Open access • Proceedings Article • DOI:10.23919/EUSIPCO.2019.8903163

Cardiac Motion Estimation Using Convolutional Sparse Coding — Source link

Nelson Diaz, Adrian Basarab, Jean-Yves Tourneret, Henry Arguello Fuentes

Institutions: Industrial University of Santander, University of Toulouse

Published on: 02 Sep 2019 - European Signal Processing Conference

Topics: Motion estimation, Sparse approximation and Neural coding

Cardiac Motion Estimation Using Convolutional Sparse Coding

Nelson Diaz[1], Adrian Basarab[2], Jean-Yves Tourneret[3], and Henry Arguello Fuentes[4]

[1]Department of Electrical Engineering
Universidad Industrial de Santander, 680002 Bucaramanga, Colombia
Email: nelson.diaz@correo.uis.edu.co

[2]University of Toulouse, IRIT, CNRS UMR 5505, F-31062 Toulouse, France
Email: adrian.basarab@irit.fr

[3]University of Toulouse, IRIT/INP-ENSEEIHT/TéSA, 31071 Toulouse, France
Email: jean-yves.tourneret@enseeiht.fr

[4]Department Computer Science
Universidad Industrial de Santander, 680002 Bucaramanga, Colombia
Email: henarfu@uis.edu.co

***Abstract*—This paper studies a new motion estimation method based on convolutional sparse coding. The motion estimation problem is formulated as the minimization of a cost function composed of a data fidelity term, a spatial smoothness constraint, and a regularization based on convolution sparse coding. We study the potential interest of using a convolutional dictionary instead of a standard dictionary using specific examples. Moreover, the proposed method is evaluated in terms of motion estimation accuracy and compared with state-of-the-art algorithms, showing its interest for cardiac motion estimation.**

***Index Terms*—Ultrasound imaging, cardiac motion estimation, Convolutional dictionary, sparse representation**

I. Introduction

Ultrasound imaging (UI) is a high temporal resolution imaging modality used in many clinical applications such as echocardiography due to its low cost, non-ionization characteristics, and comfort for the patient. An active research area in UI is tissue motion estimation. In particular, the problem of cardiac motion estimation in UI has been addressed with different approaches based on block-matching [1], the monogenic signal [2], and B-splines [3]. More recently, Ouzir et al. have introduced in [4] an energy minimization problem to estimate the motion of the heart. The proposed energy was a linear combination of a data fidelity term, a smoothness term and a regularization constructed from a sparse decomposition in a dictionary of motions determined using a standard patch-based dictionary learning method. The resulting motion estimation algorithm showed better results than classical methods such as block-matching, the monogenic signal, and B-splines.

This paper investigates a new motion estimation method based on convolutional dictionary learning (CDL). The motivation of this study is to estimate the heart motion by using consecutive frames of ultrasound images as inputs to build a convolutional dictionary [5]. Particularly, the k^{th} cardiac motion image $\mathbf{s}_k$ is modeled as a convolution between the coefficient maps $\mathbf{x}_{m,k}$ and a set of M filters $\mathbf{d}_m$. The coefficient maps indicate where the filters are activated, and the filters are supposed to model specific structures contained in the images of interest. A particular example is displayed in Fig. 1, where Fig. 1(a) displays one frame of the heart motion, Fig. 1(b) shows the estimated filters for the image and Fig. 1(c) shows the map of the cardiac motions associated with the red patch of the image. Note that the convolutional dictionary of Fig. 1(b) was obtained using $M = 32$ filters of size $L \times L$ with $L = 8$. In Fig. 1(c), the cardiac motions of the red patch are written as the linear combination of 10 filters convolved with a respective set of coefficient maps. Note that only 10 filters are required to represent this patch and that the 22 remaining filters are inactive, i.e., with zero coefficients. To improve the quality of the visualization, only the non-zero values of the coefficient maps have been shown. The key advantage of using a convolutional sparse model is its translation-invariant property which may offer a better representation in comparison with standard dictionary learning strategies. Indeed, each patch of the image can be sparsely represented with the proposed model by a single shift-invariant local dictionary [6].

