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Center of mass perception: Perturbation of symmetry

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Bingham and Muchisky (1993) found that observers were very accurate in determining the location of the center of mass in planar objects. Systematic errors were affected primarily by object orientation, while random errors varied with the amount of symmetry. Radial and axial reflective symmetry affected errors in different ways. In the current study, we investigated the different effects of axial reflective versus rotational symmetry. All random errors decreased with increasing rotational symmetry. Axial reflective symmetry further reduced errors in the direction perpendicular to the axis. We replicated the effect on systematic error of orientation. However, we also found an effect of the perturbation of symmetry that suggested that observers used an approximation to symmetry. To investigate this possibility, we constructed a series of objects in which axial reflective symmetry was established and then perturbed by varying amounts. We found that systematic errors were structured by the underlying approximate symmetries, and we discuss the problem of quantifying symmetry.

The role of visual perception in the act of reaching to grasp an object is not only in guiding the appropriately configured hand to the object, but also in selecting target locations on the surface of the object where the grasp is to be established (Bingham & Muchisky, 1993; Iberall, Bingham, & Arbib, 1986). In the context of assembly tasks where "precision grasps" (Napier, 1980) are used, grasps often are established with respect to the center of mass in an object. In a precision grasp, an object is pinched between finger and thumb pads at two points of contact. The stability of such a grasp is determined by whether the grasp is at, above, or below the center of mass.

Bingham and Muchisky (1993) investigated whether observers could determine the location of the center of mass on the basis of object shape when given objects composed of a single material.[1] Participants were asked to indicate with a pair of tongs where a precision grasp should be located on planar objects so that the object would remain at the same orientation at which it was placed within the grasp.

Because the center of mass is itself a symmetry property, Bingham and Muchisky (1993) hypothesized that increased symmetry in object shape would enable more accurate estimates. Seven planar object shapes were used to vary the amount of symmetry in the plane. As shown in Figure 1, the shapes were quadrilaterals, right triangles, isosceles triangles, parallelograms, rectangles, equilateral triangles, and squares. These shapes exhibit qualitatively distinct types of planar symmetry, including reflection through a line (axial reflective symmetry), reflection through a point (radial symmetry), and congruences with rotation in the plane about a point (rotational symmetry).

Given these qualitatively distinct types of symmetry, establishing a way to quantify symmetry is difficult. Symmetry can be indexed within a type—one can count, for instance, the number of reflective symmetry axes, or the number of congruences within a 360° rotation, or the simple presence or absence of radial symmetry. The question is, how should symmetry be counted *across* types? Rosen (1983) has suggested that symmetry should be quantified in terms of corresponding symmetry groups. Objects with different types of symmetry may be isomorphic to the same symmetry group, as are, for instance, the isosceles triangle and the parallelogram. Both can be represented in terms of the order-2 group, C_2, which consists of the identity transformation (that is, not doing anything) and one other transformation which, when applied twice, yields the identity. For the isosceles triangle the additional transformation is reflection through an axis, while for the parallelogram it consists of rotations in the plane by 180°. Rosen has suggested that amount of symmetry should be counted in terms of the order of the group representation. Thus, according to Rosen, the equilateral triangle, which is isomorphic to the order-6 group, D_3, would have 3 times the symmetry of the isosceles triangle or the parallelogram.

In Figure 1, the ordering of the shapes produced by Rosen's (1983) scheme is the same as that used by Bingham and Muchisky (1993) who computed "total symmetry" by adding the number of reflective symmetry axes to the number of congruences in 360° of rotation in the plane, finally incrementing the sum by 1 if radial sym-

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metry also was present.[2] Bingham and Muchisky found that random errors generally decreased with increases in total symmetry. However, total symmetry (or the order of the symmetry groups) was an imperfect predictor of random error.

Judgments of equilateral triangles exhibited more random error than did those of shapes with less total symmetry, including rectangles and parallelograms. The key difference between these figures was the presence or absence of radial symmetry. Thus, qualitatively distinct types of symmetry appeared to have had different effects on the accuracy of judgments, in which case reduction to the underlying group structure would be an inappropriate way to quantify symmetry in this application. Radial symmetry determined larger differences in random error within which the effect of other forms of symmetry could be discerned. Increased amounts of both rotational and axial reflective symmetry appeared to have reduced random errors. However, we had difficulty in distinguishing the respective effects of these symmetries because they were confounded in the shapes that we had used.

