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CDMA and IDMA: Iterative Multiuser Detections for Near-Far Asynchronous Communications

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Abstract—**This paper provides comprehensive comparison of the iterative multiuser detection techniques for *code devision multiple access* (CDMA) and *interleave division multiple access* (IDMA). We investigate the performance in various scenarios such as user-asynchronism, multipath channels, near-far problem, and overloaded scenarios. We develop our system model which is general enough to take into account the above mentioned scenarios as well as being capable of incorporating aspects relevant to these access schemes. We provide formal descriptions of both schemes using our system model that illuminates the similarities and differences of the two different schemes. Computer simulations are performed in a variety of scenarios. It is observed that IDMA performs better than or as good as CDMA despite its simplicity.**

I. Introduction

In light of great success in turbo coding and decoding [1], the iterative decoding approach ("turbo principle") has been applied in a wide variety of detection and decoding problems such as equalizations and multiuser detections. Our focus in this paper is a *multiuser detection* (MUD) which, together with *maximum a posteriori probability* (MAP) decoder, iteratively mitigates *multiple access interference* (MAI) and also *inter symbol interference* (ISI), if the channel is frequency selective. MUD techniques have been intensively studied, especially in the area of CDMA systems. In [2] an optimal iterative MUD for synchronous CDMA has been derived. However, the complexity of this method is prohibitive for medium to large number of users. In [3] an iterative MUD for asynchronous CDMA has been proposed. This MUD applies an instantaneous *minimum mean square error* (MMSE) filter which is computed based on channel state information and *a priori* information provided from the previous decoding stage. This scheme realizes a good trade-off for the performance and its complexity.

Yet another attractive multiple access scheme was recently proposed, so called IDMA [4], [5]. In contrast to CDMA, which separates users by user specific signatures or spreading codes, distinct interleavers are the only means to separate users for IDMA. The detection algorithm for IDMA requires complexity significantly less than that of [3] for CDMA. The performance reported, e.g. in [4]–[7], is surprisingly good despite its simplicity. However, there has been no serious comparison between CDMA and IDMA. We are particularly interested in the performance for asynchronous communications (user asynchronous and multipath channel) and in near-far scenarios. These aspects are important for systems where perfect synchronization and power control are difficult to achieve or too costly. The uplink of cellular systems and decentralized systems are a few examples. We will also investigate the bandwidth efficiency of both systems in overloaded scenarios where the number of users exceeds the spreading factor for CDMA.

Our investigation is based on [3] which provides us with a useful system model for algorithm development. However, for multipath channels it restricts us to use spreading codes which are constant over symbols within every transmission frame (cf. Section V.A. in [3]). Therefore, in Section II we start our discussion with constructing our system model which can be commonly used for CDMA with scrambling code capability and also for IDMA. Then, we briefly review the iterative MUD for CDMA in Section III. In Section IV we re-derive the iterative MUD for IDMA in [8] using our system model in a similar way in Section III in order to enlighten differences and similarities of IDMA to CDMA. In Section V CDMA and IDMA are compared by means of computer simulations in various scenarios. This paper is summarized in Section VI.

II. System Model

We consider the discrete-time baseband system model illustrated for CDMA and IDMA in Fig. 1. Information bits $\boldsymbol{b}_k = [b_k[0], \ldots, b_k[N_\mathrm{b}-1]]^\mathrm{T} \in \{0,1\}^{N_\mathrm{b}}$ of user k, $k = 1, \ldots, K$, are encoded by the rate R_c convolutional channel encoder which gives $\boldsymbol{c}'_k = [c'_k[0], \ldots, c'_k[N_\mathrm{c}-1]]^\mathrm{T} \in \{0,1\}^{N_\mathrm{c}}$ coded bits where $(\bullet)^\mathrm{T}$ denotes transposition. For CDMA the coded bits are interleaved by the interleaver Π_k whose outputs $\boldsymbol{c}_k = [c_k[0], \ldots, c_k[N_\mathrm{c}-1]]^\mathrm{T}$ are mapped to BPSK symbols $\boldsymbol{s}_k = [s_k[0], \ldots, s_k[N_\mathrm{c}-1]]^\mathrm{T} \in \{+1,-1\}^{N_\mathrm{c}}$, which are then spread by the spreading code $\boldsymbol{u}_k[i] = [u_{k,0}[i], \ldots, u_{k,N_\mathrm{u}-1}[i]]^\mathrm{T} \in \{+1,-1\}^{N_\mathrm{u}}$. Note that $\boldsymbol{u}_k[i]$ has the symbol index i so that it may vary over symbols within each frame. The spread signals, i.e. chips, are multiplied with the scalar $a_k \in \mathbb{R}$, which controls the transmit power in order to simulate the near-far scenarios. Before the signal $\boldsymbol{x}_k = [x_k[0], \ldots, x_k[N-1]]^\mathrm{T}$ ($N_\mathrm{c}N_\mathrm{u} = N$) being transmitted to the channel, it is delayed by the user specific delay $\tau_k T_\mathrm{c}$ where T_c denotes chip time and τ_k is a nonnegative integer. This delay accounts for user-asynchronous scenarios.

