Causality of the Einstein-Israel-Stewart Theory with Bulk Viscosity

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We prove that Einstein's equations coupled to equations of the Israel-Stewart-type, describing the dynamics of a relativistic fluid with bulk viscosity and nonzero baryon charge (without shear viscosity or baryon diffusion) dynamically coupled to gravity, are causal in the full nonlinear regime. We also show that these equations can be written as a first-order symmetric hyperbolic system, implying local existence and uniqueness of solutions to the equations of motion. We use an arbitrary equation of state and do not make any simplifying symmetry or near-equilibrium assumption, requiring only physically natural conditions on the fields. These results pave the way for the inclusion of bulk viscosity effects in simulations of gravitational-wave signals coming from neutron star mergers.

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Introduction.—The recent detection of a binary neutron star merger using gravitational waves [1] and electromagnetic signals [2] marked the dawn of the multimessenger astronomy era [3]. Such events are expected to provide key information about the properties of matter at extreme densities and temperatures [4], as the density of the inner region of the object left over after the merger can be several times larger than the nuclear saturation density ($\sim 0.16\ \mathrm{fm}^{-3}$) while still subject to temperatures of the order of tens of MeV.

Even though the properties of the equation of state of the highly dense matter formed after the merger are still uncertain [5], for many years it was assumed that this system could be reasonably described as an ideal fluid (coupled to Einstein's equations) since the timescales for viscous transport to set in were previously determined [6] to be outside the 10 msec range, which is the typical timescale associated with damping due to gravitational wave emission. These estimates were recently revisited in Ref. [7] using state-of-the-art merger simulations where it was concluded that, while neutrino-driven thermal transport and shear dissipation remain unlikely to affect the post-merger gravitational wave signal (unless turbulent motion occurs), damping of high-amplitude oscillations due to bulk viscosity is likely to be relevant if direct Urca processes remain suppressed. Therefore, bulk viscosity is expected to play an important role in gravitational wave emission and, as such, it should be taken into account and thoroughly investigated in merger simulations.

However, as stressed in Ref. [7], the effects of bulk viscosity have not yet been included in merger simulations because this requires a formulation of relativistic fluid dynamics including bulk viscosity that is compatible with the underlying causality property of relativity theory in the strong nonlinear regime probed by the mergers (see also Refs. [8–10] for further discussions regarding viscous effects).

In this Letter we solve this issue by proving that the equations of motion of the Israel-Stewart (IS) type [11–13], describing a relativistic fluid with bulk viscosity and baryon charge (without shear viscosity or baryon diffusion) dynamically coupled to gravity, are causal in the full nonlinear regime when

$$\left[\frac{\zeta}{\tau_\Pi} + n\left(\frac{\partial P}{\partial n}\right)_\varepsilon\right]\frac{1}{\varepsilon + P + \Pi} \leq 1 - \left(\frac{\partial P}{\partial \varepsilon}\right)_n, \qquad (1)$$

where $\zeta = \zeta(\varepsilon, n)$ is the bulk viscosity, $\tau_\Pi = \tau_\Pi(\varepsilon, n)$ is the bulk relaxation time, Π is the bulk scalar, ε is the energy density, $P = P(\varepsilon, n)$ is the equilibrium pressure defined by the equation of state, and n is the baryon density. Requirement (1) is a simple nonlinear generalization of the known condition ensuring the linear stability of the IS equations around equilibrium. In fact, we note that near equilibrium, where $|\Pi/(\varepsilon + P)| \ll 1$, and at zero baryon density where the speed of sound squared is $c_s^2 = dP/d\varepsilon$, Eq. (1) reduces to $[\zeta/\tau_\Pi(\varepsilon + P)] + \mathcal{O}(\Pi/\varepsilon + P) \leq 1 - c_s^2$, which is the standard condition for causality and stability in the linearized regime around equilibrium [14–16].

We also show how to express the equations of motion of this theory that describes a bulk viscous relativistic fluid coupled to gravity as a first-order symmetric hyperbolic (FOSH) system. This immediately implies local existence and uniqueness of its solutions, and sets the equations in a form for which known numerical algorithms can be applied [17,18]. We stress that our results remain valid if one considers solely the fluid dynamic equations in a fixed (e.g., Minkowski) background. To the best of our knowledge, this is the first time that such statements (causality, local existence, uniqueness) have been proven for IS-like theories in the nonlinear regime.