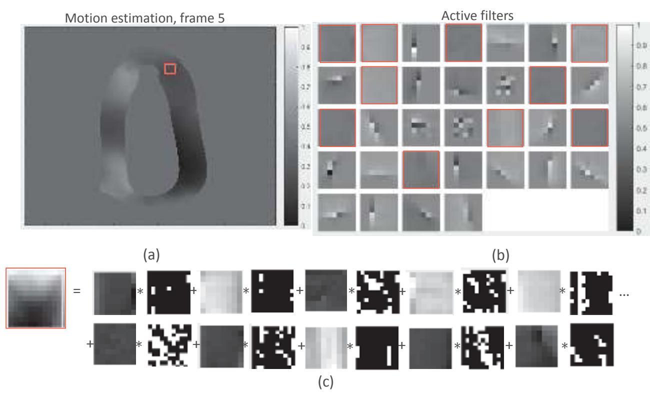


Fig. 1. Example of a sparse representation of a cardiac motion patch using a convolutional dictionary. The filter size is $L \times L$ with $L = 8$ and the number of filters is $M = 32$.

This paper is organized as follows. Section II introduces the motion estimation problem along with the proposed solution based on sparse regularization. Section III summarizes some key elements of CDL and its applications to cardiac motion estimation. Section IV evaluates the performance of the proposed motion estimation method highlighting the interest of CDL for UI. Conclusions and future work are reported in Section V.

II. Motion estimation

The 2D motions for a pair of successive frames $(\mathbf{r}_k, \mathbf{r}_{k+1}) \in \mathbb{R}^{J\times N}$ acquired at time instants k and $k+1$ are denoted as $(\mathbf{s}_{k,x}, \mathbf{s}_{k,y}) \in \mathbb{R}^{J\times N}$ where $\mathbf{s}_{k,x}$ and $\mathbf{s}_{k,y}$ are the motions along the x and y axes. Since the motion estimation problem is considered independently the displacement vector is equal to $\mathbf{s}_k = \mathbf{s}_{k,x}$ or $\mathbf{s}_k = \mathbf{s}_{k,y}$. The motion estimation field is formulated as the minimization of a cost function with energy $E_{\text{data}}(\mathbf{s}_k)$ penalized by spatial and sparse regularizations, i.e.,

$$\underset{\mathbf{x},\mathbf{s}_k}{\operatorname{argmin}} \left\{ E_{\text{data}}(\mathbf{s}_k) + \lambda_d E_{\text{sparse}}(\mathbf{s}_k, \mathbf{x}) + \lambda_s E_{\text{spatial}}(\mathbf{s}_k) \right\} \quad (1)$$

where $(\lambda_d, \lambda_s) \in \mathbb{R}^2$ are two parameters balancing the importance of the data fidelity and regularization terms

A. Data Fidelity Term

The maximum likelihood (ML) method is a well accepted technique for motion estimation [7]. It maximizes the conditional probability density function of the measurement vector $\mathbf{r}_{k+1}$ given $\mathbf{r}_k$ and s. The ML estimator is classically computed in the negative log-domain

$$\underset{\mathbf{s}}{\operatorname{argmin}} - \ln\left[p(\mathbf{r}_{k+1})|\mathbf{r}_k(n), \mathbf{s}\right]. \quad (2)$$

Straightforward computations exploiting the Rayleigh statistics of ultrasound images detailed in [4] lead to the following data fidelity term

$$E_{\text{data}}(\mathbf{s}) = -2d(\mathbf{s}) + 2\log[e^{2d(\mathbf{s})} + 1] + C \quad (3)$$

where

$$d(\mathbf{s}) = \frac{1}{b}\sum_{n=1}^{N}[\mathbf{r}_{k+1}(n + \mathbf{s}(n)) - \mathbf{r}_k(n)]$$

n indicates the pixel index, $\mathbf{s} = [s(1), \ldots, s(N)]^T$ is the vectorized motion, $\mathbf{r}_k = [r_k(1), \ldots, r_k(N)]^T$ is the vectorized ultrasound image in frame k, and $C = -\log(2\sigma^4/b)$ is a known constant (depending on a scale parameter $\sigma \in \mathbb{R}^+$ and on the linear gain associated with the formation of the log-compressed B-mode image).

