Center-Piece Subgraphs: Problem Definition and Fast Solutions

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ABSTRACT

Given Q nodes in a social network (say, authorship network), how can we find the node/author that is the center-piece, and has direct or indirect connections to all, or most of them? For example, this node could be the common advisor, or someone who started the research area that the Q nodes belong to. Isomorphic scenarios appear in law enforcement (find the master-mind criminal, connected to all current suspects), gene regulatory networks (find the protein that participates in pathways with all or most of the given Q proteins), viral marketing and many more.

Connection subgraphs is an important first step, handling the case of Q=2 query nodes. Then, the connection subgraph algorithm finds the b intermediate nodes, that provide a good connection between the two original query nodes.

Here we generalize the challenge in multiple dimensions: First, we allow more than two query nodes. Second, we allow a whole family of queries, ranging from 'OR' to 'AND', with 'softAND' in-between. Finally, we design and compare a fast approximation, and study the quality/speed trade-off.

We also present experiments on the DBLP dataset. The experiments confirm that our proposed method naturally deals with multi-source queries and that the resulting subgraphs agree with our intuition. Wall-clock timing results on the DBLP dataset show that our proposed approximation achieve good accuracy for about 6 : 1 speedup.

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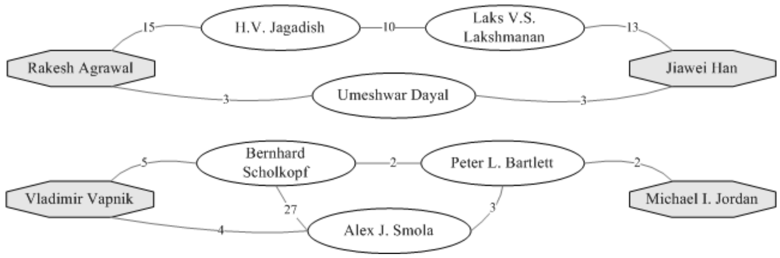
1. INTRODUCTION

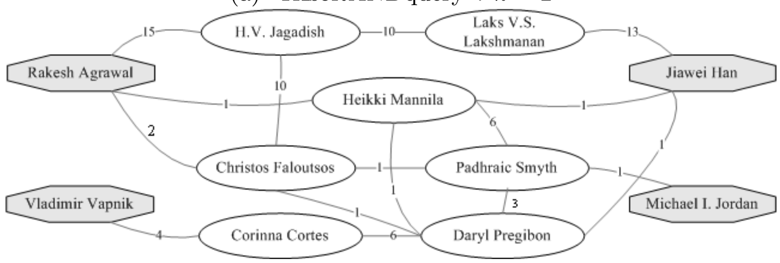
Graph mining has been attracting increasing interest recently, for community detection, partitioning, frequent subgraph discovery and many more. Here we introduce and solve a novel problem, the "*center-piece subgraph*" (*CEPS*) problem: Given Q query nodes in a social network (e.g., co-authorship network), find the node(s) and the resulting subgraph, that have strong connections to all or most of the Q query nodes. The discovered nodes could contain a common advisor, or other members of the research group, or an influential author in the research area that the Q nodes belong to. As mentioned in the abstract, there are multiple alternative applications (law enforcement, gene regulatory networks).

Earlier work [6] focused on the so-called "connection subgraphs". Although the inspiration for the current work, the connection subgraph algorithm can only handle the case of Q=2. This is exactly the major contribution of our work: we allow not only pairs of query nodes, but any arbitrary number Q of them.

Figure 1 gives screenshots of our system, showing our solution on a DBLP graph, with Q=4 query nodes. All 4 researchers are in data mining, but the first two (Rakesh Agrawal and Jiawei Han) are more on the database side, while Michael Jordan and Vladimir Vapnik are more on the machine learning and statistical side. Figure 1(b) gives our *CEPS* subgraph, when we request nodes with strong ties to all four query nodes. The results make sense: researchers like Daryl Pregibon, Padhraic Smythe and Heikki Mannila are vital links, because of their cross-disciplinarity and their strong connections with both the above sub-areas. Figure 1(a) illustrates an important aspect of our work, the $K\_softAND$ feature, which we will discuss very soon. In a nutshell, in a $K\_softAND$ query, our method finds nodes with connections to at least k of the query nodes ($k = 2$ in Figure 1(a)).



(a) "K_softANDquery": $k = 2$



(b) "AND query"

Figure 1: Center-piece subgraph among Rakesh Agrawal, Jiawei Han, Michael I. Jordan and Vladimir Vapnik.

Thus, we define the center-piece subgraph problem, as follows:

PROBLEM 1. *Center-Piece Subgraph Discovery(*CEPS*)*

Given: *an edge-weighted undirected graph* $\mathbf{W}$, *Q nodes as source queries* $\mathcal{Q} = \{q_i\}$ $(i = 1, ..., Q)$, *the softAND coefficient k and an integer budget b*

Find: *a suitably connected subgraph* $\mathcal{H}$ *that (a) contains all query nodes* q_i *(b) at most b other vertices and (c) it maximizes a "goodness" function* $g(\mathcal{H})$.

Allowing Q query nodes creates a subtle problem: do we want the qualifying nodes to have strong ties to all the query nodes? to at least one? to at least a few? We handle all of the above cases with our proposed $K\_softAND$ queries. Figure 1(a) illustrates the case where we want intermediate nodes with good connections to at least $k = 2$ of the query nodes. Notice that the resulting subgraph is much different now: there are two disconnected components, reflecting the two sub-communities (databases/statistics).

The contributions of this work are the following

- The problem definition, for arbitrary number Q of query nodes, with careful handling of a lot of the subtleties.
- The introduction and handling of $K\_softAND$ queries.
- $EXTRACT$, a novel subgraph extraction algorithm.
- The design of a fast, approximate method, which provides a 6 : 1 speedup with little loss of accuracy.

