Capture Effects of Wireless CSMA/CA Protocols in Rayleigh and Shadow Fading Channels

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***Abstract*— We investigate the capture effects for a wireless local area network (LAN) system in the presence of multipath, shadowing, and near–far effects. The performance of wireless media access control (MAC) protocols with Rayleigh fading, shadowing, and capture effect are analyzed. We consider carrier-sense multiple-access/collision-avoidance (CSMA/CA) protocols as the wireless MAC protocols, since CSMA/CA protocols are based on the standard for wireless LAN's IEEE 802.11. We analyze and compare the channel throughput and packet delay for three types of CSMA/CA protocols: basic CSMA/CA, stop-and-wait automatic repeat request (SW ARQ) CSMA/CA, and four-way handshake (4-WH) CSMA/CA. We calculate the capture probability of an access point (AP) in a channel with Rayleigh fading, shadowing, and near–far effects, and we derive the throughput and packet delay for the various protocols. We have found that the performance of CSMA/CA in a radio channel model is 50% less than in an error-free channel model in low-traffic load, while the throughput and packet delay of CSMA/CA in a radio channel model show better performance than in an error-free channel model in high-traffic load. We also found that the 4-WH CSMA/CA protocol is superior to the other CSMA/CA protocols in high-traffic load.**

***Index Terms*— Capture effect, CSMA/CA, MAC protocol, Rayleigh fading, wireless LAN.**

I. Introduction

MORE AND MORE stations connect to wireless local area networks (LAN's) and demand for various wireless services, which support data, voice, and video, has rapidly increased. The costs for installation and relocation for cable LAN's have increased. However, wireless LAN's offer many advantages in installation, maintenance, and relocation from the viewpoint of cost and efficiency. Wireless LAN manufacturers currently offer a number of nonstandardized products based on conventional radio modem technology, spread-spectrum technology in industrial, scientific, and medical (ISM) bands, and infrared technology [1]. Since 1990, the IEEE Project 802.11 Committee has worked to establish a universal standard for wireless LAN protocol for interoperability between competing products [2], and the European Telecommunications Standards Institute (ETSI) set up an *ad hoc* group to investigate radio LAN's in 1991 [3]. One of the important research issues in wireless LAN's is the design and analysis of medium access control (MAC) protocols. In this paper, we consider a carrier-sense multiple-access/collision-avoidance (CSMA/CA) protocol, which is a basic mechanism of the IEEE 802.11 MAC protocol, and analyze the performance of CSMA/CA protocols by using a mathematical method based on renewal theory.

MAC protocols for wireless communications have been widely studied. There are some analytical studies for CSMA/CA protocols and some simulation studies [4], [5]. However, Chen [6], assumes that CSMA/CA is a nonpersistent CSMA, and Chhaya [7] calculates the throughput of CSMA/CA with a simple model. Other studies do not present analytical approaches. There are also many studies for ALOHA family protocols in a fading channel and with shadowing [8]–[11]. However, the characteristics of the CSMA/CA cannot be described by ALOHA protocols and has not yet been analyzed in a fading channel model. In this paper, we present an exact analytical approach for the channel throughput and the normalized packet delay of CSMA/CA protocols in Rayleigh fading, shadowing, and with the near–far effect. We consider a centralized wireless LAN configuration and focus on the performance of an access point (AP) in a wireless LAN. We analyze the performances of three types of CSMA/CA protocols and compare the throughput and normalized packet delay with each other.

This paper is organized as follows. In Section II, the propagation model and system model for CSMA/CA protocols is described. The throughput of three types of CSMA/CA is analyzed in Section III, and in Section IV, packet delay is calculated. In Section V, some numerical results are reported. Finally, we give concluding remarks in Section VI.

II. System Descriptions

A. Propagation Model

We focus on the performance of AP in the infrastructure networks. We consider that the AP is located in the center of the infrastructure configuration, and the other terminals are distributed in the basic service area (BSA) with the given spatial distribution density function. The radio channel can be characterized statistically by three independent multiplicative propagation mechanisms, namely, multipath fading, shadowing, and groundwave propagation [12]. The groundwave propagation gives rise to the near–far effect and determines the area-mean power w_a, which means the received power averages over an area. Therefore, the normalized area-mean power received from a wireless terminal at a distance r_i from

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the AP is taken to have the form

$$w_a = r_i^{-\xi} \tag{1}$$

with the exponent ξ typically taking values of three–four. We assume that shadowing is superimposed on the near–far effect. This fluctuation is described by a lognormal distribution of the local-mean power w_L about the area-mean power w_a with logarithmic standard deviation σ_s. We also assume that power control is not used and that Rayleigh fading is an accurate characterization of the link fading process. While Rician fading is also of interest, it is a much more difficult type of fading process to treat analytically. See [12] and [13] for a more detailed indoor propagation model. Thus, the instantaneous received power w_s of a signal from a wireless terminal is exponentially distributed about the local-mean power w_L. Taking into account Rayleigh fading, lognormal shadowing, and near–far effects, the unconditional probability density function (pdf) of the instantaneous power w_s of a received packet is [9], [12]

$$f_{w_s}(w_s) = \int_0^\infty \int_0^\infty \frac{1}{w_L} \exp\left(-\frac{w_s}{w_L}\right) \frac{f(r_i)}{\sqrt{2}\sigma_s w_L} \cdot \exp\left\{-\frac{\ln^2 (r_i^\xi w_L)}{2\sigma_s}\right\} dr_i\, dw_L \tag{2}$$

where $f(r_i)$ is the pdf of the propagation distance describing the spatial distribution. We consider the uniform spatial distribution in which we assume that the wireless terminals are uniformly distributed in a circle of unit radius around the AP. In this case, the pdf of the propagation distance is given by $f(r_i) = 2r_i,\ r_i \in (0, 1)$ [12], [14].

B. Power Capture Model

The test packet can be received successfully—that is, it captures the receiver in the presence of other overlapping or interfering packets—if its instantaneous power is larger than the instantaneous joint interference power by a minimum certain threshold factor z. This effect is the capture effect, and the threshold factor is called the capture ratio [15], [16]. As is often done in the literature on the subject, instantaneous power is assumed to remain approximately constant for the time interval of packet duration. To use a convenient method of analyzing capture probabilities, we consider the weight function approach for the Rayleigh fading channels [9] based on Laplace transforms [17].

We consider that a wireless network consists of M terminals $(s_1, s_2, \cdots, s_i, \cdots, s_M)$. We define $\mathcal{R}_n$ as the set of terminals, which means n terminals transmit a packet at the same time. The s_1 denotes a receiver which wants to receive the packet from a certain transmitter $s_i (s_i \in \mathcal{R}_n)$. If the s_1 receives the packet successfully from s_i, the instantaneous signal power w_s should exceed the joint interference signal power w_L from $n-1$ terminals $(\mathcal{R}_n - \{s_i\})$ by the capture ratio z. However, the s_1 does not receive the packet successfully from s_i if only w_s is captured from w_L, since the w_s also includes the joint interference signal with multipath fading, shadowing, and near–far effect by itself. Let w_f be the joint interference signal for only s_i and w_0 denote the desired signal power of a packet, then w_0 and w_f are included in the w_s.

