capital.

rent by trading on his private information. The following proposition characterizes the trader's optimal trading strategy for this case.

Proposition 10 *Under mandatory loss recognition disclosures, suppose the manager offers the investor the security* $(\psi_l, \Delta_m, \Delta_h)$ *where* $\Delta_m > \Delta_h$ *and hence induces the disclosure policy* $d(\theta_l) = \theta_l$, $d(\theta_m) = \emptyset$, $d(\theta_h) = \theta_h$. *Then, the trader's optimal trading strategy is* $y(\emptyset, s_{withheld}) = s_{withheld}$ *and* $y(d, s_{withheld}) = 0$ *when* $d \neq \emptyset$.

That is, the trader submits a buy order when he knows that the reason the manager did not disclose information is that he is hiding information, and otherwise, the trader stays out of the market. Thus, the trader's decision to participate in the market for security $\tilde{\psi}$ is the opposite of that described in Proposition 2 above where all disclosures were voluntary. The reason for this radically changed trading behavior follows from inequality (34) which establishes that the security's payoff when the reason the manager did not make a disclosure is that he is hiding information is higher than its expected value when the reason the manager did not make a disclosure is that he did not receive information. Thus, while the investor still incurs positive expected trading losses to the informed trader when the firm is subject to the mandatory loss recognition requirements, these expected trading losses occur in different states of the world as compared to when there are no mandatory disclosure requirements.

Case (ii) of Proposition 9 identifies a restriction on securities that result in the manager engaging in full disclosure of his private information. When these restrictions hold, the market maker and the investor are at informational parity with the informed trader, so: the security will always be correctly priced; the investor will not incur any expected trading losses; the firm's cost of capital will be zero; and the first-best outcome will be achieved. The next proposition shows that the manager can exploit this result and achieve first-best by issuing an equity security.[38]

Proposition 11 *Under mandatory loss recognition disclosures, for all* $e \in (0, \mu)$, *the equity security* $\tilde{\psi} = \frac{\tilde{\theta}}{\mu}$ *(i.e.,* $(\psi_l, \Delta_m, \Delta_h) = (1 - \frac{e}{\mu}, \frac{e}{\mu}, \frac{e}{\mu})$*) induces the manager to engage in full disclosure and achieves the first-best outcome.*[39]

The equity security identified in the proposition entails that the manager sells fraction $1/\mu$ of the firm to the investor. To confirm that this equity security achieves first-best, notice that the fraction of the firm's cash flows that the manager retains is worth $(1 - 1/\mu) \times E[\tilde{\theta}] = \mu - 1$, the first-best amount.

Notice that no risky debt security will induce the manager to fully disclose his private information, because no risky debt security satisfies inequality (32) (either as a weak or strict inequality).[40] Thus, no

[38]It may be worthwhile to point out that Proposition 3, which characterizes a firm's cost of capital associated with issuing a particular security when there are no mandatory disclosure requirements, does not characterize the firm's cost of capital in the presence of mandatory disclosure requirements. Hence, the fact that the equity security in Proposition 11 has the property that $\Delta_m > 0$ and $\Delta_h > 0$ is not inconsistent with the firm's cost of capital being zero in the presence of the mandatory loss recognition disclosure requirements.

[39]The proof of the proposition is immediate: since $\Delta_m = \Delta_h$, condition (32) holds (weakly) for the equity security $(\psi_l, \Delta_m, \Delta_h) = \left(1 - \frac{e}{\mu}, \frac{e}{\mu}, \frac{e}{\mu}\right)$, and so the manager (weakly) prefers full disclosure.

[40]Recall that risky debt entails $\Delta_m > 0$ and that, if $\Delta_h > 0$, then $\Delta_m = e$. So, risky debt never can satisfy (32) as a strict inequality, and it could only satisfy (32) as a weak inequality if $\Delta_h = \Delta_m = e$, which is ruled out by the monotonicity requirement (4), according to Lemma 1.

risky debt security gets the manager to engage in a policy of full disclosure; hence no risky debt security completely eliminates the informed trader's informational advantage over the market maker, and hence no risky debt security yields the first-best outcome.[41]

We conclude this subsection with two further comments concerning Proposition 11. First, the proposition illustrates how what are considered to be optimal securities can change with the introduction of mandatory disclosure requirements. As we noted in earlier sections of the paper, equity securities are typically not optimal when all disclosures are voluntary, though they are optimal here in the presence of mandatory loss recognition disclosures. Second, the proposition illustrates how precommitting to a disclosure policy may lead to an increase in the manager's maximum expected profits, as first-best outcomes are achievable over the full spectrum of the volatility of the firm's cash flows here, whereas first-best outcomes are achievable only if risk-free debt is a feasible security when precommitment to a disclosure policy is impossible. We shall see in the next subsection, however, that the ability to make precommitments to adhere to a disclosure policy does not always increase the manager's maximum expected profits.

4.2.2 Optimal security design under mandatory gains recognition disclosures

In this subsection we discuss the mandatory disclosure regime referred to as "gains recognition" above, which entails that the manager must disclose the medium outcome (θ_m) when he knows it.

Under gains recognition, the manager will always keep silent when he learns that $\tilde{\theta} = \theta_l$ occurs – as he cannot possibly benefit by revealing that the firm's cash flows are the lowest possible level. Since the manager will always disclose $\tilde{\theta} = \theta_h$ when he learns it, and he necessarily discloses $\tilde{\theta} = \theta_m$ when he learns it to be in conformity with the gains recognition mandatory disclosure requirement, it follows that the disclosure policy the manager adopts in the presence of mandatory gains recognition is tantamount to the manager implementing the expansive disclosure policy. From this observation, it is immediate that mandatory gains recognition disclosures sometimes reduces the manager's maximum expected profits relative to the situation where all disclosures are voluntary because we know from Proposition 7 that, when all disclosures are voluntary, the manager's maximum expected profits are sometimes higher when he selects a security that induces him to adopt the limited disclosure policy rather than the expansive disclosure policy. These remarks prove:

Proposition 12 *Under mandatory gains recognition disclosures, the disclosure policy the manager is induced to adopt is always the expansive disclosure policy, and the manager's maximum expected profits are always weakly (in fact, strictly for sufficiently high levels of cash flow volatility) below what the manager could achieve when all disclosures are voluntary.*

Viewing mandatory disclosures as a commitment to a disclosure policy, this result shows that commit-

[41]But, risk-free debt would continue to yield the first-best even in the presence of mandatory loss recognition disclosure requirements, but only over the limited set of volatilities $e \in [0, \mu - 1]$. In contrast, the equity security $\tilde{\psi} = \frac{\tilde{\theta}}{\mu}$ yields first-best over the entire spectrum (recall assumption (1) above) of volatilities $e \in (0, \mu)$.

ment to a disclosure policy can reduce the manager's maximum expected profits. While it is commonplace to observe that mandatory disclosure requirements may be undesirable because they may generate proprietary costs of disclosure, Propositions 11 and 12 combined indicate that unqualified statements about the desirability or undesirability of expanded mandatory disclosure requirements, or unqualified statements about the desirability of committing to expanded disclosures, typically are invalid even absent concerns about disclosing proprietary information.

