CENTRAL BANK COMMUNICATION AND EXPECTATIONS STABILIZATION

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ABSTRACT

This paper analyzes the value of communication in the implementation of monetary policy. The central bank is uncertain about the current state of the economy. Households and firms are uncertain about the statistical properties of aggregate variables, including nominal interest rates, and must learn about their dynamics using historical data. Given these uncertainties, when the central bank implements optimal policy, the Taylor principle is not sufficient for macroeconomic stability: for reasonable parameterizations self-fulfilling expectations are possible. To mitigate this instability, three communication strategies are contemplated: i) communicating the precise details of the monetary policy -- that is, the variables and coefficients; ii) communicating only the variables on which monetary policy decisions are conditioned; and iii) communicating the inflation target. The first two strategies restore the Taylor principle as a sufficient condition for stabilizing expectations. In contrast, in economies with persistent shocks, communicating the inflation target fails to protect against expectations driven fluctuations. These results underscore the importance of communicating the systematic component of monetary policy strategy: announcing an inflation target is not enough to stabilize expectations -- one must also announce how this target will be achieved.

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A central bank that is inscrutable gives the markets little or no way to ground these perceptions [about monetary policy] in any underlying reality — thereby opening the door to expectational bubbles that can make the effects of its policies hard to predict. (Blinder, 1999)

1   Introduction

Since the 1990's, central banking practice has shifted from secrecy and opaqueness towards greater transparency about monetary policy strategy and objectives. At the same time, an increasing number of central banks have adopted an inflation targeting framework for monetary policy. One potential benefit from a successful implementation of inflation targeting is the anchoring of expectations, with its stabilizing effect on macroeconomic activity. Failing to anchor expectations might result in undesired fluctuations and economic instability.

Given the role of expectations, a central bank's communication strategy is a crucial ingredient of inflation targeting. Yet despite its importance, relatively little formal analysis in the context of dynamic stochastic general equilibrium models has been done on the mechanisms by which communication might prove beneficial. The analysis here addresses this hiatus. Using a simple model of output gap and inflation determination — of the kind used in many recent analyses of monetary policy — a number of communication strategies are considered which vary the kinds of information the central bank communicates about its monetary policy deliberations.

Motivated by Friedman (1947, 1968), a model is developed in which monetary policy stabilization is conducted in the presence of two informational frictions. First, the central bank has imperfect information about the current state of the economy and must forecast the current inflation rate and output gap when setting the nominal interest rate in any period. Because of this observation lag the central bank responds to information and the state of the economy with a delay: policy is implementable in the sense of McCallum (1999) and Orphanides (2003). Second, households and firms have an incomplete model of the macroeconomy, knowing only their own objectives, constraints and beliefs. Consequently, they do not have a model of how aggregate state variables, including nominal interest rates, are determined. They forecast

exogenous variables relevant to their decision problems by extrapolating from historical patterns in observed data. Such beliefs capture uncertainty about the future path of nominal interest rates that is not present in a rational expectations analysis of the model and creates a delay in the transmission of monetary policy: because beliefs take time to adjust to new information, policy changes affect the macroeconomy only gradually. Under these assumptions, expectations may become unanchored and need not be consistent with the central bank's monetary policy strategy, with adverse consequences for economic stability. These two frictions combined present a challenge for stabilization policy and permit examination of the role of communication in policy design.[1]

Communication is modeled as providing agents with certain types of information about how the central bank determines its nominal interest rate setting. This information serves to simplify agents' forecasting problem and to coordinate expectations about various macroeconomic variables in a desirable way. Worth underscoring is that uncertainty about the path of nominal interest rates is only one of several sources of uncertainty present in this economy. Indeed, households and firms are similarly unsure about how aggregate output and inflation are determined. The central question is whether uncertainty about the determination of interest rates is an especially important source of uncertainty and whether additional knowledge about the future path of nominal interest rates helps anchor expectations, assisting macroeconomic stabilization.[2]

Three communication strategies are considered. In the benchmark strategy the central bank discloses, under full credibility, the policy rule employed to set nominal interest rates. Agents therefore know which variables appear in the policy rule and the precise restriction that holds among these endogenous variables at all points in time in the forecast horizon. An alternative interpretation of this communication strategy is that the bank discloses its forecasts of the entire future path of its policy instrument. A consequence of knowing the

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[1]These frictions also formed the basis of Friedman's (1968) critique of nominal interest rate rules as a means to implement monetary policy. The analysis here evaluates the verity of this claim, building on the seminal analysis of Howitt (1992), and explores the value of communication in macroeconomic stabilization policy.

[2]On a technical level, the analysis is concerned with the question of whether communication assists convergence to the underlying rational expectations equilibrium of the model.

policy rule is that agents need not independently forecast the path of nominal interest rates — it is sufficient to forecast the set of variables upon which nominal interest rates depend. Because this relation is one of the many equilibrium restrictions agents are attempting to learn, by imposing this restriction on their regression model, a more efficient forecast obtains.

The second communication strategy makes available less information. Rather than conveying the precise policy rule, the central bank only announces the set of variables upon which nominal interest rates are conditioned. This strategy might reflect partial central bank credibility or the inability to accurately communicate the complexities of the decision making process: market participants use available data and the information about the policy rule to verify the reaction function used to set the nominal interest rate — see Mishkin (2004).

Finally, motivated by the inflation targeting literature, which emphasizes the potential benefits of announcing an inflation target for anchoring inflation expectations, we explore the advantages of only communicating the central bank's desired average outcomes for inflation, nominal interest rates and the output gap. Here the only information that is communicated is the central bank's commitment to conduct policy in such a way as to achieve the target for inflation on average. No information on how the central bank will achieve this objective is given.

The central results are as follows. First, in the case of no communication, policy rules that implement optimal policy under rational expectations frequently lead to self-fulfilling expectations. An aggressive response to inflation expectations — as adherence to the Taylor principle prescribes — does not guarantee stability. On the contrary, it is likely to further destabilize expectations. Importantly the Taylor principle is not sufficient for stability under learning dynamics in contrast to a rational expectations analysis of the model.

Second, communicating the entire policy decision process — that is, the relevant conditioning variables and policy coefficients — mitigates instability and allows successful implementation of optimal policy by stabilizing expectations. Hence, communicating accurate information about the systematic component of current and future monetary policy decisions anchors expectations and promotes macroeconomic stability. Since our approach to modeling household and firm beliefs represents a small departure from the rational expectations

assumption — indeed this assumption is nested as a special limiting case — this result underscores the value of communication. These stabilization benefits can also be fully captured by a communication strategy that only conveys the set of endogenous variables on which monetary policy decisions are conditioned, as proposed by the second communication strategy. This information, combined with knowledge that nominal interest rates are a linear function of these objects, delivers convergence to rational expectations equilibrium and protects against expectations driven instability.

Furthermore we show the importance of incomplete information for the role of communication. We demonstrate that if the central bank has perfect information about the state of the economy, then communication is not required for expectations stabilization. Indeed, policy conditioned on the current inflation rate and output gap restores the Taylor principle. Because the central bank promptly responds to contemporaneous developments in the economy, large departures of expectations from equilibrium values are prevented. Thus it is the interaction of the two frictions that leads to instability. However, in practice the current state will never be accurately observed, making transparency and communication of monetary policy desirable.

An implication of these results is that it is precisely when the central bank is uncertain about the state that communication is effective. That is, when the central bank faces a difficult prediction problem regarding the state of the economy, the benefits of communication are high. By announcing the monetary policy strategy, the central bank can better control the economy even though the near term evolution of the economy is highly uncertain.

Third, communication strategies that only announce an inflation target and the associated average long-run values of the nominal interest rate and output gap frequently lead to expectations driven instability. In an economy with persistent shocks, the conditions for convergence are identical to those for the benchmark no communication case where these quantities must be learned. Hence, in such economies, communicating the inflation target does little to help anchor expectations.

It is clear then that communication helps by providing information about the systematic component of policy and importantly by giving information on how the central bank intends to

achieve its announced objectives. Credibility about the future conduct of policy matters not only because of the stabilization bias that emerges from a rational expectations equilibrium analysis, as is well known from Kydland and Prescott (1977), but also because it helps protect against departures from rational expectations equilibrium that arise from small expectational errors on the part of households and firms.

Related literature: The analysis builds on an earlier literature commencing with Cukierman and Meltzer (1986) and more recently Faust and Svensson (2001, 2002). These papers consider models in which the central bank has an idiosyncratic employment target which is imperfectly observed by the public. Fluctuations in this target lead to central bank temptation to deviate from pre-announced inflation goals. In this framework, transparency entails costs and benefits. In Cukierman and Meltzer (1986) and Faust and Svensson (2002), when a central bank has a low average inflation bias, lack of transparency helps surprise the public, making monetary policy more effective in stabilizing output fluctuations. The central bank fully understands the private sector's expectations formation mechanism and can exploit it to stabilize output. However, if the central bank has an high inflation bias, increased transparency allows the private sector to observe the employment target with greater precision and therefore raises the costs to the central bank of deviating from its announced objectives. Transparency is therefore desirable as it provides a commitment mechanism.[3]

This literature assumes rational expectations on the part of the central bank and the public. Here we assume that the central bank does not have complete information on private sector expectations formation and therefore cannot manipulate agents' beliefs to its own advantage. We therefore exclude strategic interaction between the central bank and the private sector. Furthermore, in our model, agents have incomplete information about the policy reaction function, unlike the papers above where agents have imperfect information about specific variables that appear in the reaction function.

