CETA: Designing Mixed-reality Tangible Interaction to Enhance Mathematical Learning

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ABSTRACT

The benefits of applying technology to education have been often questioned. Learning through digital devices might imply reducing the children's physical interaction with the real world, when cognitive theories hold that such interaction is essential to develop abstract concepts in Mathematics or Physics. However, conflicting reports suggest that tangible interaction does not always improve engagement or learning. A central question is how cognitive theories can be successfully applied to the design of interactive systems in order to achieve enhanced learning experiences. In this paper we discuss the interaction design of a mixed-reality system for mathematics learning for school-aged children. Our design approach combines inspiration from previous frameworks with a user-centered design process with early prototype evaluations. As a result of this process we have created a mixed-reality environment for low-cost tablets and an augmented version of the Cuisenaire rods, a milestone of the manipulatives for mathematics learning.

ACM Classification Keywords

H.5.2 Information interfaces and presentation: User interfaces; H.5.1 Multimedia Information Systems: Artificial, augmented, and virtual realities

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INTRODUCTION

Nowadays children have earlier access to digital technology. Specifically, programs such as *One Laptop Per Child* (OLPC) have provided computers and/or tablets to school children worldwide. Educational content is continuously being developed and easily spread through online platforms able to reach the remotest locations. As this technology is already deployed in the classrooms, it is reasonable to devote efforts to create content that encourages the learning process.

However, some question the learning benefits of applying digital technology in education [9]. A potential problem is that some physical interaction with the environment is replaced by mouse-keyboard or multitouch interaction without considering the impact it may have. Several theories such as constructivism, embodied cognition [5, 42] and physically distributed learning [29], support the idea that physical interaction plays a key role in the learning process [6]. The general aim of this paper is to discuss how these theories can ground the design of interactive systems to enhance learning.

Conceptually, Tangible User Interfaces (TUIs) augment everyday physical objects and environments with digital information to become interaction devices [23]. Throughout this paper we refer to this meaning, 'where physical interaction goes beyond touching a mouse, a keyboard or a (touch) screen'. Tangibility might have a different impact depending on the learning task; physics and mathematics are subjects where tangible interaction and real world observation seems to play an important role [46, 44, 28, 27]. However, there is still little formal evidence of tangibles enhancing learning, and how cognitive theories might be translated into the design of interactive systems to support an enhanced learning. To fill this gap, this paper focuses on mathematics learning and discusses the design of CETA (Ceibal Tangible), a mixed-reality system with tangible interaction for school-aged children.

We need to understand the relation between physical actions and cognitive processes to successfully design a tangible interactive system supporting learning. This means to understand and trace the relation between the physical and digital elements through actions, the system feedback and the impact of these elements on the problem solving processes. Such elements are identified and described as physical objects, digital objects, actions, informational relations and learning activities in the Tangible Learning Design Framework (TLDF) [6]. Along this paper we use this terminology and framework. Then, we can formally specify the role of each interaction element and argue its inclusion in the system, i.e., how we envision that specific design features will help users to achieve specific goals. Besides the framework the research questions proposed in [6] inspire us in the application to a mathematics learning context. We aim to answer the following questions:

Q1: In a mathematics learning activity: *how can we shape the level of abstraction by changing the actions and informational relations between physical and digital objects?*

Q2: Regarding physical actions and objects: *Which actions (such as 'pick up' or 'group') are relevant and desirable in this specific mathematical learning activity? How can we promote these actions through the design of particular affordances? Which complementary epistemic actions might be supported in order to enhance the problem solving process?*

Q3: Considering a mixed-reality system, *which is the most effective and less disruptive way to slow down the interaction pace and encourage reflection?*

We address these questions following a research through design approach. Our design was grounded in previous research results related to tangible interaction design for mathematics learning [28, 27] and informed by literature related to the use of classical manipulatives in mathematics learning [11, 12]. When theories or previous evidence were inconclusive, our design explored different possibilities following a user centered design with user tests. We discuss the CETA system and design decisions, including the application of some of the TLDF [6] guidelines. As a result of the user tests, we were able to validate previous research. Indeed, from the evidence gathered, manipulation itself is not enough to enhance learning: the modulation of the interaction pace is essential to encourage reflection between the children's actions and the system feedback. We addressed this issue through what we call 'action submit' and observed that pacing down the interaction reduced trial and error strategy and encouraged reflection. The main resulting artifact of the whole process is the CETA mixed-reality environment and the digital augmented version of the the Cuisenaire rods, which aims to the inclusion of low-cost tangible and meaningful technology in classrooms.

BACKGROUND

In this section we introduce some cognitive concepts relevant to ground our design: cognition offloading, physically distributed learning, image schemas, conceptual metaphors, and epistemic actions.

Cognitive offloading:

Operations with concepts such as mathematical ones involve the elaboration of mental representations of both abstract and concrete objects. For instance, a group of items should be conceived by assembling different elements in a joint group, being the group itself a mental representation that must be stored during a mathematical operation. Also an abstract concept such as the addition of two new units must be conceptualized demanding increasing cognitive resources (keeping in mind the meaning of this operation). Cognitive offloading refers to the possibility of lightening these cognitive demands by the inclusion of actual objects representing abstract concepts. Since these objects are available to the perceptual system they release working memory load [18, 29].

In the case of operations, actual actions over objects aid in the realization of abstract relationships facilitating mathematical thinking [19]. That is how manipulatives help to decrease cognitive load by giving place to external representations of objects and operations [27, 34].

