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CATALYTIC IGNITION BY EXTERNAL ENERGY FLUX: STEADY STATE ANALYSIS

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This paper deals with the steady state catalytic ignition of premixed gases over a flat plate of finite thickness. An external heat flux is added to the plate to facilitate the ignition process. The influence of the finite thermal conductivity of the plate on the catalytic ignition is evaluated in the two distinct limits of the reduced Damköhler number of order of unity and much less than unity. In the case of the reduced Damköhler number of order of unity, the chemical reaction is important over the whole length of the plate. The results obtained previously without external energy flux are reproduced. In the case of the reduced Damköhler number much less than unity, which is of practical interest, the chemical reaction is important only in a narrow zone close to the trailing edge, except for very high thermal conductivity of the plate. Analytical expressions are obtained for the critical external heat flux required to produce the catalytic ignition.

Introduction

In recent years experimental and theoretical works on catalytic combustion have received attention in the literature. This combustion process represents an alternative to conventional burners, especially in gas turbine combustion chambers. NO_x emissions can be greatly reduced using catalytic combustion. There are two types of catalytic combustion, depending on the onset of the gas-phase reaction. In diffusive combustion, species conversion is produced only by the surface reaction and thus is controlled by diffusion. In both types of combustion, it is necessary to achieve catalytic ignition. Catalytic ignition on a flat plate has been the object of several studies. Artyuk *et al.*[1] using the local similarity concept, solved the governing equations for an adiabatic plate numerically. Lindbergh and Schmitz[2,3] studied numerically the surface ignition process for a flat plate and wedge type boundary layer flows. These analyses were made for the limiting cases of adiabatic and perfectly conducting plates. Mihail and Teodorescu[4] used a refined numerical analysis to solve the integral governing equations applying the Lighthill approximation for high Prandtl and Schmidt numbers.[5] Ahluwalia and Chung[6] analyzed the same problem but they solved the integral governing equations through the erroneous utilization of the Laplace method. These studies for an adiabatic plate show the transition from a kinetically controlled process close to the leading edge to a diffusion controlled process downstream. Liñán[7] showed that this transition occurs abruptly at a well defined distance if the ratio of the activation energy of the catalytic reaction to the thermal energy is large enough. This transition has a universal character. In a very recent paper, Liñán and Treviño,[8] using asymptotic methods, studied the ignition and extinction of the catalytic reaction in the flow of a reacting mixture over a flat plate, with inclusion of longitudinal heat transfer through the plate due to finite values of the thermal conductivity. It was found that the critical Damköhler number for ignition is not strongly affected by this axial heat conduction. However the finite thermal conductivity has a strong influence on the extinction process. In general, the ratio of the Damköhler number for ignition to that of extinction is a very large number for the high activation energy of the catalytic surface reaction. Thus, it is convenient to work with surface Damköhler numbers well below that for ignition, and therefore requiring some external heat sources in order to produce catalytic ignition.

The objective of this paper is the study of the catalytic steady state ignition process in a flat plate boundary layer flow with an external heat flux applied to the plate.

Formulation

The physical model analyzed is the following. A gaseous combustible mixture flows parallel to a catalytic plate of finite thickness and thermal conductivity. The catalytic plate is heated externally to achieve catalytic ignition. Using the Lighthill approximation[5] for high Prandtl (Pr) and Schmidt (Sc) numbers (this gives good results for Pr and Sc of order of unity) and assuming no chemical reactions and quasisteady behavior in the gas phase, the governing integro-differential equations are given by

$$-k_0 C^n \exp\left(-\frac{Ta}{T}\right) = 0.332D\sqrt{\frac{u_\infty}{\nu x}}\,Sc^{1/3}\int_{C_\infty}^{C} K(\bar{x},x)d\bar{C} \quad (1)$$

$$-\rho_p c_p d\frac{\partial T}{\partial t} + \lambda_p d\frac{\partial^2 T}{\partial x^2} + (-\Delta H)k_0 C^n \exp\left(-\frac{Ta}{T}\right) + q_e = 0.332\lambda\sqrt{\frac{u_\infty}{\nu x}}\,Pr^{1/3} \cdot \left\{\int_{T_l}^{T} K(\bar{x},x)d\bar{T} + T_l - T_\infty\right\} \quad (2)$$

with the kernel $K(\bar{x},x)$ given as

$$K(\bar{x},x) = \left\{1 - \left(\frac{\bar{x}}{x}\right)^{3/4}\right\}^{-1/3}$$