All of the shapes, with the exception of the isosceles triangle and the parallelogram, were equivalent in the amounts of rotational and axial reflective symmetry. We measured systematic and random error independently along orthogonal x and y-axes. The introduction of an axis of reflective symmetry in the isosceles triangle as compared with the right triangle produced a reduction in the random error along the y direction perpendicular to the symmetry axis, but not in the x direction along the axis. Loss of the axis but gain in rotational (and radial) symmetry in the parallelogram left the y random error equal to that of the isosceles triangle, but reduced the x random error below that of the isosceles triangle. Further addition of axes of reflective symmetry in the rectangle again failed to affect y random error, but further reduced x random error beyond that for the parallelogram. Although rotational and axial reflective symmetry seemed to have had independent effects on random errors, these effects were impossible to sort out. The effects were confounded further by changes in the lengths of the x and y-axes. The lengths of the axes were also found to contribute to random error patterns.

EXPERIMENT 1
Reflective Versus Rotational Symmetry

To eliminate these confounds, we used a different set of objects to manipulate shape and symmetry. To produce shapes with the desired symmetry properties, we began with a shape that has infinite symmetry of both types. A circle is continuously self-congruent under rotation about its center in the plane and has an infinite number of reflective symmetry axes. We perturbed the symmetries of the circle by appending smaller circular arcs to its perimeter. To construct a series of objects varying in the number of reflective symmetry axes, circular arcs 2 cm in radius were centered on the perimeter of a 5-cm circle. As shown in Figure 1, the axial reflective symmetries of the quadrilateral, parallelogram, and right triangle (0 axes), the isosceles triangle (1 axis), the rectangle (2 axes), the equilateral triangle (3 axes), and the square (4 axes) were reproduced.

These circular shapes also exhibited the same number of rotational symmetries as did their corresponding polygonal shapes, as follows: quadrilateral, right, and isosceles triangles—1 self-congruence in 360°; parallelogram and rectangle—2 self-congruences; equilateral triangle—3 self-congruences; and square—4 self-congruences. As shown in Figure 1, additional 2×1 cm elliptical arcs were added to the circular figures to produce a new series of shapes in which the axial reflective symmetries were eliminated, but the corresponding rotational symmetries were preserved. x and y-axes were of equivalent lengths in corresponding objects from the two series. The question was whether a perturbation that eliminated axial reflective symmetry but preserved rotational symmetry would eliminate or preserve associated reductions in random errors.

Method

Participants. Ten undergraduates at Indiana University participated in the experiment for credit in an introductory psychology course. All of the participants had normal or corrected-to-normal vision and reported no motor disabilities.

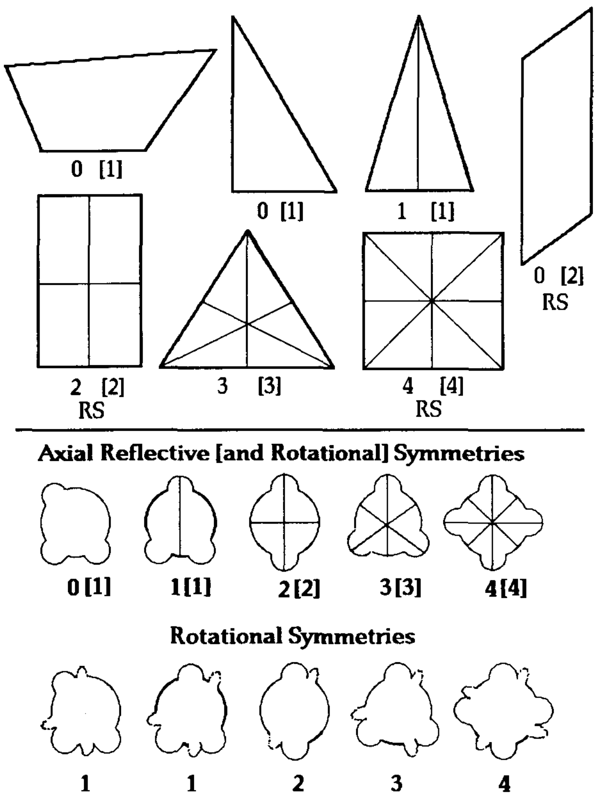

Figure 1. Top: The 7 shapes used by Bingham and Muchisky (1993), showing reflective symmetry axes and listing the number of axes under each shape, together with the number of self-congruences with 360° of rotation in the plane and the presence of radial symmetry (RS). Bottom: The 10 shapes used in the current experiments.