For IDMA the output $\boldsymbol{c}'_k$ from the convolutional encoder is further encoded by the rate R_r simple repetition code to

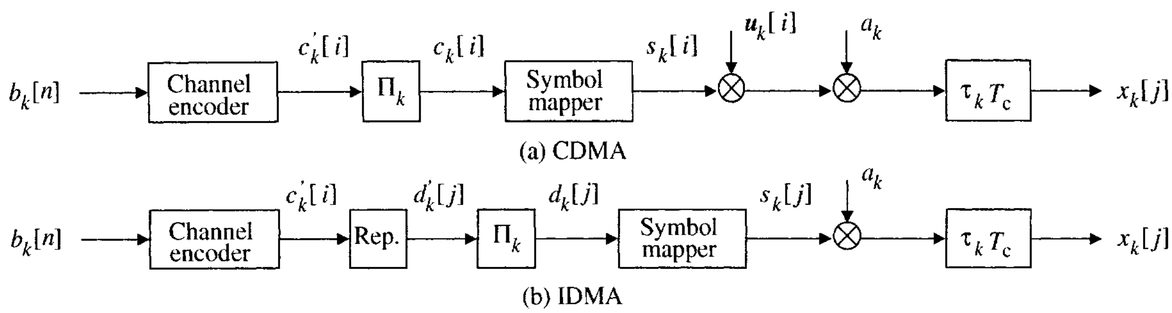

Fig. 1. Transmitter structures of CDMA and IDMA.

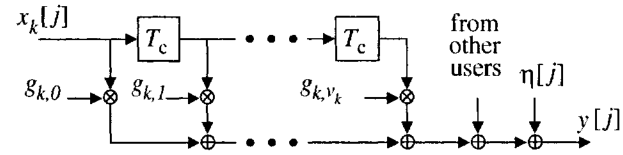

Fig. 2. Multiple access multipath channel.

get $\boldsymbol{d}'_k = [d'_k[0], \ldots, d'_k[N-1]]^{\mathrm{T}} \in \{0,1\}^N$. Throughout this paper we consider the same bandwidth efficiency for both CDMA and IDMA using the same convolutional encoder that means $R_\mathrm{r} = 1/N_\mathrm{u}$ so that $N = N_\mathrm{c}/R_\mathrm{r}$. The interleaver Π_k permutes $\boldsymbol{d}'_k$ to get $\boldsymbol{d}_k = [d_k[0], \ldots, d_k[N-1]]^{\mathrm{T}}$ which is mapped to BPSK symbol $\boldsymbol{s}_k = [s_k[0], \ldots, s_k[N-1]]^{\mathrm{T}}$. Note that the interleaver size is different for CDMA and IDMA except for $R_\mathrm{r} = N_\mathrm{u} = 1$ as long as the same bandwidth efficiency is maintained. The BPSK symbols, or chips, $\boldsymbol{s}_k$ are multiplied with a_k and delayed by $\tau_k T_\mathrm{c}$ in the same way as CDMA. Then the signal $\boldsymbol{x}_k$ is transmitted over the channel.

The channel is modeled with the finite-length impulse response filter $\sum_{\ell=0}^{\nu_k} g_{k,\ell}\delta[j-\ell]$ as illustrated in Fig. 2. The channel has the normalized energy of $\sum_{\ell=0}^{\nu_k} \mathrm{E}[g_{k,\ell}^2] = 1$. The coefficients $g_{k,\ell} \in \mathbb{R}, \forall k, \forall \ell$, are assumed to be time-invariant and known to the receiver. The channel memory ν_k is a nonnegative integer, and together with τ_k, the total delay of user k becomes $(\tau_k + \nu_k)T_\mathrm{c}$. We define the maximum total delays plus one over all users as $D_\mathrm{c} = \max_k(\tau_k + \nu_k + 1)$ and $D_\mathrm{s} = \lceil (D_\mathrm{c} - 1)/N_\mathrm{u} \rceil + 1$, in number of chips and in symbols, respectively, where $\lceil \bullet \rceil$ rounds the argument to the nearest larger integer. The received signal $y[j]$ comprises of the signals from K users propagated over the respective channel and it is corrupted by the noise $\eta[j] \in \mathbb{R}$ with the variance $\sigma_\eta^2 = N_0/2$. The noise process is assumed to be independent and identically distributed and independent of the data.

By denoting $\boldsymbol{s}[i] = [s_1[i], \ldots, s_K[i]]^{\mathrm{T}} \in \mathbb{R}^K$ and the *effective channel* $\boldsymbol{H}^{(i)}[\ell] = [\boldsymbol{h}_1^{(i)}[\ell], \ldots, \boldsymbol{h}_K^{(i)}[\ell]]$ of dimension $N_\mathrm{u} \times K$, the received signal vector may be expressed as

$$\boldsymbol{y}[i] = \sum_{\ell=0}^{D_s-1} \boldsymbol{H}^{(i-\ell)}[\ell]\boldsymbol{s}[i-\ell] + \boldsymbol{\eta}[i], \tag{1}$$

where $\boldsymbol{\eta}[i]$ is the noise vector of dimension N_u. Note that the effective channel, which takes into account the channel and the spreading code, may be seen time-varying over symbols if the spreading code is dependent on the symbol index i, e.g. using scrambling code for CDMA. The k-th column of $\boldsymbol{H}^{(i)}[\ell]$, i.e. $\boldsymbol{h}_k^{(i)}[\ell]$, can be determined from the discrete convolution

$$\boldsymbol{f}_k^{(i)} = a_k \boldsymbol{u}'_k[i] * \boldsymbol{g}'_k,$$

where $\boldsymbol{g}'_k = [\boldsymbol{0}_{\tau_k}^{\mathrm{T}}, g_{k,0}, \ldots, g_{k,\nu_k}, \boldsymbol{0}_{D_c-\tau_k-\nu_k-1}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{D_c}$ takes into account the channel and delay of user k and $\boldsymbol{u}'_k[i] = [\boldsymbol{u}_k^{\mathrm{T}}[i], \boldsymbol{0}_{N_\mathrm{u}(D_s-1)}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{N_\mathrm{u}D_s}$ the spreading code of user k for symbol i where $\boldsymbol{0}_i$ denotes zero vector of dimension i. Then, $\boldsymbol{h}_k^{(i)}[\ell]$ is defined as

$$\boldsymbol{h}_k^{(i)}[\ell] = [\boldsymbol{f}_k^{(i)}[\ell N_\mathrm{u} + 1], \ldots, \boldsymbol{f}_k^{(i)}[(\ell+1)N_\mathrm{u}]]^{\mathrm{T}}.$$