In the above equations, a one-step catalytic reaction of the Arrhenius type is assumed. A uniform plate temperature in the transverse direction is also assumed, with the analysis being valid for values of the plate thickness, d, such as

$$d << 3\lambda_p\sqrt{L}Pr^{2/3}(c_p\sqrt{u_\infty\rho_\infty\mu_\infty})^{-1}.$$

k_0 represents the pre-exponential term of the catalytic reaction; n is the reaction order; Ta is the activation temperature of the reaction; T and C correspond to the temperature and the reactant concentration on the surface of the plate, respectively; $(-\Delta H)$ is the heat released per unit mole of fuel consumed; λ and λ_p correspond to the thermal conductivities of the gas and the plate, respectively; D is the mass diffusion coefficient; ν is the kinematic coefficient of viscosity; d corresponds to the thickness of the plate; u_∞, T_∞ and C_∞ are the velocity, temperature and reactant concentration of the free stream, respectively; T_l is the temperature of the plate at the leading edge; x corresponds to the longitudinal coordinate with the origin at the leading edge; ρ_p and c_p correspond to the density and the specific heat of the plate, respectively; $q_e(x,t)$ is the external heat flux per unit surface.

The initial and boundary conditions associated with equations (1) and (2) for adiabatic leading and trailing edges, are given by

$$T(x,0) = T_0(x)$$

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0 \text{ and } x = L \text{ for all } t. \quad (3)$$

Here L corresponds to the length of the plate.

Introducing the following non-dimensional variables and coordinates

$$\theta = \frac{T - T_\infty}{T_\infty\beta_1} \text{ with } \beta_1 = \frac{(-\Delta H)DC_\infty Sc^{1/3}}{T_\infty\lambda Pr^{1/3}};$$

$$Y = \frac{C}{C_\infty};\ \tau = \frac{0.332\lambda\sqrt{u_\infty}\,Pr^{1/3}}{\rho_p c_p d\sqrt{\nu L}}\,t;\ \chi = \frac{x}{L}, \quad (4)$$

the governing equations reduce to

$$-\frac{\delta}{\Gamma}Y^n \exp\left(\frac{\Gamma\theta}{1+\beta_1\theta}\right) = \frac{1}{\sqrt{\chi}}\int_1^Y K(\bar{\chi},\chi)d\bar{Y} \quad (5)$$

$$-\frac{\partial\theta}{\partial\tau} + q(\chi,\tau) + \alpha\frac{\partial^2\theta}{\partial\chi^2} + \frac{\delta}{\Gamma}Y^n\exp\left(\frac{\Gamma\theta}{1+\beta_1\theta}\right) = \frac{1}{\sqrt{\chi}}\left\{\int_{\theta_l}^{\theta}K(\bar{\chi},\chi)d\bar{\theta} + \theta_l\right\}, \quad (6)$$

where the non-dimensional parameters are defined as

$$\delta = \frac{k_0 C_\infty^{n-1}\sqrt{\nu L}\Gamma\exp(-Ta/T_\infty)}{0.332D\sqrt{u_\infty}Sc^{1/3}}$$

$$\Gamma = \frac{Ta\beta_1}{T_\infty}$$

$$\alpha = \frac{\lambda_p d\sqrt{\nu}}{0.332Pr^{1/3}\sqrt{u_\infty}L^{3/2}\lambda}$$

$$q = \frac{q_e\sqrt{\nu L}}{0.332Pr^{1/3}\sqrt{u_\infty}\lambda T_\infty\beta_1}. \quad (7)$$

The non-dimensional initial and boundary conditions are then reduced to

$\theta(\chi,0) = \theta_0(\chi)$

$$\frac{\partial\theta}{\partial\chi} = 0 \text{ at } \chi = 0 \text{ and } \chi = 1 \text{ for all } \tau. \quad (8)$$

The parameter β_1 gives the ratio of the energy released by the chemical reaction and the thermal energy of the free stream and in general is of the order of unity: Γ is the ratio of the activation energy of the catalytic reaction and the thermal energy of the mixture, and in combustion it is a large number; δ is the Damköhler number and represents the ratio of the residence time to the reaction time; α represents the ratio of the ability of the plate to carry heat in the streamwise direction and the ability of the gas to carry heat from the plate; q is the ratio of the added external heat flux and the heat flux transmitted to the gas-phase.

