

# Challenges for $\Lambda$ CDM: An update

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A number of challenges to the standard  $\Lambda$ CDM model have been emerging during the past few years as the accuracy of cosmological observations improves. In this review we discuss in a unified manner many existing signals in cosmological and astrophysical data that appear to be in some tension ( $2\sigma$  or larger) with the standard  $\Lambda$ CDM model as specified by the Cosmological Principle, General Relativity and the Planck18 parameter values. In addition to the well-studied  $5\sigma$  challenge of  $\Lambda$ CDM (the Hubble  $H_0$  tension) and other well known tensions (the growth tension, and the lensing amplitude  $A_L$  anomaly), we discuss a wide range of other less discussed less-standard signals which appear at a lower statistical significance level than the  $H_0$  tension some of them known as ‘curiosities’ in the data) which may also constitute hints towards new physics. For example such signals include cosmic dipoles (the fine structure constant  $\alpha$ , velocity and quasar dipoles), CMB asymmetries, BAO Ly $\alpha$  tension, age of the Universe issues, the Lithium problem, small scale curiosities like the core-cusp and missing satellite problems, quasars Hubble diagram, oscillating short range gravity signals etc. The goal of this pedagogical review is to collectively present the current status (2022 update) of these signals and their level of significance, with emphasis on the Hubble tension and refer to recent resources where more details can be found for each signal. We also briefly discuss theoretical approaches that can potentially explain some of these signals.

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## I Introduction

The concordance or standard  $\Lambda$  Cold Dark Matter ( $\Lambda$ CDM) cosmological model (Carroll 2001; Peebles 1984; Peebles and Ratra 2003) is a well defined, predictive and simple cosmological model (see Bull *et al.* 2016, for a review). It is defined by a set of simple assumptions:

- The Universe consists of radiation (photons, neutrinos), ordinary matter (baryons and leptons), cold (non-relativistic) dark matter (CDM) (Bertone *et al.* 2005; Bosma 1981; Freeman 1970; Rubin *et al.* 1980; Rubin and Ford 1970; Zwicky 1933, 1937) being responsible for structure formation and cosmological constant  $\Lambda$  (Carroll 2001; Carroll *et al.* 1992), a homogeneous form of energy which is responsible for the late time observed accelerated expansion. The cosmological constant is currently associated with a dark energy or vacuum energy whose density remains constant even in an expanding background (see Padmanabhan 2003, 2005; Peebles and Ratra 2003; Weinberg 1989, for a review).
- General Relativity (GR) (Einstein 1917) is the correct theory that describes gravity on cosmological scales. Thus, the action currently relevant on cos-

mological scales reads

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} (R - 2\Lambda) + \frac{1}{4\alpha} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_m(\psi, A) \right] \quad (1.1)$$

where  $\alpha$  is the fine structure constant,  $G$  is Newton's constant,  $F_{\mu\nu}$  is the electromagnetic field-strength tensor and  $\mathcal{L}_m$  is the Lagrangian density for all matter fields  $\psi_m$ .

- The Cosmological Principle (CP) states that the Universe is statistically homogeneous and isotropic in space and matter at sufficiently large scales ( $\gtrsim 100 \text{ Mpc}$ ).
- There are six independent (free) parameters: the baryon  $\omega_b = \Omega_{0b} h^2$  and cold dark matter  $\omega_c = \Omega_{0c} h^2$  energy densities (where  $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  is the dimensionless Hubble constant and  $\Omega_X \equiv \rho_X/\rho_{crit}$  is the density of component  $X$  relative to the critical density,  $\rho_{crit} = 3H^2/8\pi G$ ), the angular diameter distance to the sound horizon at last scattering  $\theta_s$ , the amplitude  $A_s$  and tilt  $n_s$  of primordial scalar fluctuations and the reionization optical depth  $\tau$ .
- The spatial part of the cosmic metric is assumed to be flat described by the Friedmann-Lemaître-Roberson-Walker (FLRW) metric

$$ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (1.2)$$

which emerges from the CP.

Assuming this form of the metric and Einstein's field equations with a  $\Lambda$ -term we obtain the Friedmann equations which may be written as

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \quad (1.3)$$

$$\ddot{a} = -\frac{4\pi G}{3} (\rho + \frac{3p}{c^2}) + \frac{\Lambda c^2}{3} \quad (1.4)$$

where  $a$  is the scale factor  $a = \frac{1}{1+z}$  (with  $z$  the redshift). The cosmological constant may also be viewed as a cosmic dark energy fluid with equation of state parameter

$$w = \frac{p_\Lambda}{\rho_\Lambda} = -1 \quad (1.5)$$

where  $\rho_\Lambda$  and  $p_\Lambda$  are the energy density and the pressure of the dark energy respectively.

- A primordial phase of cosmic inflation (a period of rapid accelerated expansion) is also assumed in order to address the horizon and flatness problems (Albrecht and Steinhardt 1987; Guth 1981; Linde

(1982; Starobinsky 1987). During this period, Gaussian scale invariant primordial fluctuations are produced from quantum fluctuations in the inflationary epoch.

Fundamental generalizations of the standard  $\Lambda$ CDM model may be produced by modifying the defining action (1.1) by generalizing the fundamental constants to dynamical variables in the existing action or adding new terms. Thus the following extensions of  $\Lambda$ CDM emerge:

- Promoting Newton's constant to a dynamical degree of freedom by allowing it to depend on a scalar field  $\Phi$  as  $G \rightarrow G(\Phi(r,t))$  where the dynamics of  $\Phi$  is determined by kinetic and potential terms added to the action. This class of theories is known as 'scalar-tensor theories' with its most general form with second order dynamical equations the Horndeski theories (Deffayet *et al.* 2011; Horndeski 1974) (see also Kase and Tsujikawa 2019; Kobayashi 2019, for a comprehensive review).
- Promoting the cosmological constant to a dynamical degree of freedom by the introduction of a scalar field (quintessence) with  $\Lambda \rightarrow V(\Phi(r,t))$  and the introduction of a proper kinetic term.
- Allowing for a dynamical Fine Structure Constant (Maxwell Dilaton theories) with  $\alpha \rightarrow \alpha(\Phi(r,t))$  (Barrow and Graham 2013; Barrow and Lip 2012; Barrow *et al.* 2002; Bekenstein 1982; Sandvik *et al.* 2002) (see also Martins 2017, for a review).
- Addition of new terms to the action which may be functions of the Ricci scalar, the torsion scalar or other invariants ( $(f(R), f(T), \dots)$ ) (Cai *et al.* 2016; Capozziello 2002; De Felice and Tsujikawa 2010; Ferraro and Fiorini 2007; Nesseris *et al.* 2013; Nojiri and Odintsov 2006, 2011; Sotiriou and Faraoni 2010; Starobinsky 1987).

The  $\Lambda$ CDM model has been remarkably successful in explaining most properties of a wide range of cosmological observations including the accelerating expansion of the Universe (Perlmutter *et al.* 1999; Riess *et al.* 1998), the power spectrum and statistical properties of the cosmic microwave background (CMB) anisotropies (Page *et al.* 2003), the spectrum and statistical properties of large scale structures of the Universe (Bernardeau *et al.* 2002; Bull *et al.* 2016) and the observed abundances of different types of light nuclei hydrogen, deuterium, helium, and lithium (Cyburt *et al.* 2016; Iocco *et al.* 2009; Schramm and Turner 1998; Steigman 2007).

Despite of its remarkable successes and simplicity, the validity of the cosmological standard model  $\Lambda$ CDM is currently under intense investigation (see Abdalla *et al.* 2022; Anchordoqui *et al.* 2021; Buchert *et al.* 2016; Di Valentino *et al.* 2021c; Schmitz 2022; Schöneberg *et al.* 2021, for a review). This is motivated by a range of profound theoretical and observational difficulties of the model.

The most important theoretical difficulties that plague  $\Lambda$ CDM are the fine tuning (Burgess 2015; Martin 2012; Weinberg 1989) and coincidence problems (Steinhardt 1997; Velten *et al.* 2014). The first fundamental problem is associated with the fact that there is a large discrepancy between observations and theoretical expectations on the value of the cosmological constant  $\Lambda$  (at least 60 orders of magnitude) (Copeland *et al.* 2006; Martin 2012; Sola 2013; Weinberg 1989) and the second is connected to the coincidence between the observed vacuum energy density  $\Omega_\Lambda$  and the matter density  $\Omega_m$  which are approximately equal nowadays despite their dramatically different evolution properties. The anthropic principle has been considered as a possible solution to these problems. It states that these 'coincidences' result from a selection bias towards the existence of human life in the context of a multiverse (Susskind 2003; Weinberg 1987).

In addition to the above theoretical challenges, there are signals in cosmological and astrophysical data that appear to be in some tension ( $2\sigma$  or larger) with the standard  $\Lambda$ CDM model as specified by the Planck18 parameter values (Aghanim *et al.* 2020a,e). The most intriguing large scale tensions are the following<sup>1</sup> (Abdalla *et al.* 2022) (see also Di Valentino *et al.* 2021f,g, for a recent overview of the main tensions):

- **The Hubble tension ( $> 4\sigma$ ):** (see Section II) Using a distance ladder approach, the local (late or low redshift) measurements of the Hubble constant  $H_0$  are measured to values that are significantly higher than those inferred using the angular scale of fluctuations of the CMB in the context of the  $\Lambda$ CDM model. Combined local direct measurements of  $H_0$  are in  $5\sigma$  tension (or more if combinations of local measurements are used) with CMB indirect measurements of  $H_0$  (Di Valentino 2021; Riess 2019; Wong *et al.* 2020). The Planck/ $\Lambda$ CDM best fit value is  $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Aghanim *et al.* 2020e) while the local measurements using Cepheid calibrators by the SH0ES Team indicate  $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$  ( $\sim 5\sigma$ ) (Riess *et al.* 2021b) (see Di Valentino *et al.* 2021c; Saridakis *et al.* 2021; Shah *et al.* 2021, for a review). In the previous analysis by the SH0ES Team (Riess *et al.* 2021a) using the Gaia Early Data Release 3 (EDR3) parallaxes (Gaia Collaboration *et al.* 2020) a value of  $H_0 = 73.2 \pm 1.3$  is obtained, at a  $4.2\sigma$  tension with the prediction from Planck18 CMB observations. A wide range of local observations appear to be consistently larger than the Planck/ $\Lambda$ CDM measurement of  $H_0$  at various levels of statistical significance (Di Valentino 2021; Riess 2019; Wong *et al.* 2020). Theoretical models

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<sup>1</sup> We use the term 'curiosity' as a term describing a discrepancy between datasets in  $\Lambda$ CDM best fit parameter values at a level with a statistical significance  $\lesssim 3\sigma$ .

addressing the Hubble tension utilize either a recalibration of the Planck/ΛCDM standard ruler (the sound horizon) assuming new physics before the time of recombination (Agrawal *et al.* 2019; Karwal and Kamionkowski 2016; Poulin *et al.* 2019) or a deformation of the Hubble expansion rate  $H(z)$  at late times (Alestas *et al.* 2020a; Di Valentino *et al.* 2016b) or a transition/recalibration of the SNIa absolute luminosity due to late time new physics (Marra and Perivolaropoulos 2021) (see in Perivolaropoulos 2021a, for a relevant talk). Also, for more detailed discussions of the proposed new-physics models see in (Anchordoqui *et al.* 2021; Di Valentino *et al.* 2021c; Schöneberg *et al.* 2021; Verde *et al.* 2019).

- **The growth tension ( $2 - 3\sigma$ ):** (see Subsection III.1) Direct measurements of the growth rate of cosmological perturbations (Weak Lensing, Redshift Space Distortions (peculiar velocities), Cluster Counts) indicate a lower growth rate than that indicated by the Planck/ΛCDM parameter values at a level of about  $2 - 3\sigma$  (Abbott *et al.* 2018d; Basilakos and Nesseris 2017; Joudaki *et al.* 2018b). In the context of General Relativity such lower growth rate would imply a lower matter density and/or a lower amplitude of primordial fluctuation spectrum than that indicated by Planck/ΛCDM (Kazantzidis and Perivolaropoulos 2018; Macaulay *et al.* 2013; Nesseris *et al.* 2017; Skara and Perivolaropoulos 2020).
- **CMB anisotropy anomalies ( $2 - 3\sigma$ ):** (see Subsection III.2) These anomalies include lack of power on large angular scales, small vs large scales tension (different best fit values of cosmological parameters), cold spot anomaly, hints for a closed Universe (CMB vs BAO), anomaly on super-horizon scales, quadrupole-octopole alignment, anomalously strong ISW effect, cosmic hemispherical power asymmetry, lensing anomaly, preference for odd parity correlations, parity violating rotation of CMB linear polarization (cosmic birefringence) etc. (see Akrami *et al.* 2020b; Schwarz *et al.* 2016, for a review).
- **Cosmic dipoles ( $2 - 5\sigma$ ):** (see Subsection III.3) The large scale velocity flow dipole (Kashlinsky *et al.* 2009; Watkins *et al.* 2009), the Hubble flow variance in the cosmic rest frame (Wiltshire *et al.* 2013), the dipole anisotropy in radio source count (Bengaly *et al.* 2018), the quasar density dipole (Sectrest *et al.* 2021) and the fine structure constant dipole (quasar spectra) (King *et al.* 2012; Webb *et al.* 2011) indicate that the validity of the cosmological principle may have to be reevaluated.
- **Baryon Acoustic Oscillations (BAO) curiosities ( $2.5 - 3\sigma$ ):** (see Subsection III.4) There is a discrepancy between galaxy and Lyman- $\alpha$  (Ly $\alpha$ ) BAO at an effective redshift of  $z \sim 2.34$  (Addison *et al.* 2018; Cuceu *et al.* 2019; Evslin 2017).
- **Parity violating rotation of CMB linear polarization (Cosmic Birefringence):** (see Subsection III.5) The recent evidence of the non zero value of birefringence poses a problem for standard ΛCDM cosmology and indicates a hint of a new ingredient beyond this standard model. In particular using a novel method developed in Minami (2020); Minami and Komatsu (2020b); Minami *et al.* (2019), a non-zero value of the isotropic cosmic birefringence  $\beta_a = 0.35 \pm 0.14$  deg (68% C.L) was recently detected in the Planck18 polarization data at a  $2.4\sigma$  statistical significance level by Minami and Komatsu (2020a).
- **Small-scale curiosities:** (see Subsection III.6) Observations on galaxy scales indicate that the ΛCDM model faces several problems (core-cusp problem, missing satellite problem, too big to fail problem, angular momentum catastrophe, satellite planes problem, baryonic Tully-Fisher relation problem, void phenomenon etc.) in describing structures at small scales (see Bullock and Boylan-Kolchin 2017; Del Popolo and Le Delliou 2017, for a review).
- **Age of the Universe:** (see Subsection III.7) The age of the Universe as obtained from local measurements using the ages of oldest stars in the Milky Way (MW) appears to be marginally larger and in some tension with the corresponding age obtained using the CMB Planck18 data in the context of ΛCDM cosmology (Verde *et al.* 2013).
- **The Lithium problem ( $2 - 4\sigma$ ):** (see Subsection III.8) Measurements of old, metal-poor stars in the Milky Way's halo find 5 times less lithium than that BBN predicts (Fields 2011).
- **Quasars Hubble diagram ( $\sim 4\sigma$ ):** (see Subsection III.9) The distance modulus-redshift relation for the sample of 1598 quasars at higher redshift ( $0.5 < z < 5.5$ ) is in some tension with the concordance ΛCDM model indicating some hints for phantom late time expansion (Banerjee *et al.* 2021b; Lusso *et al.* 2019; Risaliti and Lusso 2019).
- **Oscillating signals in short range gravity experiments:** (see Subsection III.10) A reanalysis of short range gravity experiments has indicated the presence of an oscillating force signal with sub-millimeter wavelength (Antoniou and Perivolaropoulos 2017; Perivolaropoulos 2017b).
- **Anomalously low baryon temperature ( $\sim 3.8\sigma$ ):** (see Subsection III.11) The Experiment to Detect the Global Epoch of Reionization Signature (EDGES) collaboration (Bowman *et al.* 2018)

using global (sky-averaged) 21-cm absorption signal, reports anomalously low baryon temperature  $T_b \approx 4K$  at  $z \approx 17$  (half of its expected value).

- **Colliding clusters with high velocity ( $\sim 6\sigma$ ):** (see Subsection III.12) The El Gordo (ACT-CL J0102-4915) galaxy cluster at  $z = 0.87$  is in its formation process which occurs by a collision of two subclusters with mass ratio 3.6 merging at a very high velocity  $V_{infall} \simeq 2500\text{km/s}$ . Such cluster velocities at such a redshift are extremely rare in the context of  $\Lambda\text{CDM}$  as demonstrated by [Asencio et al. \(2020\)](#) using the estimation of [Kraljic and Sarkar \(2015\)](#) for the expected number of merging clusters from interrogation of the DarkSky simulations.

The well known Hubble tension and the other less discussed curiosities of  $\Lambda\text{CDM}$  at a lower statistical significance level may hint towards new physics (see [Huterer and Shafer 2018](#), for a review).

In the context of the above observational puzzles the following strategic questions emerge

- What are the current cosmological and astrophysical datasets that include the above non-standard signals?
- What is the statistical significance of each signal?
- Is there a common theoretical framework that may explain simultaneously many non-standard signals?

These questions will be discussed in what follows. There have been previous works ([Perivolaropoulos 2008, 2011](#)) collecting and discussing signals in data that are at some statistical level in tension with the standard  $\Lambda\text{CDM}$  model but these are by now outdated and the more detailed and extended update provided by the present review may be a useful resource.

The plan of this review is the following: In the next section (II) we focus on the Hubble tension. We provide a list of observational probes that can lead to measurements of the Hubble constant, point out the current tension level among different probes and discuss some of the possible generic extensions of  $\Lambda\text{CDM}$  model that can address this tension. In section III we present the current status of other less significant tensions, their level of significance and refer to recent resources where more details can be found for each signal. We also discuss possible theoretical approaches that can explain the non-standard nature of these signals. Finally, in section IV we conclude and discuss potential future directions of the reviewed research.

In Table VII of the Appendix A we list the acronyms used in this review.

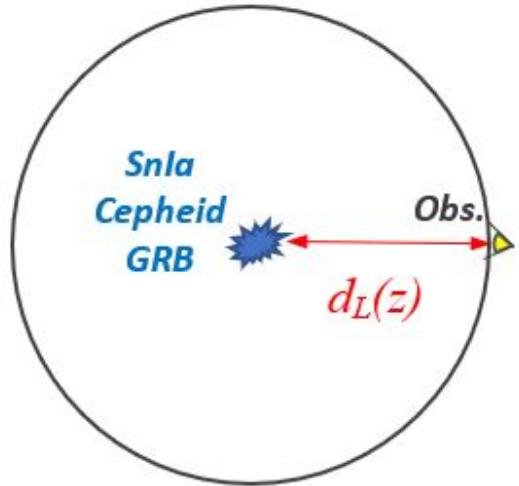


FIG. 1. The luminosity distance is obtained from the apparent and absolute luminosities.

## II Hubble tension

### II.1 Methods for measuring $H_0$ and data

The measurement of the Hubble constant  $H_0$  which is the local expansion rate of the Universe, is of fundamental importance to cosmology. This measurement has improved in accuracy through number of probes (see [Weinberg et al. 2013](#), for a review of most well established probes).

Distances to cosmological objects constitute the most common way to probe the cosmic metric and the expansion history of the Universe. In this subsection we review the use of the two main cosmological distances used to probe the cosmic expansion history.

#### • Luminosity distance

Consider a luminous cosmological source of absolute luminosity  $L$  (emitted power) and an observer (Fig. 1) at a distance  $d_L$  from the luminous source. In a static cosmological setup, the power radiated by the luminous source is conserved and distributed in the spherical shell with area  $4\pi d_L^2$  and therefore the apparent luminosity  $l$  (energy flux) detected by the observer is

$$l = \frac{L}{4\pi d_L^2} \quad (2.1)$$

Eq. (2.1) defines the quantity  $d_L$  known as *luminosity distance*. It is straightforward to show that in an expanding flat Universe, where the energy is not conserved due to the increase of the photon wavelength and period with time, the luminosity distance can be expressed as ([Dodelson 2003; Perivolaropoulos 2006](#))

$$d_L(z)_{th} = c(1+z) \int_0^z \frac{dz'}{H(z')} \quad (2.2)$$

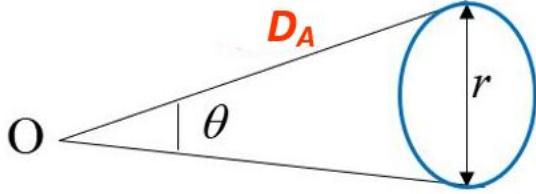


FIG. 2. The angular diameter distance is obtained from the angular and physical scales.

The luminosity distance is an important cosmological observable that is measured using standard candles (see Subsection II.1.1)

### • Angular diameter distance

Consider a source (standard ruler) with a physical scale  $r$  that subtends an angle  $\theta$  in the sky (Fig. 2). In Euclidean space, assuming that  $\theta$  is small, the physical angular diameter distance  $D_A$  is defined as (e.g. [Dodelson 2003; Hobson et al. 2006](#))

$$D_A(z) = \frac{r}{\theta} \quad (2.3)$$

A particularly useful standard ruler is the sound horizon at recombination calibrated by the peaks of the CMB anisotropy spectrum and observed either directly through the CMB anisotropies or through its signatures in the large scale structure (Baryon Acoustic Oscillations (BAO)) (see Subsection II.1.2).

It is straightforward to show that in an expanding flat Universe the physical angular diameter distance can be expressed as (e.g. [Dodelson 2003](#))

$$D_A(z)_{th} = \frac{c}{(1+z)} \int_0^z \frac{dz'}{H(z')} \quad (2.4)$$

The luminosity and angular diameter distances can be measured using standard candles and standard rulers thus probing the cosmic expansion rate at both the present time ( $H(z=0) \equiv H_0$ ) and at higher redshifts ( $H(z)$ ).

#### II.1.1 Standard candles as probes of luminosity distance

The luminosity distance to a source may be probed using standardizable candles like Type Ia supernovae (SNIa) ( $z < 2.3$ ) ([Betoule et al. 2014; Perlmutter et al. 1999; Riess et al. 1998; Scolnic et al. 2018](#)) and gamma-ray bursts (GRBs) ( $0.1 < z \lesssim 9$ ) ([Amati et al. 2019, 2008; Cao et al. 2022a,b; Cardone et al. 2010; Cucchiara et al.](#)

2011; [Dai et al. 2021; Dainotti and Del Vecchio 2017; Dainotti et al. 2013; Demianski et al. 2017, 2021; Dirirsa et al. 2019; Hu et al. 2021; Khadka et al. 2021; Khadka and Ratra 2020a; Liu et al. 2022; Luongo and Muccino 2021; Luongo et al. 2021; Salvaterra et al. 2009; Samushia and Ratra 2010; Schaefer 2007; Tang et al. 2019; Tanvir et al. 2009; Wang et al. 2016c](#).

Surveys can indicate the distance-redshift relation of SNIa by measuring their peak luminosity that is tightly correlated with the shape of their characteristic light curves (luminosity as a function of time after the explosion) and the redshifts of host galaxies. The latest and largest SNIa dataset available that incorporates data from six different surveys is the Pantheon sample consisting of a total of 1048 SNIa in the redshift range  $0.01 < z < 2.26$  (the number of SNIa with  $z > 1.4$  is only six) ([Scolnic et al. 2018](#)). More recently, the Pantheon+ sample which comprises 18 different samples has been released ([Brout et al. 2022; Scolnic et al. 2021](#)) (see also [Brownsberger et al. 2021; Peterson et al. 2021](#)). [Brout et al. \(2022\); Scolnic et al. \(2021\)](#) present 1701 light curves of 1550 distinct SNIa in the redshift range  $0.001 < z < 2.26$  including SNIa which are in very nearby galaxies ( $z \lesssim 0.01$ ) with measured Cepheid distances. For determination of  $H_0$  the SH0ES team ([Riess et al. 2021b](#)) use as calibrator sample 42 SNIa in the 37 Cepheid hosts and 277 SNIa in the Hubble flow ( $0.0233 < z < 0.15$ ) from the Pantheon+ sample.

The apparent magnitude<sup>2</sup>  $m_{th}$  of SNIa in the context of a specified form of  $H(z)$ , is related to their luminosity distance  $d_L(z)$  of Eq. (2.2) in Mpc as

$$m(z)_{th} = M + 5 \log_{10} \left[ \frac{d_L(z)}{\text{Mpc}} \right] + 25 \quad (2.6)$$

Using now the dimensionless Hubble free luminosity distance

$$D_L(z) = \frac{H_0 d_L(z)}{c} \quad (2.7)$$

the apparent magnitude can be written as

$$m(z)_{th} = M + 5 \log_{10} [D_L(z)] + 5 \log_{10} \left[ \frac{c/H_0}{\text{Mpc}} \right] + 25 \quad (2.8)$$

The use of Eq. (2.8) to measure  $H_0$  using the measured apparent magnitudes of SNIa requires knowledge of the value of the SNIa absolute magnitude  $M$  which can be obtained using calibrators of local SNIa at  $z < 0.01$  (closer

<sup>2</sup> The apparent magnitude  $m$  of an astrophysical source detected with flux  $l$  is defined as

$$m = -2.5 \log_{10} \left( \frac{l}{l_0} \right) \quad (2.5)$$

where  $l_0$  is a reference flux (zero point). The absolute magnitude  $M$  of an astrophysical source is the apparent magnitude the source would have if it was placed at a distance of 10 pc from the observer.

than the start of the Hubble flow) in the context of a distance ladder (e.g. [Sandage et al. 2006](#)) using calibrators like Cepheid stars.

In the cosmic distance ladder approach each step of the distance ladder uses parallax methods and/or the known intrinsic luminosity of a standard candle source to determine the absolute (intrinsic) luminosity of a more luminous standard candle residing in the same galaxy. Thus highly luminous standard candles are calibrated for the next step in order to reach out to high redshift luminosity distances.

### II.1.1.1 SnIa standard candles and their calibration

- **SnIa-Cepheid:** Geometric anchors may be used to calibrate the Cepheid variable star standard candles at the local Universe (primary distance indicators) whose luminosities are correlated with their periods of variability<sup>3</sup>. The MW, the Large Magellanic Cloud (LMC) and NGC 4258 are used as distance geometric anchor galaxies. For Cepheids in the anchor galaxies there are three different ways of geometric distance calibration of their luminosities: trigonometric parallaxes in the MW ([Benedict et al. 2007](#); [Casertano et al. 2016](#); [van Leeuwen et al. 2007](#); [Lindegren et al. 2016](#); [Riess et al. 2014, 2021a, 2018a,b](#)), Detached Eclipsing Binary Stars (DEBs) in the LMC ([Pietrzynski et al. 2019](#)) and water masers (see Subsection II.1.5) in NGC 4258 ([Reid et al. 2019](#); [Yuan et al. 2022](#)). The DEBs technique relies on surface-brightness relations and is a one-step distance determination to nearby galaxies independent from Cepheids ([Pietrzynski et al. 2013](#)).

Using the measured distances of the calibrated Cepheid stars the intrinsic luminosity of nearby SnIa residing in the same galaxies as the Cepheids is obtained. This SnIa calibration which fixes  $M$  is then used for SnIa at distant galaxies to measure  $H_0$  ( $z \in [0.01, 0.1]$ ) and  $H(z)$  ( $z \in [0.01, 2.3]$ ).

- **SnIa-TRGB:** Instead of Cepheid variable stars, the Tip of the Red Giant Branch (TRGB) stars in the Hertzsprung-Russell diagram ([Beaton et al. 2016](#); [Freedman et al. 2020](#)) and Miras ([Huang et al. 2019, 2018](#)) (see also [Czerny et al. 2018](#), for a review) can be used as calibrators of SnIa. The Red Giant stars have nearly exhausted the hydrogen in their cores and have just began helium burning (helium flash phase). Their brightness can be standardized using parallax methods and they can serve as bright standard candles visible in the local Universe for the subsequent calibration of SnIa.

- **SnIa-Miras:** Miras (named for the prototype star Mira) are highly evolved low mass variable stars at the tip of asymptotic giant branch (AGB) stars (e.g. [Iben and Renzini 1983](#)). The water megamaser as distance indicator (see Subsection II.1.5) can be used to calibrate the Mira period-luminosity (PL) relation ([Huang et al. 2018](#)). Miras with short period ( $< 400$  days) have low mass progenitors and are present in all galaxy types or in the halos of galaxies, eliminating the necessity for low inclination SnIa host galaxies.

- **SBF:** Another method to determine the Hubble constant based on calibration of the peak absolute magnitude of SnIa is the Surface Brightness Fluctuations (SBF) method ([Cantiello et al. 2018](#); [Jensen et al. 2001](#); [Khetan et al. 2021](#)). SBF is a secondary<sup>4</sup> luminosity distance indicator that uses stars in the old stellar populations (II) and can reach larger distances than Cepheids even inside the Hubble flow region where the recession velocity is larger than local peculiar velocities ( $z > 0.01$ ) ([Biscardi et al. 2008](#); [Blakeslee 2012](#); [Blakeslee et al. 1999, 2009](#); [Mei et al. 2005](#); [Tonry and Schneider 1988](#); [Tonry et al. 1997](#)). For SBF calibration [Blakeslee et al. \(2021\)](#) use both Cepheids and TRGB demonstrating that these calibrators are consistent with each other.

Assume that a galaxy includes a finite number of stars covering a range of luminosity. Using SBF in the galaxy image for the determination of its distance, the ratio  $\bar{L}$  of the second and first moments of the stellar luminosity function in the galaxy is used along with the mean flux per star  $\bar{l}$  as follows ([Blakeslee et al. 1999](#); [Tonry and Schneider 1988](#))

$$d^2 = \frac{\bar{L}}{4\pi\bar{l}} \quad (2.9)$$

where

$$\bar{L} = \frac{\int n(L)L^2 dL}{\int n(L)L dL} = \frac{\sigma_L^2}{\langle L \rangle} \quad (2.10)$$

where  $n(L)$  is the expectation number of stars with luminosity  $L$ . Thus SBF can be viewed as providing an average brightness. A galaxy with double distance appears with double smoothness due to the effect of averaging.

### II.1.1.2 Alternative cosmological standard candles

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<sup>3</sup> The period–luminosity (PL) relation is also called the Leavitt law ([Leavitt 1908](#); [Leavitt and Pickering 1912](#)).

<sup>4</sup> Nearby Cepheids or stellar population models are used for the empirical or theoretical calibration of the SBF distances respectively.

- **SneII:** An independent method to determine the Hubble constant utilizes Type II supernovae (SneII) as cosmic distance indicators (de Jaeger *et al.* 2020a). SneII are characterised by the presence of hydrogen lines in their spectra (Filippenko 1997, 2000). This feature distinguishes SneII from other types of supernovae. Their light curve shapes include a plateau of varying steepness and length differ significantly from those of SnIa. The use of SneII as standard candles is motivated by the fact that they are more abundant than SnIa (Graur *et al.* 2017; Li *et al.* 2011) (although 1-2 mag fainter Richardson *et al.* 2014) and are produced by different stellar populations than SnIa which are more difficult to standardize. The SneII progenitors (red super giant stars) however are better understood than those of SnIa.

Different SneII distance-measurement techniques have been proposed and tested. These include, the expanding photosphere method (Dessart and Hillier 2005; Eastman *et al.* 1996; Kirshner and Kwan 1974), the spectral-fitting expanding atmosphere method (Baron *et al.* 2004; Dessart *et al.* 2008), the standardized candle method (Hamuy and Pinto 2002), the photospheric magnitude method (Rodríguez *et al.* 2014) and the photometric color method (de Jaeger *et al.* 2015). For example, the standardized candle method is based on the relation between the luminosity and the expansion velocity of the photosphere (Hamuy and Pinto 2002; de Jaeger *et al.* 2017, 2020b; Olivares E. *et al.* 2010).

- **GRBs:** In addition to SnIa and SneII, GRBs are widely proposed as standard candles to trace the Hubble diagram at high redshifts (Basilakos and Perivolaropoulos 2008; Khadka *et al.* 2021; Khadka and Ratra 2020a; Lamb and Reichart 2000; Wang *et al.* 2015). However GRBs distance calibration is not easy and various cosmology independent methods (e.g. Liu and Wei 2015) or phenomenological relations (e.g. Amati *et al.* 2002; Ghirlanda *et al.* 2004) have been proposed for their calibration.

Furthermore GRBs can be combined with other probes to study the redshift evolution of Hubble constant (Dainotti *et al.* 2022) (see Moresco *et al.* 2022, for a review).

### II.1.1.3 Using SnIa to measure $H_0$ and $H(z)$ :

The best fit values of the parameter  $H_0$  and the deceleration parameter  $q_0$  may be obtained<sup>5</sup> (Camarena and Marra 2020b) using local distance ladder measurements (e.g.

Cepheid calibration up to  $z \simeq 0.01$ ) to measure directly  $M$ , low  $z$  measurements of the SnIa apparent magnitude  $m(z)$  and a kinematic local expansion of  $D_L(z)$  ( $z < 0.1$ ) as (e.g. Weinberg 2008)

$$D_L(z, q_0) = z \left[ 1 + \frac{1}{2}(1 - q_0)z \right] \quad (2.11)$$

Alternatively,  $q_0$  may be fixed to its  $\Lambda$ CDM value  $q_0 = -0.55$  and  $H_0$  may be fit as a single parameter (Riess *et al.* 2019, 2011, 2016).

Using higher  $z$  SnIa the best fit parameters of  $\Lambda$ CDM may be obtained by fitting the  $\Lambda$ CDM expansion rate  $H(z)$

$$H^2(z) = H_0^2 [\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})] \quad (2.12)$$

where  $\Omega_{0m}$  is the matter density parameter today. Using Eqs. (2.2), (2.7) and (2.12), the Hubble free luminosity distance can be written as

$$D_L(z, \Omega_{0m}) = (1+z) \int_0^z \frac{dz'}{[\Omega_{0m}(1+z')^3 + (1-\Omega_{0m})]^{1/2}} \quad (2.13)$$

A key assumption in the use of SnIa in the measurement of  $H_0$  and  $H(z)$  is that they are standardizable and after proper calibration they have a fixed absolute magnitude independent of redshift<sup>6</sup>. This assumption has been tested in Colgáin (2019); Drell *et al.* (2000); Kazantzidis *et al.* (2021); Kazantzidis and Perivolaropoulos (2019, 2020); Koo *et al.* (2020); Luković *et al.* (2020); Sapone *et al.* (2021); Tutusaus *et al.* (2019, 2017).

Using the degenerate combination

$$\mathcal{M} = M + 5 \log_{10} \left[ \frac{c/H_0}{Mpc} \right] + 25 \quad (2.14)$$

into Eq. (2.8) we obtain

$$m(z, M, H_0, \Omega_{0m})_{th} = \mathcal{M}(M, H_0) + 5 \log_{10} [D_L(z, \Omega_{0m})] \quad (2.15)$$

The theoretical prediction (2.15) may now be used to compare with the observed  $m_{obs}$  data and to obtain the best fits for the parameters  $\mathcal{M}$  and  $\Omega_{0m}$ . Using the maximum likelihood analysis the best fit values for these parameters may be found by minimizing the quantity

$$\chi^2(\mathcal{M}, \Omega_{0m}) = \sum_i \frac{[m_{obs,i} - m_{th}(z_i; \mathcal{M}, \Omega_{0m})]^2}{\sigma_i^2} \quad (2.16)$$

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<sup>5</sup>  $q_0$  is the deceleration parameter  $q_0 \equiv -\frac{1}{H_0^2} \frac{d^2 a(t)}{dt^2} \Big|_{t=t_0}$

<sup>6</sup> The possibility for intrinsic luminosity evolution of SnIa with redshift was first pointed out by Tinsley (1968). Also, the assumption that the luminosity of SnIa is independent of host galaxy properties (e.g. host age, host morphology, host mass) and local star formation rate has been discussed in Jones *et al.* (2018); Kang *et al.* (2020); Kim *et al.* (2018); Rigault *et al.* (2020); Rose *et al.* (2019).

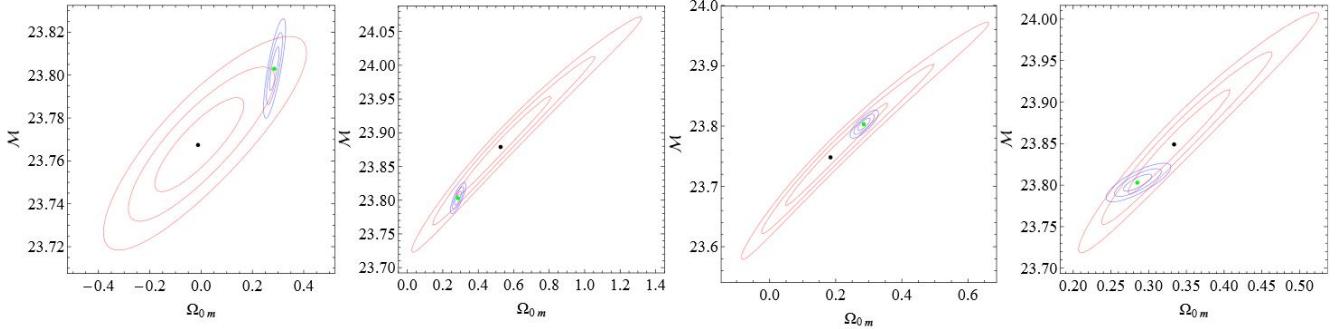


FIG. 3. The  $1\sigma - 3\sigma$  confidence contours in the parametric space  $(\Omega_{0m}, \mathcal{M})$ . The blue contours correspond to the  $1\sigma - 3\sigma$  full Pantheon dataset (1048 SNIa datapoints) best fit, while the red contours describe the  $1\sigma - 3\sigma$  confidence contours of the four bins (from left to right). The black points represent the best fit of each bin, while the green dot represents the best fit value indicated by the full Pantheon dataset ( $\Omega_{0m} = 0.285$  and  $\mathcal{M} = 23.803$ ) (from Kazantzidis and Perivolaropoulos 2020).

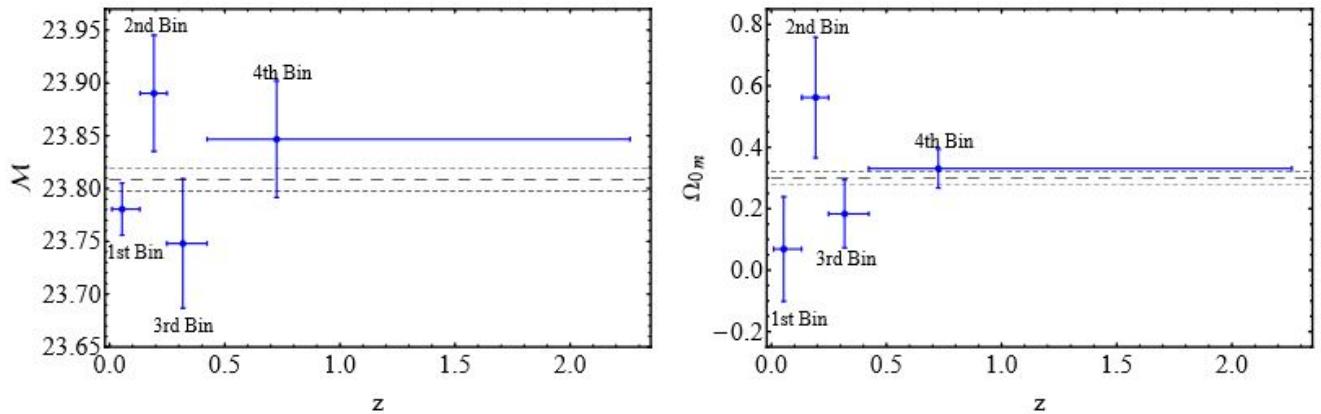


FIG. 4. The best fit values of  $\mathcal{M}$  (left panel) and  $\Omega_{0m}$  (right panel) as well as the  $1\sigma$  errors for the four bins, including the systematic uncertainties. This oscillating behaviour is relatively improbable in the context of constant underlying  $\mathcal{M}$  and  $\Omega_{0m}$  (from Kazantzidis and Perivolaropoulos 2020).

The results from the recent analysis by Kazantzidis and Perivolaropoulos (2020) using the latest SNIa (Pantheon) data (Scolnic *et al.* 2018) (consisting of 1048 datapoints in the redshift range  $0.01 < z < 2.3$  sorting them from lowest to highest redshift and dividing them in four equal uncorrelated bins) in the context of a  $\Lambda$ CDM model are shown in Figs. 3 and 4<sup>7</sup>. An oscillating signal for  $\mathcal{M}$  and  $\Omega_{0m}$  ( $2\sigma$ ) is apparent in Fig. 4 and its statistical significance may be quantified using simulated data (Kazantzidis *et al.* 2021; Koo *et al.* 2020).

The presence of large scale inhomogeneities at low  $z$  including voids or a supercluster (Grande and Perivolaropoulos 2011) can be a plausible physical ex-

planation for this curious behavior. In the context of a local void model the analysis by Kazantzidis and Perivolaropoulos (2020) indicated that the value of  $H_0$  increases by  $2 - 3\%$  which is less than the  $9\%$  required to address the Hubble tension. The bias and systematics induced by such inhomogeneities on the Hubble diagram within a well-posed fully relativistic framework (light cone averaging formalism Gasperini *et al.* 2011) has been discussed in Fanizza *et al.* (2020).