The system is operational, with careful design and numerous optimizations, like alternative normalizations of the adjacency matrix, a fast algorithm to compute the scores for $K\_softAND$ queries.

Our experiments on a large real dataset (DBLP) show that our method returns results that agree with our intuition, and that it can be made fast (a few seconds response time), while retaining most of the accuracy (about 90%).

The rest of the paper is organized as follows: in Section 2, we review some related work; Section 3 provides an overview of the proposed method: *CEPS*. The goodness score calculation is proposed Section 4 and its variants are presented in the Appendix. The "$EXTRACT$" algorithm and the speeding up strategy are provided in Section 5 and Section 6, respectively. We present experimental results in Section 7; and conclude the paper in Section 8.

2. RELATED WORK

In recent years, there is increasing research interest in large graph mining, such as pattern and law mining [2][5][7][20], frequent substructure discovery [27], influence propagation [18], community mining [9][11][12] and so on. Here, we make a brief review of the related work, which can be categorized into four groups: 1) measuring the goodness of connection; 2) community mining; 3) random walk and electricity related methods; 4) graph partition.

The goodness of connection. Defining a goodness criterion is the core for center-piece subgraph discovery. The two most natural measures for "good" paths are shortest distance and maximum flow. However, as pointed out in [6], both measurements might fail to capture some preferred characteristics for social network. The goodness function for survivable network [13], which is the count of edge-disjoint or vertex-disjoint paths from source to destination, also fails to adequately model social relationship. A more related distance function is proposed in [19] [23]. However, It cannot describe the multi-faceted relationship in social network since center-piece subgraph aims to discover collection of paths rather than a single path.

In [6], the authors propose an delivered current based method. By interpreting the graph as an electric network, applying +1 voltage to one query node and setting the other query node 0 voltage, their method proposes to choose the subgraph which delivers maximum current between the query nodes. In [25], the authors further apply the delivered current based method to multi-relational graph. However, the delivered current criterion can only deal with pairwise source queries. Moreover, the resulting subgraph might be sensitive to the order of the query nodes (See Figure 2 for an example). On the other hand, as we will show very soon, connection subgraph can actually be viewed as a special case of the proposed center-piece subgraph ("AND query" with pair source nodes).

Random walk related methods. The proposed importance score calculation is based on random walk with restart. There are many applications using random walk and related methods, including PageRank [22], personalized PageRank [14], SimRank [16], neighborhood formulation in bipartite graph [26], content-based image retrieval [15], cross modal correlation discovery [24], BANKS system [1], ObjectRank [3], RalationalRank [10] and so on.

Community detection. Center-piece subgraph discovery is also related with community detection, such as [9][11][12]. However, we cannot directly apply community detection to subgraph discovery especially when the source queries are remotely related or they lie in different communities.

Graph partition and clustering. There are a bunch of graph partition and clustering algorithms proposed in the literature, e.g. METIS [17], spectral clustering [21], flow simulation [8], co-clusterfing [4], betweenness based method [12]. It is worth pointing out that the proposed method is orthogonal to the specific graph partition algorithms.

3. PROPOSED METHOD: OVERVIEW

Let us first define the goodness score for nodes. For a given node j, we have two types of goodness score for it:

- Let $r(i,j)$ be the goodness score of a given node j wrt the query q_i;
- Let $r(\mathcal{Q},j)$ be the goodness score of a given node j wrt the query set $\mathcal{Q}$.

A natural way to measure the goodness of the subgraph $\mathcal{H}$ is to measure the goodness of the nodes it contains: the more 'good'/important nodes (wrt the source queries) it contains, the better $\mathcal{H}$ is. Thus, the goodness criterion of $\mathcal{H}$ can be defined as:

$$g(\mathcal{H}) = \sum_{j \in \mathcal{H}} r(\mathcal{Q}, j) \tag{1}$$

With the above goodness criterion, a straightforward way to choose the "best" subgraph should be the one which maximizes $g(\mathcal{H})$:

$$\mathcal{H}^* = argmax_{\mathcal{H}} g(\mathcal{H}) \tag{2}$$

However, no connection is guaranteed in this way and the resulting subgraph $\mathcal{H}$ might be a collection of isolated nodes. Thus, there are two basic problems in center-piece subgraph discovery: 1) how to define a reasonable goodness score $r(\mathcal{Q},j)$ for a given node j; 2): how to quickly find a connection subgraph maximizing $g(\mathcal{H})$. Moreover, since it might be very difficult to directly calculate the goodness score $r(\mathcal{Q},j)$, we further decompose it into two steps. The pseudo code for the proposed method (*CEPS*) is listed as follows:

4. GOODNESS SCORE CALCULATION

There are two basic concepts in goodness score calculation:

Table 1: *CEPS*

Input:	the weighted graph $\mathbf{W}$, the query set $\mathcal{Q}$, $K\_softAND$ coefficient k and the budget b
Output:	the resulting subgraph $\mathcal{H}$
Step 1:	**Individual Score Calculation**. Calculate the goodness score $r(i,j)$ for a single node j wrt a single query node q_i
Step 2:	**Combining Individual Scores.** Combine the individual score $r(i,j)$ to get the goodness score $r(\mathcal{Q},j)$ for a single node j wrt the query set $\mathcal{Q}$
Step 3:	**"*EXTRACT*".** Extract quickly a connection subgraph $\mathcal{H}$ with budget b maximizing the goodness criteria $g(\mathcal{H})$

- Let $r_{i,j}$ be the *steady-state probability* that a particle will find itself at node j, when it does random walk with restarts (RWR) from query node q_i.
- Let $r(\mathcal{Q},j,k)$ be the *meeting probability*, that is, the steady-state probability that at least k-out-of-Q particles, doing RWR from the query nodes of $\mathcal{Q}$, will all find themselves at node j in the steady state; k is the K_softAND coefficient.

