Watanabe, K.
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CERTAIN INVARIANT SUBRINGS ARE GORENSTEIN I

KEIICHI WATANABE

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Introduction

Let $R=k[X_1, \cdots, X_n]$ be a polynomial ring over a field k and G be a finite subgroup of $GL(n, k)$. We assume that $|G|$, the order of G, is not zero in k. Then G acts on 1-forms of R and thus G can be considered as an automorphism group of R. We want to investigate the invariant subring R^G. We have two theorems concerning R^G already.

Theorem ([4], Théorème 1) *R^G is again a polynomial ring if and only it G is generated by pseudo-reflections.* (We call $g \in G$ a pseudo-reflection if rank $(g-I) \leqq 1$, where I is the unit matrix).

Theorem ([2], Proposition 13) *R^G is a Macaulay ring.*
After these theorems, we ask:
"When is R^G a Gorenstein ring?"
We prove in this paper the following theorems.

Theorem 1. *If $G \subset SL(n, k)$, then R^G is a Gorenstein ring.*
We apply this theorem to the case of regular local rings. If (R, m) is a regular local ring and if G is a finite subgroup of Aut (R), then G acts linearly on m/m^2. Thus we have the canonical homorphism $\lambda: G \rightarrow GL(m/m^2)$. We also assume that $|G|$ is a unit in R. Then applying Theorem 1, we get the following theorem.

Theorem 3. *If $\lambda(G) \subset SL(m/m^2)$, then R^G is Gorenstein.*
To reduce the case of regular local rings to the case of polynomial rings, we use the following theorem.

Theorem 4. *Let (A, m) be a local ring. (We assume always the Noetherian property.) We suppose that A has a filtration $F=(F_i)_{i \geqq 0}$ satisfying the following conditions.*
(i) $F_0=A$ and $F_1=m$.
(ii) $(F_i)_{i \geqq 0}$ *defines the same topology as the m-adic topology on A. We put $R=Gr^{\cdot}(A)=\bigoplus_{i \geqq 0} F_i/F_{i+1}$ the associated graded algebra and $M=R_+=\bigoplus_{i \geqq 1} F_i/F_{i+1}$ the*

canonical maximal ideal of R. *Then,*

(1) *If* R_M *is Macaulay, then* A *is Macaulay.*

(2) *If* R_M *is Gorenstein, then* A *is Gorenstein.*

1. Preliminaries

The contents of this section can be found elsewhere. But for the convenience of the readers, I put the proofs. As for the definition and the fundamental properties of Gorenstein rings, see [1].

In this section, R is a Noetherian ring and G is a finite group acting on R. We assume that $|G|$, the order of G, is a unit in R. We denote by R^G the invariant subring of R by G and by ρ the Reynolds operator $R\rightarrow R^G$ defined by $\rho(r)=\frac{1}{|G|}\sum_{g\in G} g(r)$ for $r\in R$.

Lemma 1. *If* $f_1, \cdots, f_s$ *are elements in* R^G *which form an* R*-regular sequence, then they form also an* R^G*-regular sequence and* $R^G/(f_1, \cdots, f_s)\cong(R/(f_1, \cdots, f_s))^G$.

Proof. It suffices to show the latter part. Let's put $a=(f_1, \cdots, f_s)R$. If $h\in R$ and $h-g(h)\in a$ for all $g\in G$, $h-\rho(h)\in a$ and $\rho(h)\in R^G$ obtaining that $R^G/(f_1, \cdots, f_s)R^G\rightarrow(R/(f_1, \cdots, f_s))^G$ is surjective. Since injectivity is clear, we are done.

Lemma 2. *If* R *is Macaulay, then* R^G *is Macaulay.*

Proof. If $(f_1, \cdots, f_s)$ is a parameter system of R^G, it is also a parameter system for R. Since R is Macaulay, $(f_1, \cdots, f_s)$ forms an R-regular sequence and by Lemma 1, it forms an R^G-regular sequence. So R^G is Macaulay.

Lemma 3. *If* (A, m) *is an Artinian local ring, the following conditions are equivalent.*

(a) *A is Gorenstein.*

(b) $length_A(0: m)=1$.

