Central bank collateral, asset fire sales, regulation, and liquidity

by U. Bindseil*

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This paper analyses the potential roles of bank asset fire sales and recourse to central bank credit to ensure banks' funding liquidity and solvency. Both asset liquidity and central bank haircuts are modeled as power functions within the unit interval. Funding stability is captured as strategic bank run game in pure strategies between depositors. Asset liquidity, the central bank collateral framework and regulation determine jointly the ability of the banking system to deliver maturity transformation and financial stability. The model also explains why banks tend to use the least liquid eligible assets as central bank collateral and why a sudden non-anticipated reduction of asset liquidity, or a tightening of the collateral framework, can destabilize short term liabilities of banks. Finally, the model allows discussing how the collateral framework can be understood, beyond its essential aim to protect the central bank, as financial stability and non-conventional monetary policy instrument.

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Key words: Asset liquidity, liquidity regulation, bank run, central bank collateral framework, unconventional monetary policy

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Non-technical summary

This paper provides a simple model of the interaction between the asset liquidity of banks, the central bank collateral framework, and liquidity regulation, such as to better understand the effects of these factors on financial stability and the ability of the banking system to deliver maturity transformation, which is one of its key functions for society.

Stability of short term bank funding is modeled as strategic game of short term depositors who have the option to keep their deposits with the bank or to "run". After establishing the conditions for stability of short term funding, the effects of the key exogenous variables (asset liquidity, central bank collateral framework, regulation) on the endogenous variables are derived, namely on (i) the liability structure (the mix of short term funding, long term funding, and equity), (ii) the relative reliance on the two emergency funding sources (fire sales vs. central bank credit), and on (iii) the funding costs of banks, as proxy of the banks' ability to deliver maturity transformation. For example, the model shows that (absent regulation) the tighter the central bank collateral framework, the higher the equilibrium shares of bank equity and long term funding in the total length of bank balance sheets, and the higher the average funding costs of banks, which is a proxy for the lower ability of the banking system to deliver maturity transformation (as in our model, the asset composition of the bank is given). The identification of these equilibrium effects also allows to better understand and to revisit the likely effectiveness of liquidity regulation with regard to achieving certain goals, such as e.g. "reduce reliance on central banks as source of emergency liquidity", "avoid that in a crisis banks fire sale non-liquid assets", etc.

The issues captured are close to the ones of Rochet and Vives (2004). The model here is simpler and more narrow in terms of equilibrium concept, but refined in terms of capturing asset liquidity and access to the central bank. Both are modeled as power functions within the unit interval, while Rochet and Vives (2004) segregate assets simply into "liquid" and "non-liquid" and do not capture the potential access to central bank credit by post-haircut collateral availability. Another paper of relevance is Ashcraft et al (2011), who indeed assume differentiated central bank collateral haircuts across assets and note, similar to the present paper, that the haircut policy of central banks is also monetary policy. Again, while Ashcraft et al (2011) is an encompassing general equilibrium model, it does not capture the role of asset fire sales in banks' liquidity management and in its implications for the ability of banks to undertake maturity transformation (and through this channel on effective monetary conditions).

An example is provided in which indeed liquidity regulation improves social welfare by imposing excess liquidity buffers in normal times beyond those that banks would hold voluntarily, such that sudden declines of asset liquidity trigger with less probability a destabilization of short term liabilities and the associated negative externalities (of course only if the buffers established by liquidity regulation can be used smoothly in case of a liquidity crisis).

The paper also illustrates how changes of the collateral framework can be understood as policy tool to maintain financial stability, explaining why most central banks tended to extend collateral eligibility in 2007 and 2008. The challenge for the central bank is to built-in the possibility to extend collateral buffers, without inviting banks to factor this in. In a crisis characterized by a drop in asset liquidity, a widening of central bank collateral buffers can contribute to preserve market access of banks and hence

to prevent large recourse to the central bank. In this sense, the paper provides further illustration of Bagehot's (1873) conjecture that only the "brave plan" of the 19th century Bank of England would be a "safe" plan.

It is also shown that the identified impact of asset liquidity and of the central bank collateral framework on funding costs of banks is relevant for monetary policy for at least two reasons. First, policy makers need to be aware that a tightening of any of the two emergency liquidity sources also tightens, everything else unchanged, monetary conditions. Second, when the central bank has reached the zero lower bound, and therefore cannot use standard interest rate policies any longer to lower the money rate, it could consider to use its collateral framework to ease monetary conditions. In this sense, the model fits into the program as defined e.g. by Woodford (2010) or Friedman (2013) to better understand the role of financial intermediation in determining the relationship between central bank lending rates and the actual funding conditions of the real economy, i.e. the financial intermediation spread. The use of the collateral framework for policy purposes (financial stability and monetary policy) has of course to take place with due consideration to the original purpose of the collateral framework, which is the protection of the central bank.

1. Introduction

The model proposed in this paper sheds new light on how asset liquidity, Basel III type liquidity regulation, and the central bank collateral framework affect financial stability and monetary policy. In January 2013, the Basle Committee on Banking Supervision has issued again, after a number of adaptations, a document describing *Basle III liquidity regulation*, and in particular the so-called Liquidity Coverage Ratio (LCR) and the related concept of High Quality Liquid Assets (HQLA). The Financial crisis of 2007/2008 is said to also have been triggered by the insufficient asset liquidity buffers of banks relative to their short term liabilities. These insufficient buffers would have led to an (at least temporarily) excessive reliance on central bank funding. In the words of Basel Committee (2013, 1):

> *2. During the early "liquidity phase" of the financial crisis that began in 2007, many banks – despite adequate capital levels – still experienced difficulties because they did not manage their liquidity in a prudent manner. The crisis drove home the importance of liquidity to the proper functioning of financial markets and the banking sector. Prior to the crisis, asset markets were buoyant and funding was readily available at low cost. The rapid reversal in market conditions illustrated how quickly liquidity can evaporate, and that illiquidity can last for an extended period of time. The banking system came under severe stress, which necessitated central bank action to support both the functioning of money markets and, in some cases, individual institutions.*

This excessive reliance on central bank funding as alluded to in the last sentence is considered to have constituted a form of moral hazard. The LCR requires a certain amount of HQLAs to be maintained by banks relative to possible liquidity outflows in a one month stress scenario. The Basel Committee (2013, 7) considers as constituting characteristic of HQLAs that they can be fire-sold without large losses even under stressed circumstances:

> *24. Assets are considered to be HQLA if they can be easily and immediately converted into cash at little or no loss of value. The liquidity of an asset depends on the underlying stress scenario, the volume to be monetised and the timeframe considered. Nevertheless, there are certain assets that are more likely to generate funds without incurring large discounts in sale or repurchase agreement (repo) markets due to fire-sales even in times of stress*

As recalled by e.g. Brunnermeier et al (2009), negative externalities are key to justify regulation of financial markets. In case of liquidity regulation, these negative externalities relate in particular to the asset fire sales spiral and more generally to various forms of negative contagion. According to the Financial Services Authority (2009, 68), "liquidity risk has inherently systemic characteristics, with the reaction of one bank to liquidity strains capable of creating major liquidity strains for others." Also the Basel Committee (2013, 8) refers to the negative externalities of asset fire sales:

> *25. … An attempt by a bank to raise liquidity from lower quality assets under conditions of severe market stress would entail acceptance of a large fire-sale discount or haircut to compensate for high market risk. That may not only erode the market's confidence in the bank, but would also generate mark-to-market losses for banks holding similar instruments and add to the pressure on their liquidity position, thus encouraging further fire sales and declines in prices and market liquidity.*

Finally, the welfare economic tradeoffs between the efficiency of the banking system in delivering maturity transformation and financial stability is also crucial when assessing the net benefits of regulation for society. In the words of the Turner review (Financial Services Authority, 2009, 68):

[T]here is a tradeoff to be struck. Increased maturity transformation delivers benefits to the non-bank sectors of the economy and produces term structures of interest rates more favourable to long-term investment. But the greater the aggregate degree of maturity transformation, the more the systemic risks and the greater the extent to which risks can only be offset by the potential for central bank liquidity assistance.

This trade-off, and the revealed reservations of regulators regarding large central bank reliance of banks under liquidity stress, and at the same time the regulators awareness that asset fire sales of imperfectly liquid assets create negative externalities (that could be avoided by the recourse to central bank credit), suggests that analyzing the impact of liquidity regulation must integrate the role of central bank funding.[1]

While the *central bank collateral framework* got relatively limited attention in academic writing, it is one of the most complex and economically significant elements of monetary policy implementation. Unencumbered central bank eligible collateral is potential liquidity, as it can, in principle, be swapped into central bank money. It is therefore not exaggerated to argue that the collateral framework must be an important ingredient of any theory of liquidity crises (as noted by Bagehot, 1873), and of any monetary theory. A survey of current central bank practice in G20 countries is provided by Markets Committee (2013). Section 1.3 of this report also summarizes the various measures taken during the financial crisis by central banks (p 8-9):

"During the height of the financial crisis in 2008–09, a number of central banks introduced, to varying degrees, crisis management measures such as a temporary acceptance of additional types of collateral, a temporary lowering of the minimum rating requirements of existing eligible collateral or a temporary relaxation of haircut standards. Many of these temporary changes have expired."

Changes to the collateral framework also seem to play a role as monetary policy instrument (i.e. beyond being an instrument to address a liquidity crisis) when central banks approach the zero lower interest rate bound. For example, the Bank of England's "Funding for lending" scheme (FLS) also relies on a widening of the collateral framework (see Churm and Radia, 2012, 317).

"A broad range of collateral is eligible for use in the FLS, so that, as far as possible, the availability of collateral does not constrain banks' ability to use the FLS. Therefore eligible collateral in the FLS… includes portfolios of loans, various forms of asset-backed securities and covered bonds, and sovereign and central bank debt."[2]

Four strands of academic literature are relevant to the present paper.

First, the interaction between liquidity regulation and monetary policy implementation has been analyzed by e.g. Bindseil and Lamoot (2011), Bech and Keister (2012), ECB (2013a), and Körding and Scheubel (2013). However, this literature does not model in detail the role of the central bank collateral framework, nor does it draw concrete conclusions on the financial stability implications of this interaction.

[1] See also Bank of England (2013) for a recent explanation of its lender of last resort (LOLR) policies.
[2] Portfolios of loans and ABS are not eligible for ordinary Bank of England credit open market operations.

Second, Rochet and Vives (2004) is close to the present paper in the sense that it also models the role of fire sales and the central bank lender of last resort (LOLR) function for banks' funding stability, liquidity and solvency. Rochet and Vives (2004) obtain partially similar results, including with regard to the merits of liquidity regulation. The model of Rochet and Vives (2004) is more sophisticated in terms of generality of equilibrium. On the other side, it takes strong simplifying assumptions regarding asset liquidity (only two types of assets are distinguished: cash and non-liquid assets). Also the logic and limits of access to central bank credit are modeled in a way that seems more remote to central bank practice, as indeed central bank credit is primarily restricted by collateral availability. Beyond Rochet and Vives (2004), there is an extensive more general multiple funding equilibrium literature such as represented by e.g. Morris and Shin (2000) under the headline of "global games". This literature uses more general and sophisticated equilibrium concepts than the present paper, which limits itself to pure and dominant strategies of investors/depositors, and to the existence or not of a Strict Nash Equilibrium in the sense of Fudenberg and Tirole (1991, 11). These simple concepts appear sufficient to progress in a relevant way with regard to understanding the interrelation between asset liquidity, the central bank collateral framework, and liquidity regulation on one side, and the stability of bank funding and the effective monetary conditions on the other side. [3]

Third, Ashcraft et al (2011) relates to the present model in the sense that central bank haircut policies are identified and modeled as a monetary policy instrument (see also Chapman et al, 2011). Ashcraft et al (2011) assume that banks refinance assets at the central bank and that the haircut determines the leverage ratio and thus the funding costs of assets, being a weighted average of the risk free rate and the shadow cost of equity (see also Brunnermeier and Pedersen, 2009). Again, Ashcraft et al (2011) offer a general equilibrium model, which the present paper does not. The present model however avoids the strong assumption that central bank haircuts determine leverage ratios and hence funding costs of assets. Indeed, central banks do not refinance the majority of bank assets, but only a small fraction. Normally, the length of the central bank balance sheet is determined by the amount of banknotes in circulation, while the size of the banking system and of financial intermediation is much bigger. For example, in the case of the euro area, banknotes are around EUR 1 trillion, while the length of the aggregate banking system balance sheet is EUR 32 trillion. In the present model, the role of haircuts for monetary conditions does not stem from leverage ratios being the inverse of haircuts, but from the role of haircuts for the cheapest sustainable funding structure of banks.