Marra and Perivolaropoulos (2021) have pointed out that this  $H_0$  tension is related to the mismatch between the SNIa absolute magnitude calibrated by Cepheids at  $z < 0.01$  (Camarena and Marra 2020b, 2021)

$$M^< = -19.2334 \pm 0.0404 \quad (2.17)$$

and the SNIa absolute magnitude using the parametric free inverse distance ladder calibrating SNIa absolute magnitude using the sound horizon scale (Camarena and Marra 2020a)

$$M^> = -19.401 \pm 0.027 \quad (2.18)$$

<sup>7</sup> For  $M = -19.24$  as indicated by Cepheid calibrators (Camarena and Marra 2020b) of SNIa at  $z < 0.01$  and the SNIa local determination  $H_0 = 74 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Riess *et al.* 2019) Kazantzidis and Perivolaropoulos (2020) find  $\mathcal{M} = 23.80$  which is consistent with the full Pantheon SNIa best fit shown in Fig. 3.

Thus a transition in the absolute magnitude with amplitude  $\Delta M \simeq 0.2$  may provide a solution to this tension (see Subsection II.2.4 and in Alestas *et al.* 2021b; Marra and Perivolaropoulos 2021). Alternatively if this discrepancy is not due to systematics (Follin and Knox 2018; Verde *et al.* 2019), it could be an indication of incorrect estimate of the sound horizon scale due e.g. to early dark energy (Chudaykin *et al.* 2020) or to late phantom dark energy (Alestas *et al.* 2020a).

Note also that Rameez and Sarkar (2021) find discrepancies between ‘Joint Light-curve Analysis’ (JLA) SnIa and Pantheon SnIa datasets which imply an uncertainty in the calibration of the absolute magnitude or equivalently of the Hubble constant which is large enough to undermine the claim for Hubble tension.

#### II.1.1.4 Observational data - Constraints:

- **SnIa-Cepheid:** Using the analysis of the Hubble Space Telescope (HST) observations (Sandage *et al.* 2006) the Hubble constant  $H_0$  value has been measured from Cepheid-calibrated supernovae (using 70 long-period Cepheids in the LMC) by the Supernovae  $H_0$  for the Equation of State (SH0ES) of dark energy collaboration (Riess *et al.* 2019, 2011, 2016). The analysis by the SH0ES Team using this local model-independent measurement refers  $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Riess *et al.* 2021b), which results in  $5\sigma$  tension with the value estimated by CMB Planck18 (Aghanim *et al.* 2020e) assuming the  $\Lambda\text{CDM}$  model while in previous analysis by SH0ES Team (Riess *et al.* 2021a) using the Gaia Early Data Release 3 (EDR3) parallaxes (Gaia Collaboration *et al.* 2020) and reaching 1.8% precision by improving the calibration a value of  $H_0 = 73.2 \pm 1.3$  is obtained, a  $4.2\sigma$  tension with the prediction from Planck18 CMB observations. Riess *et al.* (2018b) analysing the HST data, using Cepheids as distance calibrators reports  $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{Mpc}^{-1}$ . A reanalysis of the SH0ES collaboration results using a cosmographic method allowing also the deceleration parameter  $q_0$  to be a free parameter by Camarena and Marra (2021) leads to  $H_0 = 74.30 \pm 1.45 \text{ km s}^{-1} \text{Mpc}^{-1}$ .

Breuval *et al.* (2020) considered companion and average cluster parallaxes instead of direct Cepheid parallaxes and obtained  $H_0 = 72.8 \pm 1.9 \text{ (statistical + systematics)} \pm 1.9 \text{ (ZP)} \text{ km s}^{-1} \text{Mpc}^{-1}$  when all Cepheids are considered and  $H_0 = 73.0 \pm 1.9 \text{ (statistical + systematics)} \pm 1.9 \text{ (ZP)} \text{ km s}^{-1} \text{Mpc}^{-1}$  for fundamental mode pulsators only (where ZP is the second Gaia data release (GDR2)) (Brown *et al.* 2018) parallax zero point).

Various other previous estimates of  $H_0$  have been obtained by treatments of the distance ladder (Burns *et al.* 2018; Dhawan *et al.* 2018; Feeney

*et al.* 2018). In particular, Dhawan *et al.* (2018) find  $H_0 = 72.8 \pm 1.6$  (statistical) $\pm 2.7$  (systematic)  $\text{km s}^{-1} \text{Mpc}^{-1}$  using SnIa as standard candles in the near-infrared (NIR), Burns *et al.* (2018) find  $H_0 = 73.2 \pm 2.3 \text{ km s}^{-1} \text{Mpc}^{-1}$  analysing the final data release of the Carnegie Supernova Project<sup>8</sup> (CSP) I (Krisciunas *et al.* 2017) and Feeney *et al.* (2018) find  $H_0 = 73.15 \pm 1.78 \text{ km s}^{-1} \text{Mpc}^{-1}$  using a Bayesian hierarchical model of the local distance ladder.

• **SnIa-TRGB:** The Carnegie–Chicago Hubble Program<sup>9</sup> (CCHP) (Beaton *et al.* 2016) using calibration of SnIa with the TRGB method estimates  $H_0 = 69.8 \pm 0.8 (\pm 1.1\% \text{stat}) \pm 1.7 (\pm 2.4\% \text{sys}) \text{ km s}^{-1} \text{Mpc}^{-1}$  (Freedman *et al.* 2019) and a revision of their measurements has lead to  $H_0 = 69.6 \pm 0.8 (\pm 1.1\% \text{stat}) \pm 1.7 (\pm 2.4\% \text{sys}) \text{ km s}^{-1} \text{Mpc}^{-1}$  (Freedman *et al.* 2020). Recently, the updated TRGB calibration applied to a distant sample of SnIa from the CCHP lead to a value of the Hubble constant of  $H_0 = 69.8 \pm 0.6 \text{ (stat)} \pm 1.6 \text{ (sys)} \text{ km s}^{-1} \text{Mpc}^{-1}$  (Freedman 2021). Using the LMC and the NGC 4258 as TRGB calibration of the SnIa distance ladder, the SH0ES team finds  $H_0 = 72.4 \pm 2 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Yuan *et al.* 2019) and  $H_0 = 71.1 \pm 1.9 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Reid *et al.* 2019) respectively. Freedman (2021); Freedman *et al.* (2020); Hoyt (2021) argue that the difference in the derived value of  $H_0$  by SH0ES team compared to CCHP was due to incorrect assumptions regarding calibration of the TRGB in the LMC made by Yuan *et al.* (2019). A value of  $H_0 = 65.8 \pm 3.5 \text{ (stat)} \pm 2.4 \text{ (sys)} \text{ km s}^{-1} \text{Mpc}^{-1}$  is obtained by Kim *et al.* (2020) using peculiar velocities and TRGB distances of 33 galaxies located between the Local Group and the Virgo cluster ( $\sim 16.5 \text{ Mpc}$ ) (mainly the sample of Virgo infall galaxies from Karachentsev *et al.* 2018).

More recently, Soltis *et al.* (2021) have reported a measurement of  $H_0 = 72.1 \pm 2.0 \text{ km s}^{-1} \text{Mpc}^{-1}$  using the TRGB distance indicator calibrated from the European Space Agency (ESA) Gaia mission Early Data Release 3 (EDR3) trigonometric parallax of Omega Centauri (Gaia Collaboration *et al.* 2020). Anand *et al.* (2021) find  $H_0 = 71.5 \pm 1.8 \text{ km s}^{-1} \text{Mpc}^{-1}$  combining TRGB measurements with either the Pantheon or CSP samples of supernova. Finally, Jones *et al.* (2022) using NIR only cosmological analysis and TRGB distances to calibrate the SnIa luminosity of the CSP and RAISIN (an anagram for “SnIa in the IR”) samples (Brout *et al.* 2019; Jones *et al.* 2017) and

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<sup>8</sup> <https://csp.obs.carnegiescience.edu>

<sup>9</sup> <https://carnegiescience.edu/projects/carnegie-hubble-program>

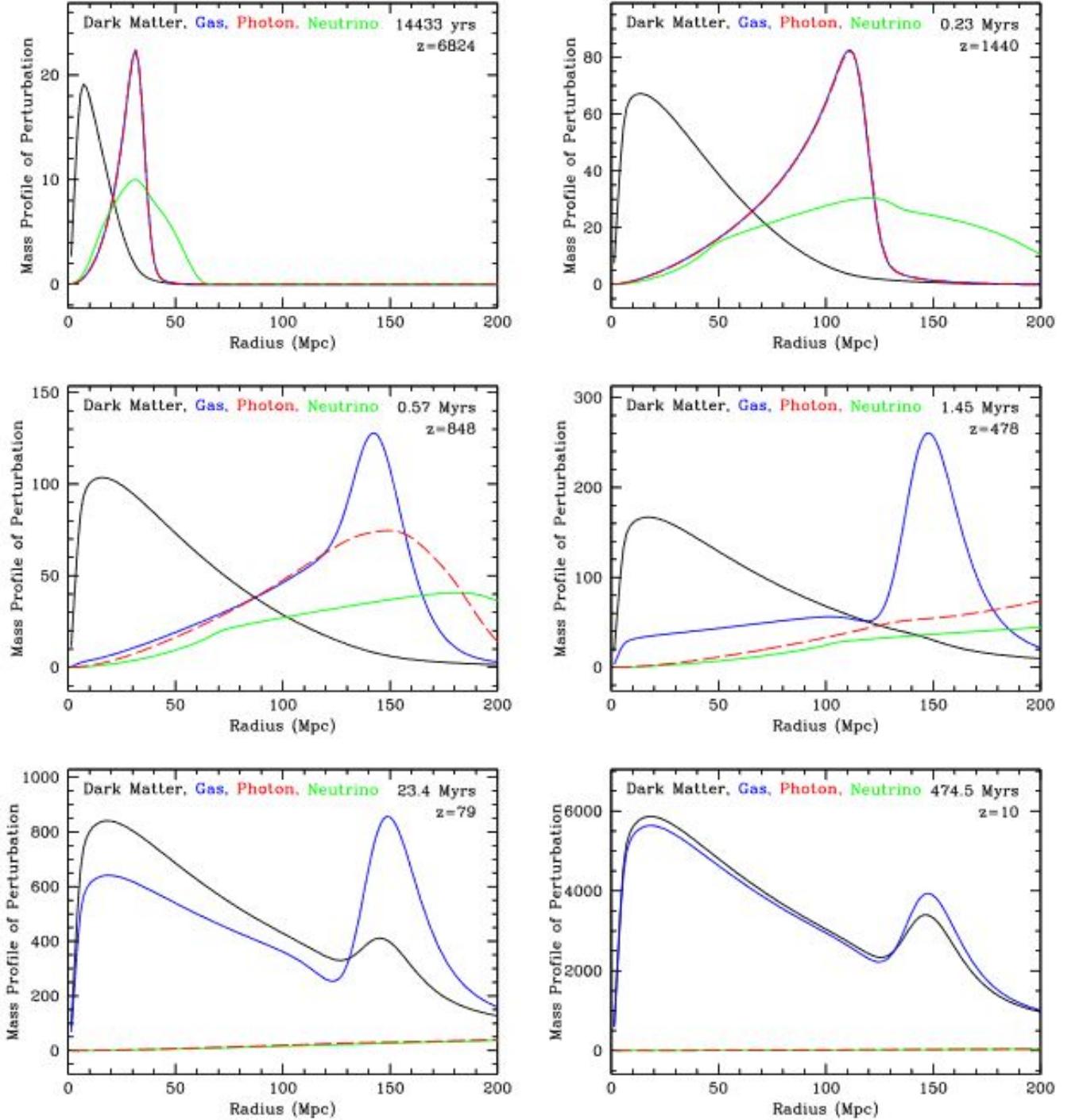


FIG. 5. The snapshots show the radial mass profile of perturbation as a function of the comoving radius of an initially point-like overdensity located at the origin for redshifts  $z = 6824, 1440, 848, 478, 79, 10$ . The time after the Big Bang are given in each snapshots. The black, blue, red, and green lines correspond to the dark matter, baryons, photons, and neutrinos (all perturbations are fractional for that species), respectively. The top snapshots are for the early time before recombination where the overdensities in photons and baryons evolve together, the middle snapshots for soon after but close to recombination where the baryons freeze at the location reached with the photons forming a thick spherical shell, and the bottom snapshots are for long after recombination where the baryon overdensities start to gravitationally grow like dark matter overdensities (from Eisenstein *et al.* 2007).

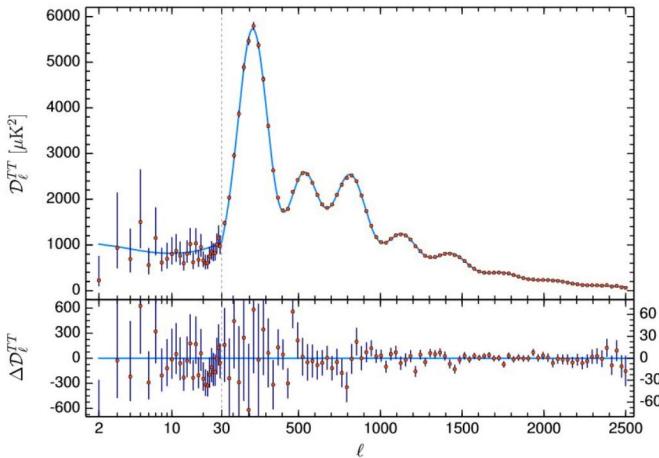


FIG. 6. The Planck18 CMB angular power spectrum  $D_l^{TT} \equiv l(l+1)/(2\pi)C_l^{TT}$  (top) and residual angular power spectrum (bottom) of temperature fluctuations as a function of multipole moment  $l$ . The light blue line in the upper panel is the best-fitting to the Planck TT, TE, EE+lowE+lensing likelihoods assuming the base- $\Lambda$ CDM cosmology. The red points correspond to the binned Planck data. The lowest multipole range ( $l \leq 30$ ) is dominated by cosmic variance (approximated as Gaussian), while positions and amplitudes of the acoustic peaks are accurately constrained (from Aghanim *et al.* 2020e).

Dhawan *et al.* (2022) using TRGB calibration of SNIa observed by the Zwicky Transient Facility (ZTF) (Bellm *et al.* 2018; Graham *et al.* 2019) report  $H_0 = 72.4 \pm 3.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $H_0 = 76.94 \pm 6.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$  respectively.

- **SNIa-Miras:** Calibration of SNIa in the host NGC 1559 galaxy with the Miras method using a sample of 115 oxygen-rich Miras<sup>10</sup> discovered in maser host NGC 4258 galaxy, has lead to a measurement of the Hubble constant as  $H_0 = 73.3 \pm 4 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Huang *et al.* 2019).
- **SBF:** Calibrating the SNIa luminosity with SBF method and extending it into the Hubble flow by using a sample of 96 SNIa in the redshift range  $0.02 < z < 0.075$ , extracted from the Combined Pantheon Sample has lead to the measurement  $H_0 = 70.50 \pm 2.37 \text{ (stat)} \pm 3.38 \text{ (sys)} \text{ km s}^{-1} \text{ Mpc}^{-1}$  by Khetan *et al.* (2021). Previously Cantiello *et al.* (2018) combining distance measurement with the corrected recession velocity of NGC 4993 reported a Hubble constant  $H_0 = 71.9 \pm 7.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . A new measurement of the Hubble constant  $H_0 = 73.3 \pm 0.7 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$  has recently been obtained based on a set of 63 SBF (Blakeslee *et al.* 2021) distances extending out to 100 Mpc.

- **SneII:** SNeII have also been used for the determination of  $H_0$ . Using 7 SNeII as cosmological standardisable candles with host-galaxy distances measured from Cepheid variables or the TRGB the Hubble constant was measured to be  $H_0 = 75.8_{-4.9}^{+5.2} \text{ km s}^{-1} \text{ Mpc}^{-1}$  (de Jaeger *et al.* 2020a). More recently, de Jaeger *et al.* (2022) find  $H_0 = 75.4_{-3.7}^{+3.8} \text{ km s}^{-1} \text{ Mpc}^{-1}$  using 13 SNeII.

### II.1.2 Sound horizon as standard ruler: early time calibrators

Before recombination ( $z > 1100$ ), the primeval plasma of coupled baryons to photons (baryon-photon fluid) oscillates as spherical sound waves emanating from baryon gas perturbations are driven by photon pressure. At recombination when the Universe has cooled enough the electrons and protons combine to form hydrogen (e.g. Peebles 1968), photons decouple from baryons and propagate freely since the pressure becomes negligible. Thus the spherical sound wave shells of baryons become frozen. This process which was first detected in the galaxy power spectrum by Cole *et al.* (2005); Eisenstein *et al.* (2005) is illustrated in Fig. 5. It inflicts a unique Baryon Acoustic Oscillations (BAO) scale on the CMB anisotropy spectrum peaks shown in Fig. 6 and on the matter large scale structure (LSS) power spectrum on large scales at the radius of the sound horizon (the distance that the sound waves have traveled before recombination). This scale emerges as a peak in the correlation function  $\xi(s)$ <sup>11</sup> as illustrated in Fig. 7 or equivalently as damped oscillations in the LSS power spectrum (Eisenstein and Hu 1998; Eisenstein *et al.* 2005; Matsubara 2004; Meiksin *et al.* 1999). The characteristic BAO scale is also imprinted in the Lyman- $\alpha$  (Ly $\alpha$ ) forest absorption lines of neutral hydrogen in the intergalactic medium (IGM) detected in quasar (QSO) spectra.

The measured angular scale of the sound horizon  $\theta_s$  at the drag epoch when photon pressure vanishes can be used to probe the Hubble expansion rate using the standard ruler relation (e.g. Amendola and Tsujikawa 2015; Peebles 1980)

$$\theta_s = \frac{r_s}{d_A} \quad (2.20)$$

where  $d_A \equiv \frac{D_A}{a} \equiv (1+z)D_A = c \int_0^z \frac{dz'}{H(z')}$  is the comoving angular diameter distance to last scattering (at

<sup>11</sup> The correlation function is defined as the excess probability of one galaxy to be found within a given distance of another. Using the Landy-Szalay estimator (Landy and Szalay 1993) this function can be computed (Eisenstein *et al.* 2005)

$$\xi(s) \equiv \langle \delta(x)\delta(x+s) \rangle = \frac{DD(s) - 2DR(s) + RR(s)}{RR(s)} \quad (2.19)$$

where  $s$  is the comoving galaxy separation distance and  $DD(s)$ ,  $RR(s)$  and  $DR(s)$  correspond to the number of galaxy pairs with separations  $s$  in real-real, random-random and real-random catalogs, respectively.

<sup>10</sup> Miras can be divided into oxygen- and carbon-rich Miras.

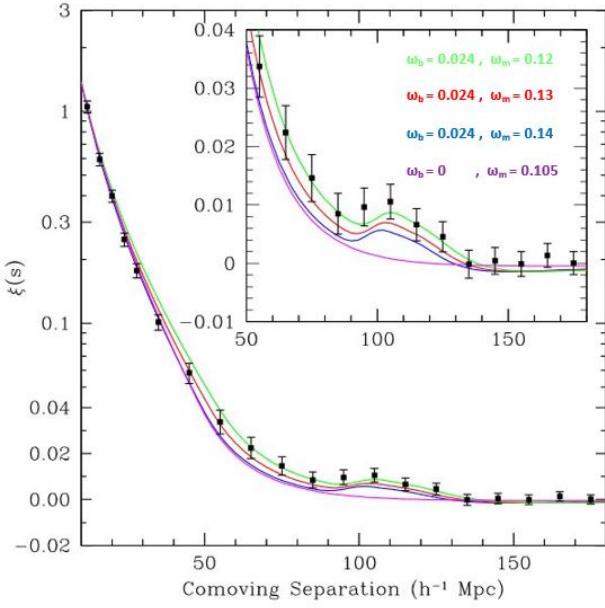


FIG. 7. The signature of baryonic acoustic oscillations in galaxy two-point correlation function  $\xi(s)$  as measured by Eisenstein *et al.* (2005) using the luminous red galaxy samples of the Sloan Digital Sky Survey. The data show the existence of a baryonic acoustic peak in the galaxy correlation function  $\xi(s)$  around the comoving separation scale  $100 h^{-1} Mpc$ . The solid green, red, and blue lines correspond to model predictions with  $\Omega_{0m}h^2 = 0.12, 0.13$  and  $0.14$ , respectively. All models are taken to have the same  $\Omega_{0b}h^2 = 0.024$  and  $n = 0.98$ . The magenta line corresponds to a model with no baryons and  $\Omega_{0m}h^2 = 0.105$ , which has no acoustic peaks (from Eisenstein *et al.* 2005).

redshift  $z \approx 1100$ ) and  $r_s$  is the radius of sound horizon at last scattering.

The radius  $r_s$  of the sound horizon at last scattering can be calculated by the distance that sound can travel from the Big Bang,  $t = 0$ , to time  $t_d$  at the drag epoch when the photon pressure can no longer prevent gravitational instability in baryons. This happens shortly after the time  $t_s$  of the last scattering when the optical depth due to Thomson scattering reaches unity (Eisenstein and Hu 1998). Thus (Aubourg *et al.* 2015)

$$\begin{aligned} r_s &= \int_0^{t_d} \frac{c_s(a)}{a(t)} dt = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z; \rho_b, \rho_\gamma, \rho_c)} dz = \\ &= \int_0^{a_d} \frac{c_s(a)}{a^2 H(a; \rho_b, \rho_\gamma, \rho_c)} da \end{aligned} \quad (2.21)$$

where the drag redshift  $z_d$  corresponds to time  $t_d$ ,  $\rho_b$ ,  $\rho_c$  and  $\rho_\gamma$  denote the densities for baryon, cold dark matter and radiation (photons) respectively and  $c_s$  is the sound speed in the photon-baryon fluid given by (Efstathiou

and Bond 1999; Komatsu *et al.* 2009)

$$c_s = \frac{c}{\sqrt{3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right)}} = \frac{c}{\sqrt{3 \left( 1 + \frac{3\omega_b}{4\omega_\gamma} a \right)}} \quad (2.22)$$

The expansion rate  $H(z)$  depends on the ratio of the matter density to radiation density and the sound speed determined by the baryon-to-photon ratio. Both the matter-to-radiation ratio and the baryon-to-photon ratio can be estimated from the details of the acoustic peaks in CMB anisotropy power spectrum (e.g. Hu and Dodelson 2002). Thus the CMB is possible to lead to an independent determination of the radius of sound horizon. Alternatively an independent determination of the radius of sound horizon can be obtained using primordial deuterium measurements (Addison *et al.* 2018, 2013). Now using the Eqs. (2.4) and (2.20) we can write the angular size of the sound horizon as

$$\theta_s = \frac{H_0 r_s}{c \int_0^{z_d} \frac{dz'}{E(z')}} \quad (2.23)$$

where  $E(z)$  is the dimensionless normalized Hubble parameter and for a flat  $\Lambda$ CDM model is given by

$$E(z) \equiv \frac{H(z)}{H_0} = [\Omega_{0m}(1+z)^3 + (1-\Omega_{0m})]^{1/2} \quad (2.24)$$

Eq. (2.23) indicates that there is a degeneracy between  $r_s$ ,  $H_0$  and  $E(z)$ . Thus  $H_0$  can not be derived using the BAO data alone which constrain  $E(z)$  and the degeneracy is broken when  $r_s$  is fixed using either CMB power spectra (Zarrouk *et al.* 2018) or deuterium abundance (Addison *et al.* 2018, 2013).

For example  $r_s = 147.05 \pm 0.30 Mpc$  is inferred from Planck18 TT,TE,EE+lowE CMB data (Aghanim *et al.* 2020e). Using the independent determination of  $r_s$ , measuring the angular acoustic scale  $\theta_s$  from the location of the first acoustic peak in the CMB spectrum and fitting the integral in Eq. (2.23) using low  $z$  BAO or SnIa data, the Hubble constant  $H_0$  can be derived. This is the ‘inverse distance ladder’ approach (Aubourg *et al.* 2015; Cai *et al.* 2022a; Cuesta *et al.* 2015) which uses the sound horizon scale calibrated by the CMB peaks or by Big Bang Nucleosynthesis (BBN) (Schöneberg *et al.* 2019) instead of the SnIa absolute magnitude  $M$  calibrated by Cepheid stars to obtain  $H_0$ .

The deformation of the expansion rate  $H(a)$  before recombination using additional components like early dark energy that increase  $H(a)$  in Eq. (2.21) and thus decrease  $r_s$  and increase the predicted value of  $H_0$  for fixed measured  $\theta_s$  in Eq. (2.23), has been used as a possible approach to the solution of the Hubble tension. A challenge for this class of models is the required fine-tuning so that the evolution of  $H(z)$  returns quickly to its standard form after recombination for consistency with lower  $z$  cosmological probes and growth measurements (Pogosian *et al.* 2010). The assumed increase of  $H(z)$

at early times has been claimed to lead to a worsened growth tension (Jedamzik *et al.* 2021) as discussed below even though the issue is under debate (Chudaykin *et al.* 2021; Smith *et al.* 2021).

### II.1.2.1 Observational data - Constraints

- **CMB:** The measurement of the Hubble constant  $H_0$  using the sound horizon at recombination as standard ruler calibrated by the CMB anisotropy spectrum is model dependent and is based on assumptions about the nature of dark matter and dark energy as well as on an uncertain list of relativistic particles (see Chang *et al.* 2022b, for a review). The best fit value obtained by the Planck18/ΛCDM CMB temperature, polarization, and lensing power spectra is  $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Aghanim *et al.* 2020e). The measurements of the CMB from the combination Atacama Cosmology Telescope (ACT)<sup>12</sup> and Wilkinson Microwave Anisotropy Probe (WMAP) estimated the Hubble constant to be  $H_0 = 67.6 \pm 1.1 \text{ km s}^{-1} \text{Mpc}^{-1}$  and from ACT alone to be  $H_0 = 67.9 \pm 1.5 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Aiola *et al.* 2020). Note that the analysis of the nine-year data release of WMAP (Hinshaw *et al.* 2013) alone prefers a value for the Hubble constant  $H_0 = 70.0 \pm 2.2 \text{ km s}^{-1} \text{Mpc}^{-1}$ . More recently, Balkenhol *et al.* (2021) obtain CMB-based constraints on Hubble parameter  $H_0 = 67.49 \pm 0.53 \text{ km s}^{-1} \text{Mpc}^{-1}$  using combined South Pole Telescope<sup>13</sup> (SPT), Planck, and ACT DR4 datasets. Dutcher *et al.* (2021) find  $H_0 = 68.8 \pm 1.5 \text{ km s}^{-1} \text{Mpc}^{-1}$  using SPT-3G data alone, while a previous analysis of SPT data by Henning *et al.* (2018) results in  $H_0 = 71.3 \pm 2.1 \text{ km s}^{-1} \text{Mpc}^{-1}$ .
- **BAO:** The analysis of the wiggle patterns of BAO is an independent way of measuring cosmic distance using the CMB sound horizon as a standard ruler. This measurement has improved in accuracy through a number of galaxy surveys which detect this cosmic distance scale: the Sloan Digital Sky Survey (SDSS) supernova survey (Tegmark *et al.* 2006; York *et al.* 2000) encompassing the Baryon Oscillation Spectroscopic Survey (BOSS) which has completed three different phases (Dawson *et al.* 2013). Its fourth phase (SDSS-IV) (Blanton *et al.* 2017) encompasses the Extended Baryon Oscillation Spectroscopic Survey (eBOSS) (Dawson *et al.* 2016) (see also Alam *et al.* 2021a; Gil-Marin *et al.* 2020; Hou *et al.* 2020; Neveux *et al.* 2020; Raichoor *et al.* 2020), the WiggleZ Dark Energy Survey (Blake *et al.* 2011a, 2012; Drinkwater *et al.*

2010), the 2-degree Field Galaxy Redshift Survey (2dFGRS) (Cole *et al.* 2005; Colless *et al.* 2001), the 6-degree Field Galaxy Survey (6dFGS) (Beutler *et al.* 2011, 2012; Jones *et al.* 2009).

More recently, BAO measurements have been extended in the context of quasar redshift surveys and Ly $\alpha$  absorption lines of neutral hydrogen in the IGM detected in QSO spectra using the complete eBOSS survey. The measurement of BAO scale using first the auto-correlation of Ly $\alpha$  function (Bautista *et al.* 2017; Delubac *et al.* 2015; de Sainte Agathe *et al.* 2019) and then the Ly $\alpha$ -quasar cross-correlation function (Blomqvist *et al.* 2019; Font-Ribera *et al.* 2014) or both the auto- and cross-correlation functions (du Mas des Bourboux *et al.* 2020) pushed BAO measurements to higher redshifts ( $z \sim 2.4$ ). Recent studies present BAO measurements from the Ly $\alpha$  using the eBOSS sixteenth data release (DR16) (Ahumada *et al.* 2020) of the SDSS IV (e.g. du Mas des Bourboux *et al.* 2020).

As discussed in subsection II.1.2 BAO data alone cannot constrain  $H_0$  because BAO observations measure the combination  $H_0 r_s$  rather than  $H_0$  and  $r_s$  individually (where  $r_s$  is the radius of sound horizon). Using the CMB calibrated physical scale of the sound horizon and the combination of BAO with SnIa data (i.e inverse distance ladder) the value  $H_0 = 67.3 \pm 1.1 \text{ km s}^{-1} \text{Mpc}^{-1}$  was reported which is in agreement with the value obtained by CMB data alone (Aubourg *et al.* 2015). The analysis by Wang *et al.* (2017) using a combination of BAO measurements from 6dFGS (Beutler *et al.* 2011), Main Galaxy Sample (MGS) (Ross *et al.* 2015), BOSS DR12 and eBOSS DR14 quasar sample in a flat ΛCDM cosmology reports  $H_0 = 69.13 \pm 2.34 \text{ km s}^{-1} \text{Mpc}^{-1}$ . Using BAO measurements and CMB data from WMAP, Zhang and Huang (2019) reported the constraints of  $H_0 = 68.36^{+0.53}_{-0.52}$ . The analysis by Addison *et al.* (2018) combining galaxy and Ly $\alpha$  forest BAO with a precise estimate of the primordial deuterium abundance (BBN) results in  $H_0 = 66.98 \pm 1.18 \text{ km s}^{-1} \text{Mpc}^{-1}$  for the flat ΛCDM model. Alam *et al.* (2021b) find  $H_0 = 67.35 \pm 0.97 \text{ km s}^{-1} \text{Mpc}^{-1}$  using BOSS galaxy and eBOSS, with the BBN prior independent from the CMB anisotropies. D'Amico *et al.* (2020) obtain  $H_0 = 68.5 \pm 2.2 \text{ km s}^{-1} \text{Mpc}^{-1}$  performing a analysis for the cosmological parameters of the DR12 BOSS data using the Effective Field Theory of Large-Scale Structure (EFTofLSS) formalism<sup>14</sup> and Colas *et al.* (2020) obtain  $H_0 =$

<sup>12</sup> <https://act.princeton.edu>

<sup>13</sup> <https://pole.uchicago.edu>

<sup>14</sup> The EFTofLSS formalism can provide a prediction of the LSS clustering in the mildly non-linear regime (Baumann *et al.* 2012; Carrasco *et al.* 2012; D'Amico *et al.* 2021a; Perko *et al.* 2016; Porto *et al.* 2014).

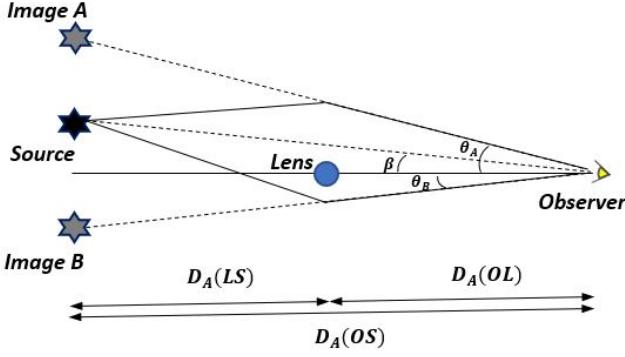


FIG. 8. Schematic illustration of a typical gravitational lens system.

$68.7 \pm 1.5 \text{ km s}^{-1} \text{Mpc}^{-1}$  assuming a BBN prior on the baryon fraction of the energy density instead of the baryon/dark-matter ratio.

Recently, Pogosian *et al.* (2020) report the constraints of  $H_0 = 69.6 \pm 1.8 \text{ km s}^{-1} \text{Mpc}^{-1}$  using BAO data, including the released eBOSS DR16, and CMB data from Plank. Zhang *et al.* (2022) infer  $H_0 = 68.19 \pm 0.99$  imposing BBN priors on the baryon density and combining the BOSS Full Shape with the BAO measurements from BOSS and eBOSS. Also, a new analysis of galaxy 2-point functions in the BOSS survey, including full-shape information and post-reconstruction BAO by Chen *et al.* (2022) results in  $H_0 = 69.23 \pm 0.77 \text{ km s}^{-1} \text{Mpc}^{-1}$  and a full-shape analysis of BOSS DR12 by Philcox and Ivanov (2022) results in  $H_0 = 68.31^{+0.83}_{-0.86} \text{ km s}^{-1} \text{Mpc}^{-1}$ . A previous analysis of BOSS DR12 on anisotropic galaxy clustering in Fourier space by Ivanov *et al.* (2020c) gives  $H_0 = 67.9 \pm 1.1 \text{ km s}^{-1} \text{Mpc}^{-1}$ . Finally, analyzing the BOSS DR12 galaxy power spectra using a new approach based on the horizon scale at matter-radiation equality Farren *et al.* (2021) find  $H_0 = 69.5^{+3.0}_{-3.5} \text{ km s}^{-1} \text{Mpc}^{-1}$  and adding Planck lensing Philcox *et al.* (2021) find  $H_0 = 70.6^{+3.0}_{-5.4} \text{ km s}^{-1} \text{Mpc}^{-1}$ .

### II.1.3 Time delays: gravitational lensing

Gravitational lensing time-delay cosmography can be used to measure  $H_0$ . This approach was first proposed by Refsdal (1964) and recently implemented by Birrer *et al.* (2020); Shajib *et al.* (2020); Wong *et al.* (2020) (see also Suyu *et al.* 2018; Treu and Marshall 2016, for clear reviews). Strong gravitational lensing (Refsdal 1964) arises from the gravitational deflection of light rays of a background source when an intervening lensing mass distribution (e.g. a massive galaxy or cluster of galaxies) exists along the line of sight. The light rays go through different paths such that multiple images of the background

source appear around the intervening lens (Oguri 2019).

The time delay  $\Delta t_{AB}$  between two images  $\theta_A$  and  $\theta_B$  by a single deflector originating from the same source at angle  $\beta$  shown in Fig. 8 is given as (Suyu *et al.* 2010)

$$\Delta t_{AB} = \frac{1 + z_L}{c} \frac{D_A(OL)D_A(OS)}{D_A(LS)} [\phi(\theta_A, \beta) - \phi(\theta_B, \beta)] \quad (2.25)$$

where  $z_L$  is the lens redshift,  $D_A(OL)$  is the angular diameter distance to the lens,  $D_A(OS)$  is the angular diameter distance to the source,  $D_A(LS)$  is the angular diameter distance between the lens and the source and  $\phi(\theta, \beta)$  is the Fermat potential (e.g. Suyu *et al.* 2010)

$$\phi(\theta, \beta) = \frac{(\theta - \beta)^2}{2} - \psi(\theta) \quad (2.26)$$

with  $\psi(\theta)$  the lensing potential at the image direction. The time delay  $\Delta t_{AB}$  in Eq. (2.25) is thus connected to the time delay distance defined as (e.g. Suyu *et al.* 2010, 2017)

$$D_{\Delta t} = \frac{1 + z_L}{c} \frac{D_A(OL)D_A(OS)}{D_A(LS)} \quad (2.27)$$

This distance is inversely proportional to  $H_0$

$$D_{\Delta t} \propto \frac{1}{H_0} \quad (2.28)$$

and thus its measurement constrains  $H_0$ . Strongly lensed quasars (bright and time variable sources) lensed by a foreground lensing mass are used to measure the above observable time delay on cosmologically interesting scales (Bonvin *et al.* 2017; Keeton and Kochanek 1997; Kochanek 2003; Oguri 2007; Wong *et al.* 2020). Active galactic nuclei (AGN) constitute another background source which may be used to measure the time delay (Eigenbrod *et al.* 2006; Fassnacht *et al.* 1999; Kochanek *et al.* 2006). Recently, Qi *et al.* (2022) proposed the strongly lensed SnIa as a precise late-universe probe to improve the measurements on the Hubble constant and cosmic curvature. The inference of  $H_0$  from  $D_{\Delta t}$  is relatively insensitive to the assumed background cosmology.

Note that a source of systematic effects in time delay cosmography is the uncertainty of the mass along the line of sight modeling with respect to the mass sheet transformation (MST). This is a mathematical degeneracy (e.g. Falco *et al.* 1985; Gorenstein *et al.* 1988; Kochanek 2002, 2020; Saha and Williams 2006; Schneider and Sluse 2014) and can bias the strong lensing determination of Hubble constant (Kochanek 2021).

#### II.1.3.1 Observational data - Constraints:

Strong gravitational lensing time delay measurements of  $H_0$  are consistent with the local measurements using late time calibrators and in mild tension with Planck (e.g. Bonvin *et al.* 2017). The method of the measurement of  $H_0$  Lenses in COSMOGRAIL's Wellspring (H0LiCOW) collaboration (Wong *et al.* 2020) is independent of the

cosmic distance ladder and is based on time delays between multiple images of the same source, as occurs in strong gravitational lensing.

Using joint analysis of six gravitationally lensed quasars with measured time delays from the COSmological MOnitoring of GRAvItational Lenses (COSMOGRAIL) project, the value  $H_0 = 73.3^{+1.7}_{-1.8} \text{ km s}^{-1} \text{Mpc}^{-1}$  was obtained which is in  $3.1\sigma$  tension with Planck CMB. Assuming the Universe is flat and using lensing systems from the lensing program H0LiCOW and the Pantheon supernova compilation a value of  $H_0 = 72.2 \pm 2.1 \text{ km s}^{-1} \text{Mpc}^{-1}$  was reported by the analysis of Ref. (Liao *et al.* 2019). A similar value of  $H_0 = 72.8^{+1.6}_{-1.7} \text{ km s}^{-1} \text{Mpc}^{-1}$  was found using updated H0LiCOW dataset consisting of six lenses (Liao *et al.* 2020). The reanalysis of the four publicly released lenses distance posteriors from the H0LiCOW by Yang *et al.* (2020b) leads to  $H_0 = 73.65^{+1.95}_{-2.26} \text{ km s}^{-1} \text{Mpc}^{-1}$ . The analysis of the strong lens system DES J0408 – 5354 by Shajib *et al.* (2020) for strong lensing insights into dark energy survey collaboration (STRIDES), infers  $H_0 = 74.2^{+2.7}_{-3.0} \text{ km s}^{-1} \text{Mpc}^{-1}$  in the  $\Lambda$ CDM cosmology. The analysis by Birrer *et al.* (2020) based on the strong lensing and using Time-Delay COSMOgraphy (TDCOSMO<sup>15</sup>, <sup>16</sup>) data set alone infers  $H_0 = 74.5^{+5.6}_{-6.1} \text{ km s}^{-1} \text{Mpc}^{-1}$  and using a joint hierarchical analysis of the TDCOSMO and Sloan Lens ACS (SLACS) (Bolton *et al.* 2006) sample reports  $H_0 = 67.4^{+4.1}_{-3.2} \text{ km s}^{-1} \text{Mpc}^{-1}$ . Chen *et al.* (2019) based on a joint analysis of 3 strong lensing system, using ground-based adaptive optics (AO) from SHARP AO effort and the HST find  $H_0 = 76.8 \pm 2.6 \text{ km s}^{-1} \text{Mpc}^{-1}$ . A reanalysis of six of the TDCOSMO lenses using a power-law mass profile model results in  $H_0 = 74.2 \pm 1.6 \text{ km s}^{-1} \text{Mpc}^{-1}$  (Millon *et al.* 2020). Analysing 8 strongly, quadruply lensing systems Denzel *et al.* (2021) present a determination of the Hubble constant  $H_0 = 71.8^{+3.9}_{-3.3} \text{ km s}^{-1} \text{Mpc}^{-1}$  which is consistent with both early and late Universe observations. The value  $H_0 = 73.6^{+1.8}_{-1.6} \text{ km s}^{-1} \text{Mpc}^{-1}$  was reported by Qi *et al.* (2021) by combining the observations of ultra-compact structure in radio quasars and strong gravitational lensing with quasars acting as background source.

#### II.1.4 Standard sirens: gravitational waves

An independent and potentially highly effective approach for the measurement of  $H(z)$  and the Hubble constant is the use of gravitational wave (GW) observations and in particular those GW bursts that have an electromagnetic counterpart (standard sirens) (Dalal *et al.* 2006; Holz and Hughes 2005; Nissanke *et al.* 2013, 2010; Schutz 1986). In analogy with the traditional standard candles, it is

possible to use standard sirens to directly measure the luminosity distance  $d_L$  of the GW source.

Standard sirens involve the combination of a GW signal and its independently observed electromagnetic (EM) counterpart. Such counterpart may involve short gamma-ray bursts (SGRBs) signal from binary neutron star mergers (Eichler *et al.* 1989) or associated isotropic kilonova emission (Coulter *et al.* 2017; Soares-Santos *et al.* 2017) and enables the immediate identification of the host galaxy. In contrast to traditional standard candles such as SnIa calibrated by Cepheid variables, standard sirens do not require any form of cosmological distance ladder. Instead they are calibrated in the context of general relativity through the observed GW waveform.

The simultaneous observations of the GW signal and its EM counterpart (multi-messenger observations) of nearby compact-object merger leads to a measurement of the luminosity distance which depends on the inclination angle of the binary orbit with respect to the line of sight and the redshift (measured using photons) of the host galaxy respectively. An EM counterpart detected with a GW observation can further constrain the inclination angle and may also indicate the source's sky position and the GW merger's time and phase (e.g. Nissanke *et al.* 2013).