5 Conclusion

The owners of a firm use all of the instruments at their disposal, e.g., their operating choices, financing choices, investment choices, and disclosure choices, to maximize the expected value of their residual stake in their firm. Because the array of such instruments available to a firm's owners is in practice so large, any model of a firm must inevitably put some restrictions on the range of endogenous variables the owners of a firm are posited to select. While it is perhaps natural in studying a firm's choices among financial accounting and disclosure policies to take the firm's investment and operating decisions as given, the present paper demonstrates that, for purposes of studying a firm's disclosure policy, we have shown that it is undesirable to take the firm's financing decisions as given, because as the firm's financing decisions change, the residual stake the owners of the firm have claim over changes, and the latter change alters the owners' preferred voluntary disclosure policy. We have also shown that the reverse is true too: what capital structure the owners of a firm prefer also depends on the firm's disclosure policy. These interdependencies imply that, in equilibrium, a firm's capital structure and disclosure policy are jointly determined, and that together they determine the firm's cost of capital.

This paper studies the equilibrium specification of a firm's capital structure and disclosure policy. We establish that the effect of issuing a security on a firm's cost of capital depends separately on the security's upside potential (the difference in the security's payoff depending on whether the firm's cash flows assume high or median values) and the security's downside risk (the difference in the security's payoff depending on whether the firm's cash flows assume median vs. low values), with these separate contributions varying with the firm's disclosure policy. We show that, holding the security the firm issues fixed, the contribution of the security's upside potential to the firm's cost of capital decreases with the extent of its voluntary disclosures and, perhaps surprisingly, the contribution of a security's downside risk to the firm's cost of capital increases with the extent of its voluntary disclosures.

Notwithstanding this last result, the model predicts that, in equilibrium - when the firm optimally changes its capital structure as its environment changes - there is a negative association between a firm's cost of capital and the extent of its voluntary disclosures. This negative association is attributable to the fact that both the firm's equilibrium cost of capital and its disclosure policy are linked to the volatility of the firm's cash flows. In fact, we established that there is a hierarchical ranking of the firm's preferred capital structure and disclosure policies: firms with the lowest amount of cash flow volatility prefer to issue risk-free debt and adopt an expansive disclosure policy. As the volatility of firms' cash flows progressively increases, firms initially prefer investment-grade debt and expansive disclosures, then they prefer investment-grade debt and limited disclosures, and finally, at very high volatility levels, they prefer "junk" debt and limited disclosures.

We also studied the effects of imposing mandatory disclosure requirements. The importance of endogenizing a firm's capital structure when studying its disclosure policies was strikingly illustrated by the demonstration that imposing certain mandatory ("loss recognition") disclosure requirements inhibits the firm from making voluntary supplementary disclosures when the firm's capital structure is held fixed, whereas imposing these same mandatory disclosure requirements encourages the firm to make voluntary supplementary disclosures when the firm's capital structure is allowed to change optimally to reflect these mandatory disclosure requirements. The changes in a firm's optimal capital structure in response to the imposition of mandatory disclosure requirements was also shown sometimes to be striking. Equity securities, which are typically not optimal when all disclosures are voluntary, were demonstrated sometimes to become optimal in the presence of mandatory disclosure requirements.

Appendix

Proof of Proposition 1

Part (i). In view of the monotonicity conditions (2)-(4), the unique optimality of $d(\theta_h) = \theta_h$ and $d(\theta_l) = \emptyset$ is clear. Consequently, the proof focuses on the determination of $d(\theta_m)$. If Part (i) describes the manager's disclosure policy then $E[\tilde{\theta} - \psi(\tilde{\theta})|d]$ yields

$$E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \theta_h\right] = \theta_h - \psi_h$$

and, by Bayes' rule,

$$\begin{aligned} & E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \emptyset\right] \\ = \; & \frac{(1-p)\left(\mu - E[\widetilde{\psi}]\right) + p\left((1-\eta)(\mu - \psi_m) + \frac{\eta}{2}(\theta_l - \psi_l)\right)}{1 - p + p\left(1 - \frac{\eta}{2}\right)} \\ = \; & \mu - E[\widetilde{\psi}] - \frac{p\left((1-\eta)\left(\psi_m - E[\widetilde{\psi}]\right) + \frac{\eta}{2}\left(e - E[\widetilde{\psi}] + \psi_l\right)\right)}{1 - p\frac{\eta}{2}} \end{aligned}$$

where

$$E[\widetilde{\psi}] = (1-\eta)\,\psi_m + \frac{\eta}{2}(\psi_l + \psi_h) = \psi_m + \frac{\eta}{2}\left[(\psi_h - \psi_m) - (\psi_m - \psi_l)\right].$$

Given the restriction to truth-telling, were the manager to disclose $d = \mu$, then necessarily:

$$E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \mu\right] = \mu - \psi_m.$$

So, for the disclosure policy in part (i) to be an equilibrium, we require:

$$E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \mu\right] \leq E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \emptyset\right],$$

i.e.,

$$\mu - \psi_m \leq \mu - E[\widetilde{\psi}] - \frac{p\left((1-\eta)\left(\psi_m - E[\widetilde{\psi}]\right) + \frac{\eta}{2}\left(e - E[\widetilde{\psi}] + \psi_l\right)\right)}{1 - p\frac{\eta}{2}}$$

which simplifies to

$$pe \leq (\psi_m - \psi_l) - (1-p)(\psi_h - \psi_m)$$

Part (ii). If part (ii) describes the manager's disclosure policy then $E\left[\tilde{\theta} - \psi(\tilde{\theta})|d\right]$ yields

$$\begin{aligned}
E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \theta_h\right] &= \theta_h - \psi_h \\
E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \mu\right] &= \mu - \psi_m \\
E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \emptyset\right] &= \frac{(1-p)\left(\mu - E[\widetilde{\psi}]\right) + p\frac{\eta}{2}(\theta_l - \psi_l)}{1 - p + p\frac{\eta}{2}}
\end{aligned}$$

For the disclosure policy in part (ii) to be an equilibrium, we need

$$E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \mu\right] \geq E\left[\tilde{\theta} - \psi(\tilde{\theta})|d = \emptyset\right],$$

i.e.,

$$\mu - \psi_m \geq \frac{(1-p)\left(\mu - E[\widetilde{\psi}]\right) + p\frac{\eta}{2}(\theta_l - \psi_l)}{1 - p + p\frac{\eta}{2}}$$

which simplifies to

$$pe \geq (\psi_m - \psi_l) - (1-p)(\psi_h - \psi_m). \blacksquare$$

Proof of Proposition 2

First suppose that the security induces an expansive disclosure policy, so the only time the manager withholds information is when the information is θ_l. Then it is trivial to confirm that the only equilibrium trading strategy for the informed trader is to trade when he knows that the reason the manager did not disclose information is that he did not receive information and to not trade otherwise.