In order to find the capture probability, denote $q(n|z)$, for n colliding packets. We first consider that the $q(s_1|z)$ denotes the probability of capture for a packet from the s_i with a distance of r_i. Given the local mean power (w_f), it can be expressed as

$$q(s_1|z) = \int_0^\infty \int_0^\infty \frac{f(r_i)}{\sqrt{2}\sigma_s w_f} \exp\left\{-\frac{\ln^2 (r_i^\xi w_f)}{\sqrt{2}\sigma_s}\right\} \cdot \left\{\phi_{w_f}\left(\frac{z}{w_f}\right)\right\} dr_i\, dw_f. \tag{3}$$

Here, $\phi(*)$ is the one-sided Laplace image of the pdf of the instantaneous joint interference power w_f, defined as

$$\phi_{w_f}(s) \triangleq \int_0^\infty \exp(-sx) f_{w_f}(x\, dx). \tag{4}$$

Using (2), $\phi(*)$ can be expressed as

$$\phi_{w_f}(s) = \int_0^\infty \int_0^\infty \frac{1}{1+sw_f} \frac{f(r)}{\sqrt{2}\sigma_s w_f} \cdot \exp\left\{-\frac{\ln^2 (r^\xi w_f)}{\sqrt{2}\sigma_s}\right\} dr\, dw_f. \tag{5}$$

Furthermore, if the interference received power w_L is due to incoherent accumulation of n independent fading signals for n wireless terminals, the joint pdf of the received power is the n-fold convolution of the pdf of the individual signal power [8], [15]. Thus, the probability of capture, given that n terminals transmit packets at the same time, can be expressed as

$$q(n|z) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f(r_1) f(s_1|r_1, z) \cdot \phi_{w_f}\left(\frac{z}{w_f}\right) f(r_2) f(S_n|r_2, z) \cdot \left\{\phi_{w_L}\left(\frac{z}{w_L}\right)\right\}^{n-1} dw_f\, dw_L\, dr_1\, dr_2 \tag{6}$$

where

$$f(s_1|r_1, z) \triangleq \frac{1}{\sqrt{2\pi}\sigma_s w_f} \exp\left\{-\frac{\ln^2 (r_1^\xi w_f)}{\sqrt{2}\sigma_s}\right\}$$

$$f(S_n|r_2, z) \triangleq \frac{1}{\sqrt{2\pi}\sigma_s w_L} \exp\left\{-\frac{\ln^2 (r_2^\xi w_L)}{\sqrt{2}\sigma_s}\right\}$$

$$f(r_1) = \begin{cases} 2r_1, & \text{if } 0 < r_1 < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$f(r_2) = \begin{cases} 2r_2, & \text{if } 0 < r_2 < 1 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where the $f(*)$ is the pdf of the random distance of a transmitting terminal from the receiving terminal, and this is assumed by uniform spatial distribution. Note that $f(s_1|r_1, z)$ and $f(S_n|r_2, z)$ are statistically independent. Finally, the capture probability, conditional on the number of n interferers, is the three-fold integral form as

$$
\begin{aligned}
q(n|z) = &\frac{2}{\sqrt{\pi}} \int_0^1 \int_{-\infty}^{\infty} r_1 \exp(-x_1^2) \\
&\cdot \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x_1, y_1) \exp(-y_1^2) \right] dx_1\, dr_1 \\
&\cdot \frac{2}{\sqrt{\pi}} \int_0^1 \int_{-\infty}^{\infty} r_2 \exp(-x_2^2) \\
&\cdot \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x_2, y_2) \exp(-y_2^2) \right]^{(n-1)} dx_2\, dr_2
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
f(x_1, y_1) \triangleq &\left[\sqrt{z} r_1^2 \exp\left\{ \frac{\sqrt{2}}{2} \sigma_s (y_1 - x_1) \right\} \right] \\
&\cdot \arctan\left[\frac{1}{z r_1^4} \exp\left\{ \frac{\sqrt{2}}{2} \sigma_s (x_1 - y_1) \right\} \right] \\
f(x_2, y_2) \triangleq &\left[\sqrt{z} r_2^2 \exp\left\{ \frac{\sqrt{2}}{2} \sigma_s (y_2 - x_2) \right\} \right] \\
&\cdot \arctan\left[\frac{1}{z r_2^4} \exp\left\{ \frac{\sqrt{2}}{2} \sigma_s (x_2 - y_2) \right\} \right].
\end{aligned} \tag{9}
$$

The $q(n|z)$ can be obtained using the Hermite polynomial methods [17]. The probability q_n that one out of n packets captures the AP is found from

$$q_n = n q(n|z). \tag{10}$$

C. CSMA/CA Protocol Model

The IEEE 802.11 MAC protocol supports coexisting asynchronous and time-bounded services using different priority levels with different interframe space (IFS) delay controls. Three kinds of IFS are used to support three backoff priorities such as a short IFS (SIFS), a point coordination function IFS (PIFS), and distributed coordination function IFS (DIFS) [2]. Wireless packet transmission suffers from "the hidden terminal effect," so IEEE 802.11 MAC protocol provides alternative ways of packet transmission flow control. We consider three types of CSMA/CA according to the packet transmission flow control in this paper. First, an actual data packet is used only for a packet transmission. This is called basic CSMA/CA. Second, immediate positive acknowledgments are employed to confirm the successful reception of each packet. We call this scheme stop-and-wait automatic repeat request (SW ARQ) CSMA/CA. The last is four-way handshake (4-WH) CSMA/CA which uses request to send (RTS) and clear to send (CTS) packets prior to the transmission of the actual data packet.