Physically distributed Learning:

As stated above, manipulatives can aid abstract thinking when objects work as external representations of the learning concepts. For the Physically Distributed Learning theory (PDL) [29], it is crucial for the learners to have a deep understanding of the way in which concrete objects represent abstract entities. A single one to one correspondence between an object and a concept would not be sufficient. Instead, knowledge about how different objects relate to each other and how they can be rearranged would be required to represent the conceptual structure behind mathematical operations. Indeed, for PDL, a richer understanding is achieved when children are allowed to rearrange the environment (i.e., a group of objects) in order to represent the solution to a posed problem (i.e., select the fourth of the group) [29]. Thus, the environment is reinterpreted in order to reflect the abstract structure of the operation to be performed. Therefore, PDL goes beyond simple cognitive offloading, demanding a deeper comprehension of the link between an abstract structure and the structure of an interactive environment. The exploitation of such structures in a stable form has been studied under the labels of image schemas and conceptual metaphors.

Image schemas and conceptual metaphors: Some specific spatial configurations of objects and actions performed over them are typically found when abstract operations are carried out. For instance, the action of taking apart a subgroup of objects within a bigger group will be linked to the operation of subtraction [20]. These spatial arrangements and actions give place to stable external representations, which are stored in memory and can be recovered to aid the accomplishment of symbolic operations as mathematical.

Conceptual metaphors enable the understanding of abstract concepts in terms of more concrete concepts, by providing a cognitive mechanism that enables us to translate inferences made in one domain to another one. For instance, to group and to count small collections of objects can result in neural connections deriving from sensory-motor physical operations (like adding (n+1) or subtracting (n-1)), which, in turn, may result in conceptual metaphors at the neural level: from physical objects to mental operations with numbers [25]. Collections (of objects) with different magnitudes help to learn that numbers also have magnitude; bigger collections of objects represent a metaphor for bigger numbers, the smallest collection represents the number one; taking out a collection from another collection represents subtracting and so forth. These kind of analogies have been proven to be useful for intuitive interaction design [**?**].

Epistemic Actions:

Defined as complementary actions on objects that make problems easier to solve but are not necessarily part of the solution [6]. These actions are performed to exploit the advantages of offloading cognition and conceptual metaphors. Moreover, these actions may reveal information that is hidden or that is hard to compute mentally [24]. For example, rotating a *Tetris* block while we are developing a solution or rotating a map in the mobile phone to follow directions. Research in the use of manipulatives for math learning showed that concrete material fosters the discovery of more strategies to solve mathematical problems [28].

RELATED WORK

In this section we present a selection of studies that are related with the design of CETA regarding the use of tangibles or digital manipulatives and concretely, the use of TUIs in education, and the Cuisenaire rods.

TUIs or digital manipulatives Similar to the concept of TUI [23], digital paradigms and technologies applied to traditional manipulatives are known as *digital manipulatives* [33]. In [33] four computationally-augmented versions of traditional manipulatives are discussed (blocks, beads, balls and badges). Beyond the intrinsic value of the traditional manipulatives, digital manipulatives enable children to familiarize in advance with concepts related with dynamic systems.

Virtual and physical manipulatives were compared in a number partitioning task [28], making efforts to determine which is the role of the physical representation. On the one hand, benefits of virtual manipulatives are: potential to link representations, audiovisual feedback, tracking of the past actions, adaptability and availability. On the other hand, physical manipulatives offers unique benefits such as tactile feedback (size, shape and quantity up to certain limit) and proprioception which allows children to know the position of the block in relation with their body just by touching them [27]. In the case of a mixed-reality system it is possible to exploit benefits from both worlds.

Digital tangibles for education can be distinguished between "Froebel-inspired Manipulatives" (FiMs) and "Montessori-inspired Manipulatives" (MiMs) [47]. The former are building toys that enables children to design real world objects while the later are focused in the modeling of more abstract structures. According to them, TUI are useful for learning abstract concepts in the sense that they provide: sensory engagement (multimodal), accessibility (easier for younger children, novices and people with learning disabilities), and group learning (multi-hand interface enabling natural group interaction).

In TUI there could be sensible, sensable and desirable movements. "Sensible movements are those that users naturally perform; sensable are those that can be measured by a computer; and desirable movements are those that are required by a given application" [8]. In TUI design, this classification is useful in order to detect interaction conflicts and opportunities.

Augmenting the Cuisenaire rods Cuisenaire rods were created in 1952 by educator Georges Cuisenaire [13]. He was inspired in Friedrich Fröbel who had previously designed a set of wooden building blocks [15], but Cuisenaire's design consisted on smaller rods incorporating different colors for each length. However they are considered MiMs, as they allow to model abstract structures related with numbers [47]. He showed that some students who had learned using traditional methods and were rated as 'weak', when they later changed to use the manipulative rods they became 'very good' at traditional arithmetic [16]. Cuisenaire rods supports children's mathematics learning, for example, allowing them to explore and discover the concept of additive composition joining smaller rods to form larger ones.

With respect to cuisenaire rods, most of the digital approaches are *virtual manipulatives* [2, 30, 1], i.e., traditional GUI based programs where the rods are represented with graphics and children manipulate them through mouse-keyboard based interaction or in the best case using multitouch screens. Otherwise, TICLE (Tangible Interfaces for Collaborative Learning) table [35], use a mixed-reality environment that enables tangible interaction with real objects on a table and provide audiovisual feedback on a side monitor. An augmented version of Tangram (an old Chinese geometry puzzle composed by seven pieces) [37] was implemented using this device, where many children can collaborate having equal access to the device at the same time [36]. Also an application to work the concepts of odd and even numbers through Cuisenaire rods was developed [36]. While the tangible interaction proposed by this system it is valuable and allows to explore mathematical concepts in a collaborative way, the main drawback it is the size (a big table and a computer) and probably the cost of producing it and its mobility. To the best of our knowledge, this is the closest approach to develop an augmented cuisenaire rod.