More recently, a literature has emerged focusing on the question of whether transparency

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[3]Svensson (1999) further argues on the ground of this result that for inflation targeting central banks it is generally desirable to publish detailed information on policy objectives, including forecasts. Such transparency enhances the public's understanding of the monetary policy process and raises the cost to a central bank from deviating from its stated objectives.

of central bank forecasts of exogenous state variables is desirable. In these models, the public correctly understands central bank preferences but has imperfect information about the central bank's forecast of the aggregate state. Building on Morris and Shin (2002), Amato and Shin (2003), Hellwig (2002) and Walsh (2006), among others, show that full transparency about the central bank forecast is not always desirable because private agents may overreact to noisy public signals and under react to more accurate private information. More generally, Geraats (2002) argues that models based on diverse private information often have the property that pronouncements by the central bank may lead to frequent shifts in expectations leading to increased economic volatility. In contrast, Roca (2006) shows that some of these conclusions depend on the postulated objectives of the central bank. Similarly, Svensson (2006) and Woodford (2005) argue that the conclusions of Morris and Shin (2002) depend on implausible parameter assumptions.[4]

Our analysis departs from this literature by analyzing the value of communicating information about current and future nominal interest rate decisions of the central bank. Like Walsh (2006), the present analysis considers a theory of price setting that is consistent with recent New Keynesian analyses of monetary policy. Unlike Walsh, we propose a fully articulated dynamic stochastic general equilibrium model, and, rather than assuming that the central bank and private agents have asymmetric information about the kinds of disturbances that affect the economy, we consider a framework in which these actors have symmetric information about shocks. The asymmetry instead lies in knowledge about how nominal interest rates are determined — that is, monetary policy strategy. This permits a tractable analysis of communication about endogenous decision variables of the central bank — that is the sequence of choices about the path of nominal interest rates — rather than announcements about exogenous state variables.[5]

Finally, the paper is related to Orphanides and Williams (2005) which presents a reduced

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[4]See also Woodford (2005) and Geraats (2002) for a review of the benefits of central bank communication and transparency.

[5]Rudebusch and Williams (forthcoming) present an analysis that is similar in spirit, analyzing the consequences of asymmetric information about future policy actions. One of the contributions of our paper is to build on their analysis by developing microfoundations which imply asymmetric information about the economy.

form model in which announcing the inflation target achieves a better inflation-output trade-off. Because it reduces the amplitude of macroeconomic fluctuations the announcement of the inflation target is welfare enhancing. However, in their model, regardless of whether or not the inflation target is announced, expectations are well anchored: self-fulfilling expectations cannot arise. The improvement in welfare results from agents having a more accurate forecast of future policy decisions. In contrast, this paper presents a model in which self-fulfilling expectations emerge even if the inflation target is announced and credible. However, consistent with Orphanides and Williams (2005), we show that even when expectations are well anchored in the case of announcing an inflation target, variations in expectations lead to greater business cycle fluctuations in response to shocks than in the case where all details of the monetary policy strategy are known to households and firms.

The paper proceeds as follows. Section 2 delineates a simple model of the macroeconomy. Section 3 details private agents' expectations formation and the adopted criterion to assess macroeconomic stability. Section 4 provides foundational results. Section 5 explores the role of communication in stabilization policy. Section 6 provides graphical analysis of the dynamics of expectations under communication and provides some extensions to the core analytical results. Section 7 concludes.

2   A Simple Model

The following section details a simple model of output gap and inflation determination that is similar in spirit to Goodfriend and King (1997), Rotemberg and Woodford (1999) and Svensson and Woodford (2005). A continuum of households face a canonical consumption allocation problem and decide how much to consume of available differentiated goods and how much labor to supply to firms for the production of such goods. A continuum of monopolistically competitive firms produce differentiated goods using labor as the only input and face a price setting problem of the kind proposed by Rotemberg (1982).[6] The major difference is the incorporation of non-rational beliefs. The analysis follows Marcet and Sargent

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[6] An analysis of price setting of the kind proposed by Calvo (1983), as implemented by Yun (1996), would lead to similar conclusions.

(1989a) and Preston (2005), solving for optimal decisions conditional on current beliefs. We abstract from various mechanisms of persistence, such as habit formation, price indexation and inertial monetary policy. This provides sharp, perspicuous analytical results.[7] However, we demonstrate that our conclusions regarding the value of communication in policy design remain pertinent in models with such frictions.

2.1 Microfoundations

Households. Households maximize their intertemporal utility derived from consumption and leisure

$$\hat{E}_{t-1}^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\ln C_T^i - h_T^i \right]$$

subject to the flow budget constraint

$$B_t^i \le R_{t-1} B_{t-1}^i + W_t h_t^i + P_t \Pi_t - P_t C_t^i - T_t^i$$

where B_t^i denotes holdings of the one period riskless bond, R_t denotes the gross interest paid on the bond, W_t the nominal wage, h_t^i labor supplied by household i and T_t^i lump-sum taxes and transfers for household i. Financial markets are assumed to be incomplete and Π_t denotes profits from holding shares in an equal part of each firm. Nominal income in any period t is $P_t Y_t^i = W_t h_t^i + P_t \Pi_t$ and P_t is the aggregate price level defined below. $\hat{E}_t^i$ denote the beliefs at time t held by each household i, which satisfy standard probability laws. Section 3 describes the precise form of these beliefs and the information set available to agents in forming expectations. However, two points are worth noting. First, in forming expectations, households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true. Second, given the assumed conditioning information for expectations formation, consumption plans are made one period

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[7]It is also motivated by Milani (2004) which suggests that a purely forward looking model of this kind with learning dynamics provides a superior characterization of U.S. macroeconomic time series than does a rational expectations model with these persistence mechanisms.

in advance and therefore predetermined.[8] Labor supply decisions are not predetermined and are conditioned on period t information.[9]

Each household consumes a composite good

$$C_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\theta_t - 1}{\theta_t}} \, dj \right]^{\frac{\theta_t}{\theta_t - 1}}$$

which is made of a continuum of differentiated goods, $c_t^i(j)$, each produced by a monopolistically competitive firm j. The elasticity of substitution among differentiated goods, θ_t, is time-varying, with $E[\theta_t] = \theta > 1$. This is a simple way of modeling time-varying mark-ups, introducing a trade-off between inflation and output stabilization relevant to optimal policy design.

A log-linear approximation to the first order conditions of the household problem provides the household Euler equation

$$\hat{C}_t^i = \hat{E}_{t-1} \left[\hat{C}_{t+1}^i - (\hat{\imath}_t - \pi_{t+1}) \right] \tag{1}$$

and the intertemporal budget constraint

$$\hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \omega_{t-1}^i + \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \hat{Y}_T^i \tag{2}$$

where

$$\hat{Y}_t \equiv \ln(Y_t/\bar{Y}); \;\; \hat{C}_t \equiv \ln(C_t/\bar{C}); \;\; \hat{\imath}_t \equiv \ln(R_t/\bar{R}); \;\; \pi_t = \ln(P_t/P_{t-1}) \;\; \text{and} \;\; \omega_t^i = B_t^i/\bar{Y}$$

and $\bar{z}$ denotes the steady state value of any variable z.

Solving the Euler equation recursively backwards, taking expectations at time $t-1$ and substituting into the intertemporal budget constraint gives

$$\hat{C}_t^i = (1 - \beta) \omega_{t-1}^i + \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1 - \beta) \hat{Y}_T^i - \beta (i_T - \pi_{T+1}) \right]. \tag{3}$$

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[8]We consider a model with pricing and spending decisions determined one period in advance so as to put households, firms and policymakers on an identical informational footing. This could similarly be achieved by the alternative assumption that the central bank has a policy reaction function that responds to one period ahead expectations of inflation and agents condition decisions on period t information. All results continue to hold.

[9]This assumption ensures markets clear in equilibrium.

Optimal consumption decisions depend on current wealth and on the expected future path of income and the real interest rate.[10] The optimal allocation rule is analogous to permanent income theory, with differences emerging from allowing variations in the real rate of interest, which can occur either due to variations in the nominal interest rate or inflation. Importantly, nominal interest rates affect consumption demand only through expectations of their future evolution. This property underscores the role of managing expectations in policy design. Note also, that as households become more patient, current consumption demand is more sensitive to expectations about future macroeconomic conditions.

Firms. There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function

$$Y_{j,t} = A_t h_{j,t}$$

where A_t denotes an aggregate technology shock. Each firm chooses a price P_{jt} in order to maximize its expected discounted value of profits

$$\hat{E}_{t-1}^j \sum_{T=t}^{\infty} Q_{t,T} P_T \Pi_{j,T}$$

where

$$\Pi_{j,t} = (1 - \tau) \frac{P_{j,t}}{P_t} Y_{j,t} - \frac{W_t}{P_t} h_{jt} - \frac{\psi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2$$

denotes period profits and the quadratic term the cost of adjusting prices as in Rotemberg (1982).[11] The tax, τ, on revenues is chosen to eliminate the steady state distortion arising from monopolistic competition. Given the incomplete markets assumption it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income

$$Q_{t,T} = \beta^{T-t} \frac{P_t Y_t}{P_T Y_T}$$

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[10]Using the fact that total household income is the sum of dividend and wage income, combined with the first order conditions for labor supply and consumption, delivers a decision rule for consumption that depends only on forecasts of prices: that is, goods prices, nominal interest rates, wages and dividends. However, we make the simplifying assumption that households forecast total income, the sum of dividend payments and wages received.

[11]The results are similar to the case of a Calvo pricing model.

for $T \geq t$.[12]

The intratemporal consumer problem implies aggregate demand for each differentiated good is

$$Y_{jt} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_t} Y_t$$

where Y_t denotes aggregate output and

$$P_t = \left[\int_0^1 (P_{j,t})^{1-\theta_t} \, dj\right]^{\frac{1}{1-\theta_t}}$$

is the associated price index. Summing up, the firm chooses a sequence for $P_{j,t}$ to maximize profits, given the constraint that demand should be satisfied at the posted price, taking as given P_t, Y_t, and W_t. Again, given the information upon which expectations are conditioned, prices are determined one period in advance.

In a symmetric equilibrium, all firms set the same price, so that $P_{j,t} = P_t$. Log-linearizing the first order condition for the optimal price we obtain

$$\hat{P}_t - \hat{P}_{t-1} = \pi_t = \hat{E}_{t-1}^i \beta \pi_{t+1} + \xi \hat{E}_{t-1}^i (\hat{s}_t + \hat{\mu}_t)$$

where, $\hat{P}_t = \log P_t$, $\xi = \theta \bar{Y}/\psi$ is inversely related to the cost of adjusting the prices, $\mu_t = \theta_t (\theta_t - 1)^{-1}$ denotes the mark-up; $\hat{s}_t \equiv \ln(s_t/\bar{s})$ marginal costs defined below; and $\hat{\mu}_t \equiv \ln(\mu_t/\bar{\mu})$. Solving forward and making use of the transversality condition we obtain

$$\hat{P}_t = \hat{P}_{t-1} + \hat{E}_{t-1}^i \sum_{T=t}^{\infty} (\beta)^{T-t} \xi (\hat{s}_T + \hat{\mu}_T) \qquad (4)$$

which states that each firm's current price depends on the expected future path of real marginal costs and cost-push shocks.