B. Spatial Regularization

The spatial regularization term promotes the smoothness of the motion estimation field and is defined as

$$E_{\text{spatial}}(\mathbf{s}) = \|\nabla \mathbf{s}\|_2^2 \quad (4)$$

where ∇ denotes the gradient operator, and $\|.\|_2^2$ is the squared ℓ_2 norm which promotes low spatial gradients. This constraint imposes smooth fluctuations of the motion field which corresponds to a first-order spatial regularization [8].

C. Sparse Regularization

The proposed sparse regularization determines the motion $\mathbf{s}_k$ that is best represented as a convolution between M filters $\mathbf{d}_m$ and the sparse coefficient maps $\mathbf{x}_m$, i.e,

$$E_{\text{sparse}}(\mathbf{s}_k, \mathbf{x}_k) = \left\| \mathbf{s}_k - \sum_{m=1}^{M} \mathbf{x}_m * \mathbf{d}_m \right\|_2^2 \quad (5)$$

with a sparse constrain on $\mathbf{x}_m$ (which will appear in (8)). The filters are composed of specific patterns contained in the training motions and $\mathbf{x}_m$ are the activation maps of each atom. Motivations for using this kind of regularization include the fact that convolutional sparse representations are invariant to translations contrary to standard dictionary learning techniques [5]. Combining linearly (3), (4) and (5) yields to the proposed energy which is minimized for motion estimation in (1). This energy exploits the Rayleigh distribution of the noise and the spatial and sparse regularizations. The next section introduces the algorithm proposed to solve (1).

III. Optimization strategy

The regularization (5) assumes that an image frame of cardiac motion can be well represented by the sum of M convolutions between the coefficient maps and the corresponding filters. More precisely, the kth cardiac frame image $\mathbf{s}_k \in \mathbb{R}^{J\times N}$ is approximated as follows

$$\mathbf{s}_k \approx \sum_{m=1}^{M} \mathbf{d}_m * \mathbf{x}_m \quad (6)$$

where $*$ denotes the two-dimensional convolution. In order to solve (1), we propose to use Algorithm 1 which consists of three steps: 1) dictionary learning (see line 2), sparse coding (see line 3), and cardiac motion estimation (see line 6). These three steps are detailed below

Algorithm 1 Motion estimation field for a pair of images using convolutional dictionary learning.

Input: $\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho,$
$\widetilde{\mathbf{s}}$ = campe2 motions, $\hat{\mathbf{s}}$ = campe1 motions
Output: s
1: **function** MEFCDL($\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho, \widetilde{\mathbf{s}}_0, \hat{\mathbf{s}}_0$)
2: $\mathbf{d}_m \leftarrow$ Computes the dictionary by solving (7)
3: $\mathbf{x}_m \leftarrow$ Computes the coefficient maps by solving (8)
4: **for** $k \leftarrow 1, K$ **do**
5: **for** $j \leftarrow 1, J$ **do**
6: $\operatorname{argmin}_{\mathbf{s}}\big\{E_{\text{data}}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \mathbf{s}_{j-1}) +$
$\lambda_s\|\nabla \mathbf{s}_{j-1}\|_2^2 + \lambda_d(k)\|\mathbf{s}_{j-1} - \sum_m \mathbf{d}_m * \mathbf{x}_m\|_2^2\big\}$
s.t. $\|\mathbf{d}_m\| = 1\ \forall m$ ▷ Motion estimation
7: **return** s ▷ (Estimated motion field)