Second, suppose that the security induces a limited disclosure policy, so the manager does not disclose either θ_m or θ_l. In order for it to be optimal for the trader not to purchase the security if the trader knows that the manager withheld information, the expected purchase price of the security (with no purchasing by the informed trader) must exceed the expected payoff from the security, $E\left[\psi(\tilde{\theta})|\tilde{\theta} \in \{\mu, \theta_l\}\right]$. That is, the following inequality must hold:

$$E\left[P(\emptyset, \widetilde{X})\middle| y(\emptyset, 1) = 0\right] \geq E\left[\widetilde{\psi}|\tilde{\theta} \in \{\mu, \theta_l\}\right] \tag{37}$$

It must also be optimal for the trader to purchase the security if $s_{withheld} = 0$. This is the case if the expected purchase price (with purchasing by the informed trader) is less than the unconditional expected value of the security, $E[\widetilde{\psi}]$, i.e.,

$$E[\widetilde{\psi}] > E\left[P(\emptyset, \widetilde{X})\middle| y(\emptyset, 0) = 1\right] \tag{38}$$

When the trader purchases the security, he submits a market order, and he sometimes winds up paying $P(\emptyset, 1)$ for the security (when the investor is not subject to a liquidity shock), and at other times he winds up paying $P(\emptyset,0)$ (when the investor is subject to a liquidity shock). $P(\emptyset,1)$ equals $E[\widetilde{\psi}]$ (since the market maker can infer that only the investor submitted a market order, and that, since the investor traded, it must have been the case that the trader knew that the manager made no disclosure only because the manager received no information), while $P(\emptyset,0)$ is a weighted average of $E[\widetilde{\psi}]$ and $E\left[\widetilde{\psi}|\tilde{\theta}\in\{\mu,\theta_l\}\right]$. Since $E[\widetilde{\psi}] > E\left[\widetilde{\psi}|\tilde{\theta}\in\{\mu,\theta_l\}\right]$ always holds, we have

$$E[\widetilde{\psi}] > E\left[P(\emptyset,\widetilde{X})\Big| y(\emptyset,0)=1\right] \geq E\left[\widetilde{\psi}|\tilde{\theta}\in\{\mu,\theta_l\}\right]. \tag{39}$$

(37) and (38) now follow from (39). ■

Proof of Proposition 3

The investor's expected payoff is

$$\begin{aligned} I &= (1-\upsilon)E[\widetilde{\psi}] + \upsilon\begin{pmatrix} \Pr(d=\emptyset,X=-1|q=1)P(\emptyset,-1) \\ +\Pr(d=\emptyset,X=0|q=1)P(\emptyset,0) \\ +\Pr(d=\theta_h)\psi_h+\Pr(d=\mu)\psi_m \end{pmatrix} \\ &= E[\widetilde{\psi}] + \upsilon\begin{pmatrix} \Pr(d=\emptyset,X=-1|q=1)P(\emptyset,-1) \\ +\Pr(d=\emptyset,X=0|q=1)P(\emptyset,0) \\ +\Pr(d=\theta_h)\psi_h+\Pr(d=\mu)\psi_m - E[\widetilde{\psi}] \end{pmatrix}. \end{aligned}$$

Note that

$$\Pr(d=\emptyset,X=-1|q=1)+\Pr(d=\emptyset,X=0|q=1)+\Pr(d=\theta_h)+\Pr(d=\mu)=1.$$

The price $P(\emptyset,X)$ can be written as:

$$\begin{aligned} P(\emptyset,X) &= \psi_m + \Pr(\theta_h|\emptyset,X,d_m)(\psi_h-\psi_m) - \Pr(\theta_l|\emptyset,X,d_m)(\psi_m-\psi_l) \\ &= E[\widetilde{\psi}] - \left(\psi_m + \frac{\eta}{2}\left[(\psi_h-\psi_m)-(\psi_m-\psi_l)\right]\right) + \psi_m \\ &\quad + \Pr(\theta_h|\emptyset,X,d_m)(\psi_h-\psi_m) - \Pr(\theta_l|\emptyset,X,d_m)(\psi_m-\psi_l) \\ &= E[\widetilde{\psi}] + \left(\Pr(\theta_h|\emptyset,X,d_m)-\frac{\eta}{2}\right)(\psi_h-\psi_m) - \left(\Pr(\theta_l|\emptyset,X,d_m)-\frac{\eta}{2}\right)(\psi_m-\psi_l). \end{aligned}$$

Then, I can be written as:

$$\begin{aligned} I &= E[\widetilde{\psi}] + \upsilon\begin{pmatrix} \Pr(d=\emptyset,X=-1|q=1,d_m)\begin{pmatrix} E[\widetilde{\psi}] + \left(\Pr(\theta_h|\emptyset,X=-1,d_m)-\frac{\eta}{2}\right)(\psi_h-\psi_m) \\ -\left(\Pr(\theta_l|\emptyset,X=-1,d_m)-\frac{\eta}{2}\right)(\psi_m-\psi_l) \end{pmatrix} \\ +\Pr(d=\emptyset,X=0|q=1,d_m)\begin{pmatrix} E[\widetilde{\psi}] + \left(\Pr(\theta_h|\emptyset,X=0,d_m)-\frac{\eta}{2}\right)(\psi_h-\psi_m) \\ -\left(\Pr(\theta_l|\emptyset,X=0,d_m)-\frac{\eta}{2}\right)(\psi_m-\psi_l) \end{pmatrix} \\ +\Pr(d=\theta_h)\left(E[\widetilde{\psi}] + \left(1-\frac{\eta}{2}\right)(\psi_h-\psi_m)+\frac{\eta}{2}(\psi_m-\psi_l)\right) \\ +\Pr(d=\mu)\left(E[\widetilde{\psi}] - \frac{\eta}{2}\left((\psi_h-\psi_m)-(\psi_m-\psi_l)\right)\right) - E[\widetilde{\psi}] \end{pmatrix} \\ &= E[\widetilde{\psi}] + \upsilon\begin{pmatrix} \Pr(d=\emptyset,X=-1|q=1,d_m)\begin{pmatrix} \left(\Pr(\theta_h|\emptyset,X=-1,d_m)-\frac{\eta}{2}\right)(\psi_h-\psi_m) \\ -\left(\Pr(\theta_l|\emptyset,X=-1,d_m)-\frac{\eta}{2}\right)(\psi_m-\psi_l) \end{pmatrix} \\ +\Pr(d=\emptyset,X=0|q=1,d_m)\begin{pmatrix} \left(\Pr(\theta_h|\emptyset,X=0,d_m)-\frac{\eta}{2}\right)(\psi_h-\psi_m) \\ -\left(\Pr(\theta_l|\emptyset,X=0,d_m)-\frac{\eta}{2}\right)(\psi_m-\psi_l) \end{pmatrix} \\ +\Pr(d=\theta_h)\left(1-\frac{\eta}{2}\right)(\psi_h-\psi_m)+\frac{\eta}{2}(\psi_m-\psi_l) \\ -\Pr(d=\mu)\frac{\eta}{2}\left((\psi_h-\psi_m)-(\psi_m-\psi_l)\right) \end{pmatrix}. \end{aligned}$$

and therefore

$$A_h^{d_m} = \upsilon \begin{pmatrix} \Pr(d=\emptyset, X=-1|q=1,d_m)\left(\frac{\eta}{2}-\Pr(\theta_h|\emptyset, X=-1,d_m)\right) \\ +\Pr(d=\emptyset, X=0|q=1,d_m)\left(\frac{\eta}{2}-\Pr(\theta_h|\emptyset, X=0,d_m)\right) \\ -\left(1-\frac{\eta}{2}\right)\Pr(d=\theta_h)+\frac{\eta}{2}\Pr(d=\mu) \end{pmatrix}$$

$$A_l^{d_m} = \upsilon \begin{pmatrix} \Pr(d=\emptyset, X=-1|q=1,d_m)\left(\Pr(\theta_l|\emptyset, X=-1,d_m)-\frac{\eta}{2}\right) \\ +\Pr(d=\emptyset, X=0|q=1,d_m)\left(\Pr(\theta_l|\emptyset, X=0,d_m)-\frac{\eta}{2}\right) \\ -\frac{\eta}{2}\Pr(d=\theta_h)-\frac{\eta}{2}\Pr(d=\mu) \end{pmatrix}$$

The two parameters $A_h^{d_m}$ and $A_l^{d_m}$ capture the fact that when the investor incurs a liquidity shock it forces him to sometimes sell at the price $P(\emptyset,0)$ which is not the security's expected value based on the trader's information. Specifically, $A_h^{d_m}$ captures by how much the likelihood of the high payoff ψ_h as reflected in $P(\emptyset,0)$ is lower than the prior likelihood of $\frac{\eta}{2}$. Similarly, $A_l^{d_m}$ captures by how much the likelihood of the low payoff ψ_l as reflected in $P(\emptyset,0)$ is higher than the prior likelihood of $\frac{\eta}{2}$.