In the case of GW events with small enough localization volumes without an observed EM counterpart (dark sirens) (Chen *et al.* 2018) a statistical analysis over a set of potential host galaxies within the event localization region may provide redshift information. A candidate for such statistical method is a merger of stellar-mass binary black holes<sup>17</sup> (BBH) which is usually not expected to result in bright EM counterparts unless it takes place in significantly gaseous environment (Graham *et al.* 2020). For example, GW190521 (Abbott *et al.* 2020c) is a possible candidate with EM counterpart corresponding to a stellar-origin BBH merger in active galactic nucleus (AGN) disks (McKernan *et al.* 2019) detected by ZTF (Bellm *et al.* 2018; Graham *et al.* 2019).

Alternatively, in the absence of an EM counterpart the redshift can be determined by exploiting information on the properties of the source (e.g. the knowledge of neutron star equation of state) to derive frequency-dependent features in the waveform (Messenger and Read 2012) or using the gravitational waveform to determine the redshift of the mass distribution of the sources (Farr *et al.* 2019; Taylor and Gair 2012). Also, Leandro *et al.* (2022) use an alternative method, presented in Ding *et al.* (2019), for redshift determination by the statistical knowledge of the redshift distribution of sources. Trott and Huterer (2021) argue that any absolute determination of  $H_0$  may be biased due to the fundamental

<sup>15</sup> TDCOSMO collaboration (Millon *et al.* 2020) was formed by members of H0LiCOW, STRIDES, COSMOGRAIL and SHARP.

<sup>16</sup> <http://www.tdcosmo.org/>

<sup>17</sup> The stability analysis of the structures around black holes have been widely employed in the literature (see e.g. Alestas *et al.* 2020b; Aretakis 2011; Belczynski *et al.* 2016; Cunha *et al.* 2017; Kiuchi *et al.* 2011).

degeneracy between redshift and  $H_0$  and therefore can not lead to reliable determination of  $H_0$ . According to Trott and Huterer (2021) the reliable determination of  $H_0$  with GW can only be achieved using standard sirens.

The luminosity distance-redshift relation Eq. (2.2) determines the Universe's expansion history and the associated cosmological parameters including the Hubble constant  $H_0$  (Abbott et al. 2017a; Fishbach et al. 2019). In particular using the mergers of binary neutron stars (BNS), or a binary of a neutron star with a stellar-mass black hole (NS-BH), which are excellent standard sirens, both the luminosity distance (from the gravitational wave waveform) and redshift of the host galaxy (from the electromagnetic counterpart) can be measured.

Using a BNS or a NS-BH merger, the distance to the source can be estimated from the detected amplitude  $\langle h \rangle$  (r.m.s. - averaged over detector and source orientations) of the GW signal by the expression (Andersson and Kokkotas 1996; Jaranowski et al. 1996; Kokkotas 2008; Kokkotas and Stergioulas 2005; Schutz 1986)

$$d = Cf^{-2}\langle h \rangle^{-1}\tau^{-1} \quad (2.29)$$

where  $f$  is the gravitational wave frequency,  $\tau \equiv f/\dot{f}$  is the timescale of frequency change,  $C$  is a known numerical constant. Assuming a flat<sup>18</sup> Universe the luminosity distance can then be obtained from the relation

$$d(z) = \frac{1}{1+z}d_L(z) \quad (2.30)$$

For nearby sources, the recession velocity using the Hubble's law is determined by the expression

$$v(z) = H_0 d(z) \quad (2.31)$$

and using Eqs. (2.2), (2.7) and (2.30) is given by

$$v(z) = \frac{H_0 d_L(z)}{1+z} = \frac{c D_L(z)}{1+z} = c H_0 \int_0^z \frac{dz'}{H(z')} \quad (2.32)$$

At low redshifts using the local expansion Eq. (2.11) we obtain

$$v(z) = \frac{cz}{1+z} \left[ 1 + \frac{1}{2}(1-q_0)z \right] \quad (2.33)$$

which is approximated for  $d \leq 100 \text{ Mpc}$  (or  $z \leq 0.03$ ) as

$$v(z) = cz = H_0 d \quad (2.34)$$

Using Eqs. (2.31) and (2.33), the equation for the determination of  $H_0$  as a function of observables,  $z$  and  $d$  is (e.g. Zhang et al. 2017)

$$H_0 = \frac{cz}{d(1+z)} \left[ 1 + \frac{1}{2}(1-q_0)z \right] \quad (2.35)$$

where the deceleration parameter may be set by a fit to the GW data or may be fixed to its Planck/ $\Lambda$ CDM best fit form ( $q_0 = -0.55$ ).

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<sup>18</sup> In an open (closed) Universe the distance in Hubble's law is given  $d(z) = \frac{1}{1+z} \frac{\chi}{\sinh \chi} d_L(z)$  ( $d(z) = \frac{1}{1+z} \frac{\chi}{\sin \chi} d_L(z)$ )

**II.1.4.1 Observational data - Constraints:** The first multi-messenger detection of a BNS merger, GW170817, by LIGO (Aasi et al. 2015) and Virgo (Acernese et al. 2015) interferometers enabled the first standard siren measurement of the Hubble constant  $H_0$ . Using the BNS merger GW170817, the distance to the source was estimated to be  $d = 43.8^{+2.9}_{-6.9} \text{ Mpc}$  (i.e. at redshift  $z \sim 0.01$ ) from the detected amplitude  $\langle h \rangle$  (r.m.s. - averaged over detector and source orientations) of the GW signal by the Eq. (2.29) (Abbott et al. 2017a). Also using the Hubble flow velocity  $v_H = 3017 \pm 166 \text{ km s}^{-1}$  inferred from measurement of the redshift of the host galaxy, NGC 4993 (NGC 4993 was identified as the unique host galaxy), the Hubble constant was determined to be  $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Abbott et al. 2017a) (see Fig. 9) by using Eq. (2.34).

Using continued monitoring of the the radio counterpart of GW17081 combining with earlier GW and EM data Hotokezaka et al. (2019) obtain a improved measurement of  $H_0 = 68.9^{+4.7}_{-4.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Note that using the BNS merger GW170817 in Fishbach et al. (2019) and a statistical analysis (as first proposed in Schutz (1986)) over a catalog of potential host galaxies, the Hubble constant was determined to be  $H_0 = 77.0^{+37.0}_{-18.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Using density-estimation Likelihood-Free Inference (LFI) Gerardi et al. (2021) focused on the inference of the cosmological expansion  $H_0$  from GW-selected catalogues of BNS mergers with EM counterparts.

Also using the BBH merger GW170814 as a standard (dark) siren in the absence of an electromagnetic counterpart, combined with a photometric redshift catalog from the Dark Energy Survey (DES) (Abbott et al. 2018c) the analysis by Soares-Santos et al. (2019) results in  $H_0 = 75^{+40}_{-32} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Using multiple GW observations (the BNS event GW170817 and the BBH events observed by advanced LIGO and Virgo in their first and second observing runs) in Abbott et al. (2021a) the Hubble constant was constrained as  $H_0 = 69.0^{+16.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Using the event GW190814 from merger of a black hole with a lighter compact object the Hubble constant was measured to be  $H_0 = 75^{+59}_{-13} \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Abbott et al. 2020a). In Mukherjee et al. (2020) the BBH merger GW190521 was analysed choosing the NR-Sur7dq4 waveform<sup>19</sup> for the estimation of luminosity distance, after marginalizing over matter density  $\Omega_{0m}$  when the  $\Lambda$ CDM model is considered and using its EM counterpart ZTF19abanhr<sup>20</sup> as identified in Graham et al. (2020) the Hubble constant was measured to be  $H_0 =$

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<sup>19</sup> NR-Sur7dq4 is a numerical relativity surrogate 7-dimensional approximate waveform model of binary black hole merger with mass ratios  $q \equiv \frac{m_1}{m_2} \leq 4$  (Varma et al. 2019). This model is made publicly available through the gwsurrogate (see <https://pypi.org/project/gwsurrogate>) and surfinBH (see <https://pypi.org/project/surfinBH>) Python packages.

<sup>20</sup> The ZTF19abanhr event was reported by ZTF (Bellm et al. 2018). This candidate EM counterpart is flare after a kicked BBH merger in the accretion disk of an AGN (McKernan et al.

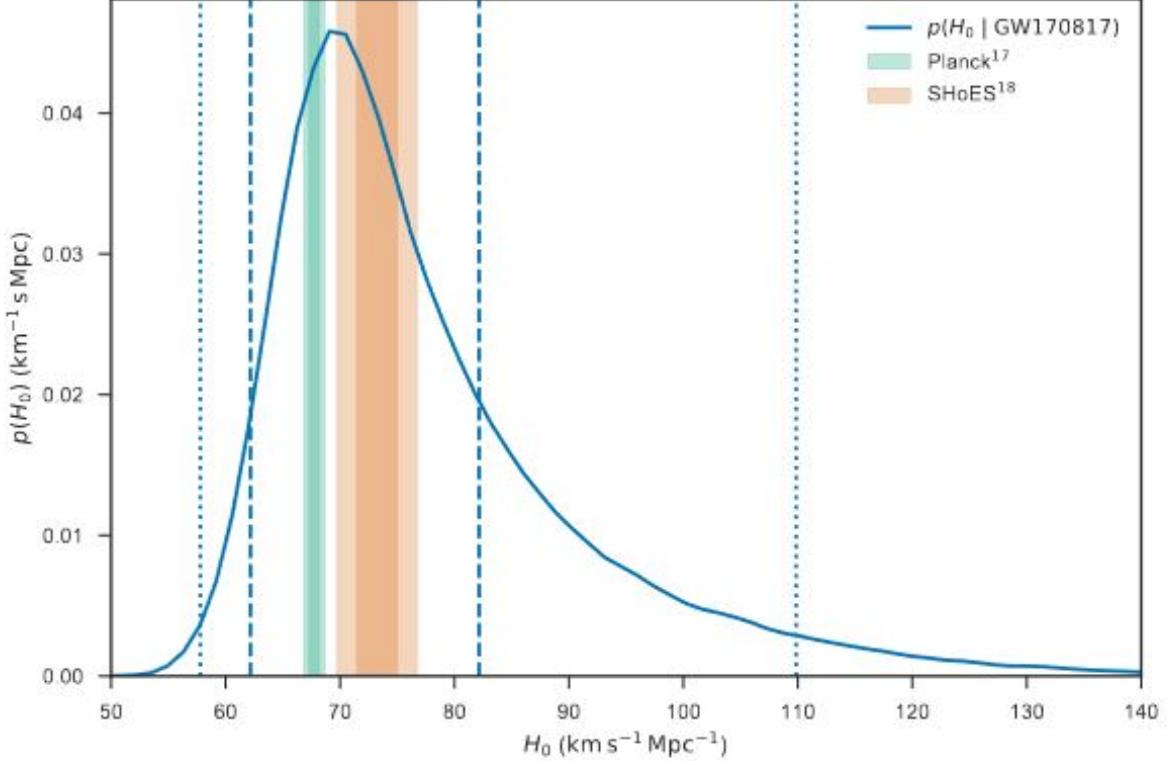


FIG. 9. The probability of different values of  $H_0$  with the maximum at  $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{Mpc}^{-1}$  (solid blue curve) derived by BNS event GW170817. The dashed and dotted lines show minimal 68.3% (1 $\sigma$ ) and 95.4% (2 $\sigma$ ) credible intervals. The shaded green and orange bands show the 1 $\sigma$  and 2 $\sigma$  constraints from the analysis of the CMB data obtained by the Planck (Ade *et al.* 2016a) and from the analysis of the SNe Ia data obtained by SHoES (Riess *et al.* 2016) respectively (from Abbott *et al.* 2017a).

$50.4^{+28.1}_{-19.5} \text{ km s}^{-1} \text{Mpc}^{-1}$ . The same study (Mukherjee *et al.* 2020) choosing different types of waveform finds  $H_0 = 43.1^{+24.6}_{-11.4} \text{ km s}^{-1} \text{Mpc}^{-1}$  and  $H_0 = 62.2^{+29.5}_{-19.7} \text{ km s}^{-1} \text{Mpc}^{-1}$ . Combining their results with the binary neutron star event GW170817 leads to  $H_0 = 67.6^{+4.3}_{-4.2} \text{ km s}^{-1} \text{Mpc}^{-1}$ . In Chen *et al.* (2020) for the same GW-EM event, assuming a flat wCDM model has obtained  $H_0 = 48^{+23}_{-10} \text{ km s}^{-1} \text{Mpc}^{-1}$ .

The analysis by Abbott *et al.* (2021b) using 47 gravitational-wave sources from the Third LIGO–Virgo–KAGRA Gravitational-Wave Transient Catalog (GWTC-3), infers  $H_0 = 68^{+12}_{-7} \text{ km s}^{-1} \text{Mpc}^{-1}$ . Palmese *et al.* (2021) find  $H_0 = 72.77^{+11.0}_{-7.55} \text{ km s}^{-1} \text{Mpc}^{-1}$  using the best available gravitational wave events, uniform galaxy catalog from the Dark Energy Spectroscopic

Instrument (DESI) (Aghamousa *et al.* 2016a,b) Legacy Survey and combining with the GW170817. The value  $H_0 = 88.6^{+17.1}_{-34.3} \text{ km s}^{-1} \text{Mpc}^{-1}$  for GW190521 event was reported, and  $H_0 = 73.4^{+6.9}_{-10.7} \text{ km s}^{-1} \text{Mpc}^{-1}$  was obtained when combining the GW190521 with the results of the neutron star merger GW170817 (Gayathri *et al.* 2021). More recently, Mukherjee *et al.* (2022) report  $H_0 = 67^{+6.3}_{-3.8} \text{ km s}^{-1} \text{Mpc}^{-1}$  combining the bright standard siren measurement from GW170817 with a better measurement of peculiar velocity.

### II.1.5 Megamaser technique

Observations of water megamasers which are found in the accretion disks around supermassive black holes (SMBHs) in active galactic nuclei (AGN) have been demonstrated to be powerful one-step geometric probes for measuring extragalactic distances (Herrnstein *et al.* 1999; Humphreys *et al.* 2013; Reid *et al.* 2013).

Assuming a Keplerian circular orbit around the SMBH, the centripetal acceleration and the velocity of a masing

2019) with peak luminosity occurred 50 days after the BBH event GW19052. The ZTF19abanrhr was first observed after 34 days from the GW detection at the sky direction ( $RA = 192.42625^0$ ,  $Dec = 34.82472^0$ ) and was associated with an AGN J124942.3 + 344929 at redshift  $z = 0.438$  (Graham *et al.* 2020).

cloud are given as (Reid *et al.* 2013)

$$A = \frac{V^2}{r} \quad (2.36)$$

$$V = \sqrt{\frac{GM}{r}} \quad (2.37)$$

where  $G$  is the Newton's constant,  $M$  is the mass of the central supermassive black hole, and  $r$  is the distance of a maser cloud from the supermassive black hole.

The angular scale  $\theta$  subtended by  $r$  is given by

$$\theta = \frac{r}{d} \quad (2.38)$$

where  $d$  is the distance to the galaxy.

Thus, from the velocity and acceleration measurements obtained from the maser spectrum, the distance to the maser may be determined

$$d = \frac{V^2}{A\theta} \quad (2.39)$$

where  $A$  is measured from the change in Doppler velocity with time by monitoring the maser spectrum on month timescales. Using Hubble's law the Hubble constant may be approximated as (Reid *et al.* 2013)

$$H_0 \approx \frac{v}{d} \quad (2.40)$$

where  $v$  is the measured recessional velocity.

In order to constrain the Hubble constant the Megamaser Cosmology Project (MCP) (Reid *et al.* 2009) uses angular diameter distance measurements to disk megamaser-hosting galaxies well into the Hubble flow ( $50 - 200 \text{ Mpc}$ ). These distances are independent of standard candle distances and their measurements do not rely on distance ladders, gravitational lenses or the CMB (Pesce *et al.* 2020). Early measurements of  $H_0$  using masers tended to favor lower values of  $H_0 \simeq 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$  while more recent measurements favor higher values  $H_0 \simeq 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$  as shown e.g. in Table I.

**II.1.5.1 Observational data - Constraints:** Recently, the Megamaser Cosmology Project (MCP) (Reid *et al.* 2009) using geometric distance measurements to megamaser-hosting galaxies and assuming a global velocity uncertainty of  $250 \text{ km s}^{-1}$  associated with peculiar motions of the maser galaxies constrains the Hubble constant to be  $H_0 = 73.9 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Pesce *et al.* 2020). Previously the MCP reported results on galaxies, UGC 3789 with  $H_0 = 68.9 \pm 7.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Reid *et al.* 2013), NGC 6264 with  $H_0 = 68.0 \pm 9.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Kuo *et al.* 2013), NGC 6323 with  $H_0 = 73_{-22}^{+26} \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Kuo *et al.* 2015) and NGC 5765b with  $H_0 = 66.0 \pm 6.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Gao *et al.*

2016). Reid *et al.* (2019) use a improved distance estimation of the maser galaxy NGC 4258 (also known as Messier 106) to calibrate the Cepheid-SN Ia distance ladder combined with geometric distances from MW parallaxes and DEBs in the LMC. The measured value of the Hubble constant is  $H_0 = 73.5 \pm 1.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

### II.1.6 Tully-Fisher relation (TFR) as distance indicator

The Tully-Fisher (TF) method is a historically useful distance indicator based on the empirical relation between the intrinsic total luminosity (or the stellar mass) of a spiral galaxy<sup>21</sup> and its rotation velocity (or neutral hydrogen (HI) 21 cm emission line width) (Tully and Fisher 1977). This method has been used widely in measuring extragalactic distances (e.g. Sakai *et al.* 2000).

The Baryonic Tully Fisher relation (BTFR) (Gurovich *et al.* 2004; McGaugh 2005; McGaugh *et al.* 2000; Verheijen 2001) connects the rotation speed  $V_c$  and total baryonic mass  $M_b$  (stars plus gas) of a spiral galaxy as

$$M_b = A_c V_c^s \quad (2.41)$$

where  $s$  (with  $s \approx 3-4$  Lelli *et al.* 2016b; McGaugh 2005; McGaugh *et al.* 2000) is a parameter and  $\log A_c$  is the zero point in a log-log BTFR plot. This relation has been measured for hundreds of galaxies. The rotation speed  $V_c$  can be measured independently of distance while the total baryonic mass  $M_b$  may be used as distance indicator since it is connected to the intrinsic luminosity. Thus, the BTFR is a useful cosmic distance indicator approximately independent of redshift and thus can be used to obtain  $H_0$ .

The BTFR has a smaller amount of scatter with a corresponding better accuracy as a distance indicator than the classic TF relation (Lelli *et al.* 2016b). In addition the BTFR recovers two decades in velocity and six decades in mass (Iorio *et al.* 2016; Lelli *et al.* 2019; McGaugh 2012, 2005; McGaugh *et al.* 2000; Schombert *et al.* 2020).

A simple heuristic analytical derivation for the BTFR is obtained (Aaronson *et al.* 1979) by considering a star rotating with velocity  $v$  in a circular orbit of radius  $R$  around a central mass  $M$ . Then the star velocity is connected with the central mass as

$$v^2 = G M_b / R \implies v^4 = (G M_b / R)^2 \sim M_b S G^2 \quad (2.42)$$

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<sup>21</sup> Similarly, in the case of a elliptical galaxy the Faber-Jackson (FJ) empirical power-law relation  $L \propto \sigma^{\gamma_{FJ}}$  (where  $L$  is the luminosity of galaxy,  $\sigma$  the velocity dispersion of its stars and  $\gamma_{FJ}$  is a index close to 4) (Faber and Jackson 1976) can be used as a distance indicator. The FJ relation is the projection of the fundamental plane (FP) of elliptical galaxies which defined as  $R_{eff} \propto \sigma^{s_1} I_{eff}^{s_2}$  (where  $R_{eff}$  is the effective radius and  $I_{eff}$  is the mean surface brightness within  $R_{eff}$ ) (Djorgovski and Davis 1987).

where  $G$  is Newton's constant and  $S$  the surface density  $S \equiv M/R^2$  which may be shown to be approximately constant (Freeman 1970). From Eqs. (2.41) and (2.42) we have

$$A_c \sim G^{-2} S^{-1} \quad (2.43)$$

which indicates that the zero point intercept of the BTFR can probe both galaxy formation dynamics (through e.g.  $S$ ) and possible fundamental constant dynamics (through  $G$ ) (Aleñas *et al.* 2021a).

**II.1.6.1 Observational data - Constraints:** The analysis by Kourkchi *et al.* (2020) using infrared (IR) data of sample galaxies and the Tully Fisher relation determined the value of Hubble constant to be  $H_0 = 76.0 \pm 1.1(\text{stat.}) \pm 2.3(\text{sys.}) \text{ km s}^{-1} \text{Mpc}^{-1}$ . In Schombert *et al.* (2020) a value of  $H_0 = 75.1 \pm 2.3(\text{stat.}) \pm 1.5(\text{sys.}) \text{ km s}^{-1} \text{Mpc}^{-1}$  was found using Baryonic Tully Fisher relation for 95 independent Spitzer photometry and accurate rotation curves (SPARC) galaxies<sup>22</sup> (up to distances of  $\sim 130 \text{ Mpc}$ ).

### II.1.7 Extragalactic background light $\gamma$ -ray attenuation

This method is based on the fact that the extragalactic background light (EBL) which is a diffuse radiation field that fills the Universe from ultraviolet (UV) through infrared wavelength induces opacity for very high energy (VHE) photons ( $\geq 30 \text{ GeV}$ ) induced by photon-photon interaction (Hauser and Dwek 2001). In this process a  $\gamma$ -ray and an EBL photon in the intergalactic medium may annihilate and produce an electron-positron pair (Gould and Schréder 1966). The induced attenuation in the spectra of  $\gamma$ -ray sources is characterized by an optical depth  $\tau_{\gamma\gamma}$  that scales as  $n\sigma_T l$  (where  $n$  is the photon density of the EBL,  $\sigma_T$  is the Thomson cross section, and  $l$  is the distance from the  $\gamma$ -ray source to Earth). The cosmic evolution and the matter content of the Universe determine the  $\gamma$ -ray optical depth and the amount of  $\gamma$ -ray attenuation along the line of sight (Domínguez *et al.* 2019; Zeng and Yan 2019). Thus a derivation of  $H_0$  can be obtained by measuring the  $\gamma$ -ray optical depth with the  $\gamma$ -ray telescopes (Domínguez and Prada 2013). This derivation is independent and complementary to that based on the distance ladder and cosmic microwave background (CMB) and seems to favor lower values of  $H_0$  as shown in Table I.

**II.1.7.1 Observational data - Constraints:** The analysis by Domínguez *et al.* (2019) using extragalactic background light  $\gamma$ -ray attenuation data

from Fermi Large Area Telescope (Fermi-LAT) derives  $H_0 = 67.4^{+6.0}_{-6.2} \text{ km s}^{-1} \text{Mpc}^{-1}$  and  $\Omega_{0m} = 0.14^{+0.06}_{-0.07}$ . The analysis by Zeng and Yan (2019) fitting the  $> 10 \text{ GeV}$  extragalactic background data with modeled extragalactic background spectrum results in  $H_0 = 64.9^{+4.6}_{-4.3} \text{ km s}^{-1} \text{Mpc}^{-1}$  and  $\Omega_{0m} = 0.31^{+0.13}_{-0.14}$ .

### II.1.8 Cosmic chronometers

Cosmic chronometers are objects whose evolution history is known. For instance such objects are some types of galaxies. The observation of these objects at different redshifts and the corresponding differences in their evolutionary state has been used to obtain the value of  $H(z)$  at each redshift  $z$ .

The cosmic chronometer technique for the determination of  $H_0$  was originally suggested in Jimenez and Loeb (2002) and is based on the quasi-local ( $0.07 \lesssim z \lesssim 2.36$ ) measurements along the Hubble flow of the Hubble parameter expressed as

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} \quad (2.44)$$

Thus, the expansion rate may be obtained by measuring the age difference  $\Delta t$  between two old and passively evolving galaxies<sup>23</sup> which are separated by a small redshift interval  $\Delta z$ , to infer the  $dz/dt$  (Moresco *et al.* 2016, 2012).

This approach determines the  $H_0 = H(z=0)$  independent of the early-Universe physics and is not based on the distance ladder (e.g. Chen *et al.* 2017; Farooq *et al.* 2017; Gómez-Valent and Amendola 2018; Jimenez and Loeb 2002; Yu *et al.* 2018). The estimated  $H_0$  values are more consistent with the values estimated from recent CMB and BAO data than those values estimated from SNIa. The value of  $H_0$  can not be derived using the cosmic chronometers observations alone because there is a background degeneracy between  $H_0$  and  $\Omega_{0m}$  and this degeneracy is broken when these observations are combined.

**II.1.8.1 Observational data - Constraints:** In Chen *et al.* (2017) the value of Hubble constant was found to be  $H_0 = 68.3^{+2.7}_{-2.6}$  in the flat  $\Lambda$ CDM model relying on 28  $H(z)$  measurements and their extrapolation to redshift zero. Analysing 31  $H(z)$  data determined by the cosmic chronometric (CCH) method, and 5  $H(z)$  data by BAO observations and using the Gaussian Process (GP) method (Joudaki *et al.* 2018a; Seikel *et al.* 2012; Shafieloo *et al.* 2012; Yahya *et al.* 2014) to determine a continuous  $H(z)$  function the Hubble constant is estimated to be  $H_0 \sim 67 \pm 4 \text{ km s}^{-1} \text{Mpc}^{-1}$  by Yu *et al.* (2018). Also

<sup>22</sup> The SPARC catalogue contains 175 nearby (up to distances of  $\sim 130 \text{ Mpc}$ ) late-type galaxies (spirals and irregulars) (Lelli *et al.* 2016a,b). The SPARC data are publicly available at <http://astroweb.cwr.edu/SPARC>.

<sup>23</sup> These galaxies form only a few new stars and become fainter and redder with time. The time that has elapsed since they stopped star formation can be deduced.

using the GP an extension of this analysis by [Gómez-Valent and Amendola \(2018\)](#), including the  $H(z)$  measurements obtained from Pantheon compilation and HST CANDELS and CLASH Multi-Cycle Treasury (MCT) programs, finds  $H_0 = 67.06 \pm 1.68 \text{ km s}^{-1} \text{ Mpc}^{-1}$  which is more consistent again with the lower range of values for  $H_0$ . The GP method ([Rasmussen and Williams 2005](#)) is used as a ‘non-parametric’ technique which does not assume any parametrization or any cosmological model (see [Ó Colgáin and Sheikh-Jabbari 2021](#), for a discussion about GP as model independent method). The GP modeling approach has been performed by several authors to reconstruct cosmological parameters and thus to extract cosmological information directly from data (see e.g [Avila et al. 2021](#); [Belgacem et al. 2020b](#); [Bengaly 2022, 2020](#); [Bengaly et al. 2020a](#); [Benisty 2021](#); [Benisty et al. 2022b](#); [Bonilla et al. 2021a,b](#); [Briffa et al. 2020](#); [Busti et al. 2014](#); [Cai et al. 2017b](#); [Dhawan et al. 2021](#); [Escamilla-Rivera et al. 2021,?](#); [Haridasu et al. 2018](#); [Holsclaw et al. 2010a,b](#); [Keeley et al. 2020b, 2021](#); [L’Huillier and Shafieloo 2017](#); [L’Huillier et al. 2018, 2020](#); [Li et al. 2021a](#); [Marques et al. 2019](#); [Mukherjee and Banerjee 2021](#); [Nunes et al. 2020](#); [Pinho et al. 2018](#); [Ren et al. 2022](#); [Renzi et al. 2021](#); [Renzi and Silvestri 2020](#); [Ruiz-Zapatero et al. 2022](#); [Sahni et al. 2014](#); [Seikel and Clarkson 2013](#); [Shafieloo et al. 2018](#); [Sharma et al. 2022](#); [Sun et al. 2021](#); [Wang and Meng 2017b](#); [Zhang and Li 2018](#); [Zhang and Xia 2016](#)).

Recently, a analysis by [Moresco et al. \(2022\)](#) reports  $H_0 = 67.8^{+8.7}_{-7.2} \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $H_0 = 66.5 \pm 5.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$  for a generic open wCDM and for a flat  $\Lambda$ CDM respectively. The analysis by [Moresco et al. \(2022\)](#) examine the possible effects that can systematically bias the measurement and can affect the CC method. It should be pointed out however that the quality and reliability of cosmic chronometer data has been challenged by some authors. This is partly due to the fact that these datapoints are not model independent and are obtained by combining several datasets ([Gómez-Valent and Amendola 2018](#)). This has improved significantly in the context of the aforementioned analysis by [Moresco et al. \(2022\)](#) where a detailed study of the covariance matrix and the effects of systematics has been implemented.

### II.1.9 HII galaxy measurements

The ionized hydrogen gas (HII) galaxies (HIIG) emit massive and compact bursts generated by the violent star formation (VSF) in dwarf irregular galaxies. The HIIG measurements can be used to probe the background evolution of the Universe. This method of  $H_0$  determination is based on the standard candle calibration provided by a  $L - \sigma$  (luminosity-velocity dispersion) relation. This relation exists in HIIGs and Giant extragalactic HII regions (GEHR) in nearby spiral and irregular galaxies. The turbulent emission line ionized gas velocity dispersion  $\sigma$  of

the prominent Balmer lines<sup>24</sup> H-alpha ( $H\alpha$ ) and H-beta ( $H\beta$ ) relates with its integrated emission line luminosity  $L$  ([Chávez et al. 2016, 2012, 2014](#); [González-Morán et al. 2019](#); [Melnick et al. 2000](#); [Plionis et al. 2011](#); [Siegel et al. 2005](#); [Terlevich et al. 2015](#); [Wei et al. 2016](#); [Yennapureddy and Melia 2017](#)). The relationship between  $L(H\beta)$  and  $\sigma(H\beta)$  has a small enough scatter to define a cosmic distance indicator (that can be utilized out to  $z \sim 4$ ) independently of redshift and can be approximated as ([Chávez et al. 2016, 2012, 2014](#); [González-Morán et al. 2019](#); [Leaf and Melia 2018](#); [Ruan et al. 2019a](#); [Terlevich et al. 2015](#); [Wei et al. 2016](#); [Yennapureddy and Melia 2017](#))

$$\log L(H\beta) = \nu \log \sigma(H\beta) + \kappa \quad (2.45)$$

where  $\nu$  and  $\kappa$  are constants representing the slope and the logarithmic luminosity at  $\log \sigma(H\beta) = 0$ .

From Eq. (2.1) the luminosity  $L(H\beta)$  is given by

$$L(H\beta) = 4\pi d_L^2 l(H\beta) \quad (2.46)$$

Thus using Eq. (2.45), the distance modulus  $\mu \equiv m - M$  of an HIIG can be obtained ([Chávez et al. 2016](#); [González-Morán et al. 2019](#); [Leaf and Melia 2018](#); [Ruan et al. 2019a](#); [Wei et al. 2016](#); [Yennapureddy and Melia 2017](#))

$$\mu_{obs} = 2.5 [\nu \log \sigma(H\beta) + \kappa - \log l(H\beta)] - 100.2 \quad (2.47)$$

This observational distance modulus can be compared with the theoretical distance modulus. From the Eq. (2.6) this is given

$$\mu(z)_{th} = 5 \log_{10} \left[ \frac{d_L(z)}{\text{Mpc}} \right] + 25 \quad (2.48)$$

Using now the dimensionless Hubble free luminosity distance Eq. (2.7) this can be written as

$$\mu(z)_{th} = 5 \log_{10} [D_L(z)] + 5 \log_{10} \left[ \frac{c/H_0}{\text{Mpc}} \right] + 25 \quad (2.49)$$

In order to obtain the best fit values for the parameters  $\Omega_{0m}$  and  $H_0$  this theoretical prediction may now be used to compared with the observed  $\mu_{obs}$  data. Using the maximum likelihood analysis the best fit values for these parameters may be found in the usual manner by minimizing the quantity

$$\chi^2(H_0, \Omega_{0m}) = \sum_i \frac{[\mu_{obs,i} - \mu_{th}(z_i; H_0, \Omega_{0m})]^2}{\varepsilon_i^2} \quad (2.50)$$

where  $\varepsilon_i$  is the uncertainty of the  $i$ th measurement.

<sup>24</sup> The Balmer series, or Balmer lines is one of a set of six named series describing the spectral line emissions of the hydrogen atom. This is characterized by the electron transitioning from  $n \geq 3$  to  $n = 2$  (where  $n$  is the principal quantum number of the electron. The transitions  $n = 3$  to  $n = 2$  and  $n = 4$  to  $n = 2$  are called H-alpha and H-beta respectively.

**II.1.9.1 Observational data - Constraints:** Using 156 HII galaxy measurements as a new distance indicator and implementing the model-independent GP, the Hubble constant was found to be  $H_0 = 76.12^{+3.47}_{-3.44} \text{ km s}^{-1} \text{Mpc}^{-1}$  which is more consistent with the recent local measurements (Wang and Meng 2017a). Using data of 130 giant HII regions in 73 galaxies with Cepheid determined distances the best estimate of the Hubble parameter is  $H_0 = 71.0 \pm 2.8 (\text{random}) \pm 2.1 (\text{systematic}) \text{ km s}^{-1} \text{Mpc}^{-1}$  (Fernández Arenas *et al.* 2018).

### II.1.10 Combinations of data

The Hubble constant  $H_0$  values at 68% CL through direct and indirect measurements obtained by the different methods described in subsection II.1 are shown in Table I and described in more detail below in Fig. 10. Also the relative probability density value of  $H_0$  was derived by recently published studies in the literature are shown in Fig. 11.

Cosmological parameter degeneracies from each individual probe can be broken using combination of probes. The multi-probe analysis are crucial for independent  $H_0$  determination and are required in order to reduce systematic uncertainties (Chen and Ratra 2011; Suyu *et al.* 2012) (see Moresco *et al.* 2022, for a review).

The analysis by Wong *et al.* (2020) using a combination of SH0ES and H0LiCOW results reports  $H_0 = 73.8 \pm 1.1 \text{ km s}^{-1} \text{Mpc}^{-1}$  which raises the Hubble tension to  $5.3\sigma$

between late Universe determinations of  $H_0$  and Planck. This has been discredited by Kochanek (2021) who points out that an artificial reduction of the allowed degrees of freedom can lead to very precise but inaccurate estimates of  $H_0$  based on gravitational lens time delays.

The analysis by Abbott *et al.* (2018b) using a combination of the Dark Energy Survey (DES) (Abbott *et al.* 2018d; Krause *et al.* 2017; Troxel *et al.* 2018) clustering and weak lensing measurements with BAO and BBN experiments assuming a flat  $\Lambda\text{CDM}$  model with minimal neutrino mass ( $\Sigma m_\nu = 0.06 \text{ eV}$ ) finds  $H_0 = 67.2^{+1.2}_{-1.0} \text{ km s}^{-1} \text{Mpc}^{-1}$  which is consistent with the value obtained with CMB data.

Using an extension of the standard GP formalism, and a combination of low-redshift expansion rate data (SnIa+BAO+CC) the Hubble constant was estimated to be  $H_0 = 68.52^{+0.94+2.51(\text{sys})}_{-0.94} \text{ km s}^{-1} \text{Mpc}^{-1}$  by Haridasu *et al.* (2018). Using an alternative method Baxter and Sherwin (2021) analysing the current CMB lensing data from Planck combined with Pantheon supernovae and using conservative priors, find an  $r_s$  independent constraint of  $H_0 = 73.5 \pm 5.3 \text{ km s}^{-1} \text{Mpc}^{-1}$ . Analysing low-redshift cosmological data from SnIa, BAO, strong gravitational lensing,  $H(z)$  measurements using cosmic chronometers and growth measurements from LSS observations for  $\Lambda\text{CDM}$  model Dutta *et al.* (2019) find  $H_0 = 70.30^{+1.36}_{-1.35} \text{ km s}^{-1} \text{Mpc}^{-1}$  which is in  $\sim 2\sigma$  tension with various low and high redshift observations.

TABLE I: The Hubble constant  $H_0$  values at 68% CL through direct and indirect measurements by different methods.