The equations of motion.—IS theories of relativistic fluid dynamics [11–13] were proposed decades ago as a way to solve the long-standing acausality and instability problem of the relativistic Navier-Stokes (NS) equations [19,20]. The basic idea is that a dissipative current such as the bulk scalar Π does not instantaneously take its NS form [We use units $c = \hbar = k_B = 1$. The spacetime metric signature is $(-{+}{+}{+})$. Greek indices run from 0 to 3, Latin indices from 1 to 3.] $\Pi_{\mathrm{NS}} = -\zeta \nabla_\mu u^\mu$ (where u_μ is the fluid 4-velocity which obeys $u_\mu u^\mu = -1$) during the fluid evolution; rather, Π obeys a relaxation-type equation that describes how it may relax to Π_{NS} within the relaxation timescale τ_Π. While such theories were originally [12] derived by imposing the second law of thermodynamics for a judicious choice of the out-of-equilibrium entropy density, their modern versions in Refs. [21] and [22] have focused on different aspects of relativistic hydrodynamics. Reference [21] emphasized the effective field theory (EFT) character of relativistic hydrodynamics and its applicability in the strong coupling regime (see Ref. [23] for a detailed discussion), elucidating the role played by conformal invariance and how that requires the presence of additional terms in the equations of motion that were not usually taken into account in the original IS theory. In Ref. [22] a new moment expansion in relativistic kinetic theory [24] was employed, together with a power counting scheme involving Knudsen and inverse Reynolds numbers, to derive the equations of motion of hydrodynamics and obtain their corresponding transport coefficients. These new values for the transport coefficients led to an overall improvement with respect to the original IS theory when comparing hydrodynamic calculations to exact solutions of the Boltzmann equation [25–27].

The aforementioned versions of the IS theory differ from their original counterpart while retaining its basic physical insights. As described in the previous paragraph, there are different theories representing an improved, albeit different, formulation of the IS equations. These theories will be referred to here as generalized IS theories.

With applications to neutron star mergers in mind, here we only consider the dissipative effects coming from bulk viscosity. The fluid energy-momentum tensor is given by $T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu}$, where $\Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projector orthogonal to u_μ and $g_{\mu\nu}$ is the spacetime metric. Using $u^\mu u_\mu = -1$, energy-momentum conservation, $\nabla_\mu T^{\mu\nu} = 0$, is equivalent to the equations

$$u^\alpha \nabla_\alpha \varepsilon + (\varepsilon + P + \Pi)\nabla_\alpha u^\alpha = 0, \tag{2}$$

$$(\varepsilon + P + \Pi)u^\beta \nabla_\beta u_\alpha + \alpha_1 \Delta_\alpha^\beta \nabla_\beta \varepsilon + \alpha_2 \Delta_\alpha^\beta \nabla_\beta n + \Delta_\alpha^\beta \nabla_\beta \Pi = 0, \tag{3}$$

where $\alpha_1 = (\partial P/\partial \varepsilon)_n$ and $\alpha_2 = (\partial P/\partial n)_\varepsilon$. Equations (2)–(3) are supplemented by the following bulk scalar relaxation equation

$$\tau_\Pi u^\alpha \nabla_\alpha \Pi + \Pi + \lambda \Pi^2 + \zeta \nabla_\alpha u^\alpha = 0, \tag{4}$$

where $\lambda = \lambda(\varepsilon, n)$ is a transport coefficient. Equation (4) corresponds to Eq. (63) in Ref. [22] without shear viscosity or baryon diffusion and with $\delta_{\Pi\Pi} = 0$. It is common practice to omit the term proportional to $(\nabla_\alpha u^\alpha)^2$ in Ref. [22] and, therefore, we have also done so here [28]. We choose to work with this specific generalized IS equation because it contains all the relevant physics while making the interpretation of our results clear. In fact, although it is possible to include other terms in the bulk channel [22,29,30], the above equation contains the essential terms for our discussion: τ_Π is associated with causality, λ parametrizes the presence of nonlinear terms that do not contain derivatives of the fields, and the term $\zeta \nabla_\alpha u^\alpha$ ensures that the NS limit can be recovered. In Sec. 2.3 we explain how our methods apply with no change to other generalized IS theories.