By defining the symbol vector

$$\boldsymbol{s} = [\boldsymbol{s}^{\mathrm{T}}[i - D_\mathrm{s} + 1], \ldots, \boldsymbol{s}^{\mathrm{T}}[i], \ldots, \boldsymbol{s}^{\mathrm{T}}[i + D_\mathrm{s} - 1]]^{\mathrm{T}},$$

of dimension $K(2D_\mathrm{s} - 1)$, and the channel matrix

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}^{(i-D_s+1)}[D_\mathrm{s}-1] \cdots & \boldsymbol{H}^{(i)}[0] & & \\ & \ddots & \vdots & \ddots & \\ & & \boldsymbol{H}^{(i)}[D_\mathrm{s}-1] \cdots & \boldsymbol{H}^{(i+D_s-1)}[0] \end{bmatrix},$$

of dimension $N_\mathrm{u}D_\mathrm{s} \times K(2D_\mathrm{s} - 1)$, the received signal vector

$$\boldsymbol{y} = [\boldsymbol{y}^{\mathrm{T}}[i], \ldots, \boldsymbol{y}^{\mathrm{T}}[i + D_\mathrm{s} - 1]]^{\mathrm{T}},$$

of dimension $N_\mathrm{u}D_\mathrm{s}$ can be concisely expressed as follows

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\eta}, \tag{2}$$

where $\boldsymbol{\eta} = [\boldsymbol{\eta}^{\mathrm{T}}[i], \ldots, \boldsymbol{\eta}^{\mathrm{T}}[i + D_\mathrm{s} - 1]]^{\mathrm{T}} \in \mathbb{R}^{N_\mathrm{u}D_\mathrm{s}}$.

Note that the above development is also applicable for IDMA by replacing the symbol index i with the chip index j, and setting $N_\mathrm{u} = 1$ and $\boldsymbol{u}_k[j] = 1$. Since symbol is equivalent to chip for IDMA, the symbol delay is equal to the chip delay, i.e. $D_\mathrm{s} = D_\mathrm{c}$. Because there is no spreading for IDMA, the effective channel is constant over chips, i.e. there is no dependency of the effective channel on the chip index j. The received signal vector in (1) may be written as $\boldsymbol{y}[j] = \sum_{\ell=0}^{D_c-1} \boldsymbol{H}[\ell]\boldsymbol{s}[j-\ell] + \boldsymbol{\eta}[j]$ and the effective channel matrix $\boldsymbol{H}$ can be found accordingly.

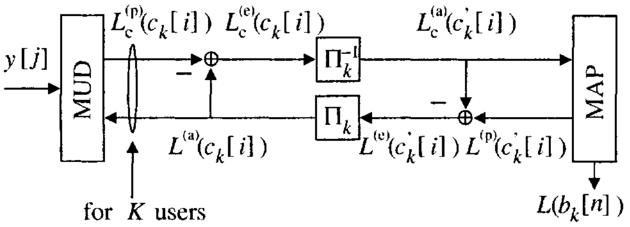

Fig. 3. Receiver structure of iterative detection for CDMA.

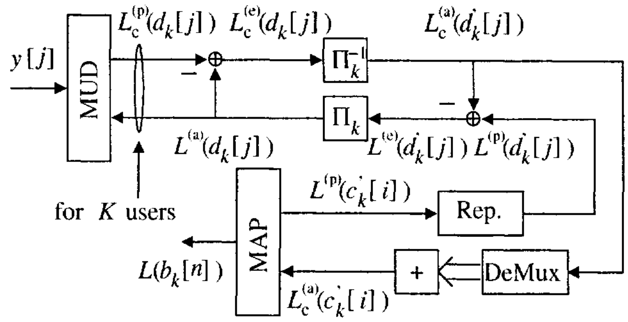

Fig. 4. Receiver structure of iterative detection for IDMA.

III. ITERATIVE MULTIUSER DETECTION FOR CDMA

We briefly review the iterative multiuser detection for CDMA based on MMSE filter proposed in [3]. The receiver structure of the iterative detection for CDMA is illustrated in Fig. 3. The MUD computes *a posteriori* log-likelihood ratio (LLR) $L_c^{(p)}(c_k[i])$ about the coded bits $c_k[i]$ based on its inputs of the received signal $\boldsymbol{y}$ and *a priori* LLR $L^{(a)}(c_k[i])$ from the decoder that is initialized to be zero before any iteration. The *a posteriori* LLR is calculated as

$$\begin{aligned} L_c^{(p)}(c_k[i]) &= \log\frac{P\{c_k[i]=0 \mid \boldsymbol{y}\}}{P\{c_k[i]=1 \mid \boldsymbol{y}\}} \\ &= \underbrace{\log\frac{P\{\boldsymbol{y} \mid c_k[i]=0\}}{P\{\boldsymbol{y} \mid c_k[i]=1\}}}_{L_c^{(e)}(c_k[i])} + \underbrace{\log\frac{P\{c_k[i]=0\}}{P\{c_k[i]=1\}}}_{L^{(a)}(c_k[i])}. \end{aligned}$$