In this paper, for simplicity, only the steady state q constant case is analyzed. An extension to other cases can be easily made. The transient analysis is left for a forthcoming paper. As shown by Liñán and Treviño,[8] for zero external heat flux, the critical Damköhler number for ignition is 0.711 for $\alpha = 0$ and 0.736 for $\alpha \to \infty$. In general, for α of the order of unity, the critical Damköhler number for extinction is very small compared to unity. Therefore it is convenient to work in practical cases with values of $\delta << 1$. To achieve ignition, it is necessary to add a non-dimensional external heat flux q of the order of unity. However to study the transition from zero external heat flux, the analysis for values of δ of the order of unity is included.

Ignition Regime under Steady State Conditions

In the ignition regime, Y and θ differ little from the respective frozen values, $Y_f = 1$ and $\theta_f = q\phi$ where ϕ is derived from the energy equation without the reaction term

$$1 + \alpha\frac{d^2\phi}{d\chi^2} = \frac{1}{\sqrt{\chi}}\left\{\int_{\phi_l}^{\phi} K(\bar{\chi},\chi)d\bar{\phi} + \phi_l\right\} \quad (9)$$

with the associated boundary conditions

$$\frac{d\phi}{d\chi} = 0 \text{ at } \chi = 0 \text{ and } \chi = 1.$$

In the following subsections three specific cases for α are analyzed: perfectly conducting plate ($\alpha \to \infty$), adiabatic plate ($\alpha = 0$) and α of the order of unity.

Perfectly Conducting Plate ($\alpha \to \infty$)

In this particular case, the temperature of the plate is uniform over its whole length. After integrating equation (6) and applying the adiabatic boundary conditions at both edges, we obtain in a first approximation the following relation:

$$q + \frac{\delta}{\Gamma}\exp\left(\frac{\Gamma\theta}{1 + \beta_1\theta}\right) = 2\theta. \quad (10)$$

Integration of the frozen energy equation (9) with the associated boundary conditions gives $\phi = \phi_l = \theta_f/q = 1/2$. For the high activation energy of the catalytic reaction ($\Gamma >> 1$), an increase of the non-dimensional plate temperature of order $1/\Gamma_c$ above its frozen value is sufficient to achieve catalytic ignition, where Γ_c is the relevant non-dimensional activation energy defined as

$$\Gamma_c = \Gamma\left(1 + \frac{\beta_1 q}{2}\right)^{-2}. \quad (11)$$

Thus, we assume a solution to equation (10) of the form

$$\theta = \frac{q}{2} + \frac{\psi}{\Gamma_c} + 0\left(\frac{1}{\Gamma_c}\right)^2 \quad (12)$$

where ψ is of the order of unity. Introducing equation (12) in equation (10), we obtain

$$2\psi\exp(-\psi) = \frac{\delta\Gamma_c}{\Gamma}\exp\left(\frac{\Gamma q}{2 + \beta_1 q}\right) \quad (13)$$

giving the temperature of the plate as a function of a modified Damköhler number. Clearly, ignition occurs for $\psi = 1$. Therefore equation (13) gives the critical value q_I of q that leads to ignition for a given δ as

$$\frac{4\delta}{(2 + \beta_1 q_I)^2}\exp\left(\frac{\Gamma q_I}{2 + \beta_1 q_I}\right) = \frac{2}{e}. \quad (14)$$

If δ is of the order of unity, q_I must be of order $1/\Gamma$, whereas q_I rises to values of the order of unity for $\delta << 1$. For $\delta = 2/e$, equation (14) gives $q_I = 0$ as found by Liñán and Treviño.[8]

Adiabatic Plate ($\alpha = 0$)

In this case the energy balance equation (6) reduces to

$$q + \frac{\delta}{\Gamma}\exp\left(\frac{\Gamma\theta}{1 + \beta_1\theta}\right) = \frac{1}{\sqrt{\chi}}\int_0^{\theta} K(\bar{\chi},\chi)d\bar{\theta}. \quad (15)$$