Fourth, the present paper contributes to explain the spread between the risk free rate (which is close to the rate of central bank credit operations and the rate of remuneration of overnight deposits of households with the banking system) and the actual funding costs of the real economy (or the effective monetary conditions for the economy). In this sense, the paper contributes to the program such as defined for instance by Woodford (2010) or Friedman (2013) to enrich the analysis of monetary policy in particular by capturing more explicitly the spread between the central bank credit operations rate and the actual monetary conditions as they are felt by the real economy when seeking bank or market funding. Indeed, the present paper shows that a drop in the liquidity of bank assets and/or an increase in central bank haircuts both tighten effective monetary conditions in the sense that they reduce the ability of the banking system to undertake maturity transformation, and hence, everything else equal, will increase the share of "expensive" bank funding sources such as long term bonds and equity, implying that also the lending rates that a competitive banking system is able to offer, have to increase.

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[3] Other relevant papers that model funding liquidity, leverage and asset liquidity are Brunnermeier and Pedersen (2009), Acharya, Gale and Yorulmazer (2011), and Acharya and Viswanathan (2011).

2. A model of funding stability with continuous asset liquidity and haircuts

Throughout this paper, we consider the following stylized bank balance sheet. The total length of the balance sheet has been set to unity. Assets are heterogeneous in a continuous sense, while there are three types of liabilities which are each separately homogenous (with $e \in [0,1], t \in [0,1], e + t \in [0,1]$)

Figure 1: A stylised bank balance sheet to analyse funding stability of a bank

Assets		Liabilities	
Assets	1	Short term debt 1	(1-t-e)/2
		Short term debt 2	(1-t-e)/2
		Long term debt ("term funding")	t
		Equity	e

The stylized balance sheet is sufficient to capture one key issue of banking: how to ensure the confidence of short term creditors of the bank such that they do not easily switch to a fear mode in which they start withdrawing deposits, triggering self-fulfilling destructive dynamics ending in bank default. This is the well-known bank run problem, as analyzed for instance by Diamond and Dybvig (1983). Confidence may be sustained in particular by two means. *First*, the bank may limit the role of short term funding. However, in general, investors prefer to hold short term debt instruments over long term debt instruments, and request a higher interest rate on long term debt. In other words, long term debt is associated with higher funding costs for the bank, or, put differently, maturity transformation is one of the key contributions of banking to society (see Financial Services Authority, 2009, 68). *Second*, the bank may aim at holding sufficient amounts of liquid assets, both in the sense of being able to liquidate these assets in case of need, and in order to be able to pledge them with the central bank at limited haircuts. However, on average, liquid assets generate lower return than illiquid ones (e.g. Houweling et al, 2005, or Chen et al, 2007, for recent empirical studies).

Consider now in more detail the different balance sheet positions of the representative bank.

2.1 Bank assets

The total assets of the banks have initial value 1. Assets are not homogeneous, but can be differentiated as follows: (i) Asset liquidity, as measured by the "fire sale" discount to be accepted if an asset is to be sold in the short run; (ii) Eligibility and haircut if submitted to the central bank as collateral. Assets are either central bank eligible or ineligible, and if they are eligible they are accepted at a certain haircut. This is simplified in the model through the assumption that all assets are eligible, but have varying haircuts between 0 and 1; (iii) Finally, assets have a differentiated treatment in liquidity regulation, as they are accepted or not as "high quality liquid asset" (HQLA; see annex 4 of Basle Committee, 2013).

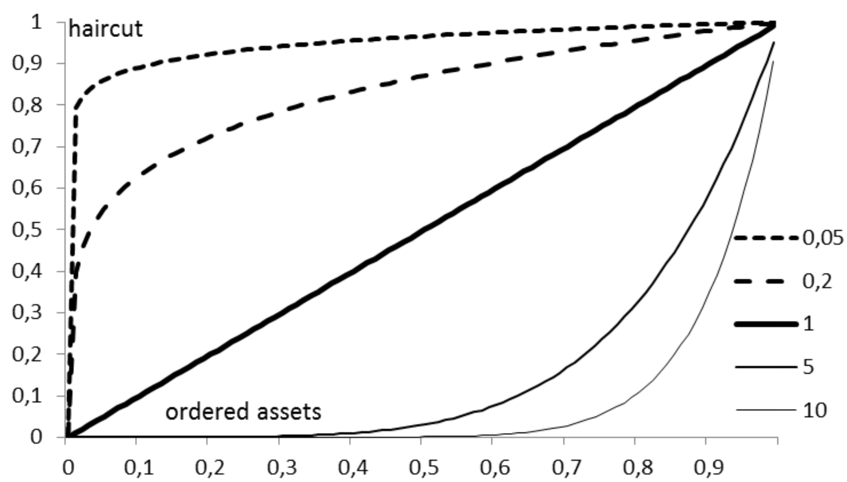
Treatment of bank assets as central bank collateral

Assume that there is a continuum of assets and that assets are ranked from those which the central bank considers the best quality collateral to the ones that it considers the least suitable collateral. The central bank collateral haircut function is then a function from the assets unity interval [0,1] into the haircut unity interval [0,1]. Assume that this function has the following form with $\delta \geq 0$:

$$h(x)=x^{\delta} \quad (1)$$

The power function in the unit interval captures broadly the properties of a typical central bank haircut framework: haircuts for the most liquid assets will be close to zero, while haircuts for the least liquid assets accepted will be very high, and an often significant part of assets will not be accepted at all, which is broadly equivalent to a 100% haircut (see also below the example of the ECB haircut schedule). If δ is close to 0, then the haircuts increase and converge quickly towards 1. If in contrast δ is large (say 10) then haircuts stay at close to zero for a while and only start to increase in a convex manner when approaching the least liquid assets. The total haircut (and the average haircut) if all assets are pledged is 1/(δ+1), and potential central bank credit is δ/(δ+1). This is obtained from the integration rule $\int x^{\delta} = x^{\delta+1}/(\delta+1)$. The following figure illustrates the haircut function h(x) for various values of δ.

Figure 2: the exponential haircut function $h(x)=x^{\delta}$ in the unity interval for different values of δ



For example, in the case of the Eurosystem, out of EUR 32 trillion of aggregated bank assets, the value of central bank eligible collateral after haircuts that could be used at any moment in time is around EUR 5 trillion. The eligibility criteria and haircut matrices are provided by the ECB and one can match this information in principle with an informed guess of banks' assets holdings. This implies that the effective average haircut applied by the Eurosystem to (the entirety of) bank assets is around 84%, and central bank refinancing power is 16% of eligible assets, which approximately implies a parameter value δ=0.2. Table 1 below presents an excerpt of the ECB haircut scheme, showing 3 out of the 5 maturity buckets. The haircut scheme is a mapping of three features of each security into a haircut, namely (see ECB Press Release of 18 July 2013; ECB, 2011, Chapter 6):

- Rating: BBB rated assets have higher haircuts than A-AAA rated ones (assets with ratings below BBB are normally not eligible at all);
- Residual maturity: the longer the residual maturity of bonds, the higher the price volatility and hence the higher the haircut;
- Institutional liquidity category of assets: The ECB has established six such categories, which are supposed to group assets into homogenous institutional groups in terms of liquidity. Of course any such grouping will be a simplification. For instance, Government bonds of a relatively small euro area country with relatively little debt outstanding (e.g. Slovakia, classified in category I) may be less liquid than the bonds of a large Government linked issuer in category II, such as the German development bank KfW. Therefore, the approach taken reflects the need for central banks to establish sufficiently simple, transparent and manageable frameworks.

Table 1: ECB's haircuts (in %) for different securities classified in liquidity categories according to issuer types, for three buckets of residual maturity and for two rating classes, as published by ECB on 18 July 2013 (annex to press release of that date)

	Category I	Category II	Category III	Category IV	Category V	Categ.VI*
Issuer types → Assets ↓	Central Government debt	Local Gvt debt; Jumbo covered bonds	Covered bonds; corp. bonds	Unsecured bank debt instruments	ABS	Credit claims to corporates
0-1Y; A-AAA	0.5	1.0	1.0	6.5	10.0	12.0
3-5Y; A-AAA	1.5	2.5	3.0	11.0	10.0	21.0
≥ 10Y; A-AAA	5.0	8.0	9.0	17.0	10.0	45.0
0-1Y; BBB	6.0	7.0	8.0	13.0	22.0	19.0
3-5Y; BBB	9.0	15.5	22.5	32.5	22.0	46.0
≥ 10Y; BBB	13.0	22.5	27.5	37.5	22.0	65.0

*For credit claims: referring to nominal value, as applicable to most credit claims accepted by Eurosystem

Moreover, the ECB announced through a press release of 9 February 2012 a framework to accept additional credit claims as collateral. National central banks of the Eurosystem establish the haircuts for these additional credit claims, which typically also include credit claims towards obligors with a BB equivalent rating. For example the Banca d'Italia (2012, 2) Provided the following haircuts for the maturity buckets mentioned above: 0-1Y: 42%; 3-5Y 70%; above 10Y: 80%.[4]

As far as marketable assets (securities) are concerned, the following two further remarks are relevant:

- First, it should be noted that *valuation below nominal* also has effects similar to haircuts in terms of reducing the potential recourse to central bank credit. For example, a BBB rated bank bond with 4 years residual maturity may, due to a widening of credit spreads or due to a downgrade of the issuer, be valued by the ECB at 90%, instead of the nominal 100%. This implies that the ratio between its central bank refinancing power and its nominal value will be: 90%*(1-32.5%)= 60.75%. In this sense the effective overall markdown (due to valuation and haircut) will be 39.25%.
- Second, in principle, a large part of asset in the balance sheet of banks could be made central bank eligible through *securitisation* (i.e. packaging into Asset-Backed Securities, ABS). Banks however do not undertake such a general securitisation of large parts of their assets, as it is costly to securitise, as it may stigmatise the bank (why is it so desperate to maximise its potential recourse to the central bank?) and because the effectiveness of this in terms of added potential central bank credit may be low. Indeed, there are three factors that reduce the central bank refinancing power of an ABS relative to the value of underlying assets: (i) the fact that the ECB accepts only the senior tranche, i.e. the junior and/or mezzanine tranches issued are worthless from the perspective of central bank funding; (ii) valuation of the ABS below par; (iii) haircut.