These two kinds of steady probability ($r_{i,j}$ and $r(\mathcal{Q},j,k)$) are the base of our goodness score calculation (for both $r(i,j)$ and $r(\mathcal{Q},j)$). It's basic idea is that: suppose there are Q random particles doing RWR from each query node independently; then after convergency, each particle has some *steady-state probability* staying at the node j; and different particles have some *meeting probability* at the node j. The *steady-state probability* and the *meeting probability* provide some hints on how the node j is related with the source queries, and are used to compute the goodness score of node j. Moreover, by designing different *meeting probability*, we can get the specific type of goodness score tailored for the specific query scenario. Table 2 lists all the symbols and definitions used throughout this paper.

4.1 Individual score calculation

Here we want to compute the goodness score $r(i,j)$ of a single node j, for a single query node q_i. We propose to use random walks with restart, from the query node q_i.

Suppose a random particle starts from query q_i, the particle iteratively transmits to its neighborhood with the probability that is proportional to the edge weight between them, and also at each step, it has some probability c to return to node q_i. $r(i,j)$ is defined as the *steady-state probability* $r_{i,j}$ that the particle will finally state at node i:

$$r(i,j) \triangleq r_{i,j} \tag{3}$$

More formally, if we put all the $r_{i,j}$ probabilities into matrix form $\mathbf{R} = [r_{i,j}]$, then

$$\mathbf{R}^T = c\mathbf{R}^T \times \tilde{\mathbf{W}} + (1-c)\mathbf{E} \tag{4}$$

where $\mathbf{E} = [\vec{e}_i](i = 1, ..., Q)$ is the $N \times Q$ matrix, c is the fly-out probability, and $\tilde{\mathbf{W}}$ is the adjacency matrix $\mathbf{W}$ appropriately normalized, say, column-normalized:

$$\tilde{\mathbf{W}} = \mathbf{W} \times \mathbf{D}^{-1} \tag{5}$$

The problem can be solved in many ways - we choose the iteration method, iterating Eq. 4 until convergence. For

Table 2: Symbols

Symbol	Description
N	total number of nodes in the weighted graph
m	iteration step
c	fly-out probability for random walk with restart
$\vec{e}_i$	$N \times 1$ unit query vector, with all zeros except one at row q_i
$\mathbf{W} = \{w_{i,j}\}$	the edge weighted matrix $(i, j = 1, ..., N)$
$\mathbf{D} = \{d_{i,j}\}$	$N \times N$ matrix, $d_{i,i} = d_i$, and $d_{i,j} = 0$ for $i \neq j$
d_i	the sum of the i^{th} row of W
$\mathcal{H}$	the chosen center-piece subgraph
Q	number of source query nodes
$\mathcal{Q} = \{q_i\}$	set of query nodes $(i = 1, ..., Q)$
$\acute{\mathcal{Q}}$	the first $(Q-1)$ query nodes of query set $\mathcal{Q}$, $\acute{\mathcal{Q}} = \{q_i\}, (i = 1, .., (Q-1))$
$\varnothing$	null query set, which contains no query node
$r(i, j)$	goodness score for a single node j wrt query node q_i
$r(\mathcal{Q}, j)$	goodness score for a single node j wrt query set $\mathcal{Q}$
$r(\mathcal{Q}, (j, l))$	goodness score for a single edge (j, l) wrt query set $\mathcal{Q}$
$r_{i,j}$	*steady-state probability* of a single node j wrt query node q_i
$\mathbf{R}$	$Q \times N$ matrix of $[r_{i,j}]$
$r(\mathcal{Q}, j, k)$	*meeting probability* of a single node j, wrt $k(k = 1, .., Q)$ or more of the query nodes of $\mathcal{Q}$
$r(i, (j, l))$	*meeting probability* of a single edge (j, l), wrt query node q_i
$r(\mathcal{Q}, (j, l), k)$	*meeting probability* of a single edge (j, l), wrt $k(k = 1, .., Q)$ or more of the query nodes of $\mathcal{Q}$

simplicity, in this paper, we iterate Eq. 4 m times, where m is a pre-fixed iteration number.

4.2 Combining individual scores

Here we want to combine the individual score $r(i, j)(i = 1, ..., Q)$ to get $r(\mathcal{Q}, j)$, the goodness score for a single node j wrt the query set $\mathcal{Q}$. We propose to use the *meeting probability* $r(\mathcal{Q}, j, k)$ of random walk with restart. Furthermore, by using different softAND coefficient k, we can deal with different types of query scenario.

The most common query scenario might be that "given Q query nodes, find the subgraph $\mathcal{H}$ the nodes of which are important/good wrt ALL queries". In this case, $r(\mathcal{Q}, j)$ should be high if and only if there is a high probability that ALL particles will finally meet at node j:

$$r(\mathcal{Q}, j) \triangleq r(\mathcal{Q}, j, Q) = \prod_{i=1}^{Q} r(i, j) \tag{6}$$

Eq. 6 actually defines a logic AND operation in terms of individual goodness scores: the node j is important wrt the query set $\mathcal{Q}$ if and only if it is important wrt every query node. Thus, we refer such query type as "AND query".

A complemental query scenario is "OR query": "given Q queries, find the subgraph $\mathcal{H}$ the nodes of which are important wrt at least ONE query". In this case, $r(\mathcal{Q}, j)$ should be high if and only if there is a high probability that at least one particle will finally stay at node j:

$$r(\mathcal{Q}, j) \triangleq r(\mathcal{Q}, j, 1) = 1 - \prod_{i=1}^{Q} (1 - r(i, j)) \tag{7}$$

Eq. 7 defines a logic OR operation in terms of individual importance scores: the node j is important wrt the source queries if and only if it is important wrt at least one source query.