We assume that the time is slotted with slot size a (propagation delay/packet transmission time). To analyze better the exact throughput of the CSMA/CA, we use a finite population (M terminals). A terminal generates a new packet with probability g, which includes new arrival and rescheduled

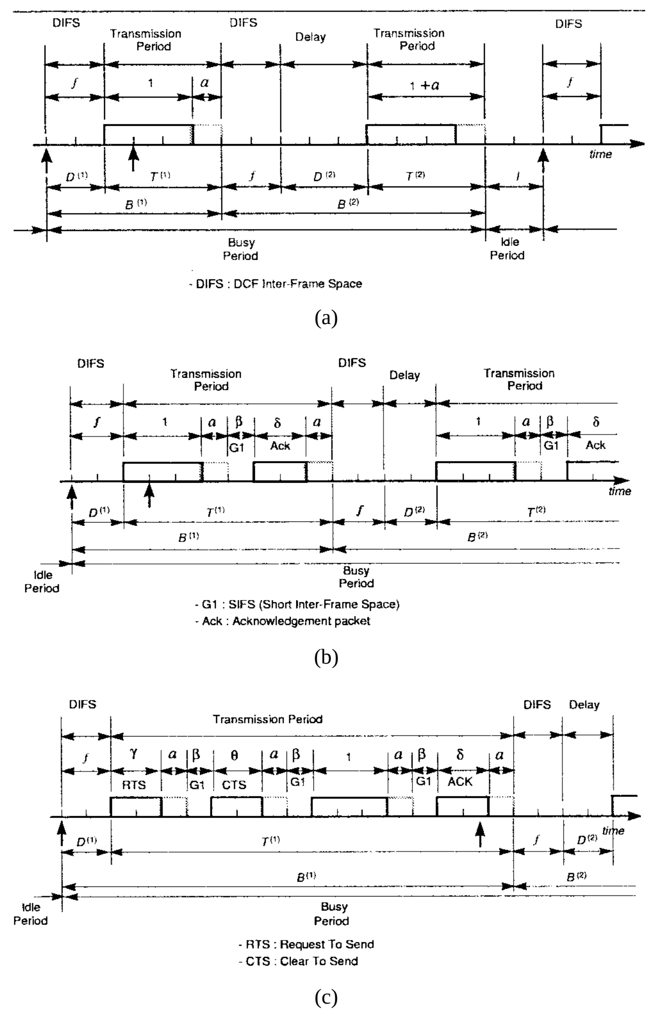


Fig. 1. Channel model in the three types of CSMA/CA. (a) Basic CSMA/CA, (b) SW ARQ CSMA/CA, and (c) 4-WH CSMA/CA.

packets during a slot. We assume that each ready station starts a packet transmission with probability p, and this p is related to the backoff delay in the IEEE 802.11 standard. The duration of the packet transmission period is assumed to be fixed to unit of time one. We consider the CSMA/CA as a hybrid protocol of slotted one-persistent CSMA and p-persistent CSMA. We assume that a channel state consists of a sequence of regeneration cycles composed of idle (I) and busy periods (B). Let U be the time spent in useful transmission during a regeneration cycle and S be the channel throughput. The throughput S can be obtained by the above three terms, and the normalized packet delay is also calculated using the throughput.

III. Throughput Analysis

A. Basic CSMA/CA

In the following, we consider the basic CSMA/CA protocol and calculate the expected value of the idle period, the busy period and the useful transmission period. The throughput of CSMA/CA is then derived. In CSMA/CA, channel states are illustrated in Fig. 1(a). Let us introduce notations which define the channel states. In Fig. 1(a), the busy period is divided into several subbusy periods such that the jth subbusy period, which is denoted by $B^{(j)}$, is composed of a transmission delay (denoted by $D^{(j)}$) and transmission time (denoted by $T^{(j)}$).

In the subbusy period $B^{(1)}$, $D^{(1)}$ is a DIFS delay. However, $D^{(j)}$ is a stochastic random variable, if $j \geq 2$. $B^{(j)}$ is composed of a DIFS delay, $D^{(j)}$ and $T^{(j)}$. The DIFS delay is assumed to have l slots, and the size of DIFS is $f\ (= l \times a)$. In the case of the basic CSMA/CA model, transmission period $T^{(j)}$ is fixed at $1+a$, whether the transmission is successful or not. Let J be the number of subbusy periods in a busy period. The busy period B and the useful transmission period U are simply given by

$$B = \sum_{j=1}^{J} B^{(j)} \quad U = \sum_{j=1}^{J} U^{(j)}. \tag{11}$$

Next, we have to find the number of subbusy periods in a busy period. In CSMA/CA every terminal transmits a pending packet after it detects the free medium for greater than or equal to a DIFS. Therefore, the busy period continues in the event that a packet is generated during the last transmission period as well as the last DIFS delay. Let TP be the sum of the last transmission period and the last DIFS delay. Then TP is $1+a+f$ in the basic CSMA/CA model. Since J is geometrically distributed, the distribution and the expectation of J are

$$\begin{aligned} \Pr\,[J=j] &= [1-(1-g)^{(TP/a)M}]^{j-1}\cdot(1-g)^{(TP/a)M} \\ \overline{J} &= \frac{1}{(1-g)^{(TP/a)M}}, \qquad j=1,2,\cdots. \end{aligned} \tag{12}$$

$B^{(1)}$ occurs when one or more packets arrive in the last slot of the idle period, and $B^{(2)}$ occurs when one or more packets arrive in $T^{(1)}$. Since the length of $B^{(j)} (j \geq 3)$ is independent of $B^{(2)}$ and is identically distributed, the expectation of $B^{(j)} (j \geq 2)$ is $(\overline{J}-1) \times E[B^{(2)}]$. In the same manner, we get $U^{(j)}$. Thus, the expectation of a busy period and useful transmission time is given by

$$\begin{aligned} \overline{B} &= E[B^{(1)}] + (\overline{J}-1)E[B^{(2)}] \\ \overline{U} &= E[U^{(1)}] + (\overline{J}-1)E[U^{(2)}]. \end{aligned} \tag{13}$$

Since the idle period is geometrically distributed, distribution and expectation of the duration for an idle period are given by

$$\begin{aligned} \Pr\,[I=ka] &= (1-g)^{M(k-1)}\cdot[1-(1-g)^{M}] \\ \overline{I} &= \frac{a}{[1-(1-g)^{M}]}, \qquad k=1,2,\cdots. \end{aligned} \tag{14}$$

To find $E[D^{(j)}]$ and $E[U^{(j)}]$, let $P_n(X)$ be the probability that n packets arrive among M users during X slots. $P_n(X)$ is expressed as

$$P_n(X) = \frac{\binom{M}{n}[1-(1-g)^{X/a}]^n(1-g)^{X(M-n)/a}}{1-(1-g)^{XM/a}}, \quad n=1,2,\cdots,M. \tag{15}$$

Furthermore, let $N_0^{(j)}$ be the number of packets accumulated at the end of a transmission period. Then the distribution of $N_0^{(j)}$ is expressed as

$$\Pr\,[N_0^{(j)}=n] = P_n(TP), \qquad j=2,3,\cdots. \tag{16}$$

In order to find the distribution of $D^{(j)}$ when $N_0^{(j)}=n$ and $j \geq 2$, we consider k, $k=0,1,2,\cdots$, to be the number of slot boundaries as $D^{(j)}$ is greater than or equal to k slots in the following cases: n terminals, which are already scheduled to transmit a packet, do not transmit a packet, and $(M-n)$ empty terminals generate no packets during k slots. Thus, we have

$$\Pr[D^{(j)} \geq ka | N_0^{(j)}=n] = (1-p)^{kn}(1-g)^{k(M-n)}. \tag{17}$$

We can derive the expectation of $D^{(j)}$, given that $N_0^{(j)}=n$, by unconditioning on $N_0^{(j)}$ in (17). The expectation of $D^{(j)} (j \geq 2)$ can then be calculated

$$E[D^{(j)}] = \begin{cases} f[1-(1-g)^M], & j=1 \\ \dfrac{a}{1-(1-g)^{(TP/a)M}}\left(\displaystyle\sum_{k=1}^{\infty}\{(1-p)^k - (1-g)^{(TP/a)}[(1-p)^k-(1-g)^k]\}^M \cdot (1-g)^{(TP/a)M}\sum_{k=1}^{\infty}(1-g)^{kM}\right), & j=2,3,\cdots. \end{cases} \tag{18}$$