For $\alpha = 0$, the frozen solution can be obtained from equation (9) and is given by

$$\phi = \frac{\theta_f}{q} = b\sqrt{\chi} \text{ with}$$

$$b = \frac{\sqrt{3}}{2\pi}\frac{\Gamma(4/3)\Gamma(1/3)}{\Gamma(5/3)} \doteq 0.73. \quad (16)$$

As expected, the non-dimensional frozen temperature on the plate increases from zero at the leading edge to its maximum value at the trailing edge. For $\delta << 1$, that is $q_I \sim 1$, the catalytic reaction will take place only in a narrow zone close to the trailing edge where the temperature is of the order of unity. On the other hand, for values of $\delta \sim 1$, that is q_I of order $1/\Gamma$, the frozen non-dimensional temperature at the trailing edge is also of the order of $1/\Gamma$. The chemical reaction therefore will take place over the whole length of the plate. Thus, there are two distinguished limits which are analyzed in the following.

$\delta << 1$

In this case there are two different regions. An inner narrow reactive zone close to the trailing edge and an outer zone where the temperature is given by the frozen solution (16). In the reactive zone, a temperature profile given by

$$\theta = qb\sqrt{\chi} + \frac{\psi(\chi)}{\Gamma_c} + 0\left(\frac{1}{\Gamma_c^2}\right) \quad (17)$$

is assumed, where Γ_c now is given by

$$\Gamma_c = \Gamma(1 + \beta_1 qb)^{-2}$$

Introducing an inner stretching coordinate of the form

$$\zeta = (\chi - 1)\frac{\Gamma qb}{2}, \quad (18)$$

equation (15) then transforms to

$$\Delta \exp(\psi + \zeta) = \int_0^{\psi} \frac{d\bar{\psi}}{\sqrt[3]{\zeta - \bar{\zeta}}} \quad (19)$$

where Δ is the relevant Damköhler number defined as

$$\Delta = \frac{\delta}{(1 + \beta_1 qb)^2} \exp\left(\frac{\Gamma qb}{1 + \beta_1 qb}\right)\left\{\frac{3}{2\Gamma_c qb}\right\}^{1/3}. \quad (20)$$

Equation (19) can be rewritten as

$$\exp(\psi + \xi) = \int_0^{\psi} \frac{d\bar{\psi}}{\sqrt[3]{\xi - \bar{\xi}}} \quad (21)$$

where $\xi = \zeta + \text{Ln}\Delta$. Equation (21) shows that the ignition process in this case, in a first approximation, has a universal character. Ignition is obtained when $\psi \rightarrow \infty$ for a given value of ξ, denoted as ξ_I. The numerical calculation using equation (21) gives a value of $\xi_I = -.315$. Ignition occurs at the trailing edge $\zeta = 0$, thus giving the value of the critical reduced Damköhler number $\Delta_I = \exp(\xi_I) = 0.73$.

$\delta \sim 1$

As mentioned before, the analysis made in the previous subsection cannot give us the external heat flux needed for ignition for values of δ of the order of unity. The catalytic reaction takes place over the whole length of the plate. It is convenient to define q_I of the order of unity as $q_I = q\Gamma$. Equation (15) therefore transforms to

$$q_I + \delta \exp\left(\frac{\Gamma\theta}{1 + \beta_1\theta}\right) = \frac{\Gamma}{\sqrt{\chi}}\int_0^{\theta} K(\bar{\chi},\chi)d\bar{\theta}. \quad (22)$$

Eq. (22) can be solved by assuming the following expansion for θ

$$\theta = q_I b\sqrt{\chi} + \frac{\psi}{\Gamma} + O\left(\frac{1}{\Gamma^2}\right). \quad (23)$$

Equation (22) then takes the form

$$\exp\left(\frac{q_I b}{\delta}\sqrt{\xi}\right)\exp(\psi) = \frac{1}{\sqrt{\xi}}\int_0^{\psi} K(\bar{\xi},\xi)d\bar{\psi}, \quad (24)$$

where $\xi = \chi\delta^2$. Numerical solution of equation (24) shows that $\psi \rightarrow \infty$ for a given value of ξ, denoted as $\xi_I = \delta^2$. Figure 1 shows how q_{II} varies with δ for values of δ of the order of unity. The value of $\delta = 0.711$ for $q_{II} = 0$ is obtained as shown in a previous paper by Liñán and Treviño.[8] Figure 1 also shows q_{II} plotted for a perfectly conducting plate, using equation (14). As expected, for a given value of δ, the critical value for ignition q_{II} increases as α increases. This is because the heat conducted upstream through the plate can be easily transferred to the gas phase in regions close to the leading edge, thus causing the maximum temperature to decrease as α increases. For high activation energies, the chemical reaction is very sensitive to this maximum temperature.