Liquidity of bank assets

Now consider asset liquidity in the sense of the ability of banks to sell assets in the short term without this inflicting value losses and hence a loss for the bank. Assume again that assets are ranked from the

[4] Also the Bank of England and the Fed are highly transparent on their haircuts (see Bank of England: "Sterling Monetary Framework - Summary of haircuts for securities eligible for the Bank's lending operations, 02 October 2012", US Fed: "Federal Reserve Discount Window & Payment System Risk Collateral Margins Table1 - Effective Date: October 19, 2009 (updated January 3, 2011)" - both found on the central banks' websites).

most liquid to the least liquid, and that the fire sale discount function is a function from [0,1] into [0,1] with the following function form, with $\theta \geq 0$.

$$d(x)=x^{\theta} \tag{2}$$

If θ is close to 0, then the fire sale discounts increase and converge quickly towards 1. If in contrast θ is large (say 10), then discounts are close to zero for most assets and only start to increase in a convex manner when approaching the least liquid assets. If a certain share x of the bank's assets has to be sold, then the fire sale discounts will have to be booked as a loss and reduce equity. Assuming that the bank starts with the most liquid assets, the loss will be $x^{\theta+1}/(\theta+1)$. Empirical estimates of default costs in the corporate finance literature vary between 10% and 44% (see e.g. Glover, 2011, and Davydenko et al, 2012). In fact this cost can be interpreted as the liquidation cost of assets, captured in the parameter θ. Liquidation of all assets will lead to a damage of $1/(1+\theta)$, such that remaining asset value will be $\theta/(1+\theta)$. If default cost is 10%, this would mean that $\theta = 9$, and if default cost is 44%, then $\theta = 1.27$. For a value of default costs in the middle of the empirical estimates of say 25%, one obtains $\theta=3$.

It is important to note that in the model proposed, exogenous shocks to asset values are not considered and therefore are not the cause of financial instability. In the model, instability is triggered by liquidity issues, and specifically the possibility of a bank run affecting short term liabilities of the bank. This will have a potential effect on asset values only in the sense that asset fire sales lead to the necessity to reduce sales prices relative to fair prices.[5]

2.2 Bank liabilities

Four types of liabilities are distinguished: (i) *Short term liabilities* are equally split to two ex-ante identical depositors; (ii) *Long term debt* does not mature within the period considered, and are ranked pari passu with short term debt; (iii) *Equity* is junior to all other liabilities, and is also a stable funding source; (iv) *Central bank borrowing* is zero initially, but can substitute for outflows of short term liabilities in case of need. It is collateralized and therefore the central bank acquires in case of default ownership of the assets pledged as collateral. Apart from this, the central bank claim ranks pari passu, i.e. remaining claims after collateral liquidation are treated in the same way as an initial unsecured deposit. The sum of liabilities obviously has to be one. In the balance sheet above, short term debt is presented as the residual item, but this is an arbitrary choice.

2.3 Time line

The model is based on the following time line:

- Initially, the bank has the balance sheet composition as shown in figure 1.
- Short term depositors/investors play a strategic game with two alternative actions: to run or not to run. "Running" means to withdraw the deposits and to transfer them to another account, accepting a small cost capturing the transaction cost of withdrawing the deposits, which we notate by ε.
- It is not to be taken for granted that depositors can withdraw all their funds. If one or both of the depositors run, then at least one or several of the following will apply: (i) Substitution of deposit outflows with central bank credit, assuming that the bank has sufficient eligible collateral. (ii) Liquidation of assets: if central bank collateral is insufficient to completely substitute short term

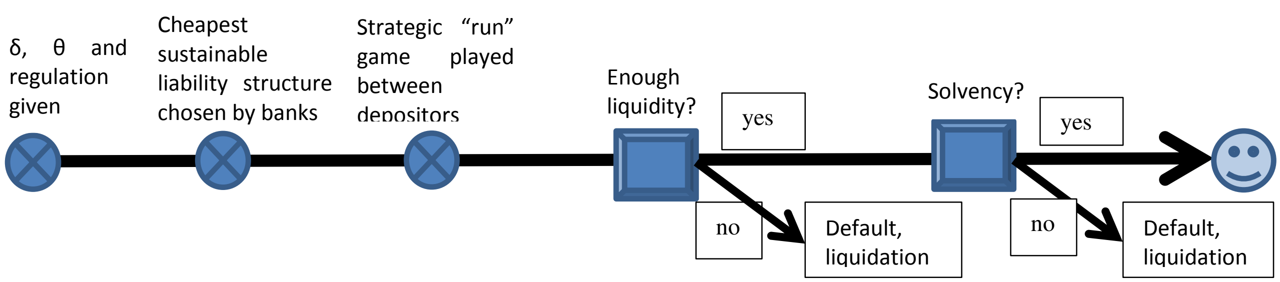
[5] The model could also be extended by adding exogenous asset value shocks. The effect on equity of such shocks could destabilize short term funding within the model.

funding withdrawals, then the bank will also do asset fire sales. (iii) If it is impossible to pay out the depositors that want to withdraw their deposits, Illiquidity induced default will occur. If illiquidity induced default occurs, all (remaining) assets need to be liquidated, and corresponding default related losses occur.

- If the bank was not closed due to illiquidity in the previous stage, still its solvency is analyzed and if capital is negative, the bank is resolved. Again, it is assumed in this case that the full costs of immediately liquidating all assets materialize.

The full timeline is summarized in the following figure. Sections 3 and 4 take the first two nodes as given and start directly with the third, i.e. with the strategic game. Sections 5 and 6 work backwards to the beginning of this time line.

Figure 3: Time-line of bank liability structure model



2.4 Strict Nash No-Run (SNNR) equilibrium

The decision set of depositor i (i=1,2) from which he will choose his decision D_i consists in $\{K_i, R_i\}$, whereby "K" stands for "keeping" deposits and "R" stands for "run". Call the pay-off function of depositor i: $U_i=U_i(D_1,D_2)$. Note that the strategic game is symmetric, i.e. $U_1(K_1,K_2)=U_2(K_1,K_2)$, $U_1(K_1,R_2)=U_2(R_1,K_2)$, $U_1(R_1,K_2)=U_2(K_1,R_2)$, $U_1(R_1,R_2)=U_2(R_1,R_2)$. This allows to express in the rest of the paper conditions only with reference to one of the two players, say depositor 1.

A *Strict Nash equilibrium* is defined as a strategic game in which each player has a unique best response to the other players' strategies (see Fudenberg and Tirole, 1991, 11). A *Strict Nash No-Run (SNNR) equilibrium* in the run game is therefore one in which the no-run choice dominates the "run" choice regardless of what the other depositors decide, i.e. an SNNR equilibrium is defined by

$$U_1(K_1,K_2)>U_1(R_1,K_2) \cap U_1(K_1,R_2)>U_1(R_1,R_2) \quad (3)$$

3. Pure reliance on either central bank funding or on asset fire sales

3.1 Pure reliance on central bank funding

Assume first that asset liquidation is not an option, say because markets are totally frozen, i.e. $\theta=0$. Therefore, the analysis can focus on the sufficiency or not of potential recourse to central bank credit. The following proposition states the necessary condition for funding stability of banks in this case.

Proposition 1: If $\theta = 0$, a Strict Nash No-Run (SNNR) equilibrium prevails if and only if $\delta/(\delta+1) \geq (1-t-e)/2$, i.e. the liquidity buffer based on recourse to central bank credit is not smaller than one half of the short term deposits.

To prove proposition 1 (and similar subsequent propositions), it is sufficient to calculate through the pay-offs for the alternative decisions of depositors under the possible parameter combinations and establish the frontiers of parameter combinations under which the conditions of an SNNR equilibrium apply. Distinguish now the three possible cases $(1-t-e) \leq \delta/(\delta+1)$; $(1-t-e)/2 \leq \delta/(\delta+1) < (1-t-e)$; and $\delta/(\delta+1) < (1-t-e)/2$, respectively, i.e. the cases in which liquidity buffers provided by central bank collateral are sufficient to compensate for the withdrawal of all deposits; for the withdrawal of (not more than) one depositor; and not even for the withdrawal of one depositor. It may be noted that in the case assumed here that $\theta=0$, illiquidity of the bank means complete destruction of asset value as the liquidation value of assets is assumed to be zero. Recall also that ε is the small transaction cost of transferring deposits out of the account with the bank (e.g. to the account with some other bank), i.e. the cost of "running" assuming the bank can pay out. The proposition follows from verifying the condition $U_1(K_1,K_2)>U_1(R_1,K_2) \cap U_1(K_1,R_2)>U_1(R_1,R_2)$ for the three different cases distinguished above. One obtains the pay-offs shown in Table 2.

Table 2: Pay-offs for depositor 1 for the four possible combinations of depositors' decisions, and for three different sizes of liquidity buffers

Liquidity buffer size:	$U_1(K_1,K_2)$	$U_1(R_1,K_2)$	$U_1(K_1,R_2)$	$U_1(R_1,R_2)$
$(1-t-e)\leq\delta/(\delta+1)$	$(1-t-e)/2$	$(1-t-e)/2-\varepsilon$	$(1-t-e)/2$	$(1-t-e)/2-\varepsilon$
$(1-t-e)/2\leq\delta/(\delta+1)<(1-t-e)$	$(1-t-e)/2$	$(1-t-e)/2-\varepsilon$	$(1-t-e)/2$	$(\delta/(\delta+1))/2$
$\delta/(\delta+1)<(1-t-e)/2$	$(1-t-e)/2$	$\delta/(\delta+1)$	0	$(\delta/(\delta+1))/2$

It is easily verified that the conditions for an SNNR equilibrium are given if and only if $(1-t-e)/2<\delta/(\delta+1)$. This concludes the proof.

The key problem with the case $\delta/(\delta+1)<(1-t-e)/2$ is that $U_1(K_1,R_2)<U_1(R_1,R_2)$. If the other depositor runs, then it is better to run to get some money out of the bank before the bank defaults, i.e. it is better to get out $(\delta/(\delta+1))/2$ and thereby to contribute to the bank default than to let the other depositor gets out $\delta/(\delta+1)$, triggers bank default, and have yourself a payoff of zero because the remaining liquidation value of the bank's assets is zero.

3.2 Pure reliance on asset fire sales

Now consider the case in which the central bank accepts no collateral at all (or just does not offer any credit operations with banks as it prefers to implement monetary policy exclusively through outright operations, following e.g. the advice of Friedman, 1982). In this case $\delta=0$, such that addressing deposit outflows will have to rely exclusively on asset liquidation. Assume that the bank does whatever it takes in terms of asset liquidation to avoid illiquidity induced default. The total amount of liquidity that the bank can generate through asset fire sales is $\theta/(\theta+1)$. Therefore, illiquidity induced default will materialise only if deposit withdrawals eventually exceed this amount. While with full reliance on central bank lending, the question was whether the related liquidity buffers would be sufficient (and if not, who would recover what), in the present case, two default triggering events need to be considered. Indeed, even if the bank has survived a liquidity withdrawal, it may afterwards be assessed as insolvent and thus be liquidated at the request of the bank supervisor. As noted above, for a given liquidity withdrawal x, the fire sale related loss is $x^{\theta+1}/(\theta+1)$. Default due to insolvency occurs if this loss exceeds initial equity.[6]

Proposition 2: If $\delta=0$, a SNNR equilibrium exists if and only if $(1-t-e)/2\leq\theta/(\theta+1)$ and $e\geq((1-t-e)/2)^{\theta+1}/(\theta+1)$.

The proposition can be verified by again establishing the strategic game pay-offs and showing under which circumstances the SNNR conditions are met. The proof is provided in the annex. In sum, to ensure financial stability in the case of absence of central bank credit, minimum liquidity and capital buffers are needed in some appropriate combination to ensure the stability of a given amount of short term funding. The lower the asset liquidity, the lower the amount of short term funding that can be sustained for a given level of equity.

4. Cases in which the bank relies on both types of liquidity buffers

Now consider the cases in which both $\theta>0$ and $\delta>0$. It is assumed that the ordering of assets is the same for both forms of liquidity generation, i.e. if asset i is subject to lower fire sale discounts than asset j, then also asset i will have a lower central bank collateral haircut than asset j. Proposition 3 narrows down the actual range of mixed cases, i.e. cases in which both liquidity sources play a role in the planning of the bank.