Besides the above two typical scenarios, the user might also ask "given Q queries, find the subgraph $\mathcal{H}$ the nodes of which are important wrt at least $k(1 \leq k \leq Q)$ queries". We refer such query type as "$K\_softAND$ query". In this case, $r(\mathcal{Q}, j)$ should be high if and only if there is a high probability that at least k-out-of-Q particles will finally meet at node j.

$$r(\mathcal{Q}, j) \triangleq r(\mathcal{Q}, j, k) \tag{8}$$

To avoid exponential enumeration (which is $O(2^k)$), Eq. 8 can be computed in a recursive manner:

$$r(\mathcal{Q}, j, k) = r(\acute{\mathcal{Q}}, j, k-1) \cdot r(Q, j) + r(\acute{\mathcal{Q}}, j, k) \tag{9}$$

where $r(\varnothing, j, 0) = 1 (j = 1, ..., Q)$.

Intuitively, Eq. 8 defines a logic operation in terms of individual importance scores that is between logic AND and logic OR. In this paper, we refer it as logic K_softAND: the node j is important wrt the source queries if and only if it is important wrt at least k-out-of-Q source queries.

It is worth pointing out that both "AND query" and "OR query" can be viewed as special cases of "$K\_softAND$ query": "AND query" is actually "Q_softAND query"; while "OR query" is actually "1_softAND query"

4.3 Variation: normalization on W

To compute the goodness score $r(i, j)$ and $r(\mathcal{Q}, j)$, we need to construct the transition matrix $\tilde{\mathbf{W}}$ for random walk with restart. A direct way is to normalize $\mathbf{W}$ by column as Eq. 5. However, as pointed out in [6], there might be the so called "pizza delivery person" problem, that is, the node with high degree is prone to receive too much attention (receiving too high individual goodness score in our case). To deal with this problem, we propose to normalize $\mathbf{W}$ as Eq. 10. The normalized weighted graph $\mathbf{W}$ will be further used to formulate the transition matrix $\tilde{\mathbf{W}}$ by Eq. 5.

$$w_{j,l} \leftarrow w_{j,l} / (d_j)^{\alpha} \tag{10}$$

for all $j, l = 1, ..., N$.

The motivation of normalization is as follows: for the high degree node j, every edge $(j, l)(l = 1, ..., N)$ is penalized by

$(d_i)^\alpha$ and vice versa. The coefficient α control the penalization strength: bigger α indicates stronger penalization. Note that the idea of penalizing the node with high degree is similar with that of setting a universal sink node in [6].

5. THE "EXTRACT" ALGORITHM

The "EXTRACT" algorithm takes as input the weighted graph $\mathbf{W}$, the importance scores on all nodes, the budget b and the softAND coefficient k; and produces as output a small, unweighted, undirected graph $\mathcal{H}$. The basic idea is similar with the display generation algorithm in [6]: 1) instead of trying to find an optimal subgraph maximizing $g(\mathcal{H})$ directly, we decompose it into finding key paths incrementally; 2) by sorting the nodes in order, we can quickly find the key paths by dynamic programming in the acyclic graph.

However, we cannot directly apply the original display generation algorithm since it can only deal with pair source queries (and also the resulting subgraph is sensitive to the order of the source queries). To deal with this issue, we extend the original algorithm in the following aspects:

(1) Instead of finding a source-source path, at each step, the algorithm will pick up a most promising destination node *pd*; and try to find a source-destination path for each source query node.

(2) The order (which will be used in the dynamic programming) is specified with each source query node.

(3) Key path discovery differs with the different query types: for "AND query" the algorithm will discover Q paths for all source nodes at each step; for "K_softAND query", it only discovers k paths for the first k source nodes; while for "OR query", the algorithm will only find 1 path at each step.

Before presenting the algorithm, we require the following definitions:

- *SPECIFIED DOWNHILL NODE.* Node u is downhill from node v wrt source q_i ($v \to d_i, u$) if $r(i,v) > r(i,u)$;
- *SPECIFIED PREFIX PATH.* A specified prefix path $P(i,u)$ is any downhill path that starts from source q_i and ends at node u; that is, $P(i,u) = (u_0, u_1, ..., u_n)$ where $u_0 = q_i, u_n = u$, and $u_j \to d_i, u_{j+1}$;
- *EXTRACTED GOODNESS.* The extracted goodness is the total goodness score of the nodes within the subgraph $\mathcal{H}$: $CF(\mathcal{H}) = \sum_{j\in\mathcal{H}} r(\mathcal{Q}, j)$.
- *EXTRACTED MATRIX.* $C_s(i,u)$ is the extracted goodness score from source node q_i to node u along the prefix path $P(i,u)$ so that:
    1. $P(i,u)$ has exactly s nodes not in the present output graph $\mathcal{H}$
    2. $P(i,u)$ extracts the highest goodness score among all such paths that start from q_i and end at u.
- *ACTIVE SOURCE.* For $K\_softAND$, the source node q_i is active wrt destination node pd if $r(i,pd) \geq r^{(k)}(i,pd)$, where $r^{(k)}(i,pd)$ is the k^{th} largest value among $r(i,pd)$, $(i = 1, ..., Q)$. Note that the number of active source differs with the query type[1]: for "OR query", there is only one active source while for "AND query", all sources are active. For a specific query type, an active source q_i might turn into inactive when the destination node pd changes and vice versa.

The destination node pd can be decided by Eq. 11:

$$pd = argmax_{j \notin \mathcal{H}} r(\mathcal{Q}, j) \tag{11}$$

where $\mathcal{H}$ is the partially built output subgraph.