Market maker's beliefs if the manager implements the limited disclosure policy

Suppose the manager does not disclose the median value and the market maker conjectures that the trader does not buy the security if he sees $s_{withheld}=1$ and purchases the security if he sees $s_{withheld}=0$. Then, the market maker's beliefs are as follows.

If $X=1$ and $d=\emptyset$, the trader must have purchased the security because $s_{withheld}=0$. Hence, the market maker beliefs equal the prior beliefs.

$$\begin{aligned} \Pr(\theta_h|\emptyset, X=1, d_m=0) &= \Pr(\theta_h|s_{withheld}=0)=\frac{\eta}{2} \\ \Pr(\theta_l|\emptyset, X=1, d_m=0) &= \Pr(\theta_l|s_{withheld}=0)=\frac{\eta}{2} \end{aligned}$$

If $X=0$ and $d=\emptyset$, either (1) the trader purchased the security because $s_{withheld}=0$ and there was a liquidity shock (which occurs with probability $(1-p)\upsilon$) or (2) the trader did not purchase the security because $s_{withheld}=1$ (which implies $s_{knowcash}\in\{\mu,\theta_l\}$) and there was no liquidity shock (which occurs with probability $p\left(1-\frac{\eta}{2}\right)(1-\upsilon)$). Hence, the market maker's beliefs are

$$\begin{aligned} \Pr(\theta_h|\emptyset, X=0, d_m=0) &= \frac{\Pr(\theta_h, d=\emptyset, X=0)}{\Pr(d=\emptyset, X=0)} = \frac{\frac{\eta}{2}(1-p)\upsilon}{(1-p)\upsilon+\left(1-\frac{\eta}{2}\right)p(1-\upsilon)} < \frac{\eta}{2} \\ \Pr(\theta_l|\emptyset, X=0, d_m=0) &= \frac{\Pr(\theta_l, d=\emptyset, X=0)}{\Pr(d=\emptyset, X=0)} = \frac{\frac{\eta}{2}((1-p)\upsilon+p(1-\upsilon))}{(1-p)\upsilon+\left(1-\frac{\eta}{2}\right)p(1-\upsilon)} > \frac{\eta}{2} \end{aligned}$$

If $X=-1$ and $d=\emptyset$. From $X=-1$, the market maker can infer that the trader did not purchase the security because $s_{withheld}=1$. Hence,

$$\begin{aligned} \Pr(\theta_h|\emptyset, X=-1, d_m=0) &= 0 \\ \Pr(\theta_l|\emptyset, X=-1, d_m=0) &= \frac{\frac{\eta}{2}}{1-\eta+\frac{\eta}{2}} = \frac{\eta}{2-\eta} \end{aligned}$$

Based on these beliefs and

$$\begin{aligned} \Pr(d=\emptyset, X=-1|q=1, d_m=0) &= \left(1-\frac{\eta}{2}\right)p \\ \Pr(d=\emptyset, X=0|q=1, d_m=0) &= (1-p) \\ \Pr(d=\theta_h) &= \frac{\eta}{2}p \\ \Pr(d=\mu) &= 0 \end{aligned}$$

we can compute

$$
\begin{aligned}
A_h^0 &= \upsilon \begin{pmatrix} \Pr(d=\emptyset, X=-1|q=1, d_m=0)\left(\frac{\eta}{2} - \Pr(\theta_h|\emptyset, X=-1, d_m=0)\right) \\ +\Pr(d=\emptyset, X=0|q=1, d_m=0)\left(\frac{\eta}{2} - \Pr(\theta_h|\emptyset, X=0, d_m=0)\right) \\ -\left(1-\frac{\eta}{2}\right)\Pr(d=\theta_h) + \frac{\eta}{2}\Pr(d=\mu) \end{pmatrix} \\
&= \upsilon\left(\left(1-\frac{\eta}{2}\right)p\left(\frac{\eta}{2}-0\right) + (1-p)\left(\frac{\eta}{2} - \frac{\frac{\eta}{2}(1-p)\upsilon}{(1-p)\upsilon + \left(1-\frac{\eta}{2}\right)p(1-\upsilon)}\right) - \left(1-\frac{\eta}{2}\right)\frac{\eta}{2}p\right) \\
&= \upsilon(1-p)\frac{\eta}{2}\frac{\left(1-\frac{\eta}{2}\right)p(1-\upsilon)}{(1-p)\upsilon + \left(1-\frac{\eta}{2}\right)p(1-\upsilon)}
\end{aligned}
$$

and

$$
\begin{aligned}
A_l^0 &= \upsilon \begin{pmatrix} \Pr(d=\emptyset, X=-1|q=1, d_m=0)\left(\Pr(\theta_l|\emptyset, X=-1, d_m=0) - \frac{\eta}{2}\right) \\ +\Pr(d=\emptyset, X=0|q=1, d_m=0)\left(\Pr(\theta_l|\emptyset, X=0, d_m=0) - \frac{\eta}{2}\right) \\ -\frac{\eta}{2}\Pr(d=\theta_h) - \frac{\eta}{2}\Pr(d=\mu) \end{pmatrix} \\
&= \upsilon\left(\left(1-\frac{\eta}{2}\right)p\left(\frac{\eta}{2-\eta} - \frac{\eta}{2}\right) + (1-p)\left(\frac{\frac{\eta}{2}((1-p)\upsilon + p(1-\upsilon))}{(1-p)\upsilon + \left(1-\frac{\eta}{2}\right)p(1-\upsilon)} - \frac{\eta}{2}\right) - \frac{\eta}{2}\frac{\eta}{2}p\right) \\
&= \upsilon(1-p)\frac{\eta}{2}\frac{\frac{\eta}{2}p(1-\upsilon)}{(1-p)\upsilon + \left(1-\frac{\eta}{2}\right)p(1-\upsilon)}
\end{aligned}
$$

Market maker's beliefs if the manager implements the expansive disclosure policy

Following the same logic as before, we can calculate

$$
\begin{aligned}
\Pr(\theta_h|\emptyset, X=0, d_m=1) &= \frac{\Pr(\theta_h, d=\emptyset, X=0)}{\Pr(d=\emptyset, X=0)} = \frac{\frac{\eta}{2}(1-p)\upsilon}{(1-p)\upsilon + \frac{\eta}{2}p(1-\upsilon)} < \frac{\eta}{2} \\
\Pr(\theta_l|\emptyset, X=0, d_m=1) &= \frac{\Pr(\theta_l, d=\emptyset, X=0)}{\Pr(d=\emptyset, X=0)} = \frac{\frac{\eta}{2}((1-p)\upsilon + p(1-\upsilon))}{(1-p)\upsilon + \frac{\eta}{2}p(1-\upsilon)} > \frac{\eta}{2}.
\end{aligned}
$$

