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Cascading Failures in Interdependent Infrastructures: An Interdependent Markov-Chain Approach

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Abstract
Many critical infrastructures are interdependent networks in which the behavior of one network impacts those of the others. Despite the fact that interdependencies are essential for the operation of critical infrastructures, such interdependencies can negatively affect the reliability and fuel the cascade of failures within and across the networks. In this paper, a novel interdependent Markov-chain framework is proposed that enables capturing interdependencies between two critical infrastructures with the ultimate goal of predicting their resilience to cascading failures and characterizing the effects of interdependencies on system reliability. The framework is

sufficiently general to model cascading failures in any interdependent networks; however, this paper focuses on the electric-cyber infrastructure as an example. Using this framework it is shown that interdependencies among reliable systems, i.e., systems with exponentially distributed failure sizes, can make the individually reliable systems behave unreliably as a whole with power-law failure-size distributions.

SECTION I. Introduction

Critical infrastructures such as electricity, communication, transportation, gas and water networks use services from one another and influence each other due to their interdependencies. Although many interdependencies are essential for the operation of infrastructures, negative interdependencies affect the reliability of interdependent networks and can fuel cascade of failures within and across the networks. For instance, in the 2003 blackout in Italy, unplanned shutdown of a power station led to failures of communication network nodes and the supervisory control and data acquisition (SCADA) system of the power grid. This event, in turn, led to further failures in the power grid resulting in a large cascading failure in the system [1]. Another example of such interdependency is observable in the 2003 blackout in the Northeast United States, where the combination of power-component failures as well as computer and human events contributed to the cascading failures that ultimately led to the large blackout. In light of the above, interdependencies and extensive integration of critical infrastructures, such as the power grid and the communication network, mandate that they should be studied as a single coupled system specially when the reliability is of concern.

Here, we present an Inter-Dependent Markov Chain (IDMC) model, which provides a probabilistic framework to capture the effects of interdependencies among physical networks on the stochastic dynamics of cascading failures in an abstract setting. The idea of this approach is to build an integrated probabilistic framework consisting of a system of interdependent heterogeneous Markov chains (MCs)–one chain for each physical system. The interdependencies are captured such that a transition in a MC affects the transition probabilities of other MCs. We consider discrete-time MCs and model the interaction between interdependent systems by characterizing the transition probabilities of the IDMC based on individual chains and their interdependencies. In this paper, we first present the general IDMC framework and then we derive an IDMC model for cascading failures in electric-cyber infrastructure.

A naïve approach for coupling MCs would rely on the generation of the Cartesian product of the individual state spaces, which results in state-space explosion, deeming the approach to be intractable. Another shortcoming of the latter approach is that the new transition probabilities among the states of the coupled MC cannot be readily derived from the transition probabilities of the individual MCs. An even more serious flaw of the naïve approach is that it is based on the false and built-in assumption that combining MCs results in a new MC. In general, the interdependencies between MCs will result in memory in the combined process, which prevents the combined chain from being Markov. The proposed IDMC approach alleviates the flaws associated with the naïve approach for combining MCs and provides a minimal MC framework that encompasses the individual chains while capturing the interdependencies. Moreover, based on the IDMC framework, we introduce metrics to quantify the strength of interdependency between systems.

The presented IDMC framework enables system-level prediction of the behavior of the interdependent systems while overcoming the complexity of tracking the details of the system by means of meaningful abstractions. Here, by abstraction we mean identifying few key variables and parameters to represent the fine-grained states of the system with less complexity while implicitly capturing the effects of the rest of the parameters in the parameters and variables of the IDMC framework to approximate the behavior of the whole system.

A key insight obtained from the application of the IDMC model to the electric-cyber infrastructure is that interdependencies among reliable systems, i.e., systems with exponentially distributed failure sizes, can make them behave unreliably as a whole, as evidenced by power-law distributions for the size of the failures within each system.

SECTION II. Related Work

The majority of research in cascading failures in critical infrastructures has been focused on single, non-interacting systems. Among such works is the category of probabilistic models, which, for instance, include models based on branching processes [3], Markov chains [4], [5], regeneration theory [6], and so forth. For instance, in [6] we developed a scalable probabilistic approach based on regeneration theory and a reduced state space of the power grid to model the dynamics of cascading failures in time. The transition rates among the states of the model are defined to be state and age dependent, and their functional forms are calculated empirically from power-system simulations. The regeneration-based approach can collapse to a Markov process when the time between successive events are independent and exponentially distributed. However, the model can also capture the stochastic events when the underlying events are non-Markovian. We also developed a scalable probabilistic model for cascading failures based on a continuous-time MC in [5] that captures key physical attributes of the power grid through parametric transition rates.

Aside from efforts aiming to study the reliability of single systems, there has been great interest in understanding the behavior of interdependent systems. The general concepts of interdependent infrastructures and the challenges in modeling such systems have been discussed in [7] and [8]. Graph-based analyses of interdependent networks have also emerged. For instance, the work of Buldyrev *et al.* [9] considers a graph-based approach that utilizes percolation theory for modeling cascading failures in interdependent networks and provides an analytical formulation of the percentage of failed nodes in the steady state while identifying the role of the coupling between the networks. Another problem that has been considered in evaluating the reliability of interdependent networks is the characterization of the minimum number of nodes/links whose removals will disrupt the functionality of the entire network [1], [10]. Probabilistic models for studying interdependent networks has also been proposed. For instance, models based on the theory of branching processes and mean-field theory have been presented in [11] and [12] to model cascading failures in coupled infrastructure systems in an abstract setting.

It is also important to clarify that the IDMC model, presented in this paper, is different from the influence model [13]. In the latter model, a MC is associated with each node of the network and the evolution of each chain can be influenced both by nodes' internal dynamics as well as the evolution of the chains at their neighboring sites. While the idea of the current paper (that transitions in one MC is affected by transitions in other MCs) is similar to that of the influence model, the focus of the influence model is on the effects of network topology on the overall stochastic dynamics based on linear interactions among large number of network nodes; on the other hand, the IDMC framework focuses on the stochastic dynamics of systems, instead of individual nodes, by associating a MC to each system and allowing for linear and non-linear interactions of few non-networked (but interdependent) MCs.

SECTION III. Review of MC Model for Cascading Failures

Due to its relevance to the IDMC development, we begin by reviewing germane aspects of a MC model for cascading failures that was introduced in our earlier work [5]. This MC has state-dependent transition probabilities that are parameterized by certain physical and operating characteristics of the system. Although the model in [5] was presented in the context of power grids, we believe this model is sufficiently general to model epidemic spread and cascading failures in a high-level setting in various complex systems beyond power systems. In its simplified form considered in this paper, the state space of the MC aggregately represents the state of a physical system (e.g., a power grid) using two state variables: number of failures in the system, F , and the susceptibility (alternatively, stability) of the system to further failures, I . We term the latter binary state variable the *cascade-stability variable*, where $I = 1$ indicates a cascade-stable state and $I = 0$ indicates otherwise. The state variable I can collectively capture various physical attributes of the system affecting cascading failures beyond the number of failures. For instance, in a power grid the physical and control attributes of the system specify whether a power-grid state is cascade-stable or not. The state variable I also serves to specify the *absorbing* ($I = 1$) and *non-absorbing* (or transitory) (I=0) states of the MC. Presence of multiple absorbing states in the MC enables the modeling of various sizes of failures. Here, cascading failures are thought of as sequences of transitions in the state of the system, each due to a single failure. The single-failure-per-transition assumption is justified whenever time is divided into small intervals each allowing at most a single failure event.