(c) *There exists an element* z *in* A, $z\neq 0$, *such that for every* $x\neq 0$ *in* A *there exists an element* y *in* A *satisfying* $xy=z$.

Proof. (a)$\Leftrightarrow$(b) is almost the definition itself. (b)$\Leftrightarrow$(c) is straightforward.

Lemma 4. *Let* (A, m) *be an Artinian local Gorenstein ring,* G *a finite group acting on* A. *We assume that* $|G|$ *is a unit in* A *and we denote by* z *an element in* A *satisfying the condition (c) of Lemma 3. If* z *is invariant under* G, *then* A^G *is Gorenstein.*

Proof. We check the condition (c) of Lemma 3 for A^G. Take $x\neq 0$ in A^G. By assumption, there exists y in A satisfying $xy=z$. Then $x\rho(y)=z$ and $\rho(y)$

is in A^G.

Lemma 5. *Let A be a ring which contains a field k and let k' be an extension field of k. If a group G acts on A and G acts trivially on k, we can extend the action of G to $A'=A\otimes_k k'$ naturally. Then $(A')^G=A^G\otimes_k k'$. Thus $(A')^G$ is faithfully flat over A^G and if $(A')^G$ is Gorenstein, A^G is Gorenstein.*

Proof. We write elements of A' in the form $x'=\sum_{i=1}^{n} x_i c_i$ where $x_i\in A$, $c_i\in k'$ and c_i's are linearly independent over k. For any $g\in G$, $g(x')=\sum_{i=1}^{n} g(x_i)\otimes c_i$ and if x is G-invariant, all x_i's are G-invariant. Thus we have $(A')^G=A^G\otimes_k k'$ and so $(A')^G$ is faithfully flat over A^G. The latter part holds by [5], Theorem 1'.

2. The case when G is cyclic

In this section, we use the following notations.
$R=k[X_1, \cdots, X_n]$. the polynomial ring over a field k.
G is finite cyclic subgroup of $GL(n, k)$. We asume that $(ch(k), |G|)=1$.
g is a generator of G. We put $|G|=m$ and we denote by ε a primitive m-th root of unit.
$n=R^G\cap(X_1, \cdots, X_n)$ and $\mathcal{O}=(R^G)_n$.

By Lemma 5, we may assume that k is algebraically closed and that g is in a diagonal form, $g=\begin{bmatrix} e_1 & & \\ & \ddots & \\ & & e_n \end{bmatrix}$, where e_i's are m-th roots of unity. We write $e_i=\varepsilon^{a_i}$.

Lemma 6. *If $det(g)=1$, then $\mathcal{O}$ is Gorenstein.*

Proof. $X_1^m, \cdots, X_n^m$ are in R^G and by Lemma 1, we have $\mathcal{O}/(X_1^m, \cdots, X_n^m)\mathcal{O} \cong (R/(X_1^m, \cdots, X_n^m)R)^G$. $A=R/(X_1^m, \cdots, X_n^m)R$ is an Artinian local ring. As A is a complete intersection, A is Gorenstein. In A, $z=(X_1\cdot\cdots\cdot X_n)^{m-1}$ satisfies the condition of Lemma 3 (c). If det $(g)=1$, $z\in A^G$ and by Lemma 4, A^G is Gorenstein. Thus $\mathcal{O}$ is Gorenstein.

Before proving the converse of Lemma 6, we need to fix some terminology.

DEFINITION 1. m and a_i are as in the beginning of this section. We put
$I=\{(r_1, \cdots, r_n)\,|\,r_i\text{'s are integers and } 0\leqq r_i<m \text{ for } i=1, \cdots, n\}$
$J=\{(r_1, \cdots, r_n)\in I\,|\,\sum_{i=1}^{n} r_i a_i\equiv 0 \pmod m\}$.

We define an order in I and J. Namely, $(r_1, \cdots, r_n)\geqq(s_1, \cdots, s_n)$ if $r_i\geqq s_i$ for $i=1, \cdots, n$. We call an element of J minimal if it is minimal among the elements of J which are not $(0, \cdots, 0)$.