A. Dictionary Learning

In the first step of Algorithm 1, a dictionary is estimated off-line by using a set of training cardiac motions denoted as $\widetilde{\mathbf{s}}$. The dictionary is obtained by solving the following optimization problem

$$\underset{\mathbf{d}_m, \mathbf{x}_{k,m}}{\operatorname{argmin}} \frac{1}{2} \sum_k \left\| \sum_m \mathbf{x}_{k,m} * \mathbf{d}_m - \widetilde{\mathbf{s}}_k \right\|_2^2 + \lambda \sum_m \sum_k \|\mathbf{x}_{k,m}\|_1$$
$$\text{s.t.} \quad \|\mathbf{d}_m\| = 1 \ \forall m = 1, ..., M \tag{7}$$

which was solved by using the alternating direction method of multipliers (ADMM) [9].

B. Sparse Coding

In the second step of Algorithm 1, the coefficient maps $\mathbf{x}_m$ are computed from cardiac motions $\hat{\mathbf{s}}_k$. More precisely, $\mathbf{x}_m$ is estimated by using the dictionary $\mathbf{d}_m$ obtained in (7) (see line 2) and by solving the following problem using ADMM

$$\underset{x_m}{\operatorname{argmin}} \frac{1}{2} \left\| \sum_m \mathbf{x}_m * \mathbf{d}_m - \hat{\mathbf{s}}_k \right\|_2^2 + \lambda \sum_m \|\mathbf{x}_m\|_1. \tag{8}$$

C. Estimation Motion

In the last step of Algorithm 1, the cardiac motion estimation are estimated using an algorithm similar to the one proposed in [4]. In order to take into account the modified dictionary learning regularization, we consider the following optimization problem

$$\underset{\mathbf{s}}{\operatorname{argmin}} \Big\{ E_{\text{data}}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \mathbf{s}_{j-1}) + \lambda_s \|\nabla \mathbf{s}_{j-1}\|_2^2 + \lambda_d$$
$$\times \|\mathbf{s}_{j-1} - \sum_m \mathbf{d}_m * \mathbf{x}_m\|_2^2 \Big\} \text{ s.t. } \|\mathbf{d}_m\| = 1 \ \forall m. \tag{9}$$

The minimization problem (9) can be solved by setting the gradient of the cost function to zero and following the approach in [3]. Note that the horizontal and vertical motions $\mathbf{s}_x$ and $\mathbf{s}_y$ are computed independently.

IV. Experimental results

This section analyzes the performance of the proposed motion estimation based on CDL and compares it with standard dictionary learning [4] and with other state-of-the-art methods. For this comparison, we consider highly realistic simulations performed using data with a controlled ground-truth generated using the method studied in [10]. The proposed approach is compared with the recent method of [4] (which showed very competitive results when compared to block-matching [1], the monogenic signal [2], and B-splines [3]).

A. Simulation Scenario

Filters: The filters were computed with the pathological sequence LADdist by solving (7) with 500 iterations. The number of filters was set to $M = 32$, and the filter size was $L \times L$ with $L = 48$. The regularization parameter was $\lambda = 0.001$. Fig. 2 shows the corresponding set of convolutional dictionaries trained with horizontal, and vertical cardiac motions. Note that the filters trained with horizontal motions are depicted in Fig. 2 (a), (c), (e), (g), whereas the filters trained with vertical motions are shown in Fig. 2 (b), (d), (f), (h). Three scenarios were investigated: 1) one dictionary