We also include a nonzero baryon current $J_\mu = n u_\mu$, whose conservation gives (our results can be easily adapted for $n = 0$),

$$u^\mu \nabla_\mu n + n \nabla_\mu u^\mu = 0. \tag{5}$$

The fluid is dynamically coupled to gravity via Einstein's equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{6}$$

where $R_{\mu\nu}$ is the Ricci tensor, $R = g_{\mu\nu} R^{\mu\nu}$, Λ is the cosmological constant, and G is Newton's gravitational constant. Equations (2)–(6), with $u^\mu u_\mu = -1$, define the generalized Einstein-Israel-Stewart (EIS) theory considered in this work.

Causality: Causality is the idea that no information propagates faster than the speed of light and that no closed timelike curves exist in spacetime; i.e., the future cannot influence the past (see the Supplemental Material [31] for the precise definition of causality, which includes Refs. [32–36]). This concept lies at the core of relativity theory and, therefore, the dynamics of the fluid sector must be compatible with it. Despite its importance, causality has not been established in the nonlinear regime for the IS theory or its variations (see Refs. [37–39] for a discussion). While causality is known to hold in ideal relativistic hydrodynamics at the nonlinear level [40–43], only statements valid in the linearized regime around equilibrium are known for the IS theory [14,15,19,44,45]. Attempts to go beyond the linear regime, although restricted to $1 + 1$ dimensions or assuming very strong symmetry conditions, and in flat spacetime, appeared, respectively, in Refs. [14] and [46] [compare (1) with the causality condition found in Ref. [14]]. Thus, a general proof of causality of IS-like systems (coupled to gravity) is so far lacking.

We show that if $[\zeta/\tau_\Pi(\varepsilon+P+\Pi)]+\alpha_1+(\alpha_2 n/\varepsilon+P+\Pi)\geq 0$ and condition (1) is satisfied, then the generalized EIS system is causal. Causality of Eqs. (2)–(5) in a fixed background also holds under the same assumptions.

We refer to Theorem 1 in the Supplemental Material [31] for a formal statement of the causality of the EIS system as well as its proof. In a nutshell, in the Supplemental Material [31] we establish that the values of the fields $\varepsilon, u_\alpha, \Pi$, and $g_{\alpha\beta}$ at a point p are influenced only by the dynamics of such fields in the causal past of p. In Minkowski space, the causal past of p is simply the bottom half of the light cone with vertex at p. This generalizes to curved spacetimes, where the half-bottom of the light cone is replaced by a curved conelike region with vertex at p.

Condition $[\zeta/\tau_\Pi(\varepsilon+P+\Pi)]+\alpha_1+(\alpha_2 n/\varepsilon+P+\Pi)\geq 0$ is physically very natural as $\alpha_1+(\alpha_2 n/\varepsilon+P)$ is the speed of sound squared in equilibrium and ζ is non-negative.

The EIS equations as a FOSH system, existence, and uniqueness: A system of first order partial differential equations for an unknown vector Φ is said to be a FOSH system if it can be written as $\mathcal{A}^\mu(\Phi)\partial_\mu\Phi+\mathcal{B}(\Phi)=F$, where $\mathcal{A}^\mu$ are matrices and $\mathcal{B}$ is a vector, all possibly depending on Φ but not on its derivatives, $\mathcal{A}^\mu$ are symmetric, $\mathcal{A}^0$ is positive definite, and F is a given source term.

Several important properties are readily available for FOSH systems [47–49]. One such property is that for these systems the initial-value problem admits existence and uniqueness of solutions. Besides assuring a firm theoretical basis for the system, knowing that the equations admit existence and uniqueness of solutions is helpful to ensure the reliability of many numerical schemes; see Ref. [50] for examples of the potential pitfalls of simulating equations for which no existence and uniqueness results are available, and [37] for a complementary discussion. While for most of standard physical theories, existence and uniqueness of solutions had long been settled [41], for theories of relativistic fluids with viscosity very few results are available [37,51,52], and none so far for the generalized IS theory in the nonlinear regime.