Proposition 3: if either $\delta \geq \theta$ or $(\theta > \delta \cap \delta/(\delta+1) \geq (1-t-e)/2)$, then banks will only rely on central bank credit to address possible deposit withdrawals, and hence the conditions established in proposition 1 apply to the existence of an SNNR equilibrium.

[6] Note that it is assumed that equity is never sufficient to absorb the losses resulting from a bank default, i.e. it is assumed that $e \leq 1/(\theta+1)$. Of course one could also calculate through the opposite case, but it is omitted here as it does not seem to match reality.

Taking recourse to the central bank does not cause a loss, while fire sales cause one. If in addition, central bank recourse yields more liquidity (as by assumption $\delta>\theta$), then central bank credit strictly dominates asset fire sales as a source of emergency liquidity. If $\theta>\delta$ and central bank liquidity buffers allow to address liquidity outflows relating to one depositor, i.e. $\delta/(\delta+1) \geq (1-t-e)/2$, which, as shown previously, allows to sustain the SNNR equilibrium, then again relying only on central bank credit dominates strategies to rely on both sources.

The cases in which the bank wants to rely potentially on both funding sources therefore appear to be limited to the ones in which $\theta>\delta$ and $(1-t-e)/2>\delta/(\delta+1)$. Again a number of cases have to be distinguished. There will generally be a trade-off between the maximum liquidity generation and the ability to avoid losses, under the optimal use of the two funding sources. For example, the maximum generation of liquidity is achieved through fire sales only, and will be equal to $\theta/(1+\theta)$. However, this also leads to the highest possible fire sales losses $1/(1+\theta)$, and it is realistic to assume that this extent of losses would exceed equity, and anyway if all of the assets of the bank are sold, it has ceased to exist. The lowest generation of liquidity is achieved if all assets are pledged for central bank credit, and in this case liquidity generation is $\delta/(1+\delta)$ and fire sale losses are 0. Between these two extreme pairs of liquidity generation and fire sale losses, the set of efficient combinations of the two variables can be calculated. The following proposition addresses the question whether the bank's strategy should foresee to fire sale the most liquid assets and pledge the rest with the central bank, or the other way round.

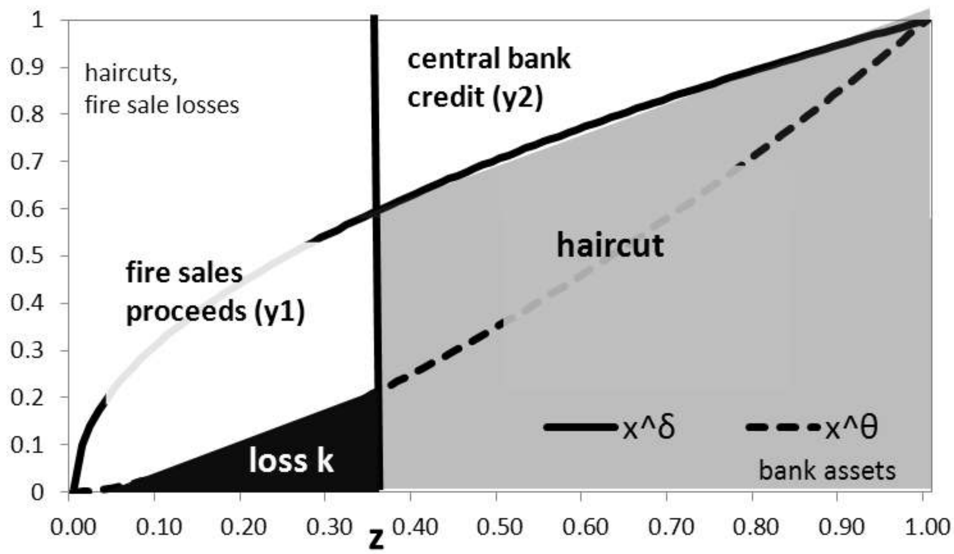
Proposition 4: In funding strategies to address withdrawals of short term deposits relying on both funding sources, the bank should always foresee to fire sale the most liquid assets and pledge the rest as collateral with the central bank (and not the other way round).

The proof of this proposition is provided in the annex. The proof relies on showing that with the strategy to fire sale the most liquid assets and pledge the rest, the bank can achieve combinations of liquidity generation and fire sale cost, which are always superior to the combinations under the reverse strategy. The following Proposition 5 provides the condition in the case of strategies relying on both funding sources for a SNNR equilibrium, depending on the initial liability structure of the bank and the parameters θ and δ.

Proposition 5: Let $z \in [0,1]$ determine which share of its assets is foreseen by the bank to be used for fire sales (i.e. the less liquid share 1-z of assets are foreseen for pledging with the central bank). Let k=h(z) be the fire sale losses from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z). Then a SNNR equilibrium exists if and only if $\exists\, z \in [0,1]$: $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1-t-e)/2$ and $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1} \leq e$.

The proof of proposition 5 is provided in the annex. Figure 4 illustrates the generation of liquidity and fire sale losses under strategy z. The figure reflects that the bank plans to fire sale the most liquid part of its assets z, and pledge with the central bank the least liquid part of assets (1-z). Therefore, total liquidity y that could be generated corresponds to the sum of y_1, the surface above the fire sale loss curve x^θ up to z, and y_2, the surface above the haircut curve x^δ, starting at z. Fire sale losses k will be equal to the surface below the fire sale loss curve between 0 and z.

Figure 4: Use of bank assets according to strategy z (y_1 is liquidity and k the loss generated from fire-selling the most liquid assets z; y_2 is the liquidity generated from pledging with the central bank the least liquid assets (1-z))



The following proposition 6 describes the nature of the combinations of y and k that can be achieved by varying z between 0 and 1. This proposition will be the basis for finding an optimal liability structure of the bank as discussed in the subsequent two sections. The optimal strategy z is determined by the idea to start from the least liquid assets and pledge with the central bank everything as collateral until one needs to switch in order to achieve the necessary total liquidity y. One should switch as late as possible such as to minimise fire sale losses. If one never switches then as it was assumed (to achieve a true mixed case in terms of emergency liquidity sources) that (1-t-e)/2>δ/(δ+1) one will not get enough liquidity. If one switches too early one does not minimise fire sale losses and hence one needs more equity to sustain the strategy.

Proposition 6: Let k=h(z) be the fire sales from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z), whereby h(0)=0, h(1)= 1/(1+θ) and dh/dz >0 and f(0)= δ/(δ+1), f(1)= θ/(1+θ) and df/dz < 0. Then there is a non-decreasing function k=g(y): [0, θ/(1+θ)] → [0, 1/(1+θ)] with:

$$\forall\ y \in \left[0, \frac{\delta}{\delta+1}\right]: g(y) = 0\ ; \qquad \forall\ y \in \left[\frac{\delta}{\delta+1}, \frac{\theta}{1+\theta}\right]: \frac{dg}{dy} > 0; \qquad g\left(\frac{\theta}{1+\theta}\right) = \frac{1}{1+\theta}.$$

In the proof of proposition 5, it was shown that the total liquidity generated by strategy z was $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1}$. This is a monotonously declining function since the second term is smaller than the third term and the third term grows faster than the second term. It therefore can be inverted into the function $z=f^{-1}(y)$. It had also been shown in the proof of proposition 5 that the total of fire sales k resulting from strategy z, are: $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1}$. Again, this function is monotonously increasing and can be inverted z = $h^{-1}(k)$= $\left((\theta+1)k\right)^{\frac{1}{\theta+1}}$. Inserting this into y=f(z) generates the monotonous and invertible relationship between liquidity provision and fire sales:

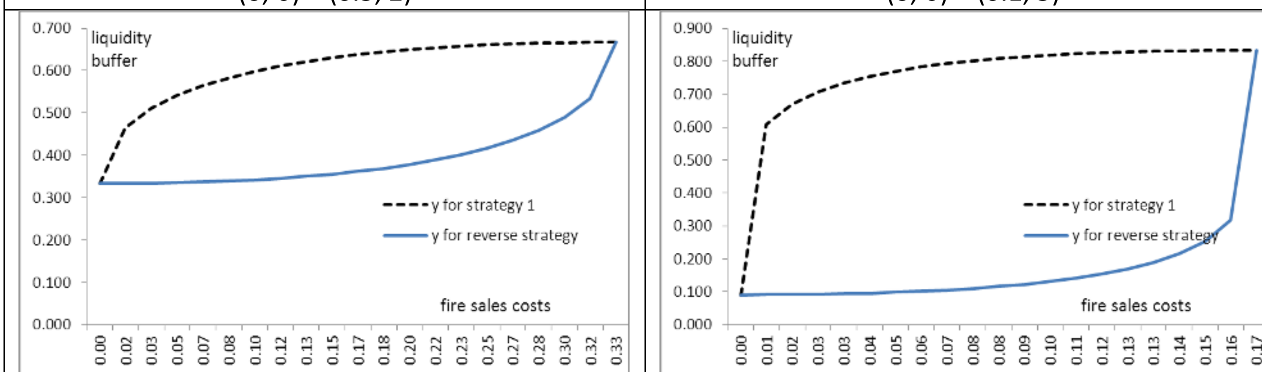
$$y = g^{-1}(k) = \frac{\delta}{\delta+1} + \frac{\left(\left((\theta+1)k\right)^{\frac{1}{\theta+1}}\right)^{\delta+1}}{\delta+1} - k \tag{4}$$

The rest of the statements made in proposition 6 can be shown easily by inserting the relevant parameter values in the functional forms derived.

The following figure shows the liquidity possibility sets under the two strategies (i.e. strategy to fire sale the most liquid assets, and pledging the rest, and the reverse order) for (δ, θ) = (0.5, 2), (0.1, 5). The horizontal axis contains the fire sale costs k, while the vertical-axis maps the liquidity provision. This also illustrates proposition 4 in the sense that indeed the reverse strategy is dominated.

Figure 5: liquidity generation / fire sales trade-offs for the efficient and the reverse strategy and for two alternative parameter combinations

(δ, θ) = (0.5, 2)	(δ, θ) = (0.1, 5)



5. Stable funding structure with the lowest possible cost

In the previous sections, it was assumed that the initial bank balance sheet was given, and the conditions for stability of short term funding were established. It was shown that depending on its liability mix, the haircut (δ) and asset liquidity (θ), bank funding was stable or not. This section will make the liability structure endogenous in a very simple setting. In a full equilibrium model, one would also need to model household preferences and the decisions on investment projects. For the purposes of this paper, one can simplify and assume instead that different liabilities require different remuneration rates but are at these rates perfectly elastic.[7] This is a plausible assumption in normal times (in crisis times, it may no

[7] Although this is not done here, one could also apply the model to endogenize the liquidity parameters of banks' assets. One could imagine that the bank has a "production function" to improve asset liquidity, and that this function can be expressed as the cost to increase δ, θ. The investment into improved asset transparency / liquidity would be v, and one could assume two functions δ(v) and θ(v) with δ(0)= δ0 and θ(0)= θ0 and dδ/dv >0 and dδ/dv < 0, as well as dθ/dv >0 and dθ/dv < 0. Ways to improve the liquidity of assets are for instance to (i) standardise assets (e.g. by standardising claims at origination); (ii) originate claims only to standard and transparent projects (foregoing the higher return properties of idiosyncratic, very information intense projects) (ii) securitise assets; (iii) develop information systems that capture asset characteristics and risk factors; (iv) allow for third party asset

longer hold, and indeed the model foresees that in crisis times access to short term funding is completely lost). In this setting, total bank funding cost will be a proxy for the ability of banks to deliver maturity transformation (as the bank asset composition in our model is given), which is an essential contribution of banking to society.