Apparatus. The 10 circular shapes shown in Figure 1 were cut from 1-cm-thick plywood. The objects were presented to each observer with the plane of the figure parallel to gravity. Objects were held upright in a transparent spring-loaded clamp affixed to a wooden base. The side of the figure facing observers was an unfinished smooth wood surface. The side facing the experimenter had polar-coordinate paper, in millimeters, attached to it, with the origin of the coordinates fixed at the center of mass. The Archimedean method was used to determine the location of the center of mass and the origin for the polar coordinates on each object. Each object was suspended alternatively from two different points along its perimeter. At each point, a plumb line was hung and marked on the object. The intersection of the two lines marked the location of the center of mass. The tongs used to indicate judgments were held and manipulated in one hand like a large pair of scissors. The point of the tongs that contacted the surface viewed by the observer was padded to prevent indentation of the surface.

Procedure. The observers were asked to judge where they felt the "stable point" was, having been told that this was the point at which an object would remain stable without rotating about the point of contact when held upright with the thumb and index finger; furthermore, if the object were to be rotated to another orientation, it would remain in the new orientation in which it had been placed. This was demonstrated using a different object from those used in the judgment trials. During each trial, the observers were asked to close their eyes while the object in the clamp was changed. In this way, they were prevented from obtaining information about the center of mass by observing the experimenter handling the objects.

The participants indicated their judgments by lightly grasping the object with the tongs at the appropriate location, but never lifting it. The experimenter measured the error in estimation by noting the angle and the radial distance of the point of contact on the polar coordinates on the back of the object.

Objects were presented in four orientations, determined by starting at 180° from those shown in Figure 1 and rotating in successive 90° increments. The resulting orientations were at 0°, 90°, 180°, and 270°. This reproduced the orientations used by Bingham and Muchisky (1993). All participants were presented with each of the 10 objects in 4 orientations 4 times each in 4 blocks of trials, for a total of 160 trials. Presentation order within each block was randomized. Each participant's session lasted 1.5 h, with a 5-min break in the middle of the session.

Results and Discussion

The results revealed that rotational and axial reflective symmetry had distinct effects on judgment accuracy. Accuracy increased with increasing rotational symmetry, and additional increases in accuracy occurred along directions perpendicular to reflective axes.

Coordinates were transformed from polar to Cartesian, with the positive x-axis corresponding to the 0° radial. Systematic errors, represented by x and y means, were analyzed by performing repeated measures analyses of variance (ANOVAs) on the x and y data, with shape (or amount of symmetry within type), orientation, and type of symmetry as factors. Most of the variation in systematic error was produced by changes in orientation. Orientation was significant [$F(3,117) = 13.0, p < .001$]. The shape × orientation interaction was not significant, and although the symmetry × orientation interaction was significant [$F(3,117) = 4.6, p < .01$], means for the two symmetry levels were all within 0.6 mm of each other. x means for each orientation are plotted in Figure 2a.

Along the y-axis, again, most of the variation in systematic error occurred with changes in orientation. Orientation was significant [$F(3,117) = 16.6, p < .001$]. The orientation × symmetry and orientation × shape interactions were significant, but in all cases means were within 0.5 mm of one another. The y means for each orientation are plotted in Figure 2a.

