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CARTESIAN CLOSED TOPOLOGICAL HULLS

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ABSTRACT. It is shown in this paper that if a concrete category $\mathfrak{A}$ admits embedding as a full finitely productive subcategory of a cartesian closed topological (CCT) category, then $\mathfrak{A}$ admits such embedding into a smallest CCT category, its CCT hull. This hull is characterized internally by means of density properties and externally by means of a universal property. The problem is posed of whether every topological category has a CCT hull.

Introduction. The category **Top** of topological spaces and continuous maps is not cartesian closed, hence inconvenient for many purposes in homotopy theory, topological algebra and functional analysis. Fortunately it can be fully embedded into some cartesian closed topological categories (briefly, CCT categories) such as the category **Conv** of convergence spaces (cf. D. C. Kent [21] and L. D. Nel [27]), the smaller category **Lim** of limit spaces (cf. H. J. Kowalsky [23], H. R. Fischer [14], A. Bastiani [2], C. H. Cook and H. R. Fischer [12], E. Binz and H. H. Keller [6], E. Binz [4], [5], A. Machado [24] and others) and the even smaller category **Pstop** of pseudotopological ($= L^*$ -) spaces (cf. G. Choquet [11], H. Poppe [29], A. Machado [24], L. D. Nel [27]). Recent work of P. Antoine [1] and A. Machado [24] has brought to light the existence of a smallest full CCT subcategory of **Lim** which contains **Top**. G. Bourdaud [7], [8], [9] studied smallest full CCT subcategories of **Lim** containing the categories of pretopological spaces and of uniformisable topological spaces respectively.

In this paper we will show that any concrete category $\mathfrak{A}$ with finite products which can be fully embedded in some CCT category such that the embedding is dense or preserves finite products, can be likewise embedded in a smallest CCT category, called the cartesian closed topological hull of $\mathfrak{A}$. This hull will be characterized internally by means of suitable density properties and externally by means of a universal property.

Preliminaries. All categories in this paper are supposed to be concrete, i.e., equipped with an underlying (faithful and amnestic[2]) functor into **Set**. A

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[2] A faithful functor $U: \mathfrak{A} \to \mathfrak{X}$ is called *amnestic* if an $\mathfrak{A}$-isomorphism f is an $\mathfrak{A}$-identity whenever Uf is an $\mathfrak{X}$-identity.

functor will be called an *embedding* if it is full, faithful, injective on objects and preserves underlying sets. All subcategories are full and isomorphism-closed; thus typically their embedding functors are embeddings in the above sense. A category $\mathfrak{A}$ is called *topological* if it satisfies the following conditions:

$\mathfrak{A}$ is initially complete[3];

$\mathfrak{A}$ has small fibres;

The fibre of a set with cardinality one consists of just one object.

A category $\mathfrak{A}$ is called *cartesian closed* provided:

$\mathfrak{A}$ has finite products and, for any $\mathfrak{A}$-object A the functor $A \times -: \mathfrak{A} \to \mathfrak{A}$ has a right adjoint. It is known (H. Herrlich [17]) that a topological category is cartesian closed iff for any pair (A, B) of $\mathfrak{A}$-objects the morphism set $\mathfrak{A}(A, B)$ can be equipped with the structure of an $\mathfrak{A}$-object (called a power and denoted by $\mathfrak{A}[A, B]$) such that:

The evaluation map $e: A \times \mathfrak{A}[A, B] \to B$ is an $\mathfrak{A}$-morphism;

for any $\mathfrak{A}$-morphism $f: A \times C \to B$ there exists a unique $\mathfrak{A}$-morphism $f^*: C \to \mathfrak{A}[A, B]$ with $e \circ (\mathrm{id}_A \times f^*) = f$.

If a category is said to have *finite products* then we will assume that these are preserved by the underlying functor (hence they are initial sources). A subcategory $\mathfrak{A}$ of $\mathfrak{B}$ is called *dense* (resp. *codense*) in $\mathfrak{B}$ provided that for any $\mathfrak{B}$-object B there exists a final epi-sink $(f_i: A_i \to B)_I$ (resp. an initial source $(f_i: B \to A_i)_I$) with all A_i belonging to $\mathfrak{A}$.