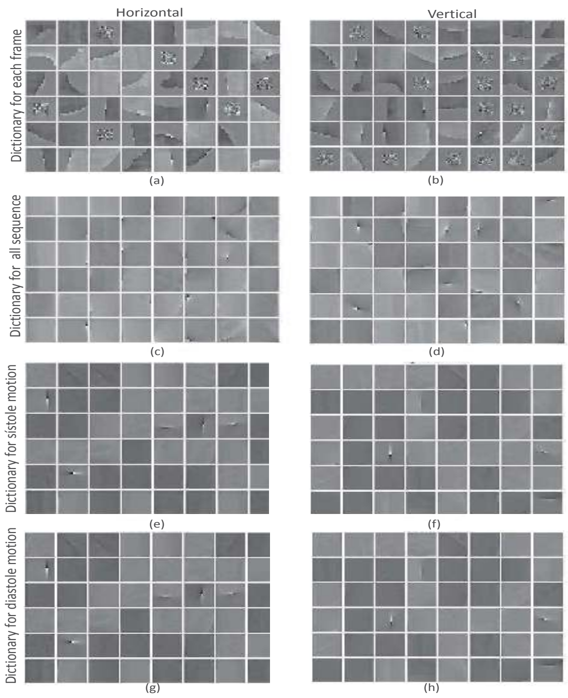


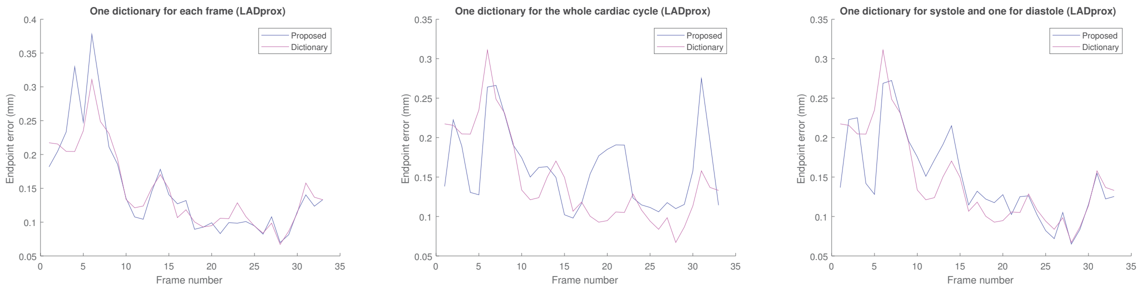
Fig. 2. Dictionaries obtained using LADdist motions (with a filter size $L \times L$ with $L = 32$, and a number of filters $M = 48$): (top row) An example of filters for the 1st frame. (second row) A dictionary estimated for the whole cardiac cycle. (third row) A dictionary for systole. (bottom row) A dictionary for diastole. The first columns (a),(c),(e),(g) show dictionaries trained with horizontal motions, (b),(d),(f),(h) are for dictionaries trained with vertical motions. All the dictionaries are obtained with 500 iterations.

of filters for each frame of motions (top-row and Fig. 2(a), (b)), 2) one dictionary of filters for the whole cardiac cycle (second row and Fig. 2(c), (d)), 3) one dictionary for systole frames (frames 1-12) (third-row and Fig. 2(e), (f)) and one dictionary for diastole frames (frames 13-33) (bottom-row and Fig. 2(g), (h)). Finally, the coefficient maps were obtained from (8) with the pathological sequence of LADprox displacements (500 iterations) and a fixed dictionary of filters (see Section IV-A).

Motions: The regularization parameter was set to $\lambda_s = 0.75$ and λ_d was logarithmically increased from 1×10^{-9} to 1×10^{-3} in 12 iterations. The parameters of the three steps of algorithm 1 are summarized in Table I. In the table II is depicted a detailed cross-validation to select the filter size and filter number.