If $X=-1$ and $d=\emptyset$. From $X=-1$, the market maker can infer that the trader did not purchase the security because $s_{withheld}=1$ and $s_{knowcash}=\mu$. Hence,

$$
\begin{aligned}
\Pr(\theta_h|\emptyset, X=-1, d_m=1) &= 0 \\
\Pr(\theta_l|\emptyset, X=-1, d_m=1) &= 1.
\end{aligned}
$$

Based on these beliefs and

$$
\begin{aligned}
\Pr(d=\emptyset, X=-1|q=1, d_m=1) &= \frac{\eta}{2}p \\
\Pr(d=\emptyset, X=0|q=1, d_m=1) &= (1-p) \\
\Pr(d=\theta_h) &= \frac{\eta}{2}p \\
\Pr(d=\mu) &= (1-\eta)p
\end{aligned}
$$

we can compute

$$
A_h^1 = \upsilon(1-p)\frac{\eta}{2}\frac{\frac{\eta}{2}p(1-\upsilon)}{(1-p)\upsilon + \frac{\eta}{2}p(1-\upsilon)}
$$

and

$$
A_l^1 = \upsilon(1-p)\frac{\eta}{2}\frac{\left(1-\frac{\eta}{2}\right)p(1-\upsilon)}{(1-p)\upsilon + \frac{\eta}{2}p(1-\upsilon)}. \blacksquare
$$

Proof of Proposition 5

The constraint set for the OSD0 Program was described in the text. The only other point to make about the constraint set is that, in all non-first best cases (i.e., when $1 + e - \mu > 0$), the investor's "supply financing" line (22) intersects the positive orthant in (Δ_m, Δ_h)-space.

The key to determining the smallest value attained by the objective function (17) in the constraint set is the observation that the (downward-sloping) iso-cost lines of the objective function (17) are flatter than the slope of the line (22), as the following lemma reports.

Lemma 2 $\frac{A_l^0}{A_h^0} < \frac{1-\eta/2-A_l^0}{\eta/2-A_h^0}$.

Proof. Rewrite A_l^0 and A_h^0 as $A_l^0 = \frac{\eta}{2}A^0$ and $A_h^0 = \left(1 - \frac{\eta}{2}\right)A^0$, where

$$A^0 = \upsilon\,(1-p)\,\frac{\eta}{2}\,\frac{p\,(1-\upsilon)}{(1-p)\,\upsilon + \left(1-\frac{\eta}{2}\right)p\,(1-\upsilon)}.$$

With this notation,

$$\frac{A_l^0}{A_h^0} = \frac{\frac{\eta}{2}A^0}{\left(1-\frac{\eta}{2}\right)A^0} = \frac{\frac{\eta}{2}}{1-\frac{\eta}{2}} \tag{40}$$

and and $\frac{1-\eta/2-A_l^0}{\eta/2-A_h^0} = \frac{1-\frac{\eta}{2}-\frac{\eta}{2}A^0}{\frac{\eta}{2}-\left(1-\frac{\eta}{2}\right)A^0}$. Then it is easy to confirm that the inequality in the statement of the lemma is equivalent to $0 < 1 - \eta$, which always holds. ■

Given this fact, it follows that the optimal security is defined by the "most southwest" point of the constraint set (18), (19), (20). We identify what that "most southwest" point is by studying the following four mutually exclusive and exhaustive cases (for $e > \mu - 1$).

- In case 1 ("low volatility"), the point on the line (22) lying on the Δ_m-axis, i.e., the point $(\Delta_m^0, \Delta_h^0) \equiv \left(\frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0}, 0\right)$, lies to the left of $(\Delta_m, \Delta_h) = (pe, 0)$.[42] See Figure 2. This case occurs when $\frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0} \leq pe$, i.e., when $e \leq \frac{\mu-1}{1-\left(1-\frac{\eta}{2}-A_l^0\right)p} \equiv \underline{e}_{ID}^0$. In this case, the "southwest-most" (Δ_m, Δ_h) in the feasible set is $(pe, 0)$. This corresponds to the security $(\psi_l, \Delta_m, \Delta_h) = (\mu - e, pe, 0)$. This security is investment-grade debt with face value $\mu - (1-p)e$.

- In case 2 ("medium volatility"), (Δ_m^0, Δ_h^0) lies to the right of $(\Delta_m, \Delta_h) = (pe, 0)$ but to the left of $(\Delta_m, \Delta_h) = (e, 0)$. See Figure 2. This case occurs when $e > \underline{e}_{ID}^0$ and $\frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0} \leq e$, i.e., when $\underline{e}_{ID}^0 < e \leq \frac{\mu-1}{\frac{\eta}{2}+A_l^0} \equiv \underline{e}_{SD}^0$. In this case, the "southwest-most" (Δ_m, Δ_h) in the feasible set is (Δ_m^0, Δ_h^0). This corresponds to the security $(\psi_l, \Delta_m, \Delta_h) = \left(\mu - e, \frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0}, 0\right)$. This security is investment-grade debt with face value $\mu - e + \frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^0}$.

- In case 3 ("high volatility"), (Δ_m^0, Δ_h^0) lies to the right of $(\Delta_m^0, \Delta_h) = (e, 0)$ and the point (Δ_m, Δ_h) on the line (22) evaluated at $\Delta_m = e$ lies inside the box $[0, e] \times [0, e]$. This case occurs when $e > \underline{e}_{SD}^0$ and $\frac{1+e-\mu-\left(1-\frac{\eta}{2}-A_l^0\right)e}{\left(\frac{\eta}{2}-A_h^0\right)} \leq e$, so $\underline{e}_{SD}^0 < e < \frac{\mu-1}{A_l^0+A_h^0} \equiv \bar{e}_{SD}^0$. See Figure 2. In this case, the "southwest-most" (Δ_m, Δ_h) in the feasible set is $\left(e, \frac{1+e-\mu-\left(1-\frac{\eta}{2}-A_l^0\right)e}{\frac{\eta}{2}-A_h^0}\right)$. This corresponds to the

[42] Also note that $(\Delta_\mu^0, \Delta_h^0)$ lies to the right of $(\Delta_\mu, \Delta_h) = (0, 0)$ whenever first-best is not attainable.

security $(\psi_l, \Delta_m, \Delta_h) = (\mu - e, e, \frac{1+e-\mu-\left(1-\frac{\eta}{2}-A_l^0\right)e}{\frac{\eta}{2}-A_h^0})$. This security is speculative debt with face value $\frac{\mu(\frac{\eta}{2}-A_h^0-1)+1+(\frac{\eta}{2}+A_l^0)e}{\frac{\eta}{2}-A_h^0}$.

- In case 4 ("extremely high volatility"), (Δ_m^0, Δ_h^0) lies to the right of $(\Delta_m, \Delta_h) = (e, 0)$ but the point (Δ_m, Δ_h) on the line (22) evaluated at $\Delta_m = e$ lies outside the box $[0, e] \times [0, e]$. This case occurs when $e \geq \bar{e}_{SD}^0$. In this case, no feasible security can implement the limited disclosure policy.

Substituting the values of the downside risk and upside potential of the securities that have been identified to be optimal in each of these four cases into the expression for the firm's cost of capital in Proposition 3 completes the proof of Proposition 5.■

Proof of Proposition 6

We proceed, as we did in the case of the limited disclosure policy above, to identify the solution to the OSD1 Program geometrically. The next lemma establishes that the iso-cost lines for the objective function (23) and the investor's "supply financing" line (27) are parallel.