Tangibles for education Other tangible interaction approaches, have been applied in learning contexts using tablets or laptops. They use a mirror in the front camera to redirect the camera vision and computer vision techniques to enable objects detection. Osmo [3] is a mixed-reality play system for iPads. It is used for different learning fields such as mathematics, physics, geometry (also through a Tangram activity) and programming. Strewbies is an Osmo based tangible game for learning programming [22]. They used the topcode vision library [21] to detect real objects, as we did later in CETA. A similar approach had been previously used together with laptops to design tangible educational contents for children with motor impairments [10].

Two previous researches conducted through the design and evaluation of educational interactive systems are especially relevant to CETA. The first one is "Towards Utopia" [7] a tangible environment to enable children to learn concepts related to land use planning and sustainable development, whose design was informed with cognitive load theory and constructivist learning theories. The thorough evaluation of the environment showed that it supports learning; and the paper provides a set of design guidelines that were included and discussed in the TLDF [6].

The second is the mixed-reality system EarthShake designed to support children's learning of physics principles [45]. It was evaluated [46, 44] through a 2x2 experiment, crossing mixed-reality vs screen-only (pure virtual) with physical or without physical control. It was concluded that the real world physical observation supported learning while the simple hands-on control (pressing a physical button or shaking a tablet) did not add learning, and the authors hypothesized that this could be because these physical controls were not relevant to the learning objectives [46, 44]. They also explain that a key component for the success of the mixed-reality system for learning enhancement is the interactive feedback. This feedback was developed as guides and a self-explanation menu synchronized with the physical world [44].

We use the TLD framework [6] to conceptualize and describe how the tangible interaction supports cognition, going from the design of the learning activity, physical and digital objects, to actions and the relation between them. It provides a taxonomy of system elements: Physical Objects, which are used to interact with the system, and have visual, haptic and optionally auditory attributes. Digital Objects with visual and auditory attributes too, and a temporal property that makes their attributes dynamically change over time. Actions, which are the set of input manipulations that users perform on physical objects or on digital objects in particular cases (e.g. multitouch) whose discoverability by users is important. Informational Relations, the mappings between physical objects, digital objects and actions, which can be perceptual (physical objects representing digital objects) or behavioral (specific actions on physical objects impacts on digital objects), and whose structures, for example the cardinalities (one-to-one or many-to-one), must be considered. And Learning Activities which frame the learner interaction with the system.

SYSTEM OVERVIEW

CETA is a mixed-reality environment inspired in OSMO. It is composed by an Android low cost tablet, a mirror, a holder and a set of wooden blocks, which play the role of manipulatives (see Figure 1). Blocks have rectangular shape and are ranged in length and divided in square sub-elements going from one to five per block. To "see" and detect the blocks on the table, the camera is redirected towards it using the mirror. The blocks become digital manipulatives through markers, which are recognized through the use of the TopCode vision library [21], which works in Android under flexible light conditions. To deal with partial occlusions, which will happen when children manipulate blocks, we implemented a cache. The cache stores the markers detected in the previous five frames, using the visible markers to infer the position and orientation of the non-visible ones and estimate the position and orientation of the entire block. We included one marker per sub-element within the blocks, e.g., block 1 has just one marker, while block 5 has five. This strategy matches the number of markers with the value of the blocks (see Figure 4-b) and detecting one marker is enough to estimate the position of a block. The software architecture is divided in three layers (Figure 2) splitting the game logic from the object detection module. At the same time, the object detection module could use any computer vision library and, for example, detect objects by color or shape instead of using a marker-based approach. All the technical description, software design and implementation are discussed in depth in [?].

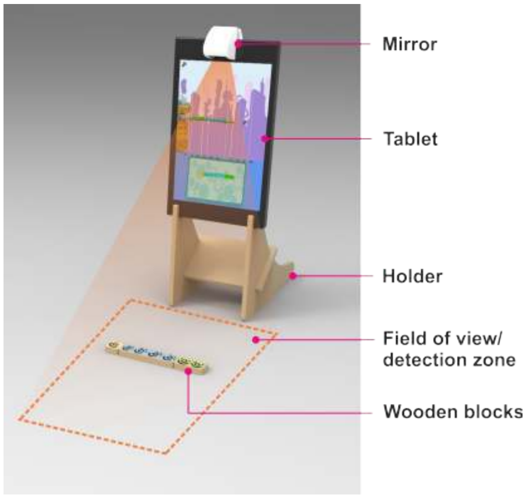

Figure 1. CETA environment setup.

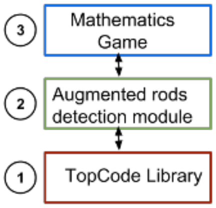

Figure 2. CETA software architecture in three layers: (1) Application, in CETA Game it is an Android Activity (2) Augmented rods detection module (3) Vision library, we used TopCode to detect markers.

DESIGN CONCEPTUALIZATION

In this section we discuss the design of CETA in terms of the five element taxonomy proposed in the TLDF [6].

Learning Activity

The goal of the game is to learn the concepts of additive composition and the number line representation. The additive composition implies understanding how numbers can be composed by smaller numbers in different ways (4=2+2, 4=1+1+1+1,

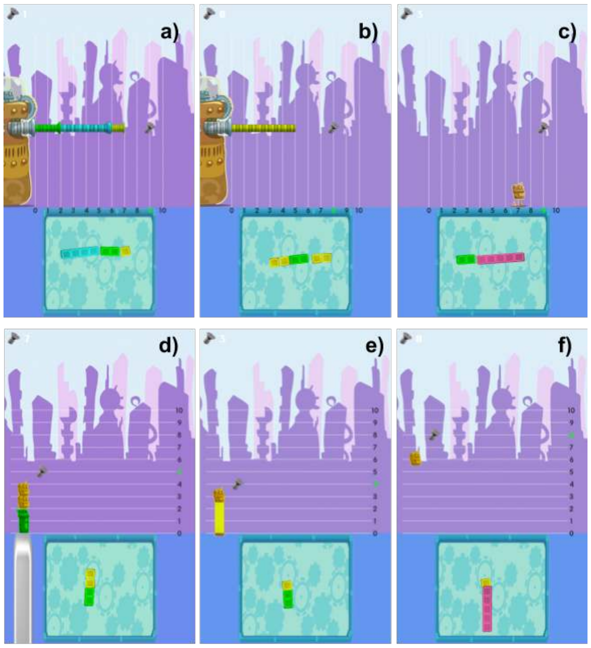

Figure 3. Stages of the CETA game. (a) Bruno composes blocks and creates a long arm to reach the screw (b) Bruno enlarges its arm to reach the screw (c) Bruno moves forward to reach the screw (d) Bruno and friends make a tower to reach the screw (e) Bruno grows to reach the screw (f) Bruno flies to reach the screw