Of particular importance is the assumption that the transition probabilities of the MC are state dependent. This enables modeling various operating scenarios for the system when failures accumulate; it also facilitates capturing different phases of cascading failures such as the escalation and onset phases with different rates of failures. Specifically, we define the cascade-stop probability, P_{stop} , as the probability of transiting from a state with F_i failures and $I = 0$ to a state with the same number of failures and $I = 1$. The cascade-stop probability is a function of F_i , i.e., $P_{\text{stop}}(F_i)$, and it completely characterizes the MC and the cascading-failure behavior of the system. In [5], we estimated $P_{\text{stop}}(F_i)$ using power-system simulations. In Fig. 1, we show the structure of the MCs for each of the two interdependent systems.

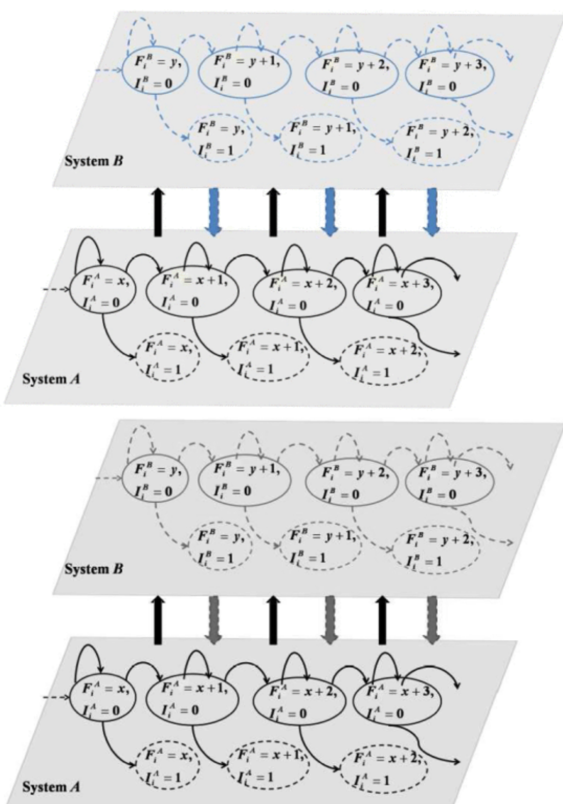


Fig. 1. Two MCs representing the stochastic dynamics of cascading failures in each of the interdependent systems and the coupling effect between MCs.

SECTION IV. Interdependent Markov Chains (IDMC)

For simplicity, we describe the IDMC approach using two interdependent systems, A and B , with the understanding that the same approach can be applied to any finite number of interdependent systems. We assume that the stochastic dynamics of cascading failures in each of the systems is modeled by a MC similar to that described in Section III and shown in Fig. 1. In these MCs, the state of each system at discrete time $n \geq 0$ is denoted by the number of failures and the stability variable, e.g., $S_n^A = (F_n^A, I_n^A)$ for system A . The state-space of the MC for system A is denoted by $\mathcal{S}_A$, where $\mathcal{S}_A = \{1, \dots, m_A\} \times$ {absorbing,non-absorbing} and m_A is the number of components in system A . For simplicity, the state space of the MC for system B is assumed to have the same structure as that for system A ; however, in general the state spaces of the individual MCs can be different. The random processes X_n and Y_n represent the state of the systems A and B , respectively, at discrete time $n \geq 0$.

As alluded to in the Introduction Section, a naïve and incorrect way to couple two MCs is to develop a *combined MC* with a state space $\mathcal{S}_C$ formed by the Cartesian product of the state spaces of the MCs associated with the individual systems, namely $\mathcal{S}_C = \mathcal{S}_A \times \mathcal{S}_B$. The shortcoming of this construction is that the transition probabilities among the states in $\mathcal{S}_C$ are not determined solely by the transition probabilities of the individual MCs, and the combined process is not guaranteed to be a MC as seen from the following example.
Let $X_1, X_2, \dots$ be an independent and identically distributed (i.i.d.) sequence, and let the process Y_n be defined as $Y_n = X_{n-1} + X_{n-2}$ for $n > 2$ and $Y_2 = Y_1 = X_1$. The process Y_n is Markov because

$$\begin{aligned} \mathrm{P}\{Y_n | Y_{n-1}, \dots, Y_1\} &= \mathrm{P}\{X_{n-1} + X_{n-2} | X_{n-2} + X_{n-3}, \dots, X_1\} \\ &= \mathrm{P}\{X_{n-1} + X_{n-2} | X_{n-2} + X_{n-3}\} = \mathrm{P}\{Y_n | Y_{n-1}\}. \end{aligned} \quad (1)$$

However, the process (X_n, Y_n) is not Markovian because

$$\begin{aligned} &\mathrm{P}\{Y_n, X_n | Y_{n-1}, \dots, Y_1, X_{n-1}, \dots, X_1\} \\ &= \mathrm{P}\{X_{n-1} + X_{n-2}, X_n | X_{n-2} + X_{n-3}, \dots, X_1, X_{n-1}, \dots, X_1\} \\ &= \mathrm{P}\{Y_n, X_n | Y_{n-1}, X_{n-1}, X_{n-2}\}. \end{aligned} \quad (2)$$

The extra term X_{n-2} in the last line cannot be dropped, since Y_n is not independent of X_{n-1}, X_{n-2} and Y_{n-1} due to their common terms. In particular, the process (X_n, Y_n) is not Markov because of the interdependency between the individual processes. This means that even if we are able to model systems A and B individually by MCs, the interdependencies between the two systems can result in memory in the combined process, which prevents the combined chain from being Markov. Putting this observation in the context of interdependent systems, we observe that the stochastic dynamics of one system is affected by the dynamics of the other system, and as such, the one-step transitions in the whole system can be generally dependent on multiple previous transitions of its constituent subsystems. As a result, defining the new state space simply by the Cartesian product of the state spaces does not provide sufficient information to fully capture the interdependency between the systems and to characterize the transition probabilities of the combined process. The challenge in combining MCs to represent interdependent systems is to incorporate sufficient *memory* in each MC while keeping the complexity of the combined chain to a minimum.

A. State Space and Transition Probabilities

Based on the discussion in the previous subsection, where we pointed out that the interdependencies between two systems generally depend on the history of their dynamics, the stochastic dynamics of system A may in general depend on M_1 -step memory of system B and similarly system B may in general depend on M_2 -step memory of system A . For simplicity of notation and without loss of generality, we assume that $M_1 = M_2 = M$.