For given, deterministic δ and θ, competing banks will always go to the limit in terms of the cheapest possible liability structure as determined by the conditions in the strategic depositor game, such that the no-run equilibrium is still maintained as SNNR equilibrium. Assume that the cost of remuneration of the three asset types are r_e for equity, r_t for term funding, and 0 for short term deposits. Also assume that $r_e > r_t > 0$, and that θ > δ. What will in this setting be the composition of the banks' liabilities? The objective of choosing a liability composition will be to minimize the average overall remuneration rate subject to maintaining a stable short term funding basis. One strategy could be to aim at δ/(δ+1) ≥ (1-e-t)/2, such that fire sales will not be needed at all as backstop. If fire sales are not needed, then term funding is superior to equity and equity will be set to zero, i.e. liabilities will consist only in term funding t and in the two short term deposits (1-t)/2. Therefore the condition for stable short term funding will be δ/(δ+1) ≥ (1-t)/2 => t* = 1-2δ/(δ+1). The average remuneration rate of bank funding would be t* r_t. A second strategy would be to rely only on the fire sales approach but to hold the necessary equity. This would mean that the two minimum conditions to be fulfilled are θ/(1+θ) = (1-t-e)/2 and e =$((1-t-e)/2)^{(1+\theta)}$ /(1+θ). These conditions can be solved for a unique optimum t* and e*, and hence for the average necessary remuneration rate of bank liabilities t* r_t + e* r_e.

The general problem of optimal liquidity management is to *minimise through the choice of* $t \in [0,1], e \in [0,1], t+e \in [0,1]$ $z \in [0,1]$ *the average remuneration rate of the banks' liabilities t*r_t + e*r_e, subject to the conditions* $\frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1-t-e)/2$ *and* $\frac{z^{(\theta+1)}}{\theta+1} \leq e.$

Below some illustrative results of this optimisation problem are presented for different values of the liquidity and haircut parameter. Table 3 varies the collateral framework of the central bank as captured through the parameter δ. All other exogenous variables are kept constant (the assumption of a shadow cost of equity of 10% is in line with e.g. Ashcraft et al, 2011, 5). The table reveals that in this example, the *less* restrictive the central bank collateral framework (i.e. the higher δ):

- The higher the equilibrium share of short term funding
- The lower the equilibrium share of long term funding
- The lower the equilibrium share of equity
- The lower the equilibrium ratio between equity and term funding
- The more limited the potential role of asset fire sales relative to central bank credit
- The lower the funding costs of banks and hence the lower, in competitive equilibrium, the costs of bank funding to the real economy.

Normally, one would assume that the central bank collateral framework is designed on the basis of considerations outside the present model (in particular risk protection). However, the results above illustrate that whatever the reasons for the design of the framework, the framework will matter for (i) the equilibrium liability structure of banks, (ii) the role of central bank funding in case of a bank run; (iii) the likelihood of negative asset fire sales externalities in case of a bank run; (iv) the effective monetary conditions; (v) the efficiency of bank intermediation as measured by funding costs of the real economy.

review exercises; etc. This idea, that would be an ingredient of a full equilibrium model, is not taken further here, and hence the focus in the rest of the paper is only on the cheapest stable liability structure.

In sum, the central bank collateral framework seems to be important not only for the state of the banking system, but also for monetary policy and optimal banking regulation.

One may also note that a sudden non-anticipated tightening of the collateral framework can cause a bank run as it may repeal the fulfilment of the conditions for an SNNR equilibrium. The triggering of bank runs by central banks who suddenly limit the borrowing potential of banks is illustrated for example by Bagehot (1873) and King, (1936) for the 19th century Bank of England, or by Priester (1932), for the Reichsbank decision of 13 July 1931.

Table 3: Impact of central bank collateral framework on banks' liability structure and cost

Exogenous parameters					
δ	**0.01**	**0.1**	**0.2**	**0.5**	**1**
Θ	1				
r_t	2%				
r_e	10%				
Results					
t	0.39	0.39	0.38	0.29	0.00
e	0.05	0.04	0.03	0.01	0.00
Implied short term funding (1-t-e)	0.56	0.57	0.59	0.70	1.00
Share of assets foreseen for fire sales (z)	0.33	0.29	0.25	0.11	0.00
Refinancing costs of bank	1.32%	1.21%	1.08%	0.64%	0.00%

Table 4 varies the parameter capturing asset liquidity. Asset liquidity is likely to vary over time with the liquidity cycle, and will deteriorate in particular in a liquidity crisis. Moreover, the liquidity of assets may change structurally over time with changes of market infrastructure and IT systems, asset standardisation, new securitisation techniques etc. In this example, the higher θ, i.e. the better the asset liquidity:

- The higher the equilibrium share of short term funding
- The lower the equilibrium share of long term funding
- The lower the funding costs of banks and hence the lower, in competitive equilibrium, the costs of bank credit to the real economy.[8]

Interestingly, the equilibrium share of equity first increases, and then decreases again. Also the share of assets foreseen for fire sales first increases and then decreases again. The increase may be seen positive if one would like to see independence from the central bank. However, it could also create problems if it is tested in combination with a crisis related deterioration of asset liquidity, as it may lead to particular asset fire sales dynamics.

The non-monotonous behaviour of the equilibrium equity share and the share of assets foreseen for fire sales shows how complex, even in a simple model, the relationship between the various variables can be, and hence how careful one needs to be in interpreting developments and designing a regulatory

[8] This can be understood as a pro-cyclical element. In a downturn, when credit risk and information asymmetries increase, asset liquidity will decrease and will thus lead to an increase of bank equilibrium funding costs, even when the concrete occurrence of a bank run can be avoided as the bank could adjust its funding structure sufficiently early.

framework. Table 4 also confirms that a decline in asset liquidity can reduce the sustainable size of short term funding (and hence trigger a bank run).

Table 4: Impact of asset liquidity on banks' liability structure and cost

Exogenous parameters						
δ	0.1					
θ	**0.4**	**0.7**	**1**	**1.5**	**2**	**4**
r_t	2%					
r_e	10%					
Results						
t	0.79	0.60	0.39	0.11	0.00	0.00
e	0.00	0.03	0.04	0.06	0.04	0.01
Implied short term funding (1-t-e)	0.21	0.37	0.57	0.84	0.96	0.99
Share of assets foreseen for fire sales (z)	0.03	0.16	0.30	0.46	0.51	0.49
Refinancing costs of bank	1.62%	1.47%	1.21%	0.78%	0.43%	0.05%

6. The impact of regulation on the liability structure and on funding cost

Now the model and bank optimization problem developed in the previous section is used to understand the effects of minimum capital and an LCR type liquidity regulation. Table 5 shows the impact of various levels of minimum capital requirements (simple minimum levels of e) starting again from the standard parameter set. It may be noted that without capital requirements, the equilibrium equity level is 0.04, and hence only capital requirements above 0.04 are binding. In this example, the higher the capital requirement:

- The lower the equilibrium share of long term funding
- The higher z, the role of asset fire sales relative to the role of central bank lending. This may be considered positive if one would like to see independence from the central bank, but it may be seen as negative because of higher fire sale externalities.

The equilibrium share of short term funding increases when equity levels are pushed higher due to capital adequacy requirements. However of course it starts to fall again once long term funding has been completely crowded out and hence the increase of equity must be at the expense of the share of short term funding.

Table 5: Impact of minimum capital requirements on banks' liability structure and cost

Exogenous parameters					
Δ	**0.1**				
Θ	1				
r_t	2%				
r_e	10%				
Minimum capital requirements	**0.00**	**0.05**	**0.10**	**0.20**	**0.30**
Optimization parameters					
T	0.39	0.36	0.17	0.00	0.00
E	0.04	0.05	0.10	0.20	0.30
Implied short term funding (1-t-e)	0.57	0.59	0.73	0.80	0.70
Share of assets foreseen for fire sales (z)	0.29	0.32	0.45	0.52	0.52
Refinancing costs of bank	1.21%	1.21%	1.34%	2.00%	3.00%

To interpret the impact of minimum liquidity requirements in the form of the LCR, the concept of HQLA needs to be specified within the model. One can for instance define that an HQLA is an asset for which fire sale losses are not higher than 10%. For the chosen case θ=1 this means that the first 10% of assets are HQLAs. Therefore, an LCR requirement of 0.25 means that short term funding can be up to 0.4, an LCR requirement of 1 means short term funding of a maximum of 0.1 etc. In fact, in the assumed case with a given asset composition, the LCR becomes equivalent to a regulation that sets a maximum share of short term liabilities. Table 6 varies the level of the minimum LCR.

Table 6: Impact of LCR requirement on bank liability structure and cost

Exogenous parameters					
δ	**0,1**				
θ	1				
r_t	2%				
r_e	10%				
Minimum LCR	**-**	**0.25**	**0.50**	**1**	**2**
Implied maximum of short term funding	**1**	**0.40**	**0.20**	**0.10**	**0.06**
Optimization parameters					
t	0.39	0.59	0.80	0.90	0.95
e	0.04	0.01	0.00	0.00	0.00
Implied short term funding (1-t-e)	0.57	0.40	0.20	0.10	0.05
Share of assets foreseen for fire sales (z)	0.29	0.16	0.02	0.00	0.00
Refinancing costs of bank	1.21%	1.30%	1.60%	1.80%	1.90%

The liquidity regulation is binding already with an LCR requirement of 0.25. Tightening the LCR requirement (i.e. increasing the minimum LCR = reducing the maximum share of short term assets) has the following effects:

- It increases the equilibrium share of long term funding
- It reduces the equilibrium level of equity (absent binding capital adequacy regulation)
- It decreases z, the role of fire sales relative to the role of central bank pledging of assets
- It increases refinancing costs of banks (and hence, in a competitive equilibrium of the banking system, the refinancing costs of the real economy)

The counterintuitive result that liquidity regulation strengthens the role of potential reliance on central bank funding relative to the potential role of fire sale can be interpreted as follows: since liquidity regulation for a given set of assets leads only to an increased share of long term borrowing, the total need for liquidity buffers decreases relative to the available buffers, and therefore to save on the most expensive type of liabilities, namely equity, the bank can rely more and more on central bank credit alone as source of liquidity.

To revisit more generally the rationale of liquidity regulation requires obviously going beyond the equilibrium impact as described above. The impact on the equilibrium liability structure and on bank funding costs in itself is relevant in assessing the overall merits of regulation, but is not sufficient to justify regulation. What the equilibrium model presented above does not really capture is how the liquidity buffers foreseen to sustain short term funding are actually tested. In equilibrium, they would according to the model never be tested, and hence in fact no liquidity regulation would be needed, allowing for the lowest cost of bank intermediation for a given θ, δ.

One approach to justify regulation would be some ad hoc assumption on the existence of exogenous *non-anticipated shocks* that will lead to an actual testing of buffers. Assume for instance that once in a while, asset liquidity as captured by θ suddenly declines, and neither banks nor the short term creditors can anticipate this for some reason. If the decline of θ is sufficiently strong, it can change the strategic game between the two short term bank creditors in a way to lead to a bank run and socially undesirable fire sale losses. *Alternatively*, and maybe more convincingly, one may assume that banks anticipate shocks to the liquidity parameters including the fact that these shocks can lead to bank runs and trigger costly asset fire sales. For instance, if the maximum stable short term funding that can be sustained for a given combination of θ, δ is $s^*(\theta, \delta)$, then banks would instead choose $s^\# < s^*$, whereby the exact $s^\#$ would be based on an optimization calculus in view of the probability distribution assigned to future values of θ, δ. However, the problem would still be that banks would not factor in the negative externalities of fire sales, or the dislike of public authorities regarding large recourse to central banks. Through liquidity regulation, an even lower $s^\Diamond < s^\#$ could be imposed, that would reduce the probability of bank runs, fire sales and large central bank reliance even further and towards the social optimum.