Dataset	$H_0 [\text{km s}^{-1} \text{Mpc}^{-1}]$	Year	Refs.
<b>Planck CMB</b>	<b><math>67.27 \pm 0.60</math></b>	<b>2020</b>	(Aghanim <i>et al.</i> 2020e)
Planck CMB+lensing	$67.36 \pm 0.54$	2020	(Aghanim <i>et al.</i> 2020e)
Planck+SPT+ACT CMB	$67.49 \pm 0.53$	2021	(Balkenhol <i>et al.</i> 2021)
eBOSS+Planck CMB	$69.6 \pm 1.8$	2020	(Pogosian <i>et al.</i> 2020)
SPT-3G CMB	$68.8 \pm 1.5$	2021	(Dutcher <i>et al.</i> 2021)
ACT CMB	$67.9 \pm 1.5$	2020	(Aiola <i>et al.</i> 2020)
ACT+WMAP CMB	$67.6 \pm 1.1$	2020	(Aiola <i>et al.</i> 2020)
SPT CMB	$71.3 \pm 2.1$	2018	(Henning <i>et al.</i> 2018)
WMAP9 CMB	$70.0 \pm 2.2$	2013	(Hinshaw <i>et al.</i> 2013)
BAO+WMAP CMB	$68.36^{+0.53}_{-0.52}$	2019	(Zhang and Huang 2019)
BOSS correlation function+BAO+BBN	$68.19 \pm 0.99$	2022	(Zhang <i>et al.</i> 2022)
P+BAO+BBN	$69.23 \pm 0.77$	2022	(Chen <i>et al.</i> 2022)
P+Bispectrum+BAO+BBN	$68.31^{+0.83}_{-0.86}$	2022	(Philcox and Ivanov 2022)
BAO+BBN	$66.98 \pm 1.18$	2018	(Addison <i>et al.</i> 2018)
BOSS DR12+BBN	$68.5 \pm 2.2$	2020	(D'Amico <i>et al.</i> 2020)
BOSS DR12+BBN	$68.7 \pm 1.5$	2020	(Colas <i>et al.</i> 2020)
BOSS DR12+BBN	$67.9 \pm 1.1$	2020	(Ivanov <i>et al.</i> 2020c)
BOSS+eBOSS+BBN	$67.35 \pm 0.97$	2020	(Alam <i>et al.</i> 2021b)
LSS $t_{eq}$ standard ruler	$69.5^{+3.0}_{-3.5}$	2022	(Farren <i>et al.</i> 2021)
LSS $t_{eq}$ standard ruler+lensing	$70.6^{+3.0}_{-5.4}$	2020	(Philcox <i>et al.</i> 2021)
BAO+RSD	$69.13 \pm 2.34$	2017	(Wang <i>et al.</i> 2017)

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TABLE I – continued from previous page

Dataset	$H_0$ [ $km\ s^{-1}\ Mpc^{-1}$ ]	Year	Refs.
SnIa-Cepheid	$73.04 \pm 1.04$	2022	(Riess <i>et al.</i> 2021b)
SnIa-Cepheid	$74.30 \pm 1.45$	2021	(Camarena and Marra 2021)
SnIa-Cepheid	$73.20 \pm 1.30$	2021	(Riess <i>et al.</i> 2021a)
SnIa-Cepheid	$74.03 \pm 1.42$	2019	(Riess <i>et al.</i> 2019)
SnIa-Cepheid	$73.48 \pm 1.66$	2018	(Riess <i>et al.</i> 2018b)
SnIa-Cepheid	$72.80 \pm 2.70$	2020	(Breuval <i>et al.</i> 2020)
SnIa-Cepheid	$73.00 \pm 2.70$	2020	(Breuval <i>et al.</i> 2020)
SnIa-TRGB	$76.94 \pm 6.4$	2022	(Dhawan <i>et al.</i> 2022)
SnIa-TRGB	$72.4 \pm 3.3$	2022	(Jones <i>et al.</i> 2022)
SnIa-TRGB	$71.5 \pm 1.8$	2021	(Anand <i>et al.</i> 2021)
SnIa-TRGB	$69.8 \pm 1.7$	2021	(Freedman 2021)
SnIa-TRGB	$65.8 \pm 4.2$	2021	(Kim <i>et al.</i> 2020)
SnIa-TRGB	$72.10 \pm 2.10$	2020	(Soltis <i>et al.</i> 2021)
SnIa-TRGB	$69.60 \pm 1.90$	2020	(Freedman <i>et al.</i> 2020)
SnIa-TRGB	$69.80 \pm 1.90$	2019	(Freedman <i>et al.</i> 2019)
SnIa-TRGB	$71.1 \pm 1.9$	2019	(Reid <i>et al.</i> 2019)
SnIa-TRGB	$72.40 \pm 2.00$	2019	(Yuan <i>et al.</i> 2019)
SnIa-Miras	$73.30 \pm 4.00$	2020	(Huang <i>et al.</i> 2019)
SBF	$73.30 \pm 2.50$	2021	(Blakeslee <i>et al.</i> 2021)
SBF	$70.50 \pm 4.10$	2020	(Khetan <i>et al.</i> 2021)
SBF	$71.90 \pm 7.10$	2018	(Cantiello <i>et al.</i> 2018)
SneII	$75.4^{+3.8}_{-4.9}$	2022	(de Jaeger <i>et al.</i> 2022)
SneII	$75.8^{+3.2}_{-4.9}$	2020	(de Jaeger <i>et al.</i> 2020a)
Time-delay (TD) lensing	$71.8^{+3.3}_{-3.2}$	2021	(Denzel <i>et al.</i> 2021)
TD lensing	$73.3^{+1.7}_{-1.8}$	2020	(Wong <i>et al.</i> 2020)
TD lensing	$72.8^{+1.6}_{-1.7}$	2020	(Liao <i>et al.</i> 2020)
TD lensing	$72.2 \pm 2.1$	2020	(Liao <i>et al.</i> 2019)
TD lensing	$73.65^{+1.95}_{-2.26}$	2020	(Yang <i>et al.</i> 2020b)
TD lensing	$74.2 \pm 1.6$	2020	(Millon <i>et al.</i> 2020)
TD lensing	$73.6^{+1.8}_{-1.6}$	2021	(Qi <i>et al.</i> 2021)
TD lensing	$74.2^{+2.7}_{-3.0}$	2020	(Shajib <i>et al.</i> 2020)
TD lensing	$74.5^{+5.6}_{-6.1}$	2020	(Birrer <i>et al.</i> 2020)
TD lensing+SLACS	$67.4^{+4.1}_{-3.2}$	2020	(Birrer <i>et al.</i> 2020)
TD lensing+SLACS	$76.8 \pm 2.6$	2019	(Chen <i>et al.</i> 2019)
GW Standard Sirens	$67^{+6.3}_{-3.8}$	2022	(Mukherjee <i>et al.</i> 2022)
GW Standard Sirens	$68^{+12}_{-7}$	2021	(Abbott <i>et al.</i> 2021b)
GW Standard Sirens	$72.77^{+11.0}_{-7.55}$	2021	(Palmese <i>et al.</i> 2021)
GW Standard Sirens	$73.4^{+6.9}_{-10.7}$	2021	(Gayathri <i>et al.</i> 2021)
GW Standard Sirens	$75^{+59}_{-13}$	2020	(Abbott <i>et al.</i> 2020a)
GW Standard Sirens	$50.4^{+28.1}_{-19.5}$	2020	(Mukherjee <i>et al.</i> 2020)
GW Standard Sirens	$67.6^{+4.3}_{-4.2}$	2020	(Mukherjee <i>et al.</i> 2020)
GW Standard Sirens	$48^{+23}_{-10}$	2020	(Chen <i>et al.</i> 2020)
GW Standard Sirens	$69.0^{+16.0}_{-8.0}$	2019	(Abbott <i>et al.</i> 2021a)
GW Standard Sirens	$75^{+32}_{-32}$	2019	(Soares-Santos <i>et al.</i> 2019)
GW Standard Sirens	$68.9^{+4.7}_{-4.6}$	2019	(Hotokezaka <i>et al.</i> 2019)
GW Standard Sirens	$77.00^{+37.00}_{-18.00}$	2019	(Fishbach <i>et al.</i> 2019)
GW Standard Sirens	$70.0^{+12.0}_{-8.0}$	2017	(Abbott <i>et al.</i> 2017a)
Masers	$73.90 \pm 3.00$	2020	(Pesce <i>et al.</i> 2020)
Masers	$73.50 \pm 1.40$	2019	(Reid <i>et al.</i> 2019)
Masers	$66.0 \pm 6.0$	2016	(Gao <i>et al.</i> 2016)
Masers	$73.0^{+26.0}_{-22.0}$	2015	(Kuo <i>et al.</i> 2015)
Masers	$68.0 \pm 9.0$	2013	(Kuo <i>et al.</i> 2013)
Masers	$68.9 \pm 7.1$	2013	(Reid <i>et al.</i> 2013)
Tully Fisher	$76.00 \pm 2.60$	2020	(Kourkchi <i>et al.</i> 2020)
Tully Fisher	$75.1 \pm 2.80$	2020	(Schombert <i>et al.</i> 2020)
$\gamma$ -ray attenuation	$67.4^{+6.0}_{-6.2}$	2019	(Domínguez <i>et al.</i> 2019)

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TABLE I – continued from previous page

Dataset	$H_0 [km s^{-1} Mpc^{-1}]$	Year	Refs.
$\gamma$ -ray attenuation	$64.9^{+4.6}_{-4.3}$	2019	(Zeng and Yan 2019)
HII galaxy	$71.00 \pm 2.8$	2018	(Fernández Arenas <i>et al.</i> 2018)
HII galaxy	$76.12^{+3.47}_{-3.44}$	2017	(Wang and Meng 2017a)
Cosmic chronometers, flat $\Lambda$ CDM with systematics	$66.5 \pm 5.4$	2022	(Moresco <i>et al.</i> 2022)
Cosmic chronometers, open $w$ CDM with systematics	$67.8^{+8.7}_{-7.2}$	2022	(Moresco <i>et al.</i> 2022)
Cosmic chronometers, without systematics	$67.06 \pm 1.68$	2018	(Gómez-Valent and Amendola 2018)
Cosmic chronometers, without systematics	$67.00 \pm 4.00$	2018	(Yu <i>et al.</i> 2018)
Cosmic chronometers, without systematics	$68.3^{+2.7}_{-2.6}$	2017	(Chen <i>et al.</i> 2017)
$H(z)$ +BAO+SN-Pantheon+SN-DES+QSO+HIIG+ GRB	$69.7 \pm 1.2$	2022	(Cao and Ratra 2022)
CMB ( $r_s$ -independent)+lensing+Pantheon	$73.5 \pm 5.3$	2021	(Baxter and Sherwin 2021)
SnIa-Cepheid and TD lensing	$73.8 \pm 1.1$	2020	(Wong <i>et al.</i> 2020)
SnIa+BAO+TD lensing+cosmic chronometers+ LSS	$70.30^{+1.36}_{-1.35}$	2019	(Dutta <i>et al.</i> 2019)
BAO+BBN+WL-CC	$67.20^{+1.2}_{-1.0}$	2018	(Abbott <i>et al.</i> 2018b)
SnIa+BAO+CC	$68.52^{+0.94+2.51(sys)}_{-0.94}$	2018	(Haridasu <i>et al.</i> 2018)

More recently, the joint analysis of lower-redshift, non-CMB, data such as BAO,  $H(z)$ , SnIa, QSO, HII and GRBs by Cao and Ratra (2022) gives a model-independent determinations of the Hubble constant,  $H_0 = 69.7 \pm 1.2 km s^{-1} Mpc^{-1}$  (see also Cao *et al.* 2021a, 2020, 2021b, 2022c, for previous joint analyses).

Many other estimates of  $H_0$  have been obtained in the literature within the standard  $\Lambda$ CDM model or in alternative scenarios by using joint analysis (Bonilla *et al.* 2021a; Renzi and Silvestri 2020). In addition, many analyses using various combinations of data assuming a  $\Lambda$ CDM model or a extended model beyond  $\Lambda$ CDM cosmology investigate whether the  $H_0$  tension persists (or not). For example Okamatsu *et al.* (2021) use non-CMB data and specifically adopt the data from BAO, BBN, and SnIa to study the  $H_0$  tension. They show that this tension exists in a broad framework beyond the standard  $\Lambda$ CDM model.

### II.1.11 The current status - Historic evolution

Hubble's initial value in 1929 for the expansion rate, now called the Hubble constant, was approximately  $500 km s^{-1} Mpc^{-1}$ . From the 1970s, through the 80s and into the 90s the value of  $H_0$  was estimated to be between 50 and  $100 km s^{-1} Mpc^{-1}$  (Tully 1988). Of interest is the historical Hubble constant debate between, for example, long series of papers by Gérard de Vaucouleurs, who claimed that the value of  $H_0$  is  $90 < H_0 < 100 km s^{-1} Mpc^{-1}$  (e.g. de Vaucouleurs 1986; de Vaucouleurs and Peters 1985), and Allan Sandage, who claimed the value is  $50 < H_0 < 55 km s^{-1} Mpc^{-1}$  (Sandage and Tammann 1975, 1984) (see Turner 2022, for a historical review).

During the last decades there has been remarkable progress in measuring the Hubble constant. The avail-

able technology and measurement methods determine the accuracy of this quantity. The Hubble constant as a function of publication date, using a set of different methods is shown in Fig. 12. The values of  $H_0$  determined in the late Universe with a calibration based on the Cepheid distance scale and the derived values of  $H_0$  from analysis of the CMB anisotropy spectrum data are shown. The uncertainties in these values have been decreasing for both methods and the recent measurements disagree beyond  $4\sigma$ .

Furthermore the comoving Hubble expansion rate as a function of redshift obtained from the Planck18 CMB is shown in Fig. 13 along with a few relevant data-points demonstrating the Hubble tension.

The basic strategic questions emerge

- How can  $H(z)$  derived from Cepheid late time calibrators (blue point in Fig. 13) become consistent with  $H(z)$  derived from the sound horizon early time calibrator (black line in Fig. 13)?
- What type of systematics could move the blue point down or shift black line up in Fig. 13 in early and late time calibrators?
- To what extend can dynamical dark energy address the Hubble tension by distorting the black line in Fig. 13?

These important Hubble tension questions will be discussed in the next subsection.

## II.2 Theoretical models

A wide range of models have been used to address the  $H_0$  tension by introducing additional degrees of freedom to  $\Lambda$ CDM model where additional parameters are allowed to vary such as quintessence (Caldwell *et al.* 1998; Chiba *et al.* 1997; Copeland *et al.* 1998; Ferreira and Joyce 1997,

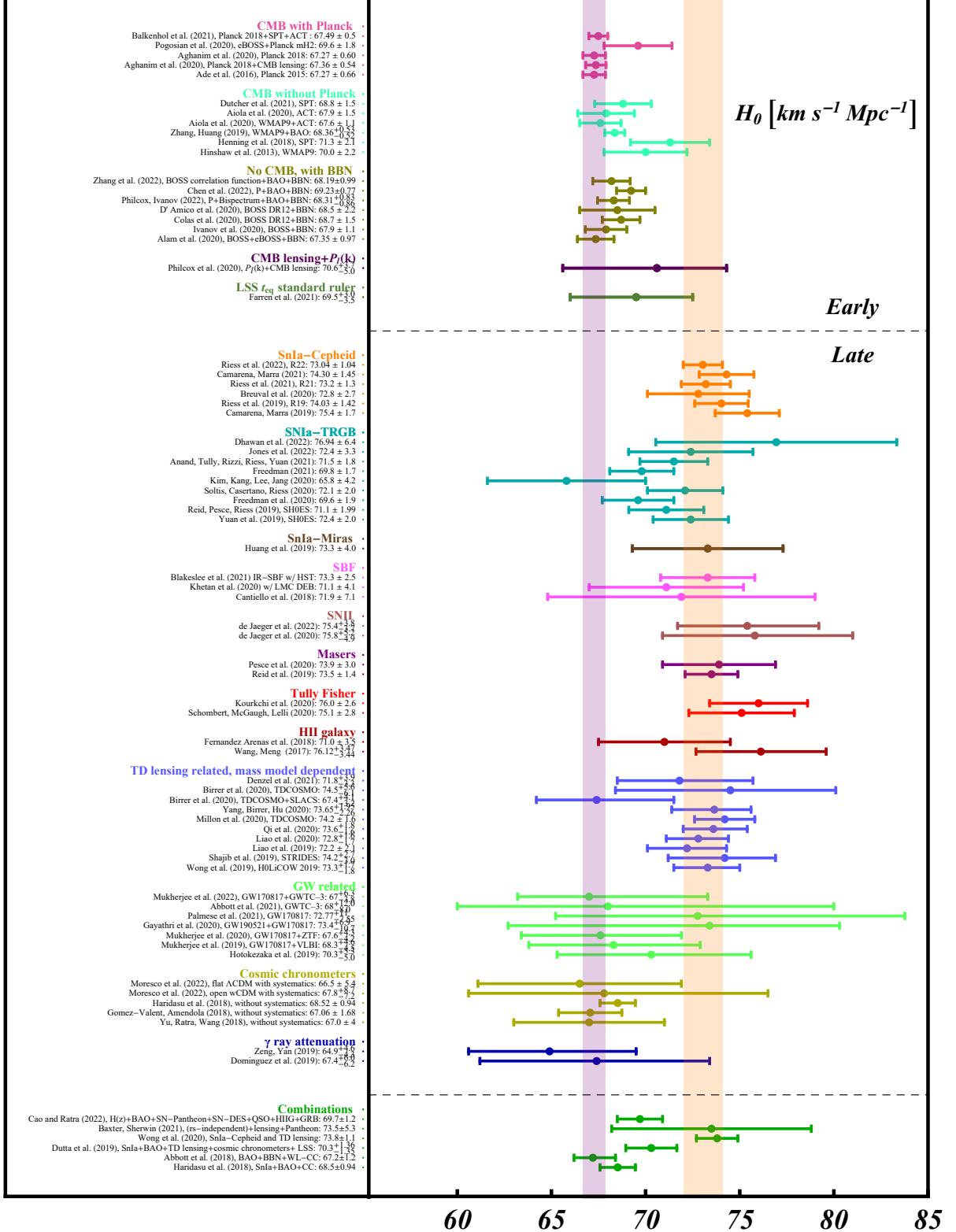


FIG. 10. The Hubble constant  $H_0$  values with the 68% CL constraints derived by recent measurements. The value of the Hubble constant  $H_0$  is derived by early time approaches based on sound horizon, under the assumption of a  $\Lambda$ CDM background.

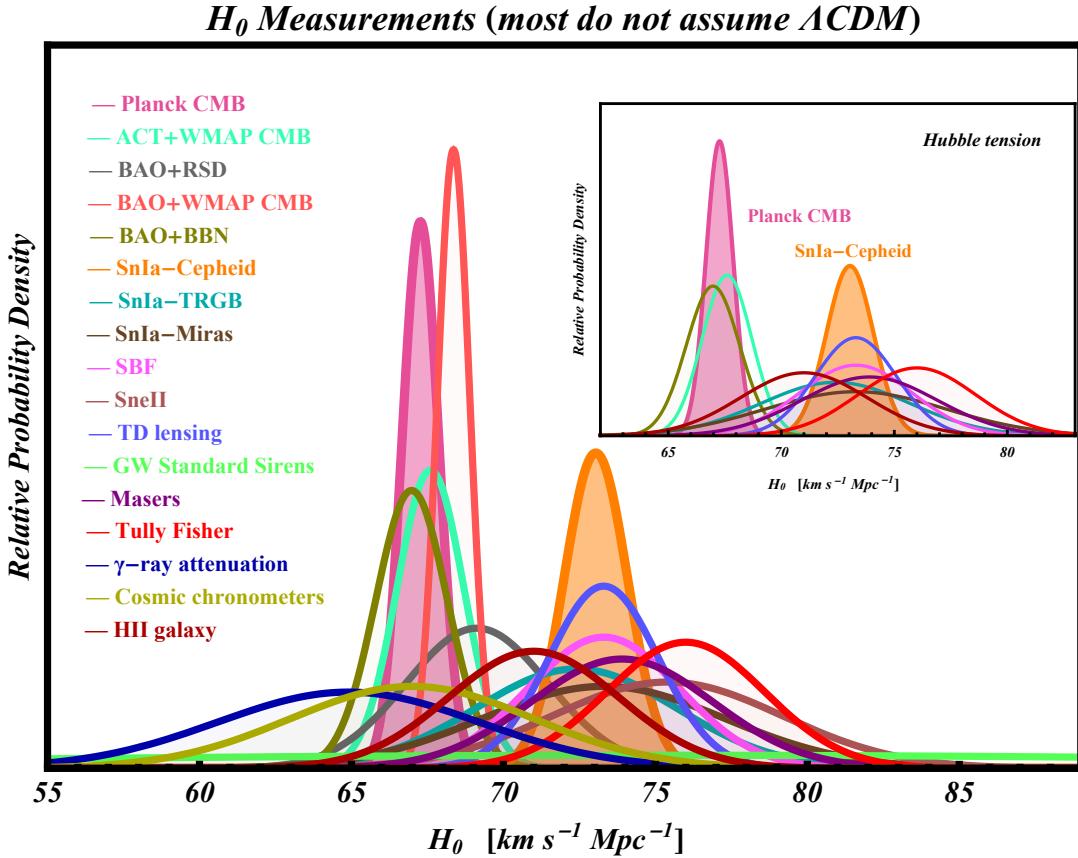


FIG. 11. The one dimensional relative probability density value of  $H_0$  derived by recent measurements (Planck CMB (Aghanim *et al.* 2020e), ACT+WMAP CMB (Aiola *et al.* 2020), BAO+RSD (Wang *et al.* 2017), BAO+WMAP CMB (Zhang and Huang 2019), BAO+BBN (Addison *et al.* 2018), SnIa-Cepheid (Riess *et al.* 2021b), SnIa-TRGB (Jones *et al.* 2022), SnIa-Miras (Huang *et al.* 2019), SBF (Blakeslee *et al.* 2021), SneII (de Jaeger *et al.* 2022), TD lensing (Wong *et al.* 2020), GW Standard Sirens (Abbott *et al.* 2020a), Masers (Pesce *et al.* 2020), Tully Fisher (Kourkchi *et al.* 2020),  $\gamma$ -ray attenuation (Zeng and Yan 2019), cosmic chronometers (Yu *et al.* 2018), HII galaxy (Fernández Arenas *et al.* 2018)). All measurements are shown as normalized Gaussian distributions. Notice that the tension is not so much between early and late time approaches but more between approaches that calibrate based on low  $z$  ( $z \lesssim 0.01$ ) gravitational physics and those that are independent of this assumption. For example cosmic chronometers and  $\gamma$ -ray attenuation which are late time but independent of late gravitational physics are more consistent with the CMB-BAO than with late time calibrators.

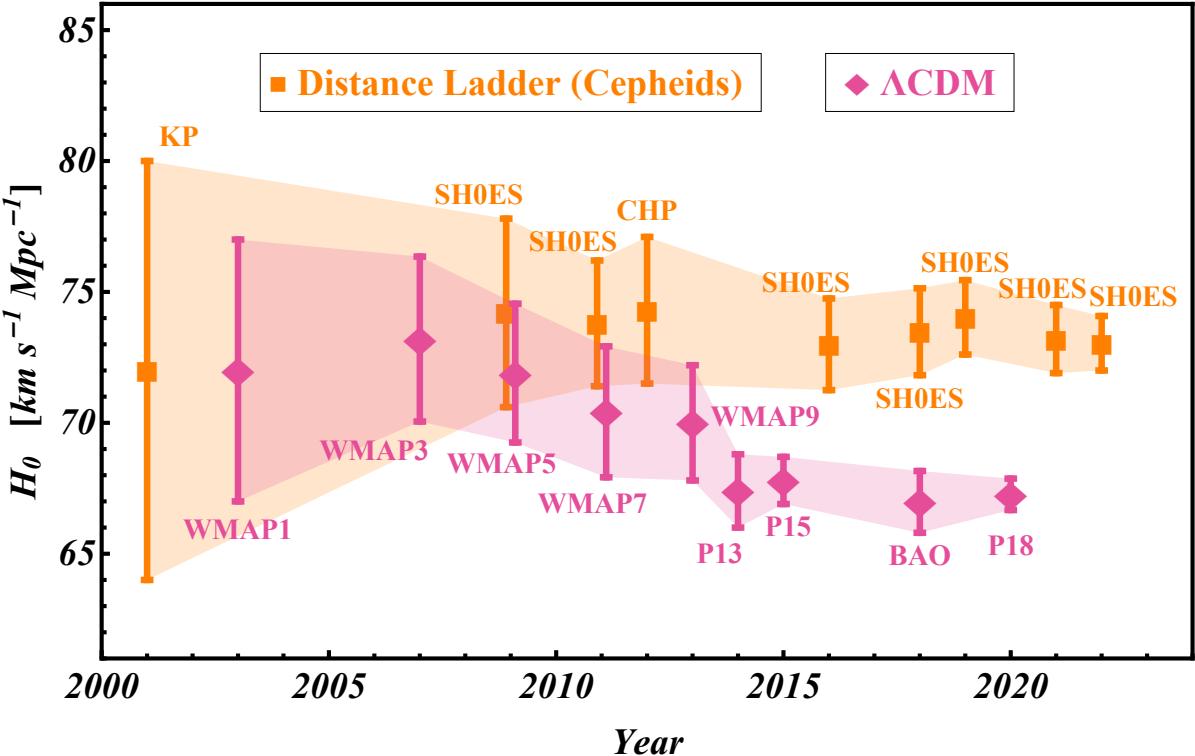


FIG. 12. The Hubble constant as a function of publication date, using a set of different tools. Symbols in orange denote values of  $H_0$  determined in the late Universe with a calibration based on the Cepheid distance scale (Key Project (KP)) (Freedman *et al.* 2001), SH0ES (Riess *et al.* 2021a, 2019, 2011, 2009, 2016, 2018b, 2021b), Carnegie Hubble Program (CHP) (Freedman *et al.* 2012)). Symbols in purple denote derived values of  $H_0$  from analysis of the CMB data based on the sound horizon standard ruler (First Year WMAP (WMAP1)) (Spergel *et al.* 2003), Three Year WMAP (WMAP3) (Spergel *et al.* 2007), Five Year WMAP (WMAP5) (Dunkley *et al.* 2009), Seven Year WMAP (WMAP7) (Komatsu *et al.* 2011), Nine Year WMAP (WMAP9) (Bennett *et al.* 2013), Planck13 (P13) (Ade *et al.* 2014a), Planck15 (P15) (Ade *et al.* 2016a), Planck18 (P18) (Aghanim *et al.* 2020e), BAO (Addison *et al.* 2018)). The orange and purple shaded regions demonstrate the evolution of the uncertainties in these values which have been decreasing for both methods. The most recent measurements disagree at greater than  $4\sigma$ .

1998; Ford 1987; Fujii 1982; Liddle and Scherrer 1999; Ratra and Peebles 1988; Scherrer 2022; Steinhardt *et al.* 1999; Tsujikawa 2013; Wetterich 1988, 1995; Zlatev *et al.* 1999), in which a scalar field plays the role of dark energy or modified gravity (Amendola *et al.* 2007; Boisseau *et al.* 2000; Esposito-Farese and Polarski 2001; Fay *et al.* 2007; Nojiri and Odintsov 2007; Perivolaropoulos 2005), in which General Relativity is modified on cosmological scales (see Bamba *et al.* 2012; Copeland *et al.* 2006, for a review).

The extensions of  $\Lambda$ CDM model which can be used to resolve the Hubble constant  $H_0$  tension fall into two categories: models with late time and models with early time modification (in the epoch before the recombination) (see Di Valentino *et al.* 2021c; Schöneberg *et al.* 2021; Verde *et al.* 2019, for a review).

The models with late time modification can be divided in four broad classes: deformations of the Hubble expansion rate  $H(z)$  at late times (e.g. late time phantom dark energy Alestas *et al.* 2020a; Di Valentino *et al.* 2016b), deformations of the Hubble expansion rate  $H(z)$  with additional interactions/degrees of freedom (e.g. interact-

ing dark energy Di Valentino *et al.* 2017b; Yang *et al.* 2020f and decaying dark matter Berezhiani *et al.* 2015), deformations of the Hubble expansion rate  $H(z)$  due to inhomogeneous/anisotropic modifications (e.g. inhomogeneous causal horizons Fosalba and Gaztañaga 2020) and transition/recalibration of the SNIa absolute luminosity (Kazantzidis and Perivolaropoulos 2020) (or combination of the previous classes e.g. late  $w - M$  phantom transition Alestas *et al.* 2021b).

Model selection statistical tools and approaches include the Akaike Information Criterion (AIC) (Akaike 1974), the Bayesian Information Criterion (BIC) (Schwarz 1978) and the Deviance Information Criterion (DIC) (Spiegelhalter *et al.* 2002) and Bayesian model comparison (e.g. Keeley and Shafieloo 2021; Koo *et al.* 2022; Mehrabi and Said 2022; Nesseris and Garcia-Bellido 2013; Saini *et al.* 2004). These tools have been developed and used to test, discriminate and compare the proposed models (Arevalo *et al.* 2017; Kerscher and Weller 2019; Liddle 2004, 2007) (see also Abdalla *et al.* 2022, for a list of statistical tools).

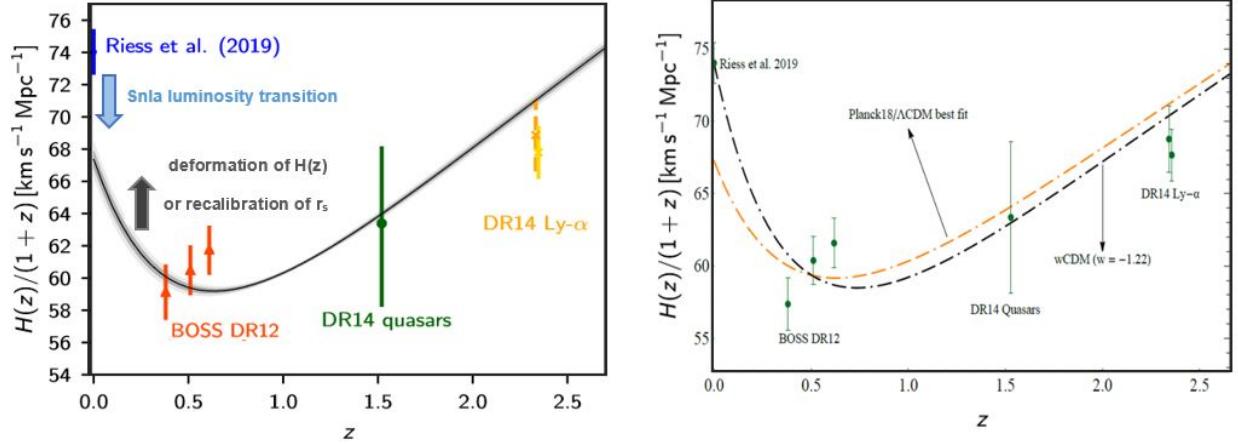


FIG. 13. Left panel: The comoving Hubble parameter as a function of redshift. The black line corresponds to the best fit obtained from the Planck18 CMB when the  $\Lambda$ CDM model is considered, while the grey areas are the  $1\sigma$  regions. The blue point at redshift zero denotes the inferred Hubble measurement by HST survey (Riess *et al.* 2019). The orange points, green point, and yellow points correspond to BAO data from BOSS DR12 survey (Alam *et al.* 2017a), BOSS DR14 quasar sample (Zarrouk *et al.* 2018), and SDSS DR12 Ly $\alpha$  sample (Riess *et al.* 2018b) respectively. The arrows indicate approaches for the resolution of the Hubble tension: Down arrow (blue) corresponds to decrease of the Riess *et. al.* datapoint due to systematics or transition of the absolute magnitude  $M$  (light blue arrow). Up arrow (black) corresponds to recalibration of  $r_s$  which shifts the whole curve up or and late time deformation of  $H(z)$  (adapted from Aghanim *et al.* 2020e). Right panel: The comoving Hubble parameter as a function of redshift for a wCDM phantom modification of  $\Lambda$ CDM model which drives upward the low  $z$  part of the  $H(z)$  curve shown in left panel. Thus it brings the  $z = 0$  prediction of the CMB closer to the  $H_0$  result of the local measurements (late time  $H(z)$  deformation).

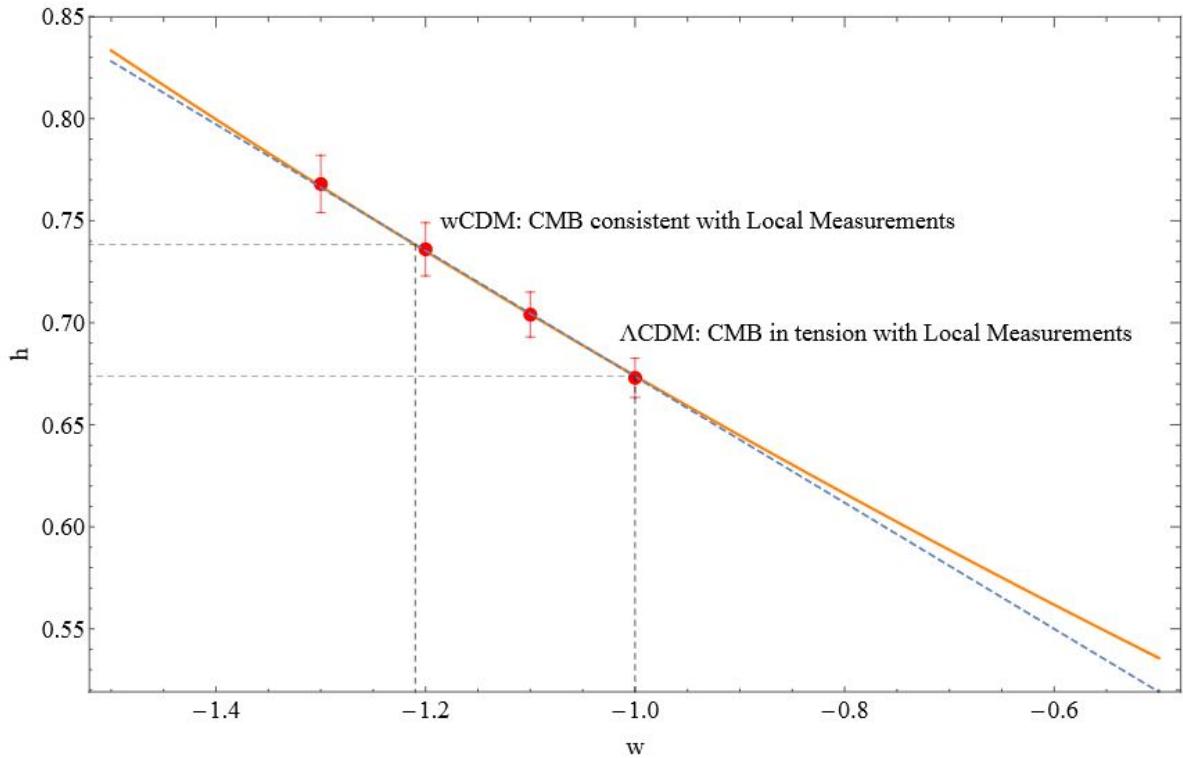


FIG. 14. The predicted value of  $h$  as a function of the fixed  $w$  assuming one parameter dark energy (wCDM) model. The theoretically predicted best fit values of  $h$  for different values of  $w$  in the case of the wCDM model (orange line), whereas the linear fitting that has been made (dashed blue line). The redpoints correspond to the actual best fit values, including the errorbars, of  $h$  for specific values of  $w$  obtained by fitting these models to the CMB TT anisotropy (from Alestas *et al.* 2020a).

### II.2.1 Late time deformations of the Hubble expansion rate $H(z)$

These late time models for the solution of the Hubble tension use a late time smooth deformation of the Hubble expansion rate Planck18/ΛCDM  $H(z)$  so that it can match the locally measured value of  $H_0$  while keeping the radius  $r_s$  of the sound horizon at the last scattering surface (see Subsection II.1.2). Many of these models effectively fix the comoving distance to the last scattering surface and the matter energy density  $\omega_m = \Omega_{0m} h^2$  to values consistent with Planck/ΛCDM to maintain consistency with the CMB anisotropy spectrum while introducing late time phantom dark energy to deform  $H(z)$  so that it matches the local measurements of  $H(z)$ . The required phantom behavior of such  $H(z)$  deformations can not be provided by minimally coupled quintessence models and therefore such models have been shown to be unable to resolve the Hubble tension (Banerjee *et al.* 2021a; Lee *et al.* 2022). These models have three problems

- They tend to worsen the fit to low z distance probes such as BAO and SnIa (e.g. Alestas *et al.* 2020a)
- They tend to worsen level of the growth tension (Alestas and Perivolaropoulos 2021).
- They tend to predict a lower value of SnIa absolute magnitude than the one determined by local Cepheid calibrators shown in Eq. (2.6) (Camarena and Marra 2020b, 2021; Marra and Perivolaropoulos 2021).

Thus, these models can not fully resolve the Hubble tension (see also Alestas *et al.* 2021a, 2022a; Arendse *et al.* 2020; Benevento *et al.* 2020; Benisty *et al.* 2022b; Bernal *et al.* 2021; Cai *et al.* 2022a,b; Efstathiou 2021; Escamilla-Rivera *et al.* 2021; Krishnan *et al.* 2021b; Theodoropoulos and Perivolaropoulos 2021; Vagnozzi *et al.* 2021c; Yang *et al.* 2021d).

Physical models where the deformation of  $H(z)$  may be achieved include the following: phantom dark energy (e.g. Alestas *et al.* 2020a) (see in Paragraph II.2.1.1), running vacuum model (e.g. Sola and Stefanic 2006) (see in Paragraph II.2.1.2), phenomenologically emergent dark energy (Li and Shafieloo 2019) (see in Paragraph II.2.1.3), vacuum phase transition (e.g. Di Valentino *et al.* 2018b) (see in Paragraph II.2.1.4), phase transition in dark energy (e.g. Khosravi *et al.* 2019) (see in Paragraph II.2.1.5). Plethora of late dark energy models with an equation of state  $w \neq -1$  ( $w < -1$  or  $w > -1$ ) both constant or dynamical with redshift (e.g. Martinelli and Tutusaus 2019) were proposed to address the Hubble tension. Recently, using a model-independent approach and a fully analytical analysis Heisenberg *et al.* (2022a,b) derive a set of necessary conditions that any late dark energy model must satisfy in order to potentially address both the Hubble and the growth tensions. In particular, solving the  $H_0$  tension requires  $w(z) < -1$  at some  $z$

and solving both the  $H_0$  and  $\sigma_8$  tensions demands time-varying dark energy equation of state which cross the phantom divide. However Alestas and Perivolaropoulos (2021) have shown that  $H(z)$  deformation approaches to the Hubble tension tend to worsen the  $\sigma_8$  growth tension.

The following models may be classified in this class of theories: the holographic dark energy (Adhikary *et al.* 2021; Colgáin and Sheikh-Jabbari 2021; Dai *et al.* 2020; Guo *et al.* 2019; Hernández-Almada *et al.* 2021; van Putten 2017, 2019; da Silva and Silva 2021a), the considering Chevallier - Polarski - Linder (CPL) (Chevallier and Polarski 2001; Kitazawa 2020; Linder 2003) parameterization (Yang *et al.* 2019c), the considering  $w$  dependence on non-vanishing spatial curvature (Miao and Huang 2018), the phantom brane dark energy (Alam *et al.* 2017b; Bag *et al.* 2021), the negative cosmological constant (Calderón *et al.* 2021; Sen *et al.* 2021; Visinelli *et al.* 2019), the negative dark energy (Dutta *et al.* 2020), the graduated dark energy (Akarsu *et al.* 2020), the simple-graduated dark energy (Acquaviva *et al.* 2021), the  $\Lambda_s$ CDM model (sign-switching) (Akarsu *et al.* 2021), the transitional dark energy (Keeley *et al.* 2019), the frame dependent dark energy (Adler 2019), the running  $H_0$  with redshift (Dainotti *et al.* 2021; Krishnan *et al.* 2021a), the varying gravitational constant (Sakr and Sapone 2022), the deviation from the cold dark matter (Elizalde *et al.* 2021) and the phantom crossing (Di Valentino *et al.* 2021d). For example in the case of the holographic dark energy model (Dai *et al.* 2020) and phantom crossing (Di Valentino *et al.* 2021d) models the tension on  $H_0$  appears to be significantly alleviated within  $1\sigma$  even though the three problems mentioned above do remain.

#### II.2.1.1 Phantom dark energy:

The deformation of  $H(z)$  through the implementation of late time phantom dark energy (Alestas *et al.* 2020a; Di Valentino *et al.* 2017a, 2016b, 2020e; Huang and Wang 2016; Vagnozzi 2020) can address the Hubble tension as shown in Fig. 13.

The analysis by Alestas *et al.* (2020a) indicates that mildly phantom models with mean equation of state parameter  $w = -1.2$  have the potential to alleviate this tension. It was shown that the best fit value of  $H_0$  in the context of the CMB power spectrum is degenerate with a constant equation of state parameter  $w$ . The CMB anisotropy spectrum was shown to be unaffected when changing  $H(z)$  provided that specific parameter combinations remain unchanged. These cosmological parameters fix to high accuracy the form of the CMB anisotropy spectrum. The values of these parameters as determined by the Planck/ΛCDM CMB temperature power spectrum are the following (Aghanim *et al.* 2020e).

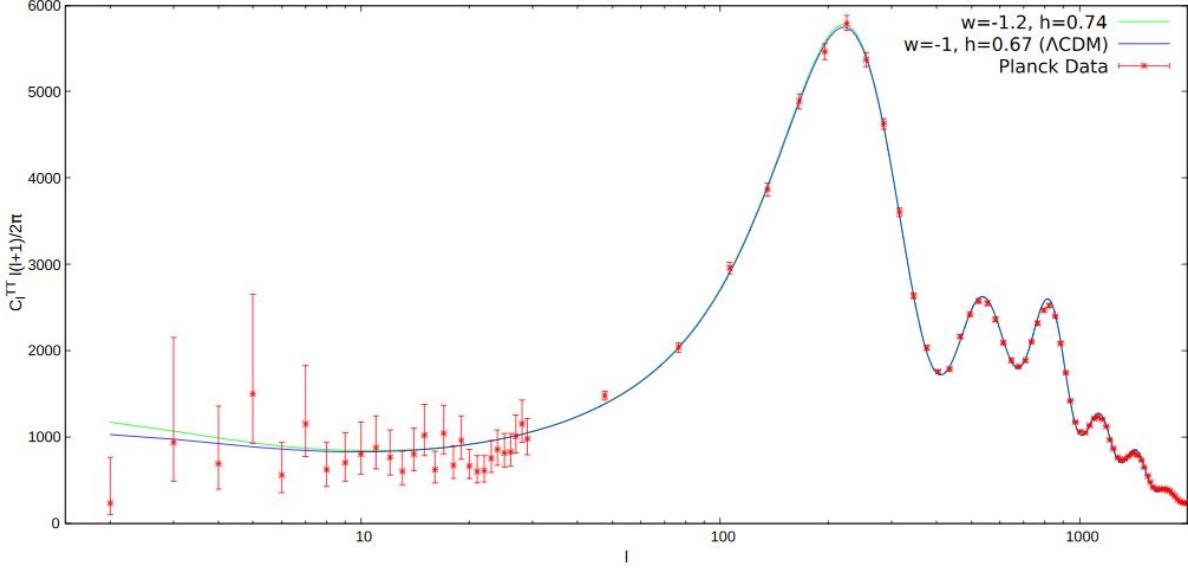


FIG. 15. The predicted form of the CMB TT anisotropy spectrum with  $w = -1$ ,  $h = 0.67$ ,  $\Omega_{0m} = 0.314$  for  $\Lambda$ CDM (blue line) and with  $w = -1.2$ ,  $h = 0.74$ ,  $\Omega_{0m} = 0.263$  (green line). Red points correspond to the binned high- $l$  and low- $l$  Planck data (from Alestas *et al.* 2020a).

$$\omega_{m,Planck} = 0.1430 \pm 0.0011 \quad (2.51)$$

$$\omega_{b,Planck} = 0.02237 \pm 0.00015 \quad (2.52)$$

$$\omega_{r,Planck} = (4.64 \pm 0.3) 10^{-5} \quad (2.53)$$

$$\omega_{k,Planck} = -0.0047 \pm 0.0029 \quad (2.54)$$

$$d_A,Planck = (4.62 \pm 0.08) (100 \text{ km s}^{-1} \text{Mpc}^{-1})^{-1} \quad (2.55)$$

where  $\omega_{i,Planck} = \Omega_{0i,Planck} h^2$  is the energy density of component  $i$  and  $d_A,Planck$  is the comoving angular diameter distance.

Using the Eq. (2.4) the comoving angular diameter distance  $d_A$  to the recombination surface is ( $c = 1$ )

$$d_A = \int_0^{z_r} \frac{dz}{H(z)} = \int_0^{z_r} \frac{dz}{h(z) 100 \text{ km s}^{-1} \text{Mpc}^{-1}} \quad (2.56)$$

where  $z_r \simeq 1100$  is the redshift of recombination and  $h(z) = H(z)(100 \text{ km s}^{-1} \text{Mpc}^{-1})^{-1}$  is the dimensionless Hubble parameter which in general takes the form

$$h(z) = [\omega_r(1+z)^4 + \omega_m(1+z)^3 + (h^2 - \omega_r - \omega_m)f_{DE}(z)]^{1/2} \quad (2.57)$$

where  $h = h(z=0)$  and  $f_{DE}(z)$  determines the evolution of dark energy.

In the context of a simple one parameter parametrization where the equation of state  $w$  remains constant in time and redshift (wCDM model),  $f_{DE}(z)$  takes the simple form

$$f_{DE}(z) = (1+z)^{3(1+w)} \quad (2.58)$$

If the four energy densities Eqs. (2.51), (2.52), (2.53) and (2.54) and the observed value of the comoving angular diameter Eq. (2.55) are fixed then they provide the analytically predicted best fit value of the Hubble parameter  $H_0$  (or  $h$ ) given the dark energy equation of state parameter  $w(w_0, w_1, \dots, z)$  where  $w_0, w_1, \dots$  are the parameters entering the  $w(z)$  parametrization. Thus assuming a flat Universe ( $\omega_k = 0$ ) and solving the following equation with respect to  $h$

$$d_A(\omega_{m,Planck}, \omega_{r,Planck}, \omega_{b,Planck}, h = 0.674, w = -1) = d_A(\omega_{m,Planck}, \omega_{r,Planck}, \omega_{b,Planck}, h, w) \quad (2.59)$$

it is straightforward to derive the degeneracy function  $h(z=0, w) \equiv h$  shown in Fig. 14 (continuous orange line). In the range  $w \in [-1.5, -1]$ ,  $h(w)$  is approximated as a straight line (dashed blue line in Fig. 14)

$$h(w) \approx -0.3093w + 0.3647 \quad (2.60)$$

For  $w = -1$ , this linear degeneracy equation leads to the best fit dimensionless Hubble constant  $h = 0.674$  as expected while for  $w = -1.217$  the corresponding predicted CMB best fit is  $h = 0.74$  which is consistent with the value obtained by local distance ladder measurements. The invariance of the CMB power spectrum when the cosmological parameters are varied along the above described degeneracy directions is shown in Fig. 15. This method of Alestas *et al.* (2020a) can be used to find general degeneracy relations between  $f_{DE}(z)$  and  $H_0$  and fixing  $h = 0.74$  gives infinite  $f_{DE}(z)$ ,  $w(z)$  forms that can potentially resolve the  $H_0$  problem if they can also properly fit the low  $z$  date (e.g. BAO, SNIa, Cepheid

value of absolute luminosity  $M$ ). Low  $z$  distance data (BAO and SnIa) will determine which one of these forms is observationally favored. However, none of these forms can provide a quality of fit to low  $z$  data equally good or better than  $\Lambda$ CDM despite the introduced additional parameters. In addition, these models suffer from the other two problems mentioned above (worse growth tension and lower value of SnIa absolute magnitude).