Using (13), (14), and (18), we obtain the sum of the expectations of the busy and the idle period as

$$\begin{aligned} \overline{B}+\overline{I} = {} & f[1-(1-g)^M] + 1 + a + \frac{1}{(1-g)^{(TP/a)M}} \\ & \cdot \Bigg((f+1+a)[1-(1-g)^{(TP/a)M}] \\ & + a\sum_{k=1}^{\infty}\{(1-p)^k - (1-g)^{(TP/a)}[(1-p)^k \\ & -(1-g)^k]\}^M - a(1-g)^{(TP/a)M}\sum_{k=1}^{\infty}(1-g)^{kM}\Bigg) \\ & + \frac{a}{1-(1-g)^M}. \end{aligned} \tag{19}$$

We now calculate the expected value of useful transmission time $E[U^{(j)}]$. In order to calculate $E[U^{(j)}]$, we consider the condition when $N_0(j)=n$ and $D^{(j)} \geq ka$. Then we have

$$E[U^{(j)} | D^{(j)} \geq ka, N_0^{(j)}=n] = \begin{cases} \displaystyle\sum_{i=1}^{n}\binom{M}{n}p^i(1-p)^{n-i}iq(i-1|z), & k=0 \\ \displaystyle\sum_{i=1}^{n}\binom{M}{n}p^i(1-p)^{n-i}\sum_{l=1}^{n}\binom{M-n}{l} \cdot g^l(1-g)^{M-n-l}(i+l)q(i+l-1|z), & k>0 \end{cases} \tag{20}$$

where $i, i \in \{1,2,\cdots,n\}$ is the number of backlogged terminals which will transmit a packet with probability p, and l is the number of terminals which generate the a packet with probability g. Using the conditional expectation in (20), we can obtain the mean successful transmission period. Since $U^{(1)}$ is the useful transmission time when one or more packets arrive during the last slot of the previous idle period, it is equal to $P_1(1)$ in (15). Thus, we have

$$\begin{aligned}\overline{U} &= E[U^{(1)}] + (\overline{J}-1)E[U^{(2)}]\\ &= \frac{1}{1-(1-g)^M}\sum_{i=1}^{M}\binom{M-n}{l}[1-(1-g)]^i\\ &\quad\cdot(1-g)^{M-i} iq(i|z) + \left(\frac{1}{(1-g)^{(TP/a)M}}\right)\\ &\quad\cdot\sum_{n=1}^{M}\left[\left\{\sum_{i=1}^{n}\binom{n}{i}p^i(1-p)^{n-i}\right.\right.\\ &\quad\left.\cdot\sum_{l=1}^{n}\binom{M-n}{l}g^l(1-g)^{M-n-l}(i+l)q(i+l|z)\right\}\\ &\quad\cdot\left(\frac{(1-p)^n(1-g)^{M-n}}{1-(1-p)^n(1-g)^{M-n}}\right)\\ &\quad\left.+\sum_{i=1}^{n}\binom{n}{i}p^i(1-p)^{n-i}iq(i|z)\right]\\ &\quad\cdot\left\{\frac{\binom{M}{n}[1-(1-g)^{(TP/a)}]^n(1-g)^{(TP/a)(M-i)}}{1-(1-g)^{(TP/a)M}}\right\}.\end{aligned} \tag{21}$$

Dividing (21) by (19), we obtain the throughput of a slotted basic CSMA/CA system composed of M identical users, each user having the geometric arrival rate g, slot time a, and DIFS delay f.

B. Stop-and-Wait ARQ CSMA/CA

In the following, we consider the SW ARQ CSMA/CA protocol and calculate its throughput. For SW ARQ CSMA/CA, whose channel states are illustrated in Fig. 1(b), β is the normalized time of SIFS, and δ is the normalized time of an ACK packet. Here the parameters and assumptions are the same as in the basic CSMA/CA protocol except that the successful transmission period (TP_S) is given by $1+\beta+\delta+2a+f$. Note that (TP_S) includes the DIFS delay since packets, generated in the period of the last DIFS delay, have to wait for the channel to become idle. When a packet transmission is unsuccessful, the ACK packet transmission period is omitted, and the unsuccessful transmission period (TP_F) is $1+a+f$. Let TP denote the duration of the jth transmission period in the busy period, then the $(j+1)$th transmission period depends only on TP. This is why the success of the $(j+1)$th transmission is determined by the number of arrivals during the jth transmission period. Hence, given a transmission period (TP), the length of the remainder of the busy period is a function of TP, and its average period is denoted by $B(TP)$. Similarly the average useful transmission period in the remainder of the busy period is denoted by $U(TP)$

$$\begin{aligned}B(TP) &= d(TP) + \{(TP_S + [1-(1-g)^{(TP_S/a)}]\\ &\quad\cdot B(TP_S)\}u(TP)\\ &\quad+\{TP_F + [1-(1-g)^{(TP_F/a)}]B(TP_F)\}\\ &\quad\cdot[1-u(TP)]\\ U(TP) &= \{1+[1-(1-g)^{(TP_S/a)}]U(TP_S)\}u(TP)\\ &\quad+\{[1-(1-g)^{(TP_F/a)}]U(TP_F)\}\\ &\quad\cdot[1-u(TP)]\end{aligned} \tag{22}$$

where

$$\begin{aligned}d(1) &= f[1-(1-g)^M]\\ d(TP) &= \frac{a}{1-(1-g)^{(TP/a)M}}\left(\sum_{k=1}^{\infty}\{(1-p)^k\right.\\ &\quad-(1-g)^{(TP/a)}[(1-p)^k-(1-g)^k]\}^M\\ &\quad\left.\cdot(1-g)^{(TP/a)M}\sum_{k=1}^{\infty}(1-g)^{kM}\right)\\ u(1) &= \frac{1}{1-(1-g)^M}\sum_{i=1}^{M}\binom{M-n}{l}[1-(1-g)]^i\\ &\quad\cdot(1-g)^{M-i}iq(i|z)\\ u(TP) &= \sum_{n=1}^{M}\left[\left\{\sum_{i=1}^{n}\binom{n}{i}p^i(1-p)^{n-i}\sum_{l=1}^{n}\binom{M-n}{l}\right.\right.\\ &\quad\left.\cdot g^l(1-g)^{M-n-l}(i+l)q(i+l|z)\right\}\\ &\quad\cdot\left(\frac{(1-p)^n(1-g)^{M-n}}{1-(1-p)^n(1-g)^{M-n}}\right)\\ &\quad\left.+\sum_{i=1}^{n}\binom{n}{i}p^i(1-p)^{n-i}iq(i|z)\right]\\ &\quad\cdot\left\{\frac{\binom{M}{n}[1-(1-g)^{(TP/a)}]^n(1-g)^{(TP/a)(M-i)}}{1-(1-g)^{(TP/a)M}}\right\}\end{aligned} \tag{23}$$