α of the Order of Unity

In this case the frozen solution can be obtained numerically from the energy equation (9) with the adiabatic boundary conditions at both edges of the plate. Figure 2 shows the ϕ profiles for different values of α. For values of $\delta << 1$, the chemical reaction will take place only in a thin region close

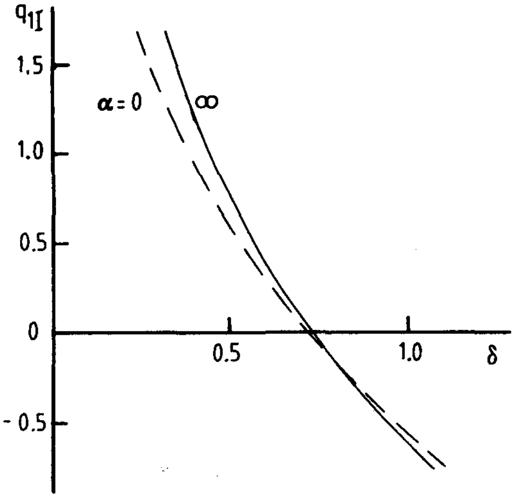

FIG. 1. Critical external heat flux for ignition as a function of δ for an adiabatic plate ($\alpha = 0$) and for a perfectly conducting plate ($\alpha \to \infty$).

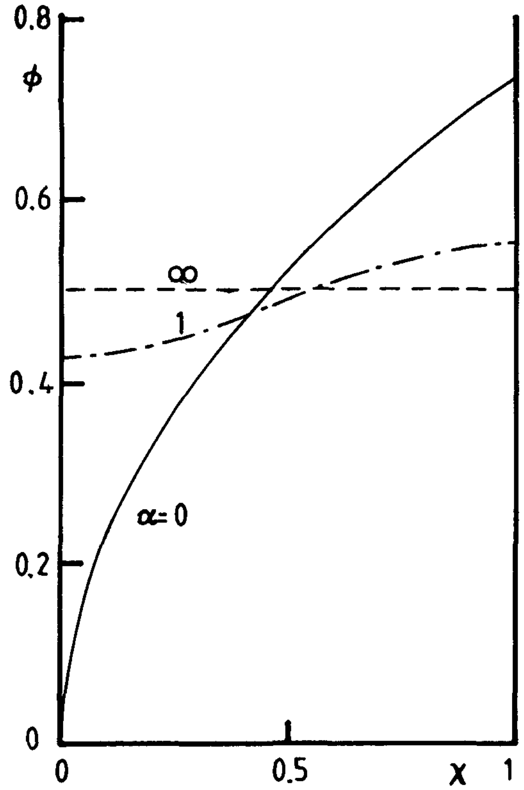

FIG. 2. Non-dimensional frozen temperature profiles $\phi(\chi)$ for different values of α.

to the trailing edge, where the temperature reaches the highest value. In this region we can write the expansion

$$\phi(\alpha) = \phi_t(\alpha) - \gamma(\alpha)(1 - \chi)^2 + \ldots, \text{ as } \chi \to 1 \quad (25)$$

where ϕ_t corresponds to the value of ϕ at the trailing edge and γ is defined as

$$\gamma(\alpha) = \frac{1}{2}\left(\frac{d^2\phi}{d\chi^2}\right)_{\chi=1}. \quad (26)$$

Close to the trailing edge, the temperature increases due to the chemical reaction only in a thin zone of the order of $(1 + \beta_1 q\phi_t)/\sqrt{\Gamma\gamma q}$, for values of $\delta << 1$. On the other hand, for values of δ of the order of unity, the reaction zone grows and thus the reaction will be important over the whole length of the plate. Both distinguished limits are analyzed in the following subsections.

