

## CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTIONS SATISFYING SUBORDINATE CONDITIONS

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*Abstract.* In this paper, we introduce and investigate each of the following subclasses:

$$\mathcal{S}_\Sigma(\lambda, \gamma, \varphi), \mathcal{HS}_\Sigma(\alpha), \mathcal{B}_\Sigma(\eta, \gamma, \varphi) \text{ and } \mathcal{B}_\Sigma(\mu; \varphi)$$

$$(0 \leq \lambda \leq 1; \gamma \in \mathbb{C} \setminus \{0\}; \alpha \in \mathbb{C}; 0 \leq \eta < 1; \mu \geq 0)$$

of bi-univalent functions,  $\varphi$  is an analytic function with positive real part in the unit disk  $\mathbb{D}$ , satisfying  $\varphi(0) = 1$ ,  $\varphi'(0) > 0$ , and  $\varphi(\mathbb{D})$  is symmetric with respect to the real axis. We obtain coefficient bounds involving the Taylor-Maclaurin coefficients  $|a_2|$  and  $|a_3|$  of the function  $f$  when  $f$  is in these classes. The various results, which are presented in this paper, would generalize and improve those in related works of several earlier authors.

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