To capture the effects of the M -step memory in each of the systems, the transition probability function must be of the form $f: (\mathcal{S}_C)^M \times \mathcal{S}_C \rightarrow [0,1]$, where $(\mathcal{S}_C)^M$ captures the information from the current state (1-step memory) as well as the previous M -1 states. The last $\mathcal{S}_C$ in the domain of transition probability function f captures the destination space of transitions. To build the equivalent MC for the finite state machine with the state space $\mathcal{S}_C$ and transition probabilities that are functions of the previous $M-1$ states of the system, we need to extend the state space $\mathcal{S}_C$ to incorporate the memory, i.e., $\mathcal{S}_I = (\mathcal{S}_A \times \mathcal{S}_B)^M$. Due to the embedded memory in the definition of the states the size of the state space $\mathcal{S}_I$ can become prohibitively large in general. In the next subsection, we introduce a quantization approach to reduce the size of the state space of an M -step MC while capturing only the necessary memory.

B. Memory Quantization

We introduce a memory quantization approach for reducing the size of the state space of the IDMC by defining equivalence classes of behaviors for the dynamics of the systems denoted by $\mathcal{H}_I$. For instance, if we can deduce that a system is stable or unstable based on the history of its dynamics then we can categorize the history of the dynamics of the system into two classes of behaviors, i.e., $\mathcal{H}_I = \{\text{stable, unstable}\}$. We define a quantization function, $g: (\mathcal{S}_A \times \mathcal{S}_B)^M \rightarrow \mathcal{H}_I$, where $\mathcal{H}_I$ is a low-cardinality set comprising the equivalent classes of past behavior. Hence, the function g compresses the memory from the past into a small number of equivalence classes. With such quantization of memory, we can compressively represent the state space of the IDMC by $\mathcal{S}_I = \mathcal{H}_I \times (\mathcal{S}_A \times \mathcal{S}_B)$.

C. Capturing the Impact of Interdependencies

Up to this point, we have discussed capturing the knowledge of the state and memory of the system in an IDMC state in a compressive fashion. The next step is to propose a simple method for describing how a specific behavior of one system affects the behavior of the other system. At a coarse level, the effects of the behavior of one system on another system can be divided into: (1) improve, (2) worsen, and (3) do not change, where the precise meaning of these terms in the MC framework, can be identified with: (1) reducing the probability of an extra failure, (2) increasing the probability of an extra failure, and (3) not changing the probability of an extra failure. Hence, the impact of interdependencies can be captured in the transition probabilities by enforcing the mentioned effects based on the state of the system and the compressed history.

D. Interdependency Strength

We define the strength of interdependency between two systems based on two factors: (1) how much the knowledge of the behavior of a system affects the dynamics of the other system (e.g., the relative change in transition probabilities), and (2) how much memory is required to capture the interdependency effects. One way to quantify the first factor is presented in (3), which characterizes the maximum influence of system B with variable Y on system A with variable X and memory M .

$$\delta_X^{(M)}(Y) = \sup_{x_i, x_{i+1} \in \mathcal{S}_A, y_j \in \mathcal{S}_B, i-M+1 \leq j \leq i} \{|\mathrm{P}\{X_{i+1} = x_{i+1} | X_i = x_i\} - \mathrm{P}\{X_{i+1} = x_{i+1} | X_i = x_i, Y_i = y_i, \dots, Y_{i-M+1} = Y_{i-M+1}\}|\}. \quad (3)$$

Note that when δ is large the dependency is strong and when it is zero then system A is independent of system B . We define the strength of the interdependency based on the first measure to be $max(\delta_X^{(M)}(Y), \delta_Y^{(M)}(X))$. To quantify the second factor, we introduce the quantity $\mathcal{K}_X^{\epsilon}(Y)$as the smallest integer M such that for all $i, j > M$, $|\delta_X^{(i)}(Y) - \delta_X^{(j)}(Y)| < \epsilon$, where ϵ is a small positive number representing the sensitivity threshold for interdependency. Thus, for a fixed pre-specified ϵ , $\mathcal{K}_X^{\epsilon}(Y)$ is the minimum memory required to be considered for system A to capture its dependency on system B . We can simply define the interdependency between the two systems by the quantity $\mathcal{K}^{\epsilon} \triangleq max(\mathcal{K}_X^{\epsilon}(Y), \mathcal{K}_Y^{\epsilon}(X))$. Note that when $\mathcal{K}^{\epsilon}$ is large the interdependency is strong and $\mathcal{K}^{\epsilon} = 0$ means that the knowledge of the current state of system A and B would be enough in modeling the interdependencies. However, the latter does not imply that the two systems are independent.

E. Interleaving Approach

In this subsection, we present a refinement to the IDMC framework, termed the interleaving framework, which enables us to directly capture the "cause-and-effect" attributes in interactions between the two systems. The interleaving approach simplifies the modeling of interdependencies between the systems by capturing the immediate effects of each transition on the other system. In particular, in the interleaving approach we assume that the two systems take turns in changing their states. We call this approach interleaving as it interleaves the transitions in system A and B in this specific order. The interleaving approach adds an extra level of detail to the basic IDMC model. For example, consider a basic two-step memory IDMC, which is partially shown in Fig. 2. In this example we observe that in the sequence of transitions, a failure in system A and B is followed by two failures in system B and then a failure in system A again. Here, the effects of individual failures in system B on system A are not clear. For instance, it is possible that the first failure caused system A to become vulnerable while the second and third failure in system B added to this vulnerability, and therefore resulted in the failure in system A two steps down the line. Meanwhile, one can also conjecture that the first and second failures did not have any effect on system A but instead it was the third failure (in system B) that triggered the failure in system A . Note that the basic IDMC model cannot resolve such granularity; however, the interleaving approach allows us to access the scenarios described earlier by breaking the combined effects into a cause-and-effect scenarios, and hence, it is more informative. Note that any interleaving model can be reduced to a basic IDMC model. All the definitions for the IDMC model presented in Section IV are valid for the interleaving framework. However, to keep track of the transitions we use an auxiliary variable to describe which system is undergoing a transition at any given time (e.g., a binary variable for two interdependent systems).

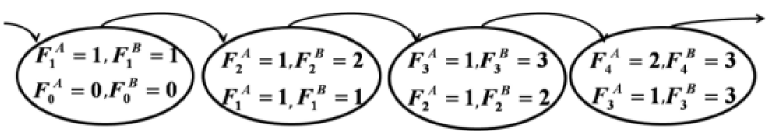


Fig. 2. An example of sequence of transitions in an IDMC model with two-step memory. In this example, we have only considered non-absorbing states.

SECTION V. An IDMC Model for Cascading Failures in Electric-Cyber Infrastructures

In this section, we develop an IDMC model using the interleaving approach, for the electric-cyber infrastructure with a focus on the interdependencies that fuel the propagation of failures. We refer to the power system by system A and the communication system by system B . To simplify the notation, we represent the number of failures in the power grid and in the communication system by the variables x and y , respectively. Further, we denote the probability of transiting to a stable power-grid state in the MC by $p(x)$ (i.e., $p(x) = P_{\text{stop}}(x)$ if we use the terminology of Section III). Also, in the MC representing the communication system, we denote the probability of transiting to a state with an extra failure by $q(y)$. Considering probabilistic failures in the interdependent communication network is specifically important to account for cases that failures in the power grid does not necessarily result in failures in the communication network (e.g., when islands form in the grid or the communication system has additional power supplies). Without loss of generality, we assume that the first failure occurs in the power grid and that cascading failures terminate only when the power grid enters a stable state (only the power-grid MC has the absorbing states).