etc.), while the number line representation requires the understanding of the order of numbers represented on a line that in general is vertical or horizontal. Both concepts are taught in the first year of elementary school to 5-6 year old children. The game narrative is about a robot called Bruno that needs to collect some screws appearing at a certain distance from it. Using the blocks, children must compose the number that matches this distance. Once they put the blocks on the table the robot will perform an action to pick the screw (see Figure 3). Horizontal and vertical orientations of the number line are used (see Figure 3 a-c, d-f). Bruno also changes its actions to reach the screws, going from more concrete to more abstract ones. This is discussed in detail in the informational relations section.

Physical Objects

We detail the physical objects design specifying which are the relevant actions in this context and how, through our design, we can promote them.

Blocks: Our block design is inspired in Cuisenaire rods [13] (Figure 4). Each rod represents a different number and has different length and color. In the original set, the smallest cuisenaire rod represents 1 and the largest 10; this mapping is linked with the image schema "shorter is less". Our design also includes sub-elements representing units, i.e., block N is composed by N sub-elements (see Figure 4-b), this variation is also popular and commonly used. Due to the interaction space constraints given by the field of view of the camera, we only included blocks from 1 to 5. Information is distributed across visual and haptic channels using different (arbitrary) color and (meaningful) size for each block. The multimodality should enhance the learning process by increasing effective working memory capacity, while conceptual metaphors based on image schemas should support learning as it is suggested in the TDLF guideline 7 [6].

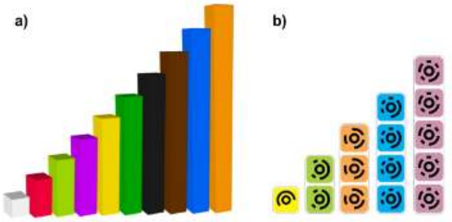

Figure 4. a)Original cuisenaire rods. b)Final design of CETA blocks

Different contemporary approaches to the use of manipulatives tend to highlight the number composition by making explicit the presence of sub units. This trend reflects a modern debate about how number is instantiated in the mind. The theories proposing a general system for magnitude, irrespective of spatial or temporal modality [41] favor analogies linking size and number, and recommend the use of manipulatives based on size. On the other hand, theoretical approaches advocating for the existence of an approximate number system [17] propose that numbers are understood as a group of items, very early in life, and recommend representations that explicitly highlight the composition of units. We followed this second approach, including a counting affordance in the composition task within the game designed.

Blocks contain magnets in their extremities providing an affordance that increases the probability of joining blocks imitating the number line representation. Physical manipulatives not only represent the object itself but also actions are required to be performed with them [32] and their design should be combined with programs to foster certain strategies [26]. Thus, the magnets play the role of facilitators of the representation. They also decrease the probability that children put blocks on top of each other, a sensible movement for some [26] but neither sensable (this action would occlude the markers of the blocks that are not on top of the stack) nor desirable for our learning activity (we encourage children to create linear representation imitating the number line) [8]. We also expected that magnets, as a novelty for children, would increase their enjoyment and engagement.

Digital Objects

The interaction zone (sector of the table) is virtually represented as a colored square. Each physical block is virtually represented through a virtual block with the same color and shape on the screen (Figure 1) below the number line. It is a scaled representation of the reality, included to help children understand how the system is interpreting their actions in a fluent and continuous way [14], not competing for user's attention and allowing him/her to focus on the consequences of the actions, and also inspired in full body interactive research [38] where it is argued that: "In unmediated full-body interactive experiences, objects should respond continuously and directly to the changing full-body gestures of users, rather than restrict the body to act as a pointer that activates buttons and widgets".

The robot itself is the most relevant digital object, it is the main character of the game and children control his actions and movements combining the blocks. In order to increase the engagement and joy we provided him a name, Bruno, and a friendly and funny appearance. Actions taken on the blocks are mapped to its shape and movements, along the levels of the game it will perform different actions in order to reach the rewards (screws), for example stretch, fly and skate. The details of this mapping are discussed in the Informational Relations section.

Actions

In CETA, children can move the blocks freely, although not all the sensible actions for them are sensable or desirable for the system [8]. Below we present the most relevant actions that may be taken with the blocks, just a subset of them are effectively interpreted as actions in the sense of TLDF and have impact on the digital objects.

The action of joining blocks has two main meanings: Group and Align. Grouping objects is related with the conceptual metaphor that putting objects close somehow adds, composes, creating a new object. Through this action children adapt and reinterpret the environment, supporting Physically Distributed Learning [29]. They also might be offloading cognition by taking action on objects [6] and by making external representation of groups [28]. This is a sensible, sensable and desirable action, and it is the most significant in our system since it represents the addition (group) and number line (align) concepts.

When the blocks are joined it is easy to visualize the result as a new block composed by smaller blocks, while at the same time each block is also composed of units. This might be interesting in order to play with the composition concept, children may visualize the result as the composition of the blocks or as the composition of the units considering the result as a big block without paying attention to the subdivisions given by the union of the blocks (Figure 6). When children align the blocks and then count the sub-units to calculate the addition, the action is considered as an epistemic action since they change the world to make the task easier, i.e., it is easier to count elements aligned than dispersed on the table. This specific action reduces the memory involved and the probability of error in mental computation[24]. This type of interaction enhances children's conceptual learning possibilities [**?**, **?**].