Recall that, if (A, m) is an n-dimensional local Macaulay ring, the 'type' of A is defined by the number $[\mathrm{Ext}_A^n(A/m, A): A/m]$. To say that A is Gorenstein is equivalemtn to say that A is Macaulay and type$(A)=1$. We denote by emb(A) the embedding dimension of A. emb$(A)=[m/m^2: A/m]$.

Lemma 7. *If the number of minimal element of J is E and the number of of maximal element of J is r, then emb$(\mathcal{O}/(X_1^m, \cdots, X_n^m))=E$ and type $(\mathcal{O})=r$.*

Proof. $X_1^{r_1}\cdots X_n^{r_n}\neq 0 \bmod(X_1^m, \cdots, X_n^m)\Leftrightarrow(r_1, \cdots, r_n)\in I$, and $X_1^{r_1}\cdots X_n^{r_n}\in R^G \Leftrightarrow(r_1, \cdots, r_n)\in J$, and type $(\mathcal{O})$=type $(\mathcal{O}/(X_1^m, \cdots, X_n^m))$. From these facts, the conclusion is immediate.

DEFINITION 2. We call an element g of $GL(n, k)$ a pseudo-reflection if the order of g is finite and rank $(g-I_n)\leqq 1$. (Where I_n denotes the unit matrix).

Proposition 1. *If R^G is Gorenstein and if G does not contain any pseudo-reflections other than the unity, then $G\subset SL(n, k)$.*

Proof. It is clear that $(m, a_1, \cdots, a_n)=1$. Since type $(\mathcal{O})=1$, J must have unique maximal element $(r_1, \cdots, r_n)$. It is sufficient to prove that $(r_1, \cdots, r_n)=(m-1, \cdots, m-1)$. If this is not the case, we may assume that $r_1<m-1$. Since $(r_1, \cdots, r_n)$ is the unique maximal element of J, for any s_i, $0\leqq s_i\leqq m-1$ $(i=2, \cdots, n)$, $(m-1, s_2, \cdots, s_n)\notin J$. If $(a_2, \cdots, a_n, m)=1$, this can not happen and so $d=(a_2, \cdots, a_n, m)>1$. Then if we put $m'=m/d$, $g^{m'}\neq 1$ and $g^{m'}$ is a pseudo-reflection. This contradicts the hypothesis that G does not contain any pseudo-reflections other than the unity.

EXAMPLE 1. If ε is a primitive 6–th root of unity and if we put $g=\begin{bmatrix}\varepsilon & & \\ & \varepsilon^2 & \\ & & \varepsilon^2\end{bmatrix}$, R^G is Gorenstein but $\det(g)\neq 1$. This is due to the fact that $g^3=\begin{bmatrix}-1 & & \\ & 1 & \\ & & 1\end{bmatrix}$ is a pseudo-reflection. If we put $H=\{1, g^3\}$, $R^G=(R^H)^{G/H}$, $R^H=k[X^2, Y, Z]$. The action of $\bar{g}=g \bmod H$ on $k[X^2, Y, Z]$ is represented by $\begin{bmatrix}\varepsilon^2 & & \\ & \varepsilon^2 & \\ & & \varepsilon^2\end{bmatrix}$ and det $(\bar{g})=1$. More generally (we don't suppose that G is cyclic), let H be the subgroup of G generated by all its pserdo-reflections. Then H is a normal subgroup of G and R^H is again a polynomial ring over k (Serre [4], Théorème 1). Thus the hypothesis "G does not contain any pseudo-reflections" is quite natural.

3. R^G is Gorenstein at the origin

Theorem 1a. *If a finite group $G\subset SL(n,k)$ acts on $R=k[X_1, \cdots, X_n]$ naturally and if $(|G|, ch(k))=1$, then R^G is Gorenstein 'at the origin'. Namely, if we put $n=R^G\cap(X_1, \cdots, X_n)$ and $\mathcal{O}=(R^G)_n$, then $\mathcal{O}$ is Gorenstein.*

Proof. We take a parameter system $(f_1, \cdots, f_n)$ of $\mathcal{O}$ as follows;

1. Each f_i is homogenous of the same degree m.
2. m is a multiple of $|G|$.

We put $A=R/(f_1, \cdots, f_n)R$ and we want to apply Lemma 4. For this purpose we notice the following fact.