The *extrinsic* LLR $L_c^{(e)}(c_k[i])$ is sent to the decoder after it is deinterleaved by Π_k^{-1}. Since the exact computation of $L_c^{(e)}(c_k[i])$ is too complex, an approximated equivalent channel model is considered as,

$$z_k[i] = \mu_k[i]s_k[i] + \zeta_k[i],$$

where $z_k[i]$ is to be computed from $\boldsymbol{y}$ and $L^{(a)}(c_k[i])$. That is explained in the next paragraph. Let us assume, for a moment, that $z_k[i]$ is given and also that $\zeta_k[i] \sim \mathcal{N}(0, \sigma^2_{\zeta_k[i]})$, then the computation of $L_c^{(e)}(c_k[i])$ is greatly simplified to

$$\begin{aligned} L_c^{(e)}(c_k[i]) &\approx \log\frac{P\{z_k[i] \mid s_k[i]=+1\}}{P\{z_k[i] \mid s_k[i]=-1\}} \\ &= \frac{2\mu_k[i]z_k[i]}{\sigma^2_{\zeta_k[i]}}, \end{aligned} \quad (3)$$

where the second line is obtained by using Gaussian probability density function. The value of $L_c^{(e)}(c_k[i])$ is merely determined by $z_k[i]$, its mean $\mu_k[i]$, and variance $\sigma^2_{\zeta_k[i]}$.

The computation of $z_k[i]$ is done in two steps: soft interference cancellation and MMSE filtering. By defining the soft symbol vector $\tilde{\boldsymbol{s}}[i] = [\tilde{s}_1[i], \ldots, \tilde{s}_K[i]]^{\mathrm{T}}$ and

$$\tilde{\boldsymbol{s}} = [\tilde{\boldsymbol{s}}^{\mathrm{T}}[i-D_s+1], \ldots, \tilde{\boldsymbol{s}}^{\mathrm{T}}[i], \ldots, \tilde{\boldsymbol{s}}^{\mathrm{T}}[i+D_s-1]]^{\mathrm{T}}, \quad (4)$$

the soft interference cancellation for i-th symbol of user k is performed as

$$\boldsymbol{y}_k[i] = \boldsymbol{y} - \boldsymbol{H}(\tilde{\boldsymbol{s}} - \tilde{s}_k[i]\boldsymbol{e}_\kappa),$$

where $\boldsymbol{e}_\kappa$ is the κ-th column of identity matrix of dimension $K(2D_s-1)$ and $\kappa = (D_s-1)K + k$. The soft symbol is calculated from the *a priori* LLR for $c_k[i]$ that reads as

$$\tilde{s}_k[i] = \tanh\{L^{(a)}(c_k[i])/2\}. \quad (5)$$

This well-known result can be easily obtained by solving $\tilde{s}_k[i] = \sum_{s_j=+1,-1} s_j P\{s_k[i]=s_j\}$ and $L^{(a)}(c_k[i]) = \log\frac{P\{s_k[i]=+1\}}{P\{s_k[i]=-1\}}$ for $\tilde{s}_k[i]$. The second step is to apply MMSE filter that is

$$z_k[i] = \boldsymbol{w}_k^{\mathrm{T}}[i]\boldsymbol{y}_k[i],$$

where $\boldsymbol{w}_k[i]$ is computed from the following optimization:

$$\begin{aligned} \boldsymbol{w}_k[i] &= \underset{\boldsymbol{w}'_k[i]}{\operatorname{argmin}}\, \mathrm{E}[(s_k[i] - \boldsymbol{w}'^{\mathrm{T}}_k[i]\boldsymbol{y}_k[i])^2] \\ &= \boldsymbol{R}^{-1}_{\boldsymbol{y}_k[i]}\boldsymbol{r}_{\boldsymbol{y}_k[i]s_k[i]}, \end{aligned}$$

where

$$\boldsymbol{R}_{\boldsymbol{y}_k[i]} = \mathrm{E}[\boldsymbol{y}_k[i]\boldsymbol{y}_k^{\mathrm{T}}[i]] = \boldsymbol{H}(\boldsymbol{I} - \tilde{\boldsymbol{S}}^2 + \tilde{s}_k^2[i]\boldsymbol{e}_\kappa\boldsymbol{e}_\kappa^{\mathrm{T}})\boldsymbol{H}^{\mathrm{T}} + \sigma_\eta^2\boldsymbol{I},$$

$$\boldsymbol{r}_{\boldsymbol{y}_k[i]s_k[i]} = \mathrm{E}[\boldsymbol{y}_k[i]s_k[i]] = \boldsymbol{H}\boldsymbol{e}_\kappa,$$

and $\tilde{\boldsymbol{S}} = \mathrm{diag}\{\tilde{\boldsymbol{s}}\}$. What remains is the computation of mean and variance of $z_k[i]$ which reads as

$$\mu_k[i] = \mathrm{E}[z_k[i]s_k[i]] = \boldsymbol{r}^{\mathrm{T}}_{\boldsymbol{y}_k[i]s_k[i]}\boldsymbol{R}^{-1}_{\boldsymbol{y}_k[i]}\boldsymbol{r}_{\boldsymbol{y}_k[i]s_k[i]},$$

$$\sigma^2_{\zeta_k[i]} = \mathrm{E}[(z_k[i] - \mu_k[i]s_k[i])^2] = \mu_k[i] - \mu_k^2[i].$$

Then the *extrinsic* LLR $L_c^{(e)}(c_k[i])$ can be calculated from (3) for N_c coded bits of K users. The MAP decoder is a standard function (e.g. [9]) which we do not describe in this paper.

IV. ITERATIVE MULTIUSER DETECTION FOR IDMA

The multiuser detection for IDMA can be found, e.g. in [8]. Here, we re-formulate it following the development for CDMA in the previous section in order to show its difference and similarity to CDMA. The receiver structure of the iterative detection for IDMA is illustrated in Fig. 4. As for CDMA, the MUD for IDMA computes *a posteriori* LLR $L_c^{(p)}(d_k[j])$ about the coded bits $d_k[j]$ based on its inputs of the received signal $\boldsymbol{y}$ and *a priori* LLR $L^{(a)}(d_k[j])$ from the decoder. The *extrinsic* LLR $L_c^{(e)}(d_k[j])$, which is given by subtracting $L^{(a)}(d_k[j])$ from $L_c^{(p)}(d_k[j])$, is defined as $L_c^{(e)}(d_k[j]) = \log\frac{P\{\boldsymbol{y}|d_k[j]=0\}}{P\{\boldsymbol{y}|d_k[j]=1\}}$.