**II.2.1.2 Running vacuum model:** The running vacuum models (RVM) (Banerjee *et al.* 2021c; Basilakos *et al.* 2016, 2019, 2020a,b, 2012; Farrugia *et al.* 2020; Lima *et al.* 2013; Mavromatos 2021a; Mavromatos and Solà Peracaula 2021a,b; Moreno-Pulido and Solà 2020; Perico *et al.* 2013,?; Rezaei *et al.* 2021; Shapiro and Sola 2004, 2008, 2009; Shapiro *et al.* 2005; Sola and Stefancic 2005, 2006; Tsiapi and Basilakos 2019) (see Mavromatos 2021b, 2022; Peracaula 2022; Sola 2011, 2013, 2015; Solà 2016; Solà and Gómez-Valent 2015, for a review) attempts to address both the Hubble constant  $H_0$  tension (Solà *et al.* 2017b) and the  $\sigma_8$  growth tension using a mechanism that has common features with the IDE models (e.g. Gómez-Valent and Solà 2017; Gómez-Valent and Solà Peracaula 2018; Solà *et al.* 2015, 2017a; Solà Peracaula 2021; Solà Peracaula *et al.* 2021) (for relaxing the growth tension, see Subsection III.1.2).

The RVM of the cosmic evolution is well motivated by the generic idea of renormalization group formalism which is used in Quantum Field Theory (QFT) (Shapiro and Sola 2000, 2002; Sola 2008) (see also Moreno-Pulido and Peracaula 2022; Moreno-Pulido and Solà 2020, for a approach using adiabatic regularization and renormalization techniques). In the RVM the cosmological constant, the corresponding vacuum energy density and pressure are assumed to be functions of the Hubble rate e.g. a power series of the Hubble rate and its cosmic time derivative with even time derivatives of the scale factor (Gómez-Valent *et al.* 2015)

$$\Lambda = a_0 + \sum_{k=1} a_k H^{2k} + \sum_{k=1} b_k \dot{H}^k, \quad (2.61)$$

$$\rho_\Lambda = \rho_\Lambda(H) = \frac{\Lambda(H)}{8\pi G} \text{ and } p_\Lambda = p_\Lambda(H) = -\rho_\Lambda(H) \text{ respectively.}$$

For the current Universe the vacuum energy density can be written in the relatively simple form (e.g. Gómez-Valent *et al.* 2015; Solà *et al.* 2017b; Solà Peracaula *et al.* 2018; Solà and Gómez-Valent 2015)

$$\rho_\Lambda(H) = \frac{\Lambda(H)}{8\pi G} = \frac{3}{8\pi G}(c_0 + \nu H^2) \quad (2.62)$$

where  $c_0 = H_0^2(\Omega_{0\Lambda} - \nu) \simeq \frac{\Lambda_0}{3}$  (with  $\Lambda_0$  the current value) is an integration constant which is fixed by the boundary condition  $\rho_\Lambda(H_0) = \rho_{\Lambda,0}$  (with  $\rho_{\Lambda,0}$  the current value) and  $\nu$  is a dimensionless running parameter which characterizes the dynamics of the vacuum at low energy. For  $\nu = 0$  the vacuum energy remains constant at all times and for  $\nu > 0$  the vacuum energy density decreases

with the time. In QFT the running parameter is  $|\nu| \simeq 10^{-6} - 10^{-3}$  (Sola 2008) but in RVM it has been treated as a free parameter by fitting to the observational data (e.g. Gómez-Valent *et al.* 2015; Solà Peracaula *et al.* 2018).

**II.2.1.3 Phenomenologically emergent dark energy:** Phenomenologically emergent dark energy (PEDE) is a zero freedom dark energy scenario proposed by Li and Shafieloo (2019). In this model the dark energy density has the following form

$$\tilde{\Omega}_{DE}(z) = \Omega_{0DE}[1 - \tanh(\log_{10}(1+z))] \quad (2.63)$$

$$\text{where } \Omega_{0DE} = 1 - \Omega_{0m} - \Omega_{0r}.$$

The dark energy in this model has no effective presence in the past and emerges at the later times and with the same number (six) of parameters compared to the spatially flat  $\Lambda$ CDM scenario. It has the potential for alleviating the  $H_0$  tension (Di Valentino *et al.* 2021a; Li and Shafieloo 2019; Liu and Miao 2020; Pan *et al.* 2020b; Rezaei *et al.* 2020; Yang *et al.* 2021a,b). The generalised emergent dark energy (GEDE) model has one extra dimensionless free parameter  $\Delta$  including both  $\Lambda$ CDM model as well as the PEDE model as two of its special limits introduced by Li and Shafieloo (2020). In the GEDE model the dark energy density has the following form (Yang *et al.* 2021c)

$$\tilde{\Omega}_{DE}(z) = \Omega_{0DE} \frac{1 - \tanh\left(\Delta \log_{10}\left(\frac{1+z}{1+z_t}\right)\right)}{1 + \tanh\left(\Delta \log_{10}(1+z_t)\right)} \quad (2.64)$$

where  $z_t$  is the transition redshift where dark energy density equals to matter density. For  $\Delta = 0$  and  $\Delta = 1$  this model recovers  $\Lambda$ CDM and PEDE model respectively. Using the latest observational Hubble dataset Hernández-Almada *et al.* (2020) revisited and constrained the free parameters of the PEDE and GEDE models.

Other versions of the PEDE model are the Modified Emergent Dark Energy (MEDE) (Benaoum *et al.* 2022) and the Transitional Dark Energy (TDE) (Zhou *et al.* 2022) models. The MEDE model with one extra degree of freedom reduces the Hubble tension to  $2.4\sigma$  (Benaoum *et al.* 2022) even though it also suffers from the three problems of the late time  $H(z)$  deformation models.

**II.2.1.4 Vacuum phase transition:** Vacuum phase transition (Di Valentino *et al.* 2021a, 2018b, 2020b, 2021e) based on vacuum metamorphosis (VM) or vacuum cold dark matter model (VCDM) (Caldwell *et al.* 2006; Parker and Raval 2000; Parker and Vanzella 2004) has the potential to address the  $H_0$  tension. This mechanism with six free parameters as the spatially flat  $\Lambda$ CDM. It also assumes a phase transition in the nature of the vacuum similar to Sakharov's induced gravity (Sakharov 1991). The phase transition occurs when the evolving Ricci scalar curvature  $R$  becomes equal to the value of

scalar field mass squared  $m^2$  (Di Valentino *et al.* 2018b)

$$R = 6(\dot{H} + H^2) = m^2 \quad (2.65)$$

where the dot corresponds to the derivative with respect to cosmic time  $t$ . After the transition the Ricci scalar curvature remains constant with  $R = m^2$  and this changes the expansion rate below ( $z < z_t$ ) due to the phase transition

$$\begin{aligned} \frac{H^2}{H_0^2} &= \Omega_{0m}(1+z)^3 + \Omega_{0m}(1+z)^3 + \\ &+ M \left\{ 1 - \left[ 3 \left( \frac{4}{3\Omega_{0m}} \right)^4 M(1-M)^3 \right]^{-1} \right\}, \quad z > z_t \end{aligned} \quad (2.66)$$

$$\frac{H^2}{H_0^2} = (1-M)(1+z)^4 + M, \quad z \leq z_t \quad (2.67)$$

where  $M = \frac{m^2}{12H_0^2}$  and  $z_t = -1 + \frac{3\Omega_{0m}}{4(1-M)}$  is the transition redshift.

### II.2.1.5 Phase transition in dark energy:

- Phase transition in dark energy explored by Banihashemi *et al.* (2019, 2020); Khosravi *et al.* (2019); Moshafi *et al.* (2021) can address the Hubble tension. Generalizing this model by assigning a more realistic time evolution of dark energy Banihashemi *et al.* (2021) propose the critically emergent dark energy (CEDE) model.

In Farhang and Khosravi (2021) the form of phase transition parametrized phenomenologically by a hyperbolic tangent function. This scenario for dark energy is similar used independently as PEDE and GEDE.

- Late dark energy (LDE) transition (Benevento *et al.* 2020) at redshifts  $z \ll 0.1$  can reduce the Hubble tension. This class of  $H(z)$  deformation models has a more intense form of the third problem of the deformation class as they predict a significantly lower value of the SnIa absolute magnitude than the other  $H(z)$  deformation models (Alestas *et al.* 2021b; Camarena and Marra 2021).

In this scenario the true Hubble constant is given by (Benevento *et al.* 2020; Mortonson *et al.* 2009)

$$H_0^2 = \tilde{H}_0^2(1 + 2\delta) \quad (2.68)$$

where  $\tilde{H}_0$  is the prediction for a flat  $\Lambda$ CDM model in the context of a CMB sound horizon calibration.

In Benevento *et al.* (2020); Dhawan *et al.* (2020) it was shown that this model can not fully resolve the Hubble problem as it would imply a transition

in the SnIa apparent magnitude which is not observed. These models however become viable in the context of a SnIa absolute magnitude transition (Alestas *et al.* 2021b; Marra and Perivolaropoulos 2021).

### II.2.2 Deformations of the Hubble expansion rate $H(z)$ with additional interactions/degrees of freedom

There exist several varieties of the models for the solution of the Hubble tension which use deformations of the Hubble expansion rate  $H(z)$  with additional interactions/degrees of freedom. For example the interacting dark energy models (e.g. Di Valentino *et al.* 2017b; Yang *et al.* 2020f) (see in Paragraph II.2.2.1) with an extra non-gravitational interaction between the components of the Universe and the decaying dark matter models (e.g. Berezhiani *et al.* 2015) (see in Paragraph II.2.2.2) with additional degrees of freedom are able to alleviate the Hubble constant  $H_0$  tension.

The following models may be classified in this class of theories: multi-interacting dark energy (Lucia 2021), new interacting dark energy (Gao *et al.* 2021), interacting vacuum energy (Kumar 2021), metastable dark energy (Li *et al.* 2019c; Yang *et al.* 2020e), Quintom dark energy (Panpanich *et al.* 2021), cannibal dark matter (Buen-Abad *et al.* 2018a), baryons-dark energy interacting (Jiménez *et al.* 2020) (see also Ferlito *et al.* 2022; Vagnozzi *et al.* 2020), swampland conjectures (Agrawal *et al.* 2021; Colgáin and Yavartanoo 2019; Ó Colgáin *et al.* 2019), nonlocal gravity (Belgacem *et al.* 2020a, 2018), late time transitions in the quintessence field (Di Valentino *et al.* 2019a), Galileon gravity (Frusciante *et al.* 2020; Heisenberg and Villarrubia-Rojo 2021; Peirone *et al.* 2019; Renk *et al.* 2017; Zumalacárregui 2020),  $f(R)$  gravity (D'Agostino and Nunes 2020; Jiménez Cruz and Escamilla-Rivera 2021; Odintsov *et al.* 2021; Wang 2021),  $f(T)$  gravity (Bahamonde *et al.* 2022; Benisty *et al.* 2022a; Briffa *et al.* 2020; Cai *et al.* 2020; Hashim *et al.* 2021; Nunes 2018; Ren *et al.* 2021, 2022; Wang and Mota 2020),  $f(T, B)$  gravity (Escamilla-Rivera and Levi Said 2020),  $f(Q)$  gravity (Mandal *et al.* 2020), Brans-Dicke gravity (Solà Peracaula *et al.* 2019, 2020), minimal theory of massive gravity (de Araujo *et al.* 2021), scale-dependent gravity (Alvarez *et al.* 2021), unimodular gravity (Linares Cedeño and Nucamendi 2021; Perez *et al.* 2021), the screened fifth forces (Desmond *et al.* 2019; Desmond and Sakstein 2020), the minimally modified gravity (De Felice *et al.* 2021), the Lifshitz cosmology (Berechya and Leonhardt 2021), the Milne cosmology (Vishwakarma 2020), 4D Gauss-Bonnet gravity (Wang and Mota 2021), the generalized Chaplygin gas (Yang *et al.* 2019f), the unified cosmologies (Yang *et al.* 2019e), the  $\Lambda$ -gravity (Gurzadyan and Stepanian 2019, 2021), the  $\Lambda(t)$ -model (Benetti *et al.* 2021, 2019), the bulk viscous cosmology (Elizalde *et al.* 2020; Normann and Brevik 2021; da Silva and Silva 2021b; Yang *et al.* 2019b) and

the surface tension hypothesis (Ortiz 2020). For instance in the case of the metastable dark energy (Yang *et al.* 2020e), generalized Chaplygin gas (Yang *et al.* 2019f) and Galileon gravity (Renk *et al.* 2017) models the tension on  $H_0$  appears to be significantly alleviated to within about  $1\sigma$  even though the there problems of the  $H(z)$  deformation models remain to be addressed.

**II.2.2.1 Interacting dark energy:** In the cosmological interacting dark energy (IDE) models (Aljaf *et al.* 2021; Amendola 2000; Amirhashchi and Yadav 2020; An *et al.* 2019; Carrilho *et al.* 2021a,b; Costa *et al.* 2014; Di Valentino *et al.* 2017b, 2021b, 2020c; Gómez-Valent *et al.* 2020; Gonzalez *et al.* 2018; Hogg and Bruni 2021; Johnson *et al.* 2022; Kumar and Nunes 2016, 2017; Kumar *et al.* 2019; Li *et al.* 2019b; Linton *et al.* 2021; Lucca and Hooper 2020; Mancini and Poutsidou 2021; Martinelli *et al.* 2019; Nunes *et al.* 2022; Pan *et al.* 2019a,b; Pettorino 2013; Poutsidou *et al.* 2013; Salvatelli *et al.* 2013, 2014; Sharma and Sur 2022; Van De Bruck and Mifsud 2018; Wang and Meng 2005; Xia and Wang 2016; Yang *et al.* 2020c,d, 2019a, 2018a,b,c, 2017c, 2020f, 2019g; Yao and Meng 2021, 2020) (see Bolotin *et al.* 2014; Wang *et al.* 2016a, for a review) the dark components of the Universe i.e dark matter (DM) and dark energy (DE) have an extra non-gravitational interaction. The IDE model was proposed to address the coincidence problem (e.g. Berger and Shojaei 2006; Cai and Wang 2005; del Campo *et al.* 2006, 2009; Comelli *et al.* 2003; He and Wang 2008; Huey and Wandelt 2006; Pavon and Zimdahl 2005; Zhang 2005). In addition the interaction between the dark fluids has been shown to be effective in substantially alleviating the Hubble constant  $H_0$  tension (Di Valentino *et al.* 2017b, 2020d; Gariazzo *et al.* 2021; Guo *et al.* 2021; Kumar and Nunes 2016; Lucca and Hooper 2020; Nunes and Di Valentino 2021; Pan *et al.* 2019a, 2020c; Wang *et al.* 2021; Yang *et al.* 2018a,b, 2020f) or in addressing the structure growth  $\sigma_8$  tension between the values inferred from the CMB and the WL measurements (An *et al.* 2018; Barros *et al.* 2019; Camera *et al.* 2019; Poutsidou and Tram 2016) (see Subsection III.1.2) or in solving the two tensions simultaneously (Di Valentino *et al.* 2020c; Kumar and Nunes 2017; Kumar *et al.* 2019).

In IDE cosmology assuming spatially flat Friedmann-Lemaître-Roberson-Walker background and pressureless dark matter ( $w_c = 0$ ) the equations of evolution of the dark matter and dark energy densities  $\rho_c$  and  $\rho_{DE}$  respectively are given by (Gavela *et al.* 2009)

$$\dot{\rho}_c + 3H\rho_c = Q(t) \quad (2.69)$$

$$\dot{\rho}_{DE} + 3H(1+w_{DE})\rho_{DE} = -Q(t) \quad (2.70)$$

where the dot corresponds to the derivative with respect to cosmic time  $t$ ,  $w_{DE} = \frac{p_{DE}}{\rho_{DE}}$  is the equation of state of dark energy and  $Q$  represents the interaction rate between the dark sectors (i.e. the rate of energy transfer

between the dark fluids). For  $Q < 0$  energy flows from dark matter to dark energy, whereas for  $Q > 0$  the energy flow is opposite.

These models combine the deformation of  $H(z)$  with an extra modification of the growth rate of perturbations due to the tuned evolution of  $\Omega_m(z)$  induced by the interaction term  $Q$ . This additional tuning allows for a simultaneous improvement of the growth tension in contrast to models that involve a simple  $H(z)$  deformation.

Various phenomenological IDE models were proposed in the literature where the rate of the interaction  $Q$  has a variety of possible functional forms (Pan *et al.* 2020a). For example in some classes of IDE models the rate of the interaction  $Q$  is proportional to the energy density of dark energy  $Q = \delta H\rho_{DE}$  (Di Valentino *et al.* 2020c; Kumar and Nunes 2017; Kumar *et al.* 2019; Yang *et al.* 2020f) or cold dark matter  $Q = \delta H\rho_c$  (Kumar and Nunes 2016) (where  $\delta$  is a constant and  $\delta = 0$  in the  $\Lambda$ CDM cosmology), or some combination of the two. Note that in the case of functional form  $Q = \delta H\rho_c$  instabilities develop in the dark sector perturbations at early times (He *et al.* 2009).

**II.2.2.2 Decaying dark matter:** Decaying dark matter into dark radiation (i.e. an unknown relativistic species that is not directly detectable), which has been first analysed by Ichiki *et al.* (2004) and studied by Anchordoqui *et al.* (2015); Bjaelde *et al.* (2012); Choi *et al.* (2020a,b); Chudaykin *et al.* (2018); Nygaard *et al.* (2021); Wang *et al.* (2014); Xiao *et al.* (2020); ?, provides a promising scenario to relieve the Hubble constant  $H_0$  tension (e.g. Berezhiani *et al.* 2015). Also, it has been shown that this scenario can resolve the  $\sigma_8$  growth tension (Abellán *et al.* 2022; Enqvist *et al.* 2015) or the two tensions simultaneously (Pandey *et al.* 2020) by a similar mechanism as in the IDE models. However, using the Planck data the analysis of the model by Chudaykin *et al.* (2016); Poulin *et al.* (2016) has shown that the cosmological tensions are only slightly alleviated (see Bringmann *et al.* 2018, for a different result).

In these models assuming spatially flat Friedmann-Lemaître-Roberson-Walker Universe, pressureless dark matter,  $w_c = 0$  and equation of state of dark radiation  $w_{DR} = 1/3$ , the equations of evolution of the dark matter and dark radiation densities  $\rho_c$  and  $\rho_{DR}$  respectively are given by (Gavela *et al.* 2009)

$$\dot{\rho}_c + 3H\rho_c = -\Gamma\rho_c \quad (2.71)$$

$$\dot{\rho}_{DR} + 4H\rho_{DR} = \Gamma\rho_c \quad (2.72)$$

where  $\Gamma = \frac{1}{\tau}$  is the decay rate of dark matter particles (with  $\tau$  the particle's lifetime). In the literature a variety of possible functional forms of the decay rate has been explored (Bringmann *et al.* 2018; Enqvist *et al.* 2015; Lesgourges *et al.* 2016; Poulin *et al.* 2016). For example in some cases the decay rate is proportional to the Hubble rate,  $\Gamma \propto H$  (Pandey *et al.* 2020). Constraints

on the decay rate of dark matter have been obtained by the analysis of [Ando and Ishiwata \(2015\)](#); [Enqvist et al. \(2015\)](#).

A model with decaying dark matter into dark radiation in early/late Universe ( $\tau \ll t_s / \tau \gg t_s$ , where  $t_s$  is the time of last scattering) increases/decreases the expansion rate  $H(a; \rho_b, \rho_\gamma, \rho_c, \rho_{DR}, \rho_{DE})$  at high/low redshifts as it predicts a smaller matter content and a larger radiation content as time evolves (the early/late Universe is dominated by the radiation/matter and the dark radiation density decreases more rapidly than the matter density,  $\rho_{DR} \propto a^{-4}$  and  $\rho_c \propto a^{-3}$ ). In the case of  $\tau \ll t_s$ , the faster cosmological expansion  $H(z)$  decreases the scale of the sound horizon  $r_s$  in Eq. (2.21) because the baryon-to-photon ratio, and thus  $c_s$  in Eq. (2.22), is tightly constrained by CMB fluctuations and BBN ([Ade et al. 2016e](#)). In the context of the degeneracy  $H_0 r_s$  shown in Eq. (2.23) the lower scale of the sound horizon  $r_s$  yields a larger value of  $H_0$ . In the case of  $\tau \gg t_s$ , the lower dimensionless normalized Hubble rate  $E(z)$  in the late-time leads to a larger value of  $H_0$  since  $\theta_s$  and  $r_s$  must be kept fixed in Eq. (2.23). Accordingly, both early and late decaying dark matter model are able to alleviate the Hubble constant  $H_0$  tension (see [Anchordoqui 2021](#); [Anchordoqui et al. 2022](#), for a detailed discussion).

There are alternative decaying dark matter models such as the light dark matter ([Alcaniz et al. 2021](#)), the dynamical dark matter ([Desai et al. 2020](#)), the many-body or 2-body decaying cold dark matter scenarios ([Blackadder and Koushiappas 2014](#)) and the decaying warm dark matter scenario ([Blinov et al. 2020](#)). In the 2-body decaying cold dark matter scenario the decaying dark matter produces two particles, one massive warm dark matter particle and one massless relativistic particle (dark radiation). This scenario can address the Hubble constant  $H_0$  tension ([Clark et al. 2021b](#); [Haridasu and Viel 2020](#); [Vattis et al. 2019](#)) and the  $\sigma_8$  growth tension ([Abellán et al. 2021](#)).

A self-interacting dark matter model with a light force mediator coupled to dark radiation studied by [Binder et al. \(2018\)](#); [Hryczuk and Jodłowski \(2020\)](#). This model can simultaneously reduce the tension between CMB and low-redshift astronomical observations of  $H_0$  and  $\sigma_8$ .

[Jaeckel and Yin \(2021a\)](#) pointed out that a dark particle from reheating ([Jaeckel and Yin 2021b](#)) can alleviate the  $H_0$  tension through its decay to relativistic component which contributes to the dark radiation.

Recently, Ly- $\alpha$  constraints on possible models of dark-matter physics have been evaluated by [Dienes et al. \(2021\)](#). In particular the Ly- $\alpha$  bounds on different classes of dark-matter velocity distributions have been obtained.

### II.2.3 Deformations of the Hubble expansion rate $H(z)$ with inhomogeneous/anisotropic modifications

Models where the cosmological principle and the FLRW metric are relaxed by considering inhomoge-

neous/anisotropic modifications have the potential to resolve the Hubble problem ([Kasai and Futamase 2019](#)). Physical models where the deformation of  $H(z)$  may be achieved with inhomogeneous/anisotropic modifications, include the following: Chameleon dark energy (e.g. [Cai et al. 2021](#)) (see in Paragraph II.2.3.1), cosmic voids ([Wu and Huterer 2017](#)) (see also Paragraph II.2.3.2) and inhomogeneous causal horizons ([Fosalba and Gaztañaga 2020](#)) (see also Paragraph II.2.3.3), charged dark matter ([Beltran Jimenez et al. 2020, 2021](#); [Jiménez et al. 2021](#)), Bianchi type I spacetime ([Akarsu et al. 2019](#)) and emerging spatial curvature ([Bolejko 2018](#); [Heinesen and Buchert 2020](#)).

**II.2.3.1 Chameleon dark energy:** Chameleon dark energy ([Khoury and Weltman 2004a,b](#)) (see also [Banerjee et al. 2010](#); [Benisty and Davis 2022](#); [Brax et al. 2007, 2010](#); [Das and Banerjee 2008](#); [Khoury 2013](#); [Upadhye et al. 2012](#); [Vagnozzi et al. 2021d](#); [Wang et al. 2012](#)) attempts to address the Hubble constant  $H_0$  tension by introducing a cosmic inhomogeneity in the Hubble expansion rate at late-time from the chameleon field coupled to the local matter overdensities ([Cai et al. 2021](#)). This field trapped at a higher potential energy density acts as an effective cosmological constant and results in a faster local expansion rate than that of the background with lower matter density.

**II.2.3.2 Cosmic voids:** In cosmic void models the local  $H_0$  departs significantly from the cosmic mean  $H_0$  because of the presence of an under-dense region (local void) ([Lombriser 2020](#)). However in [Kenworthy et al. \(2019\)](#); [Wu and Huterer \(2017\)](#) it was shown that this alternative theory is inconsistent with current observations. The analysis was based on the assumption of the validity of standard  $\Lambda$ CDM and a study of the sample variance in the local measurements of the Hubble constant this alternative theory has been shown inconsistent with current observations. [Wu and Huterer \(2017\)](#) estimated that the required radius of void to resolve the tension in  $H_0$  is about 150 Mpc and density contrast of  $\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}} \simeq -0.8$  which is inconsistent at  $\sim 20\sigma$  with the  $\Lambda$ CDM cosmology ([Haslbauer et al. 2020](#); [Kenworthy et al. 2019](#)).

In the context of this inconsistency, [Haslbauer et al. \(2020\)](#) considered a cosmological Milgromian dynamics or modified Newtonian dynamics (MOND) model ([Milgrom 1983](#)) with the presence of  $11eV/c^2$  sterile neutrinos<sup>25</sup> to show that the Keenan-Barger-Cowie (KBC) void<sup>26</sup> has the potential to resolve the Hubble tension.

<sup>25</sup> Sterile neutrinos are a special kind of neutrino with right handed chirality that might interact only through gravity ([Boyarsky et al. 2009](#); [Dodelson and Widrow 1994](#)) (see [Abazajian et al. 2012](#); [Drewes 2013](#); [Kusenko 2009](#), for a review). They have been proposed to resolve some anomalies in neutrino data.

<sup>26</sup> The KBC void ([Keenan et al. 2013](#)) is a large local underdensity

**II.2.3.3 Inhomogeneous Causal Horizons:** Fosalba and Gaztañaga (2020) proposed a simple solution to the  $H_0$  tension based on causally disconnected regions of the CMB temperature anisotropy maps from Planck (Aghanim et al. 2020b). It was pointed out that CMB maps show ‘causal horizons’ where cosmological parameters have distinct values. This could be justified by the fact that these regions of the Universe have never been in causal contact. Thus it was shown that the Hubble constant  $H_0$  takes values which differ up to 20% among different causally disconnected regions. These cosmological parameter inhomogeneities are in agreement with the model of the Universe proposed in Gaztanaga (2020) (see also Gaztanaga 2021a, 2022, 2021b for details) where the cosmological constant is simply formulated as a boundary term in the Einstein equations and where ‘Causal Horizons’ naturally arise. Thus if there are similar ‘causal horizons’ in the local universe (i.e.  $z < 1100$ ), then 20% variations between the local and high-z measures of  $H_0$  are indeed to be expected (Fosalba and Gaztañaga 2020).

#### II.2.4 Late time modifications - Transition/Recalibration of the SNIa absolute luminosity

This class of models can address the problems of the  $H(z)$  deformation models (especially the low  $M$  problem) by assuming a rapid variation (transition) of the SNIa intrinsic luminosity and absolute magnitude due e.g. to a gravitational physics transition at a redshift  $z_t \lesssim 0.01$  (Alestas et al. 2021a,b; Marra and Perivolaropoulos 2021).

**II.2.4.1 Gravity and evolution of the SNIa intrinsic luminosity:** As shown in the recent analysis by Kazantzidis and Perivolaropoulos (2020) there are abnormal features which may be interpreted as evolution of the measured parameter combination  $\mathcal{M}$  (see Section II.1.1). This measured parameter combination  $\mathcal{M}$  in Eq. (2.14) depends on the absolute magnitude  $M$  and on the Hubble constant  $H_0$  ( $M$  and  $H_0$  are degenerate parameters). Any variation of the parameter  $\mathcal{M}$  is due to a variation of  $M$  which could be induced by a varying  $\mu_G(z) \equiv \frac{G(z)}{G_0}$  (where  $G_0$  is the local value of the Newton’s constant  $G(z)$ ). If the calibrated SNIa absolute magnitude  $M$  were truly constant then the parameter  $\mathcal{M}$  should also be constant (independent of redshift).

A possible variation of the absolute magnitude  $M$  and equivalently of the absolute luminosity

$$L \sim 10^{-2M/5} \quad (2.73)$$

could be due to a variation of the fine structure constant  $\alpha$  or the Newton’s constant  $G$ .

If the absolute luminosity is proportional to the Chandrasekhar mass  $L \sim M_{Chandr}$  we have (Garcia-Berro et al. 1999; Gaztanaga et al. 2002)

$$L \sim G^{-3/2} \quad (2.74)$$

Thus  $L$  will increase as  $G$  decreases<sup>27</sup>.

Under these assumptions, we obtain

$$M(z) - M_0 = \frac{15}{4} \log \mu_G(z) \quad (2.75)$$

where  $M_0$  corresponds to a reference local value of the absolute magnitude and  $\mu_G \equiv \frac{G}{G_0}$  is the relative effective gravitational constant (with  $G$  the strength of the gravitational interaction and  $G_0$  the locally measured Newton’s constant).

Then, the Eq. (2.14) takes the following form

$$\mathcal{M}(z) = M_0 + \frac{15}{4} \log \mu_G(z) + 5 \log_{10} \left[ \frac{c/H_0}{Mpc} \right] + 25 \quad (2.76)$$

and the apparent magnitude Eq. (2.15) can be written as

$$m(z, H_0, \Omega_{0m})_{th} = \mathcal{M}(z, H_0) + 5 \log_{10} [D_L(z, \Omega_{0m})] \quad (2.77)$$

A mild tension at  $2\sigma$  level in the best fit value of  $\mathcal{M}$  was found in between the low-redshift ( $z \lesssim 0.2$ ) data and the full Pantheon dataset in the context of a  $\Lambda$ CDM model. This tension can be interpreted as (Kazantzidis and Perivolaropoulos 2020)

- a locally higher value of  $H_0$  by about 2%, corresponding to a local matter underdensity.
- a time variation of Newton’s constant which implies an evolving Chandrasekhar mass and thus an evolving absolute luminosity  $L$  and absolute magnitude  $M$  of low  $z$  SNIa.

In addition, the oscillating features shown in Fig. 4 hint also to the possibility of evolutionary effects of  $M$ . As discussed below such evolutionary effects if they exist in the form of a transition may provide a solution to the Hubble and growth tensions.

**II.2.4.2 Transition of the SNIa absolute magnitude  $M$  at a redshift  $z \simeq 0.01$ :** Recently, Marra and Perivolaropoulos (2021) have proposed that a rapid transition (abrupt deformation) at a transition redshift

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<sup>27</sup> Adopting a semi-analytical model which takes into account the stretch of SNIa light curves but assumes fixed mass of Ni, obtains SNIa light curves in the context of modified gravity Wright and Li (2018) have claimed that  $L$  will increase as  $G$  increases.

between 40 and 300  $Mpc$  (i.e.  $0.01 \lesssim z \lesssim 0.07$ ) around the Local Group.

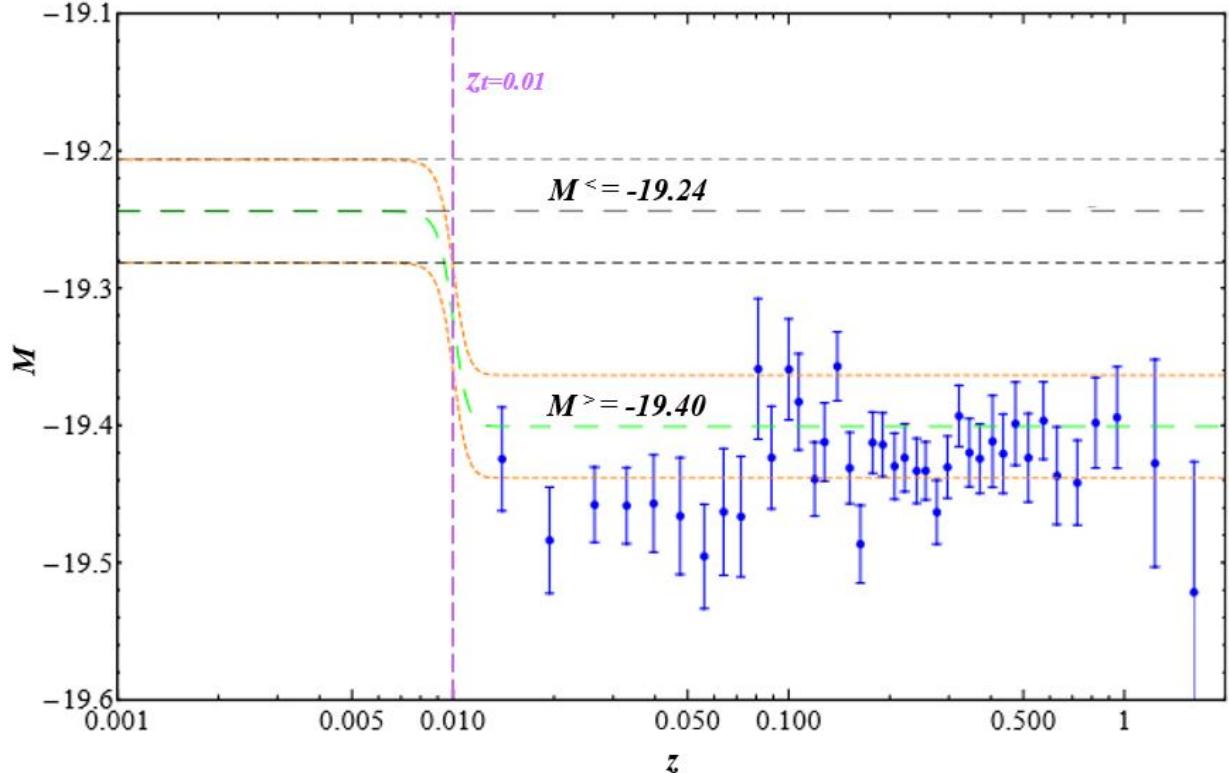


FIG. 16. The Pantheon binned SNIa absolute magnitudes Eq. (2.80)  $M_i$  (blue points) (Scolnic *et al.* 2018) for a Planck/ $\Lambda$ CDM luminosity distance. The data are inconsistent with the SNIa absolute magnitude  $M^< = -19.24$  calibrated by Cepheids but the inconsistency disappears if there is a transition in the absolute magnitude with amplitude  $\Delta M \simeq -0.2$  at redshift  $z_t \simeq 0.01$  (from Marra and Perivolaropoulos 2021).

$z_t \simeq 0.01$  in the value of the SNIa absolute magnitude  $M$  of the form

$$M^>(z) = M^< + \Delta M \Theta(z - z_t) \quad (2.78)$$

(where  $\Theta$  is the Heaviside step function) due to a rapid transition of the gravitational constant can address the Hubble tension.

In particular the analysis by Marra and Perivolaropoulos (2021) has shown that a 10% rapid transition in the value of the relative effective gravitational constant  $\mu_G$  at  $z_t \simeq 0.01$  is sufficient to induce the required reduction of  $M$

$$\Delta M \equiv M^> - M^< \simeq -0.2 \quad (2.79)$$

where  $M^<$  is the SNIa absolute magnitude of Eq. (2.17) calibrated by Cepheids at  $z < 0.01$  (Camarena and Marra 2020b, 2021) and  $M^>$  is the SNIa absolute magnitude of Eq. (2.18) using the parametric-free inverse distance ladder of Camarena and Marra (2020a). Fig. 16 shows the Pantheon SNIa absolute magnitudes for a Planck/ $\Lambda$ CDM luminosity distance (Scolnic *et al.* 2018) obtained from

$$M_i = m_i - 5 \log_{10} \left[ \frac{d_L(z_i)}{Mpc} \right] - 25 \quad (2.80)$$

where  $m_i$  are the apparent magnitude datapoints.

The data are in disagreement with the SNIa absolute magnitude  $M^<$  calibrated by Cepheids but they become consistent if there is a transition in the absolute magnitude with amplitude  $\Delta M \simeq -0.2$  (Marra and Perivolaropoulos 2021). Thus, this class of  $M$ -transition models avoids the  $M$ -problem of late time  $H(z)$  deformation models.

Assuming the power law dependence Eq. (2.74) and using RSD and WL data (Abbott *et al.* 2019b; Alestas *et al.* 2020a; Skara and Perivolaropoulos 2020) reported a best fit value  $\Delta\mu_G \equiv \mu_G^> - \mu_G^< = -0.19 \pm 0.09$  ( $\mu_G^>$  corresponds to  $z > 0.01$  and  $\mu_G^<$  corresponds to  $z < 0.01$ ) in the context of a  $\Lambda$ CDM background  $H(z)$ . The analysis by Marra and Perivolaropoulos (2021) showed that a rapid  $\sim 10\%$  increase of the effective gravitational constant roughly 150 million years ago can also solve  $\Omega_m\sigma_8$  growth tension.

Recently, Alestas and Perivolaropoulos (2021) have demonstrated that this model has an advantage over both early time and late time deformations of  $H(z)$  to fully resolve the Hubble tension while at the same time improving the level of the  $\Omega_m\sigma_8$  growth tension. In addition it has the potential to provide equally good fit to low  $z$  distance probes such as BAO and SNIa as the

Planck18/ΛCDM model.

More recently, Perivolaropoulos and Skara (2022) generalized the symmetron screening mechanism<sup>28</sup> (Hinterbichler and Khoury 2010; Hinterbichler *et al.* 2011) by allowing for an explicit symmetry  $Z_2$  breaking of the symmetron  $\phi^4$  potential. The explicit symmetry breaking can create an asymmeron wall network pinned on matter overdensities separating regions with distinct gravitational properties which could constitute a physical mechanism for the realization of gravitational transitions in redshift space that could help in the resolution of the Hubble and growth tensions. Another theoretical model leading to a gravitational transition could include a pressure non-crushing cosmological singularity in the recent past Odintsov and Oikonomou (2022).

**II.2.4.3 Late (low-redshift)  $w - M$  phantom transition:** The late (low-redshift)  $w - M$  phantom transition (Alestas *et al.* 2021b) is a late time approach involving a combination of the previous two classes: the transition of the SnIa absolute luminosity and the deformation of the Hubble expansion rate  $H(z)$ . A rapid phantom transition of the dark energy equation of state parameter  $w$  at a transition redshift  $z_t < 0.1$  of the form

$$w(z) = -1 + \Delta w \Theta(z_t - z) \quad (2.81)$$

with  $\Delta w < 0$  and a similar transition in the value of the SnIa absolute magnitude  $M$  of the form

$$M(z) = M_C + \Delta M \Theta(z - z_t) \quad (2.82)$$

with  $\Delta M < 0$  due to evolving fundamental constants can address the Hubble tension (Alestas *et al.* 2021b). Where  $\Theta$  is the Heaviside step function,  $M_C$  is the SnIa absolute magnitude Eq. (2.17) calibrated by Cepheids (Camarena and Marra 2020b, 2021) at  $z < 0.01$  and  $\Delta M$ ,  $\Delta w$  are parameters to be fit by the data. Alestas *et al.* (2021b) find  $\Delta M \simeq -0.1$ ,  $\Delta w \simeq -4$  for  $z_t = 0.02$  which imply a lower value of  $\mu_G$  at  $z > 0.02$  (about 6%) compared to the pure  $M$ -transition model.

The late (low-redshift)  $w - M$  phantom transition (LwMPT) can lead to a resolution of the Hubble tension in a more consistent manner than smooth deformations of the Hubble tension and other types of late time transitions such as the Hubble expansion rate transition (Benevento *et al.* 2020; Dhawan *et al.* 2020). Its main advantages include the consistency in the predicted value

of the SnIa absolute magnitude  $M$  and the potential for simultaneous resolution of the growth tension.

Mortsell *et al.* (2021a,b); Perivolaropoulos and Skara (2021) have analyzed the color-luminosity relation of Cepheids in anchor galaxies and SnIa host galaxies by identifying the color-luminosity relation for each individual galaxy instead of enforcing a universal color-luminosity relation to correct the NIR Cepheid magnitudes. A systematic brightening of Cepheids at distances larger than about 20  $Mpc$  which could be enough to resolve the Hubble tension was found. In addition, Perivolaropoulos and Skara (2021) investigating the effects of variation of the Cepheid calibration empirical parameters (the color-luminosity parameter or the Cepheid absolute magnitude) find hints for the presence of a fundamental physics transition taking place at a time more recent than 100 Myrs ago. The magnitude of the transition lead to value of  $H_0$  that is consistent with the CMB inferred value thus eliminating the Hubble tension. The distance range/timescale corresponding to this transition is consistent with solar system history data (Perivolaropoulos 2022) indicating an increase of the rate of impactors on the Moon and Earth surfaces by about a factor of 2-3 during the past 100Myrs which correspond to  $z < 0.008$  (Bottke *et al.* 2007; Gehrels 1995; Grier *et al.* 2001; Mazrouei *et al.* 2019; McEwen *et al.* 1997; Shoemaker 1998; Ward and Day 2007) and low redshift galaxy surveys data (Alestas *et al.* 2022b). Such a transition is also consistent with a recent analysis by Alestas *et al.* (2021a) indicating a transition in the context of the Tully-Fisher data.

