

# Carrier Frequency Offset Estimation for OFDM-Based WLANs

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**Abstract**—We present an efficient carrier frequency offset (CFO) estimation algorithm for the orthogonal frequency-division multiplexing (OFDM)-based wireless local area networks (WLANs). The packet preamble information we use is based on the high rate WLAN standards adopted by the IEEE 802.11 standardization group. Numerical results are presented to demonstrate the effectiveness of the proposed algorithm.

**Index Terms**—Carrier frequency, maximum likelihood (ML), orthogonal frequency-division multiplexing (OFDM), wireless local area network (WLAN).

## I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) has been selected as the basis for the high speed wireless local area network (WLAN) standards by the IEEE 802.11 standardization group [1]. The packet preamble specified by the IEEE standard (see Fig. 1) consists of 10 *identical* short OFDM symbols (each containing 16 data samples) and two *identical* long symbols (each containing 64 data samples). Between the short and long OFDM symbols, there is a guard interval (GI2) of length 32 that constitutes the cyclic prefix of the long symbols. These symbols can be used for the carrier frequency offset (CFO) estimation, which is important in an OFDM receiver. In [2]–[5], the CFO is estimated based on some repeated OFDM symbols with structures that are different from some of those we consider herein. For example, the approach in [5] cannot be used directly with both the short and long OFDM symbols in Fig. 1. The objective of this letter is to estimate the CFO with these OFDM symbols via a high performance nonlinear least squares (NLS) fitting approach.

## II. PROBLEM FORMATION

We consider a frequency-selective fading channel by modeling the channel impulse response as a finite impulse response (FIR) filter with the filter length assumed to be shorter than a short OFDM symbol. Hence, the first short OFDM symbol is effectively a cyclic prefix for the other nine short OFDM symbols. At an OFDM receiver, apart from a multiplicative complex

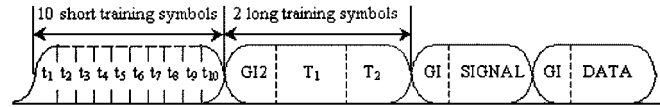


Fig. 1. OFDM preamble structure adopted by the IEEE 802.11 standardization group.

exponential accounting for the frequency offset, these nine short OFDM symbols will still be identical to each other (note that being affected by the channel-induced intersymbol interference the first OFDM symbol must be discarded). Likewise, due to GI2, the two long OFDM symbols will be identical to each other as well at the receiver. We use the subscripts  $S$  and  $L$  to differentiate the variables due to the short and long OFDM training symbols, respectively. Let  $x_S(m, n)$  ( $n = 0, \dots, N_S - 1$ , with  $N_S = 16$ , and  $m = 1, \dots, M_S - 1$ , with  $M_S = 10$ ) denote the  $n$ th sample of the  $m$ th noise-free short OFDM symbol after removing the cyclic prefix but prior to taking the FFT or estimating and equalizing the channel at the receiver. Note that  $x_S(m, n) = x_S(1, n) \exp[j2\pi(m-1)N_S\xi]$  for  $m = 1, \dots, M_S - 1$ , with  $\xi$  being the CFO. We do not require channel knowledge and treat  $x_S(1, n)$ ,  $n = 0, \dots, N_S - 1$  as a deterministic unknown. Let  $y_S(m, n)$  denote the noisy output at the receiver, i.e.,  $y_S(m, n) = x_S(m, n) + e_S(m, n)$  where  $e_S(m, n)$  is the additive zero-mean Gaussian noise. Let

$$\mathbf{y}_S(n) = [y_S(1, n) \ \dots \ y_S(M_S-1, n)]^T, \quad n = 0, \dots, N_S-1 \quad (1)$$

and

$$\mathbf{a}_S(\xi) = [1 \ e^{j2\pi\xi} \ \dots \ e^{j2\pi(M_S-2)\xi}]^T \quad (2)$$

where  $(\cdot)^T$  denotes the transpose and  $\epsilon = N_S\xi$ . Then we have

$$\mathbf{y}_S(n) = \mathbf{a}_S(\xi)x_S(1, n) + \mathbf{e}_S(n), \quad n = 0, \dots, N_S - 1 \quad (3)$$

where  $\mathbf{e}_S(n)$  is formed from  $\{e_S(m, n)\}_{m=1}^{M_S-1}$  in the same way  $\mathbf{y}_S(n)$  is formed from  $\{y_S(m, n)\}_{m=1}^{M_S-1}$ . Similar expressions can be readily obtained for the two long OFDM symbols

$$\mathbf{y}_L(n) = \mathbf{a}_L(\xi)x_L(1, n) + \mathbf{e}_L(n), \quad n = 0, \dots, N_L - 1 \quad (4)$$

where

$$\mathbf{y}_L(n) = [y_L(0, n) \ \dots \ y_L(M_L - 1, n)]^T \quad (5)$$

and

$$\mathbf{a}_L(\xi) = [1 \ e^{j2\pi(M_L-1)\mu}]^T \quad (6)$$

with  $\mu = N_L\xi$ ,  $M_L = 2$  and  $N_L = 64$ . We assume unknown  $x_L(1, n)$ ,  $n = 0, \dots, N_L - 1$ , which includes a phase shift

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$\exp\{j2\pi[(M_S - 1)N_S + N_{cp}]\xi\}$ , where  $N_{cp} = 32$  is the length of GI2.