The real marginal cost function is

$$S_t = \frac{w_t}{A_t} = \frac{C_t}{A_t}$$

where the second equality comes from the household's labor supply decision. Log-linearizing we obtain

$$\hat{s}_t = \hat{C}_t - \hat{a}_t,$$

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[12]The precise details of this assumption are not important to the ensuing analysis so long as in the log linear approximation future profits are discounted at the rate β^{T-t}.

so that current prices depend on expected future demand and technology. The responsiveness of current prices to changes in expected demand depends on the degree of nominal rigidity. A low degree of nominal rigidity implies a high value of ξ (corresponding to a low value of the cost ψ): in this case firms respond aggressively to changes in perceived demand because price changes are less costly. The opposite occurs in the case of higher costs of price adjustment. The degree of price rigidity plays a key role in the stability analysis.

2.2 Market clearing, efficient output and aggregate dynamics

The model is closed with assumptions on monetary and fiscal policy. The fiscal authority is assumed to follow a zero debt policy in every period t and this is understood to be true by agents.[13] Monetary policy is discussed in detail in the subsequent section. For now it suffices to note that a nominal interest rate rule is implemented. For a more general treatment of the interactions of fiscal and monetary policy under learning dynamics see Eusepi and Preston (2007) and Evans and Honkapohja (2006).

General equilibrium requires that the goods market clears, so that

$$A_t h_t - \frac{\psi}{2} (\Pi_t - 1)^2 = \int C_t dj = C_t. \tag{5}$$

This condition states that output net of adjustment costs is equal to aggregate consumption, determining the equilibrium demand for labor h_t at the wage $w_t = C_t$. This relation satisfies the log-linear approximation

$$\hat{h}_t + \hat{a}_t = \hat{C}_t = \hat{Y}_t.$$

For later purpose it is useful to characterize the *efficient* level of output — the level of output that would occur absent nominal rigidities and distortionary shocks under rational expectations. Under these assumptions, optimal price setting implies the log-linear approximation $E_{t-1}\hat{Y}_t^e = E_{t-1}\hat{a}_t$. Hence predictable movements in the efficient rate of output are entirely determined by the aggregate technology shock. We can use the definition of efficient output to characterize the aggregate dynamics of the economy in terms of deviations from the efficient

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[13]This implies agents do not need to forecast future tax obligations as in the analysis of Eusepi and Preston (2007).

equilibrium. Nominal bonds are also in zero net supply requiring

$$\int_0^1 B_t^i di = 0.$$

Aggregating firm and household decisions, using (3) and (4), provides

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} - (i_T - \pi_{T+1}) + \hat{r}_t^e\right] \tag{6}$$

and

$$\pi_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \xi \left(x_T + \hat{\mu}_T\right) \tag{7}$$

where $\int_0^1 \hat{E}_t^i di = \hat{E}_t$ gives average expectations; $x_t = \hat{Y}_t - E_{t-1}\hat{Y}_t^e$ denotes the log-deviation of output from its expected efficient level; and $\hat{r}_t^e = \left(\hat{Y}_{t+1}^e - \hat{Y}_t^e\right)$ the corresponding efficient rate of interest. The average expectations operator does not satisfy the law of iterated expectations due to the assumption of completely imperfect common knowledge on the part of all households and firms. Because agents do not know the beliefs, objectives and constraints of others in the economy, they cannot infer aggregate probability laws. This is the property of the irreducibility of long horizon forecasts noted by Preston (2005).

2.3   The Monetary Authority

The monetary authority minimizes a standard quadratic loss function under the assumption that agents have rational expectations. This approach follows a now substantial literature on learning dynamics and monetary policy — see Howitt (1992) for the seminal contribution and Bullard and Mitra (2002), Evans and Honkapohja (2003) and Preston (2004, 2006), inter alia, for subsequent contributions — motivated by the question of robustness of standard policy advice to small deviations from the rational expectations assumption. For alternative treatments of policy design that take into account private agent learning see Gaspar, Smets, and Vestin (2005), Molnar and Santoro (2005) and Preston (2004, 2006).

The optimal policy problem is

$$\min E_t \sum_{T=t}^{\infty} \left(\pi_T^2 + \lambda_x x_T^2\right)$$

subject to the constraints

$$x_t = E_{t-1}x_{t+1} - E_{t-1}(i_t - \pi_{t+1} - r_t^e) \tag{8}$$

$$\pi_t = \xi x_t + \beta E_{t-1}\pi_{t+1} + E_{t-1}\hat{\mu}_t \tag{9}$$

which are the model implied aggregate demand and supply equations under rational expectations.[14] Consequently E_t denotes the rational expectations operator. The weight $\lambda_x > 0$ determines the relative priority given to output gap stabilization. A second order accurate approximation to household welfare in this model can be shown to imply a specific value for λ_x. Because this is not central to our conclusions, and because this more general notation permits indexing a broader class of policy rules, we adopt this objective function unless otherwise noted.

The first order condition under optimal discretion is

$$E_{t-1}\pi_t = -\frac{\lambda_x}{\xi}E_{t-1}x_t. \tag{10}$$

Hence optimal policy dictates interest rates to be adjusted so that predictable movements in inflation are negatively related to those in the output gap.[15] This targeting rule combined with the structural relations (8) and (9) can be shown to determine the rational expectations equilibrium paths $\{i_t^*,\ \pi_t^*,\ x_t^*\}$ as linear functions of the exogenous state variables $\{r_{t-1}^e,\ \hat{\mu}_{t-1}\}$. Without loss of generality, and to make the analysis as simple and transparent as possible, we assume that the exogenous processes are determined by

$$r_t^e = \rho_r r_{t-1}^e + \varepsilon_t^r$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_t^\mu$$

where $0 < \rho_r,\ \rho_\mu < 1$ and $(\varepsilon_t^r,\ \varepsilon_t^\mu)$ are independently and identically distributed random variables, with autoregressive coefficients known to households and firms.[16] Under these

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[14]These expressions follow directly from (6) and (7) on noting that $\hat{E}_t$ satisfies the law of iterated expectations under the assumption of rational expectations — households and firms know the objectives, beliefs and constraints of other agents and can therefore determine aggregate probability laws in equilibrium.

[15]Policies under optimal commitment could similarly be analyzed without substantial differences in the conclusions of this paper. However, because such policies introduce history dependence, analytical conditions are somewhat tedious and we therefore take the case of discretion for convenience.

[16]This assumption can be dispensed with without altering results. Because these shocks are exogenous and assumed to be observed by agents, it is immediate that estimating a first order process for each shock will recover the true autoregressive coefficient with probability going to one as the sample size goes to infinity.

assumptions

$$i_t^* = \rho_r r_{t-1}^e + \frac{\rho_\mu \lambda_x + (1-\rho_\mu)\xi}{\xi^2 + \lambda_x(1-\beta\rho_\mu)}\rho_\mu \hat{\mu}_{t-1}$$

delineates the desired state contingent evolution of nominal interest rates required to implement the optimal equilibrium.

Following Svensson and Woodford (2005), rather than adopting the targeting rule (10) directly as the policy rule, we instead assume the central bank implements policy according to the nominal interest rate rule

$$i_t = i_t^* + \phi\left(\hat{E}_{t-1}\pi_t + \frac{\lambda_x}{\xi}\hat{E}_{t-1}x_t\right) \tag{11}$$

where $\phi > 0$. The serves to limit the information that the central bank requires to implement monetary policy. The central bank is assumed to observe private forecasts — through survey data — or to have an identical internal forecasting model. This rule has the property that if beliefs converge to the underlying rational expectations equilibrium then it is consistent with implementing optimal policy under a rational expectations equilibrium. This follows immediately from observing in this case that

$$\hat{E}_{t-1}\pi_t + \frac{\lambda_x}{\xi}\hat{E}_{t-1}x_t = 0$$

which in turn implies $i_t = i_t^*$ as required for optimality under rational expectations. Note also that it nests an expectations based Taylor rule as a special case, albeit with a stochastic constant.[17]

3   Learning and Central bank Communication

This section describes agents' learning behavior and the criterion to assess convergence of beliefs. Agents do not know the true structure of the economic model determining aggregate variables. To forecast state variables relevant to their decision problems, though beyond their control, agents make use of atheoretical regression models. The regression model is assumed to contain the set of variables that appear in the minimum state variable rational expectations

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[17]The stochastic constant is largely irrelevant to the stability analysis under learning dynamics. Also, if the assumption of discretionary optimization is unappealing, then a rule of this form with appropriately defined stochastic constant can implement the optimal equilibrium under commitment — see Preston (2006).

solution to the model. Each period, as additional data become available, agents re-estimate the coefficients of their parametric model.

An immediate implication is that model dynamics are self-referential: the evolution of firm and household beliefs influence the realizations of observed macroeconomic variables. In turn, changes in observed data affect agents' beliefs formation. Learning induces time variation in the data generating process describing inflation, output and nominal interest rates. The central technical question concerns the conditions under which beliefs converge to those that would obtain in the model under rational expectations, in which case the data generating process characterizing the evolution of macroeconomic variables is time invariant. Convergence is assessed using the notion of expectational stability outlined in Evans and Honkapohja (2001).

A more fundamental implication of this self-referential property is that it permits analyzing the role of communication in stabilizing expectations. In a rational expectations analysis, expectations are pinned down by construction of the equilibrium and are necessarily consistent with the adopted policy rule. By analyzing a model that permits beliefs to become unanchored from rational expectations, the value of certain types of information regarding the monetary policy process in stabilizing expectations can be clearly and fruitfully evaluated.

3.1 Forecasting

This section outlines the beliefs of agents in our benchmark analysis in the case of no communication. As additional information is communicated to households and firms, the structure of beliefs will change accordingly. These modifications will be noted as they arise, with an illustrative example given below. The agents' estimated model at date $t-1$ can be expressed as

$$Z_t = \begin{bmatrix} x_t \\ \pi_t \\ i_t \\ \hat{\mu}_t \\ \hat{r}_t^e \end{bmatrix} = \omega_{0,t-1} + \omega_{1,t-1} Z_{t-1} + \bar{e}_t \tag{12}$$

where ω_0 denotes the constant, ω_1 is defined as

$$
\omega_1 = \begin{bmatrix}
\omega_{xx} & \omega_{x\pi} & \omega_{xi} & \omega_{xu} & \omega_{xr} \\
\omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} & \omega_{\pi u} & \omega_{\pi u} \\
\omega_{ix} & \omega_{i\pi} & \omega_{ii} & \omega_{iu} & \omega_{iu} \\
0 & 0 & 0 & \rho_\mu & 0 \\
0 & 0 & 0 & 0 & \rho_r
\end{bmatrix}
$$

and $\bar{e}_t$ represents an i.i.d. estimation error. Agents are assumed to know the autocorrelation coefficients of the shocks but estimate the other parameters (with time subscripts being dropped for convenience). Hence they are attempting to learn the average value of observed macroeconomic data and also a set of slope coefficients describing the reduced form relationship between these macroeconomic objects and fundamental disturbances to the economy.