In particular, using a robust dataset of 118 Tully-Fisher datapoints Alestas *et al.* (2021a) have demonstrated that evidence for a transition in the evolution of BTFR appears at a level of more than  $3\sigma$ . Such effect could be interpreted as a transition of the effective Newton's constant. The amplitude and sign of the gravitational transition are consistent with the mechanisms for the resolution of the Hubble and growth tension discussed above (Alestas *et al.* 2021b; Marra and Perivolaropoulos 2021) (see in Perivolaropoulos 2021b, for a talk of the tensions of the ΛCDM and a gravitational transition ).

## II.2.5 Early time modifications of sound horizon

Modifying the scale of sound horizon  $r_s$  (i.e. the scale of the standard ruler) by introducing new physics before recombination that deform  $H(z)$  at prerecombination redshifts  $z \gtrsim 1100$  can increase the CMB inferred value of  $H_0$  (Aylor *et al.* 2019; Bernal *et al.* 2016; Knox and Millera 2020; Poulin *et al.* 2018a) and thus resolve the Hubble tension. Such deformation may be achieved by introducing various types of additional to the standard model components (see Asadi *et al.* 2022, for a review). These models have the problem of predicting stronger growth of perturbations than implied by dynamical probes like redshift space distortion (RSD) and weak lensing (WL) data and thus may worsen the  $\Omega_m$ - $\sigma_8$  growth tension (Alestas

<sup>28</sup> For reviews of modified gravity theories with screening mechanisms, such as the Vainshtein (Arkani-Hamed *et al.* 2003; Defayet *et al.* 2002; Vainshtein 1972) and the chameleon (Brax *et al.* 2004a,b, 2008, 2010; Gubser and Khoury 2004; Khoury and Weltman 2004a,b; Mota and Shaw 2006, 2007; Upadhye *et al.* 2006) models see (Baker *et al.* 2021; Brax *et al.* 2021, 2012; Burrage and Sakstein 2016, 2018; Davis *et al.* 2012; Hui *et al.* 2009; Jain and Khoury 2010; Joyce *et al.* 2015; Khoury 2010; Sakstein 2013, 2018) and for screening effects see (Renevey *et al.* 2021).

and Perivolaropoulos 2021; Jedamzik *et al.* 2021).

A wide range of mechanisms has been proposed for the decrease of the sound horizon scale at recombination. These mechanisms include the introduction of early dark energy, extra neutrinos or some other dark sector at recombination, features in the primordial power spectrum, modified scenarios of recombination etc. The following models and theories may be classified in this class of mechanisms: early dark energy (e.g. Poulin *et al.* 2019) (see Paragraph II.2.5.1), dark radiation (e.g. Green *et al.* 2019) (see Paragraph II.2.5.2), neutrino self-interactions (e.g. Kreisch *et al.* 2020) (see Paragraph II.2.5.3), large primordial non-Gaussianities (Adhikari and Huterer 2020) (see Paragraph II.2.5.4), Heisenberg's uncertainty principle (Capozziello *et al.* 2020) (see Paragraph II.2.5.5), early modified gravity (Braglia *et al.* 2021) (see Paragraph II.2.5.6), cosmological inflation physics (Aresté Saló *et al.* 2021; Chiang and Slosar 2018; Di Valentino and Bouchet 2016; Di Valentino *et al.* 2018c; Di Valentino and Mersini-Houghton 2017; Guo and Zhang 2017; Hazra *et al.* 2019; Keeley *et al.* 2020a; Liu and Huang 2020; Takahashi and Yin 2021a; Tanin and Tenkanen 2021; Tram *et al.* 2017; Ye *et al.* 2021a), dark matter - photon coupling (Kumar *et al.* 2018; Yadav 2019), dark matter-neutrino interactions (Paul *et al.* 2021), interacting dark radiation (Blinov and Marques-Tavares 2020), ultralight dark photon (Flambaum and Samsonov 2019), primordial black holes (Flores and Kusenko 2021; Nesseris *et al.* 2020), primordial magnetic fields (Jedamzik and Pogosian 2020; Jedamzik and Saveliev 2019; Thiele *et al.* 2021), non-standard recombination (Liu *et al.* 2020a), unparticles dark energy (Artymowski *et al.* 2021), varying fundamental constants (Franchino-Viñas and Mosquera 2021; Hart and Chluba 2018, 2020, 2022; Sekiguchi and Takahashi 2021), early-time thermalization of cosmic components (Velten *et al.* 2021), CMB monopole temperature shift (Ivanov *et al.* 2020a), open and hotter universe (Bengaly *et al.* 2020b; Bose and Lombriser 2021), Axi-Higgs cosmology (Fung *et al.* 2021a,b), string Cosmology (Anchordoqui 2020; Anchordoqui *et al.* 2020) and dark massive vector fields (Anchordoqui and Perez Bergliaffa 2019). In this list of proposed cosmological models the tension on  $H_0$  is alleviated with a significance ranging from the  $1\sigma$  to the  $3\sigma$  level.

**II.2.5.1 Early dark energy:** In the early dark energy (EDE) model (Agrawal *et al.* 2019; Alexander and McDonough 2019; Berghaus and Karwal 2020; Fondi *et al.* 2022; Gogoi *et al.* 2021; Gómez-Valent *et al.* 2021; Haridasu *et al.* 2021; Herold *et al.* 2021; Hill *et al.* 2021; Kaloper 2019; Karwal and Kamionkowski 2016; Klypin *et al.* 2021; La Posta *et al.* 2021; Lin *et al.* 2019; Lucca 2020; Moss *et al.* 2021; Mörtsell and Dhawan 2018; Murugia *et al.* 2021; Nojiri *et al.* 2021; Pettorino *et al.* 2013; Poulin *et al.* 2021, 2019; Sakstein and Trodden 2020; Smith *et al.* 2022, 2020) an additional dynamical scalar field behaves like a cosmological constant at early times

(near matter-radiation equality but before recombination). This field decays rapidly after recombination thus leaving the rest of the expansion history practically unaffected up to a rescaling which modifies  $H_0$ . This rescaling allows for the resolution of the Hubble constant tension. Using the Eq. (2.21) in a EDE model the radius of sound horizon at last scattering can be calculated by

$$\begin{aligned} r_s &= \int_0^{t_d} \frac{c_s(a)}{a(t)} dt = \\ &= \int_{z_d}^{\infty} \frac{c_s(z)}{H(z; \rho_b, \rho_\gamma, \rho_c, \rho_{DE})} dz = \\ &= \int_0^{a_d} \frac{c_s(a)}{a^2 H(a; \rho_b, \rho_\gamma, \rho_c, \rho_{DE})} da \end{aligned} \quad (2.83)$$

The baryon-to-photon ratio, and thus  $c_s$  in Eq. (2.22), is tightly constrained by CMB fluctuations and BBN (Ade *et al.* 2016e). As a consequence a EDE phase before and around the recombination epoch would increase  $H(z)$  and thus decrease the scale of the sound horizon  $r_s$  in Eq. (2.83). In the context of the degeneracy  $H_0 r_s$  shown in Eq. (2.23) this decrease of  $r_s$  leads to an increased value of  $H_0$  for a fixed measured value of  $\theta_s$ .

An EDE model can be implemented by several functional forms of scalar field which contribute to the cosmic energy shortly before matter-radiation equality. Possible functional forms of scalar field are the axion-like potential (higher-order periodic potential) inspired by string axiverse scenarios for dark energy (Kamionkowski *et al.* 2014; Karwal and Kamionkowski 2016; Marsh 2011, 2016; Poulin *et al.* 2018b), the single axion-like particle potential consisting of two cosine functions which unifies the inflaton and DM while reheating the universe (Daido *et al.* 2017, 2018), the power-law potential (Agrawal *et al.* 2019), the acoustic dark energy (Lin *et al.* 2019, 2020; Yin 2022), the  $\alpha$ -attractor-like potential (Braglia *et al.* 2020b) and others.

Poulin *et al.* (2019) consider two physical models. One that involves an oscillating scalar field and another with a slowly-rolling scalar field. In the case of the first model of the proposal of Poulin *et al.* (2019), the potential of the scalar field  $\phi$  is a generalization of the axion potential of the form

$$V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^n \quad (2.84)$$

where  $m$  is the field mass (for ultralight scalar field  $m \sim 10^{-28}$  eV) and  $f$  is a decay constant.

Consider the time evolution of the EDE scalar field which may be written as

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (2.85)$$

where the dot and the prime denote the derivatives with respect to cosmic time  $t$  and field  $\phi$  respectively.

At early times, deep in the radiation era the field  $\phi$  is initially frozen in its potential due to Hubble friction ( $H \gg m$ ) and acts as a cosmological constant with equation of state  $w_\phi = -1$  (hence the name Early Dark Energy), but when the Hubble parameter drops below some

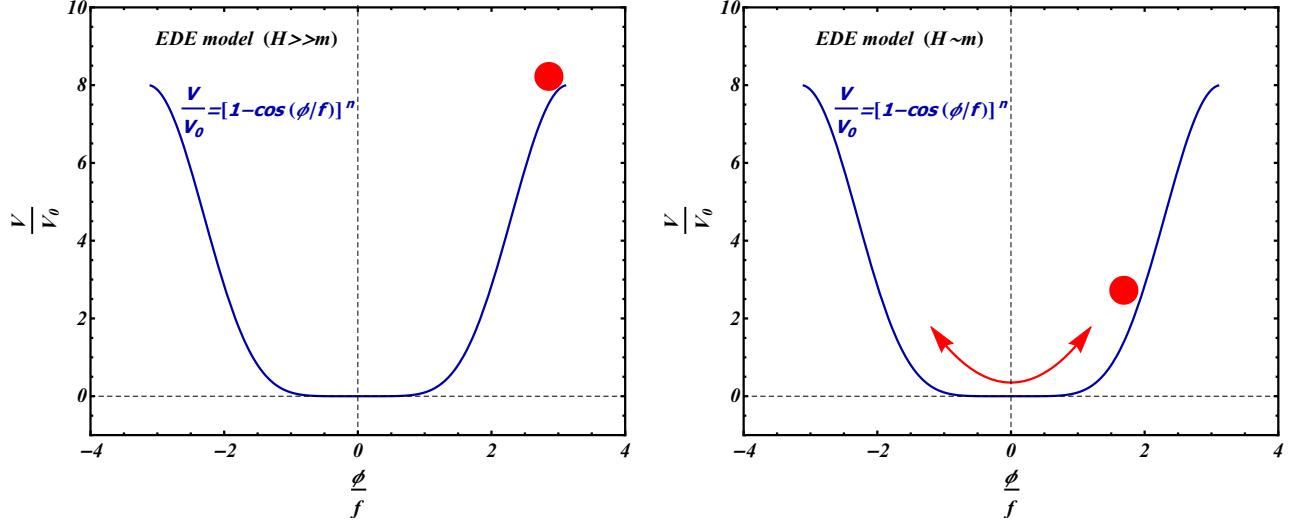


FIG. 17. The potential  $V/V_0$  (with  $V_0 = m^2 f^2$ ,  $n = 3$  in Eq. (2.84)) as a function of  $\phi/f$  at early times ( $H \gg m$ ) (left panel) when the field  $\phi$  is initially frozen in its potential due to Hubble friction and acts as a cosmological constant with equation of state  $w_\phi = -1$ , and at a critical redshift  $z_c$  when the Hubble parameter drops below some value ( $H \sim m$ ) (right panel) and the field becomes dynamical and begins to oscillate around its minimum which is locally  $V \sim \phi^{2n}$ .

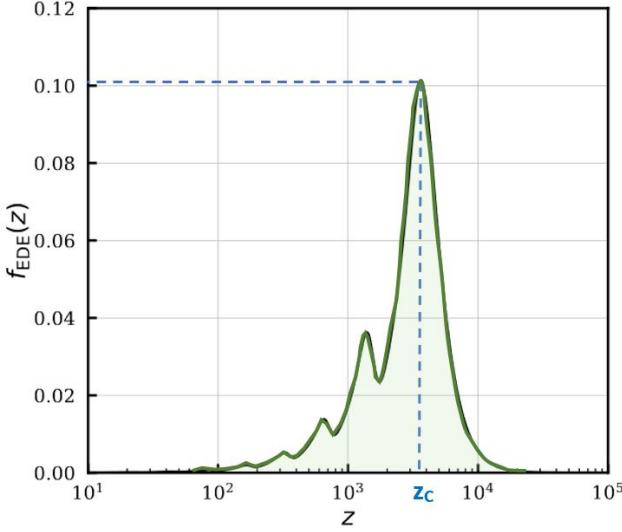


FIG. 18. Fractional contribution of EDE to the cosmic energy budget as a function of redshift (adapted from Ivanov *et al.* 2020b).

value ( $H \sim m$ ) at a critical redshift  $z_c$  (for EDE this happens when  $z_c \sim z_d$  for  $m \sim 10^{-27}$  eV) the field becomes dynamical and begins to oscillate around its minimum which is locally  $V \sim \phi^{2n}$  (Fig. 17). It thus begins to behave like a fluid with an equation of state (Turner 1983)

$$w_\phi = \frac{n-1}{n+1} \quad (2.86)$$

The energy density of the field dilutes as  $a^{-3(1+w_\phi)}$  and thus when  $n = 1$ ,  $n = 2$  and  $n \geq 3$  dilutes as cold

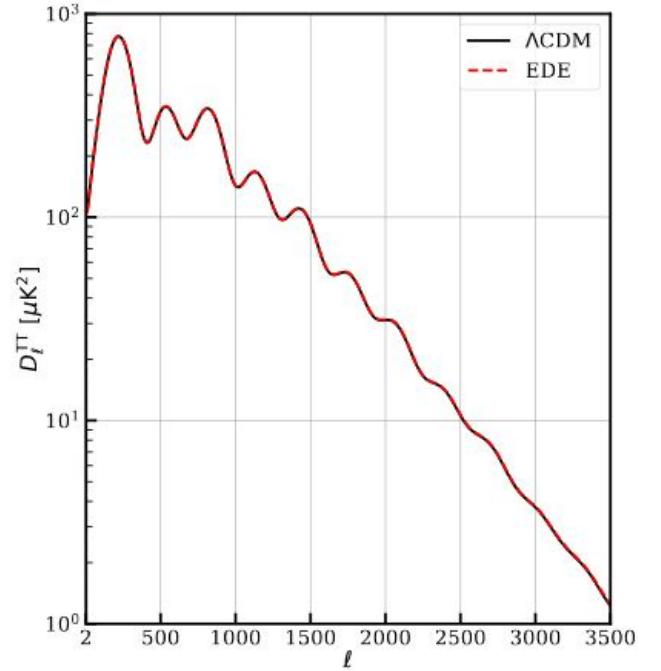


FIG. 19. CMB TT power spectrum. The black solid and the red dashed lines correspond to  $\Lambda\text{CDM}$  model with  $H_0 = 68.07 \text{ km s}^{-1} \text{Mpc}^{-1}$  and EDE model with  $H_0 = 71.15 \text{ km s}^{-1} \text{Mpc}^{-1}$  respectively (from Ivanov *et al.* 2020b).

dark matter ( $a^{-3}$ ,  $w_\phi = 0$ ), as radiation ( $a^{-4}$ ,  $w_\phi = 1/3$ ) and faster than radiation ( $a^{-x}$  with  $x > 4$ ,  $w_\phi > 1/3$ ) respectively. Also when  $n \rightarrow \infty$  the energy density dilutes as free scalar field (stiff matter (Chavanis 2015)) ( $a^{-6}$ ,  $w_\phi = 1$ ) i.e. the scalar field is fully dominated by

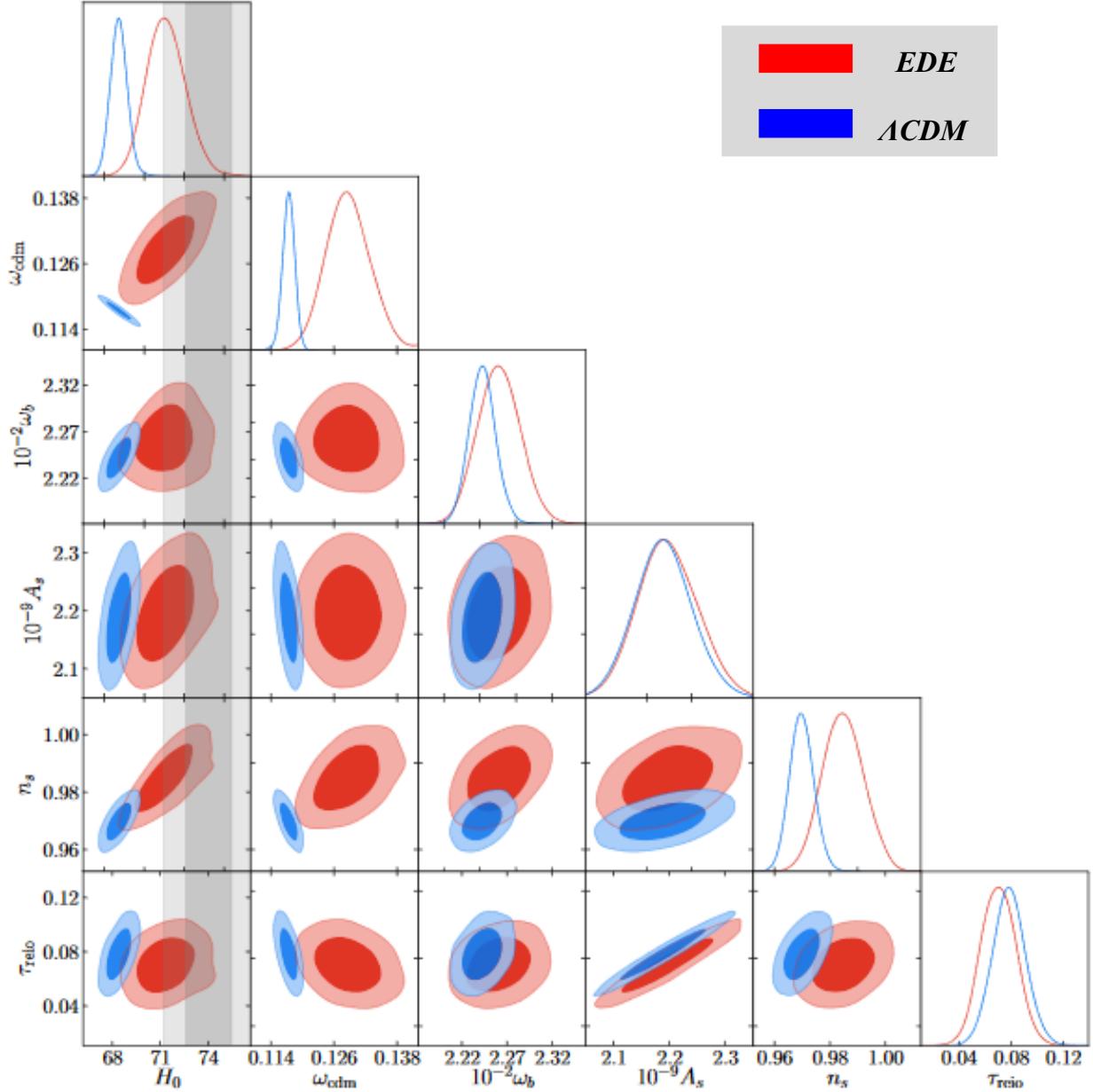


FIG. 20. Posterior 1D and 2D distributions of the cosmological  $\Lambda$ CDM parameters reconstructed from a run to all data (including Planck high  $l$  polarization) in EDE (red) and the  $\Lambda$ CDM (blue) scenario. The gray bands correspond to the SHOES determination of  $H_0$  (adapted from [Smith et al. 2020](#)).

its kinetic energy.

The EDE models are parameterized by the critical redshift  $z_c$ , the dimensionless quantity  $\theta_i = \phi_i/f$  (with  $\phi_i$  the initial value of the scalar field and  $0 < \theta_i < \pi$ ) and the peak EDE energy density fraction of the Universe  $f_{EDE}(z_c)$  which is given by

$$f_{EDE}(z_c) \equiv \frac{\Omega_\phi(z_c)}{\Omega_{\text{tot}}(z_c)} = \frac{\rho_{EDE}(z_c)}{3M_{pl}^2 H(z_c)^2} \quad (2.87)$$

where  $\Omega_\phi$  is the EDE energy density which evolves as

([Poulin et al. 2018b, 2019](#))

$$\Omega_\phi(z) = \frac{2\Omega_\phi(z_c)}{[(1+z_c)/(1+z)]^{3(w_n+1)} + 1} \quad (2.88)$$

The fractional contribution of EDE to the cosmic energy budget as a function of redshift, i.e.  $f_{EDE}(z)$ , is shown in Fig. 18 (from the analysis by [Ivanov et al. 2020b](#)). Clearly, for  $z \simeq z_c$  the EDE contributes the most to the total energy density ( $\sim 10\%$ ), for  $z > z_c$  the EDE is not dynamically important while for  $z < z_c$  decays away as radiation or faster than radiation leaving the

later evolution of the Universe relatively unchanged. By construction, the EDE models can nicely match the CMB TT power spectrum of  $\Lambda$ CDM and therefore of Planck as illustrated in Fig. 19. The black solid and the red dashed lines (almost identical) correspond to  $\Lambda$ CDM model with  $H_0 = 68.07 \text{ km s}^{-1} \text{Mpc}^{-1}$  and EDE model with  $H_0 = 71.15 \text{ km s}^{-1} \text{Mpc}^{-1}$  respectively (Ivanov *et al.* 2020b).

EDE models face the fine-tuning issues (Sakstein and Trodden 2020) and suffer from a coincidence problem (Pettorino *et al.* 2013). Carrillo González *et al.* (2021); Sakstein and Trodden (2020) proposed a natural explanation for this coincidence using the idea of neutrino-assisted early dark energy.

EDE modifies growth and  $H(z)$  at early times (around recombination) and higher matter density is required to compensate for this effect in the CMB. Higher matter density contradicts the required low value of matter density at late times from weak lensing and growth data as shown in Fig. 20. In particular the analysis by Clark *et al.* (2021a); Hill *et al.* (2020) has shown that an EDE model can not practically resolve the Hubble tension because it results in higher value of the late-time density fluctuation amplitude  $\sigma_8$  and thus the tension with LSS dynamical probes WL, RSD and CC data can get worse. In addition Jedamzik *et al.* (2021) argued that any model which attempts to reconcile the CMB inferred value of  $H_0$  by solely reducing the sound horizon results into tension with either the BAO or the galaxy weak lensing data. Thus, a compelling and full resolution of the Hubble tension may require multiple modifications (more than just the size of the sound horizon) of the  $\Lambda$ CDM cosmology.

Recent studies by Chudaykin *et al.* (2021); Smith *et al.* (2021) reexamining the above issue and using combined data method show that the EDE scenario remains a potential candidate solution to the Hubble tension. Future observations will provide data with improved quality and thus will enable more detailed tests of the EDE model.

Many alternative models have been proposed to implement the basic EDE scenario such as Chain EDE (Freese and Winkler 2021), Axion EDE (D'Amico *et al.* 2021b; Jiang and Piao 2022), Anti-de Sitter EDE (Jiang and Piao 2021, 2022; Wang and Piao 2022; Ye and Piao 2020a,b; Ye *et al.* 2021b), assisted quintessence EDE (Sabla and Caldwell 2021), EDE with extra radiation (Seto and Toda 2021a), EDE in the framework of the ultralight scalar decay to massless fields (Gonzalez *et al.* 2020) and New EDE (NEDE) (Niedermann and Sloth 2020, 2021a,b,c, 2022) which can potentially address the Hubble tension. In NEDE a vacuum first-order phase transition of the NEDE scalar field is assumed to have taken place before recombination in the early Universe. The NEDE sudden transition can be described by a scalar field whose potential at some critical point develops two non-degenerate minima (true and false vacuum)<sup>29</sup>. Al-lali *et al.* (2021) develop a phenomenological dark sector

with decaying dark energy and ultra-light axions which addresses the Hubble tension similarly to the EDE and NEDE scenarios and simultaneously can resolve the  $S_8$  tension. Karwal *et al.* (2022); McDonough *et al.* (2021); Sabla and Caldwell (2022) argue that a EDE model may require a more complicated dynamics in order to soften both the  $H_0$  and  $S_8$  tensions. In particular, McDonough *et al.* (2021) introduced the Early Dark Sector (EDS) model considering an EDE-dependence of the mass of dark matter. The considering form of the potential is given by Eq. (2.84) (with  $n = 3$ ) and the form of the field-dependent mass given by

$$m(\phi) = m_0 \exp\left(\frac{c\phi}{M_{pl}}\right) \quad (2.89)$$

as motivated by the the Swampland Distance Conjecture (SDC) (Ooguri and Vafa 2007) and its extension to axions (Baume and Palti 2016; Blumenhagen *et al.* 2017; Klaewer and Palti 2017; Scalsi and Valenzuela 2019).

**II.2.5.2 Dark radiation:** Modifications in the light relic sector can relieve the tension by changing the early-time dynamics of the Universe (Buen-Abad *et al.* 2018b; Lancaster *et al.* 2017). The dark radiation model assumes an increased number of light relics (Aboubrahim *et al.* 2022; Ackerman *et al.* 2009; Aloni *et al.* 2021; Archidiacono *et al.* 2013; Bernal *et al.* 2016; Blennow *et al.* 2012; Conlon and Marsh 2013; Dvorkin *et al.* 2014; Feng *et al.* 2017; Ghosh *et al.* 2021; Giarè *et al.* 2021; Leistedt *et al.* 2014; Seto and Toda 2021b; Vagnozzi 2020; Vogel and Redondo 2014; Wyman *et al.* 2014) which are weakly interacting components of radiation (i.e. relativistic species). For example the addition of hidden photons, sterile neutrinos (Carneiro *et al.* 2019; Feng *et al.* 2022; Gelmini *et al.* 2020, 2021), Goldstone bosons, Majoron (Fernandez-Martinez *et al.* 2021), axions (Cuesta *et al.* 2021; D'Eramo *et al.* 2018; Gu *et al.* 2021) which are predicted in many extensions of the Standard Model (SM) increases the value in the effective number of relativistic particles  $N_{eff}$  beyond its canonical expectation value  $N_{eff}^{SM} \simeq 3.044$  (Akita and Yamaguchi 2020; Bennett *et al.* 2021; Escudero Abenza 2020; Froustey *et al.* 2020; Mangano *et al.* 2005; de Salas and Pastor 2016). These extra particles modify the time of matter-radiation equality and would lead to a lower  $r_s$  sound horizon. As a consequence a lower expansion rate of the Universe and a higher value of  $H_0$  emerges from early-time physics (Green *et al.* 2019) (see Eq. (2.23)).

Another interesting approach was presented by Archidiacono *et al.* (2017, 2019); Becker *et al.* (2021); Buen-Abad *et al.* (2015, 2018b); Chacko *et al.* (2016);

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<sup>29</sup> It has recently been pointed out (Alestas *et al.* 2021a,b,

2022b; Marra and Perivolaropoulos 2021; Perivolaropoulos 2022; Perivolaropoulos and Skara 2021, 2022) that a similar mechanism in the context of the ultra late transition taking place at a redshift  $z \lesssim 0.01$  can lead to a resolution of the Hubble tension.

[Choi et al.](#) (2021); [Cyr-Racine et al.](#) (2014); [Ko et al.](#) (2017); [Ko and Tang](#) (2016, 2017); [Krall et al.](#) (2017); [Lesgourgues et al.](#) (2016); [Schewtschenko et al.](#) (2016); [Tang](#) (2016), in which dark matter (DM) interacts with a new form of dark radiation (DR) aimed at solving  $H_0$  tension. Assuming the Effective Theory of Structure Formation (ETHOS) paradigm ([Cyr-Racine et al.](#) 2016; [Vogelsberger et al.](#) 2016) the interaction between the dark matter and dark radiation components is a 2-to-2 scattering  $\text{DM} + \text{DR} \leftrightarrow \text{DM} + \text{DR}$ .

**II.2.5.3 Neutrino self-interactions:** The strong (massive) neutrino self-interactions cosmological model can provide a larger value of  $H_0$  and smaller  $\sigma_8$ , hence can resolve the tensions between cosmological datasets ([Kreisch et al.](#) 2020). The strong neutrino self-interactions were proposed in [Cyr-Racine and Sigurdson](#) (2014) and further studied in [Lancaster et al.](#) (2017); [Oldengott et al.](#) (2017). The introduction of strong self-interacting neutrinos increases the value in the effective number of relativistic particles  $N_{\text{eff}} = 4.02 \pm 0.29$  without extra neutrino species. This model modifies the standard neutrino free-streaming in the early Universe. The onset of neutrino free-streaming is delayed until close to the matter radiation equality epoch. This late-decoupling of the neutrinos shifts the CMB power spectra peaks towards smaller scales as compared to  $\Lambda\text{CDM}$  model. This shift modifies the scale of sound horizon  $r_s$  that can resolve the Hubble constant  $H_0$  tension ([Kreisch et al.](#) 2020).

Furthermore self-interactions between the neutrinos or between other additional light relics was studied by [Archidiacono et al.](#) (2016a, 2020, 2015, 2016b); [Berbig et al.](#) (2020); [Blinov et al.](#) (2019); [Brinckmann et al.](#) (2020); [Chu et al.](#) (2015); [Corona et al.](#) (2021); [Das and Ghosh](#) (2021); [Forastieri et al.](#) (2019); [Ghosh et al.](#) (2020); [Hannestad et al.](#) (2014); [He et al.](#) (2020); [Lyu et al.](#) (2021); [Mazumdar et al.](#) (2020); [Roy Choudhury et al.](#) (2021). The strong neutrino self-interactions models are basically excluded by various existing data or experimental tests ([Blinov et al.](#) 2019; [Brdar et al.](#) 2020; [Brune and Päs](#) 2019; [Deppisch et al.](#) 2020; [Lyu et al.](#) 2021). The analysis by [Brinckmann et al.](#) (2020) leads to conclusion that these models can not ease the Hubble tension more effectively than the  $\Lambda\text{CDM}+N_{\text{eff}}$  approach alone.

Models with nonstandard neutrinos - dark matter interactions were studied by [Arias-Aragon et al.](#) (2021); [Boyarsky et al.](#) (2021); [Choi et al.](#) (2019); [Di Valentino et al.](#) (2018a); [Escudero and Witte](#) (2020, 2021); [Huang and Rodejohann](#) (2021); [Mangano et al.](#) (2006); [Mosbech et al.](#) (2021); [Stadler et al.](#) (2019); [Wilkinson et al.](#) (2014). These models increase the value in the effective number of relativistic particle  $N_{\text{eff}}$  and thus can provide a solution to the Hubble problem. However, in this class of models it is not possible to solve simultaneously the Hubble and growth tensions ([Di Valentino et al.](#) 2018a).

#### II.2.5.4 Large primordial non-Gaussianities:

The presence of large primordial non-Gaussianity in the CMB can affect the higher-order  $n$ -point correlation functions statistics. A non-vanishing primordial trispectrum ( $n = 4$ ) which is the Fourier transform of the connected four-point correlation function leads to the non-Gaussian covariance of the angular power spectrum estimators ([Adhikari et al.](#) 2018; [Hu](#) 2001; [Smith et al.](#) 2015). The trispectrum is nonzero when there is a strong coupling between long-wavelength (super-CMB) modes and short-wavelength modes. The non-Gaussian covariance scenario (Super- $\Lambda\text{CDM}$  model) has two extra free parameters relative to those in  $\Lambda\text{CDM}$  and provides a larger value of  $H_0$  reducing tension with late Universe measurements of the Hubble constant ([Adhikari and Huterer](#) 2020).

**II.2.5.5 Heisenberg's uncertainty principle:** The Heisenberg's uncertainty principle ([Heisenberg](#) 1927; [Robertson](#) 1929) and the generalized uncertainty Principle ([Adler et al.](#) 2001; [Ali et al.](#) 2015; [Ashoorioon et al.](#) 2005a,b; [Ashoorioon and Mann](#) 2005a,b; [Faizal](#) 2016; [Faizal et al.](#) 2016; [Hinrichsen and Kempf](#) 1996; [Karolyhazy](#) 1966; [Kempf](#) 1996, 1997; [Kempf et al.](#) 1995; [Lake](#) 2019, 2020; [Maggiore](#) 1993a,b, 1994; [Mead](#) 1964; [Mohammadi et al.](#) 2017; [Nozari et al.](#) 2012; [Snyder](#) 1947; [Vagenas et al.](#) 2018; [Yang](#) 1947; [Zhao et al.](#) 2017b) (see [Tawfik and Diab](#) 2015, for a review) can provide constraints to the values for certain pairs of physical quantities of a particle and raise the possibility of the existence of observational signatures in cosmological data (e.g. [Perivolaropoulos](#) 2017a; [Skara and Perivolaropoulos](#) 2019). [Capozziello et al.](#) (2020) have argued that the Heisenberg's uncertainty principle can provide an explanation for the Hubble constant  $H_0$  tension. In particular the authors equate the luminosity distance (expanded for low  $z$  as in Eqs. (2.7) and (2.11)) with the photon (assumed massive) Compton wavelength

$$\lambda_C = \frac{\hbar}{mc} \quad (2.90)$$

and express the corresponding effective “rest mass” of the photon as a function of the cosmological redshift

$$m = \frac{\hbar H_0}{zc^2 [1 + \frac{z}{2}(1 - q_0)]} \quad (2.91)$$

Thus, choosing  $z = 1$ , fixing  $q_0 = -1/2$  and setting  $H_0 = 74 \text{ km s}^{-1}\text{Mpc}^{-1}$  and  $H_0 = 67 \text{ km s}^{-1}\text{Mpc}^{-1}$  in Eq. (2.91) find  $m = 1.61 \times 10^{-69} \text{ kg}$  and  $m = 1.46 \times 10^{-69} \text{ kg}$  respectively<sup>30</sup>. Thus using these results infer that the tension on the  $H_0$  measurements can be the effect of the uncertainty on the photon mass i.e.

$$\frac{\Delta m}{m} = \frac{\Delta H_0}{H_0} \simeq 0.1 \quad (2.92)$$

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<sup>30</sup> The current upper limit on the photon mass is  $m = 10^{-54} \text{ kg}$  ([Tanabashi et al.](#) 2018).

Note that the non-zero photon mass could emerge through the Heisenberg's uncertainty principle and through the recent analysis of the Standard-Model Extension<sup>31</sup> (Bonetti *et al.* 2017, 2018).

**II.2.5.6 Early modified gravity:** A scalar tensor modified gravity model can be described by the following action

$$S = \int d^4x \sqrt{-g} \left[ \frac{F(\sigma)}{2} R - \frac{g^{\mu\nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - \Lambda - V(\sigma) \right] + S_m \quad (2.93)$$

where,  $R$  is the Ricci scalar,  $\Lambda$  is the cosmological constant,  $S_m$  is the action for matter fields,  $\sigma$  is a scalar field non-minimally coupled to the Ricci scalar,  $F(\sigma)$  is the coupling to the Ricci scalar and  $V(\sigma)$  is the potential for the scalar field. A variety of possible types of the non-minimal coupling of the scalar field to the Ricci  $F(\sigma)$  and of the potential for the scalar field which can alleviate the  $H_0$  tension by reducing the sound horizon scale through modified early cosmic expansion, has been considered in the literature (Abadi and Kovetz 2021; Ballardini *et al.* 2020; Ballesteros *et al.* 2020; Braglia *et al.* 2020a; Rossi *et al.* 2019).

In particular Braglia *et al.* (2021) introduce a model of early modified gravity<sup>32</sup>. This model has a non-minimal coupling of the form (Braglia *et al.* 2021)

$$F(\sigma) = M_{pl}^2 + \xi \sigma^2 \quad (2.94)$$

and a quartic potential

$$V(\sigma) = \frac{\lambda \sigma^4}{4} \quad (2.95)$$

where  $\lambda$  and  $\xi$  are dimensionless parameters. For  $\xi = 0$  this model reduces to the EDE model of Agrawal *et al.* (2019). In the early modified gravity model, gravity changes with redshift in such a way that the  $H_0$  estimate from CMB can have larger values. Braglia *et al.* (2021) have shown that this model can resolve the Hubble tension and at the same time, in contrast to an EDE model, results in lower value of the late-time density fluctuation amplitude  $\sigma_8$  and thus the tension with LSS dynamical probes WL, RSD and CC data can be at least partially resolved. In general early modified gravity model compared to the EDE can provide a better fit to LSS data and can imply better predictions on LSS observables.

<sup>31</sup> For studies of the massive photons in the Standard-Model Extension, see Helayel-Neto and Spallicci (2019); Spallicci *et al.* (2021).

<sup>32</sup> It should not be confused with the previously introduced differed model with the same name 'Early Modified Gravity' (Brax *et al.* 2014; Lima *et al.* 2016; Pettorino and Amendola 2015). In this model gravity is allowed to be modified after BBN, before and during recombination.

### III Other Tensions - Curiosities

In this section we provide a list of the non-standard signals in cosmological data and the curiosities of the  $\Lambda$ CDM cosmology beyond the Hubble tension which is currently the most widely studied and among the most statistically significant tensions. In many cases the signals are controversial and there is currently debate in the literature on the possible physical or systematic origin of these signals. For completeness we refer to all signals we could identify in the literature referring also to references that dispute the physical origin of these signals.

#### III.1 Growth tension

The Planck/ $\Lambda$ CDM parameter values in the context of GR indicate stronger growth of cosmological perturbations than the one implied by observational data of dynamical probes. In this section we review the observational evidence for this tension also known as the  $\Omega_{0m} - \sigma_8$  tension or simply 'growth tension'.

##### III.1.1 Methods and data

The value of the growth parameter combination  $S_8 \equiv \sigma_8(\Omega_{0m}/0.3)^{0.5}$  (where  $\sigma_8$  is discussed in more detail in what follows) is found by weak lensing (WL) (Abbott *et al.* 2020d, 2018d; Asgari *et al.* 2020; Hall 2021; Hildebrandt *et al.* 2017; Joudaki *et al.* 2018b; Köhlinger *et al.* 2017), cluster counts (CC) (Ade *et al.* 2016d; Costanzi *et al.* 2019, 2021; Mantz *et al.* 2010; Rapetti *et al.* 2009; Rozo *et al.* 2010) and redshift space distortion (RSD) data (Arjona *et al.* 2020; Basilakos and Nesseris 2017; Kazantzidis and Perivolaropoulos 2018, 2019; Macaulay *et al.* 2013; Nesseris *et al.* 2017; Perivolaropoulos and Kazantzidis 2019; Sagredo *et al.* 2018; Skara and Perivolaropoulos 2020) to be lower compared to the Planck CMB (TT,TE,EE+lowE) value  $S_8 = 0.834 \pm 0.016$  (Aghanim *et al.* 2020e) at a level of about  $2 - 3\sigma$  as shown<sup>33</sup> in Table II and in Fig. 21 (see Abdalla *et al.* 2022; Di Valentino *et al.* 2021f, for a recent review of this tension). The tension is also confirmed by the latest ACT+WMAP CMB analysis (Aiola *et al.* 2020) which finds  $S_8 = 0.840 \pm 0.030$ .

This is also expressed by the fact that dynamical cosmological probes (WL, RSD, CC) favor lower value of the matter density parameter  $\Omega_{0m} \approx 0.26 \pm 0.04$  (Alam *et al.* 2021a) than geometric probes (CMB, BAO, SnIa). This

<sup>33</sup> The definition  $S_8 = \sigma_8(\Omega_{0m}/0.3)^\alpha$  with  $\alpha = 1/2$  has been uniformly used for all points. In those cases where  $\alpha \neq 1/2$  has been used in some references, the value of  $S_8$  with  $\alpha = 1/2$  was recalculated (along with the uncertainties) using the constraints on  $\sigma_8$  and  $\Omega_{0m}$  shown in those references, assuming their errors  $\sigma_{\sigma_8}$  and  $\sigma_{\Omega_{0m}}$  are Gaussian. The errors of the  $S_8$  constraints are propagated according to  $\sigma_{S_8}^2 = (\Omega_{0m}/0.3)^{2\alpha} \sigma_{\sigma_8}^2 + \sigma_8^2 \alpha^2 (\Omega_{0m}/0.3)^{2\alpha-2} \sigma_{\Omega_{0m}}^2$ , with  $\alpha = 1/2$ .

could be a signal of weaker gravity than the predictions of General Relativity in the context of a  $\Lambda$ CDM background (Kazantzidis and Perivolaropoulos 2018; Macaulay *et al.* 2013; Nesseris *et al.* 2017; Skara and Perivolaropoulos 2020; Tsujikawa 2015) (for a recent study on a weak gravity in the context of a  $\Lambda$ CDM background, see Gannouji *et al.* 2021).

The observational evidence for weaker growth indicated by the dynamical probes of the cosmic expansion and the gravitational law on cosmological scales may be reviewed as follows:

**III.1.1.1 Weak lensing:** The weak gravitational lensing from matter fluctuations along the line of sight slightly distorts the shapes (shear) and size (magnification) of distant galaxies (see Bartelmann and Schneider 2001; Kilbinger 2015; Mandelbaum 2018, for a review). This distortion is a powerful and principal cosmological probe of the mass distribution which can be predicted theoretically (Bacon *et al.* 2000; Kaiser *et al.* 2000; van Waerbeke *et al.* 2000; Wittman *et al.* 2000). Using various statistical methods shape distortions can be measured by analyzing the angular shear correlation function, or its Fourier transform, the shear power spectrum (Asgari *et al.* 2021; Hikage *et al.* 2019). A special type of WL is the galaxy-galaxy lensing (GGL) (Brainerd *et al.* 1996; Hudson *et al.* 1998) which is the slight distortion of shapes of source galaxies in the background of a lens galaxy arising from the gravitational deflection of light due to the gravitational potential of the lens galaxy along the line of sight.