Equations (2)–(5) can be written as a FOSH system with $\Phi=(\varepsilon,u^i,n,\Pi)$, a suitable $\mathcal{B}$, $F=0$, and $\mathcal{A}^0$ and $\mathcal{A}^k$ given, respectively, by

$$\begin{pmatrix} \frac{u^0}{(\varepsilon+P+\Pi)} & -\frac{u_i}{u_0} & 0 & 0 \\ -\frac{u_j}{u_0} & A_{ij}u^0\frac{(\varepsilon+P+\Pi)}{(\frac{\partial P}{\partial\varepsilon})_n} & -\frac{u_j}{u_0}\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & -\frac{u_j}{u_0}\frac{1}{(\frac{\partial P}{\partial\varepsilon})_n} \\ 0 & -\frac{u_i}{u_0}\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & \frac{u^0}{n}\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & 0 \\ 0 & -\frac{u_i}{u_0}\frac{1}{(\frac{\partial P}{\partial\varepsilon})_n} & 0 & \frac{\tau_\Pi}{\zeta(\frac{\partial P}{\partial\varepsilon})_n}u^0 \end{pmatrix}$$

and

$$\begin{pmatrix} \frac{u^k}{(\varepsilon+P+\Pi)} & \delta^k_i & 0 & 0 \\ \delta^k_j & A_{ij}u^k\frac{(\varepsilon+P+\Pi)}{(\frac{\partial P}{\partial\varepsilon})_n} & \delta^k_j\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & \frac{\delta^k_j}{(\frac{\partial P}{\partial\varepsilon})_n} \\ 0 & \delta^k_i\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & \frac{u^k}{n}\frac{(\frac{\partial P}{\partial n})_\varepsilon}{(\frac{\partial P}{\partial\varepsilon})_n} & 0 \\ 0 & \frac{\delta^k_i}{(\frac{\partial P}{\partial\varepsilon})_n} & 0 & \frac{\tau_\Pi}{\zeta(\frac{\partial P}{\partial\varepsilon})_n}u^k \end{pmatrix},$$

where $A_{ij}=g_{ij}-g_{i0}(u_j/u_0)-(u_i/u_0)g_{0j}+g_{00}(u_iu_j/u_0^2)$.

Using the above formulation of the generalized IS equations as a FOSH, we establish existence and uniqueness of solutions to the initial-value formulation for the generalized EIS system under suitable assumptions. More precisely, given an equation of state $P=P(\varepsilon,n)$, a bulk viscosity $\zeta=\zeta(\varepsilon,n)$, and a relaxation time $\tau_\Pi=\tau_\Pi(\varepsilon,n)$, consider initial conditions $\mathring{\varepsilon}\equiv\varepsilon(0)$, $\mathring{u}^i\equiv u^i(0)$, $\mathring{\Pi}\equiv\Pi(0)$, $\mathring{n}\equiv n(0)$, $g_{ij}(0)$, $\partial_0 g_{ij}(0)$ satisfying Eq. (1) along an initial surface $\Sigma\equiv\{x^0=0\}$. Assume that $\mathring{\varepsilon}+P(\mathring{\varepsilon},\mathring{n})+\mathring{\Pi}$, $\tau_\Pi(\mathring{\varepsilon},\mathring{n})$, $\zeta(\mathring{\varepsilon},\mathring{n})>0$, $(\partial P/\partial\varepsilon)(\mathring{\varepsilon},\mathring{n})+(\partial P/\partial n)(\mathring{\varepsilon},\mathring{n})\mathring{n}/(\mathring{\varepsilon}+P(\mathring{\varepsilon},\mathring{n})+\mathring{\Pi})\geq 0$, and that $\mathring{n}$, $(\partial P/\partial\varepsilon)(\mathring{\varepsilon},\mathring{n})$, and $(\partial P/\partial n)(\mathring{\varepsilon},\mathring{n})$ are nonzero. Then there exists a spacetime M containing Σ such that a unique solution to the generalized EIS equations exists on M. A similar statement, i.e., existence and uniqueness of solutions for initial conditions as above, holds for the generalized IS equations in a fixed background.

We refer to Theorem 2 in the Supplemental Material [31] for a formal statement of existence and uniqueness of solutions, its proof, and for the generalized IS as a FOSH. Here, we make some relevant remarks.