B. Performance Measure

In order to evaluate the performance of the different methods, we computed the endpoint error as in [2]. This error is defined for the nth pixel as $e_n = \sqrt{[\mathbf{s}_x(n) - \hat{\mathbf{s}}_{\mathbf{x}}(n)]^2 + [\mathbf{s}_y(n) - \hat{\mathbf{s}}_y(n)]^2}$, where $\mathbf{s}_x(n)$, $\mathbf{s}_y(n)$, $\hat{\mathbf{s}}_x(n)$, $\hat{\mathbf{s}}_y(n)$ are the true and estimated (horizontal and vertical) motions at pixel n. Fig. 3 shows the mean endpoint error for the LADprox sequence and the three scenarios. The averaged endpoint errors for the different scenarios are (1)



(a) (b) (c)

Fig. 3. Mean endpoint error (in mm) for the LADprox sequence by (a) training a convolutional dictionary for all the frames (error: 0.1556), (b) training a convolutional dictionary for each frame (error: 0.1601) and (c) training two convolutional dictionaries (one for systole and one for diastole motions) (error: 0.147). The error for the method of [4] is 0.147.

TABLE I

PARAMETERS FOR EACH STEP OF ALGORITHM 1, DICTIONARY LEARNING, SPARSE CODING, AND CARDIAC MOTION ESTIMATION.

Step	Parameters	Values
Dictionary learning	Database	LADdist
	Filter size	48×48
	Filters number	$M = 32$
	Sparsity term	$\lambda = 0.001$
	Number of iteration	500
Sparse coding	Database	LADprox
	Number of iteration	500
Cardiac motion estimation	Regularization parameter	$\lambda_s = 0.75$
	Sparsity term (Systole)	$\lambda_d = \{1 \times 10^{-6} \times 10^{-3}\}$
	Sparsity term (Diastole)	$\lambda_d = \{1 \times 10^{-9} \times 10^{-2}\}$

TABLE II

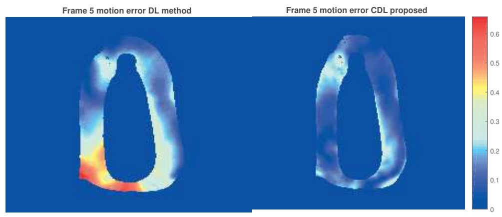
COMPARISON OF MEAN ENDPOINT ERROR VARYING THE FILTERS NUMBER AND THE FILTER SIZE FOR A ONE DICTIONARY SYSTOLE AND ONE DICTIONARY FOR DIASTOLE. NOTICE THE BEST RESULT IS ATTAINED WITH $M = 32$ AND $L = 48$.

		Filter number, M				
		8	16	24	32	48
Filter size, L	8	0.1473	0.1477	0.1477	0.1475	0.1477
	16	0.1476	0.1477	0.1478	0.1478	0.1479
	24	0.1474	0.1476	0.1469	0.1470	0.1475
	32	0.1477	0.1471	0.1477	0.1469	0.1475
	40	0.1476	0.1473	0.1478	0.1482	0.1474
	48	0.1470	0.1467	0.1466	0.1465	0.1470

0.1556 (2) 0.1601, (3) 0.147, with a preference for learning different dictionaries for the systole and diastole frames.

C. Realistic Simulations

This section considers highly realistic simulations using B-mode ultrasound data published in [11] including ground-truth of horizontal and vertical motions. More precisely, we used the LADprox sequence, which correspond to a proximal occlusion of the left anterior descending artery. Each sequence is a set of 3D images (with $224 \times 176 \times 208$ voxels, voxel size $0.7 \times 0.9 \times 0.6$ mm, frame rate $21 - 23$ Hz [11]). The sequence contains 34 images of one complete cardiac cycle. For more details, the reader is invited to consult [11] and [10].



(a) (b)

Fig. 4. Error map for the 5^{th} frame. Motion estimation using standard dictionary learning [4] (left) and proposed method (right).