This paper models communication as information about the dynamics of nominal interest rates. As an example of communication, suppose the central bank credibly announces that monetary policy will be conducted so that inflation, output and nominal interest rates will on average be zero in deviations from steady state. The model implication is that agents know this with certainty and impose this restriction on their regression model. Hence $\omega_{0,t-1} = 0$ and agents need only learn a subset of coefficients relevant to the reduced form dynamics of macroeconomic aggregates. This captures well the idea that communicating characteristics of the monetary policy strategy is an attempt to manage the evolution of expectations.

At the end of period $t-1$ agents form their forecast about the future evolution of the macroeconomic variables given their current beliefs about reduced form dynamics. Given the vector Z_{t-1}, expectations $T+1$ periods ahead are calculated as

$$
\hat{E}_{t-1} Z_{T+1} = (I_5 - \omega_{1,t-1})^{-1} \left(I_5 - \omega_{1,t-1}^{T-t+2} \right) \omega_{0,t-1} + \omega_{1,t-1}^{T-t+2} Z_{t-1}
$$

for each $T > t-1$, where I_5 is a (5×5) identity matrix. To evaluate expectations in the optimal decision rules of households and firms, note that the discounted infinite-horizon

forecasts are

$$\hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} Z_{T+1} = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(I_5 - \omega_{1,t-1})^{-1} \left(I_5 - \omega_{1,t-1}^{T-t+2} \right) \omega_{0,t-1} \right]$$

$$+ \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[\omega_{1,t-1}^{T-t+2} Z_{t-1} \right].$$

This expression can be compactly written as

$$\hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} Z_{T+1} = F_0 \left(\omega_{0,t-1}, \omega_{1,t-1} \right) + F_1 \left(\omega_{1,t-1} \right) Z_{t-1},$$

where

$$F_0 \left(\omega_{0,t-1}, \omega_1 \right) = (I_5 - \omega_{1,t-1})^{-1} \left[(1-\beta)^{-1} I_5 - \omega_{1,t-1}^2 (I_5 - \beta \omega_{1,t-1})^{-1} \right] \omega_{0,t-1}$$

$$F_1 \left(\omega_1 \right) = \omega_{1,t-1}^2 (I_5 - \beta \omega_{1,t-1})^{-1}$$

are, respectively, a (5×1) vector and (5×5) matrix.

3.2   Expectational Stability

Substituting for the expectations in the equations for the output gap, inflation and the nominal interest rate, permits writing aggregate dynamics of the economy as

$$Z_t = \Gamma_0 \left(\omega_{0,t-1}, \omega_{1,t-1} \right) + \Gamma_1 \left(\omega_{1,t-1} \right) Z_{t-1} + \Gamma_2 \bar{\varepsilon}_t \tag{13}$$

with obvious notation and where $\bar{\varepsilon}_t$ is a (5×1) vector of zeros with final two elements ε_t^{μ} and ε_t^r. This expression captures the dependency of observed dynamics on agents' beliefs about the future evolution of the economy. Moreover, it implicitly defines the mapping between agents' beliefs and the actual coefficients describing observed dynamics as

$$T \left(\omega_{0,t-1}, \ \omega_{1,t-1} \right) = \left(\Gamma_0 \left(\omega_{0,t-1}, \omega_{1,t-1} \right), \ \Gamma_1 \left(\omega_{1,t-1} \right) \right).$$

A rational expectations equilibrium is a fixed point of this mapping. For such rational expectations equilibria we are interested in asking under what conditions does an economy with learning dynamics converge to each equilibrium. Using stochastic approximation methods, Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions

for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d\left(\omega_0, \omega_1\right)}{d\tau} = T\left(\omega_0, \omega_1\right) - \left(\omega_0, \omega_1\right), \tag{14}$$

where τ denotes notional time. The rational expectations equilibrium is said to be expectationally stable, or E-Stable, when agents use recursive least squares if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[18]

4 Preliminary Foundations

This section provides the benchmark theoretical results of the paper. The model properties under both rational expectations and learning dynamics without communication are stated. The analysis of various communication strategies in the implementation of monetary policy is then explored in Section 5.

4.1 Benchmark Properties

To ground the analysis, and provide a well known comparative benchmark, the stability properties of the model under rational expectations can be summarized as follows.

Proposition 1 *Under rational expectations, the model given by equations (6), (7) and (11) has a unique bounded solution if $\phi > 1$.*

This is an example of the Taylor principle. If nominal interest rates are adjusted sufficiently to ensure appropriate variation in the real rate of interest, then expectations are well anchored. This feature along with other robustness properties noted by Levin, Wieland, and Williams (2003) and Batini and Haldane (1999), have lead to advocacy of forecast-based instrument rules for the implementation of monetary policy. Indeed, such policy rules appear in a number of central bank forecasting models — for instance, the Bank of Canada and the Bank of England. Furthermore, Clarida, Gali and Gertler (1998, 2000) adduce empirical evidence

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[18]Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D\left[T\left(\omega_0, \omega_1\right) - \left(\omega_0, \omega_1\right)\right]$ have negative real parts (where D denotes the differentiation operator and the Jacobian understood to be evaluated at the relevant rational expectations equilibrium).

for such interest rate reaction functions. Under learning dynamics the model has strikingly different predictions for the evolution of household and firm expectations.

Proposition 2 *Consider the economy under learning dynamics where the central bank does not communicate the policy rule and $\phi > 1$.*

1. *The REE is unstable under learning provided*

$$(1 + \phi) > \phi(1 - \beta)\frac{\lambda_x}{\xi^2} + \frac{\psi(\beta)}{\xi}$$

   where $\psi(\beta) > 0$, $\lim_{\beta \to 1} \psi(\beta) = 0$ and $\lim_{\beta \to 0} \psi(\beta) = \infty$. Hence:

2. *If $\beta \to 1$, then the REE is unstable under learning for every ξ and ϕ.*

3. *If $\beta \to 0$, then the REE is stable under learning for every ξ and ϕ.*

For many reasonable parameter values, the optimal policy under rational expectations cannot be implemented with learning and no communication, rendering the economy prone to self-fulfilling expectations. Indeed, standard parameterizations invariably take the household's discount rate to be near unity. In the limit $\beta \to 1$, instability occurs for all parameter values, underscoring the importance of stabilizing long-term expectations. Conversely, as β becomes small, $\psi(\beta)$ becomes unboundedly large, guaranteeing stability of the equilibrium. Intuitively, as β increases current consumption plans become more sensitive to expectations, and a correct prediction of the future path of the nominal interest rate, together with predictions about the output gap and inflation, becomes crucial for stability. Analogously, as the degree of nominal rigidity declines, goods prices become more sensitive to expectations about future marginal cost conditions. Indeed, as $\xi \to \infty$, the flexible price limit, instability obtains for all parameter values.

To give further insight to this result, consider the evolution of household beliefs in response to an inflation shock. To characterize beliefs, we study the associated ordinary differential equation of the E-Stability mapping. Figure 1 plots the local dynamics of the agents' estimates of $\omega_{x,r}$, $\omega_{\pi,r}$ and $\omega_{i,r}$ — the estimated slope coefficients on the efficient rate disturbance. Given a sufficiently large sample of data, the evolution of these belief coefficients are arbitrarily well described by the linear ordinary differential equation

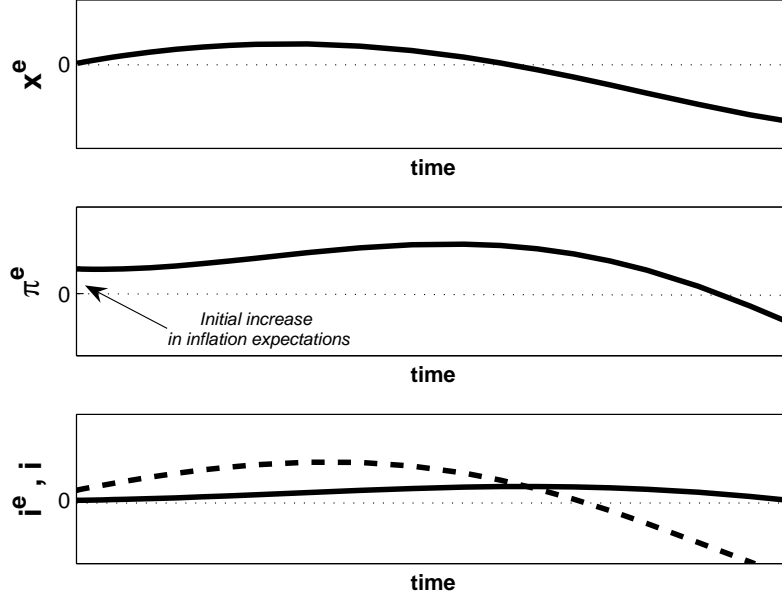$$\dot{\omega}_1 = (J^* - I_3)\,\omega_1$$

20

Figure 1: Dynamics of unanchored expectations

where

$$\omega_1 = \left(\begin{array}{ccc} \omega_{xr} & \omega_{\pi r} & \omega_{ir} \end{array} \right)'$$

and J^* is Jacobian of $T(\omega_0, \omega_1)$. This represents the first order dynamics of the ODE (14) whose eigenvalues determine E-Stability properties — see the appendix for further details. The economy is assumed to be initially in the deterministic steady state (with no shocks occurring in the simulation). We then perturb the beliefs of private agents, making the initial estimate of the inflation coefficient, $\omega_{\pi r}$, higher than its rational expectations value. This can be interpreted as an increase in inflation expectations or equivalently an expectational error on the part of agents. The model is calibrated with $\beta = 0.99$, $\theta = 10$, $\xi = 0.13$, $\phi = 2$ and $\rho_\mu = \rho_r = 0.9$.