Lemma 8. *Let $A=\bigoplus_{i\geq 0} A_i$ be a graded ring. We assme that $A_0=k$ is a filed and that each A_i is a finite dimensional vector space over k. If f is a homogenous element of A which is not a zero-divisor of A, then $\dim_k(A/fA)_n$ depends only on A, n and $deg(f)$.*

Proof. If $\deg(f)=d$, $\dim_k(A/fA)_n=\dim_k A_n-\dim_k A_{n-d}$.

We return to the proof of our theorem. By Lemma 8, for any d, $\dim_k A_d =\dim_k(R/(X_1^m, \cdots, X_n^m))_d$. If we take $z\in A$ satisfying the condition of Lemma 3 (c) (A is Gorenstein), $\deg(z)=n(m-1)$. Then we take an element $g\in G$ and assume that g is in a diagonal form. We put H the cyclic subgroup of G generated by g. Applying Lemma 8 to R^H, $\dim_k(R^H/(X_1^m, \cdots, X_n^m)R^H)_d=\dim_k(A^H)_d =\dim_k(R^H/(f_1, \cdots, f_n)R^H)_d$. As we have $(X_1, \cdots, X_n)^{m-1}\in R^H$ (g is in a diagonal form and $\det(g)=1$), $\dim_k(A^H)_{n(m-1)}=1$. As $\dim_k A_{n(m-1)}=1$, z is invariant under H. As g is arbitrary, $z\in A^G$. By Lemma 4, $A^G=\mathcal{O}/(f_1, \cdots, f_n)\mathcal{O}$ is Gorenstein. Thus $\mathcal{O}$ is Gorenstein.

4. R^G is globally Gorenstein

Theorem 1. *If a finite subgroup G of $SL(n, k)$ acts nturally on $R=k[X_1, \cdots, X_n]$ and if $(|G|, ch(k))=1$, then R^G is Gorenstein.*

Proof. By Lemma 5, we may assume that k is algebraically closed. If we take a maximal ideal n' of R^G, we can write $n'=(X_1-a_1, \cdots, X_n-a_n)R\cap R^G$ ($a_1, \cdots, a_n\in k$). We put $H=\{g\in G \mid g(a_1, \cdots, a_n)=(a_1, \cdots, a_n)\}$. We consider the diagram $R^G\rightarrow R^H\rightarrow R$. Then it is known that $R^G\rightarrow R^H$ is étale in a neighbourhood of n' (Raynaud [3], P. 103, Th. 1). Thus $(R^G)_{n'}\rightarrow(R^H)_q$ is flat (where $q=(X_1-a_1, \cdots, X_n-a_n)\cap R^H$). If $(R^H)_q$ is Gorenstein, then $(R^G)_{n'}$ is Gorenstein ([5], Theorem 1). But by the coordinate transformation $(X_1, \cdots, X_n)\rightarrow(X_1-a_1, \cdots, X_n-a_n)$, H can be regarded as a subgroup of $SL(n, k)$ and $q=(X_1, \cdots, X_n)\cap R^H$. By therorem 1a, $(R^H)_q$ is Gorenstein and we are done.

Question 1.[1] Is the converse of Theorem 1 true? Let G be a finite subgroupof $GL(n, k)$ and let us assume that $(|G|, \mathrm{ch}(k))=1$ and that G contains no pseudo-reflections other than the unity. If R^G is Gorenstein, then $G\subset SL(n, k)$?

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1) Added in proof. The statement in Question 1 has been proved by the author. The proof will appear in [6].

Question 2. Is the following statement true? Let $A=\bigoplus_{i\geq 0} A_i$ be a Northerian graded ring with A_0 a field. We put $M=A_+=\bigoplus_{i\geq 1} A_i$. If A_M is Gorenstein, is A globally Gorenstein?

5. Base extensions

Theorem 2. *Let A be a Noetherian ring and G be a finite subgroup of $SL(n, A)$. We assume that $|G|$ is a unit in A. Then G acts naturally on $R=A[X_1, \cdots, X_n]$. Then R^G is Gorenstein if and only if A is Gorenstein.*

Lemma 9. *Under the assumptions of Theorem 2, R^G is faithfully flat over A.*

Proof of Lemma 9. (i) If a is an ideal of A, then $a(R^G)=(aR)^G$. (If $\sum a_i f_i \in (aR)^G$ with $a_i \in a$ and $f_i \in R$, $\sum a_i f_i=\rho(\sum a_i f_i)=\sum a_i \rho(f_i)$ and we have $(aR)^G \subset aR^G$. The converse inclusion is clear).