Lemma 3 $\frac{A_l^1}{A_h^1} = \frac{1-\frac{\eta}{2}-A_l^1}{\frac{\eta}{2}-A_h^1}$.

Proof. We can rewrite A_l^1 and A_h^1 as $A_l^1 = \left(1 - \frac{\eta}{2}\right) A^1$ and $A_h^1 = \frac{\eta}{2} A^1$ where

$$A^1 = \upsilon (1-p) \frac{\eta}{2} \frac{p(1-\upsilon)}{(1-p)\upsilon + \frac{\eta}{2} p (1-\upsilon)}.$$

Then, the LHS of the statement in the lemma yields $\frac{\left(1-\frac{\eta}{2}\right)A^1}{\frac{\eta}{2}A^1} = \frac{1-\frac{\eta}{2}}{\frac{\eta}{2}}$ and the RHS yields $\frac{1-\frac{\eta}{2}-\left(1-\frac{\eta}{2}\right)A^1}{\frac{\eta}{2}-\frac{\eta}{2}A^1} = \frac{\left(1-\frac{\eta}{2}\right)\left(1-A^1\right)}{\frac{\eta}{2}(1-A^1)} = \frac{1-\frac{\eta}{2}}{\frac{\eta}{2}}$ which completes the proof. ■

Since the iso-cost lines of the objective function (23) are parallel to the line (27), any point (Δ_m, Δ_h) on the line (27) that satisfies (24) and (26) determines an optimal security. Specifically:

- if the point where the line (27) crosses the Δ_m-axis, $(\Delta_m^1, \Delta_h^1) \equiv (\frac{1+e-\mu}{\frac{\eta}{2}-A_h^1}, 0)$, is to the left of $(\Delta_m, \Delta_h) = (pe, 0)$, then the investment-grade debt security $(\psi_l, \Delta_m, \Delta_h) = (\psi_l, \Delta_m^1, \Delta_h^1)$ is an optimal security. See Figure 3. This case occurs when $\frac{1+e-\mu}{1-\frac{\eta}{2}-A_l^1} \leq pe$, i.e., when $e \leq \frac{\mu-1}{1-\left(1-\frac{\eta}{2}-A_l^1\right)p} \equiv \underline{e}_{ID}^1$.

- if (Δ_m^1, Δ_h^1) is to the right of $(\Delta_m, \Delta_h) = (pe, 0)$ and the line (27) intersects the trapezoid defined by (24) and (26), then no form of debt is optimal. See Figure 3. Instead, the optimal security is a hybrid security on the line (27) in the constraint set. This case occurs when $e > \underline{e}_{ID}^1$ and $\frac{1+e-\mu-\left(1-\frac{\eta}{2}-A_l^1\right)e}{\frac{\eta}{2}-A_h^1} \leq e$ (as the line (27) intersects the trapezoid defined by (24) and (26) only if the point where (27) crosses the vertical line $\Delta_m = e$ is below $\Delta_h = e$), i.e., when $\underline{e}_{ID}^1 < e < \frac{\mu-1}{A_l^1+A_h^1} \equiv \bar{e}_{ID}^1$.

- if $e \geq \bar{e}_{ID}^1$, then no security implements the expansive disclosure policy.

Having identified the optimal securities, it remains to evaluate the manager's objective function at the optimum. This task is simplified by the observation that the manager's maximum expected profits always can be determined by evaluating the firm's cost of capital at the point (Δ_m, Δ_h) the line (27) crosses the vertical axis $\Delta_m = 0$, i.e., at $(\Delta_m, \Delta_h) = (0, \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1})$, since whether or not $(0, \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1})$ satisfies all of the constraints (24), (25), (26), this point always lies on the southwest boundary of the constraint set (24), (25), (26).[43] So, for any security that optimally implements the expansive disclosure policy, the firm's cost of capital is $A_h^1 \frac{1+e-\mu}{\frac{\eta}{2}-A_h^1}$. This proves Proposition 6.■

Proof of Proposition 7

We start the proof of the proposition by proving the following Lemma.

Lemma 4 *For any $e > \underline{e}_{ID}^1$ for which there exists a feasible security – call it "security 1" – that implements the expansive disclosure policy, there exists a feasible debt security that implements the limited disclosure policy that increases the manager's expected profits relative to that obtained from security 1.*

Proof. Suppose $e > \underline{e}_{ID}^1$ and some security, say "security 1," eliciting expansive disclosure exists. Then, we are in case 3 of Proposition 6. As the proof of Proposition 6 shows, optimal securities in this case are hybrid securities determined by those securities on the line (27) that lie in the constraint set for Program OSD1. Single out the optimal security on this line that also lies on the boundary line

$$pe = \Delta_m - (1-p)\Delta_h \tag{41}$$

of the constraint set. Call this latter security "security 2," and denote its components by $(\psi_l^2, \Delta_m^2, \Delta_h^2)$. Security 2 necessarily generates at least as high expected profits for the manager as does "security 1," since security 2 is optimal for Program OSD1 and security 1 is merely feasible for this program. Recall by Proposition 1 that securities that lie on the line (41) can induce the manager to adopt either the limited or expansive disclosure policy. This is true, in particular, for security 2.

We now make two claims: security 2 is feasible for Program OSD0, and security 2 yields the manager higher expected profits under that program than under Program OSD1. This will prove that the manager is better off implementing the limited disclosure policy upon being given security 2 rather than the expansive disclosure policy. Notice that the firm's cost of capital when the manager implements the expansive disclosure policy with security 2 satisfies:

$$\begin{aligned} A_l^1\Delta_m + A_h^1\Delta_h &= A_l^1(pe + (1-p)\Delta_h) + A_h^1\Delta_h \\ &= (A_l^1 + A_h^1)\Delta_h + A_l^1 p(e - \Delta_h) \\ &> (A_l^0 + A_h^0)\Delta_h + A_l^0 p(e - \Delta_h) \\ &= A_l^0(pe + (1-p)\Delta_h) + A_h^0\Delta_h \\ &= A_l^0\Delta_m + A_h^0\Delta_h, \end{aligned} \tag{42}$$

[43] And so the value of the objective function at any feasible point will be the same as its value at this point.

where the inequality follows from Corollary 1 and the first and last equalities follow from security 2 lying on (41). Of course (42) is equivalent to:

$$\left(1 - \frac{\eta}{2} - A_l^1\right)\Delta_m + \left(\frac{\eta}{2} - A_h^1\right)\Delta_h \leq \left(1 - \frac{\eta}{2} - A_l^0\right)\Delta_m + \left(\frac{\eta}{2} - A_h^0\right)\Delta_h. \tag{43}$$

(43), along with the feasibility of $(\psi_l^2, \Delta_m^2, \Delta_h^2)$ for Program 0SD1 and the fact that $(\psi_l^2, \Delta_m^2, \Delta_h^2)$ lies on (41), implies the feasibility of $(\psi_l^2, \Delta_m^2, \Delta_h^2)$ for Program OSD0. (42) implies that the manager's expected profits are strictly higher by implementing the limited disclosure policy than the expansive disclosure policy upon being given security 2. This proves the two claims.