The solid lines represent expectations of each macroeconomic aggregate, while the dotted line gives the level of the nominal interest rate. The expectational shock engenders higher output and inflation, and gives rise to higher expectations of both these variables. The central bank's policy response is initially small, strengthening with a delay because of imperfect

information about the state of the economy. When an increase in the nominal interest rate does occur, it fails to curb the increase in expected and actual inflation because aggregate demand only depends on predictable changes in nominal interest rates, and it takes time for expectations to adjust. Private agents fail to correctly anticipate the future policy stance so that the initial increase in the policy instrument has limited effect on aggregate activity (real interest rates decline), in turn validating initial expectations of higher inflation. While not plotted, expectations about inflation, output and nominal interest rates diverge from their rational expectations values with time. Section 5 further explores the dynamics of belief formation under various assumptions about the degree of communication.

Two additional points are worth noting. First, under reasonable parameterizations an increase in ϕ renders the equilibrium less stable — for example if $\xi > (1 - \beta)$ and $\lambda_x < 1$. Moreover, a central bank that does not communicate has incentives to be less aggressive to inflation and more to output. As an example, a policy rule with $\phi < 1$ and λ_x sufficiently high will yield stability under learning. While this result may seem counterintuitive, it is a product of consumption decisions being predetermined and agents having an incorrect model of interest rates. Nominal interest rates only affect demand through their predictable movements. Changes in interest rates in period t cannot affect the current level of aggregate demand. By responding too aggressively, the central bank can lead to households forming conditional forecasts of future nominal interest rates that promote instability.[19] This finding contrasts with Ferrero (2004) and Orphanides and Williams (2005) which argue that under learning policy should be more aggressive in response to inflation. The difference in conclusion stems from the central bank's knowledge of the state of the economy and the ability to manipulate current demand through appropriate choice of the contemporaneous interest rate. In the present analysis, in absence of central bank communication the latter is not possible, and because the central bank has imperfect information about the current state it may have reason to be cautious.

Why is λ_x important for expectations stabilization? Recall that prices depend on the expected sequence of output gaps into the indefinite future. As output gap expectations

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[19]This observation explains the difference from the stability results for a similar class of policies discussed in Preston (2006).

increase, prices move accordingly, affecting *future* inflation expectations. Thus the expected output gap becomes a better indicator of future inflation expectations. By responding to expected output gap the central bank can 'move ahead' of inflation expectations, preventing instability.

Second, and related, the observation that policies giving greater weight to output gap stabilization are less likely to be prone to instability has relevance for recent debate on the merits of simple policy rules. For example, Schmitt-Grohe and Uribe (2005) demonstrate in a medium-scale model of the kind developed by Smets and Wouters (2002), that optimal monetary policy can be well approximated by a simple nominal interest rate rule that responds to contemporaneous observations of inflation. Moreover, policies that respond to the output gap are undesirable, since over-estimating the optimal elasticity by even small amounts can lead to a sharp deterioration in household welfare. What the above result demonstrates is that, in a world characterized by small departures from rational expectations, the policymaker may face a trade-off: strong responses to the output gap may reduce welfare, but they may protect against even more deleterious consequences from self-fulfilling expectations.

4.2 Eliminating Policy Delays

This striking instability result naturally raises the question of how can expectations be managed more effectively in the pursuit of macroeconomic stabilization. The model has two key information frictions. First, the central bank responds to information about the true state of the economy with a delay. This is an implication of the forecast-based monetary policy rule. Second, households and firms have an incomplete model of the macroeconomy and need to learn about the reduced-form dynamics of aggregate prices. It follows that agents are faced with statistical uncertainty about the true data generating process describing the evolution of nominal interest rates. Resolving these informational frictions may mitigate expectations driven instability.

In regards to the policymaker's uncertainty, suppose the central bank has perfect information about current inflation and the output gap. It can then implement the policy rule

$$i_t = i_t^* + \phi \left(\pi_t + \frac{\lambda_x}{\xi} x_t \right) \tag{15}$$

23

which is closer in spirit to the policy proposed by Taylor (1993). The following result obtains.

Proposition 3 *Consider the economy under learning dynamics and $\phi > 1$. If the central bank implements monetary policy with the rule (15) without communication then expectational stability obtains for all parameter values under maintained assumptions.*

Hence timely information about the state of the economy is invaluable to expectations stabilization. By responding to contemporaneous observations of the inflation rate and the output gap the Taylor principle is restored. Having perfect information about the aggregate state reduces the delay in the adjustment of monetary policy, allowing the central bank to anticipate shifts in expectations. Responding to changes in inflation in a timely fashion prevents large deviations from the rational expectations equilibrium. Comparing this result to proposition 2 underscores that instability stems from the interaction between the two sources of information frictions in the model. Given that central banks are unlikely in practice to have complete information about the current state of the economy, it is worth considering other approaches to effective management of expectations. The remainder of the paper therefore explores the role of communication.

5   The Value of Communication

Communication is modelled in a very direct and simple way. Under learning dynamics, households and firms are uncertain about the true data generating process characterizing the future path of nominal interest rates, the output gap and inflation. We can therefore ask what kinds of information about the monetary policy strategy assist in reducing the forecast uncertainty that emerges from having a misspecified model. Hence the developed framework permits a direct analysis of the benefits of communication in managing expectations.

Three communication strategies are considered. First, the central bank announces the precise details of its monetary policy, including both the variables upon which interest rate decisions are conditioned and all relevant policy coefficients. Second, the central bank communicates only the variables upon which policy decisions are conditioned. Third, the central bank communicates its inflation target. These strategies successively reduce the information

made available to the public and provide insight as to what kinds of information are conducive to macroeconomic stabilization.

5.1 Strategy 1

This communication strategy discloses all details of the monetary policy decision process. The central bank announces the precise reaction function used to determine the nominal interest rate path as a function of expectations. Agents know which variables appear in the policy rule and all relevant coefficients. Hence, agents need not forecast the path of nominal interest rates independently — they need only forecast the set of variables upon which nominal interest rates depend. An alternative, but equivalent strategy, is the central bank announces in every policy cycle t its conditional forecast path for the nominal interest rate, $\{E_{t-1}i_T\}_{T \geq t}$. Such a communication strategy might arguably characterize current practice by the Norges Bank and the Reserve Bank of New Zealand — see Norges Bank (2006). These forecasts can be used directly by the private sector in making spending and pricing decisions. Since they are by construction consistent with the adopted policy rule, if agents base decisions directly on these announced forecasts, it must be equivalent to households and firms knowing the policy rule and constructing the forecast path of nominal interest rates independently, subject to the caveats now noted.

To keep the analysis as simple as possible, we assume that the private sector and the central bank share the same expectations about the future evolution of the economy. This assumption is dispensable. Analyzing a model in which the central bank communicates its reaction function but in which there is disagreement about the forecasts is feasible though beyond the scope of this paper.[20]

Regardless of how this communication strategy is implemented, we assume that the central bank is *perfectly credible*, in the sense that the public fully incorporates announced information in their forecasts without verification. Issues related to cheap talk, as analyzed by Stein (1989) and Moscarini (2007) for example, are not considered. We assume the central bank is able

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[20]See Honkapohja and Mitra (2005) for an analysis of a New Keynesian model in which only one period ahead forecasts matter and conditions under which heterogeneous forecasts deliver the same stability results. This paper, however, does not study a model which requires agents to forecast nominal interest rates.

to fully communicate its reaction function without noise so the market fully understands its policy goals and strategy, both in the current period and into the indefinite future.

Imposing knowledge of the policy rule on households' and firms' forecasting models — or knowledge of the central bank's conditional forecast path $\{E_{t-1}i_T\}_{T\geq t}$ — is equivalent to substituting this equilibrium restriction into the aggregate demand equation to give

$$x_t = \hat{E}_{t-1} \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)x_{T+1} - (i_T^* + \phi\pi_T + \phi\frac{\lambda}{\xi}x_T - \pi_{T+1}) + \hat{r}_t^e \right].$$

The remaining model equations are unchanged with the exception of beliefs. Since nominal interest rates need not be forecast, an agent's vector autoregression model is estimated on the modified state vector

$$Z_t = \begin{bmatrix} x_t \\ \pi_t \\ \hat{\mu}_t \\ \hat{r}_t^e \end{bmatrix}.$$

Under these assumptions, uncertainty about the model concerns only the laws of motion for inflation and output, which are affected by other factors of the model beyond monetary policy decisions. Hence perfect knowledge about the central bank's policy framework does not guarantee that market participants fully understand the true model of the economy, since agents continue to face uncertainty about the objectives and constraints of other households and firms in the economy. However, it does tighten the connection between the projected paths for inflation and nominal interest rates. This property proves fundamental.

Proposition 4 *Assume the bank communicates under perfect credibility the interest rate forecast $\left\{E_{t-1}^{CB} i_T\right\}_{T=t}^{\infty}$ or, equivalently, the policy rule (11) and $\phi > 1$. Then the REE is stable for all parameter values under maintained assumptions.*

Communication of the policy rule completely mitigates instability under learning dynamics — even though the central bank and the private sector have incomplete information about the state of the economy. Indeed, given proposition 3, it is clear that communication has value precisely in circumstances where the central bank is uncertain about the current state. The result shows how communicating the reaction function helps shape beliefs about future policy,

making it possible for agents to anticipate future policy. As an example, suppose inflation expectations increase. Under full communication, agents' conditional forecasts of inflation and nominal interest rates are coordinated according to (11). Agents therefore correctly anticipate that higher inflation leads to a higher path for nominal interest rates — one that is sufficient to raise the projected path of the real interest rate. As a result, output decreases, leading to a decrease in inflation, which in turn mitigates the initial increase in expectations, leading the economy back to equilibrium. In absence of communication, an agents' conditional forecasts for nominal interest rates and inflation give rise to projected falls in future real interest rates, generating instability by validating the initial increase in inflation expectations. Section 5 discusses further intuition of how communication stabilizes expectations.

5.2 Strategy 2

Now suppose the central bank only announces the set of variables relevant to monetary policy deliberations so that agents do not know the precise restriction that holds between nominal interest rates, inflation and the output gap. Furthermore, suppose that while agents do not know the policy coefficients, they do know that nominal interest rates are set according to a linear function of these variables. By limiting knowledge of private agents about the monetary policy process relative to the benchmark full-information analysis several aspects of central bank communication can be captured. First, uncertainty about parameters and forecasts can be interpreted as a constraint on the communication ability of the central bank. This reflects the fact that the policy decision is the outcome of a complex process, the details of which are often too costly to communicate — see Mishkin (2004). Second, the central bank might face credibility issues, leading the private sector to want to verify announced policies. Third, complete announcement might not be the optimal choice for the central bank, given the agent's learning process.[21]

This partial information about the policy process can be incorporated by households and firms in the following two-step forecasting model. First, using the history of available data,

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[21] A discussion of the optimal policy under learning is left for further research.

agents run a regression of nominal interest rates on expected inflation and the output gap

$$i_t = \psi_{0,t-1} + \psi_{\pi,t-1}\hat{E}_{t-1}\pi_t + \psi_{x,t-1}\hat{E}_{t-1}x_t + e_t.$$

This yields estimates of the coefficients of the policy rule.[22] Notice that, as shown in the appendix, we consider the more general case where the regression is estimated using a recursive instrumental variable method, allowing for the possibility that private sector and central bank forecasts are different or that the central bank can communicate its expectations with noise.