To reveal the pattern of variation in systematic errors, the centroids of the distributions for each orientation were plotted (Figure 2b), with downward arrows indicating

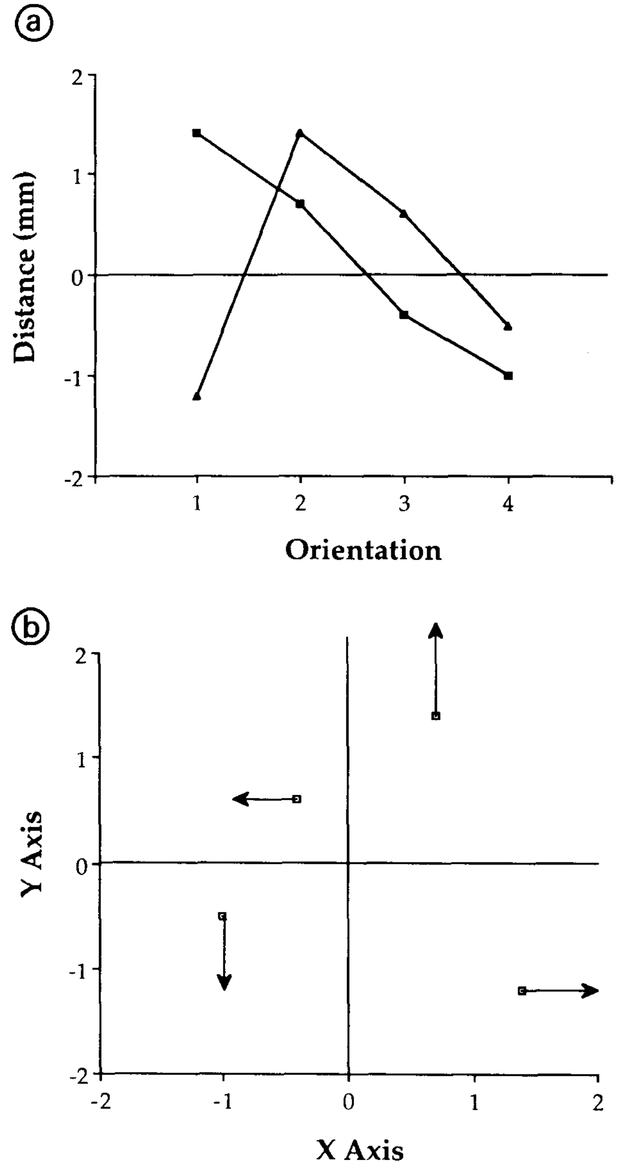

Figure 2. (a) x and y means for each orientation of the 10 objects. Means were computed across trials, participants, and objects. x = filled square, y = filled triangle. (b) Centroids of the data distributions for each orientation plotted in x and y coordinates, with the origin at the center of mass. Orientation is indicated by downward-pointing arrows.

orientation. This pattern replicated that found by Bingham and Muchisky (1993). Judgments systematically fell below and to the left of the center of mass. Because the participants were all approaching the objects from the right using the right hand, this grasp location would have yielded a stable grasp configuration. The objects would have tended to rotate into the grasp, with the object leaning against the hand between the thumb and finger. (On the other hand, the average deviation of about 1 mm was not very significant when taken against the typical 15-mm extent of the fingertip.)

Random errors were analyzed by computing x and y standard deviations for each participant across trials and orientations for each shape and symmetry type. (No significant variations in random error were found over changes in orientation when standard deviations were computed within orientations.) We performed repeated measures ANOVAs on standard deviations, with symmetry type and shape (or amount of symmetry within type) as factors. For random error along the x-axis, only shape was significant [$F(4,36) = 4.6, p < .001$]. Mean standard deviations for shapes with axial reflective versus only rotational symmetries are plotted in Figure 3. While rotational symmetry means exhibited a monotonic decrease in error with increasing amounts of symmetry, reflective symmetry means exhibited a knee in the curve similar to that appearing for the equilateral triangle in the curves of Bingham and Muchisky (1993). Random error tended to be greater for triangle-like shapes that contained at least one axis of reflective symmetry but no axis of reflective symmetry perpendicular to the x. However, this difference between the curves did not reach statistical significance. Along the y-axis, not only was shape significant [$F(4,36) = 4.7, p < .004$], but symmetry was also [$F(1,9) = 8.0, p < .02$]. Examination of the y mean standard deviations (Figure 3) revealed that random error in the y direction was less for objects with a reflective symmetry axis along the x direction than it was for those with only rotational symmetry.