In order to discover a new path between the source q_i and the promising node pd, we arrange the nodes in descending order of $r(i,j) (j = 1, ..., n)$: $\{u_1 = q_i, u_2, u_3, ..., pd = u_n\}$. (note that all nodes with smaller $r(i,j)$ than $r(i,pd)$ are ignored). Then we fill the extracted matrix C in topological order so that when we compute $C_s(t,u)$, we have already computed $C_s(t,v)$ for all $v \to d_i, u$. On the other hand, as the subgraph is growing, a new path may include nodes that are already present in the output subgraph, our algorithm will favor such paths as in [6]. The complete algorithm to discover a single path from source node q_i and the destination node pd is given in table 3.

Table 3: Single Key Path Discovery

1. Let len be the maximum allowable path length
2. For $j \leftarrow [1, ..., n]$
2.1. Let $v = u_j$
2.2. For $s \leftarrow [2, ..., len]$
If v is already in the output subgraph
$s' = s$
Else
$s' = s - 1$
Let $C_s(i,v) = max_{u\mid u \to d_i, v}(C_{s'}(i,u) + r(\mathcal{Q}, v))$
3. Output the path maximizing $C_s(i,pd)/s$, where $s \neq 0$

Based on the previous preparations, the $EXTRACT$ algorithm can be given in table 4.

Table 4: Our $\boldsymbol{EXTRACT}$ Algorithm

1. Initialize output graph $\mathcal{H}$ null
2. Let len be the maximum allowable path length
3. While $\mathcal{H}$ is not big enough
3.1. Pick up destination node pd by Eq. 11
3.2. For each active source node q_i wrt node pd
3.2.1. use table 3 to discover a key path $P(q_i, pd)$
3.2.2. add $P(q_i, pd)$ to $\mathcal{H}$
4. Output the final $\mathcal{H}$

6. SPEEDING UP CEPS

To compute $r(i,j)$, we have to solve a linear system. When the data set is large (or more precisely, when the total number of the edges in the graph is large), the processing time could be long.

Note that Eq. 4 can be solved in closed form:

$$\mathbf{R}^T = (1-c)(\mathbf{I} - c\tilde{\mathbf{W}})^{-1}\mathbf{E} \tag{12}$$

[1]Since both "AND query" and "OR query" can be viewed as special cases of "K_softAND query", the number of active sources is actually k for all query types.

Thus, an obvious way to speed up *CEPS* is to pre-compute and store the matrix $\mathbf{A} = (\mathbf{I} - c\tilde{\mathbf{W}})^{-1}$, then $\mathbf{R}^T = (1-c)\mathbf{AE}$ can be computed on-line nearly real-time. However, in this way, we have to store the whole $N \times N$ matrix A, which is a heavy burden when N is big.

As suggested by [26], the goodness score $r(i,j)(j = 1, ..., N)$ is very skewed, that is, most values of $r(i,j)$ are near zero and only a few nodes have high value. Based on this observation, we propose to pre-partition the original weighted graph $\mathbf{W}$ into several partitions and only use the partitions containing the source queries to run *CEPS*. In this paper, we use METIS [17] as the partition algorithm.

The pseudo code for the accelerated *CEPS* is summarized as follows:

Table 5: Fast *CEPS*

Input:	the weighted graph $\mathbf{W}$, the query set $\mathcal{Q}$, $K\_softAND$ coefficient k, the budget b, and the number of partitions p
Output:	the resulting subgraph $\mathcal{H}$
Step 0:	pre-partition $\mathbf{W}$ into p pieces (one-time cost)
Step 1:	pick up partitions of $\mathbf{W}$ that contain all the query nodes to construct the new weighted graph $\mathbf{nW}$
Step 2:.	run *CEPS* as in table 1 on $\mathbf{nW}$

7. EXPERIMENTAL EVALUATION

In this section, we demonstrate some experimental results. The experiments are designed to answer the following questions.

- Does the proposed goodness criterion make sense?
- Does the $EXTRACT$ algorithm capture the most goodness score?
- Does the extra normalization step really help?
- how does the pre-partition balance the quality and response time?

Data Set We use the DBLP data set to evaluate the proposed method. To be specific, the author-paper information is used to construct the weighted graph $\mathbf{W}$: every author is denoted as a node in $\mathbf{W}$; and the edge weight is the number of co-authored papers between the corresponding two authors. On the whole, there is $\approx 315K$ nodes and $\approx 1,834K$ non-zero edges in $\mathbf{W}$.

Source Queries To test the proposed algorithm, we select several people from different communities to compose the source-query repository: 13 people from database and mining; 13 people from statistical and machine learning; 11 people from information retrieval; and 11 people from computer vision. Then the source queries are generated by randomly selecting a small number of queries from the repository.

Parameter Setting The re-starting coefficient c in Eq. 4 is set 0.5 and the iteration number m is set 50 since we do not observe performance improvement with more iteration steps. The maximum allowable path length len is decided by the budget b and the number of active sources k as $[b/k]$. For normalization coefficient α, a parametric study is provided in Section 7.3. For other experiments, $\alpha = 0.5$.

Evaluation Criterion Firstly, the resulting $g(\mathcal{H})$ can be evaluated by "Important Node Ratio ($NRatio$)". That is, "how many important/good nodes are captured by $g(\mathcal{H})$?":

$$NRatio = \frac{\sum_{j \in \mathcal{H}} r(\mathcal{Q}, j)}{\sum_{j \in \mathbf{W}} r(\mathcal{Q}, j)} \tag{13}$$

Complementally, we can also evaluate by "Important Edge Ratio ($ERatio$)". That is, "how many important/good edges are captured by $g(\mathcal{H})$?":

$$ERatio = \frac{\sum_{(j,l) \in \mathcal{H}} r(\mathcal{Q}, (j,l))}{\sum_{(j,l) \in \mathbf{W}} r(\mathcal{Q}, (j,l))} \tag{14}$$

