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Certain topological indices and their polynomials of some cutting number nanostar Dendrimer $NS[n]$

P. Gladyis[1*] and G. Srividhya[2]

Abstract

Some theoretical results about nanostar Dendrimer by topological indices are explained in this research paper. This paper details with 1^{st} 2^{nd} and 3^{rd} cutting number Zagreb Index, Multiplicative and polynomial and also reverse hyper-Zagreb cutting number index multiplicative and polynomial of nanostar Dendrimer.

Keywords

Cutting number, Zagreb Polynomial, Index of Hyper-Zagreb, Reverse Index of Hyper-Zagreb , Reverse Hyper-Zagreb polynomial.

[1] *Department of Mathematics, Government Arts and Science College [Affiliated to Bharathidasan University, Trichy-620024, Tamil Nadu, India], Aranthangi-614616, Tamil Nadu, India.*

[2] *Research Department of Mathematics, Government Arts College, Trichy-620024, Tamil Nadu, India.*

***Corresponding author**: [1]gladyisp1979@gmail.com; [2]vkm292011@hotmail.com

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Contents

1. Introduction

The topological index characterized on the structure of these compound can equip to surmise physical attributes, chemical reactivity and biological action to the scientists [11, 3].

In [10] , Gutman introduced the 1^{st} and 2^{nd} Zagreb indices are

$$M_1(G)=\sum_{uv\in E(G)}(d(u)+d(v))$$

$$M_2(G)=\sum_{uv\in E(G)}(d(u)\times d(v))$$

Fath-Tabar [1,2&8] defined 1^{st} and 2^{nd} Zagreb polynomials as

$$M_1(G,x)=\sum_{uv\in E(G)}x^{d(u)+d(v)}$$

$$M_2(G,x)=\sum_{uv\in E(G)}x^{d(u)\cdot d(v)}$$

In [2], G. H. Fath-Tabar et al. defined the 3^{rd} Zagreb index as

$$M_3(G)=\sum_{uv\in E(G)}|d(u)-d(v)|$$

and the polynomial as

$$M_3(G,x)=\sum_{uv\in E(G)}x^{|d(u)-d(v)|}$$

The 1^{st} and 2^{nd} Reverse hyper-Zagreb indices was also defined in the samp paper as

$$HCM_1(G)=\sum_{uv\in E(G)}(c(u)+c(v))^2$$

$$HCM_2(G)=\sum_{uv\in E(G)}(c(u)c(v))^2$$

The 1^{st} and 2^{nd} Reverse hyper-Zagreb polynomials are defined as

$$HCM_1(G,x)=\sum_{uv\in E(G)}x^{(c(u)+c(v))^2}$$

$$HCM_2(G,x)=\sum_{uv\in E(G)}x^{(c(u)\cdot c(v))^2}$$

A cutting number [7], $c(v)$ of a vertex $v\in V(G)$ in a connected graph G is the number of pairs of vertices $\{v,w\}$ such that v and w are in different components of G.

We now consider the Nanostar Dendrimers $NS[n]$, where n is steps of growth see Fig. 1& Fig. 2.

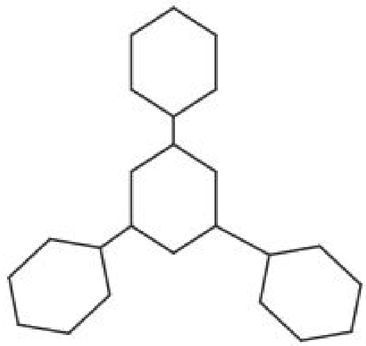

Figure 1. Nanostar Dendrimer NS[1]

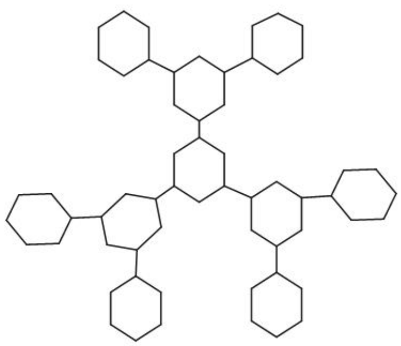

Figure 2. Nanostar Dendrimer NS[2]

In [7], we have found number of edges uv with cutting number of end vertices $(c(u),c(v))$ which is given in the Table 1.