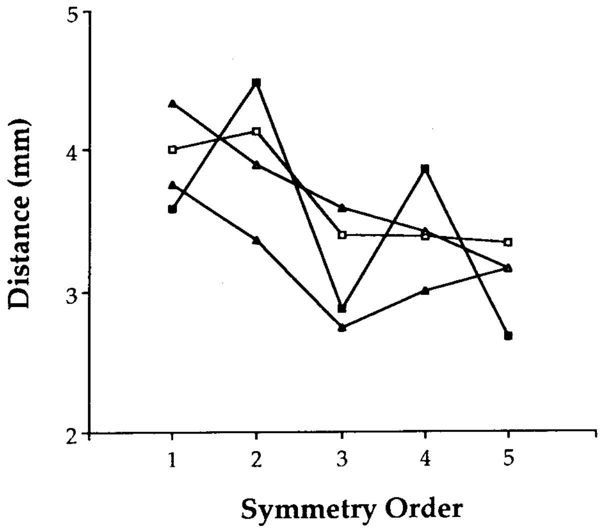

Figure 3. Mean x and y standard deviations for objects with axial reflective (and rotational) symmetry and only rotational symmetry, plotted by increasing amount of symmetry. Rotational symmetry: x = open squares, y = open triangles. Axial reflective symmetry: x = filled squares, y = filled triangles.

Thus, increasing rotational symmetry produced monotonic reductions in random error. The presence of an axis of reflective symmetry in the x direction reduced error in the y direction over and above reductions allowed by rotational symmetry. In addition, there appeared to have been a trend for error to increase somewhat along an axis of reflective symmetry unless a second reflective symmetry axis lay perpendicular to the first. The latter trend did not reach statistical significance, however.

We found that the general trends in systematic errors reflected an effect of orientation as shown in Figure 2. However, systematic errors also exhibited local increases in the first and second objects. Along the x-axis, shape was significant in the ANOVA [$F(4,156) = 11.0, p < .001$], but means for all shapes were within 1 mm of the center of mass (with the exception of Shape 1, which was at -1.17 mm). Symmetry was significant ($p < .05$), but means were within 0.5 mm of the origin. The symmetry $\times$ shape interaction was also significant [$F(4,156) = 2.9, p < .03$]. In a simple effects test, symmetry was significant at Shape 1 ($p < .01$) and marginal at Shape 2 ($p < .07$), while shape was significant at both levels of symmetry ($p < .01$). The mean for Shape 1 was farther from the center of mass for rotational symmetry (-1.7) than for reflective ($-.6$), while the mean for Shape 2 was farther from the center of mass for reflective symmetry (1.3) than for rotational (.3). Along the y-axis, shape was significant [$F(4,156) = 11.7, p < .001$], as was symmetry [$F(1,39) = 16.4, p < .001$], but in both cases means were all within 1 mm of the center of mass. The symmetry $\times$ shape interaction was significant [$F(4,156) = 18.7, p < .001$]. In a simple effects test, symmetry was significant for Shape 1 ($p < .01$) and marginal for Shape 2 ($p < .08$), while shape was significant at both levels of symmetry ($p < .01$). The mean for Shapes 1 and 2 only exceeded a distance of 1 mm with rotational symmetry, at -2.3 and 1.2, respectively.

In most of the objects, the center of mass fell on or very close to the center of the underlying circle upon which all of the objects were based. In the first and second objects, however, the center of mass was moved toward the protrusions that had been added to perturb the symmetry of the circle. With Shape 1, the participants overestimated the extent to which the center of mass was moved away from the center of the underlying circle, especially with only rotational symmetry. With Shape 2, they underestimated the extent of perturbation, especially with reflective symmetry, for which they kept inappropriately close to the center of the underlying circle. The implication was that the underlying symmetries, which we had perturbed, continued to affect the judgments. In one case, the participants overly compensated for the perturbation, whereas in the other, they appear to have used an approximation to symmetry.