Consider the following example: Assume that normally θ=1 and δ=0.1, and that r_e =10% and r_t =2% and r_s =0. As shown above, the representative bank in competitive equilibrium will then have t=0.39, e=0.04, s=0.57 z=0.29 r= 1.21%. Now assume that, in any period, with probability of 1%, θ declines suddenly for one period to 0.4, implying that the sustainable short term funding is only 0.21. Assume that indeed in these case a bank run starts leading to bank default and liquidation, and hence to costs equal to 1/(1+0.4)=0.71. Knowing this probability and the consequences, the bank could decide to always choose a liability structure such as to sustain stable short term funding even for the periods in which θ declines to 0.4. As shown, this leads to refinancing costs of 1,62%, i.e. 0.41% higher. The risk neutral bank will thus compare the 0.41% total higher funding costs with the 71% asset value destruction which occurs with probability of 1%, and will chose the more expensive liability structure that always sustains stable short term funding, i.e. the social optimum will prevail anyway, without banking regulation. If however the sudden deterioration of asset fire sales liquidity to 0.4 occurs only with probability of 0.5%, the calculus of the representative bank in competitive equilibrium leads it to choose the cheaper funding structure and to accept that once in a while bank runs will occur with bank failures. If there are additional social cost (negative externalities) of bank failures, the decisions taken by the representative bank will no longer necessary be socially optimal. If for example negative externalities of default occur which are as big as the direct damage to the bank assets, then, from a social perspective it would be desirable that the bank still chooses the more solid liquidity structure. In this case there is a rationale for liquidity regulation.[9] Of course, this does not prove the optimality of liquidity regulation as defined by Basel Committee (2013). Regulation can take a different form, as illustrated by e.g. Perotti and Suarez (2011).

Beyond this very simple example, a number of model sophistications should be considered to allow for a more qualified analysis of regulation. For example, one could assume: (i) continuous probability distributions for future values of the liquidity parameters θ and δ; (ii) that each depositor receives every

[9] Note that in this example, the losses that occur in case of default need to be allocated to the different liability holders according to the seniority of claims, and the different liability holders would require ex ante a credit risk premium relative to the basic interest rates as assumed, such that, assuming risk neutrality, all creditors of the bank are duly compensated. This does not change the eventual economic problem and the issue of externality that can justify liquidity regulation.

period a noisy independent signal on θ, which leads sometimes to the run by only one depositor, sometimes to a run by both, and mostly to a run of none; (iii) that also the central bank collateral framework can change over time, maybe with δ and θ being correlated (in fact pro-cyclical elements of collateral frameworks, such as marking-to-market or rating dependence of eligibility and haircuts, will lead to a decline of δ in a crisis).

7. The central bank collateral framework as a policy tool?

7.1 The role of the collateral framework for financial stability and monetary policy

During the financial crisis, central banks have taken a wide range of collateral measures to increase the potential recourse to central bank credit (see e.g. Markets Committee, 2009, ECB, 2013). While some measures were launched as soon as 2007 and intensified in 2008, others were taken only recently and could be interpreted as relating to the reaching of the zero lower bound, such as the Funding for Lending scheme of the Bank of England (Chrum and Radia, 2012).

As the essential purpose of the central bank collateral framework is risk protection, the observed collateral policy measures raise the question what exactly the intentions of the central banks have been to widen collateral availability (and hence potential central bank recourse) in particular in a context of deteriorating asset liquidity. The standard explanation provided by central banks and taken up in the critical perception of regulators as referred to in section 1 has been that this increase allowed the banks to substitute for the impossibility to roll over short term funding. The model proposed in this paper allows for new interpretations of collateral widening as a policy measures:

- First, when θ declines suddenly, increasing δ can be a way to preserve the no-run equilibrium (the SNNR) and thereby can be a necessary condition to *prevent* increases in central bank reliance, fire sales, and/or defaults. In this sense, it benefits all banks and financial stability in general, and not only those banks who already experience an actual run. Moreover, it may be noted that the model provides support to Bagehot's (1873) "inertia principle" according to which the central bank should not tighten its collateral framework in a financial crisis as a reaction to the deterioration of asset liquidity: "If it is known that the Bank of England is freely advancing on what in ordinary times is reckoned a good security—on what is then commonly pledged and easily convertible—the alarm of the solvent merchants and bankers will be stayed…" Lowering δ when anyway θ declines would mean to decrease particularly strongly the amount of sustainable short term funding and thereby to maximize the probability of a destabilization of bank funding, contributing, instead of preventing, large central bank recourse and fire sales of assets.
- Second, assuming that a deterioration of θ can be anticipated as a crisis is building up, one could imagine in principle *that banks can adjust their liability structure in time*. If this would indeed be possible, it is however likely that it would come at very high costs because in such a context also investors will have a strong preference for short term assets and the collective attempt of all banks to increase the maturity of their liabilities will therefore lead to a strong increase of bank funding costs (and hence to bank lending rate). This would be rather pro-cyclical, and an adjustment of the collateral framework parameter δ could be seen as a policy tool to prevent such a steep increase of funding costs.
- Third, while the effect described in the previous bullet point could at least in theory also be addressed by conventional monetary policy, i.e. a lowering of the central bank credit operations rate, this has limits as far as the zero lower bound is reached. When this limit is reached, then a widening of collateral availability may become relevant as an alternative approach to lowering effective bank funding costs or at least moderate their increase.

In this sense, the present paper contributes to the research agenda as set for instance by Woodford (2010) or Friedman (2013). Friedman (2013, 7) proposes a four-equation New Keynesian "work horse" model which in particular introduces one new equation with the role "to introduce explicitly the relationship between the policy interest rate that the central bank sets and 'the' interest rate (a metaphor for a whole constellation of interest rates) that affects the spending decisions of households and firms." Similarly, Woodford (2010, 29) notes that "instead of directly lending to ultimate borrowers themselves, savers fund intermediaries, who use these funds to lend to (or acquire financial claims on) the ultimate borrowers. Then, it becomes necessary to distinguish between the interest rate i^s (the rate paid to savers) at which intermediaries are able to fund themselves and the interest rate i^b (the borrowing or loan rate) at which ultimate borrowers are able to finance additional current expenditure…. What determines the equilibrium relationship between the two interest rates i^s and i^b?" The present paper contributes to this research program by showing how the central bank collateral framework impacts not only on financial stability, but also (and without necessarily any actual recourse to the central bank) on the average funding costs of the banking system, and hence, for given operational costs, on the lending rates of banks towards the real economy.

7.2 How to reconcile the policy role of collateral with its original aim to protect the central bank from financial risks?

A key question is how to reconcile the policy considerations above, which seem to suggest an increase of δ in a financial crisis, with the original goal of the collateral framework to control financial risk taking of the central bank. Three interpretations appear to be possible.

First, it has been argued that in a liquidity crisis, risk parameters are endogenous to central bank action, and therefore a broadening of the collateral availability through a lowering of haircuts could under some circumstances decreases the risk taking of the central bank and reduce its eventual losses (and vice versa, a tightening of the collateral framework could increase eventual central bank losses). As Bagehot (1873) formulated it, "only the brave plan" would be the "safe plan" for the Bank of England (see Bindseil and Jablecki, 2013, for a model of this conjecture).

Second, it could be argued that the central bank has multiple objectives, and that the collateral framework impacts on several of its objectives. Under some circumstances, namely when the zero lower bound (ZLB) to nominal interest rates has been reached, the central bank does no longer have enough tools to achieve all goals perfectly. In such a case, minimizing some convex loss function with regard to missing the various goals may lead the central bank to accept not achieving its normal risk control objectives through the collateral framework. For example, a central bank may have calibrated its collateral framework in a way that it is protected, in case of a counterparty default, with 99% probability against incurring a loss when liquidating the collateral portfolio. Then, under extraordinary circumstances, and taking into account the other effects of the collateral framework in achieving its goals (such as price stability and financial stability), it may accept to lower this confidence level to 95%.

Third, it could be argued that in fact a central bank can increase collateral availability in a crisis without changing the parameter δ and therefore without changing risk taking through the following approach: pre-crisis, it can declare a part of the less liquid collateral as simply non-eligible (i.e. apply a 100% haircut, above the haircut that would result from the power function with a certain δ), but remove this partial "collateral deactivation", without changing δ, when a liquidity crisis materializes. This approach, which seems to describe fairly well what many central banks have been doing during the financial crisis, has the following advantages. First, often the de-activated assets are inconvenient to use as central bank collateral, as they will often be of lower credit quality (implying the need of a more intensive due diligence work on the side of the central bank), and/or will not have the form of securities and hence their transfer as collateral will not be convenient and maybe will be subject to legal risks. Second, in view of the high haircuts they deserve to achieve ex post risk equivalence across collateral types, the relatively high administrative cost is even higher in relation to the central bank funding potentially provided. Third, keeping assets ineligible in normal times, but making them potentially eligible in times of non-anticipated crisis, provides leeway to expand collateral availability in a crisis without changing the basic risk control parameter of the central bank.

Suppose for example that a central bank considers that as a matter of principle, it will not accept assets as eligible in normal times for which haircuts deserve to be higher than 50%. Call w the asset share at which the central bank cuts off eligibility. Then w is easily obtained as $w^{\delta} = 0.5 <=> w = 0.5^{\left(\frac{1}{\delta}\right)}$. For example, if δ=0.5, then w=25%. In this case, the maximum liquidity that can be obtained from the central bank is $0.25 - \frac{0.25^{\delta+1}}{1+\delta} = 0.1667$. The same amount of potential central bank recourse has been deactivated, i.e. the collateral buffer that the central bank could activate by accepting in a crisis all assets would be $\frac{\delta}{1+\delta} - 0.1667 = 0.1667$. In other words, the central bank could double in this case central bank refinancing power from 0.1667 to 0.333 without giving up risk equivalence. When modelling the ex-ante liquidity risk management strategy of the bank, and setting an optimal strategy encompassing fire sales and central bank reliance, then, obviously one would need to assume that the counterparty would not anticipate the relaxation (otherwise, in the model it would be as if from the start the entire assets would be eligible). The ad hoc assumption that the banks do not anticipate the relaxation could appear adequate in view of the rareness of liquidity crisis and the constructive ambiguity that central banks apply ex ante to their crisis measures. It is possible to calculate through the optimisation problem to determine z (the split of asset fire sales and central bank recourse in the liquidity strategy of the bank) under the central banks' strategy to declare ineligible all assets beyond a certain w in [0,1] or beyond a certain level of haircuts. Then, it is possible to calculate the critical value $\theta^{\#}$ relative to the initial θ that can still be accommodated by the central bank making all assets eligible and hence adding an extra liquidity buffer. The idea is illustrated in the following figure. Suppose for instance that with θ=2, δ=0.5, a maximum haircut for eligible assets of 0.5 and hence w=25%, the optimal liquidity management strategy (for some assumed costs of the different liabilities) would be characterised by z = 0.15. Then, in case that due to a liquidity crisis, asset liquidity deteriorates such that θ'=0.7 implies a change of fire sale related liquidity buffer of $\frac{0.15^{2+1}}{2+1} - \left(\frac{0.15^{0.7+1}}{0.7+1}\right) = -0.0223$. This loss can be compensated by the central bank by making all assets eligible, and hence the central bank can prevent a destabilisation of deposits without putting into question its risk control parameter δ=0.5.

One can distinguish three methods of central bank to implement a framework to set a constraint in the form of w, while accepting collateral beyond w under some specific conditions which should ensure that this additional collateral is not fully factored in by banks.