The WL surveys, the Kilo Degree Survey (KiDS) (de Jong *et al.* 2013, 2015, 2017; Kuijken *et al.* 2015), the Subaru Hyper Suprime-Cam lensing survey (HSC) (Aihara *et al.* 2018; Miyazaki *et al.* 2015) and the Dark Energy Survey (DES) (Abbott *et al.* 2005; Jarvis *et al.* 2016) provide data useful for cosmic shear studies. In particular WL measurements of  $S_8$  obtained from the shear catalogues by the lensing analysis of the Canada-France-Hawaii Telescope Lensing (CFHTLens) (Erben *et al.* 2013; Heymans *et al.* 2012, 2013; Hildebrandt *et al.* 2012; Joudaki *et al.* 2017a; Miller *et al.* 2013) and the KiDS (Hildebrandt *et al.* 2017; Köhlinger *et al.* 2017) appear to be lower compared to the Planck value at a level of about  $3\sigma$ . The analysis by Hildebrandt *et al.* (2017) adopting a spatially flat  $\Lambda$ CDM model and using the KiDS-450 data reports  $S_8 = 0.745 \pm 0.039$  which results in  $2.3\sigma$  tension with the value estimated by Planck15. This KiDS-Planck discordance has also been investigated in Köhlinger *et al.* (2017) where applying the quadratic estimator to KiDS-450 shear data reports  $S_8 = 0.651 \pm 0.058$  which is in tension with the Planck2015 results at the  $3.2\sigma$  level.

Using a combination of the measurements of KiDS-450 and VISTA Kilo-Degree infrared Galaxy Survey (VIKING) (Arnaboldi *et al.* 2007), Hildebrandt *et al.* (2020) find  $S_8 = 0.737^{+0.040}_{-0.036}$  which is discrepant with measurements from the Planck analysis at the  $2.3\sigma$  level.

For the KiDS+VIKING-450 (or KV450) Wright *et al.* (2020) report an updated constraint of  $S_8 = 0.716^{+0.043}_{-0.038}$ . Meanwhile, using the DES first year (DES-Y1) data assuming a  $\Lambda$ CDM model Troxel *et al.* (2018) report  $S_8 = 0.782^{+0.027}_{-0.027}$  which is in  $\sim 2.3\sigma$  tension<sup>34</sup> with the Planck18 result. The constraint on  $S_8$  from the combined tomographic weak lensing analysis of KiDS + VIKING + DES-Y1 adopting a flat  $\Lambda$ CDM model by Joudaki *et al.* (2020) is  $S_8 = 0.762^{+0.025}_{-0.024}$  which is in  $2.5\sigma$  tension with Planck18 result and by Asgari *et al.* (2020) is  $S_8 = 0.755^{+0.019}_{-0.021}$  which is in  $3.2\sigma$  tension with Planck18 result. Analysing the most recent KiDS cosmic shear data release (KiDS-1000 Kuijken *et al.* 2019) alone and assuming a spatially flat  $\Lambda$ CDM model the value  $S_8 = 0.759^{+0.024}_{-0.021}$  was estimated by Asgari *et al.* (2021). Analysing the first-year data of HSC in the context of the flat  $\Lambda$ CDM model and using the pseudo-spectrum (pseudo- $C_l$ ) method<sup>35</sup>, Hikage *et al.* (2019) find  $S_8 = 0.780^{+0.030}_{-0.033}$  and adopting the standard two-point correlation functions (TPCF) estimators,  $\xi_{\pm}$ , Hamana *et al.* (2020) find  $S_8 = 0.804^{+0.032}_{-0.029}$ . Recently, a analysis of the KiDS-1000 data using pseudo- $C_l$  method by Loureiro *et al.* (2021) has lead to  $S_8 = 0.754^{+0.027}_{-0.029}$ . The latest cosmic shear analysis of the DES third Year (DES-Y3) (Amon *et al.* 2022; Secco *et al.* 2022) in the context of the  $\Lambda$ CDM model constrains the clustering amplitude as  $S_8 = 0.759^{+0.025}_{-0.023}$ . Also, recently Chang *et al.* (2022a) found  $S_8 = 0.73^{+0.04}_{-0.03}$  using the cross-correlations of galaxy positions and shears from DES-Y3 with CMB lensing maps from SPT and Planck.

The analysis of galaxy clustering and weak gravitational lensing of the DES-Y1 data combining three two-point functions (the so-called  $3 \times 2$ pt analysis) of gravitational lensing and galaxy positions (the cosmic shear correlation function, the galaxy clustering angular autocorrelation function, the galaxy-galaxy lensing cross-correlation function) by Abbott *et al.* (2018d) gives  $S_8 = 0.773^{+0.026}_{-0.020}$  and  $\Omega_{0m} = 0.267^{+0.030}_{-0.017}$  in flat  $\Lambda$ CDM model. This value is in  $\sim 2.3\sigma$  tension with Planck18 result. In the latest analysis by Abbott *et al.* (2022) the constraints  $S_8 = 0.776^{+0.017}_{-0.017}$  and  $\Omega_{0m} = 0.339^{+0.032}_{-0.031}$  in flat  $\Lambda$ CDM model are obtained using an improvement in signal-to-noise of the DES-Y3  $3 \times 2$ pt data relative to DES-Y1 by a factor of 2.1. Also, Heymans *et al.* (2021) using  $3 \times 2$ pt analysis of KiDS-1000+BOSS+ 2-degree Field Lensing Survey<sup>36</sup> (2dFLens) (Blake *et al.* 2016) data finds  $S_8 = 0.766^{+0.020}_{-0.014}$ . While previous analyses  $3 \times 2$ pt

<sup>34</sup> This tension was calculated by Lemos *et al.* (2021). The authors have explored a number of different methods to quantify the tension relative to the best-fit Planck2018 cosmology.

<sup>35</sup> For a realistic experiment the pseudo- $C_l$  statistics from cut-sky maps which provide incomplete data are applied in order to obtain unbiased estimates of the angular power and cross-power spectra by correcting for the convolution with the survey window (see Alonso *et al.* 2019; Brown *et al.* 2005; Hivon *et al.* 2002; Wandelt *et al.* 2001, for details of this method).

<sup>36</sup> <https://2dflens.swin.edu.au>

TABLE II. The value of the structure growth parameter combination  $S_8 \equiv \sigma_8(\Omega_{0m}/0.3)^{0.5}$ , the matter density parameter  $\Omega_{0m}$  and the power spectrum amplitude  $\sigma_8$  at 68% CL through direct and indirect measurements by different methods.

Dataset	$S_8$	$\Omega_{0m}$	$\sigma_8$	Refs.
CMB Planck TT,TE,EE+lowE	$0.834 \pm 0.016$	$0.3166 \pm 0.0084$	$0.812 \pm 0.007$	(Aghanim <i>et al.</i> 2020e)
CMB Planck TT,TE,EE+lowE+lens.	$0.832 \pm 0.013$	$0.3153 \pm 0.0073$	$0.811 \pm 0.006$	(Aghanim <i>et al.</i> 2020e)
CMB ACT+WMAP	$0.832 \pm 0.013$	$0.3153 \pm 0.0073$	$0.840 \pm 0.030$	(Aiola <i>et al.</i> 2020)
WL KiDS-1000	$0.759^{+0.024}_{-0.021}$	-	-	(Asgari <i>et al.</i> 2021)
WL KiDS + VIKING + DES-Y1	$0.755^{+0.019}_{-0.021}$	-	-	(Asgari <i>et al.</i> 2020)
WL KiDS + VIKING + DES-Y1	$0.762^{+0.025}_{-0.024}$	-	-	(Joudaki <i>et al.</i> 2020)
WL KiDS+VIKING-450	$0.716^{+0.043}_{-0.038}$	-	-	(Wright <i>et al.</i> 2020)
WL KiDS+VIKING-450	$0.737^{+0.040}_{-0.036}$	-	-	(Hildebrandt <i>et al.</i> 2020)
WL KiDS-450	$0.651 \pm 0.058$	-	-	(Köhlinger <i>et al.</i> 2017)
WL KiDS-450	$0.745 \pm 0.039$	-	-	(Hildebrandt <i>et al.</i> 2017)
WL DES-Y3	$0.759^{+0.025}_{-0.023}$	$0.290^{+0.039}_{-0.063}$	$0.783^{+0.073}_{-0.092}$	(Amon <i>et al.</i> 2022; Secco <i>et al.</i> 2022)
WL DES-Y1	$0.782^{+0.027}_{-0.027}$	-	-	(Troxel <i>et al.</i> 2018)
WL HSC-TPCF	$0.804^{+0.032}_{-0.029}$	$0.346^{+0.052}_{-0.100}$	$0.766^{+0.110}_{-0.098}$	(Hamana <i>et al.</i> 2020)
WL KiDS-1000 pseudo- $C_l$	$0.754^{+0.027}_{-0.029}$	-	-	(Loureiro <i>et al.</i> 2021)
WL HSC-pseudo- $C_l$	$0.780^{+0.030}_{-0.033}$	-	-	(Hikage <i>et al.</i> 2019)
WL CFHTLenS	$0.740^{+0.033}_{-0.038}$	-	-	(Joudaki <i>et al.</i> 2017a)
WL+CMB lensing DES-Y3+SPT+Planck	$0.73^{+0.04}_{-0.03}$	$0.25^{+0.03}_{-0.04}$	$0.82^{+0.08}_{-0.07}$	(Chang <i>et al.</i> 2022a)
WL+GC <sup>a</sup>	$0.795^{+0.049}_{-0.042}$	$0.383^{+0.028}_{-0.053}$	$0.718^{+0.044}_{-0.031}$	(Miyatake <i>et al.</i> 2021)
WL+GC+CMB lensing <sup>b</sup>	$0.7781 \pm 0.0094$	$0.305^{+0.021}_{-0.025}$	$0.774 \pm 0.033$	(García-García <i>et al.</i> 2021)
WL+GC KiDS-1000 3 × 2pt	$0.766^{+0.020}_{-0.014}$	$0.305^{+0.010}_{-0.015}$	$0.76^{+0.025}_{-0.020}$	(Heymans <i>et al.</i> 2021)
WL+GC KiDS-450 3 × 2pt	$0.742 \pm 0.035$	$0.243^{+0.026}_{-0.045}$	$0.832^{+0.086}_{-0.079}$	(Joudaki <i>et al.</i> 2018b)
WL+GC KiDS+GAMA 3 × 2pt	$0.800^{+0.029}_{-0.027}$	$0.33^{+0.05}_{-0.06}$	$0.78^{+0.06}_{-0.08}$	(van Uitert <i>et al.</i> 2018)
WL+GC DES-Y3 3 × 2pt	$0.776^{+0.017}_{-0.017}$	$0.339^{+0.032}_{-0.031}$	$0.733^{+0.039}_{-0.049}$	(Abbott <i>et al.</i> 2022)
WL+GC DES-Y1 3 × 2pt	$0.773^{+0.026}_{-0.020}$	$0.267^{+0.030}_{-0.017}$	$0.817^{+0.045}_{-0.056}$	(Abbott <i>et al.</i> 2018d)
WL+GC KiDS+VIKING-450+BOSS	$0.728 \pm 0.026$	$0.323^{+0.014}_{-0.017}$	$0.702 \pm 0.029$	(Tröster <i>et al.</i> 2020)
GC BOSS DR12 bispectrum	$0.751 \pm 0.039$	$0.32^{+0.01}_{-0.01}$	$0.722^{+0.032}_{-0.036}$	(Philcox and Ivanov 2022)
GC BOSS+eBOSS	$0.72 \pm 0.042$	-	-	(Ivanov 2021)
GC BOSS galaxy power spectrum	$0.703 \pm 0.045$	$0.293 \pm 0.012$	$0.713 \pm 0.045$	(Ivanov <i>et al.</i> 2020c)
GC BOSS power spectra	$0.736 \pm 0.051$	$0.303 \pm 0.0082$	$0.733 \pm 0.047$	(Chen <i>et al.</i> 2022)
GC BOSS DR12	$0.729 \pm 0.048$	$0.317^{+0.015}_{-0.019}$	$0.710 \pm 0.049$	(Tröster <i>et al.</i> 2020)
GC+CMB lensing DESI+Plank	$0.73 \pm 0.03$	-	-	(White <i>et al.</i> 2022)
GC+CMB lensing unWISE+Plank	$0.784 \pm 0.015$	$0.307 \pm 0.018$	$0.775 \pm 0.029$	(Krolewski <i>et al.</i> 2021)
CC AMICO KiDS-DR3	$0.78 \pm 0.04$	$0.24^{+0.03}_{-0.04}$	$0.86 \pm 0.07$	(Leschi <i>et al.</i> 2022)
CC SDSS-DR8	$0.79^{+0.05}_{-0.04}$	$0.22^{+0.05}_{-0.04}$	$0.91^{+0.11}_{-0.10}$	(Costanzi <i>et al.</i> 2019)
CC ROSAT (WtG)	$0.77 \pm 0.05$	$0.26 \pm 0.03$	$0.83 \pm 0.04$	(Mantz <i>et al.</i> 2015)
CC DES-Y1	$0.65^{+0.04}_{-0.04}$	$0.179^{+0.031}_{-0.038}$	$0.85^{+0.04}_{-0.06}$	(Abbott <i>et al.</i> 2020d)
CC XMM-XXL	$0.83 \pm 0.11$	$0.40 \pm 0.09$	$0.72 \pm 0.07$	(Pacaud <i>et al.</i> 2018)
CC SPT-tSZ	$0.749 \pm 0.055$	$0.276 \pm 0.047$	$0.781 \pm 0.037$	(Bocquet <i>et al.</i> 2019)
CC Planck tSZ	$0.785 \pm 0.038$	$0.32 \pm 0.02$	$0.76 \pm 0.03$	(Salvati <i>et al.</i> 2018)
CC Planck tSZ	$0.792 \pm 0.056$	$0.31 \pm 0.04$	$0.78 \pm 0.04$	(Ade <i>et al.</i> 2016d)
RSD+BAO+Pantheon+CC	$0.777^{+0.026}_{-0.027}$	$0.288 \pm 0.008$	$0.793^{+0.018}_{-0.020}$	(Nunes and Vagnozzi 2021)
RSD+BAO+Pantheon	$0.762^{+0.030}_{-0.025}$	$0.286 \pm 0.008$	$0.7808^{+0.021}_{-0.019}$	(Nunes and Vagnozzi 2021)
RSD	$0.739^{+0.036}_{-0.040}$	$0.254^{+0.038}_{-0.058}$	$0.804^{+0.048}_{-0.071}$	(Nunes and Vagnozzi 2021)
RSD	$0.700^{+0.038}_{-0.037}$	$0.201^{+0.036}_{-0.033}$	$0.857^{+0.044}_{-0.042}$	(Benisty 2021)
RSD	$0.747 \pm 0.029$	$0.279 \pm 0.028$	$0.775 \pm 0.018$	(Kazantzidis and Perivolaropoulos 2018)

<sup>a</sup> HSC-Y1+SDSS-III/BOSS DR11

<sup>b</sup> KiDS+DES+eBOSS+Planck

of KiDS+GAMA data and KiDS-450+BOSS+2dFLens data by [van Uitert et al. \(2018\)](#) and [Joudaki et al. \(2018b\)](#) obtained  $S_8 = 0.800^{+0.029}_{-0.027}$  and  $S_8 = 0.742 \pm 0.035$  respectively. A combined analysis of KiDS+VIKING-450+BOSS data by [Tröster et al. \(2020\)](#) resulted in  $S_8 = 0.728 \pm 0.026$ . Performing a joint analysis of galaxy-galaxy weak lensing and galaxy clustering from first-year data of HSC and SDSSS-III/BOSS DR11 [Miyatake et al. \(2021\)](#) found  $S_8 = 0.795^{+0.049}_{-0.042}$ . Also, from a combined analysis of KiDS-1000 and DES-Y1 cosmic shear and galaxy clustering, eBOSS quasars, DESI, Planck CMB lensing data [García-García et al. \(2021\)](#) obtains a constraint  $S_8 = 0.7781 \pm 0.0094$ .

Clearly, the tension between WL and CMB measurements is a level more than  $2\sigma$  as seen in Table II and in Fig. 21. In addition, the tension with more recent measurements persists at the level of  $\sim 2\sigma$ . Finally, combined analyses of WL with galaxy clustering does not change the tension level.

**III.1.1.2 Cluster counts:** Galaxy clusters which are related to peaks in the matter density field on large scales constitute a probe of the growth history of structures ([Evrard 1989; Peebles et al. 1989](#)) (see [Allen et al. 2011; Kravtsov and Borgani 2012](#), for a review). Current analyses from the number counts of galaxy clusters use catalogs from surveys at different wavelengths of the electromagnetic spectrum. Such surveys include Planck<sup>37</sup>, South Pole Telescope (SPT) and Atacama Cosmology Telescope (ACT) in the microwave (millimeter) via the thermal Sunyaev-Zel'dovich (tSZ) effect<sup>38</sup> ([Sunyaev and Zeldovich 1970, 1972, 1980](#)), extended Roentgen survey with an imaging telescope array (eROSITA<sup>39</sup>) ([Hofmann et al. 2017; Merloni et al. 2012; Pillepich et al. 2012; Predehl et al. 2010](#)) in the X-ray that finds extended sources and measures the X-ray luminosity and temperature, Sloan Digital Sky Survey<sup>40</sup> (SDSS) and Dark Energy Survey<sup>41</sup> (DES) in the optical/NIR. These surveys find peaks in the galaxy distribution and measure the richness of the corresponding clusters. The microwave/tSZ and X-ray surveys detection techniques are based on the hot ICM ([Bleem et al. 2015; Hasselfield et al. 2013](#)) and in some cases require auxiliary data to obtain useful constraints e.g. redshift estimates (see [Bleem et al. 2020; Klein et al. 2018](#), for recent methods).

The CC method is based on the predicted halo abundance (number density)  $n(M, z)$  of halos with mass less than  $M$  at redshift  $z$  which is also known as the halo mass function (HMF). This formalism was originally introduced by Press and Schechter ([Press and Schechter](#)

1974). A general mathematical form for the comoving number density expression of haloes is (e.g. [Jenkins et al. 2001; Sheth and Tormen 2002; Tinker et al. 2008; Warren et al. 2006; White 2002](#))

$$\frac{dn}{dM} = f(\sigma) \frac{\bar{\rho}_m}{M} \frac{d \ln \sigma^{-1}}{dM} \quad (3.1)$$

where  $\bar{\rho}_m = \rho_{crit} \Omega_m$  is the mean matter density of the Universe,  $\sigma$  is the rms variance of the linear density field smoothed on a spherical volume containing a mass  $M$ , and  $f(\sigma)$  is a model-dependent ‘universal’ halo multiplicity function<sup>42</sup>. There are numerous parametrizations of the multiplicity function  $f(\sigma)$  based on numerical N-body simulations or theoretical models. A popular parametrization provided by [Tinker et al. \(2008\)](#) is

$$f(\sigma) = A \left[ \left( \frac{\sigma}{b} \right)^{-\chi} + 1 \right] e^{-c/\sigma^2} \quad (3.2)$$

where  $A, \chi, b, c$  are four free parameters that depend on the halo definition and need to be calibrated.

Measurements of the abundance of galaxy clusters  $n(M, z)$  provide consistent constraints on the density of matter  $\Omega_{0m}$ , the root mean square density fluctuation  $\sigma_8$ , the parameter combination  $S_8(\alpha) = \sigma_8(\Omega_{0m}/0.3)^\alpha$  (e.g. [Asgari et al. 2021; Hikage et al. 2019; Kilbinger et al. 2013](#)) (where  $\alpha \sim 0.2 - 0.6$  and  $S_8 \equiv S_8(\alpha = 0.5)$ ), the dark energy equation-of-state  $w$  and the sum of the neutrino masses  $\sum m_\nu$  (massive neutrinos can suppress the matter power spectrum on small scales and this directly affect the growth of cosmic structure) ([Weinberg et al. 2013](#)). More recently, [Sabti et al. \(2021, 2022\)](#) used a method of clustering measurements at higher redshift ( $z = 4 - 10$ ) based on UV galaxy luminosity function data from the Hubble Space Telescope (see e.g. [Atek et al. 2018; Bouwens et al. 2015](#)). They derive the large-scale matter clustering amplitude to be  $\sigma_8 = 0.76^{+0.12}_{-0.14}$ .

Using cluster abundance analysis in the SDSS DR8 for a flat  $\Lambda$ CDM cosmological model with massive neutrinos [Costanzi et al. \(2019\)](#) find  $S_8 = 0.79^{+0.05}_{-0.04}$ . [Mantz et al. \(2015\)](#) using Weighting the Giant (WtG) ([Applegate et al. 2014; von der Linden et al. 2014](#)) lensing analysis of the X-ray ROentgen SATellite (ROSAT) cluster catalogs ([Truemper 1993](#)) find  $S_8 = 0.77 \pm 0.05$ . The analysis of the counts and weak lensing signal of the DES-Y1 dataset by [Abbott et al. \(2020d\)](#) gives  $S_8 = 0.65 \pm 0.04$  and  $\Omega_{0m} = 0.267^{+0.030}_{-0.017}$  in flat  $\Lambda$ CDM. Also, assuming a flat  $\Lambda$ CDM model and performing a galaxy cluster abundance analysis in the AMICO KiDS-DR3 catalogue [Lesci et al. \(2022\)](#) obtains  $S_8 = 0.78 \pm 0.04$ .

Using galaxy clusters observed in millimeter wavelengths through the tSZ effect [Salvati et al. \(2018\)](#) report  $S_8 = 0.785 \pm 0.038$  assuming  $\Lambda$ CDM model. The

<sup>37</sup> <https://www.cosmos.esa.int>

<sup>38</sup> The inverse Compton scattering between CMB photons and hot electrons in the intracluster medium (ICM) (see [Birkinshaw 1999; Carlstrom et al. 2002; Mroczkowski et al. 2019](#), for a review)

<sup>39</sup> <https://www.mpe.mpg.de/eROSITA>

<sup>40</sup> <https://www.sdss.org/>

<sup>41</sup> <https://www.darkenergysurvey.org>

<sup>42</sup> For a publicly available cluster toolkit Python package, see in <https://cluster-toolkit.readthedocs.io/en/latest/source/massfunction.html>.

analysis of the Planck 2015 cluster counts via the tSZ signal by Ade *et al.* (2016d) finds  $S_8 = 0.792 \pm 0.056$ . Recently, assuming a flat  $\Lambda$ CDM model, in which the total neutrino mass is a free parameter, the analysis of SPT tSZ cluster counts by Bocquet *et al.* (2019) results in  $S_8 = 0.749 \pm 0.055$ . Using X-ray clusters detected from the XMM-XXL survey (Pierre *et al.* 2016) for a flat  $\Lambda$ CDM cosmological model Pacaud *et al.* (2018) report  $S_8 = 0.83 \pm 0.10$ . Also, constraints on structure growth parameter combination  $S_8$  from cluster abundance data have been obtained by Abdullah *et al.* (2020a); de Haan *et al.* (2016); Kirby *et al.* (2019); Ntampaka *et al.* (2019); Schellenberger and Reiprich (2017); Zubeldia and Challinor (2019). For example using GalWCat19 (Abdullah *et al.* 2020b), a catalog of 1800 galaxy clusters was derived from the SDSS-DR13 (Albareti *et al.* 2017) and assuming a flat  $\Lambda$ CDM cosmology Abdullah *et al.* (2020a) measured the matter density and the amplitude of fluctuations to be  $\Omega_m = 0.310^{+0.023}_{-0.027} \pm 0.041$  (systematic) and  $\sigma_8 = 0.810^{+0.031}_{-0.036} \pm 0.035$  (systematic) respectively.

The results of  $S_8$  from all cluster count experiments as seen in Table II and in Fig. 21 are in agreement with WL measurements and similarly prefer a lower value compared to the CMB measurements.

**III.1.1.3 Redshift space distortion-Galaxy clustering:** Peculiar motions of galaxies falling towards overdense region generate large scale galaxy clustering, anisotropic in redshift space. Measuring this illusory anisotropy that distorts the distribution of galaxies in redshift space (i.e. RSD) we can quantify the galaxy velocity field. This important probe of LSS can be used to constrain the growth rate of cosmic structures (Hamilton 1992, 1997; Kaiser 1987).

In particular the RSD is sensitive to the cosmological growth rate of matter density perturbations  $f$  which depends on the theory of gravity and is defined as (Linder 2005; Polarski and Gannouji 2008; Wang and Steinhardt 1998)

$$f(a) \equiv \frac{d \ln \delta(a)}{d \ln a} \simeq [\Omega_m(a)]^{\gamma(a)} \quad (3.3)$$

where  $a = \frac{1}{1+z}$  is the scale factor,  $\delta \equiv \frac{\delta \rho_m}{\rho_m}$  is the matter overdensity field (with  $\rho_m$  is the matter density of the background) and  $\gamma$  is the growth index (e.g. Lahav *et al.* 1991). The nearly constant and scale-independent value  $\gamma \simeq \frac{6}{11} \simeq 0.545$  corresponds to General Relativity (GR) prediction in the context of  $\Lambda$ CDM (e.g. Linder 2005).

The observable combination  $f\sigma_8(a) \equiv f(a) \cdot \sigma(a)$  is measured at various redshifts by different surveys as a probe of the growth of matter density perturbations. The theoretically predicted value of this product can be obtained from the solution  $\delta(a)$  of the equation (Nesseris and Sapone 2015)

$$\delta''(a) + \left( \frac{3}{a} + \frac{H'(a)}{H(a)} \right) \delta'(a) - \frac{3}{2} \frac{\Omega_{0m} G_{eff}(a)/G}{a^5 H(a)^2 / H_0^2} \delta(a) = 0 \quad (3.4)$$

using the definition

$$\sigma(a) \equiv \sigma_8 \frac{\delta(a)}{\delta(a=1)} \quad (3.5)$$

where  $G$  is Newton's constant as measured by local experiments,  $G_{eff}$  is the effective gravitational coupling which is related to the growth of matter perturbation,  $\sigma(a)$  is the redshift dependent rms fluctuations of the linear density field within spheres of radius  $R = 8h^{-1} Mpc$  and  $\sigma_8$  is its value today.

Hence, the more robust bias free quantity  $f\sigma_8$  is given by

$$f\sigma_8(a) = \frac{\sigma_8}{\delta(a=1)} a \delta'(a) \quad (3.6)$$

RSD growth data in the form of  $f\sigma_8$ <sup>43</sup> have been provided by wide variety of surveys including the 2-degree Field Galaxy Redshift Survey (2dFGRS) (Hawkins *et al.* 2003; Peacock *et al.* 2001), VIMOS-VLT Deep Survey (VVDS) (Guzzo *et al.* 2008), SDSS (Chuang *et al.* 2013; Howlett *et al.* 2015; Reid *et al.* 2012; Samushia *et al.* 2012; Tegmark *et al.* 2006; Tojeiro *et al.* 2012), WiggleZ (Blake *et al.* 2011b), 6dFGS (Beutler *et al.* 2012; Johnson *et al.* 2014), Galaxy and Mass Assembly (GAMA) (Simpson *et al.* 2016), BOSS (Alam *et al.* 2017a; Gil-Marín *et al.* 2017; Li *et al.* 2016; White *et al.* 2015), Subaru Fiber Multi-Object Spectrograph (FMOS) galaxy redshift survey (FastSound) (Okumura *et al.* 2016), VIMOS Public Extra-galactic Redshift Survey (VIPERS) (Mohammad *et al.* 2018; Pezzotta *et al.* 2017; de la Torre *et al.* 2013), eBOSS (Alam *et al.* 2021a; Bautista *et al.* 2020; de Mattia *et al.* 2021; Tamone *et al.* 2020; Zhao *et al.* 2021, 2019, 2020), DESI (Aghamousa *et al.* 2016a,b). Using such data the  $\Omega_{0m}$  and  $\sigma_8$  parameters in the context of a  $\Lambda$ CDM background can be constrained. Thus, Kazantzidis and Perivolaropoulos (2018) using a compilation of 63 RSD datapoints find the  $\Lambda$ CDM best fit value  $\Omega_{0m} = 0.279 \pm 0.028$  and  $\sigma_8 = 0.775 \pm 0.018$ . Benisty (2021) using RSD selected data and assuming  $\Lambda$ CDM model report  $S_8 = 0.700^{+0.038}_{-0.037}$ ,  $\Omega_{0m} = 0.201^{+0.036}_{-0.033}$  and  $\sigma_8 = 0.857^{+0.044}_{-0.042}$  which are in  $3\sigma$  tension with the Planck 2018 results. Recently, using RSD data and the RSD+BAO+Pantheon and RSD+BAO+Pantheon+CC dataset combinations Nunes and Vagnozzi (2021) find  $S_8 = 0.739^{+0.036}_{-0.040}$ ,  $S_8 = 0.762^{+0.030}_{-0.025}$  and  $S_8 = 0.777^{+0.026}_{-0.027}$  respectively.

Galaxy clustering methods, such as the galaxy power spectrum and bispectrum have also been used to constrain  $S_8$ . Constraints from the BOSS galaxy power

<sup>43</sup> For an extensive compilation of RSD data points  $f\sigma_8$ , see in Skara and Perivolaropoulos (2020) and for other compilations, see in Basilakos and Nesseris (2016); Kazantzidis and Perivolaropoulos (2018); Kazantzidis *et al.* (2019); Nesseris *et al.* (2017); Sagredo *et al.* (2018). Also for a publicly available RSD likelihood for MontePython see in Arjona *et al.* (2020); Cardona *et al.* (2021).

spectrum (Ivanov *et al.* 2020c) gave  $S_8 = 0.703 \pm 0.045$  and from BOSS DR12 bispectrum (Philcox and Ivanov 2022) gave  $S_8 = 0.751 \pm 0.039$ . Previous analysis of the BOSS DR12 data by Tröster *et al.* (2020) gave  $S_8 = 0.729 \pm 0.048$ . A analysis of the power spectrum of eBOSS by Ivanov (2021) resulted in  $S_8 = 0.720 \pm 0.042$ . Recently, using the BOSS power spectra Chen *et al.* (2022) found  $S_8 = 0.736 \pm 0.051$ . Also, the combination of the auto- and cross-correlation signal of unWISE<sup>44</sup> galaxies (Schlafly *et al.* 2019) and Planck CMB lensing maps (Aghanim *et al.* 2020f) by Krolewski *et al.* (2021) gave  $S_8 = 0.784 \pm 0.015$ . Finally, using the luminous red galaxies of the DESI in combination with Planck CMB lensing maps White *et al.* (2022) found  $S_8 = 0.73 \pm 0.03$ .

Clearly, as seen in Table II and in Fig. 21 the analyses of RSD data gives  $S_8$  values in tension with CMB measurements at level more than  $2\sigma$ , in agreement with other dynamical cosmological probes (WL and CC).

### III.1.2 Theoretical models

Non-gravitational mechanisms can address the  $S_8$  tension (see Ishak 2019, for a review). Such mechanisms include the following:

- Dynamical dark energy models (Astier 2001; Barboza and Alcaniz 2008; Benisty and Staicova 2021; Chevallier and Polarski 2001; Cooray and Huterer 1999; D’Amico *et al.* 2021a; Du *et al.* 2019; Efstathiou 1999; Feng *et al.* 2006; Jassal *et al.* 2005; Joudaki *et al.* 2017b; Lambiase *et al.* 2019; Linder 2003; Melia 2017; Nesseris and Perivolaropoulos 2005; Ooba *et al.* 2019; Pan *et al.* 2018; Rezaei *et al.* 2017; Roy *et al.* 2022; Vagnozzi *et al.* 2018; Weller and Albrecht 2002; Yang *et al.* 2017b, 2019d, 2018d; Zhao *et al.* 2017a) and running vacuum models (Beltrán Jiménez *et al.* 2021; Colgáin *et al.* 2021; Gómez-Valent and Solà 2017; Gómez-Valent and Solà Peracaula 2018; Solà *et al.* 2015, 2017a; Solà Peracaula *et al.* 2021; Wang *et al.* 2018), which modify the cosmological background  $H(z)$  to a form different from  $\Lambda$ CDM (see Subsection II.2.1). This modification may involve the presence of dynamical dark energy dominant at late cosmological times or at times before recombination.
- Interacting dark energy models, which modify the equation for the evolution of linear matter fluctuations as well as the  $H(z)$  cosmological background (An *et al.* 2018; Barros *et al.* 2019; Camera *et al.* 2019; Pourtsidou and Tram 2016) as discussed in Subsection II.2.2. This class of models can address the structure growth  $\sigma_8$  tension between the values inferred from the CMB and the WL measurements.

<sup>44</sup> Wide-field Infrared Survey Explorer (WISE) (Wright *et al.* 2010) is a NASA infrared astronomy space telescope and is mapping the whole sky.

- Effects of massive neutrinos (Battye and Moss 2014; Biswas *et al.* 2019; Costanzi *et al.* 2014; Diaz Rivero *et al.* 2019; Joudaki *et al.* 2017b; Marulli *et al.* 2011; Poulin *et al.* 2018a; Zennaro *et al.* 2018) which are relativistic at early times and contribute to radiation while at late times they become non-relativistic but with significant velocities (hot dark matter) (see Lesgourgues and Pastor 2006, 2012, 2014; Wong 2011, for a review). The change of radiation to hot dark matter affects the Hubble expansion. Simultaneously the residual streaming velocities are still large enough at late times to slow down the growth of structure (Bond *et al.* 1980). This effect of massive neutrinos slows down the growth as required by the RSD data and relieves the  $S_8$  tension coming from WL data (Diaz Rivero *et al.* 2019).
- Primordial magnetic fields (Banerjee and Jedamzik 2004; Jedamzik and Saveliev 2019) (see Durrer and Neronov 2013; Subramanian 2016; Vachaspati 2021, for a review) induce additional mildly non-linear, small-scale baryon inhomogeneities present in the plasma before recombination. The required field results in a reduction of the sound horizon scale at recombination and has the potential to resolve both the  $H_0$  and  $S_8$  tension (Jedamzik and Pogosian 2020; Rashkovetskyi *et al.* 2021; Thiele *et al.* 2021).
- Non-thermal dark radiation (Das *et al.* 2021b) seems to help alleviate the  $S_8$  tension to a great extent. However, the inclusion of BAO data reduces significantly the quality of fit of this model.

In addition to these non-gravitational mechanisms discussed above that can slow down growth at low redshifts a possible interesting new fundamental physics approach can also reduce the  $S_8$  tension. Such an approach is most likely to affect three basic observable parameters: the Hubble parameter  $H(z, w)$  (with  $w$  the dark energy equation of state parameter), as well as the effective Newton constants for growth of perturbations

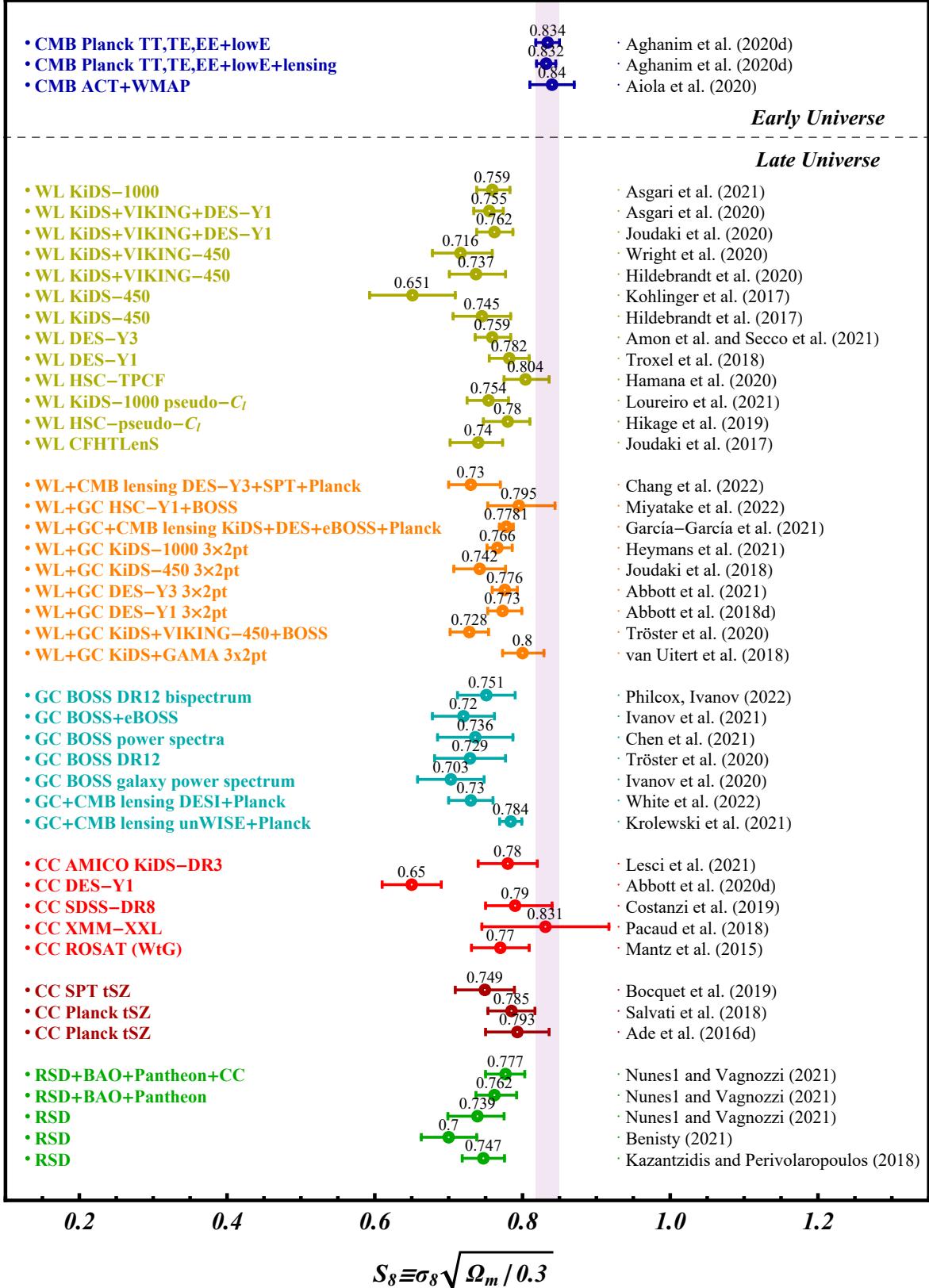
$$\mu_G(z, k) \equiv \frac{G_{eff}(z, k)}{G} \quad (3.7)$$

and lensing

$$\Sigma(z, k) \equiv \frac{G_L(z, k)}{G} \quad (3.8)$$

where  $G$  is the locally measured value of the Newton’s constant. According to  $\Lambda$ CDM  $H(z) = H(z, w = -1)$ ,  $\mu_G = 1$ ,  $\Sigma = 1$ .

The Bardeen potentials (Bardeen 1980) (the Newtonian potential  $\Psi$  and the spatial curvature potential  $\Phi$ ) appear in the scalar perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) metric in the conformal Newtonian gauge (Esposito-Farese and Polarski 2001; Ma and Bertschinger 1995; Mukhanov *et al.* 1992)

FIG. 21. The value of  $S_8$  with the 68% CL constraints derived by recent measurements.

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 - 2\Phi)d\vec{x}^2 \quad (3.9)$$

The LSS probes are sensitive to the Bardeen potentials  $\Psi$  and  $\Phi$ . In particular the WL probe is sensitive to  $\nabla^2(\Psi + \Phi)$ . The galaxy clustering arises from the gravitational attraction of matter and is sensitive only to the potential  $\Psi$ . The RSD probe is sensitive to the rate of growth of matter density perturbations  $f$  (see Eq. (3.3)) and provides measurements of  $f\sigma_8$  (see Eq. (3.6)) that depends on the potential  $\Psi$ .

At linear level, in modified gravity models, using the perturbed metric Eq. (3.9) and the gravitational field equations the following phenomenological equations in Fourier space emerge for the scalar perturbation potentials defining the functions  $\mu_G(a, k)$  and  $\Sigma(a, k)$  on sub-horizon scales (i.e.  $k/aH \gg 1$ )

$$k^2(\Psi + \Phi) = -8\pi G\Sigma(a, k)a^2\rho\Delta \quad (3.10)$$

$$k^2\Psi = -4\pi G\mu_G(a, k)a^2\rho\Delta \quad (3.11)$$

where  $\rho_m$  is the matter density of the background,  $\Delta$  the comoving matter density contrast defined as  $\Delta \equiv \delta + 3Ha(1+w)v/k$  which is gauge-invariant (Pogosian *et al.* 2010),  $w = p/\rho$  is the equation-of-state parameter and  $v^i = -\nabla^i u$  is the irrotational component of the peculiar velocity  $u$  (Boisseau *et al.* 2000).

Using the gravitational slip parameter  $\eta$  (or anisotropic stress parameter) which describes the possible inequality (Jain and Khouri 2010; Pogosian and Silvestri 2008) of the two Bardeen potentials that may occur in modified gravity theories

$$\eta(a, k) = \frac{\Phi(a, k)}{\Psi(a, k)} \quad (3.12)$$

the two LSS parameters  $\mu_G$  and  $\Sigma$  are related via

$$\Sigma(a, k) = \frac{1}{2}\mu_G(a, k)[1 + \eta(a, k)] \quad (3.13)$$

The the Hubble parameter  $H(z)$  is usually parametrized as wCDM

$$H(z) = H_0 \left[ \Omega_{0m}(1+z)^3 + (1-\Omega_{0m})(1+z)^{3(1+w)} \right]^{1/2} \quad (3.14)$$

while the two LSS parameters  $\mu_G$  and  $\Sigma$  do not have a commonly accepted parametrization. A model and scale independent parametrization for  $\mu_G$  and  $\Sigma$  which reduce to the GR value at early times and at the present time as indicated by solar system (ignoring possible screening effects) and BBN constraints ( $\mu_G = 1$  and  $\mu'_G = 0$  for  $a = 1$  and  $\mu_G = 1$  for  $a \ll 1$ ) (Gannouji *et al.* 2006; Muller *et al.* 2008; Pitjeva and Pitjev 2013) is of the form (Kazantzidis and Perivolaropoulos 2018; Nesseris *et al.* 2017; Nesseris and Perivolaropoulos 2007; Skara and Perivolaropoulos 2020)

$$\mu_G = 1 + g_a(1-a)^n - g_a(1-a)^{2n} = 1 + g_a\left(\frac{z}{1+z}\right)^n - g_a\left(\frac{z}{1+z}\right)^{2n} \quad (3.15)$$

$$\Sigma = 1 + g_b(1-a)^m - g_b(1-a)^{2m} = 1 + g_b\left(\frac{z}{1+z}\right)^m - g_b\left(\frac{z}{1+z}\right)^{2m} \quad (3.16)$$

where  $g_a$  and  $g_b$  are parameters to be fit and  $n$  and  $m$  are integer parameters with  $n \geq 2$  and  $m \geq 2$ .