where $d(TP)$ and $u(TP)$ are derived from (18) and (21), respectively. If $j \geq 2$, we have to consider that TP is a case of both TP_S and TP_F. Since a busy period is induced by the first slot before it starts, we get

$$\overline{B} = B(1) \quad \overline{U} = U(1). \tag{24}$$

Since the duration of a successful transmission is different from that of an unsuccessful transmission, $B(TP_S)$, $B(TP_F)$, $U(TP_S)$, and $U(TP_F)$ should be calculated respectively. Substituting TP by TP_S and TP_F in (17), we obtain two easily solved equations with the two unknowns $B(TP_S)$ and $B(TP_F)$. The average length of an idle period is the same as in (14). Thus, we find the throughput of SW ARQ CSMA/CA

$$S = \frac{U(1)}{B(1) + \frac{a}{[1-(1-g)^M]}}. \tag{25}$$

C. Four-Way Handshake CSMA/CA

We now proceed to calculate the throughput of the 4-WH CSMA/CA protocol. Since a packet transmission is not absolutely reliable in wireless communication environments, IEEE 802.11 provides four-way handshaking with a CSMA/CA mechanism. The carrier sense mechanism is achieved by distributing medium busy reservation information through an exchange of special small RTS and CTS frames prior to the actual data frame. If a collision occurs during the RTS packet transmission period, the packet transmission is terminated immediately and a new packet transmission is started.

We assume that packet transmission of RTS and CTS are normalized, respectively. The channel model for slotted 4-WH CSMA/CA is shown in Fig. 1(c). If the RTS packet transmission is successful, the transmission period $[T^{(j)}]$ is composed of an RTS packet transmission period (γ), CTS packet transmission period (θ), data packet transmission period (1), ACK packet transmission period (δ), 3 SIFS (3β), and four-propagation delay $(4a)$. We denote TP_{4S} as the sum of the successful transmission period and DIFS delay. Therefore, TP_{4S} is $1+\gamma+\theta+\delta+3\beta+4a+f$. In an unsuccessful case, $T^{(j)}$ is the sum of the RTS packet transmission period and an SIFS. Let TP_{4F} be the sum of the last unsuccessful transmission period and DIFS, then TP_{4F} is $\gamma+a+f$. In order to calculate the throughput of 4-WH CSMA/CA, we modify the analysis in the previous section. Substituting TP_S and TP_F with TP_{4S} and TP_{4F}, respectively, we can easily obtain $B(TP)$ and $U(TP)$. Using (22), (24), and (25) and calculating recursive forms of $B(TP_{4S})(U(TP_{4S}))$ and $B(TP_{4F})(U(TP_{4F}))$, we can obtain $B(1)$ and $U(1)$. Then we can derive the throughput of 4-WH CSMA/CA.

IV. Delay Analysis

A. Basic CSMA/CA

In a packet transmission network, the performance is usually represented by channel throughput and packet delay. We denote the expected packet delay L to be the average time between the generation and successful reception of a packet. We use, as an approximation, the average number of retransmissions for a packet. Although this approximation is known to become inaccurate if collisions occur very frequently and repeatedly involve signals from the same terminals [8]–[11], our model is believed to be reasonably accurate for the range of traffic loads that we are interested in.

In order to calculate the packet delay, we use offered traffic G and throughput S. We use the average number of retransmissions for a packet $(G/S-1)$. We now introduce the average delay R which is the time elapsed from the moment that a terminal starts sensing the channel to the moment that terminal accesses the channel. This is one of the following three cases: 1) a packet arrives and senses the channel to be in an idle period; 2) a packet arrives and senses the channel to be in a delay period D; and 3) a packet arrives and senses the channel to be in a transmission period. In the case of 1), an arbitrary packet has arrived and will find the channel idle with probability $\overline{I}/(\overline{I}+\overline{B})$. The average delay is DIFS. In the case of 2), a packet has arrived and will find the channel in the delay period with probability $\overline{D}/(\overline{I}+\overline{B})$. In this case, the average delay is also the DIFS. In the last case, a packet has arrived and will find the channel transmitting another packet with probability $(\overline{B}-\overline{D})/(\overline{I}+\overline{B})$. In this case, the packet waits for the channel to become idle and delays its transmission by the backoff algorithm. The average delay can be calculated by a residual life period in renewal theory [18]. Let T be the packet transmission period, and T is $(1+a)$ in the basic CSMA/CA model. So we can get the average delay R as

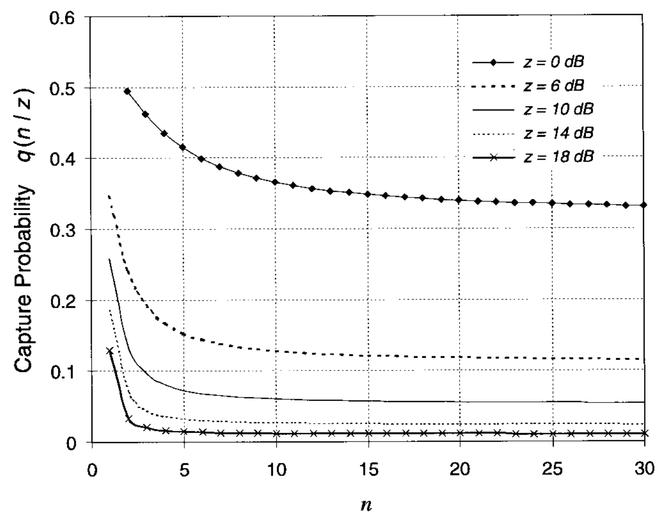


Fig. 2. Capture probability for the number of colliding terminals ($\sigma_s = 6$ dB and $\xi = 4$).

$$\overline{R} = \frac{\overline{I}}{\overline{B}+\overline{I}}f + \frac{\overline{D}}{\overline{B}+\overline{I}}f + \frac{\overline{B}-\overline{D}}{\overline{B}+\overline{I}} \cdot \left[\frac{\overline{(T+f+E[D^{(2)}])^2}}{\overline{2(T+f+E[D^{(2)}])}}\right]. \tag{26}$$