EXPERIMENT 2
Approximation to Symmetry

What do observers do in the absence of symmetry? Do they use an approximation to symmetry? The results with the perturbed circles indicate that this was not strictly the case, since approximation to symmetry would have been reflected in a tendency to err only in the direction of the center of the underlying circle. In fact, errors were sometimes in exactly the opposite direction. The results suggest, nevertheless, that the underlying approximate symmetry structured the estimates. Did the participants use the approximate symmetry as a basis for their estimates of center of mass?

An object with only one reflective symmetry axis (and no other type of symmetry) leaves the location of the center of mass along that axis strictly undetermined by symmetry. The location of the center of mass in a planar object that has two axes of reflective symmetry is more easily determined because the center of mass must lie on both axes. If a rectangular planar object (with two axes of reflective symmetry) were perturbed by the attachment of another small rectangular piece to one side, the piece could be added so as to preserve reflective symmetry about one axis while leaving apparent the near-symmetry about the remaining axis. If observers used an approximation to symmetry in judging the location of the center of mass, their judgments should tend toward a location specified by the symmetries of the original perturbed rectangular shape. If the size of the added rectangle were to be gradually increased, a new, larger rectangle with two symmetry axes would accordingly be formed. As the shape became more like the larger symmetric figure, the perturbation would effectively be away from the larger rectangle by the removal of pieces. In such circumstances, judgments based on approximation to symmetry should err in the direction of the location determined by the symmetry of the larger rectangle.

As shown in Figure 4a, we created a continuum along which the location of the center of mass changed linearly across objects. We added successively larger square figures to one side of an initially square-shaped object. The center of mass moved 1.25 cm along the x-axis in each successively larger object. The series consisted of nine objects, beginning with a square (100 cm^2) and finishing with a rectangle (300 cm^2). We expected that the occurrence of a second symmetry in the square (Object 1) and in the rectangles (Objects 5 and 9) would structure the pattern of systematic errors as shown in Figure 4a. We predicted that the symmetries would attract judgments that depended inversely on their deviation from approximate symmetries.

Method

Participants. Ten undergraduates at Indiana University participated in the experiment for credit in an introductory psychology course. Six were female and 4 were male, and all had normal or corrected-to-normal vision and reported no motor disabilities.

Apparatus. Except for the use of the objects shown in Figure 4, the apparatus was the same as in Experiment 1.