Assumptions $\mathring{\varepsilon}+P(\mathring{\varepsilon},\mathring{n})+\mathring{\Pi}$, $\tau_\Pi(\mathring{\varepsilon},\mathring{n})$, $\zeta(\mathring{\varepsilon},\mathring{n})>0$, and $(\partial P/\partial\varepsilon)(\mathring{\varepsilon},\mathring{n})+(\partial P/\partial n)(\mathring{\varepsilon},\mathring{n})\mathring{n}/(\mathring{\varepsilon}+P(\mathring{\varepsilon},\mathring{n})+\mathring{\Pi})\geq 0$ are physically natural (our results are easily adapted to the case when n is absent), while $(\partial P/\partial\varepsilon)(\mathring{\varepsilon},\mathring{n})$, $(\partial P/\partial n)(\mathring{\varepsilon},\mathring{n})\neq 0$ are very mild requirements. Note that these can be viewed as conditions on $\mathring{\varepsilon}$ and $\mathring{n}$, with the form of the equation of state remaining rather general. It is well known that because of the covariance of Einstein's equations only g_{ij} and $\partial_0 g_{ij}$, rather than $g_{\alpha\beta}$ and $\partial_0 g_{\alpha\beta}$, are given as initial data [53,54]. Similarly, only a vector field on Σ, i.e., $\mathring{u}^i$ rather than $\mathring{u}^\alpha$, is given along Σ, with the full four-velocity on Σ determined from $u^\mu u_\mu=-1$.

The proof of Theorem 2 follows from known arguments once the generalized IS equations are formulated as a FOSH system. The main difficulty to achieve the latter is that there is no method to determine whether a given set of equations can in principle be written as a FOSH system. Here, we relied on the following two ingredients to achieve this. First, computing the characteristics of the system (which is used for establishing the causality of the equations), we find them to be a combination of the flow lines of u^α and of null cones with respect to an acoustical metric. The former is associated with transport equations (which are the prototypical example of a FOSH system); the latter is associated with wave equations (which can be rewritten as FOSH). This suggests that a FOSH formulation might be possible. Second, Eqs. (2), (3), and (5) resemble the equations of an ideal fluid, for which a FOSH formulation is known [55,56]. This suggests trying to adapt the procedure that works for an ideal fluid. In this regard, it is crucial that the viscous contributions due to bulk viscosity are given by a scalar: if vector (baryon diffusion) or tensor (shear viscosity) viscous contributions are included, the characteristics become very complicated and Eqs. (2), (3), and (5) no longer resemble those of an ideal fluid. It is not clear whether causality (in the nonlinear regime) or a formulation as a FOSH system can be obtained in these cases following the approach pursued here.

We assumed that all transport coefficients depend only on ε and n (however, see the discussion below regarding other theories). Transport coefficients also generally depend on some characteristic microscopic variables [57]. In many applications, however, these microscopic variables are either neglected, treated as parameters that can be independently estimated and thus considered given (such as the viscosities in Navier-Stokes theory), or directly eliminated. Otherwise, one would not be dealing with a purely hydrodynamic theory [29]. Therefore, our assumptions fall well within the scope of applicability of hydrodynamic models. A more general setting where the dependence on the microscopic dynamics can be decoupled upon simple assumptions is explained next.

Other theories: Here we point out that our methods give, with no change, causality, existence, and uniqueness of solutions to Einstein's equations coupled to other generalized IS theories (without shear viscosity or baryon diffusion). A quick inspection in our proofs provided in the Supplemental Material [31] reveals that they remain true, with no change, if τ_Π and ζ are allowed to depend on Π. In particular, consider Eq. (63) in Ref. [22] with $\delta_{\Pi\Pi}$ not necessarily zero. Defining $\bar{\zeta} = \zeta + \delta_{\Pi\Pi}\Pi$, we obtain Eq. (4) with ζ replaced by $\bar{\zeta}$. Hence, all our results go through with $\bar{\zeta}$ in place of ζ. Our assumptions in this case, however, seem less natural since, for example, there is no reason to expect $\bar{\zeta}$ to be positive.

Finally, the methods we employ depend only on the principal part of the system (i.e., the subsystem comprised only of the terms involving derivatives in the equations.). Thus all our conclusions remain valid if Eq. (4) is replaced by

$$\tau_\Pi u^\alpha \nabla_\alpha \Pi + \zeta \nabla_\alpha u^\alpha + f(\varepsilon, n, \Pi, \tau_\Pi, \zeta) = 0,$$

where f is an arbitrary function of ε, n, Π, τ_Π, and ζ (but not depending on their derivatives) and τ_Π and ζ are allowed to depend on Π (so that, in particular, ζ can be replaced by $\bar{\zeta}$ here as well).