In (26), we can obtain $E[D^{(2)}]$ using (18) and calculate

$$\overline{D} = E[D^{(1)}] + (\overline{J}-1)E[D^{(2)}]. \tag{27}$$

We can obtain the normalized average packet delay by

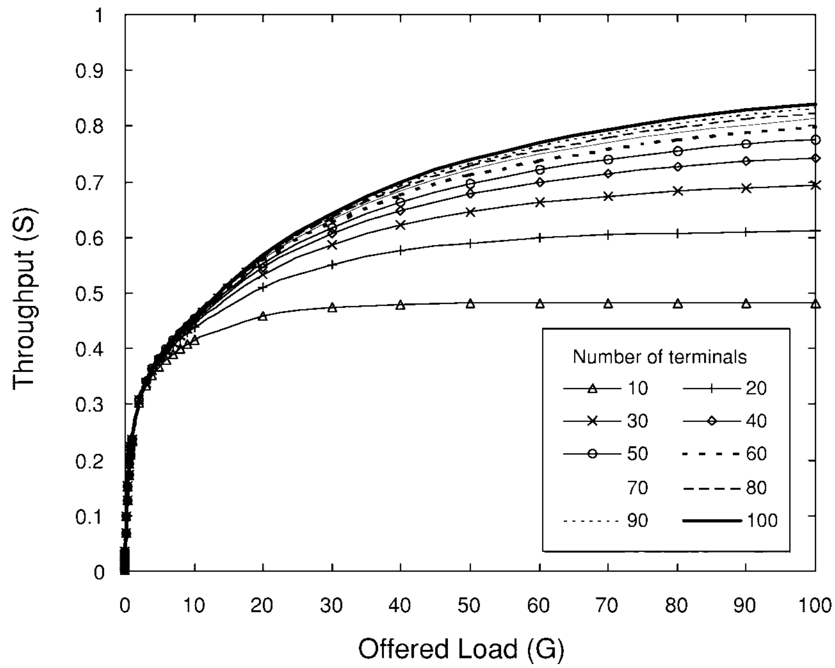
$$L = \left(\frac{G}{S}-1\right)[T+\overline{Y}+\overline{R}] + T + \overline{R} \tag{28}$$

where Y denotes random delay for a collided packet that waits for Y before sensing the channel.

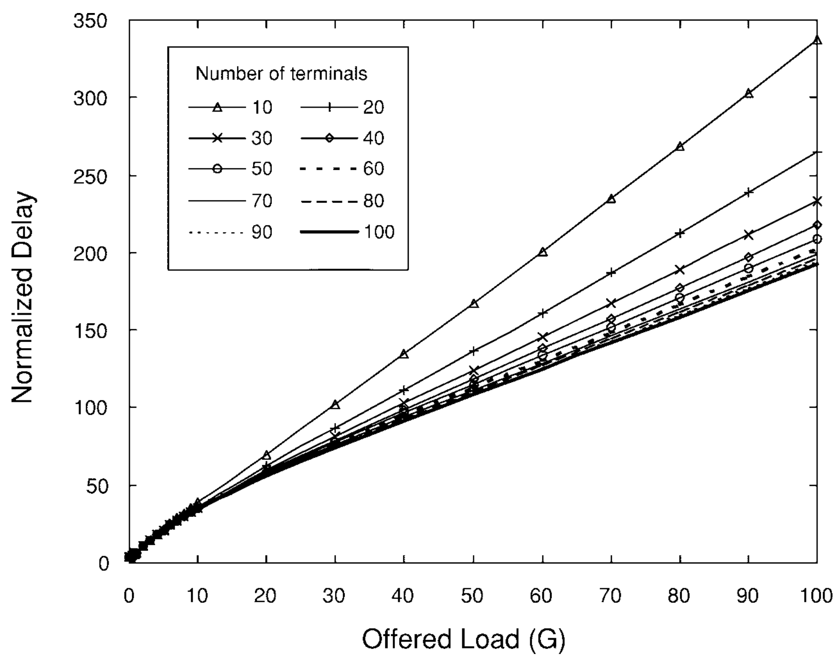
B. Stop-and-Wait ARQ CSMA/CA

As in the case of basic CSMA/CA, we calculate the average delay for the interval of successive transmissions by

$$\overline{R} = \frac{\overline{I}}{\overline{B}+\overline{I}}f + \frac{\overline{D}}{\overline{B}+\overline{I}}f + \frac{\overline{B}-\overline{D}}{\overline{B}+\overline{I}} \cdot \left\{P_{\text{Succ}}\left[\frac{\overline{(T_S+f+d(TP_S))^2}}{\overline{2(T_S+f+d(TP_S))}}\right] + P_{\text{Fail}}\left[\frac{\overline{(T_F+f+d(TP_F))^2}}{\overline{2(T_F+f+d(TP_F))}}\right]\right\} \tag{29}$$



(a)



(b)

Fig. 3. Throughput and packet delay versus offered load of basic CSMA/CA for varying of the number of terminals ($a = 0.01, p = 0.03, f = 0.06, Y = 0.06, \sigma_s = 6$ dB, $\xi = 4$, and $z = 4$). (a) Throughput versus offered load. (b) Normalized packet delay versus offered load.

where TP_S is the sum of the last successful transmission period with DIFS equal to $1+\beta+\delta+2a+f$ and TP_F is the sum of the last unsuccessful transmission period with DIFS equal to $1+a+f$. T_S is the successful transmission period $(1+\beta+\delta+2a)$, and T_F is the unsuccessful transmission period $(1+a)$. P_{Succ} denotes the probability of a successful packet transmission (G/S), and P_{Fail} is $1-P_{\text{Succ}}$. Other notations are the same as those in Section IV-A, but $\overline{D}$ has to be calculated differently. $\overline{D}$ can be obtain by $D(1)$ as follows:

$$\begin{aligned} D(1) = f &+ \{d(TP_S) + [1-(1-g)^{(TP_S/a)}]D(TP_S)\} \\ &\cdot u(1) \\ &+ \{d(TP_F) + [1-(1-g)^{(TP_F/a)}]D(TP_F)\} \\ &\cdot [1-u(1)] \end{aligned} \tag{30}$$

where $d(TP_S)$ and $d(TP_F)$ can be obtained, substituting TP with TP_S and TP_F in (23). $D(TP_S)$ and $D(TP_F)$

can be calculated by substituting one with TP_S and TP_F, respectively.