A. State Space

To define the IDMC state space we need to understand how much memory of dynamics we need to track. We assume that when the communication system becomes less stable, by experiencing a new failure, then the power-grid becomes vulnerable to extra failures. Also, we assume that when the power grid experiences a new failure, then this may trigger a failure in the communication system according to probability q . Hence, we need to capture at least the last state of the combined MC, beyond the current state, to capture the effect of interdependencies on the next transition. Therefore, we assume that the minimum memory required to capture the dynamic behavior of the system is two, i.e., $M = 2$ and $\mathcal{S}_I = (\mathcal{S}_A \times \mathcal{S}_B)^2$. However, since we are assuming a single failure per transition, many states in $\mathcal{S}_I$ are not valid; thus, a small subset of $\mathcal{S}_I$ is actually needed. For example, for every number of failures at time n , there are only two possibilities for the number of failures at time $n - 1$. As such, the size of the state-space without the disallowed states is $4(\mathcal{N}_A\mathcal{N}_B)$, where $\mathcal{N}_A$ and $\mathcal{N}_B$ represent the cardinality of the state space of systems A and B , respectively. Finally, as we described in Section IV-E, we also need an auxiliary variable to keep track of the transitions in systems A and B as they take turns. This will cause the size of the state space to be $\mathcal{S}_I = 8(\mathcal{N}_A\mathcal{N}_B)$. The concept of interleaving MC for this system based on the above assumptions is depicted in Fig. 3.

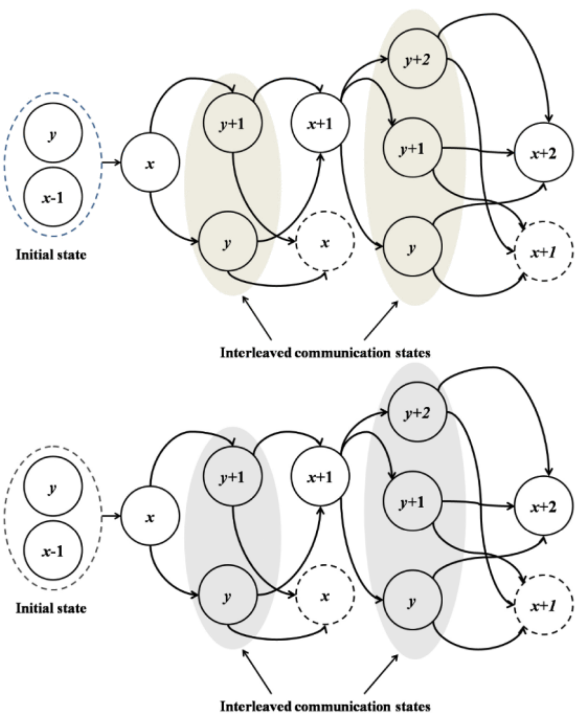


Fig. 3. The concept of the interleaving approach for coupling the MCs of the power grid and the communication system as described in Section V is depicted by interleaving the communication states among power-grid states.

Next, we define the state of the IDMC at discrete time n by $S_n = (X_n, I_n, Y_n, L_n, K_n)$, where X_n and I_n are the state variables of the power grid (respectively representing the number of failures and cascade stability indicator), and Y_n is the state of the communication system (number of failures). The variables L_n is the auxiliary variable that captures the 'transition turn' In the interleaving framework; specifically, $L_n = 0$ indicates that the last transition occurred in the power grid and and $L_n = 1$ indicates that the last transition occurred the communication system. Lastly, the binary variable K_n captures the memory of the dynamics by indicating whether any component failed in the last transition ($K_n = 1$ means a failure occurred at time $n-1$). Here, the memory quantization function g maps the history to $\mathcal{H}$ ={new failure, no new failure}. Although the size of $\mathcal{S}_I$ in this example is in the order of the size of the Cartesian product of the state-spaces we can analyze the steady-state behavior of this IDMC efficiently using difference equations as presented in Section V-D.

B. Transition Probabilities

Here, we characterize the transition probabilities of the IDMC for the electric-cyber infrastructure based on the transition probabilities of the individual MCs and the interdependencies between the two systems, as described next. In general, different power-dependency-on-communication functions and communication-dependency-on-power functions can be considered. In this paper, we consider the following dependency functions to model the interactions between the two systems.

First, we define a power-dependency-on-communication function by $d: \{0,1,2, \ldots, m_B\} \rightarrow [0,1]$, where m_B denotes the number of components in the communication system. The function d describes the dependence of the reliability of the power system on the state of the communication system. Specifically, depending on the number of failed components in the communication system (where components in communication system include routers, switches and communication links) the reliability of the power system will be reduced. As an example, if $d(y_n) = 0.5$ then the probability that the cascading failures stops in the power grid in the next transition will be reduced to half. The function d modifies the transition probabilities for the power grid based on the dynamics of the communication MC as shown in (4). There are two extreme values for d . If the failure in the communication system does not affect the power grid then $d = 1$; on the other hand, when the failure of a communication component results in a failure in the power grid deterministically then $d = 0$. In general, the closer the value of d is to zero the more reduction occurs in the reliability of the power grid. Note that function d can also depend on the state of the power grid. Similarly, to capture the communication-dependency-on-power we consider a function based on the assumption mentioned in Section V-A, which specifies that the transitions in the communication system depend probabilistically on whether or not there was a failure in the power grid in the last transition. This assumption leads to the binary dependency function, $\mathcal{J}: \{0,1\} \rightarrow \{0,1\}$, which means that the transition probabilities in the communication network will be modified to $\mathcal{J}(K_n)q(Y_n)$ and $\mathcal{J}(K_n)(1-q(Y_n)) + (1-\mathcal{J}(K_n))$ corresponding to the cases of having an extra failure and not having an extra failure, respectively. For instance, if $\mathcal{J}(K_n) = 1$ then the probability of having extra failure in the communication system is $q(Y_n)$ and zero otherwise. As such, the transition probabilities of the IDMC for the electric-cyber infrastructure from an state $S_n = (x_n, i_n, y_n, \ell_n, k_n)$ to state $S_{n+1} = (x_{n+1}, i_{n+1}, y_{n+1}, \ell_{n+1}, k_{n+1})$ are presented in (4).

$$
f(S_{n+1}|S_n) = \begin{cases}
 & \ell_{n+1} \neq \ell_n, y_{n+1} \neq y_n \\
1 & \text{(I)}: i_n = 1, x_{n+1} = x_n, \\
 & \ell_{n+1} = \ell_n, y_{n+1} = y_n \\
q(y_n) & \text{(II}.a\text{)}: i_n = i_{n+1} = 0, \\
 & \ell_n = 0, x_{n+1} = x_n \\
 & y_{n+1} = y_n + 1, \\
1 - q(y_n) & \text{(II}.b\text{)}: i_n = i_{n+1} = 0, \\
 & \ell_n = 0, x_{n+1} = x_n \\
 & y_{n+1} = y_n, \\
1 - \frac{p(x_n)d(y_n)}{k_n + d(y_n)(1-k_n)} & \\
 & \text{(III}.a\text{)}: i_n = i_{n+1} = 0, \\
 & \ell_n = 1, y_{n+1} = y_n, \\
 & x_{n+1} = x_n + 1 \\
\frac{p(x_n)d(y_n)}{k_n + d(y_n)(1-k_n)} & \\
 & \text{(III}.b\text{)}: i_n = 0, i_{n+1} = 1 \\
 & \ell_n = 1, y_{n+1} = y_n, \\
 & x_{n+1} = x_n \\
0 & \text{(IV)}: \text{otherwise.}
\end{cases} \quad (4)
$$

Next, we explain the elements of (4) individually.