Since its direct computation is too complex, an approximated equivalent channel model is considered as

$$z_k^{(\ell)}[j] = \mu_k^{(\ell)}[j]s_k[j] + \zeta_k^{(\ell)}[j],$$

for each delay component $0 \leq \ell \leq \nu_k$. Then the following approximation analogous to rake combiner is applied,

$$\begin{aligned} L_{\mathrm{c}}^{(\mathrm{e})}(d_k[j]) &= \log \frac{P\{\boldsymbol{y} \mid s_k[j] = +1\}}{P\{\boldsymbol{y} \mid s_k[j] = -1\}} \\ &\approx \log \frac{\prod_{\ell=0}^{\nu_k} P\left\{z_k^{(\ell)}[j] \mid s_k[j] = +1\right\}}{\prod_{\ell=0}^{\nu_k} P\left\{z_k^{(\ell)}[j] \mid s_k[j] = -1\right\}} \\ &= \sum_{\ell=0}^{\nu_k} \log \frac{P\left\{z_k^{(\ell)}[j] \mid s_k[j] = +1\right\}}{P\left\{z_k^{(\ell)}[j] \mid s_k[j] = -1\right\}} \\ &= \sum_{\ell=0}^{\nu_k} \frac{2\mu_k^{(\ell)}[j] z_k^{(\ell)}[j]}{\sigma^2_{\zeta_k^{(\ell)}[j]}}, \end{aligned} \tag{6}$$

where we assumed $\zeta_k^{(\ell)}[j] \sim \mathcal{N}(0, \sigma^2_{\zeta_k^{(\ell)}[j]})$ to get the last line. If we denote $L_{\mathrm{c}}^{(\mathrm{e})}(d_k^{(\ell)}[j]) = 2\mu_k^{(\ell)}[j] z_k^{(\ell)}[j] / \sigma^2_{\zeta_k^{(\ell)}[j]}$, then

$$L_{\mathrm{c}}^{(\mathrm{e})}(d_k[j]) \approx \sum_{\ell=0}^{\nu_k} L_{\mathrm{c}}^{(\mathrm{e})}(d_k^{(\ell)}[j]).$$

Its rake-like approximation is clearly seen.

In contrast to MMSE filtering for CDMA, IDMA applies only the soft interference cancellation to get $z_k^{(\ell)}[j]$:

$$\begin{aligned} \boldsymbol{y}_k[j] &= \boldsymbol{y} - \boldsymbol{H}(\tilde{\boldsymbol{s}} - \tilde{s}_k[j]\boldsymbol{e}_\kappa), \\ z_k^{(\ell)}[j] &= \boldsymbol{e}_\iota^{\mathrm{T}} \boldsymbol{y}_k[j], \end{aligned} \tag{7}$$

where $\iota = \tau_k + \ell + 1$ takes into account the user delay, $\boldsymbol{H}$ has dimension $D_{\mathrm{c}} \times (2D_{\mathrm{c}} - 1)K$ and $\kappa = (D_{\mathrm{c}} - 1)K + k$. The soft symbol $\tilde{s}_k[j]$ and vector $\tilde{\boldsymbol{s}}$ are defined in the same way as (4) where i and D_{s} are replaced by j and D_{c}, respectively. Each soft symbol is calculated from the *a priori* LLR like in (5) as $\tilde{s}_k[j] = \tanh\{L^{(\mathrm{a})}(d_k[j])/2\}$. By defining

$$\boldsymbol{\Lambda}_\kappa = \sum_{j \neq \kappa} \boldsymbol{e}_j \boldsymbol{e}_j^{\mathrm{T}},$$

the received signal $\boldsymbol{y}$ can be expressed as

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{H}(\boldsymbol{\Lambda}_\kappa + \boldsymbol{e}_\kappa \boldsymbol{e}_\kappa^{\mathrm{T}})\boldsymbol{s} + \boldsymbol{\eta} \\ &= \boldsymbol{H}\boldsymbol{e}_\kappa s_k[j] + \boldsymbol{H}\boldsymbol{\Lambda}_\kappa \boldsymbol{s} + \boldsymbol{\eta}, \end{aligned} \tag{8}$$

because $\boldsymbol{\Lambda}_\kappa + \boldsymbol{e}_\kappa \boldsymbol{e}_\kappa^{\mathrm{T}} = \boldsymbol{I}$ and $\boldsymbol{e}_\kappa^{\mathrm{T}} \boldsymbol{s} = s_k[j]$. The first component of (8) is the desired signal with the respective channel and the second one represents MAI and ISI. The third component is the noise term. From (7) and (8), $z_k^{(\ell)}[j]$ is rewritten as

$$z_k^{(\ell)}[j] = \underbrace{\boldsymbol{e}_\iota^{\mathrm{T}} \boldsymbol{H} \boldsymbol{e}_\kappa}_{\mu_k^{(\ell)}[j]} s_k[j] + \underbrace{\boldsymbol{e}_\iota^{\mathrm{T}} \boldsymbol{H}(\boldsymbol{\Lambda}_\kappa \boldsymbol{s} - \tilde{\boldsymbol{s}} + \tilde{s}_k[j]\boldsymbol{e}_\kappa) + \boldsymbol{e}_\iota^{\mathrm{T}} \boldsymbol{\eta}}_{\zeta_k^{(\ell)}[j]}.$$