Readers desiring background information may find this for categories generally in H. Herrlich and G. E. Strecker [15]; for initial sources and final sinks in H. Herrlich [16]; for CCT categories in H. Herrlich [17], [18], L. D. Nel [27], [28] and O. Wyler [32], [33]. We recall two facts for convenient reference:

0. **Proposition.** (a) *The coreflective hull of any finitely productive subcategory of a CCT category is a CCT category* (*see* [27]).

(b) *A topological category* $\mathfrak{A}$ *is a CCT category iff for any* $\mathfrak{A}$*-object A the functor* $A \times -: \mathfrak{A} \to \mathfrak{A}$ *preserves final epi-sinks* (*see* [17]).

Results. Henceforth let

$\mathfrak{D}$ be an arbitrary CCT category,

$\mathfrak{A}$ be a subcategory of $\mathfrak{D}$ closed under the formation of finite products in $\mathfrak{D}$,

$\mathfrak{C}$ be the coreflective hull of $\mathfrak{A}$ in $\mathfrak{D}$,

$\mathfrak{B}$ be the bireflective hull in $\mathfrak{C}$ of the class of all $\mathfrak{C}$-powers $\mathfrak{C}[A, A']$ of $\mathfrak{A}$-objects A, A'.

Thus $\mathfrak{A} \subset \mathfrak{B} \subset \mathfrak{C} \subset \mathfrak{D}$. Further, let $\mathfrak{X}$ be an arbitrary subcategory of $\mathfrak{C}$ containing $\mathfrak{A}$.

1. **Proposition.** $\mathfrak{A}$ *is dense in* $\mathfrak{C}$.

[3] A source $(f_i: A \to A_i)_I$ in $\mathfrak{A}$ is called *initial* provided a function $g: B \to A$ is an $\mathfrak{A}$-morphism iff all compositions $f_i \circ g$ are $\mathfrak{A}$-morphisms. $\mathfrak{A}$ is *initially complete* or is said to *have initial sources* if for any class $(A_i)_I$ of $\mathfrak{A}$-objects and any source $(f_i: X \to A_i)_I$ in **Set** there exists an initial source $(f_i: A \to A_i)_I$ in $\mathfrak{A}$ with $UA = X$.

2. Proposition. $\mathfrak{C}$ *is a CCT category.*

Proof. Proposition 0(a).

3. Remark. In general, powers in $\mathfrak{C}$ and in $\mathfrak{D}$ are formed differently. $\mathfrak{C}$ need not even be closed under the formation of $\mathfrak{D}$-powers $\mathfrak{D}[A, A']$ of $\mathfrak{A}$-objects. A counterexample is provided by $\mathfrak{A} = \mathfrak{C} =$ the category $\mathbf{Conv}_0$ of symmetric convergence spaces and $\mathfrak{D}$ the category **Grill** of grill-determined nearness spaces (see H. L. Bentley et al. [3]).

4. Proposition. *The following are equivalent*:
(a) $\mathfrak{X}$ *is a topological category,*
(b) $\mathfrak{X}$ *is a bireflective subcategory of* $\mathfrak{C}$,
(c) $\mathfrak{X}$ *is closed under formation of initial sources in* $\mathfrak{C}$.

Proof. Obviously (b) implies (a). The converse follows immediately from Proposition 1 and the result of H. Müller [26] that any topological subcategory of a topological cateogry $\mathfrak{C}$ is a bireflective subcategory of a coreflective subcategory of $\mathfrak{C}$. Equivalence of (b) and (c) is immediate.

5. Proposition. *The following are equivalent*:
(a) $\mathfrak{X}$ *is a CCT category.*
(b) $\mathfrak{X}$ *is bireflective in* $\mathfrak{C}$ *and closed under the formation of powers in* $\mathfrak{C}$.