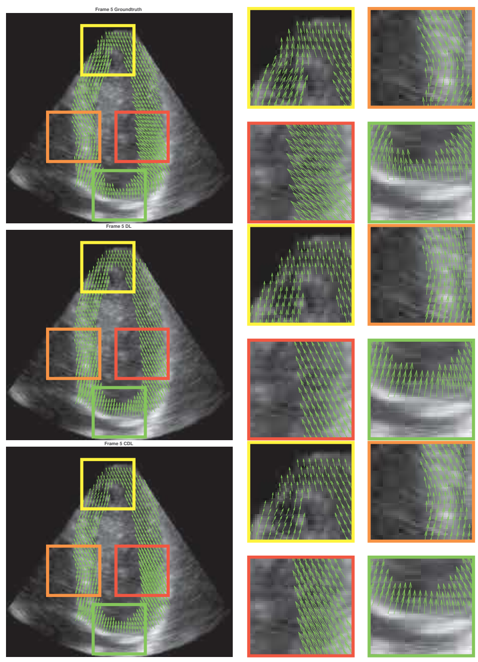


Fig. 5. Ground-truth (top) and estimated meshes of the 5^{th} frame of motion estimation using standard dictionary learning [4] (center), and motion estimation using convolutional dictionary (bottom).

In order to analyze the performance of the different algorithms, the error maps of the displacement estimates were computed. Fig. 4 displays the error maps for the 5^{th} frame of the LADprox sequence. Fig. 4 (left) shows the error maps obtained for [4] (which uses a standard dictionary learning), and Fig. 4 (right) for the algorithm 1 (which uses CDL). Fig. 5 shows a representative example of the estimated motion field for the 5^{th} frame which includes the ground-truth Fig. 5 (top), the approach in [4] with standard dictionary learning Fig. 5 (center), the proposed method in algorithm 1 by using convolutional dictionary Fig. 5 (bottom). The zoomed version displays the motion vectors obtained using the approach in [4] (standard dictionary) and Algorithm 1 (convolutional dictionary). It is clear that the vector computed using a standard dictionary looks vertical. In contrast, the vector computed using CDL better matches the ground-truth.

To show the interest of using a convolutional dictionary, the principal component analysis [12] of the coefficient maps $\mathbf{x}$ was computed, in order to highlight some correlations between these coefficients and the cardiac motions. The coefficient maps for the 5^{th} frame were projected on their two first principal components and are shown in Fig. 6(a). Clustering the projected coefficient maps using kmeans with 2 classes and comparing the resulting clusters displayed in Fig. 6(b) with the corresponding motion estimation in Fig. 6(c) shows a 94% correlation between the coefficient maps and the cardiac motions, which shows the interest of using coefficient maps to represent the motions 6(d).

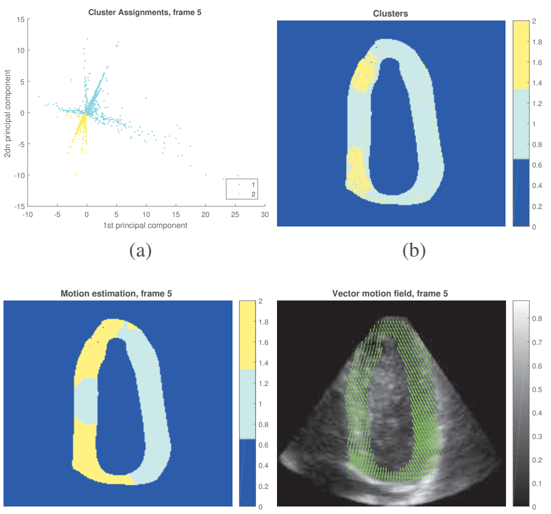


Fig. 6. Projection of the coefficient maps on their two first principal components (a), and cluster assignment of the projected coefficients using kmeans with 2 classes (b). Motions associated with the 2 clusters identified by kmeans, which can be compared with the thresholding of the true motions of Fig. 6(c). Thresholding of the motions for the 5th frame (c) and their estimates (d).