To conclude the proof, we recall that Proposition 5 established that debt securities always optimally implement the limited disclosure policy, so there is some debt security, say security 3, that is at least weakly preferable to security 2 that implements the limited disclosure policy. Since security 2 is weakly preferable to security 1, the proof of the lemma is complete. ■

Proof of Proposition 7

The proof of this proposition consists of the following three observations. First, the function defining the manager's maximum expected profits when he implements the limited disclosure policy is linear in e over the interval $e \in (\mu - 1, \underline{e}_{ID}^1]$; the same is also true of the function defining the manager's maximum expected profits when he implements the expansive disclosure policy (for this same interval $(\mu - 1, \underline{e}_{ID}^1]$).[44] Second, the manager's maximum expected profits are strictly higher under the expansive disclosure policy than under the limited disclosure policy at, and – by continuity – in the vicinity of, $e = \mu - 1$. (This follows from the discussion surrounding Proposition 4.) Third, the manager's maximum expected profits are strictly higher under the limited disclosure policy than under the expansive disclosure policy at, and – by continuity – in the vicinity of $e = \underline{e}_{ID}^1$. (This follows directly from Lemma 4 above.) Hence, these expected profit functions must cross exactly once in this interval $(\mu - 1, \underline{e}_{ID}^1]$, which defines the point e_S in the statement of the proposition. The conclusion about the global optimality of risk-free debt, investment-grade debt, or speculative debt then follows directly from Propositions 5 and 6.

Proof of Proposition 8

The idea behind the proof of this proposition is to find restrictions on both the size c of the cost the manger incurs in exerting the higher effort and the size e of the volatility of the firm's cash flows so that the securities and disclosure policies that were identified as being globally optimal in Proposition 7 in the non-moral hazard case are feasible in the moral hazard case. Once these conditions are identified, it will follow that these same securities and disclosure policies are globally optimal in the moral hazard case too, since: (a) as long as a security and disclosure policy are feasible in the moral hazard problem, the cost of capital associated with the security and disclosure policy does not change in moving from

[44]Because: (1) by Proposition 5, the manager's expected profit function when he adopts the limited disclosure policy is linear in e for e in the interval $(\mu - 1, \underline{e}_{ID}^0]$; (2) by Proposition 6, the manager's expected profit function is linear in e for e in the interval $(\mu - 1, \bar{e}_{ID}^1]$, and so, since $\underline{e}_{ID}^1 < \bar{e}_{ID}^1$, is a fortiori is linear over the interval $(\mu - 1, \underline{e}_{ID}^1]$; (3) $\underline{e}_{ID}^1 = \frac{\mu-1}{1-(1-\frac{\eta}{2}-A_l^1)p} < \frac{\mu-1}{1-(1-\frac{\eta}{2}-A_l^0)p} = \underline{e}_{ID}^0$, because $A_l^0 < A_l^1$, by Corollary 1 above; and so (4) both of these expected profit functions are linear in e for e in the interval $(\mu - 1, \underline{e}_{ID}^1]$.

the non-moral hazard case to the moral hazard case, and (b) the set of feasible securities in the moral hazard case certainly does not expand (and may shrink) relative to the set of feasible securities in the non-moral hazard case.

We start by noting that the set of securities that satisfy the incentive compatibility constraint (30) in (Δ_m, Δ_h)-space is the half-space below the strictly decreasing "boundary line" defined by (30). For future reference, we label this line "IC" and we call the set of securities that satisfy the incentive compatibility constraint as the securities "southwest" of IC.

Now, when the expansive disclosure policy is globally optimal in the non-moral hazard case, Proposition 7 proved that the associated globally optimal security is either risk-free or investment-grade debt. When the globally optimal security is risk-free debt, the IC constraint is satisfied if $c \leq e$. So, as long as $c \leq e \leq \mu - 1$, risk-free debt is feasible for the moral hazard problem.

When the expansive disclosure policy is globally optimal in the non-moral hazard case and the globally optimal security is investment-grade debt, the proof of Proposition 6 shows that the investment grade debt is of the form $(\Delta_m, \Delta_h) = (\Delta_m, 0)$ for some $\Delta_m \leq pe$. Now observe that if $(\Delta_m, \Delta_h) = (e, 0)$ lies southwest of IC, then all securities of the form $(\Delta_m, 0)$ for $\Delta_m \leq pe$ also lie southwest of IC and thus satisfy the incentive compatibility constraint. So, if this condition ($(e, 0)$ lies southwest of IC) holds, the globally optimal investment-grade debt in the non-moral hazard case is feasible in the moral hazard case. Since $(e, 0)$ is southwest of IC (by definition of IC) if and only if $\left(1 - \frac{\eta}{2}\right) e \leq e - c$, i.e., if and only if $c \leq \frac{\eta}{2} e$ and hence, since $e > \mu - 1$ when investment-grade debt is globally optimal, if and only if $c \leq \frac{\eta}{2}(\mu - 1)$.

When limited disclosure is globally optimal in the non-moral hazard case, Proposition 7 showed that the associated globally optimal security is either investment-grade debt or speculative debt. The proof of Proposition 5 showed that when the globally optimal security is investment-grade debt, it is of the form $(\Delta_m, \Delta_h) = (\Delta_m, 0)$ for some $\Delta_m \in (pe, e)$. In this case, we can repeat the argument used above concerning the expansive disclosure policy verbatim: if $(\Delta_m, \Delta_h) = (e, 0)$ lies southwest of IC, then all globally optimal investment-grade debt securities $(\Delta_m, 0)$ with $\Delta_m \in (pe, e)$ also lie southwest of IC. If $c \leq \frac{\eta}{2}(\mu - 1)$, then whenever investment-grade debt is globally optimal under limited disclosure in the non-moral hazard case, then this same investment-grade debt is feasible in the moral hazard case.

When limited disclosure is globally optimal in the non-moral hazard case and the associated globally optimal security is speculative debt, we claim that if any security is feasible in the moral hazard case, then this speculative debt security is feasible. To see this, first note that the slope of IC, $-\frac{1-\eta/2}{\eta/2}$, is strictly greater than the slope, $-\frac{1-\eta/2-A_l^0}{\eta/2-A_h^0}$, of the line (22) defining the investor's supply financing constraint (as: some simple algebra shows that $-\frac{1-\eta/2}{\eta/2} > -\frac{1-\eta/2-A_l^0}{\eta/2-A_h^0}$ if and only if

$$\frac{A_l^0}{A_h^0} < \frac{1 - \eta/2}{\eta/2}, \tag{44}$$

and this last inequality holds because, by (40) above, $\frac{A_l^0}{A_h^0} = \frac{\eta/2}{1-\eta/2}$ and $\frac{\eta/2}{1-\eta/2} < 1$ whenever $\eta < 1$). Hence, if the half-space of securities satisfying the investor supply financing constraint and the half-space of securities satisfying the incentive compatibility constraint overlap anywhere in the square $[0, e] \times [0, e]$ in (Δ_m, Δ_h)-space, they overlap somewhere along the vertical line $\Delta_m = e$ in (Δ_m, Δ_h)-space. Figure 4 illustrates the optimal security that induces limited disclosure in the presence of moral hazard by superimposing the incentive compatibility constraint on top of the constraints that determined the set of feasible securities in the nonmoral hazard sections of the paper above in the case where $c \approx 0$.