Proof. By Proposition 4, (b) implies (a). For the converse it remains to show that $\mathfrak{X}[X, X'] = \mathfrak{C}[X, X']$ for any pair (X, X') of $\mathfrak{X}$-objects. Note [17] that both objects have the same underlying set $\mathfrak{C}(X, X')$. So we need only show that the identity function on $\mathfrak{C}(X, X')$ is a morphism $\mathfrak{X}[X, X'] \to \mathfrak{C}[X, X']$ as well as a morphism $\mathfrak{C}[X, X'] \to \mathfrak{X}[X, X']$. The first morphism is obtained by taking the image of the evaluation $X \times \mathfrak{X}[X, X'] \to X'$ under the natural bijection $\mathfrak{C}(X \times \mathfrak{X}[X, X'], X') \to \mathfrak{C}(\mathfrak{X}[X, X'], \mathfrak{C}[X, X'])$. For the second observe that for any $\mathfrak{A}$-object A we have the natural bijections $\mathfrak{C}(A, \mathfrak{C}[X, X']) \simeq \mathfrak{C}(A \times X, X') = \mathfrak{X}(A \times X, X') \simeq \mathfrak{X}(A, \mathfrak{X}[X, X'])$ since $\mathfrak{X}$ is closed under the formation of products in $\mathfrak{C}$. This together with the fact that the sink $(f_i \colon A_i \to \mathfrak{C}[X, X'])_I$, consisting of all morphisms with domain A_i in $\mathfrak{A}$, is a final epi-sink in $\mathfrak{C}$ provides the second morphism. The referee pointed out that (b) follows from (a) also by virtue of the main result in B. Day [13], which moreover implies that (a) and (b) are equivalent with
(c) $\mathfrak{X}$ is bireflective in $\mathfrak{C}$ and the reflector preserves finite products.

6. Lemma. *For any* $\mathfrak{D}$*-object* D *the functor* $\mathfrak{D}[D, -] \colon \mathfrak{D} \to \mathfrak{D}$ *preserves initial sources and the contravariant functor* $\mathfrak{D}[-, D] \colon \mathfrak{D} \to \mathfrak{D}$ *transforms final epi-sinks to initial sources.*

Proof. Routine verification, using 0(b) for the second part.

7. Proposition. $\mathfrak{B}$ *is closed under the formation of powers in* $\mathfrak{C}$. *Moreover* $\mathfrak{C}[C, B]$ *belongs to* $\mathfrak{B}$ *for any* B *in* $\mathfrak{B}$, C *in* $\mathfrak{C}$.

PROOF. We have in $\mathfrak{C}$ an initial source $(f_i: B \to \mathfrak{C}[A_i, A'_i])_I$ and a final epi-sink $(g_j: A_j \to C)_J$ with all A_i, A'_i, A_j in $\mathfrak{A}$. By Lemma 6,

$$(\mathfrak{C}[C,B] \xrightarrow{\mathfrak{C}[g_j,B]} \mathfrak{C}[A_j,B])_J \quad \text{and} \quad (\mathfrak{C}[A_j,B] \xrightarrow{\mathfrak{C}[A_j,f_i]} \mathfrak{C}[A_j,\mathfrak{C}[A_i,A'_i]])_I$$

are initial sources for each j in J. The ith codomain is isomorphic to $\mathfrak{C}[A_j \times A_i, A'_i]$. By composing the above two sources we obtain an initial source of the form

$$(\mathfrak{C}[C,B] \xrightarrow{h_{ij}} \mathfrak{C}[A_j \times A_i, A'_i])_{I\times J}.$$

Since $\mathfrak{A}$ is closed under finite products, we conclude that $\mathfrak{C}[C,B]$ belongs to $\mathfrak{B}$.

By combining Propositions 4, 5 and 7 we obtain the following result.

8. THEOREM. *$\mathfrak{B}$ is the smallest CCT subcategory of $\mathfrak{C}$ containing $\mathfrak{A}$.*

By the above theorem, $\mathfrak{B}$ is uniquely determined as the smallest CCT subcategory of $\mathfrak{C}$ containing $\mathfrak{A}$. Surprisingly enough, $\mathfrak{B}$ does not depend on $\mathfrak{C}$ (resp. $\mathfrak{D}$). If we start with a different CCT supercategory of $\mathfrak{A}$ we will end up with the same $\mathfrak{B}$ (up to isomorphism). In the following we will exhibit internal conditions which characterize $\mathfrak{B}$.