- *Group I condition* implies that the IDMC is in an absorbing state because $i_n = 1$. When a system enters an absorbing state it never leaves it with probability 1.
- *Group II conditions* address the cases for which the next transition is in the communication network (as $\ell_n = 0$).
  - *Condition II.a* implies that in the previous transition there was a failure in the power system (as $k_n = 1$). Hence, the probability of a new communication failure (i.e., $\mathcal{I}(k_n)q(y_n)$) simplifies to $q(y_n)$.
  - *Condition II.b* is the complement of Condition II.a (no communication failure occurs in the next transition).
- *Group III conditions* address cases for which the next transition is in the power grid (as $\ell_n = 1$).
  - *Condition III.a* specifies the probability of having a failure in the power grid depending on the value of k_n (if there was a failure in the communication network or not). If $k_n = 1$ then the transition probability is $1 - p(x_n)d(y_n)$ and if $k_n = 0$then the transition probability is $1 - p(x_n)$.

- o *Condition III.b* addresses the case in which the next transition causes the system to enter an absorbing state. Similarly to Condition III.a, the probability of a transition to the stable state depends on the value of k_n.
- *Group IV conditions* captures transitions that are disallowed. Examples are having more than one failure in one transition or transitions between states with $\ell_{n+1} = \ell_n$ and $i_n \neq 1$ (based on the interleaving assumption).

In this setting, the strength of dependency of the power system on the communication system is $\delta_X^{(2)}(Y) = \max_{x_n \in \mathcal{S}_A, y_n \in \mathcal{S}_B} p(x_n)(1 - d(y_n))$. Similarly, the strength of dependency of the communication system on the power system is $\delta_Y^{(2)}(X) = \max_{y_n} q(y_n)$. To illustrate the IDMC model for the electric-cyber infrastructure, a portion of the MC with its transition probabilities among the states is shown in Fig. 4.

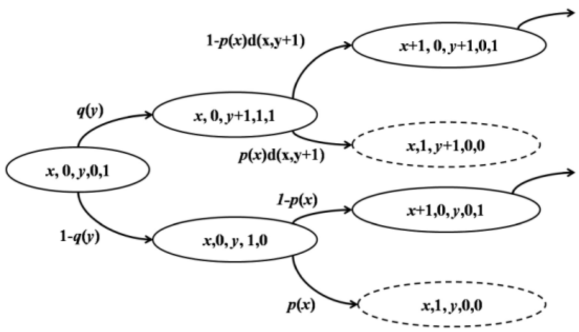


Fig. 4. Portion of the IDMC model for the power and communication system with transition probabilities among the states.

C. Steady-State Solution

In this section, we derive a system of difference equations describing the steady-state solution of the IDMC for the electric-cyber infrastructure. We begin by introducing $\mathcal{P}_{s_i}(s)$ as the asymptotic probability of reaching state $s \in \mathcal{S}_I$ from the initial state $s_i = (x_0, 0, y_0, 0,1)$; that is, the initial state has x_0 failures in the power grid and y_0 failures in the communication network and the last failure is assumed to be in the power grid. We further assume that $y_0 \leq x_0$. We are particularly interested in the probability of reaching the stable states in the power grid in which cascading failures terminate; these states have the form $\tilde{s} = (x, 1, y, 0,0)$ and it would be convenient to denote $\mathcal{P}_{s_i}(\tilde{s})$ by $\mathcal{F}(x, y)$. Similarly, for transient power-grid states, i.e., $\hat{s} = (x, 0, y, 0,1)$, we denote the probability of reaching the transient power grid state $\mathcal{P}_{s_i}(\hat{s})$ by $\mathcal{G}(x, y)$. Next, we present a characterization of $\mathcal{F}(x, y)$ and $\mathcal{G}(x, y)$.

Theorem 1:

Suppose that the initial state of the coupled system is $s_i = (x_0, 0, y_0, 0,1)$. For $x_0 \leq x \leq m_A$, $y_0 \leq y \leq m_B$, $y_0 \leq x_0$, and nonzero functions $p(.)$, $q(.)$ and $d(.)$, the following recursions hold:

$$\begin{aligned} \mathcal{F}(x,y) = \quad & \alpha_1(x,y)\mathcal{F}(x-1,y) + \alpha_2(x,y)\mathcal{F}(x-1,y-1) \\ & +\alpha_3(x,y)\mathcal{G}(x-1,y-1), \text{and} \\ \mathcal{G}(x,y) = \quad & \alpha_4(x,y)\mathcal{F}(x-1,y) + \alpha_5(x,y)\mathcal{G}(x-1,y-1), \end{aligned} \tag{5}$$

where the coefficients are given by

$$
\begin{aligned}
\alpha_1(x,y) = &\ \frac{p(x)(1-q(y))(1-p(x-1))}{p(x-1)}, \\
\alpha_2(x,y) = &\ \frac{p(x)d(y)q(y-1)(1-p(x-1)d(y-1))}{p(x-1)d(y-1)}, \\
\alpha_3(x,y) = &\ (1-q(y-1))p(x)d(y)q(y-1) \\
&\left((1-p(x-1)) - \frac{(1-p(x-1)d(y-1))}{d(y-1)}\right) \\
&+q(y-1)p(x)(1-q(y))(1-d(y)), \\
\alpha_4(x,y) = &\ \frac{(1-p(x-1))}{p(x-1)}, \text{and} \\
\alpha_5(x,y) = &\ q(y-1)(((1-p(x-1)d(y))) \\
&-\frac{d(y)p(x-1)(1-p(x-1))}{p(x-1)}),
\end{aligned}
\tag{6}
$$

and where the boundary conditions are given by

$$
\begin{aligned}
\mathcal{F}(x_0,y_0) = &\quad (1-q(y_0))p(x_0), \\
\mathcal{F}(x_0,y_0+1) = &\quad q(y_0)p(x_0)d(y_0+1), \text{and} \mathcal{G}(x_0,y_0) = 1.
\end{aligned}
\tag{7}
$$

Proof of the theorem is presented in the Appendix.

Remark:

The coefficients in (6) can be simplified in three special cases as follows.