As a second step, given these estimates, agents proceed in the same manner as strategy 1: they forecast the future paths of the output gap and inflation rate and then use the estimated policy rule to construct a set of nominal interest rate forecasts.

Proposition 5 *If households and firms understand the variables upon which nominal interest rate decisions are conditioned and $\phi > 1$, then the REE is stable under learning for all parameter values under maintained assumptions.*

Thus the central bank need not disclose all details of the monetary policy strategy. It is sufficient that information be given regarding the endogenous variables relevant to the determination of policy and the functional form of the rule — but not its parameterization. Credible public pronouncements of this kind, combined with a sufficient history of data, provide agents with adequate information to verify the implemented rule. And despite the estimation uncertainty attached to the policy coefficients, local to the rational expectations equilibrium of interest, expectations are nonetheless well anchored relative to the no communication case.[23] Indeed, this communication strategy is equally useful in protecting against instability from expectations formation as strategy 1 in which agents know the true policy coefficients. Of course, the out-of-equilibrium dynamics would differ across these two strategies — the estimation uncertainty being relevant to the true data generating process of macroeconomic

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[22]There is an important subtlety in specifying this regression. We assume that private agents include a fixed constant and do not explicitly allow for a stochastic constant as in (11). Hence the regression is misspecified, though the misspecifiction vanishes as $\rho_r, \rho_\mu \to 0$. This assumption avoids multicollinearity problems in the case of convergent learning dynamics, given the presence of only two shocks. An alternative approach, that yields that same results, is to add an additional shock to the model and allow for a stochastic.

[23]Formally, this means that agents cannot hold initial beliefs about the policy cofficients that are too different from the true values. Analyzing this possibility would require a global analysis of the model which is well beyond the scope of this paper.

variables — which in turn has welfare implications. Analyzing such implications is beyond the scope of this paper.

5.3   Strategy 3

Over the past two decades numerous countries have adopted inflation targeting as a framework for implementing monetary policy. A central part of this monetary policy strategy has been the clear articulation of a numerical target for inflation. As a final exercise, we consider a communication strategy that conveys not only the desired average outcome for inflation — that is the inflation target — but also the associated values for nominal interest rates and the output gap. Given that our analysis is in deviations from steady state, these three values are clearly zero. As discussed in section 3, given this knowledge agents no longer need to estimate a constant in their regression model, leading to more accurate forecasts of the future path of nominal interest rates.

Proposition 6 *Assume the central bank communicates only the inflation target $\bar{\pi} = 0$ and the associated values for the output gap and nominal interest rates, $\bar{x} = \bar{\imath} = 0$.*

1. *Define $\rho = \max\left(\rho_\mu, \rho_r\right)$ and let $\rho \to 1$. Then the REE is unstable under learning if (17) holds;*

2. *Let $\beta \to 1$. Then the REE is unstable under learning if*

$$\left(\phi + 2\rho\right) > 2\frac{\phi\lambda}{\xi^2}\left(1 - \rho\right) + \frac{\tilde{\psi}\left(\rho\right)}{\xi}, \tag{16}$$

where $\tilde{\psi}\left(\rho\right) > 0$, $\tilde{\psi}\left(1\right) = 0$ and $\tilde{\psi}\left(0\right) = \infty$.

Economies subject to persistent shocks may be prone to expectations driven instability. Indeed, the stability conditions for the no communication case obtain for cost-push or efficient rate disturbance processes having roots near unity. This result nicely demonstrates a fundamental insight of rational expectations analysis: it is not enough to announce an inflation target — one must also announce how one will achieve this target.[24] Only by providing

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[24]Note that if only the inflation target was announced without declaring the associated values of the long-run interest rate and output gap targets, then the stability properties can only be worse since agents must learn a greater number of coefficients.
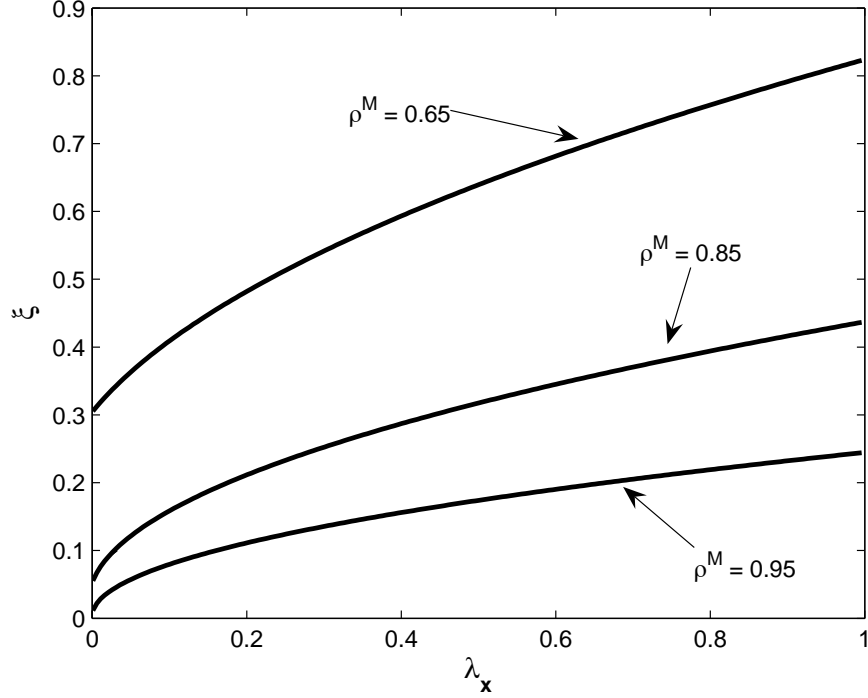
Figure 2: Announcing the inflation target is not enough

information regarding the systematic component of monetary policy can expectations be effectively managed when shocks are persistent. In contrast, as $\rho \to 0$, so that shocks have no serial correlation, there is similarly no persistence in macroeconomic aggregates. Information about the systematic component is less important as the economy has no intrinsic dynamics, making household and firm forecasting problems less complex. The result also underscores another difference to a rational expectations analysis of the model: the precise details of how exogenous disturbances are specified matters for expectational stability. This is not true for determinacy of rational expectations equilibrium.

To further interpret this condition a graphical analysis is useful. The model is calibrated s before. Figure 2 plots three contours demarcating stability and instability regions, below and above respectively, as functions of the parameters (ξ, λ_x). Each contour is indexed by the maximum autoregressive coefficient, denoted ρ^M, in the two disturbance processes. It is immediate that as the maximum eigenvalue increases the set of parameter values for which

expectations are stabilized considerably narrows. For a given degree of price stickiness, as the persistence in exogenous disturbance rises a much stronger response to the output gap is required. Similarly, for a given weight on output gap stabilization, only in economies with less flexible prices does learnability of rational expectations equilibrium obtain. Hence, the degree of nominal rigidity in price setting has important implications for stabilization policy under learning dynamics. Economies with greater rigidity tend to be conducive to expectations stabilization — current prices are less sensitive to expectations about future macroeconomic conditions. Because prices move little, inflation expectations display low volatility, in turn promoting macroeconomic stability. This is not a property of the model under rational expectations: expectations are well anchored so long as the Taylor principle is satisfied, regardless of the degree of nominal friction.

6   The Dynamics of Expectations and Extensions

So far the analysis has focused on conditions under which expectations are anchored. The following discussion demonstrates that even when expectations are anchored in the long run, communicating complete details of the monetary policy strategy greatly assists short-run stabilization policy. Hence communication matters not only for expectational stability but macroeconomic stability more generally. We then demonstrate that our insights regarding the value of communication remain pertinent in a more richly specified model that includes a number of frictions.

6.1   The Efficacy of Short-Run Stabilization Policy

Consider first the strategy in which the central bank announces the inflation target $\bar{\pi} = 0$ and $\bar{x} = \bar{\imath} = 0$. Furthermore, suppose that exogenous disturbances have sufficiently weak serial correlation, so that monetary policy induces local stability under learning — the case of nonconvergence in learning dynamics being clearly undesirable for macroeconomic stabilization. Figure 3 shows the effects of an increase in inflation expectations. The plots are generated in the way described in section 4. The two panels are distinguished by plotting the level of the nominal interest rate against inflation expectations in the first and expectations
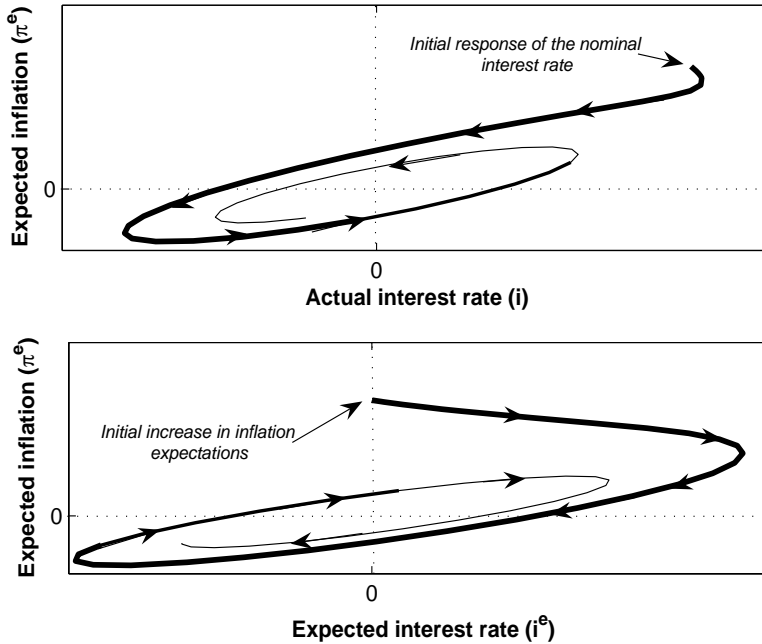
Figure 3: Dynamics under communication of the inflation target

of the nominal interest rate and inflation expectations in the second. The interest rate jumps with the rise in inflation expectations, but fails to have a strong initial impact on inflation expectations because of the absence of communication: market participants fail to anticipate correctly the future path of the policy instrument. The second panel demonstrates why: aggregate demand depends only on expectations of future nominal interest rates and it takes time for these expectations to rise.