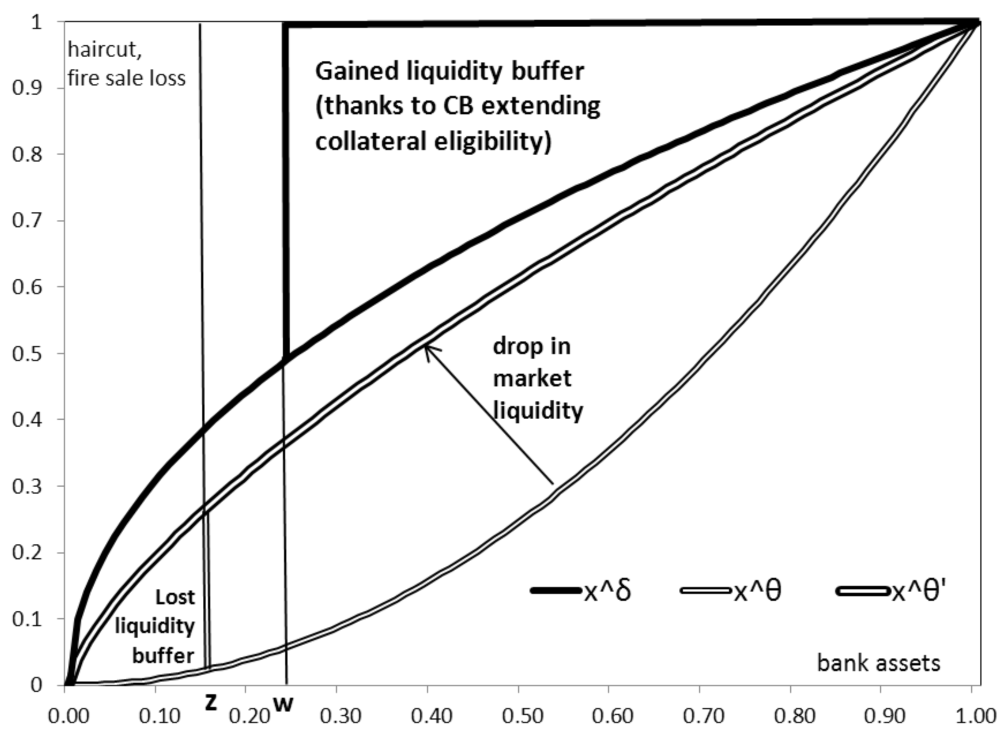
- *Lifting (or at least increase) of w for all banks for use in normal central bank credit operations,* in case of a systemic deterioration of θ. This is what e.g. the ECB announced on 15 October 2008 (See press release "Measures to further expand the collateral framework and enhance the provision of liquidity") and also many other central banks did in 2007/2008. Such a measure can *avoid* additional recourse to the central bank as it can contribute to maintain the no-run equilibrium for short term bank funding.
- Granting *"Emergency liquidity assistance" (ELA)* to a specific bank, for instance if specifically its assets were hit by a decrease of θ. Under ELA, an individual bank is granted an exception in so

far as it obtains additional central bank credit against collateral that is not eligible for the rest of the banks. This is however *not a right* to the bank (but a favor that the central bank may grant), and a higher interest rate is charged to the bank for ELA. Moreover, the bank will be asked to explain in detail what led to its problems and how it will reduce again its ELA needs to zero.

- *Segregated collateral sets in normal times*. Both the US Fed and the Bank of England apply segregated collateral sets for regular credit operations (a narrow set) and a broader set (i.e. one beyond w) for special credit facilities like the discount window. Again, access to the special credit facilities will be charged with a higher interest rates (see e.g. Bank of England, 2011). Compared to ELA, this approach to foresee a wider collateral set is rule based. It is not a substitute for the possibility to provide ELA, suggesting that while ELA foresees the possibility to pledge really the entire balance sheet of the bank, this approach foresees only a relative widening of w towards w'<1 .

The approach to set a threshold for eligible collateral at w, but to foresee a use of the collateral beyond w according to one or several of the approaches mentioned above, may allow avoiding negative externalities of liquidity crises *if it is not factored in by banks in their liquidity management* (see also Chapman et al 2011 for a similar insight). The more central banks have difficulties to achieve this, the more merits some form of liquidity regulation will have.

Figure 6: Loss of liquidity buffers reflecting deterioration of asset liquidity parameter θ=2 → θ'=0.7, more than compensated by gain of liquidity buffer through widening of eligible collateral (i.e. dropping w=25%; with unchanged δ= 0.5).



8. Conclusion

This paper provided a model of the interaction between the liquidity of bank assets, the central bank collateral framework and liquidity regulation, and of the effects of these factors on financial stability and the ability of the banking system to deliver maturity transformation, which is one of its key functions. It was assumed that liquidity and the collateral treatment of assets can be approximated by power functions, and that banks in a competitive equilibrium choose the cheapest possible liability structure that ensures stability of short term funding. Stability of short term funding was modeled in a simple way as strategic game of two short term depositors who had the options to keep their deposits with the bank or to "run". After establishing the conditions for stability of short term funding, the effects of the key exogenous variables (asset liquidity, central bank collateral framework, regulation) on the endogenous variables were derived, namely on (i) the liability structure (the mix of short term funding, long term funding, and equity), (ii) the relative reliance of the two emergency funding sources (fire sales, central bank credit), and on (iii) the funding costs of banks, as proxy of the banks' ability to deliver maturity transformation.

The model also illustrated how an extension of collateral availability can be a financial stability policy tool, matching the observation that most central banks in the current financial crisis tended to extend collateral eligibility. The challenge for the central bank is to use the tool to extend collateral buffers ex post in a liquidity crisis, without inviting banks to factor this in ex ante. The model specifically shows that in a crisis, a widening of the central bank collateral set can *prevent* large recourse to central bank credit by banks suffering from a deterioration of asset liquidity (because it preserves investor/depositor confidence, i.e. the no-run equilibrium). In this sense, the paper provides further illustration of Bagehot's (1873) conjecture that only the "brave plan" of the 19th century Bank of England would be a "safe" plan (i.e. by being "brave" and increasing collateral eligibility after an exogenous drop in asset liquidity, the central bank will contribute to preserve the market access of banks, minimize the recourse of stressed banks to central bank credit facilities, and hence be on the "safe" side also in terms of financial exposures). This contrasts with the view of e.g. Sinn and Wollmershäuser (2011, 12), who consider that extended central bank collateral eligibility and more central bank refinancing opportunities in a liquidity crisis necessarily "encourage" more actual central bank reliance.

The model also allowed to identify the impact of asset liquidity and of the central bank collateral framework on funding costs of banks is relevant for *monetary policy* for at least two reasons: first, policy makers need to be aware that a tightening of any of the two emergency liquidity sources needs to be, everything else unchanged, compensated by a lowering of the monetary policy interest rate to maintain unchanged funding costs of the real economy. Second, when the central bank has reached the zero lower bound, and therefore cannot use standard interest rate policies any longer to lower the money rate, it could consider to use its collateral framework to counteract a further increase of actual funding costs of banks (and hence of the real economy depending on banks) which would otherwise result from the deteriorated asset liquidity (because of the implied increase of the minimum share of long term debt and equity). The use of the collateral framework for policy purposes (financial stability and monetary policy) has of course to take place with due consideration to the original purpose of the collateral framework, which is the protection of the central bank.

Finally, the model also provides insights into a number of further policy issues that are currently being debated in particular in the EU (but that are universal banking topics). First, the *treatment of depositors* in case of bank resolution determines in the model the classification of deposits either into short term deposits potentially subject to a run, or "quasi-long term" deposits not subject to a run. If depositors would change their beliefs away from a perceived security of certain types of deposits, then this would put into question the no-run equilibrium, and would lead to a need for banks to adapt their liability structure towards more costly forms of liabilities (long term debt, equity). While this may be appropriate and an adjustment towards a superior financial system, it in any case would constitute a tightening in terms of funding costs of the economy. Second, in case the envisaged *EU Financial Transaction Tax* (European Commission, 2013) is implemented in its initially intended format, it would lead to a reduction of liquidity of securities (i.e. of the most liquid assets of banks). This could, according to the model, tend to destabilize short term bank liabilities, or at least lead to a need of banks to adjust their liability structure towards more long term debt and equity, implying a higher average bank (and hence real economy) funding costs.

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Annex

Proof of proposition 2

Proposition 2: If $\delta = 0$ an SNNR equilibrium exists if and only if $(1-t-e)/2 \leq \theta/(\theta+1)$ and $e \geq ((1-t-e)/2)^{\theta+1}/(\theta+1)$

To prove proposition 2, the cases as shown in figure A1-1 are examined sequentially.

Figure A1-1: Existence of a unique no-run equilibrium case when $\theta \in [0,1], \delta = 0, \ e \in [0,1], \varepsilon > 0$

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Liquidity sufficient for the withdrawal of</td></tr>
<tr><td>(A) Both depositors: $(1-t-e)<\theta/(\theta+1)$</td><td>(B) Only one depositor: $(1-t-e)/2 < \theta/(\theta+1) < (1-t-e)$</td><td>(C) Not even one depositor $\theta/(\theta+1) < (1-t-e)/2$</td></tr>
<tr><td rowspan="2">Capital sufficient for the fire sales losses associated with the withdrawal by</td><td>(i) At least one depositor:
$e >= ((1-t-e)/2)^{\theta+1}/(\theta+1)$</td><td colspan="2">Unique stable no-run equilibrium</td><td rowspan="2">No unique stable no-run equilibrium</td></tr>
<tr><td>(ii) Not even one depositor:
e >= ((1-t-e)/2)θ+1/(θ+1)</td><td colspan="2">No unique stable no-run equilibrium</td></tr>
</table>

The proposition can be verified by again establishing the strategic game pay-offs and verifying conditions $U_1(K_1,K_2) > U_1(R_1,K_2)$ and $U_1(K_1,R_2) > U_1(R_1,R_2)$ for the various cases.

In cases **(Ai)**, depositors never suffer losses and therefore the conditions for the SNNR are verified. Indeed, in this case $U_1(K_1,K_2)= (1-t-e)/2$; $U_1(R_1,K_2)=(1-t-e)/2 -\varepsilon$; $U_1(K_1,R_2)=(1-t-e)/2$; $U_1(R_1,R_2)=(1-t-e)/2 -\varepsilon$.

In the case **(Aii)** depositors face no loss when they both run, but depositor 1 faces a loss if only depositor 2 runs because of the insolvency of the bank and the pari passu ranking of all remaining depositors (short term and long term). Call RR the recovery ratio which is strictly below 1 in this case. The total asset values remaining to satisfy the other short term depositor (who have claims of (1-t-e)/2) and the long term depositors (who have claims of t) will be the liquidation value of assets minus what needed to be already paid out to the withdrawing depositor: θ/(θ+1) - (1-t-e)/2. The recovery ratio RR will therefore be:

$$RR = \frac{\frac{\theta}{\theta+1} - \frac{1-\mathrm{t}-\mathrm{e}}{2}}{\frac{1-t-e}{2}+t} = \frac{\frac{2\theta}{\theta+1} - 1+\mathrm{t}+\mathrm{e}}{1+t-e} \tag{A1-1}$$

We obtain in case (Aii) the following pay-offs: $U_1(K_1,K_2)=(1\text{-}t\text{-}e)/2$; $U_1(R_1,K_2)=(1\text{-}t\text{-}e)/2\text{-}\varepsilon$; $U_1(K_1,R_2)=RR(1\text{-}t\text{-}e)/2$; $U_1(R_1,R_2)=(1\text{-}t\text{-}e)/2\text{-}\varepsilon$. In this case, the condition $U_1(K_1,R_2) > U_1(R_1,R_2)$ is thus not satisfied and the no-run strategy is thus not dominant.