Therefore, $\mu_k^{(\ell)}[j]$ is the ℓ-th channel coefficient of user k. $\zeta_k^{(\ell)}[j]$ comprises of the noise and the soft estimation error of MAI and ISI. Using this expression and based on the assumption of $\zeta_k^{(\ell)}[j] \sim \mathcal{N}(0, \sigma^2_{\zeta_k^{(\ell)}[j]})$, the variance can be computed as

$$\sigma^2_{\zeta_k^{(\ell)}[j]} = \boldsymbol{e}_\iota^{\mathrm{T}} \boldsymbol{H}(\boldsymbol{I} - \tilde{\boldsymbol{S}}^2) \boldsymbol{\Lambda}_\kappa \boldsymbol{H}^{\mathrm{T}} \boldsymbol{e}_\iota + \sigma_\eta^2,$$

where $\tilde{\boldsymbol{S}} = \mathrm{diag}\{\tilde{\boldsymbol{s}}\}$. One can confirm that the results are essentially equivalent to Eqns. (9) and (10) in [8]. However, the relation to the MMSE approach for CDMA in the previous section can be more clearly seen with our results. From these results the *extrinsic* LLR $L_{\mathrm{c}}^{(\mathrm{e})}(d_k[j])$ can be computed from (6) for N coded bits of K users. $L_{\mathrm{c}}^{(\mathrm{e})}(d_k[j])$ is deinterleaved by Π_k^{-1} to get $L_{\mathrm{c}}^{(\mathrm{a})}(d'_k[j])$ which is then de-multiplexed to every $1/R_{\mathrm{r}}$ elements and summed up to end up with $L_{\mathrm{c}}^{(\mathrm{a})}(c'_k[i])$. The output from MAP decoder, $L^{(\mathrm{p})}(c'_k[i])$, is repeated at rate R_{r}, subtracted by $L_{\mathrm{c}}^{(\mathrm{a})}(d'_k[j])$ to get $L^{(\mathrm{e})}(d'_k[j])$, and finally sent to MUD after interleaved by Π_k.

V. Simulation Results

In this section we illustrate the performance of the iterative MUDs for CDMA and IDMA in various scenarios. Information bits of frame length $N_{\mathrm{b}} = 128$ are encoded by the rate $R_{\mathrm{c}} = 1/2$ memory 4 standard convolutional code with generator polynomial $[23, 35]_8$. The trellis of the convolutional encoder is terminated that requires 4 termination bits, resulting in 264 coded bits. For IDMA the coded bits are further encoded by the rate $R_{\mathrm{r}} = 1/4$ simple repetition code that gives 1056 coded bits. Therefore, the size of the interleaver Π_k is accordingly determined to be 264 and 1056 for CDMA and IDMA, respectively. The multiple interleavers are randomly generated from uniform distribution. The interleavers for all users are newly generated for each transmission frame and are independent of each other. Note that the user-independent interleavers are used for both IDMA and CDMA. The user spreading code for CDMA is constructed from a short code, which is multiplied with a long code if a scrambling code is applied. The user distinct short codes of spreading factor $N_{\mathrm{u}} = 4$ are taken from *orthogonal variable spreading factor* (OVSF) codes (or Hadamard codes) for $K = 4$. The long code is taken from the uplink long scrambling sequence defined in the UMTS standard specification [10]. We evaluate CDMA both with and without the scrambling sequence.

Fig. 5 shows the BER performance of CDMA with/without scrambling code and IDMA on AWGN channel where $K = 4$ users are asynchronous with the user specific delays of $\tau_k = k-1$ for $k = 1, 2, 3, 4$. The BER in this figure is averaged over 4 users and plotted for two cases: before any iteration and after 8 iterations. The performance without iteration can be regarded as the conventional non-iterative detector's performance. The single user performance is also plotted for the comparison. Without iteration the performance for all schemes is poor. Note that without iteration IDMA does not apply any interference cancellation because the soft interference cancellation is inactive (no *a priori* information from the decoder) while CDMA applies MMSE equalizer which mitigates the interference. Therefore, the performance of CDMA is much better than that of IDMA without any iteration. Although CDMA with

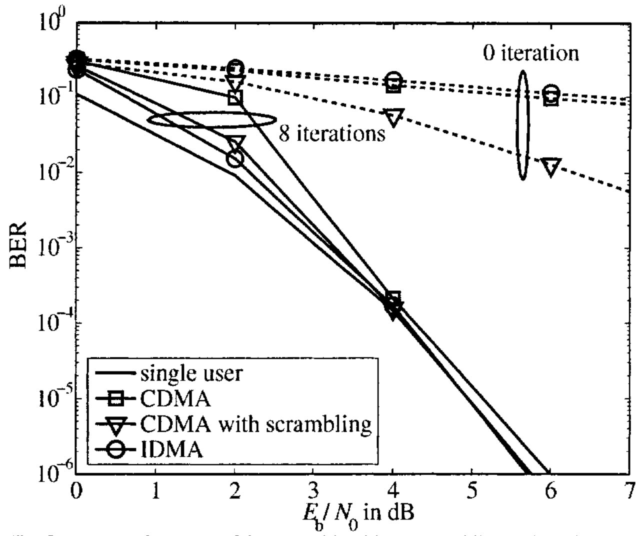

Fig. 5. BER performance of CDMA with/without scrambling code and IDMA on AWGN channel. $K = 4$ users are asynchronous with the user specific delays of $\tau_k = k - 1$ for $k = 1, 2, 3, 4$.