Given the weak initial effect on inflation, and therefore on inflation expectations, the central bank keeps increasing the nominal interest rate until inflation expectations start declining. As the response of inflation expectations is inertial, the central bank tends to overtighten. Hence inflation expectations, and as a consequence inflation, keep decreasing until they become negative, overshooting their rational expectations equilibrium values. With low interest rates and low inflation expectations a new cycle starts. The central bank eases its policy stance but expectations react with a delay, leading to excessively low nominal interest rates and high inflation expectations. Agents' beliefs eventually converge though the speed of con-
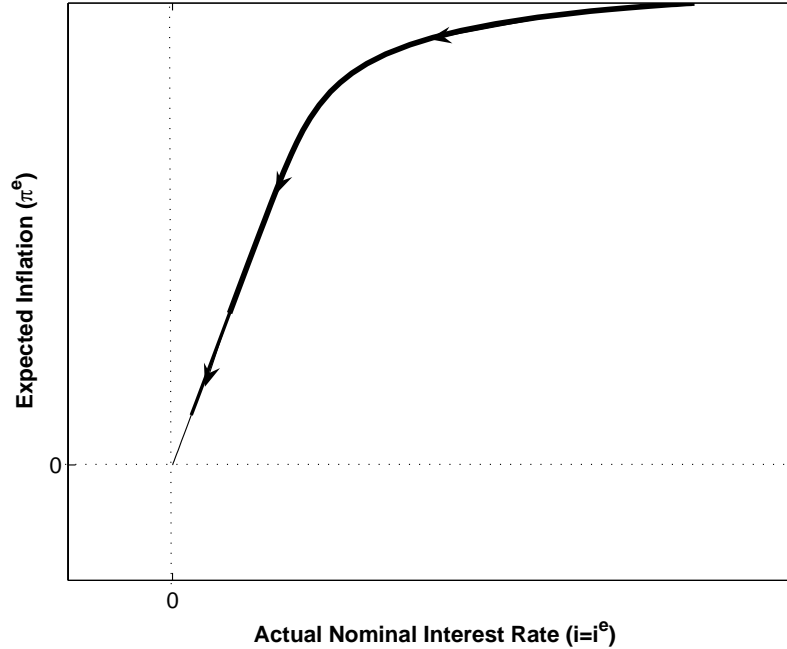
Figure 4: The Benefits of Communication

vergence depends on the chosen parameters. For example, a more aggressive policy towards inflation would magnify the oscillatory convergence back to the equilibrium. The central bank tends to over react to changes in expected inflation, amplifying expansions and recessions.

Now consider an identical analysis under full communication where market participants understand the policy rule and can correctly forecast the future path of the policy instrument. In this case, the actual and expected nominal interest rate are identical. Figure 4 shows that expected inflation and the expected interest rate rise and fall together until they converge back to the equilibrium. There is no overshooting. This is explained by market participants correctly anticipating that the interest rate will be higher in the future in response to higher inflation expectations. The anticipated positive response of the nominal interest rate increases the expected real interest rate with a reduction in output that further reduces inflation expectations. Convergence in beliefs is monotonic — there are no oscillatory dynamics in expectations. This underscores that managing expectations, even in the case of stability under learning dynamics, has stabilization benefits. This is one of the central contributions

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of Orphanides and Williams (2005), though in the context of a reduced form model of the macroeconomy.

The central bank can bring about inflation stabilization without excessive volatility of the policy instrument by fully articulating its monetary policy strategy. In the case of no communication, the central bank is more likely to over-react to changing economic conditions with the result of excessively volatile interest rates and potentially destabilizing effects on expectations.

6.2 Some Robustness Exercises

Two features of the modeling environment might give cause to question the generality of results. First, when implementing monetary policy, the central bank must estimate the current state conditional on period $t - 1$ information. Moreover, because household consumption decisions are predetermined, aggregate demand is insensitive to contemporaneous changes in the nominal interest rate. Second, the model does not include various mechanisms of persistence, such as habit formation, price indexation and inertial monetary policy. Such frictions might lead to a more stable environment if agents' expectations are conditioned on lagged state variables. Indeed, Bullard and Mitra (2000) and Preston (2006) have demonstrated that conditioning policy decisions on lagged nominal interest rates and the price level, respectively, helps promote stability under learning dynamics. This section demonstrates that models with both these features continue to exhibit instability of the kind discussed, and, therefore, that communication continues to be a desirable component of the monetary policy strategy.

Suppose households maximize

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\ln \left(C_T^i - H_T \right) - h_T^i \right]$$

subject to the same constraints specified in section 2. Consumption and labor supply plans are now both conditioned on period t information and consumption is subject to an external habit $H_T = \gamma C_{T-1}^i$, where $0 < \gamma < 1$. Similarly suppose that firms condition prices on current information. Then similar calculations determine aggregate demand and supply as

$$\tilde{x}_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1 - \beta)\tilde{x}_{T+1} - (i_T - \pi_{T+1}) + \hat{r}_t^e \right]$$

and

$$\pi_t = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \xi \left(\tilde{x}_T + \hat{\mu}_T \right)$$

where

$$\tilde{x}_t = x_t - \gamma x_{t-1}.$$

Finally, let monetary policy be determined by a nominal interest rate rule of the form

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[i_t^* + \phi \left(\hat{E}_t \pi_{t+1} + \bar{\lambda}_x \hat{E}_t x_{t+1} \right) \right]$$

for $\bar{\lambda}_x > 0$ so that current interest rates depend on expectations about the future state of the economy. The minimum state variable solution to this model under rational expectations now includes nominal interest rates and the output gap as state variables. Agents' beliefs continue to be given by (12). This closes the generalized model.

The existence of additional state variables might be thought to lead to a more stable economic environment under learning dynamics and obviate the need for communication as a prominent feature of the monetary policy strategy. Because analytical results are difficult, we here resort to a calibration study of the model. We set $\beta = 0.99$ and $\phi = 1.5$. The habit parameter is $\gamma = 0.95$, which is on the high side with respect to the recent empirical literature to emphasize the robustness of our results. Two experiments are conducted. In the first, $\rho_i = 0$ (no interest smoothing), and the combination of $\bar{\lambda}_x$ and ξ that guarantee local stability is analyzed. As in the model of section 2, a high response to output gap is needed to guarantee local stability under learning.[25] The higher ξ, the more flexible prices, and the higher the required response to the output gap. Nonetheless, instability arises for many reasonable parameters values as in the model presented in section 2. In the second experiment we introduce some degree of interest smoothing with $\rho_i = 0.7$. Perhaps surprisingly, interest smoothing increases the set of parameters for which we obtain instability. A higher weight on past interest rates implies a lower weight on the current response to expected inflation and output gap, making the path of the policy instrument flatter and weakening the response to the output gap. These experiments are repeated with date $t-1$ expectations in the monetary

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[25]We consider $\xi \in [0.01, 0.13]$ and $\bar{\lambda}_x \in [0.01, 1]$. To assist interpretation, the former interval would imply Calvo frequency of price adjustments on the interval $[0.7, 0.9]$. For $\xi > 0.13$ (or a Calvo parameter below 0.7) instability occurs for any $\bar{\lambda}_x$ in the interval.

policy rule and similar results obtain. Hence, the economy continues to be prone to instability under no communication when the economy is not purely forward looking, and, given the logic of the analysis in earlier sections, communication strategies of the kind discussed, would be expected to be equally effective in mitigating instability.

As a final remark, consider the case of $\rho = \gamma = \omega = 0$. This delivers the purely forward looking model of section 2, with all decisions and policy now conditioned on time t information. Here analogous analytical conditions are feasible for all cases analyzed in the paper. The basic insights are identical, though there are some minor differences in detail. For instance, the Taylor principle is only sufficient for determinacy of rational expectations equilibrium as too strong response to expected inflation can give rise to indeterminacy.

These exercises, by no means exhaustive, demonstrate that the basic insights of the paper are general. They do not depend on either the precise timing of information – whether expectations are conditioned on time t or $t-1$ information — or on the model being purely forward looking. The fact that both the central bank and private agents face informational problems renders communication a valuable tool in stabilization policy. By coordinating expectations the central bank can better achieve its objectives.

7   Conclusion

This paper develops a dynamic stochastic general equilibrium model for the analysis of the role of communication in a central bank's monetary policy strategy. Three communication strategies are considered when the central bank attempts to implement optimal policy. First, the central bank announces the exact details of its monetary policy decision process. This includes both the variables appearing in its policy rule and the relevant policy coefficients. Second, the central bank discloses only the variables appearing in the policy rule. This limits the information households and firms have relative to the full information case, possibly reflecting imperfect credibility of central bank announcements. Third, the central bank announces only its desired inflation target and associated long-run values of the output gap and nominal interest rates.

The central results are as follows. Under no communication the policy rule fails to sta-

bilize macroeconomic dynamics, promoting expectations driven fluctuations — self-fulfilling expectations are possible. However, by announcing the details of the policy process stability is restored. Communication permits households and firms to construct more accurate forecasts of future macroeconomic conditions, engendering greater stability in observed output, inflation and nominal interest rates.

If instead the central bank only discloses the variables upon which interest rate decisions are condition, stability still obtains for all parameter values. Even though this communication strategy imparts less information about the policy process relative to the full communication case, the resulting estimation uncertainty is small. Hence, agents once again can make more accurate forecasts which is conducive to macroeconomic stabilization.

Finally, if the central bank only announces the desired inflation target, economies with persistent shocks will frequently be prone to expectations driven fluctuations. This makes clear that it is not sufficient to announce desired objectives — one must also announce the systematic component of policy which describes how these objective will be achieved.

A Appendix

A.1 Propositions and Proofs

Proposition 2: Under no communication, the REE is unstable under learning if

$$(1+\phi)\xi > \phi(1-\beta)\frac{\lambda}{\xi} + \psi(\beta) \qquad (17)$$

where $\psi(\beta) > 0, \psi(1) = 0$.