In case **(Bi),** i.e. $(1\text{-}t\text{-}e)/2 < \theta/(\theta+1) < (1\text{-}t\text{-}e)$ and $e \geq ((1\text{-}t\text{-}e)/2)^{\theta+1}/(\theta+1)$, as long as only one depositor runs, default is avoided both from a liquidity and from the solvency perspective. If both depositors run at once, then they will each recover $(\theta/(\theta+1))/2$, but then nothing is left of the company in terms of assets and hence for the remaining deposits the recovery ratio is zero. The pay-offs are $U_1(K_1,K_2)=(1\text{-}t\text{-}e)/2$; $U_1(R_1,K_2)=(1\text{-}t\text{-}e)/2\text{-}\varepsilon$; $U_1(K_1,R_2)=(1\text{-}t\text{-}e)/2$; $U_1(R_1,R_2)=(\theta/(\theta+1))/2$. An SNNR equilibrium applies since the conditions $U_1(K_1,K_2) > U_1(R_1,K_2)$ and $U_1(K_1,R_2) > U_1(R_1,R_2)$ are both fulfilled.

In case **(Bii)**, i.e. $(1\text{-}t\text{-}e)/2 < \theta/(\theta+1) < (1\text{-}t\text{-}e)$ and $e < (((1\text{-}t\text{-}e)/2)^{\theta+1}/(\theta+1)$, if one depositor runs, this is enough to trigger solvency induced default, and hence the second depositor will face losses. The pay-offs are $U_1(K_1,K_2)=(1\text{-}t\text{-}e)/2$; $U_1(R_1,K_2)=(1\text{-}t\text{-}e)/2\text{-}\varepsilon$; $U_1(K_1,R_2)=RR(1\text{-}t\text{-}e)/2$; $U_1(R_1,R_2)=(\theta/(\theta+1))/2$. It is easy to show that $RR(1\text{-}t\text{-}e)/2 < (\theta/(\theta+1))/2$: it follows from the fact that the available total asset liquidation value $\theta/(\theta+1)$ is identical across all default scenarios, and that from the perspective of depositor 1, if only depositor 2 runs not only depositor 2 can recover more than if both run, but also the term depositors will still get their part of the share (they get nothing if both runs). Therefore, again, the condition $U_1(K_1,R_2) > U_1(R_1,R_2)$ is not satisfied and hence there is no SNNR equilibrium.

In case **(C)**, in which not even one depositor can withdraw without triggering liquidity induced default, i.e. $\theta/(\theta+1) < (1\text{-}t\text{-}e)/2$, depositors will face losses whenever one depositor runs. The question is whether the pay-off is better if both run, relative to the case that only the other depositor runs. This is the case since again the total asset liquidation value is the same across all cases, and in case the other depositor runs he is able to withdraw more value for himself, and hence leaves less value for all others. In fact in case (C), the remaining value for others is zero (RR=0) as all assets are sold to satisfy as much as possible the short term depositor(s) who withdraw. The pay-offs are $U_1(K_1,K_2)=(1\text{-}t\text{-}e)/2$; $U_1(R_1,K_2)=(\theta/(\theta+1))$; $U_1(K_1,R_2)=0$; $U_1(R_1,R_2)=(\theta/(\theta+1))/2$. Therefore, in all cases of (C), the condition $U_1(K_1,R_2) > U_1(R_1,R_2)$ is not fulfilled. This completes the verification of proposition 2.

Proof of proposition 4

Proposition 4 states that in funding strategies to address withdrawals of short term deposits relying on both funding sources, the bank should always foresee to fire sale the most liquid assets and pledge the rest. It is shown in the proof of proposition 6 that when applying the strategy to foresee fire-selling the share z of most liquid assets, and to pledge (1-z) of the least liquid assets, one obtains as possibility set the following monotonous and invertible relationship between liquidity provision and fire sales:

$$y = \frac{\delta}{\delta+1} + \frac{((\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} - k \tag{A2-1}$$

One should first derive a similar possibility set for the reverse strategy. The liquidity from fire selling assets (1-z) starting from the least liquid is:

$$y_1 = (1-z) - \int_z^1 x^\theta \, dx = (1-z) - \left(\frac{1^{\theta+1}}{\theta+1} - \frac{z^{\theta+1}}{\theta+1}\right) = 1 - z + \frac{z^{\theta+1}-1}{\theta+1} \tag{A2-2}$$

The liquidity generated from pledging to the central bank assets z starting from the most liquid is:

$$y_2 = z - \int_0^z x^\delta \, dx = z - \frac{z^{(\delta+1)}}{\delta+1}$$

(A2-3)

Total liquidity generated as a function of z is therefore:

$$y = y_1 + y_2 = f(z) = 1 + \frac{z^{\theta+1} - 1}{\theta+1} - \frac{z^{(\delta+1)}}{\delta+1}$$

(A2-4)

Total fire sale costs k resulting from strategy z, k=h(z) are:

$$k = h(z) = \int_z^1 x^\theta \, dx = \frac{1 - z^{(\theta+1)}}{\theta+1}$$

(A2-5)

This function is monotonously declining in z and can be inverted z = h^{-1}(k):

$$z = h^{-1}(k) = (1 - (\theta+1)k)^{\frac{1}{\theta+1}}$$

(A2-6)

Inserting this into y=f(z) generates the monotonous and invertible relationship between liquidity provision and fire sales:

$$y = 1 + \frac{(1-(\theta+1)k)^1 - 1}{\theta+1} - \frac{(1-(\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} = 1 - k - \frac{(1-(\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1}$$

(A2-7)

To prove proposition 4, one now has to show that the possibility set (A2-1) dominates the possibility set (A2-7), or in other words that the for a given liquidity k, the former always generates more y, i.e. that for any $k \in [0, 1/(1+\theta)]$:

$$\frac{\delta}{\delta+1} + \frac{((\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} - k - 1 + k + \frac{(1-(\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} \geq 0$$

(A2-8)

$$<=> \quad \frac{\delta}{\delta+1} - 1 + \frac{((\theta+1)k)^{\frac{\delta+1}{\theta+1}} + (1-(\theta+1)k)^{\frac{\delta+1}{\theta+1}}}{\delta+1} \geq 0$$

(A2-9)

$$<=> \quad ((\theta+1)k)^{\frac{\delta+1}{\theta+1}} + (1-(\theta+1)k)^{\frac{\delta+1}{\theta+1}} \geq 1$$

(A2-10)

Note that $(\theta+1)k \leq 1$ because $k = 1/(\theta+1)$ is the highest possible level of fire sale losses, namely the one obtained if all assets are fire sold. Second, note that the exponent $(\delta+1)/(\theta+1)$ is below one as it was assumed that $\delta < \theta$ (otherwise the trivial case applies that the bank never undertakes any fire sales but always takes recourse to the central bank). So it remains to be shown that for $x\epsilon[0,1]$ and $a\epsilon[0,1]$: $x^a + (1-x)^a \geq 1$. This is obvious since x^a is a concave function in the unity interval.

Proof of Proposition 5

***Proposition 5**: Let $z \in [0,1]$ determine which share of its assets is foreseen by the bank to be used for fire sales (i.e. the less liquid share 1-z of assets are foreseen for pledging with the central bank). Let k=h(z) be the fire sales from fire selling the z most liquid assets and let y=f(z) be the total liquidity generated from fire selling the most liquid assets z and from pledging the least liquid assets (1-z). Then a SNNR equilibrium exists if and only if* $\exists\, z \in [0,1]$: $y = f(z) = \frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1} \geq (1-t-e)/2$ and $k = h(z) = \frac{z^{(\theta+1)}}{\theta+1} \leq e$.

First, it has to be shown that a stable unique no-run equilibrium is achieved when (i) liquidity buffers generated by the envisaged strategy are at least as large as the deposits of one short term depositor *and* (ii) equity buffers are at least as large as to absorb fire sale losses resulting from the strategy. Second, it has to be shown that for a given z, i.e. the share of assets (starting from the most liquid) that is foreseen for fire sales, the liquidity buffers generated are $\frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1}$ and fire sale losses are $\frac{z^{(\theta+1)}}{\theta+1}$. Finally, it has to be shown that if $\nexists\, z \in [0,1]$: $\left(y = f(z) \geq \frac{1-t-e}{2} \cap k = h(z) =\leq e \right)$, then $(U_1(K_1, K_2) \leq U_1(R_1, K_2) \cup U_1(K_1, R_2) > U_1(R_1, R_2)$.

On the first part: Call liquidity buffers available under strategy z y(z) and fire sale losses under strategy z k(z). Assume now that y(z)≥(1-t-e) and e ≥ k(z), then the pay-offs in the strategic game between short term depositors take the following form: $U_1(K_1,K_2)$=(1-t-e)/2; $U_1(R_1,K_2)$=(1-t-e)/2-ε; $U_1(K_1,R_2)$=(1-t-e)/2-ε; $U_1(R_1,R_2)$=(1-t-e)/2-ε. This game fulfils the SNNR equilibrium conditions.

Now assume that (1-t-e)>y(z)≥(1-t-e)/2 and e ≥ k(z) and also assume (as previously) that e<1/(1+θ). It is assumed that the central bank will after the counterparty default fire sell the assets pledged with it immediately, which will generate a value as determined by the parameter θ, applied to the (1-z) less liquid part of the assets of the bank. As θ>δ, the central bank will pay back an amount to the insolvency administrator corresponding to the difference. This pay-back will be used to pay out under pari passu (i) the term depositors; (ii) the remaining claims of the short-term depositors. What is essential here is that the recovery ratio RR on these remaining claims of short term depositors will be below one since it was assumed that e < 1/(1+θ). Therefore, the pay-off to the short term depositor will be y(z)/2 + RR ((1-t-e)/2 –y(z)/2) < (1-t-e)/2- ε. The strategic game then has the following pay-offs: $U_1(K_1,K_2)$=(1-t-e)/2; $U_1(R_1,K_2)$=(1-t-e)/2-ε; $U_1(K_1,R_2)$=(1-t-e)/2-ε; $U_1(R_1,R_2)$= y(z)/2 + RR ((1-t-e)/2 –y(z)/2). This strategic game fulfils the SNNR equilibrium conditions.

Now it has to be shown that for a given z, i.e. the share of assets (starting from the most liquid) that is foreseen for fire sales, the liquidity buffers generated are $\frac{\delta}{\delta+1} + \frac{z^{\delta+1}}{\delta+1} - \frac{z^{(\theta+1)}}{\theta+1}$ and the fire sale losses generated are $\frac{z^{(\theta+1)}}{\theta+1}$. Call y_2 the liquidity that is generated through collateral pledge, and y_1 the liquidity that is generated through fire sales in the strategy described by z. The central bank liquidity from pledging assets (1-z) starting from the least liquid is:

$$y_2 = (1-z) - \int_z^1 x^\delta \, dx = (1-z) - \left(\frac{1^{\delta+1}}{\delta+1} - \frac{z^{\delta+1}}{\delta+1} \right) = \frac{\delta}{\delta+1} - z + \frac{z^{\delta+1}}{\delta+1} \tag{A3-1}$$

The liquidity generated from fire selling assets z starting from the most liquid is:

$$y_1 = z - \int_0^z x^\theta \, dx = z - \frac{z^{(\theta+1)}}{\theta + 1}$$

(A3-2)

Total liquidity generated as a function of z is therefore:

$$y = y_1 + y_2 = f(z) = \frac{\delta}{\delta + 1} + \frac{z^{\delta+1}}{\delta + 1} - \frac{z^{(\theta+1)}}{\theta + 1}$$

(A3-3)

Total fire sale costs k resulting from strategy z, k=h(z) are:

$$k = h(z) = \int_0^z x^\theta \, dx = \frac{z^{(\theta+1)}}{\theta + 1}$$

(A3-4)

Finally, it has to be shown that if $\nexists\, z \in [0,1]$: $\left(y = f(z) \geq \frac{1-t-e}{2} \cap k = h(z) \leq e \right)$, then $(U_1(K_1, K_2) \leq U_1(R_1, K_2) \cup U_1(K_1, R_2) > U_1(R_1, R_2)$. This can be shown in analogy to similar cases shown in the proof of proposition 2.