Our problem of interest herein is to estimate the CFO  $\xi$  from  $\mathbf{y}_S(n)$  and  $\mathbf{y}_L(n)$  without knowing the training symbol values or the underlying FIR channel.

### III. ALGORITHM

The last two short OFDM symbols are intended by the IEEE 802.11 standardization group for coarse CFO estimation. Yet all of the nine short OFDM symbols can be used by minimizing the following nonlinear squares (NLS) cost function

$$\begin{aligned} C_1 & \left( \xi, \{x_S(1, n)\}_{n=0}^{N_S-1} \right) \\ & = \sum_{n=0}^{N_S-1} \|\mathbf{y}_S(n) - \mathbf{a}_S(\xi)x_S(1, n)\|^2 \end{aligned} \quad (7)$$

where  $\|\cdot\|$  denotes the Euclidean norm. Note that when the additive noise is white and Gaussian, the above NLS estimates are the maximum likelihood estimates conditioned on  $x_S(1, n)$ . When the noise is not white, the NLS estimates can still have excellent accuracy [6], [7]. The minimization of  $C_1(\xi, \{x_S(1, n)\}_{n=0}^{N_S-1})$  in (7) with respect to  $x_S(1, n)$  and  $\xi$  yields

$$\hat{x}_S(1, n) = \frac{1}{M_S - 1} \mathbf{a}_S^H(\xi) \mathbf{y}_S(n) \Big|_{\xi=\hat{\xi}}, \quad n = 0, \dots, N_S - 1 \quad (8)$$

and

$$\hat{\xi} = \arg \max_{\xi} \frac{1}{M_S - 1} \sum_{n=0}^{N_S-1} |\mathbf{a}_S^H(\xi) \mathbf{y}_S(n)|^2 \quad (9)$$

which, by taking into account (2), can be computed efficiently by applying one-dimensional (1-D) FFT to each  $\mathbf{y}_S(n)$  (padding with zeros is necessary to achieve high estimation accuracy and we zero-pad to 512 in our simulations).

The two long OFDM symbols are intended by the IEEE 802.11 standardization group for fine CFO estimation, which can be obtained similar to (9). Since  $M_L = 2$ , a closed-form solution is possible

$$\hat{\xi} = -\frac{1}{2\pi N_L} \arg \left( \sum_{n=0}^{N_L-1} y_L(0, n) y_L^*(1, n) \right) \quad (10)$$

where  $\arg(\cdot)$  denotes argument and  $(\cdot)^*$  stands for complex conjugate.

However, both the short and the long OFDM symbols can be used together to obtain the NLS CFO estimate as follows:

$$\begin{aligned} \hat{\xi} = \arg \max_{\xi} & \left\{ \frac{1}{M_S - 1} \sum_{n=0}^{N_S-1} |\mathbf{a}_S^H(\xi) \mathbf{y}_S(n)|^2 \right. \\ & \left. + \frac{1}{M_L} \sum_{n=0}^{N_L-1} |\mathbf{a}_L^H(\xi) \mathbf{y}_L(n)|^2 \right\}. \end{aligned} \quad (11)$$

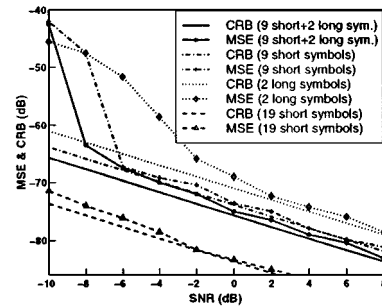


Fig. 2. Comparison of MSEs with CRBs as a function of SNR for the OFDM carrier frequency offset estimates with a frequency-selective fading channel.

### IV. NUMERICAL RESULTS

We evaluate the algorithm performance for a frequency-selective fading channel by assuming that the packet timing is available. We set  $\xi = 0.08/N_S$  and the channel impulse function is chosen as  $\mathbf{h} = [e^{j1.38} \ 0.5e^{j0.30} \ 0.3e^{-j2.02}]^T$ . The mean-squared errors (MSEs) of the CFO estimates are obtained from 200 Monte Carlo trials and are compared with the corresponding Cramér–Rao bounds (CRBs) as a function of SNR, which is defined as the ratio of the average energy per sample to the variance of the additive white Gaussian noise. Fig. 2 shows the MSEs and CRBs of the CFO estimates versus SNR when using only nine short, only 2 long, both nine short and two long OFDM symbols, and 19 short OFDM symbols (this new preamble is to replace GI2,  $T_1$ , and  $T_2$  with  $t_1 \sim t_{10}$  and eliminate the windowing between  $t_{10}$  and GI2 so that there are 20 identical short OFDM symbols in the preamble). Observe from Fig. 2 that using only the nine short OFDM symbols yields an improvement of about 2.7 dB with respect to using only the long ones. Using both the nine short and two long OFDM symbols yields an extra improvement of 1.9 dB. A performance improvement of 9.8 dB is obtained by using 19 short symbols with respect to using nine short symbols.

### V. CONCLUSIONS

We have presented an efficient CFO estimation algorithm for the OFDM based WLANs by using the packet preamble structure adopted by the IEEE 802.11 standardization group. We have found that using only the short OFDM symbols results in a 2.7 dB improvement in accuracy with respect to using only the long OFDM symbols. Using both yields the best performance with an extra improvement of 1.9 dB in accuracy. We have also found that using 19 short symbols yields a performance improvement of about 9.8 dB with respect to using nine short symbols. This new preamble may be considered if much better blind CFO frequency offset estimation is needed.

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