Consider now the case when ζ and τ_Π depend on a microscopic variable ℓ, and suppose that ℓ cannot be neglected or treated as a parameter but needs to be found by solving its own evolution equation which couples to the hydrodynamic variables. If one assumes, as it is often done [28], that the ratio ζ/τ_Π depends only on ε, n, and Π, then all our conclusions remain valid as long as f is such that $(1/\tau_\Pi)f(\varepsilon, n, \Pi, \tau_\Pi, \zeta)$ does not involve dynamical microscopic variables. Thus, our results are applicable to many cases of interest in heavy ion collisions (e.g., Ref. [58]) and neutron star mergers. In this regard, note that our condition (1) depends only on the ratio ζ/τ_Π.

Conclusions.—In this Letter we proved that the generalized EIS equations for a relativistic fluid with bulk viscosity and nonzero baryon charge (without shear viscosity or baryon diffusion) are causal in the full nonlinear regime. We also proved that these equations admit a FOSH formulation, existence, and uniqueness of solutions. Our results hold under hypothesis typically satisfied by viscous relativistic fluids. In particular, we do not require any symmetry or near-equilibrium assumption. The most immediate application of these results is in the study of neutron star mergers. In this regard, it is crucial that our results hold under general assumptions on the equation of state since the latter is not yet known from first principles.

Given that theories of IS type are the most widely used formulation of relativistic viscous fluid dynamics [59], it is crucial to put them on a firm theoretical (and mathematical) foundation. Combining our results with known properties of the generalized IS equations [13,15,16,22], the case considered here seems to be the first example in the literature of a theory of relativistic viscous fluids such that (i) causality, local existence, and uniqueness of solutions hold in the nonlinear regime with or without coupling to Einstein's equations, (ii) linear stability around equilibrium holds, (iii) the equations of motion are derivable from microscopic approaches (such as kinetic theory), and (iv) solutions are guaranteed to exist in function spaces (the Sobolev spaces, see the Supplemental Material [31]) well suited for the implementation of numerical codes. (The theory introduced in Ref. [37] also satisfies (i)–(iii), but it is applicable only to conformal fluids and its solutions exist on function spaces more restrictive than Sobolev spaces.) This is the first time that all such properties are shown to hold since Eckart's seminal work in 1940 [60].

As Theorems 1 and 2 settle the basic question of causality and existence of solutions to the EIS system, it is now possible to generalize to relativistic viscous fluids key results known to hold for ideal fluids, such as the formation of shocks [61], global-in-time results [62], or the problem of accurately describing the fluid-vacuum interface on stars [63].

These results have two further applications: the hydrodynamic evolution of the quark-gluon plasma formed in heavy ion collisions [64] and cosmology. Current state-of-the-art modeling of quark-gluon plasma dynamics involves solving IS-like equations taking into account shear and bulk viscosities and, more recently, baryon diffusion [58]. Differently than the case of neutron star mergers [7], in heavy ion collisions shear and baryon diffusion effects are not negligible though bulk viscosity also plays an important role [28,65,66]. In this regard, we note Theorem 2 guarantees the existence of solutions even in the presence of cavitation where $P + \Pi \sim 0$ [67,68] and $\varepsilon > 0$, a phenomenon whose numerical description can be quite challenging [69]. Another application of the EIS system studied here is in cosmology, where the inclusion of bulk viscosity has been widely studied [70–74]. Indeed, our results do not make any symmetry assumptions, thus allowing the study of cosmological models with viscosity outside the symmetry class of homogeneous and isotropic solutions (e.g., Refs. [75,76]).

The equations here investigated suffice for many important applications (including neutron star mergers). Recent developments in EFTs, however, indicate that a more complete description of fluid phenomena should include further terms in the equations. While such contributions are negligible in many cases of interest, they consist of a genuine prediction of EFT, and their omission can lead to inconsistencies in 2nd order Kubo formulas [29,30]. Generalizing our results to theories that include these effects is, therefore, important for a more complete description of relativistic viscous fluids. While our techniques do not apply directly to such cases, we expect them to provide a starting point for studying these scenarios.

Our results open the door for new in-depth studies of fluid dynamics under extreme conditions such as in relativistic turbulent phenomena [77] and neutron star mergers. In particular, the latter is expected to take center stage in the coming years and our causality and existence results, which remain valid in the full nonlinear regime, provide the necessary cornerstone for quantitative studies of nonequilibrium viscous fluid phenomena in strong gravitational fields.

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