2. Some cutting number Topological Indices and polynomials of Nanostar Dendrimer NS[n]

In this paper, we introduce some other cutting number based topological indices of graphs. For two-connected graphs, cutting number of each vertex is zero. So, we define these indices for graphs with cut vertices only.

We define the 1^{st}, 2^{nd} and 3^{rd} cutting number Zagreb indices of G as

$$M_{1C}(G) = \sum_{uv\in E[G]} [c(u)+c(v)]$$
$$M_{2C}(G) = \sum_{uv\in E[G]} [c(u)c(v)]$$
$$M_{3C}(G) = \sum_{uv\in E[G]} |c(u)-c(v)|$$

We define the 1^{st}, 2^{nd} and 3^{rd} cutting number Zagreb polynomial of the graph G as

$$M_{1C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)+c(v)]}$$
$$M_{2C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)c(v)]}$$
$$M_{3C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)-c(v)]},$$

where $c(u)$ and $c(v)$ are the cutting number of u and v.

We define the 1^{st} 2^{nd} and 3^{rd} cutting number Reverse hyper-Zagreb indices of G as

$$RHM_{1C}(G) = \sum_{uv\in E[G]} [c(u)+c(v)]^2$$
$$RHM_{2C}(G) = \sum_{uv\in E[G]} [c(u)c(v)]^2$$
$$RHM_{3C}(G) = \sum_{uv\in E[G]} |c(u)-c(v)|^2$$

We define the 1^{st} 2^{nd} and 3^{rd} cutting number Reverse hyper-Zagreb polynomial of the graph G as

$$RHM_{1C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)+c(v)]^2}$$
$$RHM_{2C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)c(v)]^2}$$
$$RHM_{3C}[G,x] = \sum_{uv\in E[G]} x^{[c(u)-c(v)]^2}$$

Theorem 2.1. *Consider $NS[n]$ be the Nanostar Dendrimer. Then,*

(i) $M_{1C}[NS(n)] = 1944n\times 2^{2n} - 2106\times 2^{2n} - 648\times 2^{2n+1} + 648\times 2^{n+1} + 2862\times 2^{n} - 756.$

(ii) $M_{2C}[NS(n)] = 15552\times 2^{4n+1} - 9720\times 2^{3n+1} - 34452\times 2^{2n+1} + 9180\times 2^{n+1} - 3888\times 2^{n+3} + 23328\times 2^{2n+2} + 40824\times 2^{3n} - 70308\times 2^{2n} + 34776\times 2^{n} - 75816n\times 2^{3n} + 27540n\times 2^{2n} - 5292$

(iii) $M_{3C}[NS(n)] = 1296n\times 2^{n} - 1458\times 2^{2n} - 432\times 2^{2n+1} + 432\times 2^{n+1} + 36n\times 2^{n} + 1962\times 2^{n} - 504.$

Proof.

(i) $M_1C[NS(n)]$

$$= \sum_{uv\in E[NS(n)]} [c(u)+c(v)]$$
$$= \sum_{i=0}^{n-1} (6\times 2^i)\left[\left\{\left(((18\times 2^n)-12)-(6\times 2^{n-i}-6)\right)\times(6\times 2^{n-i}-7)\right\}+0\right]$$
$$+\sum_{i=0}^{n-1} (3\times 2^i)\left[\left\{\left(((18\times 2^n)-12)-(6\times 2^{n-i}-6)\right)\times(6\times 2^{n-i}-7)\right\}\right.$$
$$\left.+\left\{\left(((18\times 2^n)-12)-(6\times 2^{n-i}-5)\right)\times(6\times 2^{n-i}-6)\right\}\right]$$
$$+\sum_{i=0}^{n-1} (6\times 2^i)\left[\left\{\left(((18\times 2^n)-12)-(6\times 2^{n-i}-5)\right)\times(6\times 2^{n-i}-6)\right\}+0\right]$$
$$= 1944n\times 2^{2n} - 2106\times 2^{2n} - 648\times 2^{2n+1} + 648\times 2^{n+1} + 2862\times 2^{n} - 756$$