1. α_1 and α_4 are the same as (6) but

$$
\begin{aligned}
\alpha_2(x,y) = &\ \frac{p(x)d(y)q(y-1)(1-p(x-1))}{p(x-1)}, \\
\alpha_3(x,y) = &\ q(y-1)p(x)\big(1-q(y)\big)\big(1-d(y)\big), \text{ and} \\
\alpha_5(x,y) = &\ 0.
\end{aligned}
\tag{8}
$$

2. all α values are zero except for α_2, which is given by

$$
\alpha_2(x,y) = \frac{d(y)p(x)q(y-1)(1-p(x-1)d(y-1))}{p(x-1)d(y-1)}. \tag{9}
$$

3. the coefficients are given by

$$
\begin{aligned}
\alpha_1(x,y) = &\ \frac{p(x)(1-q(y))(1-p(x-1)d(y))}{p(x-1)d(y)}, \\
\alpha_2(x,y) = &\ \frac{p(x)d(y)q(y-1)}{p(x-1)d(y-1)}, \\
\alpha_3(x,y) = &\ (1-q(y-1))d(y)p(x)q(y-1) \\
&\times\left((1-p(x-1)) - \frac{1}{d(y-1)})\right), \\
\alpha_4(x,y) = &\ \frac{(1-p(x-1))}{p(x-1)}, \text{ and} \\
\alpha_5(x,y) = &\ (1-d(y))q(y-1).
\end{aligned}
\tag{10}
$$

Theorem 1 enables the direct calculation of the probability of cascade size in each system. Specifically, note that $\mathcal{R}_p(x) = \sum_{y:0\le y\le x} \mathcal{F}(x,y)$ and $\mathcal{R}_c(y) = \sum_{x:y\le x\le N_1} \mathcal{F}(x,y)$ are the probabilities of the cascade with x terminal failures in the power grid and y terminal failures in the communication system, respectively. The time complexity of the numerical calculations of (5) is in the order of the number of components in the system, which makes the model scalable to large systems even though the analytical state space of the problem is large. We wish to emphasize that to apply this framework to real-world systems such as power grids, one can study historical data and simulations to extract and estimate the parameters of the model, in particular, the transition probabilities of the individual MCs and the dependencies between the systems. By estimating such parameters, the presented framework will provide reliability evaluations based on the features of the systems under study. Similar studies have been presented in Section V-D and [5].

D. Results

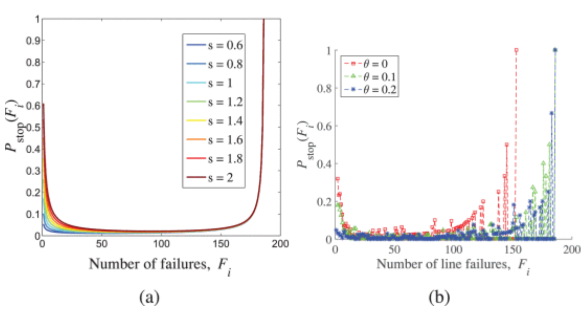
Here, we present numerical results for $\mathcal{R}_p(x)$ and $\mathcal{R}_c(y)$ and identify a function d that leads to unreliable behavior in the coupled systems. Before presenting our results, let us introduce our definition of reliable and unreliable systems.

1) Reliable and Unreliable Systems:

We term a system for which the probability distribution of the cascade size follows an exponential distribution a *reliable* system since the probability of large cascades is small compared to heavy-tail distributions. We term a system with a heavy-tail distribution of the cascade size an *unreliable* system. Particularly, we consider two types of unreliable systems: (*type* i) a system for which the probability mass function (PMF) of the cascade size follows power-law distribution, and (*type* ii) a system for which the PMF of the cascade size has a hump at the tail. Next, we introduce these systems in more details. Consider the MC of system A in Fig. 1. Let $B(n|S_i)$ represent the probability of a cascade size n , conditional on the initial state of the system. A recursion for $B(n|S_i)$ is given by:

$$B(n|S_i) = \frac{P_{\text{stop}}(n)(1-P_{\text{stop}}(n-1))}{P_{\text{stop}}(n-1)} B(n-1|S_i). \quad (11)$$

If we assume that the initial state of the power grid has one failure with $I = 0$, i.e., $S_0 = (1,0)$, then the boundary condition for (11) is $B(1|S_0) = P_{\text{stop}}(1)$. Based on (11), if the transition probabilities of the MC were constant then standard analysis of the recursion tells us that the PMF of the blackout size follows an exponential distribution. Therefore, our definition of reliable systems implies that they have constant transition probabilities. Similarly, using (11) it can be shown that in an unreliable system of type ifor which the stochastic dynamics of cascading failures are modeled by the MC in Fig. 1, the PMF of the cascade size will follow the Zipf's law (a discrete power-law distribution) whenever the transition probabilities follow specific bowl-shape forms represented in Fig. 5-a. The PMF of Zipf's law distribution is given by $P(x,s,k) = 1/\left(\sum_{i=1}^{k} i^{-s}\right)x^{-s}$, where s is a free parameter of the distribution and k is the total number of components in the system. The details of derivation of the bowl-shape functions have been discussed in [16] and [17].



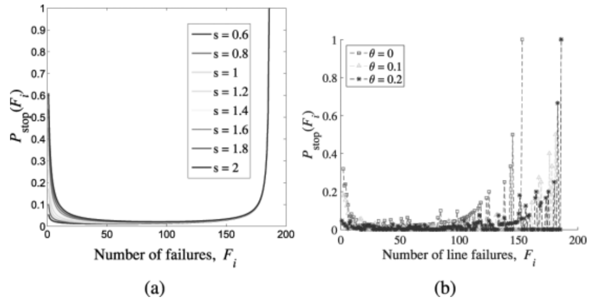


Fig. 5. (a) The bowl-shape forms for the transition probabilities of the MC in Fig. 1, i.e., $P_{\text{stop}}(F_i)$ functions, which result in the Zipf's law PMF of the cascade size for various s ; (b) the bowl-shape forms for $P_{\text{stop}}(F_i)$ based on the power-system simulations adopted from [5], which result in a hump in the tail of the distribution.

Further, using (11) we can show that an unreliable system of type *ii* for which the stochastic dynamics of cascading failures are modeled by the MC in Fig. 1 has the following property. If the transition probabilities follow the bowl-shape functions obtained using the power-system simulations then the PMF of the cascade size will have a hump at the tail, as shown in [5, Fig. 11]. The simulations are based on quasi-static simulation of the power grid using MATPOWER [15] and the results are depicted in Fig. 5-b for $P_{\text{stop}}(n)$. The described hump at the tail of PMF represents stress over the system, which leads to shift of the probability mass to the tail of the distribution. Particularly, the size of the hump and the behavior of the bowl-shape function depend on the operating characteristics of the grid. In [5], we introduced the loading level on the power system, load-shedding constraints and the transmission line-tripping threshold as examples of such operating characteristics. For instance, the load-shedding constraints, defined as the ratio of the uncontrollable loads (loads that do not participate in load shedding) to the total load in the power grid, affect $P_{\text{stop}}(n)$ as shown in three plots for three scenarios in Fig. 5-b. Certain operating settings for the system can be associated with high-level of stress, for instance, the scenarios associated with the blue plot in Fig. 5-b. For a discussion on the properties of simulation-based bowl-shape functions for $P_{\text{stop}}(n)$ refer to [16] and [17].