   Proof: The ODE (14) evaluated at the rational expectations equilibrium, can be decomposed in four independent sub-systems. The first includes the three constant terms,

$$\dot{\omega}_0 = \left(J_{\omega_0^*} - I_3\right)\omega_0 \qquad (18)$$

where $J_{\omega_0^*}$ contains a sub-matrix of the Jacobian, evaluated at the REE equilibrium. The second and the third include the coefficients to the exogenous shocks,

$$\dot{\omega}_u = \left(J_{\omega_u^*} - I_3\right)\omega_u \qquad (19)$$

where $\omega_u = \left(\begin{array}{ccc} \omega_{xu} & \omega_{\pi u} & \omega_{iu} \end{array} \right)'$ and

$$\dot{\omega}_r = \left(J_{\omega_r^*} - I_3\right)\omega_r \qquad (20)$$

where $\omega_r = \left(\begin{array}{ccc} \omega_{xr} & \omega_{\pi r} & \omega_{ir} \end{array} \right)'$. Finally, the fourth includes the coefficients on the endogenous variables

$$vec\left(\dot{\omega}_e\right) = \left(J_{\omega_e^*} - I_9\right)vec\left(\omega_e\right), \qquad (21)$$

where

$$\omega_e = \left(\begin{array}{ccc} \omega_{xx} & \omega_{x\pi} & \omega_{xi} \\ \omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} \\ \omega_{\pi x} & \omega_{\pi\pi} & \omega_{\pi i} \end{array} \right).$$

   In order to show the instability result, it is sufficient to evaluate the real parts of the eigenvalues of the matrix $\left(J_{\omega_0^*} - I_3\right)$. Necessary conditions for stability under learning are

$$\text{Trace}\left(J_{\omega_0^*} - I_3\right) < 0 \qquad (22)$$

$$\text{Determinant}\left(J_{\omega_0^*} - I_3\right) > 0 \qquad (23)$$

and

$$-\text{Sm}\left(J_{\omega_0^*} - I_3\right) * \text{Trace}\left(J_{\omega_0^*} - I_3\right) + \text{Determinant}\left(J_{\omega_0^*} - I_3\right) > 0, \tag{24}$$

where Sm denotes the sum of all principle minors of $\left(J_{\omega_0^*} - I_3\right)$. The trace can be calculated as

$$-\xi - \frac{\beta\phi\lambda}{1-\beta} < 0$$

while the determinants is

$$(\beta - 1)^{-2}\left[\xi\left(\phi - 1\right) + (1 - \beta)\frac{\phi\lambda}{\xi}\right]$$

which is positive under the assumption $\phi > 1$.

Evaluating (24) provides

$$\frac{2 - 4\beta + 2\beta^2 + \beta\left(-\xi + (1 - \beta)\phi\frac{\lambda}{\xi} - \xi\phi\right)}{(1-\beta)^2}$$

which is *negative* provided

$$(1 + \phi)\xi > \phi(1 - \beta)\frac{\lambda}{\xi} + \psi\left(\beta\right)$$

where $\psi\left(\beta\right) = \beta^{-1}\left(2 - 4\beta + 2\beta^2\right)$.

Proposition 3: Assume that the central bank has perfect information about inflation and output gap. Then the REE is stable under learning for all parameter values, independently of central bank communication.

Proof: The proof follows the logic of Proposition 2. The only condition affected by the change in policy rule is (24). For the evolution of the constant terms we get

$$\frac{(\phi\lambda\beta + \xi(1 - \beta))(-2\phi\lambda\beta(1 - \beta) - 2\xi\beta^2 - \xi^2\beta(\phi - 1) + 4\xi\beta - 2\xi)}{-(1-\beta)^3\xi^2}$$

which is positive for every parameter value. For the shock coefficients we get

$$\frac{\left(-2\beta\lambda\phi\left(1 - \rho^M\beta\right) - \beta\xi^2\left(\phi - \rho\right) - 2\xi\left(2\beta\rho - \beta - 1\right)\left(\beta\rho - 1\right)\right)\left(-2\beta\xi\rho + \beta\xi + \beta\lambda\phi + \xi\right)}{-(1-\beta)^3\xi^2} > 0.$$

It is straightforward to show that the coefficients on the endogenous variables converge to their values under rational expectations. These results are available on request.

Proposition 4: Assume the bank communicates under perfect credibility the interest rate forecast $\left\{E_{t-1}^{CB}i_T\right\}_{T=t}^{\infty}$ or, equivalently, the policy rule (11). Then the REE is stable if $\phi > 1$.

Proof: The system has lower dimensionality, since agents do not have to forecast the nominal interest rate equation. Given that they know the steady state of the system, stability under learning is governed by the dynamics of agent's estimates of the shocks' coefficients and the lagged endogenous variables coefficients (also in this case they evolve as three separate subsystems). Consider first the stability of the coefficients. We have $\left(\tilde{J}_{\omega_u^*} - I_2\right) = \tilde{F}(\rho_u)$ and $\left(\tilde{J}_{\omega_r^*} - I_2\right) = \tilde{F}(\rho_r)$ so that instability can be determined by analyzing $\tilde{F}(\rho)$. The trace of $\tilde{F}(\rho)$ is negative

$$\frac{(\beta + \frac{\beta\phi\lambda}{\xi} - 2\beta\rho + 1)}{(-1 + \beta\rho)} < 0$$

and the determinant can be expressed as

$$\beta\frac{\frac{\phi\lambda}{\xi}(1 - \beta\rho) + \xi(\phi - \rho) + (1 - \beta\rho)(1 - \rho)}{(-1 + \beta\rho)^2}$$

so that it is positive provided $\phi > 1$. Finally $\tilde{J}_{\omega_e^*} - I_6$ can be shown to have stable eigenvalue for every parameter values.

Proposition 5: If households and firms understand the variables on which nominal interest rate decisions are conditioned on, then the REE is stable under learning if $\phi > 1$.

Proof: The Actual law of Motion can be re-written as:

$$Z_t = T_0\left(\omega_0, \hat{\phi}\right) + T_1\left(\omega_1, \hat{\phi}\right)Z_{t-1}$$

The evolution of $\hat{\phi}$ is described by:

$$\hat{\phi}_t = \hat{\phi}_{t-1} + \gamma_t\tilde{R}_{t-1}^{-1}\begin{bmatrix}1\\r_{t-1}^n\\\mu_{t-1}\end{bmatrix}\left(\hat{i}_t - \hat{\phi}_{t-1}'\begin{bmatrix}1\\\hat{E}_{t-1}x_t\\\hat{E}_{t-1}\pi_t\end{bmatrix}\right)$$

where we assume agents use a Recursive Instrumental Variable estimator, to encompass the case of noise in the announced forecast:

$$\tilde{R}_t = \tilde{R}_{t-1} + \gamma_t\left(\begin{bmatrix}1\\r_{t-1}^n\\\mu_{t-1}\end{bmatrix}\begin{bmatrix}1\\\hat{E}_{t-1}x_t\\\hat{E}_{t-1}\pi_t\end{bmatrix}' - \tilde{R}_{t-1}\right)$$

so we can substitute for the correct coefficients

$$
\hat{\phi}_t = \hat{\phi}_{t-1} + \gamma_t \tilde{R}_{t-1}^{-1} \begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \left(\phi' - \hat{\phi}_{t-1}' \right) \begin{bmatrix} 1 \\ r_{t-1}^n \\ \hat{E}_{t-1}x_t \\ \hat{E}_{t-1}\pi_t \end{bmatrix}
$$

$$
\hat{\phi}_t = \hat{\phi}_{t-1} + \gamma_t \tilde{R}_{t-1}^{-1} \begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1}x_t \\ \hat{E}_{t-1}\pi_t \end{bmatrix}' \left(\phi - \hat{\phi}_{t-1} \right)
$$

and

$$
\tilde{R}_t = \tilde{R}_{t-1} + \gamma_t \left(\begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1}x_t \\ \hat{E}_{t-1}\pi_t \end{bmatrix}' - \tilde{R}_{t-1} \right)
$$

Taking limits we have

$$
\dot{\hat{\phi}} = \tilde{R}^{-1} M \left(\Omega, \hat{\phi} \right) \left(\phi - \hat{\phi} \right)
$$

and

$$
\dot{\tilde{R}} = M \left(\Omega, \hat{\phi} \right) - \tilde{R}.
$$

Assuming that

$$
M \left(\Omega, \hat{\phi} \right) = E_{t \to \infty} \left(\begin{bmatrix} 1 \\ r_{t-1}^n \\ \mu_{t-1} \end{bmatrix} \begin{bmatrix} 1 \\ \hat{E}_{t-1}x_t \\ \hat{E}_{t-1}\pi_t \end{bmatrix}' \right)
$$

is finite we get

$$
\tilde{R} \to M \left(\Omega, \hat{\phi} \right)
$$

and therefore

$$
\hat{\phi} \to \phi.
$$

The stability conditions are then the same as for the case of full communication.

Proposition 6: Assume the central bank communicates only the inflation target $\pi^* = 0$ and the associated values for the output gap and nominal interest rates $x^* = i^* = 0$.

1. Define $\rho^M = \max(\rho_u, \rho_r)$ and let $\rho^M \to 1$. Then the REE is unstable under learning if (17) holds;

2. Let $\beta \to 1$. Then the REE is unstable under learning if

$$\xi(\phi + 2\rho) > 2\frac{\phi\lambda}{\xi}(1 - \rho) + \tilde{\psi}(\rho),\tag{25}$$

where $\tilde{\psi}(\rho) > 0$, $\tilde{\psi}(1) = 0$.

Proof: In case the agents know the constant of the system ($\omega_0 = 0$), stability is determined by (19), (20) and (21). It can be shown that $(J_{\omega_u^*} - I_3) = F(\rho_u)$ and $(J_{\omega_r^*} - I_3) = F(\rho_r)$ so that instability can be determined by analyzing $F(\rho^M)$.

1. It can be shown that $F(1) = (J_{\omega_0^*} - I_3)$ and that F is continuous in ρ. The rest follows from Proposition 1.

2. Let $\beta \to 1$. We proceed as in Proposition 1. The trace of $F(\rho^M)$ is equal to -3, while the determinant is

$$(\rho - 1)^{-2}\left(g(\rho^M) + \xi(\phi - \rho^M) + \frac{\phi\lambda}{\xi}(1 - \rho^M)\right) > 0$$

given $\phi > 1$ and

$$g(\rho^M) = 1 - 2\rho^M + (\rho^M)^2 > 0.$$

Finally, the last element is

$$\frac{\tilde{\psi}(\rho) + 2\frac{\phi\lambda}{\xi}(1 - \rho) - \xi(\phi + 2\rho)}{(\rho - 1)^2}$$

which gives (25).

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