δ << 1

In the thin reaction or inner zone, the temperature increment in due to the chemical reaction can be assumed to be given by

$$\theta - q\phi = \frac{\psi_1}{\Gamma_c} + \frac{\psi_2(\zeta)}{\Gamma_c^{3/2}} + \ldots \quad (27)$$

with $\Gamma_c = \Gamma(1 + \beta_1 q\phi_t)^{-2}$ and ψ_1 as a constant, dictated by the adiabatic boundary condition at the trailing edge. The inner coordinate ζ is defined as

$$\zeta = \sqrt{\gamma q \Gamma_c}\,(1 - \chi). \quad (28)$$

In a first approximation, the energy balance equation for the inner zone reduces to

$$\frac{d^2\psi_2}{d\zeta^2} = -\Delta(\alpha)\exp(\psi_1 - \zeta^2). \quad (29)$$

Here the reduced Damköhler number is defined by

$$\Delta(\alpha) = \frac{\delta\sqrt{\Gamma_c(\alpha)q}}{\alpha\Gamma q\gamma(\alpha)}\exp\left\{\frac{\Gamma q\phi_t(\alpha)}{1 + \beta_1 q\phi_t(\alpha)}\right\}. \quad (30)$$

Integration of equation (29) and using the adiabatic boundary condition at the trailing edge, gives

$$\left(\frac{d\psi_2}{d\zeta}\right)_{\zeta\to\infty} = -\Delta(\alpha)\exp(\psi_1)\int_0^\infty \exp(-\zeta^2)d\zeta = -\frac{2\Delta(\alpha)}{\sqrt{\pi}}\exp(\psi_1). \quad (31)$$

This condition is to be matched with the outer convective-diffusive zone governed by the equation

$$\alpha \frac{d^2\psi_e}{d\chi^2} = \frac{1}{\sqrt{\chi}} \left\{ \int_{\psi_{el}}^{\psi_e} K(\bar{\chi},\chi) d\bar{\psi}_e + \psi_{el} \right\} \quad (32)$$

where $\psi_e = \Gamma_c(\theta_e - q\phi)$. The subscript e denotes the outer zone. The boundary conditions to equation (32) are given by

$$\frac{d\psi_e}{d\chi} = 0 \text{ at } \chi = 0 \text{ and } \psi_e = \psi_I \text{ at } \chi = 1. \quad (33)$$

The gradient $d\psi_e/d\chi$ at $\chi = 1$, which in fact is at the edge of the inner zone, can be written in first approximation as

$$\frac{d\psi_e}{d\chi} = \psi_I F(\alpha) \text{ at } \chi = 1 \quad (34)$$

where $F(\alpha)$ can be calculated from equations (32) and (33). F, γ and ϕ_t are plotted in Figure 3 as a function of α. Combining solutions of equations (31) and (34) gives

$$\exp(-\psi_I)\psi_I = \frac{2\Delta(\alpha)\sqrt{\gamma(\alpha)}}{\sqrt{\pi}\, F(\alpha)} \quad (35)$$

which clearly shows that ignition occurs for $\psi_I = 1$. Thus, the critical condition for ignition is

$$\frac{2\delta}{\sqrt{\Gamma \pi q_I}} \frac{\exp\left\{ \dfrac{\Gamma q_I \phi_t(\alpha)}{1 + \beta_1 q_I \phi_t(\alpha)} + 1 \right\}}{\{1 + \beta_1 q_I \phi_t(\alpha)\}\alpha F(\alpha)\sqrt{\gamma(\alpha)}} = 1. \quad (36)$$

$\delta \sim 1$

When δ is of the order of unity, the reaction will

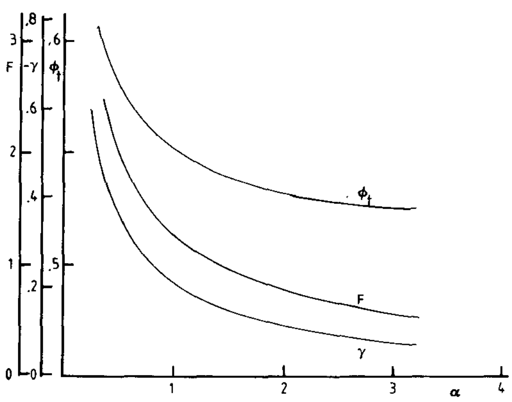

FIG. 3. Variation of the parameters F, γ and ϕ_t as a function of α.