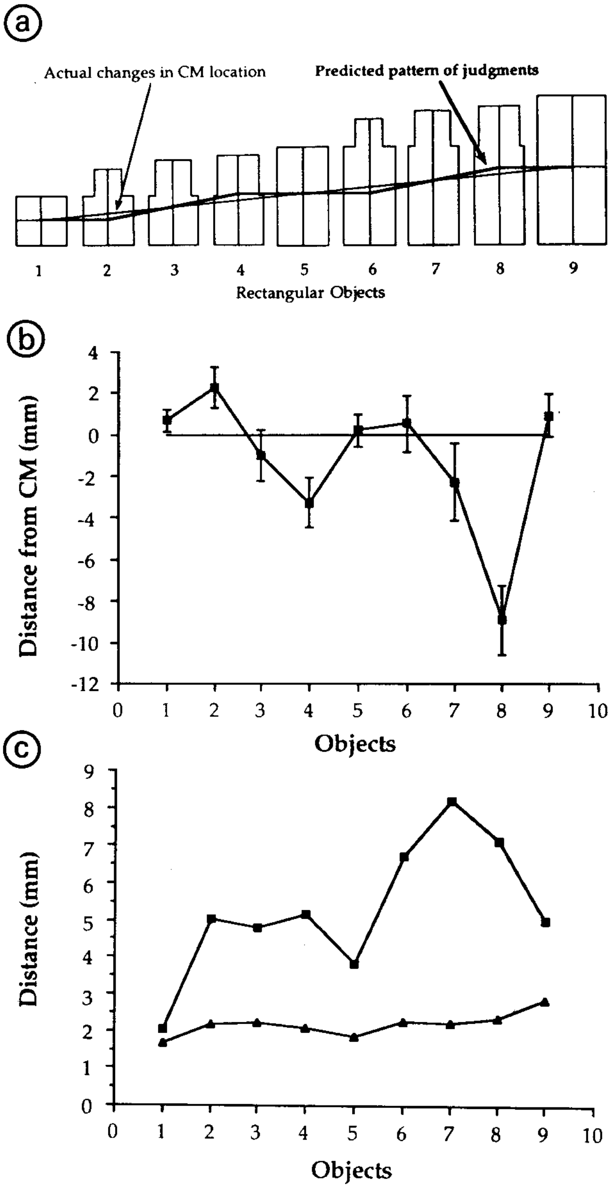

Figure 4. (a) The sequence of 9 objects used in Experiment 2, each moved the center of mass along the trajectory shown together with the predicted pattern of systematic error in judgments if participants used an approximation to the nearest symmetry. At least one reflective symmetry axis lies in each object. (A second axis [not shown] appeared perpendicular to the first in Objects 1, 5, and 9.) (b) *x* means with standard error bars for the 9 objects. (c) *x* and *y* mean standard deviations for each of the 9 objects. *x* = filled squares, *y* = filled triangles.

Procedure. The participants were required to perform the same grasping task as in Experiment 1. They viewed each of the nine objects at three orientations (0°, 135°, 270°) three times in a session lasting 1 h and including a 5-min break. In the 0° orientation, the one reflective symmetry axis was vertical, at 180° from the orientations shown in Figure 4a.

Results and Discussion

As before, coordinates were transformed from polar to Cartesian, with the x-axis along the long reflective symmetry axis and the positive direction towards the perturbation. Systematic errors were analyzed in terms of mean errors along x and y-axes. Repeated measures ANOVAs were performed on the x and y distances, with shape and orientation as factors. Along the x-axis, orientation and shape were both significant [$F(2,24) = 11.3, p < .004$, and $F(8,96) = 7.6, p < .001$, respectively]. Along the y-axis, only shape was significant [$F(8,96) = 3.3, p < .002$]. We will focus on the x means for the 0° orientation in which the symmetry axis common to all of the objects was vertical. As shown in Figure 4b, the results exhibited exactly the inverse of the pattern predicted by direct approximation to symmetry, with the x means instead following a pattern consistent with overestimation of the perturbation and a balance struck halfway between two alternative symmetries. The mean errors on the y-axis, though significant across shapes, showed too little range—within $\pm.75$ mm—to be of any real significance. Observers stayed very close to the single common symmetry axis.

Random errors were analyzed by performing repeated measures ANOVAs on x and y standard deviations calculated for each participant, with orientation and number of symmetry axes as factors. Rather than analyzing each object separately, we grouped the objects according to those that had a second reflective symmetry axis and those that did not. Along the x-axis, the amount of symmetry was significant [$F(1,12) = 53.6, p < .001$]. As shown in Figure 4c, observers were much less variable with a reflective symmetry axis in the y direction. The amount of random error also increased linearly with increases in the size of the objects, as can be seen clearly for Objects 1, 5, and 9. Amount of symmetry was not significant for y standard deviations, and random error along the y-axis was consistently low. Lack of variation was consistent with the universal presence of the symmetry axis along the x direction and the unchanging length of the y-axis.

The combined pattern in Figures 4b and 4c demonstrated that symmetries, whether exact or approximate, strongly influence judgments of center of mass location in objects. The data clearly crossed and recrossed the 0-distance axis in Figure 4b at points at which the second axis of reflective symmetry appeared, and, in addition, crossed at points midway between the symmetry points—that is to say, the crossing locations were symmetrically distributed with respect to the underlying points of symmetry in the set of objects. Rather than staying close to the center of mass location of the nearest symmetric shape, observers seem to have overestimated the strength of perturbations, for perturbations involving material both added and subtracted from a symmetric form.

GENERAL DISCUSSION

In shape perception, symmetry has been approached primarily in the context of shape recognition (Quinlan, 1991; Rock, 1973). Recognition has been cast as a matter of finding a match between current and stored descriptions of shapes. Congruence of coordinate axes has been employed as a means of establishing correspondence between descriptions to determine whether they match (Kanade & Kender, 1983; Marr, 1982; Quinlan, 1991; Rock, 1973, 1983; Wiser, 1981). This has led to the problem of how to establish the location and orientation of coordinates in shapes. Axes of reflective symmetry have been hypothesized as a possible means. Studies have focused on conditions determining the recognition of axial reflective symmetry and on those determining when axial reflective symmetry affects shape recognition (Corballis & Roldan, 1975; Palmer, 1985; Pashler, 1990; Rock, 1973, 1983).