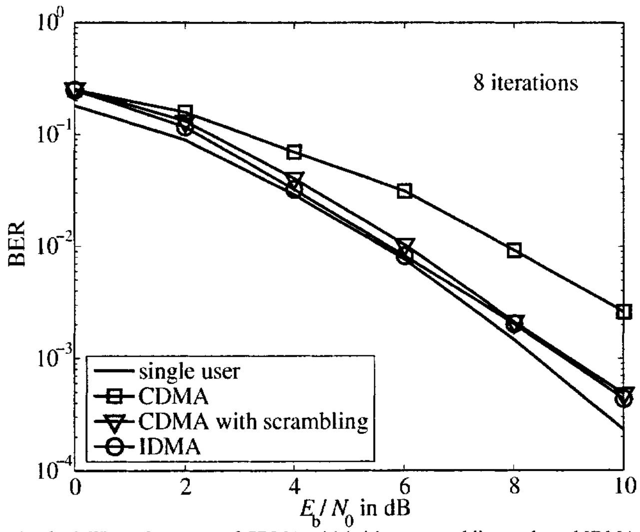

Fig. 6. BER performance of CDMA with/without scrambling code and IDMA on multipath channel ($\nu_k = 7$, $\forall k$, uniform power delay profile). $K = 4$ users are synchronous ($\tau_k = 0$, $\forall k$).

scrambling code performs best amongst all, it is still far from the single user bound. Even those small amounts of user asynchronism (maximum of 3 chips out of the frame length of 1056 chips) break the orthogonality of users for CDMA on the perfectly synchronous AWGN channel (no ISI). The performance of all the schemes, however, approaches the single user bound after 8 iterations. IDMA performs best in spite of the least amount of computational efforts required at the receiver.

Fig. 6 illustrates the BER performance on multipath channel with the setting similar to the previous scenario. The channel delay is set as $\nu_k = 7$ for all $K = 4$ users. The channel taps are generated from zero mean Gaussian distribution with uniform power delay profile, i.e. $\mathrm{E}[g_{k,\ell}^2] = 1/(\nu_k + 1) = 1/8$.

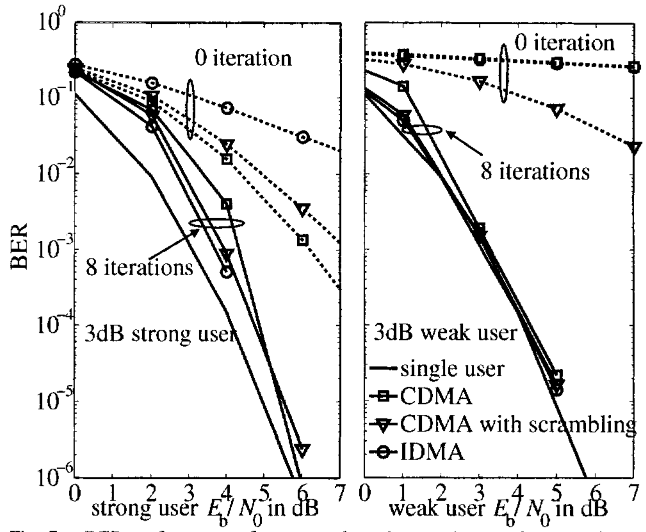

Fig. 7. BER performance of strong and weak users in near-far scenario on AWGN user-asynchronous channel ($\tau_k = k-1$ for $k = 1, 2, 3, 4$) for CDMA with/without scrambling code and IDMA. 2 users use 3 dB higher transmit power than the other 2 users.

We assume quasi static channel and ideal channel estimation at the receiver. All users transmit their data synchronously ($\tau_k = 0\ \forall k$). Plotted is the performance averaged over all users after 8 iterations. The performance of IDMA and CDMA with scrambling code converges to the single user bound. The performance of CDMA without the scrambling code is about 2 dB worse.

Next, we consider near-far scenarios where 2 users use transmit power which is 3 dB higher than the other 2 users ($a_1 = a_2 = 1, a_3 = a_4 = 10^{-3/20}$). The BER performance of the strong and weak users on AWGN user-asynchronous channel ($\tau_k = k - 1$ for $k = 1, 2, 3, 4$) is illustrated in Fig. 7. The BERs before and after 8 iterations are plotted over E_b/N_0. The single user performance is plotted for the comparison as well. It is well known that conventional non-iterative detection for multiple access schemes of CDMA-type experiences the near-far problem when the transmit power control is not performed. That can be clearly observed from the figure. The weak user heavily suffers from strong user's signal and the performance degrades severely. The performance of all the schemes drastically improves iteration by iteration and it approaches the single user bound. The iterative detection brings more benefits to the weak user than to the strong user because after every iteration the strong user's signal can be detected more reliably, and therefore can be cancelled out from the weak user's signal (soft interference cancellation) with low probability of error.

Fig. 8 shows the BER performance of the strong and weak users on multipath channel in the near-far scenario. The channel parameters are the same as described for Fig. 6. The transmit power for the first and second users is 3 dB higher than that for the third and fourth users. Similar to AWGN channel, the performance approaches the single user bound after 8 iterations and the weak users get more benefit than the strong users by the iterative detection.

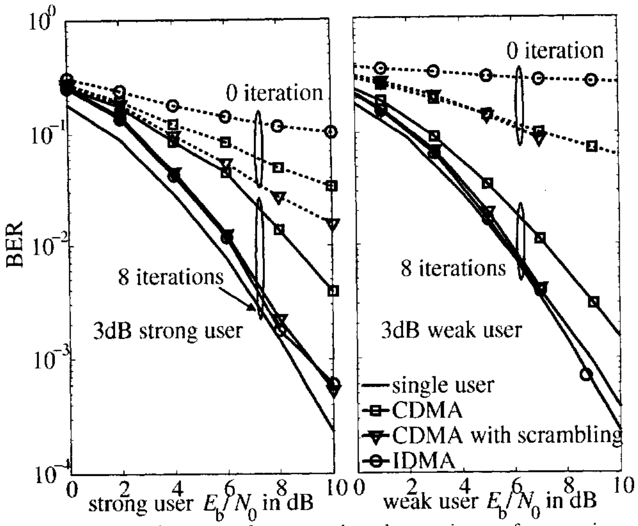

Fig. 8. BER performance of strong and weak users in near-far scenario on multipath channel ($\nu_k = 7$, $\forall k$, uniform power delay profile) for CDMA with/without scrambling code and IDMA. $K = 4$ users are synchronous ($\tau_k = 0$, $\forall k$). 2 users use 3 dB higher transmit power than the other 2 users.