V. Conclusions

This paper introduced a new method for cardiac motion estimation in 2D ultrasound images based on a sparse decomposition of the motions on convolutional dictionaries. The method exploits the noise characteristics based on the B-mode distribution of ultrasound images and regularizes the estimation problem using a smoothing term and a sparse decomposition of the motions in a convolutional dictionary. The results obtained with the proposed method compares favourably with other state-of-the-art methods. An interesting property of the coefficients of the sparse decomposition on a convolutional dictionary is a strong correlation with the corresponding motions. Future work will be devoted to the study of classification and anomaly detection methods based on the parameters resulting from this sparse decomposition on convolutional dictionaries.

Acknowledgment

Part of this work was supported by the STIC-AmSud project 18-STIC-05. HyperMed - Image reconstruction from compressed measurements: application to hyperspectral and medical Imaging.

References

[1] K. Kaluzynski, S. Y. Emelianov, A. R. Skovoroda, and M. O'Donnell, "Strain rate imaging using two-dimensional speckle tracking," *Trans. Ultrason., Ferroelectr., Freq. Control*, vol. 48, no. 4, pp. 1111–1123, July 2001.

[2] M. Alessandrini, A. Basarab, H. Liebgott, and O. Bernard, "Myocardial motion estimation from medical images using the monogenic signal," *IEEE Trans. Image Process.*, vol. 22, no. 3, pp. 1084–1095, March 2013.

[3] A. Myronenko, X. Song, and D. J. Sahn, "Maximum likelihood motion estimation in 3D echocardiography through non-rigid registration in spherical coordinates," in *Functional Imaging and Modeling of the Heart*, N. Ayache, H. Delingette, and M. Sermesant, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 427–436.

[4] N. Ouzir, A. Basarab, H. Liebgott, B. Harbaoui, and J.-Y. Tourneret, "Motion estimation in echocardiography using sparse representation and dictionary learning," *IEEE Trans. Image Process.*, vol. 27, no. 1, pp. 64–77, Jan 2018.

[5] B. Wohlberg, "Efficient algorithms for convolutional sparse representations," *IEEE Trans. Image Process.*, vol. 25, no. 1, pp. 301–315, Jan 2016.

[6] V. Papyan, Y. Romano, and M. Elad, "Convolutional neural networks analyzed via convolutional sparse coding," *J. Mach. Learn. Res.*, vol. 18, no. 1, pp. 2887–2938, Jan. 2017. [Online]. Available: http://dl.acm.org/citation.cfm?id=3122009.3176827

[7] B. Cohen and I. Dinstein, "New maximum likelihood motion estimation schemes for noisy ultrasound images," *Pattern Recognition*, vol. 35, no. 2, pp. 455 – 463, 2002. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S003132030100053X

[8] B. K. Horn and B. G. Schunck, "Determining optical flow," *Artificial Intelligence*, vol. 17, no. 1, pp. 185 – 203, 1981. [Online]. Available: http://www.sciencedirect.com/science/article/pii/0004370281900242

[9] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011. [Online]. Available: http://dx.doi.org/10.1561/2200000016

[10] M. Alessandrini, M. De Craene, O. Bernard, S. Giffard-Roisin, P. Allain, I. Waechter-Stehle, J. Weese, E. Saloux, H. Delingette, M. Sermesant, and J. D'hooge, "A pipeline for the generation of realistic 3D synthetic echocardiographic sequences: Methodology and open-access database," *IEEE Trans. Med. Imaging*, vol. 34, no. 7, pp. 1436–1451, July 2015.

[11] M. Alessandrini, B. Heyde, S. Queirs, S. Cygan, M. Zontak, O. Somphone, O. Bernard, M. Sermesant, H. Delingette, D. Barbosa, M. De Craene, M. O'Donnell, and J. D'hooge, "Detailed evaluation of five 3D speckle tracking algorithms using synthetic echocardiographic recordings," *IEEE Trans. Med. Imaging*, vol. 35, no. 8, pp. 1915–1926, Aug 2016.

[12] J. Jackson, *A User's Guide to Principal Components*, ser. Wiley Series in Probability and Statistics. Wiley, 2005. [Online]. Available: https://books.google.fr/books?id=f9s6g6cmUTUC