Object perception for the purpose of control in object manipulation has been approached indirectly, if at all, in the shape perception literature (Quinlan, 1991). The use of shape information in object handling has been treated largely as mediated by shape recognition whereby, presumably, recognition of a shape would enable one to access information about what types of manipulation an object affords and how they might best or most stably be executed. In our studies, however, we have approached shape perception directly in terms of object handling. Based on the observation that the center of mass is often used to establish grasp locations, we have investigated the ability to detect center of mass location in objects that vary in shape but are composed of a single material (Bingham & Muchisky, 1993). Because the center of mass is itself a symmetry property, we surmised that geometric symmetries of our objects might determine accuracy in locating the center of mass, and that shape recognition as such would not be required to use shape information or symmetries to guide activity. Our supposition has been confirmed: We have found that greater symmetry has yielded greater accuracy in determining center of mass location. However, the situation has been complicated by the existence of qualitative differences in symmetry.

In principle, more symmetry should enable better accuracy in locating the center of mass; but there are different types of symmetry. Do all the types affect center of mass detection? We have found that they do, but apparently not all symmetries affect accuracy in the same way or to equal extents. Rotational symmetry produced equal reduction of errors along both x and y-axes. Greater frequency of self-congruence in a 360° rotation yielded less random error. Axial reflective symmetry produced a reduction of errors only in a direction perpendicular to the axis, and the reduction was greater than that produced by equal amounts of rotational symmetry. No effect of radial symmetry was apparent in the results of either experiment, though one had been in previous experiments (Bingham & Muchisky, 1993). A somewhat similar pattern seemed to be associated with the presence of a reflective symmetry axis with or without a perpendicular symmetry axis. However, without variations in object size or axis length, we could not perform the more sensitive analysis that we had in the previous study to reveal the effect of radial symmetry. This may also account for the lack of an effect of orientation on random errors, that we had found previously with large objects.

The fact that different types of symmetry affected the accuracy of judgments in different ways prohibits the use of a reduction of the different types to isomorphic symmetry groups as a way of quantifying symmetry. Rotation in the plane about a point is qualitatively distinct from rotation out of the plane about a line in the plane. One yields a point and the other, a line. Although isomorphic from the perspective of group theory, the transformations are clearly not isomorphic. Given our data, they are certainly not isomorphic from the perspective of visual perception and object manipulation. Nevertheless, all forms of symmetry appear to be employed in perception and, in the case of center of mass perception, to a common end—namely, the reduction of error in center of mass location. The question therefore remains as to what common basis might be used to relate the effects of different types of symmetry.

Furthermore, symmetry seems to play a role in object perception even when objects are not strictly symmetric. Systematic errors in locating the center of mass appear to have been determined by the nearest approximate symmetry. Reference to "approximate symmetry" entails employing a metric to determine either the tolerance separating the symmetric from the asymmetric or the relative magnitude of a perturbation from symmetry. Echoing Plato, we note that nothing is perfectly symmetric if measured finely enough, and yet plenty of things are symmetric to the eye. What determines, in any given instance, the equivalence class of shapes that are close enough to symmetry to be treated as symmetric? Alternatively, when symmetry has been perturbed, how might we measure the strength of the perturbation?

Symmetry holds out the promise of a means of quantifying shape in a fashion directly related to the ways that shapes or objects might be used. But first, we must confront the need to quantify symmetry.

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NOTES

1. Single material composition guarantees nearly homogeneous mass distribution, which is required, in turn, if object geometry is to provide information about the center of mass.

2. Radial symmetry or reflection through a point is equivalent in outcome to rotation in the plane by 180°. The difference between "total symmetry" and group order is that, in the former, radial symmetry is counted separately, with the result that the symmetry of the rectangle and square are counted as 5 and 9, respectively, rather than 4 and 8. The ordinal relations among the shapes in Figure 1 were unaffected by this difference.

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