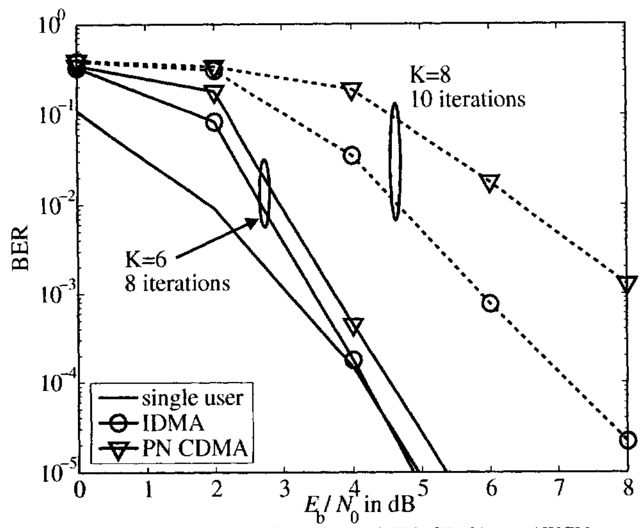

Fig. 9. BER performance of IDMA and PN CDMA on AWGN user-asynchronous channel ($\tau_k = k - 1$, $\forall k$) in overloaded scenarios ($K = 6, 8 > N_u = 1/R_r = 4$).

From the simulation results illustrated so far, we observed that in fully loaded scenarios ($K = N_u = 1/R_r = 4$) IDMA performs as good as or even slightly better than CDMA, which uses orthogonal short codes with/without long scrambling sequence. Note again that we use user-independent interleavers also for CDMA. We also compare both schemes in overloaded scenarios where the number of users exceeds the spreading factor or the repetition code length ($K = 6, 8 > N_u = 1/R_r = 4$). For the overloaded scenario we apply the common sign-alternating code $[+1, -1, +1, -1]$ for all users as the short code and the user-distinct UMTS uplink long scrambling sequences as the long code. Fig. 9 shows the performance of IDMA and random code or pseudo noise code CDMA (PN CDMA) on AWGN user-asynchronous channel. The BER is averaged over all equal power users. It can be seen that the performance still approaches the single user bound for $K = 6$ users after 8 iterations for both IDMA and PN CDMA. IDMA performs slightly better than PN CDMA, but the difference is marginal. The difference becomes more pronounced for $K = 8$ users, which is twice as large as the spreading factor and repetition code length: $N_u = 1/R_r = 4$. The performance degradation of PN CDMA against IDMA at BER=10^{-3} after 10 iterations is about 2 dB. The performance does not further improve after 10 iterations.

VI. Summary

We started our investigation with constructing our system model which is capable of using scrambling code for CDMA and also incorporating our main interests such as user asychronism, near-far scenario, and frequency selectivity of the channel in a convenient way. Using the system model the iterative MUD for CDMA applying the instantaneous MMSE filtering was briefly reviewed. Then, we re-derived the iterative MUD for IDMA in a similary way of the development for CDMA. By doing so, the similarity and difference between CDMA and IDMA were illuminated. We evaluated both techniques by means of computer simulations in various scenarios such as user asynchronism, multipath channel, near-far problem, and overloaded systems. In all the scenarios we evaluated, IDMA performs better than or as good as CDMA despite its simplicity.

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References

[1] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo-codes," *IEEE Transactions on Communications*, vol. 44, no. 10, pp. 1261–1271, October 1996.

[2] M. Moher, "An iterative multiuser decoder for near-capacity communications," *IEEE Transactions on Communications*, vol. 46, no. 7, pp. 870–880, July 1998.

[3] X. Wang and H. V. Poor, "Iterative (Turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Transactions on Communications*, vol. 47, no. 7, pp. 1046–1061, July 1999.

[4] L. Ping, L. Liu, K. Y. Wu, and W. K. Leung, "On interleave-division multiple-access," in *Proc. IEEE International Conference on Communications*, vol. 5, June 2004, pp. 2869–2873.

[5] H. Schoeneich and P. A. Hoeher, "A hybrid multiple access scheme delivering reliability information," in *Proc. 5th Int. ITG Conf. on Source and Channel Coding (SCC)*, January 2004, pp. 437–442.

[6] L. Ping, L. Liu, K. Y. Wu, and W. K. Leung, "Interleave-division multiple-access (IDMA) communications," in *Proc. 3rd International Symposium on Turbo Codes & Related Topics*, 2003, pp. 173–180.

[7] L. Ping, L. Liu, and W. K. Leung, "A simple approach to near-optimal multiuser detection: Interleave-division multiple-access," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC2003)*, vol. 1, March 2003, pp. 391–396.

[8] L. Ping, L. Liu, K. Y. Wu, and W. K. Leung, "Interleave-division multiple-access," *submitted to IEEE Transaction on Wireless Communications*.

[9] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Transactions on Information Theory*, vol. 20, no. 2, pp. 284–287, March 1974.

[10] 3G TS 25.213, "3rd generation partnership project; technical specification group radio access network; spreading and modulation (FDD) (release 4)," June 2002.