Table 1. The Nanostar Dendrimer $NS[n]$

Number of edges $\mathbf{e}=\mathbf{uv}$	Cutting number of end vertices $(\mathbf{c(u)},\mathbf{c(v)})$
$12\times 2^{n-1}$	(0,0)
$6\times 2^{n-1}$ $3\times 2^{n-1}$ $6\times 2^{n-1}$	[{ ((18 $\times 2^n$)-12)-6 } × 5,0] [{ ((18 $\times 2^n$)-12)-6 } ×5, { ((18 $\times 2^n$)-12)-7 } × 6] [{ ((18 $\times 2^n$)-12)-7 } ×6,0]
$6\times 2^{n-2}$ $3\times 2^{n-2}$ $6\times 2^{n-2}$	[{ ((18 $\times 2^n$)-12)-18 } × 17,0] [{ ((18 $\times 2^n$)-12)-18 } × 17, { ((18 $\times 2^n$)-12)-19 } × 18] [{ ((18 $\times 2^n$)-12)-19 } × 18,0]
⋮	
6×2^i 3×2^i 6×2^i	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-6) } × ((6 $\times 2^{n-i}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-6) } × ((6 $\times 2^{n-i}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-5) } × ((6 $\times 2^{n-i}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-i}$)-5) } × ((6 $\times 2^{n-i}$)-6), 0]
⋮	
6×2^2 3×2^2 6×2^2	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-6) } × ((6 $\times 2^{n-2}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-6) } × ((6 $\times 2^{n-2}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-5) } × ((6 $\times 2^{n-2}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-2}$)-5) } × ((6 $\times 2^{n-2}$)-6), 0]
6 × 2 3 × 2 6 × 2	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-6) } × ((6 $\times 2^{n-1}$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-6) } × ((6 $\times 2^{n-1}$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-5) } × ((6 $\times 2^{n-1}$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^{n-1}$)-5) } × ((6 $\times 2^{n-1}$)-6), 0]
6 × 1 3 × 1 6 × 1	[{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-6) } × ((6 $\times 2^n$)-7), 0] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-6) } × ((6 $\times 2^n$)-7), { ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-5) } × ((6 $\times 2^n$)-6)] [{ ((18 $\times 2^n$)-12)- ((6 $\times 2^n$)-5) } × ((6 $\times 2^n$)-6), 0]
Total number of edges	$21\times 2^n-15$

(ii) $M_2C[NS(n)]$

$$= \sum_{uv\in E[NS(n)]}[c(u)c(v)]$$

$$= \sum_{i=0}^{n-1}\left(3\times 2^i_{\sim}\right)\left[\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-6\right)\right)\times\left(6\times 2^{n-i}-7\right)\right\}\right.$$
$$\left.\times\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-5\right)\right)\times\left(6\times 2^{n-i}-6\right)\right\}\right]$$
$$=15552\times 2^{4n+1}-9720\times 2^{3n+1}-34452\times 2^{2n+1}+9180$$
$$\times 2^{n+1}-3888\times 2^{n+3}+23328\times 2^{2n+2}+40824\times 2^{3n}-70308$$
$$\times 22n+34776\times 2^n-75816n\times 2^{3n}+27540n\times 2^{2n}-5292$$

(iii) $M_{3C}[NS(n)] = \sum_{uv\in E[NS(n)]}|c(u)-c(v)|$

$$=\sum_{i=0}^{n-1}\left(6\times 2^i\right)\left[\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-6\right)\right)\times\left(6\times 2^{n-i}-7\right)\right\}-0\right]$$
$$+\sum_{i=0.1}^{n-1}\left(3\times 2^i\right)\left[\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-6\right)\right)\times\left(6\times 2^{n-i}-7\right)\right\}\right.$$
$$\left.-\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-5\right)\right)\times\left(6\times 2^{n-i}-6\right)\right\}\right]$$
$$+\sum_{i=0}^{n-1}\left(6\times 2^i\right)\left[\left\{\left(\left((18\times 2^n)-12\right)-\left(6\times 2^{n-i}-5\right)\right)\times\left(6\times 2^{n-i}-6\right)\right\}-0\right]$$
$$=1296n\times 2^{2n}-1458\times 2^{2n}-432\times 2^{2n+1}+432\times 2^{n+1}+36n$$
$$\times 2^n+1962\times 2^n-504$$

□

Theorem 2.2. *The Zagreb polynomial of Nanostar Dendrimer NS[n] are computed as follows:*

(i) $M_{1C}[NS(n),x]$

$$=6\sum_{i=0}^{n-1}2^i\quad x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right]}$$
$$+3\sum_{i=0}^{n-1}2^i x^{\left[216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84\right]}$$
$$+6\sum_{i=0}^{n-1}2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-1}-108\times 2^n+42\right]}$$