The goodness score $r(\mathcal{Q}, (j,l))$ of an edge (j,l) is defined similarly as the goodness score for a node: what is the probability that the specific edge (j,l) will be traversed simultaneously by all (or at least k) of the particles. Firstly, we calculate the goodness score $r(i, (j,l))$ for an edge (j,l) wrt a single query node q_i:

$$r(i, (j,l)) = \frac{1}{2} \cdot (r(i,j) \cdot \tilde{\mathbf{W}}_{l,j} + r(i,l) \cdot \tilde{\mathbf{W}}_{j,l}) \tag{15}$$

Based on Eq. 15, we can easily define $r(\mathcal{Q}, (j,l))$ according to the specific query type. For example, for "AND query", $r(\mathcal{Q}, (j,l))$ can be computed as Eq. 16; while for "OR query" and "K_softAND query", $r(\mathcal{Q}, (j,l))$ can be computed as Eq. 17 and Eq. 18, respectively.

$$r(\mathcal{Q}, (j,l)) \triangleq r(\mathcal{Q}, (j,l), Q) = \prod_{q_i=1}^{Q} r(i, (j,l)) \tag{16}$$

$$r(\mathcal{Q}, (j,l)) \triangleq r(\mathcal{Q}, (j,l), 1) = 1 - \prod_{q_i=1}^{Q} (1 - r(i, (j,l))) \tag{17}$$

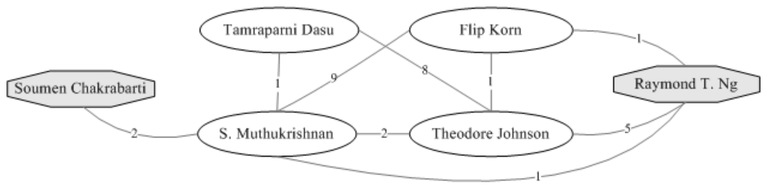
$$\begin{aligned} r(\mathcal{Q}, (j,l)) &\triangleq r(\mathcal{Q}, (j,l), k) \\ &= r(\acute{\mathcal{Q}}, (j,l), k-1) \cdot r(Q, (j,l)) + r(\acute{\mathcal{Q}}, (j,l), k) \end{aligned} \tag{18}$$

where $r(\varnothing, (j,l), 0) = 1$.

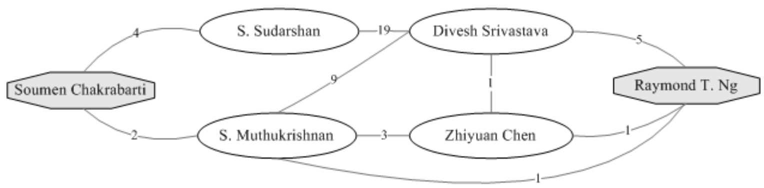
For all experiments except subsection 7.1, we run the proposed algorithm multiple times and report the mean $NRatio$ as well as mean $ERatio$.

7.1 Evaluation on the goodness $g(\mathcal{H})$: case study

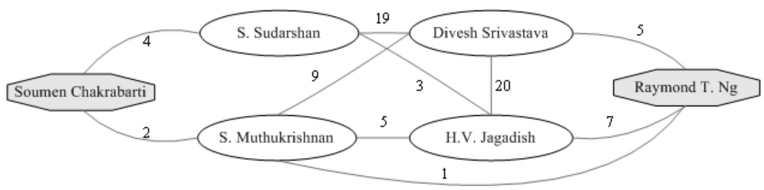
As we mentioned before, connection subgraph is a special case of center-piece subgraph ("AND query" with pair source nodes). Figure 2 shows the connection subgraph with budget 4 for "Soumen Chakrabarti" and "Raymond T. Ng". It can be seen that both our method and the delivered current method output somewhat reasonable results. It is worth pointing out that the subgraph by the delivered current method is very sensitive to the order of the source queries: comparing figure 2(a) and (b), there is only one common node ("S. Muthukrishnan"). On the other hand, if we compare figure 2(b) and (c), while most nodes are the same for the two methods, It is clear that our method captures more strong connection: compared with figure 2(b), the different node ("H.V. Jagadish") in figure 2(c), 1) has more connections (4 vs. 3) with the remaining nodes and



(a) by delivered current method (+1 voltage for Raymond and 0 voltage for Soumen)



(b) by delivered current method (+1 voltage for Soumen and 0 voltage for Raymond sink)



(c) by the proposed method

Figure 2: Connection subgraph between Soumen Chakrabarti and Raymond T. Ng.

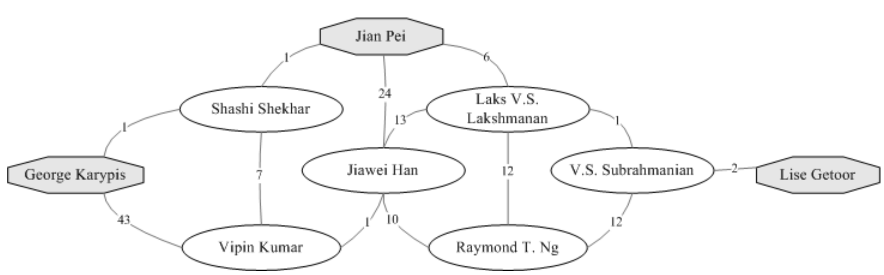


Figure 3: Center-piece subgraph among Lise Getoor, George Karypis, and Jian Pei.

2) has more co-authored papers with those connected neighbors than the corresponding node in figure 2(b) ("Zhiyuan Chen").

Figure 1 shows an example for multi-source queries. When the user asks for $2 - SoftAND$, the algorithm outputs two clear cliques (figure 1(a)), which makes some sense since "Vladimir Vapnik" and "Michael I. Jordan" belong to statistical machine learning community; while "Rakesh Agrawal" and "Jiawei Han" are database and mining people. On the other hand, if the user asks for "AND", the resulting subgraph shows a strong connection with all four queries.