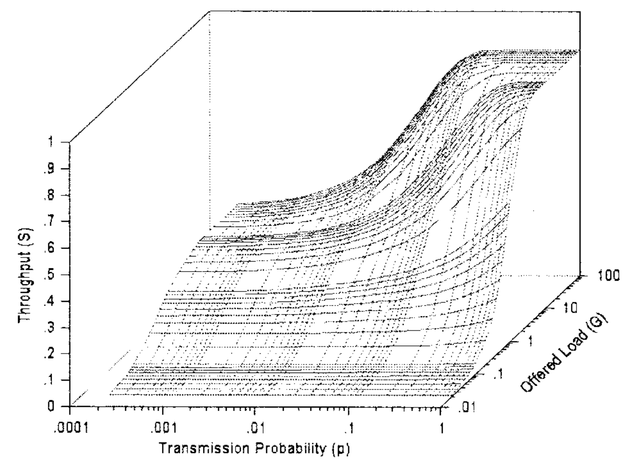
Since the backoff delay is determined by the previous transmission period, we have to calculate the backoff delay in both the cases of a successful and an unsuccessful transmission periods. Then normalized delay L in SW ARQ CSMA/CA is obtained easily by substituting the former T by T_F and the latter T by T_S in (28). In the case of the infinite population model, we can obtain the normalized delay by using a method similar to that used in calculating throughput.

C. Four-Way Handshake CSMA/CA

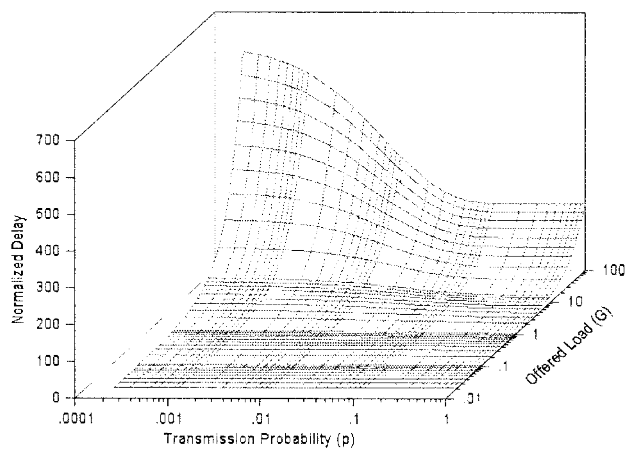
In the 4-WH CSMA/CA protocol, the packet transmission period is different from that of SW ARQ CSMA/CA. Since we have assumed that TP_{4S} is $1+\gamma+\theta+\delta+3\beta+4a+f$, TP_{4F} is $\gamma+a+f$, T_{4S} is $1+\gamma+\theta+\delta+3\beta+4a$, and T_{4F} is $\gamma+a$, we calculate the average delay for the interval of a successive transmission $(\overline{R})$ by

$$\begin{aligned}\overline{R} = &\frac{\overline{I}}{\overline{B}+\overline{I}}f + \frac{\overline{D}}{\overline{B}+\overline{I}}f + \frac{\overline{B}-\overline{D}}{\overline{B}+\overline{I}} \\ &\cdot \left\{ P_{\text{Succ}}\left[\frac{\overline{(T_{4S}+f+d(TP_{4S}))^2}}{2\overline{(T_{4S}+f+d(TP_{4S}))}}\right] \right. \\ &\left. + P_{\text{Fail}}\left[\frac{\overline{(T_{4F}+f+d(TP_{4F}))^2}}{2\overline{(T_{4F}+f+d(TP_{4F}))}}\right]\right\} \qquad (31)\end{aligned}$$

where P_{Succ} denotes the probability that a packet transmission is successful (G/S) and P_{Fail} is $1-P_{\text{Succ}}$ as in Section IV-B. $\overline{D}$ has to be calculated in a manner similar to that of SW ARQ CSMA/CA. In the case of the 4-WH CSMA/CA model, $D(1)$ is a recursive form as in (30), by substituting TP_S with TP_{4S} and TP_F with TP_{4F}. Then the normalized delay L in 4-WH CSMA/CA can be easily obtained by substituting the former T by T_{4F} and the latter T by T_{4S} in (28).



(a)



(b)

Fig. 4. Throughput and packet delay versus transmission probability p of SW ARQ CSMA/CA for varying of the offered load ($a = 0.01$, $f = 0.06$, $\delta = 0.06$, $\beta = 0.03$, $Y = 0.06$, $\sigma_s = 6$ dB, $\xi = 4$, $z = 4$, and $M = 50$). (a) Throughput versus p. (b) Normalized packet delay versus p.

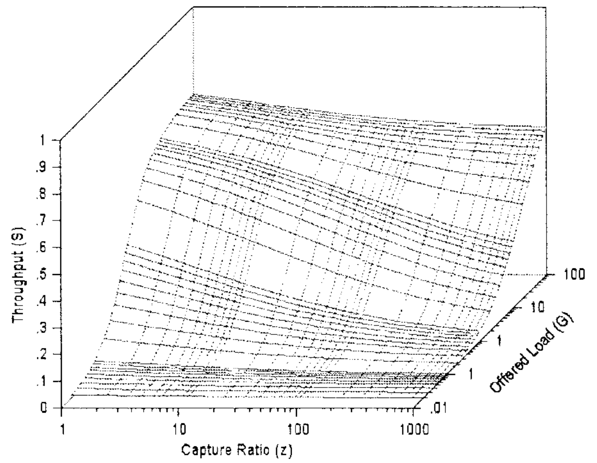
V. Numerical Results

Based on the analysis presented in the previous sections, several numerical results are shown, and the performances of three types of CSMA/CA are compared in this section. Fig. 2 plots the capture probability for the number of terminals, when the capture ratio z is varied. The capture probabilities are decreased exponentially when the number of colliding terminals are in the range of one–ten. However, if the number of colliding terminals is increased above ten, the capture probabilities converge to a finite limit. The nonzero limit of the capture probability is due to the unrealistic traffic assumed in the immediate vicinity of the AP, and the similar results are in the literature [8], [10], [12]. It also can be seen that the capture probabilities decrease exponentially as the capture ratio z increases.

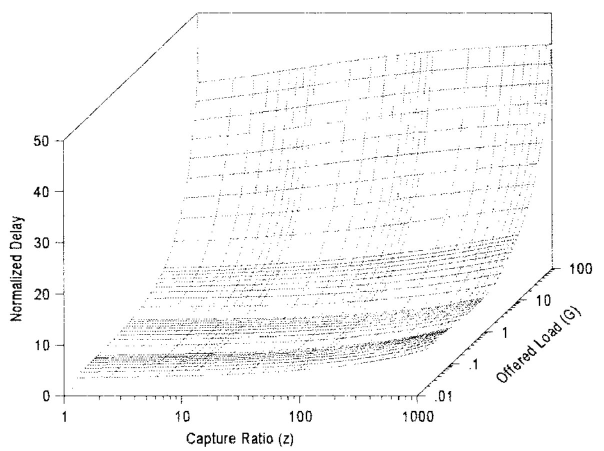
Fig. 3 shows the effect of the offered load G on the throughput and the normalized delay for basic CSMA/CA when the number of terminals varies. Note that as the number of terminals increases, the throughput does not decrease but becomes saturates asymptotically in Fig. 3(a). In the case of Fig. 3(b), the normalized packet delay is decreased as the number of terminals is increased, while it is linearly increased as the offered load is increased. To investigate the performance of SW ARQ CSMA/CA under varying transmission probability p, the throughput and normalized packet delay are represented in Fig. 4. We note that the performance of SW ARQ CSMA/CA is not degraded as the transmission probability increases.