(ii) $M_{2C}[NS(n),x]=3\sum_{i=0}^{n-1}2^i$

$$x^{\left[11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}-25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\right]}$$

(iii) $M_{3C}[NS(n),x]$

$$=6\sum_{i=0}^{n-1}2^i x^{\left[\left(108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right)\right]}$$
$$+3\sum_{i=0}^{n-1}2^i x^{\left[\left(12\times 2^{n-i}-18\times 2^n\right)\right]}$$
$$+6\sum_{i=0}^{n-1}2^i x^{\left[\left(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-1}-108\times 2^n+42\right)\right]}$$

Proof. □

(i) $M_1C[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{[c(u)+c(v)]}$

$$= \sum_{i=0}^{n-1} (6\times 2^i)\, x^{[\{\{((18\times 2^n)-12)-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)\}+0]}$$

$$+ \sum_{i=0}^{n-1} (3\times 2^i)$$

$$\times x^{[\{\{((18\times 2^n)-12)-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)\} + \{\{((18\times 2^n)-12)-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-1}-6)\}]}$$

$$+ \sum_{i=0}^{n-1} (6\times 2^i)\, x^{[\{\{((18\times 2^n)-12)-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)\}+0]}$$

$$= 6\sum_{i=0}^{n-1} 2^i x^{108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42}$$

$$+3\sum_{i=0}^{n-1} 2^i x^{216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84}$$

$$+6\sum_{i=0}^{n-1} 2^i x^{108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42}$$

(ii) $M_{2C}[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{[c(u)c(v)]}$

$$= \sum_{i=0}^{n-1} (3\times 2^i)$$

$$x^{\{\{((18\times 2^n)-12)-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7)\} \times [\{\{((18\times 2^n)-12)-(6\times 2^{n-i}-5)\times(6\times 2^{n-i}-6)\}\}}$$

$$= 3\sum_{i=0}^{n-1} 2^i$$

$$\times x^{[11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i} -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i} -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764]}$$

(iii) $M_{3C}[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{|c(u)-c(v)|}$

$$= \sum_{i=0}^{n-1} (6\times 2^i)\, x^{[\{[((18\times 2^n)-12)-(6\times 2^{n-i}-6)]\times(6\times 2^{n-i}-7)\}-0]}$$

$$+ \sum_{i=0}^{n-1} (3\times 2^i)$$

$$x^{[\{[(18\times 2^n)-12]-(6\times 2^{n-i}-6)\}\times(6\times 2^{n-i}-7) - \{[((18\times 2^n)-12)-(6\times 2^{n-i}-5)]\times(6\times 2^{n-i}-6)\}]}$$

$$\sum_{i=0}^{n-1} (6\times 2^i)\, x^{[[\{[(18\times 2^n)-12]-(6\times 2^{n-i}-5)\}\times(6\times 2^{n-i}-6)]-0]}$$

$$= 6\sum_{i=0}^{n-1} 2^i x^{[(108\times 2^{2n-1}-36\times 2^{2n-2i}+6\times 2^{n-1}-126\times 2^n+42)]}$$

$$+3\sum_{i=0}^{n-1} 2^i x^{[(12\times 2^{n-1}-18\times 2^n)]}$$

$$+6\sum_{i=0}^{n-1} 2^i x^{[(108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-1}-108\times 2^n+42)]}$$

Theorem 2.3. *Let $NS[n]$ be the molecular graph of Nanostar Dendrimer. Then*

(i) $RHM_{1C}[NS(n)]$

$$= 6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right]^2$$

$$+3\sum_{i=0}^{n-1} 2^i \left[216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84\right]^2$$

$$+6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42\right]^2$$

(ii) $RHM_{2C}[NS(n)]$

$$= 3\sum_{i=0}^{n-1} 2^i \Big[11664\times 2^{4n-2i}-7776\times 2^{4n-3i+8424\times 2^{3n-2i}}$$

$$-25272\times 2^{3n-i+9180\times 2^{2n-i}+1296\times 2^{4n-4i}}$$

$$-3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\Big]^2$$

(iii) $RHM_{3C}[NS(n)]$

$$= 6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right]^2$$

$$+3\sum_{i=0}^{n-1} 2^i \left[12\times 2^{n-1}-18\times 2^n\right]^2$$

$$+6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42\right]^2$$

Proof.