Blocks can be moved individually or in groups, using one or many fingers, or even with the edge of the hand. During the movement, occlusions can occur and therefore the system cannot momentarily detect blocks, but this is overcome when the child moves their hand releasing the block. Rotations are the most meaningful within our game since they enable to interchange the horizontal and vertical representations of the number line. The most obvious and direct impact on the digital objects is given by the virtual blocks since they are a one-to-one mapping of the physical blocks. However, more sophisticated interpretations could be done, for example constrain children to orientate the blocks with the orientation of the number line on the screen.

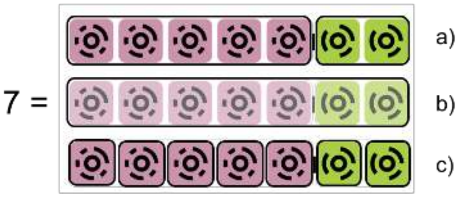

Figure 5. Different perspectives of 5 and 2 making up number 7. a) 7 = 5 + 2, b) 7 as a single bigger block, c) 7 = 1+1+1+1+1+1+1.

Putting blocks away is not sensable and the system does not interpret this action explicitly, but this is the aim of the movement. Children put pieces away to exclude them from the solution [28]. This lack of sensing is an opportunity rather than a problem. Through this action children might offload cognition since it is used to exclude blocks from the solution, and if they want they can put it back in a visible place.

Stacking blocks is a sensible movement for children [28] but given the implementation of the system are not sensable. To learn the concept of number line representation stacking objects is not desirable and, as it was explained before, magnets reduce the probability of this action being taken. However, for the addition concept this might be another way to offload cognition by making external representation of groups [28] and would be a plus if the system was able to interpret it. With the actual computer vision approach, the main drawback is that stacks tend to occlude objects behind them and children may not realize, and as a consequence the natural interaction could be affected since they could be more concerned about the camera vision than about the problem solving.

Another action is telling the system that the actual configuration of blocks on the table has to be processed and interpreted as the solution proposed by the child. In typical interfaces this is commonly carried out through clicking OK, Send, or Submit buttons. In a tangible paradigm, an approach might be to continuously process the current physical objects situation as a solution, i.e., to consider that the child is proposing a solution all the time. However, in a learning context where users have to solve problems, this strategy might not be recommendable because the reflection time is suppressed. Indeed, higher interaction pace may enable the exploration of many different solutions but reduces the reflection time [31]. Manches et al. [28] suggest that adding delays between actions is a good strategy to foster reflection, while Antle et al. [6] recommend to use spatial, physical, temporal or relational properties to slow down the interaction pace and trigger reflection.

Within our design, if the blocks on the table are not moved by the child for one second, a countdown appears on the screen and when it finishes the solution is represented. If the blocks are moved while the countdown is going on, it is automatically canceled. This strategy seems more natural and consistent with the interaction paradigm than having to touch the screen or use a special block as a "send button" as proposed in other systems [39].

Informational relations

As informational relations, we use three mappings between physical and digital objects and actions at different levels of abstraction:

A: *One to one, object to object:* Each block is represented through a single robot or a part of the robot of proportional size. For example, one block of size 2 and one of size 1 are represented through two robots, one on top of the other with size proportional to that of their physical counterparts (Figure 3-a and 3-d).

B: *Many to one, object to object:* Several blocks are mapped to the height or length of a single robot: One block of size 3 and one of size 2 resulting in a robot size 5 without visible subdivisions. This mapping is more abstract since the addition of two numbers is represented (Figure 3-b and 3-e).

C: *Many to one, object to action:* The blocks are mapped to actions of the robot, not objects. Placing one block of size 3 and one of size 1 makes the robot going up or forward 4 units. It is more abstract, as the robot moves in terms of an addition (Figure 3-c and 3-f).

We designed the game narrative through a path from more concrete to more abstract informational relations.

USER TESTS METHODS

We carried out two informal user tests with school age children in their everyday context, to validate the design of the game, as it was evolving and to test different design alternatives, in order to make more informed design decisions.

Participants: Both user tests took place in a public school in Montevideo, Uruguay, with first grade students, aged five to six. The second user test took place 6 weeks after the first one. The One Laptop Per Child program, have provided all children in this school, and all public schools in Uruguay a low end android tablet. Concretely, children from this school have the tablets since 2013. A total, of 19 children, nine girls and 10 boys participated in the study. 10 children participated in the first user test and 18 during the second one, from whom nine participated in the first user test. This difference in numbers is explained by the absence of many children on the day of the first test due to inclement weather. The user tests took place in a classroom using tables and chairs designed for children. Parents were previously informed of the activity and they provided consent for their children to participate.

Levels: During each user test, each child had a turn to play the CETA game individually. In the first user test the game had one level that used the one-to-one object-object mapping and two problems to solve. In the second user test, the game had three levels, and 6 problems to solve, each one using one of the mappings described above, and each level with one with horizontal and one vertical problem. The duration of each user test depended on the child but in average it was around 10 minutes. There was no previous training but in the second user test some of the participants had already participated in the first user test.

During the first user test, as the system was in a very early stage, we used the wizard of oz strategy. In this case, one member of the team performed as a wizard. He was situated behind the child and using a second tablet. He reproduced the same block configuration than the child had in a custom multitouch application and it was communicated to the child's tablet using the OSC protocol [43]. For the second user test we already had the computer vision system working so we used it to detect the tangible blocks.

Data collection and Analysis User test were conducted by a six-person team which organized the logistics , acted as participant observers and recorded the sessions. Personal observations were registered in the observations notes of each researcher, during and / or after the user test, depending on their role. This information was only reachable by the researchers themselves. After each user test, researchers had a debriefing meeting, to make design decisions, based on the annotations from all of them and of video analysis.