9. DEFINITION. A category $\mathfrak{B}$ is called a CCT hull of a subcategory $\mathfrak{A}$ provided the following hold:

(a) $\mathfrak{B}$ is a CCT category,

(b) $\mathfrak{A}$ is closed under the formation of finite products in $\mathfrak{B}$,

(c) $\mathfrak{A}$ is dense in $\mathfrak{B}$,

(d) powers of $\mathfrak{A}$-objects are codense in $\mathfrak{B}$.

In order to characterize those categories which have a CCT hull we need the following fact.

10. PROPOSITION. *Any dense embedding $\mathfrak{X} \to \mathfrak{Y}$ preserves initial sources.*

PROOF. Let $(f_i: X \to X_i)_I$ be an initial source in $\mathfrak{X}$, let Y be a $\mathfrak{Y}$-object and let $g: Y \to X$ be a function such that for each i in I the map $f_i \circ g: Y \to X_i$ is a $\mathfrak{Y}$-morphism. To show that g is a $\mathfrak{Y}$-morphism consider a final epi-sink $(g_j: X_j \to Y)_J$ in $\mathfrak{Y}$ with all X_j in $\mathfrak{X}$. Then all $f_i \circ g \circ g_j: X_j \to X_i$ are $\mathfrak{Y}$-morphisms and hence $\mathfrak{X}$-morphisms. Initiality of $(f_i)_I$ implies that all $g \circ g_j: X_j \to X$ are $\mathfrak{X}$-morphisms, hence $\mathfrak{Y}$-morphisms. Finality of $(g_j)_J$ implies that g is a $\mathfrak{Y}$-morphism.

11. THEOREM (EXISTENCE OF CCT HULLS). *For a category $\mathfrak{K}$ with finite products the following are equivalent*:

(a) *$\mathfrak{K}$ has a CCT hull,*

(b) *there exists a finite-product preserving embedding of $\mathfrak{K}$ into some CCT category,*

(c) *there exists a dense embedding of $\mathfrak{K}$ into some CCT category.*

PROOF. Propositions 0(a) and 10.

12. THEOREM (UNIQUENESS OF CCT HULLS). *Any two CCT hulls of $\mathfrak{A}$ are isomorphic.*

PROOF. Let $\mathfrak{B}$ and $\mathfrak{B}'$ be CCT hulls of $\mathfrak{A}$. For objects A, A', A'' of $\mathfrak{A}$ the canonical equivalences

$$\mathfrak{B}(A, \mathfrak{B}[A', A'']) \simeq \mathfrak{B}(A \times A', A'') \simeq \mathfrak{B}'(A \times A', A'') \simeq \mathfrak{B}'(A, \mathfrak{B}'[A', A''])$$

imply that for any function $f\colon A \to \mathfrak{A}(A', A'')$ the following holds:

(*) $A \xrightarrow{f} \mathfrak{B}[A', A'']$ is a $\mathfrak{B}$-morphism iff $A \xrightarrow{f} \mathfrak{B}'[A', A'']$ is a $\mathfrak{B}'$-morphism.

For each $\mathfrak{B}$-object B the sink $S(B) = (g_i\colon A_i \to B)_{I(B)}$ consisting of all $\mathfrak{B}$-morphisms with domain in $\mathfrak{A}$ and codomain B is a final epi-sink in $\mathfrak{B}$ and the source $T(B) = (f_j\colon B \to \mathfrak{B}[A_j, A_j'])_{J(B)}$ consisting of all $\mathfrak{B}$-morphisms with domain B and codomain a power of $\mathfrak{A}$-objects, is an initial source in $\mathfrak{B}$. Similarly one defines $S'(B')$ and $T'(B')$ for each $\mathfrak{B}'$-object B'. Since $\mathfrak{B}'$ is a CCT hull of $\mathfrak{A}$, (*) and 9(c), (d) imply that for each B in $\mathfrak{B}$ there exists precisely one B' in $\mathfrak{B}'$ with the same underlying set as B such that the following equivalent statements hold:

(a) $(g_i\colon A_i \to B')_{I(B)}$ is a final epi-sink in $\mathfrak{B}'$,
(b) $(g_i\colon A_i \to B')_{I(B)} = S'(B')$,
(c) $(f_j\colon B' \to \mathfrak{B}'[A_j', A_j''])_{J(B)}$ is an initial source in $\mathfrak{B}$,
(d) $(f_j\colon B' \to \mathfrak{B}'[A_j', A_j''])_{J(B)} = T'(B')$,
(e) $(g_i\colon A_i \to B')_{I(B)}$ and $(f_j\colon B' \to \mathfrak{B}'[A_j', A_j''])_{J(B)}$ are in $\mathfrak{B}'$.

It follows immediately that there exists a unique functor $H\colon \mathfrak{B} \to \mathfrak{B}'$ with $HB = B'$ for each $\mathfrak{B}$-object B. Obviously H is an isomorphism leaving A fixed.

Having established the uniqueness of CCT hulls it seems natural to characterize them by some universal property. For this we need the following result.

13. PROPOSITION. *Any dense embedding $\mathfrak{X} \to \mathfrak{Y}$ between CCT categories preserves powers.*

PROOF. Proposition 10 implies that the embedding preserves finite products. Let (X, X') be a pair of $\mathfrak{X}$-objects. To show that the canonical evaluation map $e\colon X \times \mathfrak{X}[X, X'] \to X'$ has the universal property in $\mathfrak{Y}$, let Y be a $\mathfrak{Y}$-object and $f\colon X \times Y \to X'$ be a $\mathfrak{Y}$-morphism. If $(g_i\colon X_i \to Y)_I$ is a final epi-sink in $\mathfrak{Y}$ with the X_i belonging to $\mathfrak{X}$ then, for each i in I, there exists a unique $\mathfrak{X}$-morphism $g_i^*\colon X_i \to \mathfrak{X}[X, X']$ with $e \circ (\mathrm{id}_X \times g_i^*) = f \circ (\mathrm{id}_X \times g_i)$. One easily constructs a unique function $f^*\colon Y \to \mathfrak{X}(X, X')$ such that

$$e \circ (\mathrm{id}_X \times f^*) = f \quad \text{and} \quad (\mathrm{id} \times f^*) \circ (\mathrm{id} \times g_i) = \mathrm{id} \times g_i^*$$

for all i in I. Since all $f^* \circ g_i = g_i^*$ are $\mathfrak{Y}$-morphisms, it follows that $f^*\colon Y \to \mathfrak{X}[X, X']$ is a $\mathfrak{Y}$-morphism.

14. Theorem (Universal property of CCT hulls). *Let $E: \mathfrak{A} \to \mathfrak{B}$ be an embedding of $\mathfrak{A}$ into a CCT hull, and let $F: \mathfrak{A} \to \mathfrak{D}$ be a dense embedding of $\mathfrak{A}$ into some CCT category. Then there exists a unique dense embedding $G: \mathfrak{B} \to \mathfrak{D}$ with $F = G \circ E$.*

Proof. According to Theorem 8 there exists a factorization

$$\mathfrak{A} \xrightarrow{F} \mathfrak{D} = \mathfrak{A} \xrightarrow{F'} \mathfrak{D}' \xrightarrow{F''} \mathfrak{D}$$

such that F' is an embedding of $\mathfrak{A}$ into a CCT hull $\mathfrak{D}'$ and F'' is a dense embedding of a bireflective subcategory. By Theorem 12 there exists an isomorphism $H: \mathfrak{B} \to \mathfrak{D}'$ with $F' = H \circ E$. Hence $G = F'' \circ H$ is the required dense embedding of $\mathfrak{B}$ into $\mathfrak{D}$. Uniqueness follows from the fact that G preserves initial sources (10) and powers (13).