(i) $RHM_{1C}[NS(n)]$

$$= \sum_{uv\in E[NS(n)]} [c(u)+c(v)]^2$$

$$= \sum_{i=0}^{n-1} (6\times 2^i)\, [\{((18\times 2^n)-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}+0]^2$$

$$+ \sum_{i=0}^{n-1} (3\times 2^i)\, [\{(((18\times 2^n)-12)-(6\times 2^{n-1}-6))\times(6\times 2^{n-1}-7)\}$$

$$+ \{(((18\times 2^n)-12)-(6\times 2^{n-1}-5))\times(6\times 2^{n-i}-6)\}]^2$$

$$+ \sum_{i=0}^{n-1} (6\times 2^i)\, [\{(((18\times 2^n)-12)-(6\times 2^{n-i}-5))\times(6\times 2^{n-i}-6)\}+0]^2$$

$$= 6\sum_{i=0}^{n-1} 2^i [108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42]^2$$

$$+3\sum_{i=0}^{n-1} 2^i [216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84]^2$$

$$+6\sum_{i=0}^{n-1} 2^i [108\times 2^{2n-1}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42]^2$$

(ii) $RHM_{2C}[NS(n)] = \sum_{uv\in E[NS(n)]} [c(u)c(v)]^2$

$$= \sum_{i=0}^{n-1} (3\times 2^i)\, [\{(((18\times 2^n)-12)-(6\times 2^{n-i}-6))\times(6\times 2^{n-i}-7)\}$$

$$\times \{(((18\times 2^n)-12)-(6\times 2^{n-i-5}))\times(6\times 2^{n-i}-6)\}]^2$$

$$= 3\sum_{i=0}^{n-1} 2^i \big[11664 \times 2^{4n-2i} - 7776 \times 2^{4n-3i} + 8424 \times 2^{3n-2i}$$
$$- 25272 \times 2^{3n-i} + 9180 \times 2^{2n-1} + 1296 \times 2^{4n-4i}$$
$$- 3060 \times 2^{2n-2i} + 13608 \times 2^{2n} - 9828 \times 2^n + 1764\big]_n^2$$

(iii) $RHM_{3C}[NS(n)] = \sum_{uv\in E[NS(n)]} |c(u) - c(v)|^2$

$$= \sum_{i=0}^{n-1} (6\times 2^i)\left[\left\{\left(((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right\}-0\right]^2$$
$$+\sum_{i=0}^{n-1} (3\times 2^i)\left[\left\{\left(((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right)\times\left(6\times 2^{n-i}-7\right)\right\}\right.$$
$$\left.-\left\{\left(((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right)\times\left(6\times 2^{n-i}-6\right)\right\}\right]^2$$
$$+\sum_{i=0}^{n-1} (6\times 2^i)\left[\left\{\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right]\times\left(6\times 2^{n-i}-6\right)\right\}-0\right]^2$$
$$= 6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i} - 36\times 2^{2n-2i} + 6\times 2^{n-1} - 126\times 2^n + 42\right]^2$$
$$+ 3\sum_{i=0}^{n-1} 2^i \left[12\times 2^{n-1} - 18\times 2^n\right]^2$$
$$+ 6\sum_{i=0}^{n-1} 2^i \left[108\times 2^{2n-i} - 36\times 2^{2n-2i} - 6\times 2^{n-i} - 108\times 2^n + 42\right]^2$$

□

Theorem 2.4. *Let $NS[n]$ be the Nanostar Dendrimer. The following topological indices can be Calculated:*

(i) $RHM_{1C}[NS(n),x]$

$$= 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right]^2}$$
$$+ 3\sum_{i=0}^{n-1} 2^i x^{\left[216\times 2^{2n-i}-72\times 2^{2n-2i}-234\times 2^n+84\right]^2}$$
$$+ 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42\right]^2}$$
$$+ \left[11664\times 2^{4n-2i} - 7776\times 2^{4n-3i} + 8424\times 2^{3n-2i}\right]$$
$$- 25272\times 2^{3n-i} + 9180\times 2^{2n-i} + 1296\times 2^{4n-4i}$$

(ii) $RHM_{2C}[NS(n),x] = 3\sum_{i-0}^{n-1} 2^i$

$$x^{\left[\begin{array}{l}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{array}\right]^2}$$

(iii) $RHM_{3C}[NS(n),x]$

$$= 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}+6\times 2^{n-i}-126\times 2^n+42\right]^2}$$
$$+ 3\sum_{i=0}^{n-1} 2^i x^{\left[12\times 2^{n-i}-18\times 2^n\right]^2}$$
$$+ 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-i}-108\times 2^n+42\right]^2}$$

Proof.