OBSERVATIONS

In the following lines we present the more common observations, and the design decisions made based on the two iteration user test. Following the same structure of the game presentation, we introduce the observations under the TLDF categories [6]. Some of these design decisions were implemented for the second user test, and others are to be implemented in a future version of the prototype.

Learning Activity Regarding the learning activity, which is understanding how numbers can be composed, we identified three key points that should be improved.

Identifying the goal: In both user tests children understood that they had to compose a number using blocks. In the first user test the target number was indicated with the position of a screw, and in some cases, it was not easy to identify. Considering that could be a limitation, we decided to highlight the selected number in the number line for the second user test (see Figure 3). As a result, we observed that children somehow simplified the task in two steps, they looked at the highlighted number and composed it with the blocks. We wonder if this could mean a limitation since the screw and/or number line might not be perceived, or at least not actively used when developing the solution.

Selecting vertical or horizontal arrangement: For the first user test we only considered horizontal number line arrangements and the one-to-one, object-to-object mapping. During the second user test the prototype was more advanced and we were able to test both number line orientations (horizontal and vertical) and the three mappings. In the first user test we observed that most children followed the horizontal orientation with the physical blocks. Thus, we hypothesized that this was because they were imitating the orientation of the number line on the screen. However, during the second user test we observed that even when the vertical number line was shown on the screen they still set the blocks in the horizontal orientation. Thus, it is not clear to what extent children's actions can be shaped through on-screen examples.

Closing each independent task: The game presented the problems consecutively, and sometimes children forgot that there were already blocks from the previous solution on the table. For example, in the previous problem they composed a 5 using a block of size 3 and a block of size 2, and in the new problem the system is asking for a 6, sometimes in this case they added a block of size 4 and a block of size 2 considering that just the last two blocks were going to be processed by the system, but in fact they were presenting 11 (3+2+4+2). In this case it might be desirable that the system could detect the situation and show a hint for either clean the table or re use the blocks of the previous problem. To mitigate this drawback the system might test if, the blocks of the previous solution are still in the same place and if the proposed solution is equal to the previous one plus the actual solution. When both conditions are true, the system should display a hint to suggest the removal of the previous blocks.

In accordance with previous research [28, 44] it seems that considering just the working materials is not enough, the context and how the activity is presented and guided through helps and hints play a key role. As a general implication we recommend designing the game/activity to guide and encourage the child to accomplish the goals, this might include providing hints and unlocking children when commit common errors.

Physical Objects Three features of the physical objects were tested during the user tests: Size, magnets and the subdivision in sub-elements.

Block Size: Two different block sizes were tested in the first user test but no differences were observed (unit square side: 1,5cm or 2cm). The smaller size (1.5cm side) was successfully used for the second user test. We conclude that blocks from 1,5cm (block 1) to 7,5cm (block 5) are suitable. Children can manipulate them easily and they are small enough to detect 10 units in-line (horizontal and vertical) within the field of view of the camera.

Magnets: In both user tests we observed that most children took advantage of the magnets to join blocks. Magnets suggest the in-line join of the blocks which is relevant in this context since we are working with the number line. Almost no child put blocks in a stack, which would be sensible but non sensable or desirable. The main drawback observed is that children are disturbed when magnets repel. They keep trying to join them shifting the attention focus. To overcome this drawback we might design asymmetric blocks that can only be joined by the extremities of opposite polarities (see Figure 11). In some cases children did not align the blocks using the magnets. However, it was not considered a major problem for the learning activity. Moreover, requiring a precise alignment of the blocks might reduce the enjoyment and therefore has a negative impact on the user experience.

Blocks sub-elements: Different to the Cuisenaire rods, in the first user test we introduced colored squares within the blocks as sub-elements (see Figure 8-a) and we observed that most of the children used them to count. In the second user test we included in each sub-element a marker of the TopCode computer vision library (see Figure 8-b). We observed that the inclusion of these markers has no negative effect and that all the children used the sub-elements to count. We conclude that the division of the block in sub-elements is very useful for children and that the markers have no negative effects that interfere with the task. Touching blocks is a strategy to offload cognition, and joining blocks is an epistemic action performed to solve the problem. We observed extensive use of both strategies. From this observation we might derive two general implications, the first one is the inclusion of sub-elements as a valuable feature of Cuisenaire rods, and the second one is that vision-based systems must support partial occlusions of the physical blocks since touching is a valuable offloading action while children resolve mathematical problems.

Height: In the first user test we observed that when the tablet is on the table, the children slightly tilted down their heads, causing ergonomic discomfort. To solve this problem, we lifted the tablet, with a box (see Figure 9), and this phe-nomenon was reduced. Using the tablet at a higher position, also expands the field of view of the camera. For the second user test we used a higher tablet holder. We did not observe any inconvenient related to the height of the device. The user test took place in the classroom using children's every day tables and chairs. We realized that ergonomic considerations should be taken into account to adjust the height of the tablet in relation with the height of the children and the furniture used. In this sense, it would be ok to have an adjustable tablet holder that can be adjusted if needed.

Digital Objects Observations of the digital objects might be split in those related to the virtual objects that provide continuous feedback, and the robot as the digital object where actions with blocks are mapped to reach the rewards. They are represented in the bottom and the upper area of the screen respectively.

Virtual Blocks: In both user tests we observed that children do not pay special attention to the virtual representation of the blocks on the screen. A possible explanation is that while they are manipulating the physical blocks and therefore developing the solution of the problem, they do not see the screen and most of the times they do not perceive the continuous feedback given by the virtual blocks.

Interaction Area: The virtual representation of the interaction area is included in the system in order to help children to infer the real interaction space constrained by the field of view of the camera. We tested both conditions, using a sheet to delimit the interaction space (i.e the field of view of the camera) and without the sheet (see Figure 9).