(ii) As R is A-flat, $(aR)^G \simeq (a\otimes_A R)^G$.

(iii) $(a\otimes_A R)^G \simeq a\otimes_A R^G$ (The isomorphisms is given by $\sum a_i\otimes f_i \to \sum a_i \otimes \rho(f_i)$.) By (i), (ii), (iii), $aR^G \simeq a\otimes_A R^G$ and $R^G/aR^G \simeq (R/aR)^G$. Thus R^G is faithfully flat over A.

Proof of Theorem 2. The fiber of the map f: Spec $(R^G) \to$ Spec (A) at $p\in$ Spec (A) is the Spec of $R^G\otimes_A k(p) \simeq (k(p)[X_1, \cdots, X_n])^G$ which is Gorenstein by Theorem 1. Thus f is a Gorenstein morphism in the sense of [5], Definition (1.7). The conclusion follows from [5], Theorem 1′.

REMARK. In Lemma 9, the assumption "$|G|$ is a unit in A" is essential. For example, let $A=k[e]$, k be a field of characteristic 2, $e^2=0$, $G=\langle g\rangle$, $g=\begin{bmatrix}1 & e\\ 0 & 1\end{bmatrix}$. If we put $a=eA$, then $eX_1 \in R^G$ and $e\otimes eX_1 \neq 0$ in $a\otimes_A R^G$, while $e.eX_1=0$. Thus $a\otimes_A R^G \to aR^G$ is not injective and R^G is not flat over A.

6. A theorem on the associated graded algebra of a local ring

Theorem 3. *If (A, m) is a Noetherian local ring and $(F_n)_{n\geq 0}$ be a filtration on A satisfying the two conditions.*

1. *$F_0= A$ and $F_1=m$.*
2. *$(F_n)_{n\geq 0}$ defines the same topology as the m-adic topology on A.*

We put $R=Gr^{\cdot}(A)=\bigoplus_{i\geq 0} F_i/F_{i+1}$ the associated graded algebra and $M=R_+=\bigoplus_{i\geq 1} F_i/F_{i+1}$ the canonical maximal ideal of R. Then,

(i) *if R_M is Macaulay, then A is Macaulay.*

(ii) *if R_M is Gorenstein, then A is Gorenstein.*

Proof. The proof follows immediately from the two lemmas below.

Lemma 10. *Let $f_1, \cdots, f_s$ be homogenous elements of R which make an R-sequence. If $x_1, \cdots, x_s$ are elements of A with* $\mathrm{In}(x_i)=f_i (i=1, \cdots, s)$, *then $(x_1, \cdots, x_r)$ form an A-regular sequence and $Gr^{\cdot}(A/(x_1, \cdots, x_s)) \simeq R/(f_1, \cdots, f_s)$. (If $x \in A$, $x \in F_n$ and $x \notin F_{n+1}$, then* $\mathrm{In}(x)=x \bmod F_{n+1} \in Gr^n(A)$. *The filtration of $A/(x_1, \cdots, x_s)$ is the one induced from (F_n).)*

Proof. We note the fact that if $x, y \in A$ and $\mathrm{In}(x)\,\mathrm{In}(y) \neq 0$, then $\mathrm{In}(xy)= \mathrm{In}(x)\,\mathrm{In}(y)$.

Case 1. $s=1$ (we omit the subscript 1).

If $y \in A$ and $\mathrm{In}(y) \neq 0$, by assumption $\mathrm{In}(x)\,\mathrm{In}(y) \neq 0$. Thus $\mathrm{In}(xy)=\mathrm{In}(x)$ $\mathrm{In}(y) \neq 0$ and $xy \neq 0$. On the other hand, $Gr^{\cdot}(A/xA) \simeq R/Gr^{\cdot}(xA)$ where $Gr^{\cdot}(xA)$ is the homogenous ideal of R generated by $\mathrm{In}(z)$, $z \in xA$. But if $z=xy \in xA$, then $\mathrm{In}(z)=\mathrm{In}(x)\,\mathrm{In}(y)$ and so $\mathrm{In}(z) \in fR$. Thus we have $Gr^{\cdot}(A/xA) \simeq R/fR$.