2) Impact of Interdependency on the Probability Distribution of the Cascade Size:

We consider 186 components in the power grid and in the communication system (number of links in IEEE 118 case). We only focus on the number of components and do not directly use the topological and system information in cascading failures; however, the latter information is automatically captured in the transition probabilities through the parameters [5]. The PMFs of the cascade size in the power grid for an unreliable system of type i and a reliable system are shown in Fig. 6-a as solid red and dashed lines, respectively.

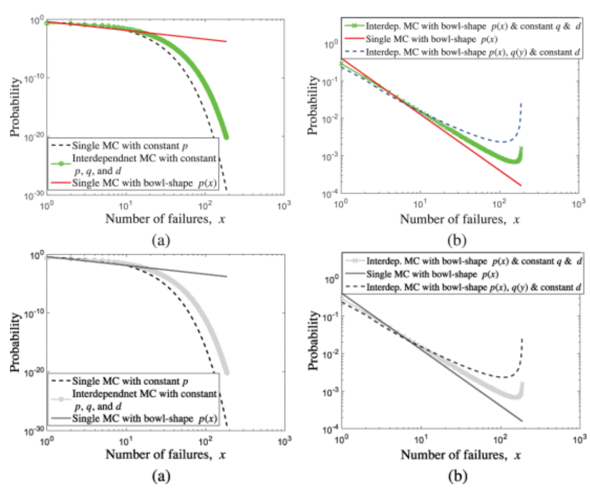


Fig. 6. PMF of the failure size (a) in the power-grid, $\mathcal{R}_p$, for a reliable power system (the dashed line), an unreliable power grid of type i (solid line) and a reliable IDMC with constant p , q , and d in (green line); (b) in the power-grid, $\mathcal{R}_p$, in an IDMC model for various scenarios of unreliable systems of type i , in a log-log scale.

In order to show that the interdependencies between two systems affect the PMF of cascade size, we use the IDMC model with constant p , q and d , which imply that the communication system and the power grid are assumed to be reliable systems individually. The PMF of the cascade size in the power grid, i.e., $\mathcal{R}_p$, for such an interdependent system is depicted in Fig. 6-a with a green line. Based on this result we observe that although both the single MC for the power grid and the IDMC model result in an exponential cascading behavior in the power grid, the probability of large cascades in the power grid intensifies in the IDMC model due to the interdependencies between the two systems.

Next, we assume that we have two interdependent systems, for which either one or both of the systems are unreliable. Recall that in Fig. 6-a we observed that an individual unreliable system of type i will have a power-law distribution represented by a solid red line. We have re-plotted the red solid line from Fig. 6-a in Fig. 6-b as a benchmark to evaluate the effects of interdependencies. We begin by assuming that the power grid is an unreliable system of type i and the communication system is a reliable system with constant q . Next, the distribution of the blackout size in the power grid, i.e., $\mathcal{R}_p$, is calculated using (5) and the results are shown in Fig. 6-b. The results suggest that due to the interdependencies the probability of large cascades in the IDMC case is elevated compared to a single MC while the heavy tail characteristic of the PMF is preserved except at the end (criticality of interdependencies).

Next, we assume that the power grid and the communication system are both unreliable systems of type i . In particular, for the communication system we consider a bowl-shape function from Fig. 5-a for $(1 - q(y))$. The results are shown by the dashed line in Fig. 6-b, and they suggest that the impact of an unreliable communication system is severe on the reliability of the whole system. In both of the interdependent examples presented here, the function d is assumed to be constant and equal to 0.4. Similarly to the results in Fig. 6-b, in Fig. 7-a we present the distribution of the failure size in the communication system, i.e., $\mathcal{R}_c$. These results also suggest that unreliable behavior of one or both of the systems intensifies the probability of large cascade sizes in the whole system.

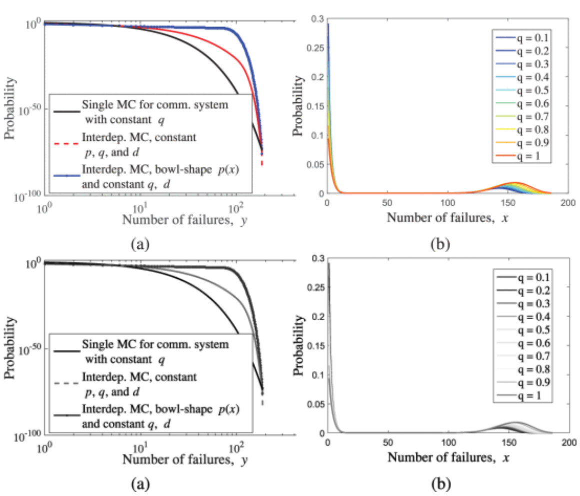


Fig. 7. PMF of the failure size, i.e., (a) $\mathcal{R}_c$, in an IDMC model for various scenarios of unreliable systems of type i ; (b) $\mathcal{R}_p$, for an IDMC model with unreliable power grid of type ii, in a log-log scale.

In Fig. 7-b, we assume that the power grid is an unreliable system of type ii, while the communication system is a reliable system with constant q . The results show that the interdependency between the two systems preserves and increases the size of the hump at the tail of the distribution as the reliability of the communication system changes through variable q . The results shown in Fig. 7-b generalize the results of our earlier work [5] (for a power system) to interdependent systems.

3) Individually Reliable Systems Can Behave Unreliably When Coupled:

We assume that we have two reliable systems with constant $p(x)$ and $q(y)$. We couple the two reliable systems using the IDMC framework. Recall that based on the green line in Fig. 6-a, two reliable systems with constant function d result in a reliable system with exponential PMF of the cascade size; however, this is not always true if function d is not constant. In this study, we want to answer the following critical question: can two coupled reliable systems form a single unreliable

system? To answer this question, we use (5) to find a function $d(y)$ such that the two reliable power and communication systems behave unreliable and result in an unreliable system, say of type i . To find such $d(y)$, we set $\mathcal{R}_p(x) = P(x, s, m_A)$, where P denotes the Zipf's law distribution with $s = 1.1$ and m_A is the total number of components in the system. The above equality means that we want that the PMF of the cascade size for the power grid follows the Zipf's law distribution for every value of $x \in \{1, \ldots, m_A\}$. This results in m_A non-linear equations for $d(y)$. We have solved this system of non-linear equations numerically using an optimization approach, which minimizes the distance between $\mathcal{R}_p(x)$ and the cascade size distribution resulted from the system of difference equations with the constraint $0 \leq d(\cdot) \leq 1$. In Fig. 8-a, we have shown the result of the distribution of the cascade size in a system, say the power grid, with 20 components when the individual systems are reliable but the distribution of the failure size in the coupled power grid approximates the power-law distribution. This result is represented by a dashed line in Fig. 8-a. We have also represented the distribution of the failure size when function d is constant and equal to 0.27 in the blue solid line, which results in exponential distribution.
The $d(y)$ values that result in the unreliable behavior for the two reliable systems are presented in Fig. 8-b. Note that the mean of these values is also equal to 0.27, which implies that not only the values of d but also their distribution in the function affect the reliability of the interdependent system. The key results in Fig. 8-a show that two individually reliable systems may behave unreliably when coupled due to interdependencies.