15. Examples. (a) The CCT hull of **Top** is the category of epitopological spaces (= espaces d'Antoine). See P. Antoine [1], A. Machado [24], G. Bourdaud [7], [9].

(b) The CCT hull of the category **PreTop** of pretopological spaces is the category of pseudotopological spaces (= L^* spaces) (see G. Bourdaud [7], [9]).

(c) The CCT hull of the category of completely regular spaces has been shown by G. Bourdaud [7], [8] to be the category of c-embedded limit spaces. Hausdorff c-embedded spaces have also been studied intensively by E. Binz [5], D. C. Kent et al. [22] and M. Schroder [30]. Hausdorff spaces do not form a topological category, only a closely related kind of category studied by L. D. Nel [27]. Fortunately most of the salient features of CCT categories remain valid.

(d) The CCT hull of the category of finite topological spaces = the coreflective hull of these spaces in **Top** = the coreflective hull of the Sierpiński space in **Top** = the category of topological spaces in which all intersections of open sets are open $\simeq$ the category of preordered sets (reflexive, transitive relations).

This is a counterexample to a result of P. Antoine [1] who claimed that any embedding F of $\mathfrak{A}$ into a CCT category $\mathfrak{K}$ which preserves initial structures and powers has a unique extension $F^*: \mathfrak{B} \to \mathfrak{K}$ preserving initial structures and powers. In fact, if $\mathfrak{K}$ is the category of sequential spaces and $\mathfrak{A}$ is the category of finite topological spaces discussed above then the natural embedding $F: \mathfrak{A} \to \mathfrak{K}$ preserves initial sources and powers, but there does not exist any extension $F^*: \mathfrak{B} \to \mathfrak{K}$ of F preserving initial sources. To see this, let R denote the reals with the usual topology and D the reals with the discrete topology, let $(f_i: D \to A_i)_I$ be the source consisting of all continuous functions from D into finite spaces and let $(g_j: R \to A_j)_J$ be the source consisting of all continuous functions from R into finite spaces. Then both sources are initial in $\mathfrak{K}$. On the other hand the sources $(f_i: D \to A_i)_I$ and $(g_j: D \to A_j)_J$ are both initial in $\mathfrak{B}$. If there were an extension $F^*: \mathfrak{B} \to \mathfrak{K}$ preserving initial sources, then we would have $D = F^* D = R$, a contradiction. The crucial difference between Theorem 14 and the claim of Antoine is that in Theorem 14 all embeddings are required to be dense.

16. REMARKS. Modifying E. Spanier's [31] construction of quasi-topological spaces, P. Antoine [1], B. Day [13] and O. Wyler [33] demonstrated that categories can be embedded into cartesian closed categories under very mild conditions. However, the resulting supercategories in general fail to have small fibres (hence are not topological in our sense). More seriously, these constructions cannot in general be carried out within a given universe. In fact, it has been shown (H. Herrlich [19]) that there exist nonfull subcategories of **Set** in which every constant map is a morphism, which cannot be fully embedded into any topological category (even if we allow large fibres) unless we are willing to leave the universe. Thus the problem of finding mild sufficient conditions under which a given $\mathfrak{A}$ will have a CCT hull remains open. Also open is the more specific question: does every topological category have a CCT hull?

By modifying the constructions of the above authors (so as to ensure small fibres) and by using the criterion 0(b) the following conditions can be shown to be sufficient for $\mathfrak{A}$ to have a CCT hull (the construction has been outlined in [28]):

(a) $\mathfrak{A}$ has small fibres and every constant map between $\mathfrak{A}$-objects is an $\mathfrak{A}$-morphism,

(b) $\mathfrak{A}$ has finite products and quotients,

(c) in $\mathfrak{A}$, finite products commute with quotients.

Special categories satisfying these conditions include **QUnif**, **Prox**, **PNear** and **SNear** (quasi-uniform, proximity, pre-nearness and seminearness spaces respectively). Since all topological categories satisfy (a) and (b) it is only (c) that has to be checked. For the cases **QUnif** and **Prox**, see Marxen [25].

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