Case 2. General case.

We assume that the assumption is true for $s=i$ and prove for $s=i+1$. As f_{i+1} is not a zero-divisor on $Gr^{\cdot}(A/(x_1, \cdots, x_i)) \simeq R/(f_1, \cdots, f_i)$, Case 1 applies.

Lemma 11. *If (A, m) is an Artinian local ring, (F_n) is a filtration on A whcih satisfies the conditions of Theorem 3 and if $R=Gr^{\cdot}(A)$ is Gorenstein, then A is Gorenstein.*

Proof. We use Lemma 3. Let h be a homogenous element of R which satisfies the condition of Lemma 3(c) for $R(O:M$ is a homogenous ideal of $R)$. Then if $z \in A$ be such that $\mathrm{In}(z)=h$, then for any $x \in A$, $x \neq 0$, there exists an element $f \in R$ such that $\mathrm{In}(x).f=h$. If we take $y \in A$ such as $\mathrm{In}(y)=f$ and if $\deg(h)=m$, $\mathrm{In}(y)\,\mathrm{In}(x)=h$ and $xy \equiv z \bmod F_{m+1}$. But as $F_{m+1}=0$, $xy=z$ and z satisfies the condition (c) of Lemma 3 for A.

7. The case of regular local rings

The statement of Theorem 4 was indicated to me by Professor M. Miyanishi with an outline of a proof. I wish to express my deep gratitude to him.

Theorem 4. *Let (R, m) be a regular local ring of dimension n and G be a finite subgroup of $Aut(R)$ satisfying the following conditions.*

1. *$|G|$ is a unit in R.*
2. *The automorphisms of $k=R/m$ inducted by the elements of G are identities.*
3. *If we denote $\lambda: G \to GL(m/m^2)$ the canonical homomorphism, then $\lambda(G) \subset SL(m/m^2)$.*

Then $S=R^G$ is Gorenstein.

The proof is divided into several steps. First we need a lemma.

Lemma 12. ([2], Proposition 10) *Let R be a commutative ring and G be a finite group acting on R. We assume that $|G|$ is a unit in R and we put $S=R^G$. Then if a is an ideal of S, then $aR \cap S=a$.*

Proof. If $\sum a_i r_i \in S$, $a_i \in a$, $r_i \in R$, then
$\sum a_i r_i = \rho(\sum a_i r_i) = \sum a_i \rho(r_i) \in a$. Thus we get the inclsuion $\subset$ and the converse is trivial.

We return to the proof of Theorem 4. From Lemma 12, we get

(1) S is a Noetherian local ring.

Proof. Since R is integral over S, S is local and by Lemma 12, S is Noetherian.

We put,

$A=Gr_m^{\cdot}(R) \simeq k[X_1, \cdots, X_n]$.

G acts naturally on A. We denote by n the maximal ideal of S and we put $F_n=S \cap m^n$. $(F_n)_{n\geqq 0}$ defines a filtration on S. We denote by B the graded ring associated to this filtration. Then we have;

(2) $B \simeq A^G$.

Proof. If $f=\mathrm{In}(x) \in A_n$ is invariant under G, then $x-\rho(x) \in m^{n+1}$ and $\rho(x) \in F_n$. Thus $A^G \subset B$. The converse implication is trivial.

(3) The filtration (F_n) defines on S the same topology as n-adic topology.

Proof. If suffices to say that for any integer $t \geqq 0$, there exists an integer t' such that $S \cap m^{t'} \subset n^t$. But as nR is m-primary, for some s, $m^s \subset nR$. Then, by Lemma 12, $m^{st} \cap S \subset (nR)^t \cap S = n^t$.

By (2), (3), Theorem 1 and Theorem 3, Theorem 4 is proved.

TOKYO METROPOLITAN UNIVERSITY

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Added in proof; Question 2 in section 4 was solved affirmatively by Y. Aoyama, S. Goto, J. Matijevic and R.C. Cowsik independently and in more general forms.