During the first user test we observed that with the sheet the children understood better the interaction area limits. As for this user test we used the wizard of oz technique, therefore, the system feedback was not as continuous and fluid as it should be. For this reason we hypothesized that with real time feedback of the virtual blocks, children would be able to infer the detection area after some tries.

For the second user test, in order to help the users to understand which are the boundaries of the detection area, we designed a fade-off behavior: when the blocks get close to the vision boundary, the virtual representation on screen starts to gradually disappear. However, despite the efforts, we still observed that without the sheet of paper on the table, most of the children do not infer the detection zone on their own, but they needed some help.

We realized that the inclusion of a physical object to delimit the working zone is required. We did it with a paper on the table. However, the main potential drawback is that the sheet could be damaged and that the position is relative to the tablet, but this can be solved by attaching the defined area to the tablet holder.

Possible improvements might be to include on-screen animations to help the children realize that the block is on the boundary (for example arrows pointing to the center) of the detection zone or explicitly explain the existence of these boundaries in a tutorial at the beginning of the game.

Auditive Feedback: We just tested auditive feedback during the second user test and it was just background music and basic sound effects when the robot reached the screws. For this reason, we did not directly observe any conclusive behavior related to the auditive channel. However, we gained some insights related to potential uses of it. For example, we observed that as the setup splits children's attention between the table and the tablet, sometimes they miss events on the screen because they are looking or manipulating the physical blocks. However, as the auditive channel is not affected it could be exploited to provide feedback reducing this negative effect. This seems to point towards a more general implication: in environments where the visual attention is split, the auditive channel might be exploited as a complementary strategy. It is expected that after some trials, children could learn that when they hear a certain sound something is happening on the screen.

Robot: The robot actions were understood by the children. Actually, these actions allowed them to realize if they had achieved the goal or not. In future versions of the game we will design a more active behavior adding animations and hints coming from the robot.

Actions Three main actions are key to achieving the objectives of the game: grouping and aligning the blocks, the third one is submit. The grouping and aligning actions are already described in the physical objects section since they are encouraged by the magnets. In the following lines we discuss the submitting action which is probably the one with the biggest impact on learning.

Action Submit (through countdown): During the first user test this property was not faithfully assessed, since with wizard mediation, countdown could have a great variation. However, we observed that without countdown the child could solve the problem without realizing it, e.g. he puts the random blocks on the table without watching the goal on the screen, if by chance this is a good solution, the system understands this is the children's answer and activate the robot movement.

In the second user test, we controlled the countdown time. Concretely, we used a two-second countdown starting after one second that the blocks are still. We observed that it takes two or three trials before children realizes that after the countdown the robot performs an action. In order to facilitate this understanding, explicit instructions could be shown in a short tutorial at the beginning of the game. In some cases, children pick the blocks before the countdown finishes. To avoid this, we can play a sound and add an animation of the robot getting ready to perform the action in order to attract their attention. This might be very useful to guide children during the first trials. In general the two second countdown is not excessive. However, shorter times might be tested in order to achieve a more fluent interaction without losing the pause for reflection. Playing a sound at the beginning of the countdown might help to highlight the event and also to attract the attention if the child is not looking at the screen.

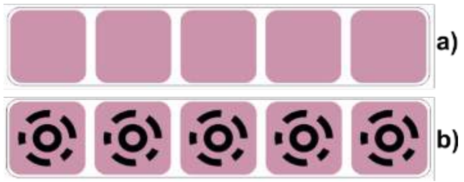

Figure 6. a) Sub-elements as colored squares b) Sub-elements including the TopCode markers.

Thus, without the action submit countdown many times children resolved the problems by chance. The system processes the blocks immediately favoring a trial and error strategy, placing different blocks until the problem is solved. However, there is no reflection in this process. This observation is consistent with previous research [31, 6, 28], although it is important to note that the delay must be introduced between children's actions and the system's response when such response is relevant for the learning goal, i.e., in our case when the system interpret the blocks and performs the addition. The continuous feedback and fluency of the system must no be affected by this delay.

Informational Relations As it was explained before, during the first user test we just had the one-to-one, object-to-object mapping. During the second user test we tested all the mappings between physical objects, actions and digital objects. In general, children understood the different shapes and movements of the robot as it is the key element that allowed them to determine if they had reached the reward or not. As a possible design issue we observed that the action of the robot is not strictly linked with the development of the solution, this means that children can reduce the task to first identifying the number and then composing it with the blocks. We do not know if the action that the robot performs to reach the reward is being perceived by children and if it has a real impact on the level of abstraction of their reasoning, which is in fact our intended purpose. In order to find out further and specific user test might be done.

DISCUSSION

As it was already suggested in previous research [6, 31, 28], the interaction pace has a determinant impact in the reflection during the learning process. In this particular context

Figure 7. Children using the sub-elements to count while they play with CETA. Left: Without delimiting the interaction area. Right: Using the sheet of paper to delimit the interaction area.

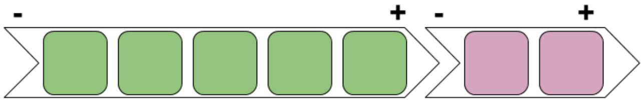

Figure 8. The block's shape constraints the way that it can be joined ensuring that magnets will not repeal.

we observed that when the system does not provide delays between children's proposed solutions (physical blocks configuration) and system evaluation (feedback), the strategy tends to be more like trial and error rather than mediated. Manipulating physical blocks might allow children to 'dive-in' and explore, while the delay gives place to 'step-out' and reflect, ideally leading them through the "ongoing dance" composed by diving-in and stepping-out described by Ackermann [4].