(i) $RHM_{1C}[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{[c(u)+c(v)]^2}$

$$= \sum_{i=0}^{n-1}\left(6\times 2^i\right) x^{\left[\left\{((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\right\}\times\left(6\times 2^{n-i}-6\right)+0\right]^2}$$
$$+ \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$\times x^{\left[\begin{array}{l}\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right\}\times\left(6\times 2^{n-i}-7\right)\right\}\\ \left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-1}-5\right)\right\}\times\left(6\times 2^{n-i}-6\right)\right\}\end{array}\right]^2}$$
$$+ \sum_{i=0}^{n-1}\left(6\times 2^i\right) x^{\left[\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-1}-5\right)\right\}\times\left(6\times 2^{n-1}-6\right)\right\}+0\right]^2}$$
$$= 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-1}-36\times 2^{2n-2i}+6\times 2^{n-1}-126\times 2^n+42\right]^2}$$
$$+ 3\sum_{i=0}^{n-1} 2^i x^{\left[216\times 2^{2n-1}-72\times 2^{2n-2i}-234\times 2^n+84\right]^2}$$
$$+ 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-i}-36\times 2^{2n-2i}-6\times 2^{n-1}-108\times 2^n+42\right]^2}$$

(ii) $\mathrm{RHM}_{2C}[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{[c(u)c(v)]^2}$

$$= \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$x^{\left[\begin{array}{l}\left\{\left[((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right\}\times\\ \left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-i}-5\right)\times\left(6\times 2^{n-i}-6\right)\right\}\right\}\end{array}\right]^2}$$
$$= 3\sum_{i=0}^{n-1} 2^i x^{\left[\begin{array}{l}11664\times 2^{4n-2i}-7776\times 2^{4n-3i}+8424\times 2^{3n-2i}\\ -25272\times 2^{3n-i}+9180\times 2^{2n-i}+1296\times 2^{4n-4i}\\ -3060\times 2^{2n-2i}+13608\times 2^{2n}-9828\times 2^n+1764\end{array}\right]^2}$$

(iii) $RHM_{3C}[NS(n),x] = \sum_{uv\in E[NS(n)]} x^{|c(u)-c(v)|^2}$

$$= \sum_{i=0}^{n-1}\left(6\times 2^i\right) x^{\left[\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-i}-6\right)\right]\times\left(6\times 2^{n-i}-7\right)\right\}-0\right]^2}$$
$$+ \sum_{i=0}^{n-1}\left(3\times 2^i\right)$$
$$\times x^{\left[\begin{array}{l}\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-1}-6\right)\right\}\times\left(6\times 2^{n-1}-7\right)\right\}\\ -\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-1}-5\right)\right\}\times\left(6\times 2^{n-1}-6\right)\right\}\end{array}\right]^2}$$
$$+ \sum_{i=0}^{n-1}\left(6\times 2^i\right) x^{\left[\left\{\left\{((18\times 2^n)-12)-\left(6\times 2^{n-1}-5\right)\right\}\times\left(6\times 2^{n-1}-6\right)\right\}-0\right]^2}$$
$$= 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-1}-36\times 2^{2n-2i}+6\times 2^{n-1}-126\times 2^n+42\right]^2}$$
$$+ 3\sum_{i=0}^{n-1} 2^i x^{\left[12\times 2^{n-i}-18\times 2^n\right]^2}$$
$$+ 6\sum_{i=0}^{n-1} 2^i x^{\left[108\times 2^{2n-1}-36\times 2^{2n-2i}-6\times 2^{n-1}-108\times 2^n+42\right]^2}$$

□

3. Conclusion

In this paper we determined cutting number Zagreb indices, Multiplicative Zagreb indices, Zagreb polynomial, Reverse hyper-Zagreb indices, Multiplicative Reverse hyper-Zagreb indices and Reverse hyper-Zagreb polynomial for Nanostar Dendrimer. It would be interesting for future study to investigate other topological indices of these Nanostar Dendrimer.

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