In usual cases of p-persistent CSMA, the performance is degraded when the transmission probability is increased above a specific value [19], [20]. The performance of CSMA/CA in the error-free channel model also shows similar drift as the transmission probability p is varied [21]. Increasing p enhances the performance, due to the beneficial effect of power capture. When p is increased, the probability of packet collision is increased, while the chance to capture a packet is increased. This is why the capture probability converges to a finite limit.

Fig. 5 reports throughput and packet delay versus capture ratio z and offered load G. In Fig. 5(a), we note that throughput decreases more rapidly in low-offered traffic while the throughput decreases less rapidly in high-offered load. This means that the power capture is more effective in high-traffic

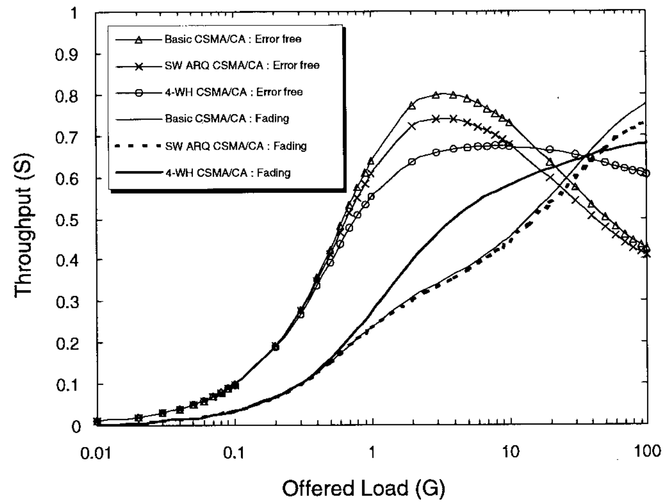


(a)

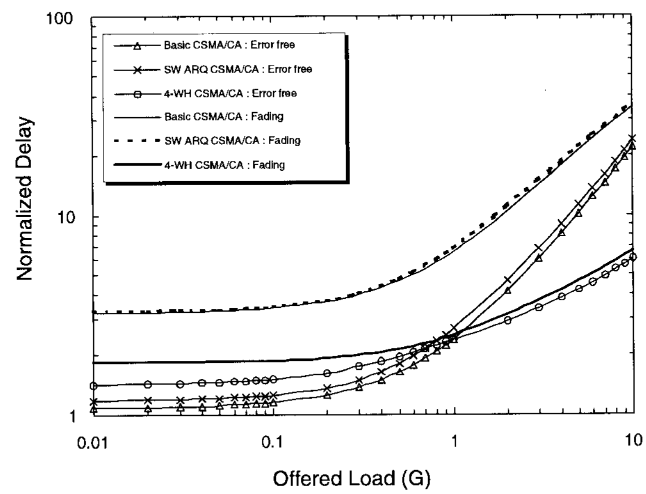


(b)

Fig. 5. Throughput and packet delay versus capture ratio z of 4-WH CSMA/CA for varying of the offered load ($a = 0.01$, $p = 0.03$, $f = 0.06$, $\delta = 0.06$, $\beta = 0.03$, $\theta = 0.06$, $\gamma = 0.1$, $Y = 0.06$, $\sigma_s = 6$ dB, $\xi = 4$, and $M = 20$). (a) Throughput versus z. (b) Normalized packet delay versus z.



(a)



(b)

Fig. 6. Performance comparison of three types of CSMA/CA protocols for error channel model and fading channel model ($a = 0.01$, $p = 0.03$, $f = 0.06$, $\delta = 0.06$, $\beta = 0.03$, $\theta = 0.06$, $\gamma = 0.1$, $Y = 0.06$, $\sigma_s = 6$ dB, $\xi = 4$, and $M = 50$). (a) Throughput comparison. (b) Normalized packet delay comparison.

loads. In the case of Fig. 5(b), the packet delay increases linearly with respect to the capture ratio z. The performance comparison of three types of CSMA/CA is represented in Fig. 6. Note that curves with polygons indicate the analytical results in the error-free channel model [21]. They can be obtained by substituting the capture ratio with zero in the presented equations, when the number of colliding terminals is above two. In the case of the error-free channel model, the basic CSMA/CA shows better performance than that of other two CSMA/CA protocols in low-traffic load, while the 4-WH CSMA/CA is superior to others in high-traffic load. In the case of the fading, shadowing, and power capture models, the performance of the 4-WH CSMA/CA is always better than that of the other two protocols. Moreover, we note that the performance of CSMA/CA in the fading channel model is worse than that in the error-free channel model when the traffic is low. This is the reason why the performance is sensitive to the multipath and shadowing environment. However, the throughput of CSMA/CA in the fading channel model increases continuously with the increase of the offered load, while the throughput in the error-free channel model decreases when the traffic increased above a specific point. This is due to the capture effect. Finally, we note that the 4-WH CSMA/CA protocol is more appropriate than the basic CSMA/CA or the SW ARQ CSMA/CA in practical wireless communication environments.

VI. Conclusions

We have investigated the capture effects for wireless LAN system in the presence of multipath, shadowing, and near–far effects. We have analyzed the performance of CSMA/CA protocols with power capture, operating on a channel impaired by Rayleigh fading, lognormal shadowing, and the near–far effect. We have considered three types of CSMA/CA protocols, including basic, SW ARQ, and 4-WH CSMA/CA, and have analyzed their throughput and packet delay.

To analyze the performance of CSMA/CA, we have considered capture probability in fading and shadowing channels. We have found that capture probability converges to a finite limit as the number of colliding terminals is increased. Furthermore, we have developed a new analytical approximation for the performance of CSMA/CA protocols with Rayleigh fading, lognormal shadowing, and power capture effect. As a result of our analysis, we have found that the throughput of CSMA/CA protocols does not decreased as the number of terminals and the offered load increases. We have also found that the performance of CSMA/CA is enhanced as transmission probability p increases and is sensitive to the capture ratio z. Extensive numerical results have been presented showing that 4-WH CSMA/CA protocol is a more attractive protocol than the other two types of CSMA/CA in practical wireless communication environments.

The main contributions of this paper are threefold: 1) the development of an analytical approach for evaluating the performance of CSMA/CA protocols in the fading and shadowing channel; 2) the performance comparison of three types of CSMA/CA protocols; and 3) the performance comparison of CSMA/CA in an error-free channel model and a fading channel.

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