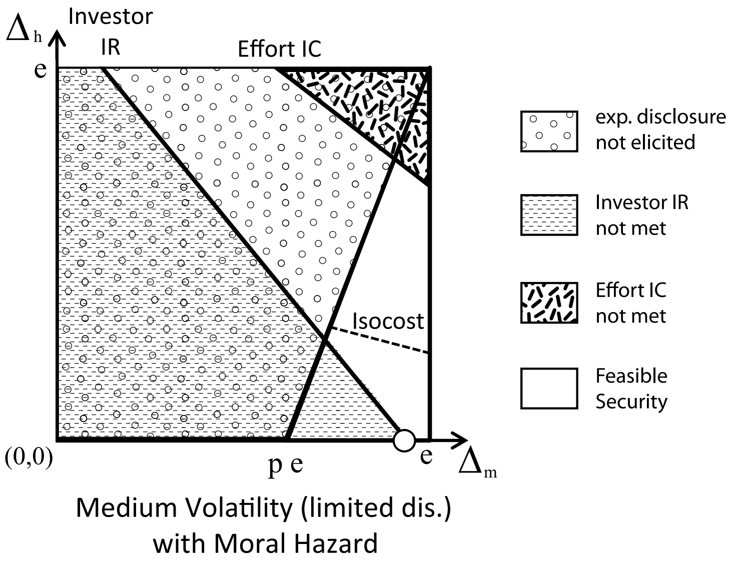

Figure 4: Security Design under Moral Hazard

It remains to determine when speculative debt is feasible for the moral hazard problem. Proposition 5(4) tells us that the face value of optimal speculative debt is

$$\frac{\mu(\frac{\eta}{2} - A_h^0 - 1) + 1 + (\frac{\eta}{2} + A_l^0)e}{\frac{\eta}{2} - A_h^0}, \tag{45}$$

when speculative debt is optimal in the non-moral problem. As speculative debt with face value (45) can be written as $\psi_l + \Delta_m + \Delta_h = (\mu - e) + e + \frac{1-\mu+(\frac{\eta}{2}+A_l^0)e}{\frac{\eta}{2}-A_h^0}$, the incentive compatibility constraint for this speculative debt is given by:

$$\left(1 - \frac{\eta}{2}\right) e + \frac{\eta}{2} \frac{1 - \mu + (\frac{\eta}{2} + A_l^0)e}{\frac{\eta}{2} - A_h^0} \leq e - c,$$

which is equivalent to

$$e \leq \frac{\mu - 1 - c\left(1 - \frac{A_h^0}{\eta/2}\right)}{A_h^0 + A_l^0} \equiv \hat{e}_{SD}^0.$$

So, for cash flow volatilities $e \leq \hat{e}_{SD}^0$, speculative debt with face value (45) is optimal for the moral hazard problem. It is easy to check that $\hat{e}_{SD}^0 \in (\underline{e}_{SD}^0, \bar{e}_{SD}^0)$ when $c \leq \frac{\eta}{2}(\mu - 1)$.[45] This completes the proof of the proposition.

[45] It is clear that

$$\frac{\mu - 1 - c\left(1 - \frac{A_h^0}{\eta/2}\right)}{A_h^0 + A_l^0} \equiv \hat{e}_{SD}^0 < \bar{e}_{SD}^0 = \frac{\mu - 1}{A_h^0 + A_l^0},$$

since the denominators of both sides of the inequality are the same, and the numerator of the LHS is clearly smaller, since $\frac{\eta}{2} > A_h^0$.

Proof of Proposition 9

Disclosure of θ_m is preferred over non-disclosure of θ_m, when the following condition is satisfied:

$$\theta_m - \psi_m \geq E[\widetilde{\theta} - \widetilde{\psi} | d = \emptyset, d(\theta_m) = \emptyset]$$

where RHS of the inequality is given by

$$E[\widetilde{\theta} - \widetilde{\psi} | d = \emptyset, d(\theta_m) = \emptyset] = \frac{p(1-\eta)(\theta_m - \psi_m) + (1-p)\left(\theta_m - \left(\frac{\eta}{2}\psi_h + (1-\eta)\psi_m + \frac{\eta}{2}\psi_l\right)\right)}{p(1-\eta) + (1-p)}.$$

Simplification yields

$$\begin{aligned}
\theta_m - \psi_m &\geq E[\widetilde{\theta} - \widetilde{\psi} | d = \emptyset, d(\theta_m) = \emptyset] \\
\theta_m - \psi_m &\geq \theta_m - \frac{\psi_h + \psi_l}{2} \\
\frac{\psi_h + \psi_l}{2} &\geq \psi_m \\
\Delta_h &\geq \Delta_m
\end{aligned}$$

The case for which non-disclosure of θ_m is preferred over disclosure of θ_m is identical.■

Proof of Proposition 10

Suppose the manager discloses both the low and high state but not the intermediate state. Then, the trader prefers to buy the security when $s_{withheld} = 0$ if and only if his expected purchasing price, $vP(\emptyset, 0) + (1-v)P(\emptyset, 1)$, is less than the expected payoff of the security, $E[\widetilde{\psi}]$. $P(\emptyset, 1)$ equals $E[\widetilde{\psi}]$ (because the market maker can infer that $s_{withheld} = 0$) and $P(\emptyset, 0)$ is a weighted average of $E[\widetilde{\psi}]$ and ψ_m. Hence, $y(\emptyset, 0) = 1$ is optimal if $E[\widetilde{\psi}] > \psi_m$ or $\Delta_h > \Delta_m$. This condition is incompatible with the manager choosing not to disclose θ_m. Hence, the trader does not buy the security when $s_{withheld} = 0$. The trader prefers to buy the security when $s_{withheld} = 1$ if and only if his expected purchasing price, $vP(\emptyset, 0) + (1-v)P(\emptyset, 1)$, is less than the payoff of the security, ψ_m. $P(\emptyset, 1)$ equals ψ_m (because the market maker can infer that $s_{withheld} = 1$) and $P(\emptyset, 0)$ is a weighted average of $E[\widetilde{\psi}]$ and ψ_m. Hence, $y(\emptyset, 1) = 1$ is optimal if $\psi_m > E[\widetilde{\psi}]$ or $\Delta_h < \Delta_m$. This condition is consistent with the manager choosing not to disclose θ_m. Hence, the trader buys the security when $s_{withheld} = 1$.■

The proof that $\frac{\mu-1}{\frac{\eta}{2}+A_l^0} \equiv \underline{e}_{SD}^0 < \hat{e}_{SD}^0$ is proven through the following sequence of equivalent inequalities:

$$\begin{aligned}
\frac{\mu - 1 - c\left(1 - \frac{A_h^0}{\eta/2}\right)}{A_h^0 + A_l^0} &> \frac{\mu - 1}{\frac{\eta}{2} + A_l^0} \\
-c\left(1 - \frac{A_h^0}{\eta/2}\right)\left(\frac{\eta}{2} + A_l^0\right) &> (\mu - 1)\left(A_h^0 + A_l^0 - \frac{\eta}{2} - A_l^0\right) \\
-\frac{c}{\eta/2}\left(\frac{\eta}{2} - A_h^0\right)\left(\frac{\eta}{2} + A_l^0\right) &> -(\mu - 1)\left(\frac{\eta}{2} - A_h^0\right) \\
\frac{c}{\eta/2}\left(\frac{\eta}{2} + A_l^0\right) &< (\mu - 1) \\
c &< \frac{\eta}{2}(\mu - 1)\frac{1}{\frac{\eta}{2} + A_l^0}
\end{aligned}$$

the last inequality is true, since $\frac{\eta}{2} + A_l^0 < 1$ and by assumption $c \leq \frac{\eta}{2}(\mu - 1)$.

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