be important over the whole length of the plate. Here, it is convenient to introduce $q_I = q\Gamma$, which is of the order of unity. The non-dimensional temperature in this case is assumed to be given by

$$\theta = \frac{q_I\phi + \psi}{\Gamma} + 0\left(\frac{1}{\Gamma}\right)^2$$

where ϕ corresponds to the frozen solution obtained from equation (9). The energy balance equation then transforms to

$$\alpha \frac{d^2\psi}{d\chi^2} + \delta \exp(q_I\phi)\exp(\psi) = \frac{1}{\sqrt{\chi}} \left\{ \int_{\psi_l}^{\psi} K(\bar{\chi},\chi) d\bar{\psi} + \psi_l \right\} \quad (37)$$

with the boundary conditions

$$\frac{d\psi}{d\chi} = 0 \text{ at } \chi = 0 \text{ and } \chi = 1.$$

The solution to these equations can be solved numerically. For a given δ, there are two solutions for each q_I below a critical q_{II}. On the other hand, for values of q_I such as $q_I > q_{II}$ there is no solution to the equations, thus denoting the ignition condition associated with q_{II}. Results obtained for this case would give curves between the two curves plotted in Figure 1.

Discussion and Conclusions

A steady state analysis was made of the catalytic ignition in a flat plate boundary layer flow. Ignition was facilitated by an external energy flux. The plate was assumed to have finite thickness and thermal conductivity. Three different types of analyses were made depending on the parameter α which denotes the ratio between the thermal resistance of the gas to that of the plate. For high thermal conductivity of the plate, $\alpha \to \infty$, the plate temperature is uniform along the longitudinal coordinate. Thus, the chemical reaction is important everywhere on the plate. In this case an analytical expression is obtained for the critical energy flux required for catalytic ignition, which is uniformly valid for all reduced Damköhler numbers. On the other hand, for values of α of the order of unity, there are two distinguished limits, depending on the Damköhler number. For values of $\delta << 1$, the chemical reaction is important only in a thin zone close to the trailing edge. Matched asymptotic methods are used to derive an analytical expression for the critical external energy flux for ignition. For values of δ of the order of unity, the reaction is to be considered over the whole length of the plate. The results obtained by this analysis give the transition between zero heat flux and heat flux for $\delta << 1$. The influence of the plate thermal conductivity is relatively important for values of the Damköhler numbers much smaller than unity. Ignition is facilitated as α decreases. This effect decreases as the Damköhler number increases, being very small for values of the Damköhler number for ignition without external energy flux.

The critical external energy flux for ignition obtained in this paper is per unit surface of the plate. From the practical point of view, it is convenient to deduce the total heat flux needed for ignition, that is,

$$Q_{eI} = q_{eI} L.$$

The non-dimensional total external heat flux for ignition is then given by

$$Q_I = \frac{\nu k_0 C_\infty^{n-1} \Gamma \exp(-Ta/T_\infty) Q_{eI}}{(0.332)^2 (PrSc)^{1/3} u_\infty D \lambda T_\infty \beta_I} = q_I \delta. \quad (38)$$

In Figure 4 typical values of Q_I are plotted as a function of the Damköhler number δ for two different sets of parameters, for $\alpha = 0$ and $\alpha \to \infty$. As expected from equation (38), Q_I has a maximum in the domain of interest of δ. For values of the Damköhler number that are small compared to unity, which are of practical interest, increasing the value of δ increases the total external energy flux needed to achieve catalytic ignition. Thus, it is convenient to work with low values of the Damköhler number, but not close to that of extinction to prevent this condition under variable working parameters.

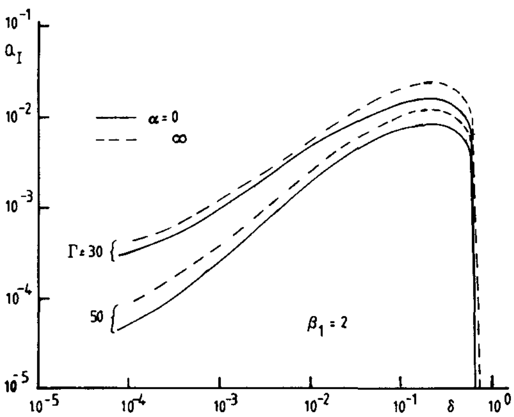

FIG. 4. Total non-dimensional external heat flux for the catalytic ignition as a function of δ for two different sets of parameters.

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