Despite having carefully conducted a theory-based design of our artifact, significant insights were gained during the user test, either validating the initial design or providing valuable feedback to improve it. In this way, we support the idea that prototyping is a fundamental practice that should not be skipped since it enables the active exploration evoking *kinaesthetic creativity* [40] in both users and researchers. It also allows us to observe how theories work when they are put in practice.

Regarding the research question Q1, we managed to design different mappings between physical objects, digital objects and actions in order to modulate the level of abstraction during the game. However, the children's strategy is not affected since the abstraction is given during the feedback phase, i.e the robot actions, and it does not require them to change their actions at anytime, i.e children might limit their strategy just by looking at highlighted number on the screen (see Figure 3) and then represent it with the blocks. As a consequence, it is not clear if children are effectively perceiving it and therefore if it has any impact in the learning experience. Further research needs to be conducted in order to determine the impact of our mappings design modulating the abstract level.

CONCLUSIONS

We have developed the CETA environment, a mixed-reality environment for low-cost tablets and used in the design of the CETA game, a mathematics learning activity for school-aged children. We took advantage of the already deployed tablets in public schools as a part of the OLPC program, aiming to enhance the learning process through the use of augmented manipulatives. The development cost is significantly low and the source and design files are open and accessible at `https://github.com/smarichal/ceta`. Therefore CETA, the environment and the game, might be a significant step towards bringing tangible technology to classrooms.

We have presented the concept of the CETA game, it is an augmentation of one of the most basic manipulatives applied in learning, the Cuisenaire rods. In the conceptualization of interaction, we have considered relevant cognitive theories, including the cognitive offloading, the conceptual metaphors and epistemic actions. In the design of the game, we slightly changed physical design of the Cuisenaire rods, including magnets in the extremities as an affordance to join the rods. The mixed-reality approach enables adding digital representations, and therefore incorporating or changing properties that could not be changed in the real object, for example adding sounds and changing sizes and colors. In addition, system feedback guides and helps children to understand the goals and if they are performing well. Lastly, digital systems and learning through games always mean an extra motivation for children, increasing the engagement and joy.

Initial prototypes of the system were tested with school-aged children in their school context, and their experiences contributed in the design of the system. Through this design we addressed three research questions related to the design of a tangible system for Mathematics learning with school-aged children. This includes the design of the learning activity, physical and digital objects, the actions and the informational relations following the TLDF [6].

Q1: *how can we shape the level of abstraction by changing the actions and informational relations between physical and digital objects?* Our approach was to change the structure of the mappings achieving three levels of abstraction altogether. The most basic and concrete one is the representation of each physical object with a single digital object, i.e., one robot per block or one subdivision of the robotic arm per block. The intermediate level is where many physical objects are represented by the shape of a single digital object, i.e., a composition of blocks determines the arm's length or height of the robot. Lastly, there is the most abstract level in which many physical objects are mapped to actions on the digital objects, i.e., a composition of blocks make the robot skate or fly the same distance as the composed number.

Q2: *Which actions are relevant and desirable in this specific mathematical learning activity?* In our case, given that the learning goal is additive composition and number line, the most relevant actions on physical objects are composing groups and align them imitating the number line. To this aim, we designed objects with magnets in the extremities, a specific affordance to create groups and align the blocks. We observed that some children first join the blocks and then count the sub-elements in order to compute the sum, this is an epistemic action supported by the system and relevant for the problem solving strategy.

Q3: *Which is the most effective and less disruptive way to slow down the interaction pace and encourage reflection?* Adding delays between children's actions and the system evaluation and response is a non disruptive strategy to slow down the interaction pace. Most children understood it and we observed that it encouraged reflection instead of trial and error strategies. These delays should not affect the continuous feedback of the system, children should realize that the system is processing the information and that they must wait.

LIMITATIONS AND FUTURE WORK

Although the user experience would benefit from a virtual representation of the blocks (see Figure 1) that would be displayed in the same space as the physical blocks in order to integrate the input and output space and support exploration [7], we used a different approach. To settle this issue, we provided continuous feedback, however, children do not perceive it until they look at the tablet screen. This requires them to lift their head and somehow "change the context", probably disrupting their active exploration. Otherwise, given the implementation of the system using a computer vision approach, we can not support the total occlusion of the blocks. As a consequence, some positive properties of manipulatives might be affected, including *proprioception* and *haptic subitizing* [27]. What we want to stress here is that the technology is not completely seamless and that it might be constraining some aspects of the natural interaction that children have with physical objects, and in some cases forcing them to adapt to the system. Although there could be better technologies to accomplish these goals, we focused on using the limited features of the low-end tablets already distributed in all public schools in Uruguay and other places where programs such as OLPC have arrived, in order to take advantage of this infrastructure.

Besides we conducted two field studies with prototypes and children, we did not follow a rigorous methodology to formally evaluate the different possibilities of the interactive system. We did a first approximation based on observation of some specific features conducting an exploratory study in real life settings. However, it was useful to make basic design decisions that complemented all the theoretical background behind each design option.

Based on the results of the two user tests, we will improve the system including features such as adding hints to guide children through the activity or when they get stuck and change the physical block design to avoid blocks to repeal (see Figure 11). In addition, we plan to make extensive use of the auditive channel to mitigate the drawback of having separated input and output spaces, and also to reinforce the sense of magnitude mapping sounds with each block following a similar strategy than with the size, but in this case through the image schema "louder is more", i.e., bigger blocks will be mapped with louder sounds.

Once the final prototype will be developed, a multimodal evaluation approach [31] would be useful to analyze the embodied interaction and formally classify and describe the interaction features of the system. In addition, an evaluation of the learning outcome using pre-test and post-test is required to compare our system with non tangible digital approaches, i.e pure virtual, or with traditional methods employed in schools, once the final prototype is developed. Such evaluations, although interesting, were beyond the purpose of this paper, which focuses on the conceptual design of the mixed reality system and the game.

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