Figure 3 shows an example for "AND query", with "George Karypis", "Lise Getoor" and "Jian Pei" as source nodes. All three researchers are working on graphs. The nodes of the retrieved "center-piece subgraph" are all database, data mining and graph mining people, forming three groups: the nodes close to "Lise Getoor" are related to the University of Maryland ("V.S. Subrahmanian" is a faculty member there and he was the advisor of "Raymond Ng"). The nodes close to "George Karypis" are faculty members at Minnesota ("Vipin Kumar", "Shashi Shekar"). The nodes close to "Jian Pei" are professors at Simon Fraser (SFU) or University of British Columbia (UBC), which are geographically nearby, both in Vancouver: "Jiawei Han" was a faculty member at SFU and thesis advisor of "Jian Pei" ; "Laks Lakshmanan" and "Raymond Ng" are faculty members at UBC. Not surprisingly, the "center-pieces" of the subgraph consist of "Raymond Ng", "Jiawei Han", "Laks Lakshmanan", which all have direct, or strong indirect connections with the three chosen query sources.

7.2 Evaluation on "$EXTRACT$" algorithm

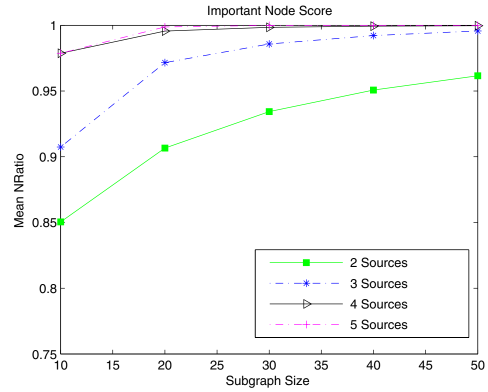
The performance of the "$EXTRACT$" algorithm is evaluated by measuring both $NRatio$ and $ERatio$ as functions of the budget b. Here, we fix the query type as "AND query".

Figure 4(a) shows the mean $NRatio$ vs. the budget b for different numbers of source queries; while figure 4(b) shows the mean $ERatio$ vs. the budget b for different numbers of source queries. Note that in both cases, our method captures most of important nodes as well as edges by a small number of budget b. For example, for 2 source queries, the resulting subgraph with budget 50 captures 95% important nodes and 70% important edges on average; for 4 source queries, the resulting subgraph with budget 20 captures 100% important nodes and 70% important edges on average. An interesting observation is that for the same budget, the subgraph with more source queries captures higher $NRatio$ as well as $ERatio$ than those with less source queries. This is consistent with the intuition: generally speaking, finding people that are important wrt all source queries becomes more difficult when the number of source queries increases. In other words, $r(\mathcal{Q}, j)$ becomes more skewed by increasing the number of source queries.

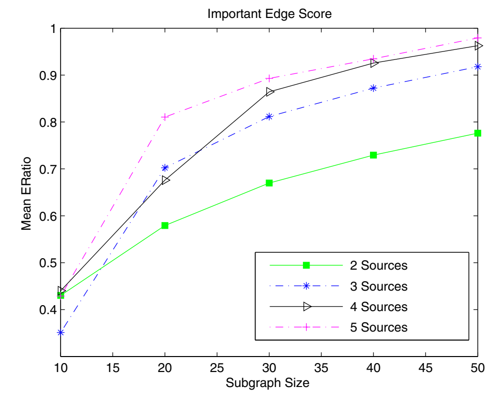
7.3 Evaluation on normalization step

Here we conduct the parametric study for normalization coefficient α. The mean $NRatio$ vs. α is plotted in figure 5(a); and the mean $iERatio$ vs. α is plotted in figure 5(b).

It can be seen that in most cases, the normalization step does help to improve the performance of the resulting subgraph $g(\mathcal{H})$. For example, the normalization with $\alpha = 0.5$ helps to capture 17.7% more important nodes and 9.1% more important edges for 2 source queries on average; while for 3



(a) Important node ratio vs. budget



(b) Important edge ratio vs. budget

Figure 4: Evaluation on "*EXTRACT*"

source queries, it captures 18.1% more important nodes and 7.6% more important edges on average.

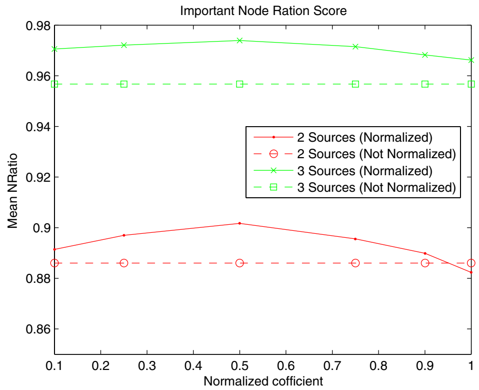
7.4 Evaluation on speedup strategy

For large graph, the response time for importance score calculation could be long. By pre-partition the original graph and performing subgraph discovery only on the partitions containing the source queries, we could dramatically reduce the response time. On the other hand, we might miss a few important nodes if they do not lie in these partitions. To measure such kind of quality loss, we use "Relative Important Node Ratio ($RelRatio$)":

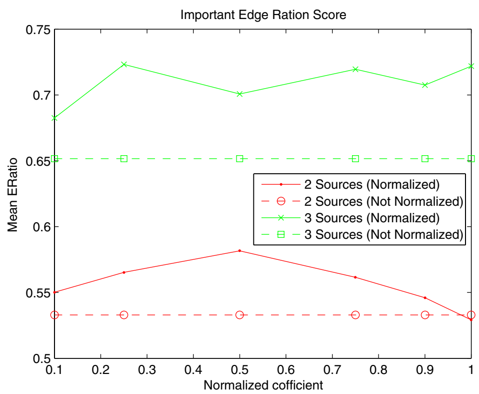
$$RelRatio = \frac{\widehat{NRatio}}{NRatio} \tag{19}$$

where $\widehat{NRatio}$ and $NRatio$ are "Important Node Ratio" for the subgraph by pre-partition and by the original whole graph, respectively.