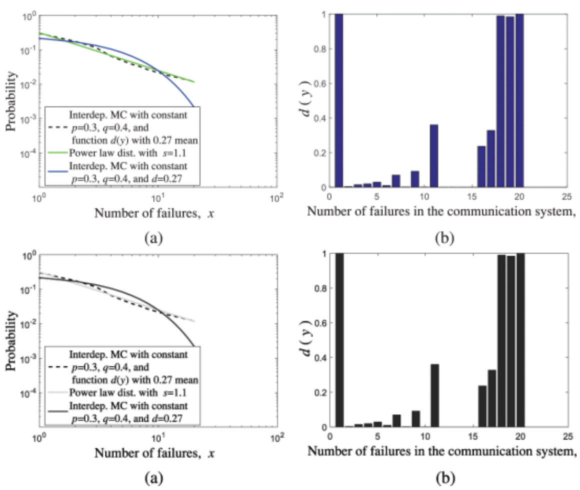


Fig. 8. (a) PMF of the failure size in the power grid, $\mathcal{R}_p$, in an IDMC model, when two reliable systems are coupled; (b) $d(y)$ values resulting in an unreliable behavior (power-law) for the two coupled reliable systems.

SECTION VI. Conclusion

We presented a novel IDMC framework for modeling cascading failures in interdependent infrastructures by developing a minimal MC that encompasses the individual MC for each physical system and their interdependencies. We presented the IDMC framework in a general setting and then, as a specific example, constructed an IDMC model for cascading failures in electric-cyber infrastructures. We studied various scenarios of reliable and unreliable systems to characterize the distribution of the failure size in coupled systems. A key insight obtained from the IDMC model is that interdependencies between two systems can make two reliable systems, i.e., systems with exponentially distributed failure sizes, behave unreliably with power-law failure-size distributions when put together.

Appendix Proof of Theorem 1

For simplicity of notation, we denote the asymptotic probability of reaching a state, say $s = (x, i, y, \ell, k)$, from the initial state s_i by $\mathcal{P}(x, i, y, \ell, k)$.

Based on the structure of the IDMC model introduced in Section V, which is partially shown in Fig. 4, as well as the transition probabilities introduced in (4), we write $\mathcal{P}(x, 1, y, 0, 0)$ as

$$\begin{aligned}
&\mathcal{P}(x,1,y,0{,}0) = p(x)\mathcal{P}(x,0,y,1{,}0) + p(x)d(y)\mathcal{P}(x,0,y,1{,}1)\\
&= p(x)(1-q(y))\mathcal{P}(x,0,y,0{,}1) + p(x)d(y)q(y)\mathcal{P}(x,0,y-1{,}0{,}1)\\
&= p(x)(1-q(y))((1-p(x-1))\mathcal{P}(x-1{,}0,y,1{,}0)\\
&+(1-p(x-1)d(y)\mathcal{P}(x-1{,}0,y,1{,}1)))\\
&+p(x)d(y)q(y)((1-p(x-1))\mathcal{P}(x-1{,}0,y-1{,}1{,}0)\\
&+(1-p(x-1)d(y-1))\mathcal{P}(x-1{,}0,y-1{,}1{,}1)),
\end{aligned} \tag{12}$$

where in the first line $\mathcal{P}(x,1,y,0{,}0)$ has been written based on the asymptotic probability of reaching to the two possible previous states and the second line is derived by repeating the previous step. We also know that

$$\begin{aligned}
\mathcal{P}(x-1{,}1,y,0{,}0) = p(x-1)\mathcal{P}(x-1{,}0,y,1{,}0)\\
+p(x-1)d(y)\mathcal{P}(x-1{,}0,y,1{,}1),
\end{aligned} \tag{13}$$

and similarly,

$$\begin{aligned}
\mathcal{P}(x-1{,}1,y-1{,}0{,}0) = p(x-1)\mathcal{P}(x-1{,}0,y-1{,}1{,}0)\\
+p(x-1)d(y-1)\mathcal{P}(x-1{,}0,y-1{,}1{,}1).
\end{aligned} \tag{14}$$

Now, if we substitute (13) and (14) in (12) then we have

$$\begin{aligned}
&\mathcal{P}(x,1,y,0{,}0) = p(x)(1-q(y))((1-p(x-1))(\mathcal{P}(x-1{,}1,y,0{,}1)\\
&-p(x-1)d(y)\mathcal{P}(x-1{,}0,y,1{,}1))/p(x)\\
&+(1-p(x-1)d(y)\mathcal{P}(x-1{,}0,y,1{,}1)))\\
&+p(x)d(y)q(y)((1-p(x-1))\mathcal{P}(x-1{,}0,y-1{,}1{,}0)\\
&+(1-p(x-1)d(y-1))(\mathcal{P}(x-1{,}1,y-1{,}0{,}1)\\
&-p(x-1)\mathcal{P}(x-1{,}0,y-1{,}1{,}0))/(p(x)d(y-1))).
\end{aligned} \tag{15}$$

Next, we simplify (15) and substitute the definition of $\mathcal{P}(x-1{,}0,y-1{,}1{,}0)$ and $P(x-1{,}0,y,1{,}1)$ based on $P(x-1{,}0,y-1{,}0{,}1)$ in (15). As mentioned earlier, we denote $P(x,1,y,0{,}0)$ by $F(x,y)$ and $P(x,0,y,0{,}1)$ by $G(x,y)$. A such, after the simplifications of (15) we can write

$$\begin{aligned}
\mathcal{F}(x,y) = \quad & \alpha_1(x,y)\mathcal{F}(x-1,y) + \alpha_2(x,y)\mathcal{F}(x-1,y-1)\\
& +\alpha_3(x,y)\mathcal{G}(x-1,y-1),\\
\mathcal{G}(x,y) = \quad & \alpha_4(x,y)\mathcal{F}(x-1,y) + \alpha_5(x,y)\mathcal{G}(x-1,y-1),
\end{aligned} \tag{16}$$

where its coefficients are functions of $p(.)$, $q(.)$ and $d(.)$. This proves the general case in Theorem 1. Based on the structure of the presented IDMC model, there are three special cases that the coefficients do not follow the general case presented in (6). This is because certain states do not have all the previous states that we used in the derivation of the above difference equations. For instance, when $y = 1$ the state $s = (x-1{,}0,y-1{,}1{,}1)$ in the above equations does not have any previous states. Similarly, the cases where $y = x$ and $y = x-1$ need to be considered as special cases due to the assumption that communication failures are triggered by power failures and thus certain states are not possible as previous states. This is because we cannot have more communication failures than power-grid failures in the system based on the assumptions of the model. The derivation of the difference equations for special cases is similar to the general case and thus have been omitted here.

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