We fix the budget 20 and the query scenario as "AND query". The mean $RelRatio$ vs. response time is shown in figure 6(a); and the mean response time vs. the number of partitions is shown in figure 6(b). It can be seen that with a little quality loss, the response process is largely speeded up. For example, with $\approx 10\%$ loss, the subgraph for 2 source queries can be generated within 5 seconds on average; with $\approx 10\%$ quality loss, the subgraph for 5 source queries can

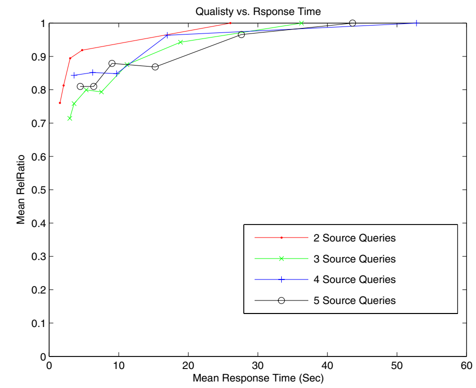


(a) Important node ratio vs. α

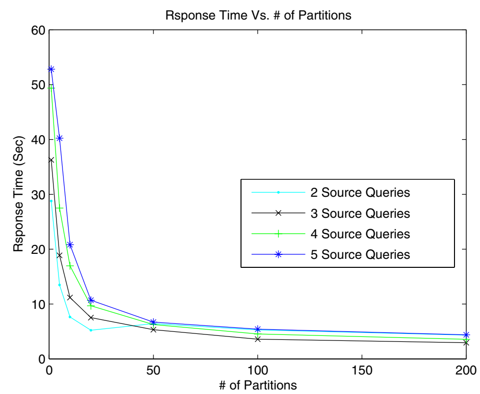


(b) Important edge ratio vs. α

Figure 5: Evaluation on normalization step



(a) Quality vs Time



(b) Time vs Number of partitions

Figure 6: Evaluation on speeding up strategy

be generated within 10 seconds on average. On the other hand, it might take $40s \sim 60s$ without pre-partition. Note that in figure 6 (b), even with a small number of partitions, we can greatly reduce the mean response time.

8. CONCLUSION AND FUTURE WORK

We have proposed the problem of "*center-piece subgraphs*", and provided fast and effective solutions. In addition to the problem definition, other contributions of the paper are the following:

- The introduction and handling of $K\_softAND$ queries, which include AND and OR queries as special cases.
- $EXTRACT$, a fast novel algorithm to quickly extract a subgraph with the appropriate connectivity and maximum "goodness" score
- The design and implementation of a fast, approximate algorithm that brings a 6:1 speedup
- Experiments on real data (DBLP), illustrating that our algorithm and "goodness score" indeed derive results that agree with intuition.

A very promising research direction is the use of parallelism, to achieve fast responses on huge graphs. Another one is to extend the concepts and algorithms to "multi-graphs", that is, graphs with different types of edges. For example, a social network, where one type of edge would indicate "e-mail correspondence", another would mean "telephone contact", and so on.

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APPENDIX

Here, we provide and discuss some variants on goodness score calculation.

- **Variant 1: calculate $r_{i,j}$ by manifold ranking**

One potential problem with Eq. 4 is that such goodness score might be asymmetric, that is $r_{i,j} \neq r_{j,i}$. For social network, this is OK since that person X is important/good for person Y does not necessarily mean that person Y is also important/good for person X. However, in some other applications, symmetry might be a desirable property for the goodness score. To deal with this problem, we can define $r_{i,j}$ as manifold ranking score [28].

Formally, $r_{i,j}$ in this case can be computed by replacing the transition matrix $\tilde{\mathbf{W}}$ in Eq. 4 by graph Laplacian S:

$$\mathbf{R}^T = c\mathbf{R}^T \times \mathbf{S} + (1-c)\mathbf{E} \tag{20}$$

where $\mathbf{S} = \mathbf{D}^{-1/2}\mathbf{W}\mathbf{D}^{-1/2}$ is graph Laplacian.

Note that since S is symmetric, the individual goodness score $r_{i,j}$ by Eq. 20 is always symmetric. That is, $r_{i,j} = r_{j,i}$. However, in this case, the resulting goodness score $r_{i,j}$ is no longer the *steady-state probability*, that is $\sum_{j=1}^{N} r_{i,j} \neq 1$. In our experiments, we find that the resulting subgraphs by Eq. 4 and Eq. 20 are actually quite similar.

- **Variant 2: calculate $r(\mathcal{Q}, j)$ by order statsitic**

Let $r^{(k)}(i,j)$ be the order statistic of $r(i,j), (i = 1, ..., Q)$. That is, $r^{(k)}(i,j)$ is the k^{th} largest value among $r(i,j), (i = 1, ..., Q)$.

Then, we can also use $r^{(k)}(i,j)$ to get $r(\mathcal{Q}, j)$. For example, we can use minimum order statistic as goodness score for "AND query":

$$r(\mathcal{Q}, j) \triangleq r^{(Q)}(i,j) = min(r(1,j), r(2,j), ..., r(Q,j)) \tag{21}$$

The probabilistic interpretation of Eq. 21 is that the node j is important wrt the source queries if and only if there is at least some high probability for every particle to finally stay at node j.

Similarly, the order statistic variants for "OR query" and "K_softAND query" can be defined as $r^{(1)}(i,j)$ and $r^{(k)}(i,j)$, respectively.