Alternatively, a rapid transition parametrization is of the form (Alesta *et al.* 2021b; Marra and Perivolaropoulos 2021)

$$\mu_G^>(z) = \mu_G^< + \Delta\mu_G \Theta(z - z_t) \quad (3.17)$$

$$\Sigma^>(z) = \Sigma^< + \Delta\Sigma \Theta(z - z_t) \quad (3.18)$$

where  $\Theta$  is the Heaviside step function,  $z_t$  is a transition redshift,  $\mu_G^>$  and  $\Sigma^>$  correspond to  $z > z_t$  and  $\mu_G^<$  and  $\Sigma^<$  correspond to  $z < z_t$ .

Various studies utilize modified gravity theories including Teleparallel theories of gravity<sup>45</sup> (D’Agostino and

Luongo 2018; Gonzalez-Espinoza *et al.* 2018) (see Bahamonde *et al.* 2021b, for a review), Horndeski theories (Kennedy *et al.* 2018; Linder 2018) or theories beyond Horndeski (D’Amico *et al.* 2017) to reduce the effective Newton’s constant  $G_{eff}$  at low redshifts and slow down growth at low redshifts. The above parametrizations can be realized in the context of physical models based on the above theories.

<sup>45</sup> Many authors have studied the extensions of the Teleparallel

gravity such as the scalar-torsion theories of gravity (Geng *et al.* 2012, 2011; Gonzalez-Espinoza *et al.* 2021; Kofinas *et al.* 2015; Leon *et al.* 2022; Paliathanasis 2021; Skugoreva *et al.* 2015) and the Teleparallel Horndeski theories (Bahamonde *et al.* 2021a, 2020, 2019; Bernardo *et al.* 2021; Dialektopoulos *et al.* 2021).

### III.2 CMB anisotropy anomalies

There is a wide range of other less discussed no-standard signals and statistical anomalies of the large angle fluctuations in the CMB (Rassat *et al.* 2014) with a typical  $2$  to  $3\sigma$  significance. As mentioned a main assumption of the  $\Lambda$ CDM model is that the fluctuations are Gaussian and statistically homogeneous and isotropic. Diverse anomalies have been noticed in the CMB at large angular scales by the space missions Cosmic Background Explorer (COBE) (Mather *et al.* 1990), Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett *et al.* 2003b) and Planck satellite (Adam *et al.* 2016), which appear to violate this assumption (see Akrami *et al.* 2020b; Schwarz *et al.* 2016, for a review). Perivolaropoulos (2014) presents possible explanations of the observed CMB anomalies and Cayuso and Johnson (2020) explore the kinetic and the polarized Sunyaev-Zel'dovich effects as potential probes of physical models of these anomalies.

In what follows we discuss some of these signals. Note that some of these may not be independent<sup>46</sup>. Some of these signals have been attributed to the look-elsewhere effect. Based on this effect any large dataset will have a small number of peculiar features when there is a careful search for such features. However, this argument may not be applicable when the considered statistics are simple and generic as are most of the signals discussed below (see Bayer and Seljak 2020; Gross and Vitells 2010; Peiris 2014, for a detailed discussion).

#### III.2.1 Hints for a closed Universe (CMB vs BAO)

The Universe under the assumption of the cosmological principle is described by the Friedmann-Lemaître-Roberson-Walker (FLRW) metric

$$ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (3.19)$$

where  $K$  characterizes the constant spatial curvature of the spatial slices with  $K = -1, 0, +1$  corresponding to open hyperbolic space (negative spatial curvature), flat Euclidean space (zero spatial curvature), and closed hyperspherical space (positive spatial curvature) respectively. The curvature density parameter is defined as  $\Omega_K \equiv -K/(Ha)^2$  so that a closed Universe corresponds to  $\Omega_K < 0$  and an open Universe to  $\Omega_K > 0$ . This parameter plays a crucial role in determining the evolution of the Universe, and is closely related with the early Universe physics.

<sup>46</sup> The covariance of CMB anomalies in the standard  $\Lambda$ CDM model has been studied by Muir *et al.* (2018). This study focusing on the correlation of observed anomalies (i.e. the relationship or connection between all of them) examines the independence of large-angle CMB feature quantities.

The Planck18 temperature and polarization data (Aghanim *et al.* 2020e) show a preference ( $\sim 3.4\sigma$ ) for a closed Universe ( $\Omega_K < 0$ ) in the context of  $\Lambda$ CDM. In particular using these data from Planck18 the curvature density parameter was constrained to be  $-0.095 < \Omega_K < -0.007$  at 99 % C.L (Aghanim *et al.* 2020d,e). This anomaly may be connected with other asymmetries of the CMB anisotropy spectrum discussed below. The preference for closed universe however disappears when the CMB data are combined with the BAO data. Di Valentino *et al.* (2019b, 2021h); Handley (2019) pointed out that Planck+BAO can give a biased result because Planck and BAO are in disagreement at more than  $3\sigma$ . Combining Planck18 data with recent BAO measurements the curvature density parameter was estimated to be  $\Omega_K = 0.0008 \pm 0.0019$  (Di Valentino *et al.* 2019b, 2021h; Handley 2019) in agreement with a spatially flat Universe. Using the full-shape galaxy power spectrum measurements  $P(k)$ , Vagnozzi *et al.* (2021a) has also confirmed that the Planck data are in tension with both the full-shape power spectrum and BAO with respect to  $\Omega_K$ . The recent study by Efstathiou and Gratton (2020) confirms the tension between Planck and BAO data in the context of cosmic curvature. Efstathiou and Gratton (2020) used a new statistical analysis (the alternative Planck CamSpec likelihood TTTEEE instead of Plik as discussed in Efstathiou and Gratton (2019)) to show that Planck favors a closed Universe at more than 99% CL. However, Planck+BAO was again found to be in agreement with a spatially flat Universe with  $\Omega_K = 0.0004 \pm 0.0018$  thus confirming previous studies by Di Valentino *et al.* (2019b); Handley (2019) .

In an effort to further investigate this tension between Planck and BAO data, the analysis of Vagnozzi *et al.* (2021b) combined Planck18 CMB temperature and polarization data with cosmic chronometer measurements and was lead to confirm that the Universe is consistent with spatial flatness to  $\mathcal{O}(10^{-2})$  level.

A positive curvature (closed Universe) may be a plausible source of the anomalous lensing amplitude (Di Valentino *et al.* 2019b, 2021h; Handley 2019) (see Subsection III.2.8).

#### III.2.2 Anomalously strong ISW effect

The decay of cosmological large-scale gravitational potential  $\Psi$  causes the integrated Sachs-Wolfe (ISW) effect (Sachs and Wolfe 1967) which imprints tiny secondary anisotropies to the primary fluctuations of the CMB and is a complementary probe of dark energy (Fosalba and Gaztanaga 2004; Giannantonio *et al.* 2008; Kable *et al.* 2021; Kofman and Starobinsky 1985). Using a stacking technique in the CMB data (see Ade *et al.* 2016c; Marcos-Caballero *et al.* 2016, for a detailed discussion) anomalously strong integrated Sachs–Wolfe (ISW) signal ( $> 3\sigma$ ) has been detected for supervoids and super-clusters on scales larger than  $100h^{-1} Mpc$  (Granett *et al.* 2008a,b). This stronger than expected within standard

$\Lambda$ CDM signal of the ISW effect first emphasised in Hunt and Sarkar (2010) has been studied by Cai *et al.* (2017c); Dong *et al.* (2020); Flender *et al.* (2013); Ilic *et al.* (2013); Kovács (2018); Kovács *et al.* (2019, 2022); Nadathur *et al.* (2012).

In particular the analysis by Kovács *et al.* (2019) for DES data alone found an excess ISW imprinted profile with  $A_{ISW} \equiv \Delta T^{data}/\Delta T^{theory} \approx 4.1 \pm 2.0$  amplitude (where  $A_{ISW} = 1$  corresponds to the  $\Lambda$ CDM prediction). Also a combination with independent BOSS data leads to  $A_{ISW} = 5.2 \pm 1.6$ . This is in  $2.6\sigma$  tension with  $\Lambda$ CDM cosmology.

The average expansion rate approximation (AvERA) inhomogeneous cosmological simulation (Rácz *et al.* 2017) uses the separate Universe conjecture to calculates the spatial average of the expansion rate of local mini-Universes predicts. It indicates under the inhomogeneity assumption, about  $\sim 2 - 5$  times higher ISW effect than  $\Lambda$ CDM depending on the  $l$  index of the spherical power spectrum (Beck *et al.* 2018). Thus large scale spatial inhomogeneities could provide an explanation to this ISW excess signal. Giannantonio *et al.* (2012) use angular cross-correlation techniques and combines several tracer catalogues to report  $A_{ISW} \approx 1.38 \pm 0.32$ .

Vagnozzi (2021) investigated the early Integrated Sachs-Wolfe (eISW) effect (see e.g. Bowen *et al.* 2002; Galli *et al.* 2010) which is assumed to occur soon after recombination ( $30 < z < 1100$ ), due to the presence of a non-negligible radiation. Constraints were thus imposed on the parameter  $A_{eISW}$  introduced by Hou *et al.* (2013). Using Planck CMB data, this parameter was constrained to  $A_{eISW} = 0.988 \pm 0.027$ , in perfect agreement with  $\Lambda$ CDM. Note that in previous studies the parameter  $A_{eISW}$  was constrained to  $A_{eISW} = 0.979 \pm 0.055$  using data from WMAP7+SPT (Hou *et al.* 2013), to  $A_{eISW} = 1.06 \pm 0.04$  from the Planck 2015 data release (Cabass *et al.* 2015), and to  $A_{eISW} = 1.064 \pm 0.042$  from the Planck 2018 temperature data alone (Kable *et al.* 2020).

In general the reported  $A_{ISW}$  amplitude varies in the literature depending on the dataset and the assumptions of the analysis. Further investigation of this issue is needed.

### III.2.3 CMB cold spot

The cold (blue) spot was first found in WMAP 1-year temperature data by Vielva *et al.* (2004) and was confirmed in Planck data (Ade *et al.* 2016c, 2014b; Akrami *et al.* 2020b) in the southern hemisphere at the galactic longitude and latitude  $(l, b) = (209^\circ, -57^\circ)$ . It is a statistical anomaly of the large-angle fluctuations in the CMB indicating non-Gaussian features. This inconsistency with Gaussian simulations has a p-value of  $\sim 1\%$ .

The cold spot is an unusually large region of low temperature with the mean temperature decrement  $\Delta T \approx -100 \mu K$  and is not consistent with the prediction of gaussianity of the standard  $\Lambda$ CDM model (Cruz *et al.*

2007, 2005, 2006).

Kovács (2018); Nadathur *et al.* (2014); Zhang and Huterer (2010) pointed out that the anomalous nature of the cold spot corresponds to a rather cold area with an angular radius in the sky of about  $5^\circ - 10^\circ$  from the centre surrounded by a hot ring.

Possible approaches for the explanation of the Cold Spot include: non-Gaussian feature due to a large statistical fluctuation (Vielva *et al.* 2004), an artifact of inflation (Cruz *et al.* 2005), the foreground (Cruz *et al.* 2006; Hansen *et al.* 2012), multiple voids (Naidoo *et al.* 2016), the imprint of a supervoid (about  $140 - 200 Mpc$  radius completely empty void at  $z \leq 1$ ) through the ISW effect (Granett *et al.* 2008a; Inoue and Silk 2006, 2007; Rudnick *et al.* 2007), the axis of rotation of the Universe (Jaffe *et al.* 2005), cosmic texture (Cruz *et al.* 2005; Zhao 2013), adiabatic perturbation on the last scattering surface (Valkenburg 2012) (see Cruz *et al.* 2009; Vielva 2010, for a review).

### III.2.4 Cosmic hemispherical power asymmetry

The cosmic hemispherical power asymmetry (or dipolar asymmetry) is a directional dependency of the CMB angular power spectrum (Eriksen *et al.* 2004, 2007; Hansen *et al.* 2004; Paci *et al.* 2010). The continuous dipolar modulation of hemispherical power asymmetry corresponds to a hemispherical temperature variance asymmetry (signal in the CMB temperature field) (Ade *et al.* 2016c; Akrami *et al.* 2014, 2020b; Bernui *et al.* 2014; Eriksen *et al.* 2004; Hansen *et al.* 2004; Hoftuft *et al.* 2009; Monteserin *et al.* 2008; O'Dwyer *et al.* 2019).

The dipolar modulated/observed CMB temperature fluctuation  $\frac{\Delta T}{T}|_{mod}$  in the direction  $\hat{n}$  which appears to extend to  $l_{max} \simeq 64$  can be expressed as (Akrami *et al.* 2014; Bennett *et al.* 2011; Gordon 2007)<sup>47</sup>

$$\frac{\Delta T}{T}|_{mod}(\hat{n}) = [1 + A_{dm}\hat{n} \cdot \hat{p}] \frac{\Delta T}{T}|_{iso}(\hat{n}) \quad (3.20)$$

where  $\frac{\Delta T}{T}|_{iso}$  is a statistically unmodulated/isotropic temperature fluctuation,  $A_{dm}$  denotes the amplitude of dipolar modulation and  $\hat{n} \cdot \hat{p}$  corresponds to the dipolar modulation between the line-of-sight (LOS) of the observer (with unit vector  $\hat{n}$ ) and the preferred dipolar direction (with unit vector  $\hat{p}$ ). The amplitude of dipolar modulation  $A_{dm}$  is large at large angular scales  $2 < l \lesssim 64$  ( $k \lesssim 0.035 Mpc^{-1}$ ), small at small angular scales  $l \gtrsim 64$  and vanishes by a multipole moment of  $\sim 500 - 600$  (Ade *et al.* 2016c, 2014b; Akrami *et al.* 2020b). The scale dependence of the hemispherical power asymmetry was suggested by Ashoorioon and Koivisto

<sup>47</sup> Note that the hemispherical dipole is distinct from the usual CMB dipole. In the former case the *power spectrum* is assumed modulated discontinuously across a circle on the sky and in the second the actual temperature map has a component modulated by a smooth cosine function across the sky (Bennett *et al.* 2011).

(2016); Byrnes *et al.* (2016a,b,c); Erickcek *et al.* (2008b); Jazayeri *et al.* (2017); Liddle and Cortès (2013); Lyth (2015); Mukherjee and Souradeep (2016); Wang *et al.* (2016b); Yang *et al.* (2017a) and was investigated by Li *et al.* (2019a); Shiraishi *et al.* (2016).

According to the hemispherical asymmetry nearly aligned with the Ecliptic, the temperature fluctuations are larger on one side of the CMB sky than on the other, resulting in an unexpected dipole configuration in the CMB power spectrum with an anomalously lower value of the variance in the northern sky compared to the southern sky (Ade *et al.* 2014b). The preferred direction for the asymmetry from the Planck18 data is  $(l, b) = (221^0, -20^0)$  in galactic coordinates and the amplitude is  $A_{dm} \sim 0.07$  with statistically significant at the  $\sim 3\sigma$  level (Akrami *et al.* 2020b). This amplitude is  $\sim 2$  times higher than expected asymmetry due to cosmic variance ( $A_{dm} \sim 0.03$ ) and it is inconsistent with isotropy ( $A_{dm} = 0$ ) at the  $\sim 3\sigma$  level. The hemispherical power asymmetry in CMB can be explained by assuming a superhorizon perturbation (Gordon 2007; Gordon *et al.* 2005) or asymmetric initial states of the quantum perturbations (Ashoorioon and Koivisto 2016).

### III.2.5 Quadrupole-octopole alignment

The fluctuations in the standard  $\Lambda$ CDM model are Gaussian and statistically isotropic. Thus in harmonic space the quadrupole ( $l = 2$ ) and octopole ( $l = 3$ ) harmonics are expected to have independent and random orientations and shapes. The quadrupole and octopole have been observed to be planar and unexpectedly aligned with each other (Copi *et al.* 2007, 2006, 2010, 2015b; de Oliveira-Costa *et al.* 2004; Schwarz *et al.* 2004). This implies a violation of statistical isotropy.

In particular in this low multipole moment anomaly the quadrupole and octopole planes are found to be mutually aligned with the direction of the cosmic dipole or CMB dipole (see Subsection III.3 and Table III) and perpendicular to the Ecliptic (Schwarz *et al.* 2016).

In order to study this large-angle anomaly one can use the maximum angular momentum dispersion (de Oliveira-Costa *et al.* 2004)

$$\langle \psi | (\hat{\mathbf{n}}_l \cdot \mathbf{L})^2 | \psi \rangle = \sum_{m=-l}^l m^2 |a_{lm}(\hat{\mathbf{n}}_l)|^2 \quad (3.21)$$

where the CMB map is represented by a wave function

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}_l) \equiv \psi(\hat{\mathbf{n}}_l) \quad (3.22)$$

Here  $a_{lm}(\hat{\mathbf{n}}_l)$  correspond to the spherical harmonic coefficients of the CMB map in a coordinate system with its  $z$ -axis in the  $\hat{\mathbf{n}}_l$ -direction.

The preferred axis  $\hat{\mathbf{n}}_l$  is the axis around which the angular momentum dispersion is maximized. The directions of the quadrupole  $\hat{\mathbf{n}}_2$  and the octopole  $\hat{\mathbf{n}}_3$  are

(de Oliveira-Costa *et al.* 2004)

$$\hat{\mathbf{n}}_2 = (-0.1145, -0.5265, 0.8424) \quad (3.23)$$

$$\hat{\mathbf{n}}_3 = (-0.2578, -0.4207, 0.8698) \quad (3.24)$$

with

$$|\hat{\mathbf{n}}_2 \cdot \hat{\mathbf{n}}_3| \simeq 0.9838 \quad (3.25)$$

This unexpected alignment of the  $\hat{\mathbf{n}}_2$  and  $\hat{\mathbf{n}}_3$  directions has only a  $1/62$  probability of happening.

An approach in the analysis of this large-angle anomaly may also involve the use the multipole vectors (Copi *et al.* 2004) (an alternative to the spherical harmonics) where each multipole order  $l$  is represented by  $l$  unit vectors i.e a dipole  $l = 1$  can be constructed by a vector, a quadrupole by the product of two vectors/dipoles, an octopole from three vectors/dipoles etc.

The alignment of low multipoles indicates the existence of a preferred direction in the CMB temperature anisotropy. Furthermore possible relation between the quadrupole-octopole alignment and the dipolar asymmetry has been investigated by Gordon (2007); Hoftuft *et al.* (2009). A negligible relation between these anomalies was reported. However the analysis by Marcos-Caballero and Martínez-González (2019) has shown that a particular dipolar modulation including the scale dependence may be connected with the quadrupole-octopole alignment.

### III.2.6 Lack of large-angle CMB temperature correlations

There is a lack of large-angle CMB temperature correlations as first was observed by COBE satellite (Hinshaw *et al.* 1996) and was confirmed by the WMAP (Bennett *et al.* 2003a; Spergel *et al.* 2003) and Planck (Ade *et al.* 2016c; Akrami *et al.* 2020b) temperature maps in the range  $l = 2$  to 32. This is in tension with the  $\Lambda$ CDM prediction.

This anomaly is directly connected to the temperature  $T$  two-point angular correlation function  $C^{TT}(\theta)$  of the CMB at large angular scale ( $\theta \gtrsim 60^0$ ) which is unexpectedly close to zero (Copi *et al.* 2009, 2010, 2015a). In angular space the two-point angular correlation function is defined as

$$C^{TT}(\theta) \equiv \langle T(\hat{\mathbf{n}}_1)T(\hat{\mathbf{n}}_2) \rangle = \frac{1}{4\pi} \sum_l (2l+1) C_l P_l(\cos \theta) \quad (3.26)$$

where the average is over all pairs of directions  $\hat{\mathbf{n}}$  with  $\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2 = \cos \theta$ ,  $P_l(\cos \theta)$  are the Legendre polynomials and  $C_l$  is the angular power spectrum

$$C_l \equiv \frac{1}{2l+1} \sum_{m=-l}^l |a_{lm}|^2 \quad (3.27)$$

with  $a_{lm}$  the spherical harmonic coefficients of the temperature fluctuations.

The simplest and most useful statistic is  $S_{1/2}$  first introduced in the WMAP first-year release (Spergel *et al.* 2003) in order to measure the deviation of the angular correlation function from zero at angular scales  $60^\circ < \theta < 180^\circ$ . It is defined as

$$S_{1/2} = \int_{\mu_2}^{\mu_1} [C^{TT}(\theta)]^2 d(\cos \theta) \quad (3.28)$$

with  $\mu_1 \equiv \cos \theta_1 = \cos 60^\circ = 1/2$  and  $\mu_2 \equiv \cos \theta_2 = \cos 180^\circ = -1$ .

A number of alternative statistics have been proposed in the literature (Akrami *et al.* 2020b; Efstathiou *et al.* 2010; Gruppuso 2014; Hajian 2007). For example a generalization of the  $S_{1/2}$  statistic suggested by Copi *et al.* (2013). This statistic known as  $S^{TQ}$  uses the two-point angular correlation function between fluctuations in the temperature  $T$  and the Stokes parameter<sup>48</sup>  $Q$ ,  $C^{TQ}(\theta)$ , which can be expressed in terms of the two-point angular power spectrum,  $C_l^{TE}$  (with  $E$  the gradient mode of polarization). The significance of a test statistic can be quantified by using the p-value<sup>49</sup>, suggested by Ade *et al.* (2014b).

No sufficient explanation has yet been suggested for this large-angle anomaly. Copi *et al.* (2016) study the ISW effect, Aurich *et al.* (2021); Bernui *et al.* (2018) explore a non-trivial spatial topology of the Universe and Pranav *et al.* (2019) study the topology of the Planck CMB temperature fluctuations in order to find a possible explanation to the suppression of large-angle CMB temperature correlations. Also the low observed power in the quadrupole is a potential explanation for the lack of correlation in the temperature maps. Copi *et al.* (2009) argue that there is a cancellation between the combined contributions of  $C_l$  with multipoles  $l \leq 5$  and the contributions of  $C_l$  with multipoles  $l \geq 6$ .

### III.2.7 Anomaly on super-horizon scales

Pranav (2021) analysed the topological characteristics of the CMB temperature fluctuation. Using mathematical investigations on persistent homology to describe the cosmic mass distribution and performing experiments on Planck 2020 data release 4 (DR4) (based on the NPIPE

<sup>48</sup> The Stokes parameters Q and U (for the Stokes parameters formalism see Jackson 1998) are used to describe the state of CMB polarization (e.g. Dodelson 2003; Zaldarriaga and Seljak 1997). These parameters are directly related to the E and B modes (Bunn *et al.* 2003; Kamionkowski *et al.* 1997; Lewis *et al.* 2002).

The polarization amplitude is given by  $P = \sqrt{Q^2 + U^2}$ .

<sup>49</sup> The probability value or p-value is the probability of measuring a test statistic equal to or more extreme as the observed one, considering that the null hypothesis is correct (Ade *et al.* 2014b). It provides the lower value of significance at which the model would be ruled out. A low p-value means that there is strong indication of new physics beyond the null hypothesis.

data processing pipeline Akrami *et al.* 2020d), Pranav (2021) claimed a detection of an anomalous topological signature in the Planck CMB maps indicating non-Gaussian fluctuations. In particular Pranav (2021) reports an anomaly in the behavior of the loops (a  $4\sigma$  deviation in the number of loops) in the observed sky compared to the analysis of the redshift evolution of structure on simulations when the  $\Lambda$ CDM model is considered.

### III.2.8 The lensing anomaly

The recent Planck18 release by Aghanim *et al.* (2020e) has confirmed the higher compared to that expected in the standard  $\Lambda$ CDM model, anomalous, lensing contribution in the CMB power spectra which is quantified by the phenomenological parameter,  $A_L$  (Calabrese *et al.* 2008; Zaldarriaga and Seljak 1998). This weak lensing parameter  $A_L$  rescales the lensing potential power spectrum as<sup>50</sup>

$$C_l^\Psi \rightarrow A_L C_l^\Psi \quad (3.29)$$

where  $A_L = 0$  corresponds to unlensed while  $A_L = 1$  is the expected lensed result (Calabrese *et al.* 2008) measuring the lensing effect in the CMB temperature power spectrum.

Since the main impacts of lensing on the CMB temperature power spectrum are to add power at small scales and to smooth the structure of the acoustic peaks and troughs (the peaks are reduced slightly, and the troughs between them filled in) (Ade *et al.* 2016b; Lewis and Challinor 2006) the adding of parameter  $A_L$  changes the amount of smoothing of the CMB primary spectra peaks and troughs. A higher lensing amplitude ( $A_L > 1$ ) than predicted in the flat  $\Lambda$ CDM cosmology ( $A_L = 1$ ) by roughly 10% (at the level of  $2.8\sigma$ ) has been found in the temperature power spectra by the Planck team (Aghanim *et al.* 2020e).

It should be noted that the oscillatory residuals between the Planck temperature power spectra and the best-fit  $\Lambda$ CDM model in the multipole range  $l \in [900, 1700]$  are in opposite phase compared to the CMB and thus phenomenologically similar to the effects of gravitational lensing (Aghanim *et al.* 2017; Motloch and Hu 2020).

A plausible explanation of the anomalous lensing amplitude is a positive curvature (closed Universe) which was investigated by Di Valentino *et al.* (2019b, 2021b); Handley (2019). Other possible sources which explain the lensing anomaly by mimicking a lensing effect are: a component of cold dark matter isocurvature (CDI) perturbation with a blue tilt (see Akrami *et al.* 2020c, for a detailed discussion) and oscillations in the primordial power spectrum which have the same frequency but opposite phase with the acoustic peaks (Aghanim *et al.*

<sup>50</sup> Note that this is not the usual  $C_l$  but it is the additional contribution due to lensing.

2020e). All these effects are degenerate with the smoothing effect of lensing.

Furthermore, the modified gravity models could be candidates for a solution of the lensing anomaly (Ade *et al.* 2016e; Di Valentino *et al.* 2020a, 2016a; Moshafi *et al.* 2021). In particular the hints for  $\Sigma_0 > 1$  (where  $\Sigma_0$  the current value of parameter  $\Sigma$  which modifies the equation for the lensing potential i.e. Eq. (3.10)) are directly connected to the lensing anomaly as characterized by  $A_L > 1$  (Di Valentino *et al.* 2020a, 2016a).

### III.2.9 High-low 1 consistency

Addison *et al.* (2016) pointed out that there are internal inconsistencies in the Planck TT power spectrum. The  $\Lambda$ CDM parameter values derived by the high  $l$  part of the CMB anisotropy spectrum ( $l > 1000$ ) are in  $2 - 3\sigma$  tension with the corresponding values of these parameters derived from the low  $l$  part of the spectrum ( $1 < 1000$ ). For example the low  $l$  multipoles predict a lower value of the cold dark matter density parameter  $\omega_c$  than the high  $l$  multipoles, with discrepancy at  $2.5\sigma$  (Addison *et al.* 2016). In addition it has been shown that the value of  $H_0$  predicted by Planck from  $l > 1000$ ,  $H_0 = 64.1 \pm 1.7 \text{ km s}^{-1} \text{Mpc}^{-1}$ , disagrees with the value predicted by Planck from  $l < 1000$ ,  $H_0 = 69.7 \pm 1.7 \text{ km s}^{-1} \text{Mpc}^{-1}$  at the  $2.3\sigma$  level. Thus it is found that the value of  $H_0$  depends on the CMB  $l$ -range examined.

This anomaly is probably related to the lensing anomaly i.e. the fact that  $\Lambda$ CDM is more consistent with the low  $l$  part of the spectrum than this not affected by the lensing anomaly (see Aghanim *et al.* 2017, 2020d; Riess *et al.* 2016, for a discussion).

### III.2.10 The preference for odd parity correlations

There is an anomalous power excess (deficit) of odd (even)  $l$  multipoles in the CMB anisotropy spectrum on the largest angular scales ( $2 < l < 30$ ), (Akrami *et al.* 2020b; Gruppuso *et al.* 2011, 2018; Kim and Naselsky 2010a,b, 2011; Land and Magueijo 2005). A map consisting of odd (even) multipoles possesses odd (even) parity thus this effect may be considered as power (spectrum) asymmetry between even and odd parity map which is known as parity asymmetry.

In order to compare even and odd multipoles Kim and Naselsky (2010a) consider the parity asymmetry statistic defined as the ratio  $P \equiv P^+/P^-$  of quantities  $P^+$  and  $P^-$  which represent the mean power in even and odd only multipoles respectively for the range  $2 \leq l \leq l_{max}$

$$P^\pm = \sum_2^{l_{max}} \frac{[1 \pm (-1)^l] l(l+1) C_l}{4\pi} \quad (3.30)$$

A different statistic to quantify the parity asymmetry has been proposed by Aluri and Jain (2012).

## III.3 Cosmic dipoles

There have been studies pointing out the presence of signals which indicate the violation of the cosmological principle. A physical mechanism producing such violation on Hubble scales is studied by Bueno Sanchez and Perivolaropoulos (2011). Various other possible mechanisms have been suggested to explain the observed violations of statistical isotropy e.g. superhorizon perturbations which introduce a preferred direction in our Universe (Erickcek *et al.* 2008a; Gordon *et al.* 2005) (see also Perivolaropoulos 2014, for a review). The dipole amplitudes and the directions ( $l, b$ ) (galactic coordinates) from the different cosmological observations described below are shown in Fig. 22 and along with the corresponding references in Table III.

The physical origin of these dipoles is described in the following subsections.

### III.3.1 Velocity radio dipole

A large scale velocity flow dipole<sup>51</sup> was pointed out in Kashlinsky *et al.* (2009); Watkins *et al.* (2009). The dipole moment of the peculiar velocity field (dipole bulk flow) which is a sensitive probe of the amplitude and growth rate of fluctuations on large scales (Koda *et al.* 2014) was investigated by Feldman *et al.* (2010); Hong *et al.* (2014); Kashlinsky *et al.* (2012, 2010, 2009); Ma and Scott (2013); Nusser and Davis (2011); Turnbull *et al.* (2012); Watkins *et al.* (2009). In many cases the results are controversial and there is a debate in the literature on the consistency with the  $\Lambda$ CDM model.

A recent detailed analysis has indicated that 'tilted observers' within the bulk flow can be misled into inferring acceleration (Tsagas 2011; Tsagas and Kadiltzoglou 2015; Tsagas *et al.* 2021) (see also Asvesta *et al.* 2022, for observational constraints of the deceleration parameter in a tilted universe).

Colin *et al.* (2019) use SNIa JLA data to demonstrate that the indications for cosmic acceleration found in the SNIa data disappears if a bulk flow induced anisotropy is allowed in the SNIa data. Thus, a bulk flow dipole (at  $3.9\sigma$ ) aligned with the local bulk flow is identified while any monopole (which can be attributed to  $\Lambda$ ) is consistent with zero (at  $1.4\sigma$ ).<sup>52</sup>

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<sup>51</sup> For peculiar velocities as variation of the Hubble expansion produced by nearby nonlinear structures see in Wiltshire *et al.* (2013) and for dipole anisotropy in radio source count, see in Bengaly *et al.* (2018).

<sup>52</sup> A recent model independent analysis of SNIa data (Pantheon) data Arjona and Nesseris (2020) implementing machine learning has confirmed a  $\sim 4.5\sigma$  detection of the accelerated expansion even though that analysis did not allow for anisotropic dipole effects.

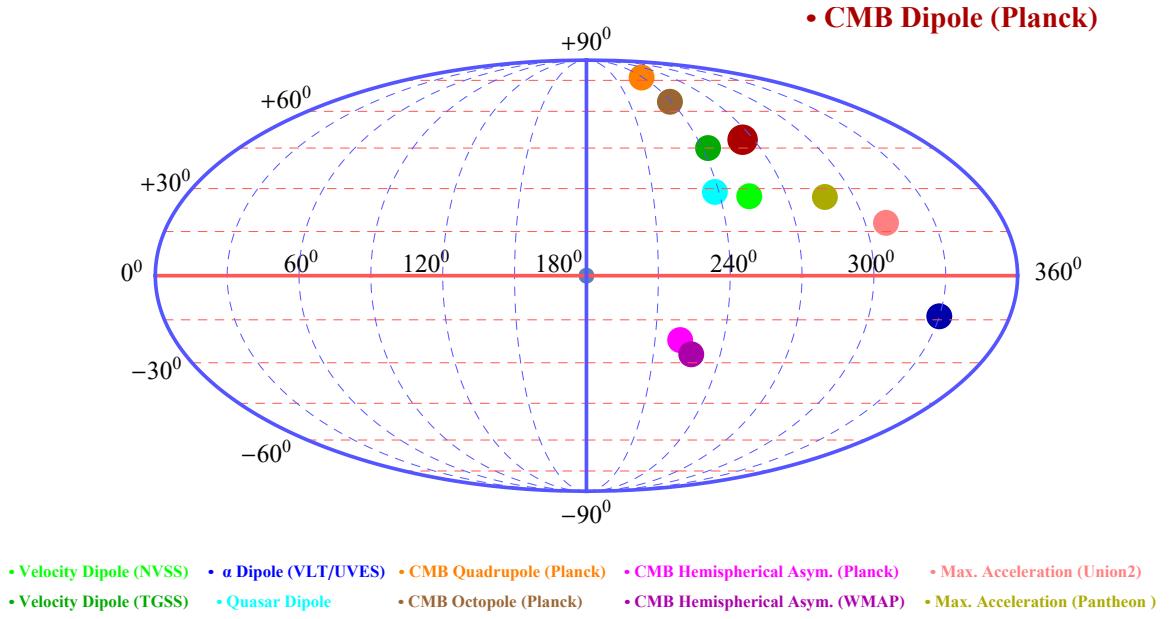


FIG. 22. Mollweide-projection view of preferred directions in galactic coordinates for different cosmological observations (see Table III).

It is usually assumed that our local (solar system) peculiar motion with respect to the CMB rest frame produces the CMB dipole anisotropy ( $l = 1$ ) (Fixsen *et al.* 1996; Kogut *et al.* 1993) (also known as solar dipole Aghanim *et al.* 2016, 2020c). In the standard model, this implies that the LSS distribution should have a similar kinematic dipole known as the velocity dipole or radio dipole which arises from the Doppler boosting of the CMB monopole and from special relativistic aberration effects (Itoh *et al.* 2010).

In order to describe the origin of this dipole, a population of sources with power-law spectra depending on frequency  $\nu$  is usually assumed

$$S_\nu \propto \nu^{-\alpha} \quad (3.31)$$

where  $S_\nu$  is the flux density and  $\alpha^{53}$  is an individual spectral index with typically assumed value  $\alpha \sim 0.75$  (Bacon *et al.* 2020). The integral source counts per unit solid angle above some limiting flux density  $S_\nu$  can be approximated by a power law

$$\frac{dN}{d\Omega}(> S) \propto S_\nu^{-x} \quad (3.32)$$

where  $x \sim 1$  and can be different for each survey. An observer moving with velocity  $v \ll c$  with respect to the frame in which these sources are isotropically distributed sees a dipole anisotropy  $1 + D \cos \theta$  over the sky with amplitude (Ellis and Baldwin 1984)

$$D = [2 + x(1 + \alpha)] \frac{v}{c} \quad (3.33)$$

According to the most recent measurements the inferred velocity of the Sun relative to the CMB rest frame is (Aghanim *et al.* 2020b,c)

$$\beta \equiv \frac{v}{c} = (1.23357 \pm 0.00036) \times 10^{-3} \quad (3.34)$$

$$\text{or } v = 369.82 \pm 0.11 \text{ km s}^{-1} \quad (3.35)$$

along the direction with galactic longitude and latitude  $(l, b) = (264.021^\circ \pm 0.011^\circ, 48.253^\circ \pm 0.005^\circ)$  or  $RA \sim 168^\circ$ ,  $Dec \sim -7^\circ$  (Aghanim *et al.* 2020b,c).

The CMB rest frame is conventionally taken to correspond to the standard of cosmic rest frame and is assumed to be statistically homogeneous and isotropic in the context of the FLRW model. In this rest frame the Hubble flow should be most uniform (minimum Hubble variation frame) and the comoving observers should not see a kinematic dipole. However it has been observed (McKay and Wiltshire 2016; Wiltshire *et al.* 2013) that

<sup>53</sup> The individual spectral index  $\alpha$  should not be confused with the fine-structure constant  $\alpha$ .

TABLE III. The amplitudes and the directions ( $l, b$ ) (galactic coordinates) from different cosmological observations (Fig. 22) along with the corresponding references . The amplitude of CMB dipole has derived using the Eq. (3.33) (e.g. Secrest *et al.* 2021).

Observations	$l$ [deg]	$b$ [deg]	Amplitude	Refs.
CMB Dipole (Planck)	$264.021 \pm 0.011$	$48.253 \pm 0.005$	$\sim 0.007$	(Aghanim <i>et al.</i> 2020b,c)
Velocity Radio Dipole (TGSS)	$243.00 \pm 12.00$	$45.00 \pm 3.00$	$0.070 \pm 0.004$	(Bengaly <i>et al.</i> 2018)
Velocity Radio Dipole (NVSS)	$253.12 \pm 11.00$	$27.28 \pm 3.00$	$0.023 \pm 0.004$	(Bengaly <i>et al.</i> 2018)
Velocity Radio Dipole (NVSS)	$253.00 \pm 2.00$	$28.71 \pm 12.00$	$0.019 \pm 0.002$	(Colin <i>et al.</i> 2017)
Velocity Radio Dipole (NVSS)	253.00	$32.00 \pm 12.00$	$0.012 \pm 0.005$	(Tiwari and Nusser 2016)
Quasar Dipole	238.20	28.80	0.01554	(Secrest <i>et al.</i> 2021)
$\alpha$ Dipole (VLT/UVES)	$330 \pm 15$	$-13 \pm 10$	$0.97^{+0.22}_{-0.20} \times 10^{-5}$	(King <i>et al.</i> 2012)
CMB Quadrupole (Planck SMICA <sup>a</sup> )	238.5	76.6		(Ade <i>et al.</i> 2014b)
CMB Octopole (Planck SMICA)	239.0	64.3		(Ade <i>et al.</i> 2014b)
CMB Hemispher. Asym. (Planck)	221	-22	0.07	(Akrami <i>et al.</i> 2020b)
CMB Hemispher. Asym. (WMAP)	227	-27	0.07	(Axelsson <i>et al.</i> 2013)
Maximum Acceleration (Pantheon)	$286.93 \pm 18.52$	$27.02 \pm 6.50$	$0.0018 \pm 0.0002$	(Kazantzidis and Perivolaropoulos 2020)
Maximum Acceleration (Union2)	$309^{+23}_{-3}$	$18^{+11}_{-10}$		(Antoniou and Perivolaropoulos 2010)

<sup>a</sup> Spectral Matching Independent Component Analysis (SMICA) is a method to extract the CMB component (Akrami *et al.* 2020a) described in Cardoso *et al.* (2008).

the dipole structure of the velocity field is less in the reference frame of the Local Group of galaxies than in the CMB frame. This persistence of the dipole structure of the velocity flow in the CMB frame at large distances is not unexpected if we are located in an underdensity (Kraljic and Sarkar 2016).

According to the standard model if the Universe is isotropic our velocity with respect to the CMB rest frame and our velocity relative to the LSS should be identical. However, as was first noted by Singal (2011), while the direction of the radio dipole is consistent with that of the CMB, the velocity of our local motion obtained from the radio dipole exceeds that obtained from the CMB dipole. Radio continuum surveys which sample the Universe at intermediate redshifts ( $z \sim 1$ ) have been used as an excellent probe to large scale isotropy and a discrepancy between the predicted and measured amplitudes of the velocity have been revealed (Bengaly *et al.* 2018; Colin *et al.* 2017; Gibelyou and Huterer 2012; Rubart and Schwarz 2013; Tiwari *et al.* 2014; Tiwari and Nusser 2016). In particular the analysis by Bengaly *et al.* (2018) has shown that the radio dipole using the sky distribution of radio sources from the NRAO VLA Sky Survey (NVSS) dataset (Condon *et al.* 1998) and TIFR GMRT Sky Survey (TGSS) dataset (Bengaly *et al.* 2018; Intema *et al.* 2017; Singal 2019a) is  $\sim 2$  and  $\sim 5$  times larger than predicted by the mock realisations within the context of  $\Lambda$ CDM cosmology respectively. The above observed discrepancy between the radio and CMB dipoles has been confirmed by independent groups and could imply the existence of an anisotropic Universe.

Possible explanations of the violation of statistical isotropy are: systematics due to the incomplete sky cov-

erage of the radio continuum surveys (Blake and Wall 2002; Gibelyou and Huterer 2012; Tiwari and Nusser 2016), intrinsic dipole in the local LSS (Fernández-Cobos *et al.* 2014), nearby nonlinear structures of voids and walls and filaments (Wiltshire *et al.* 2013), remnant of the pre-inflationary Universe (Turner 1991) and superhorizon perturbation (Das *et al.* 2021a; Ghosh 2014).

### III.3.2 Quasar dipole

The distribution of quasars across the sky may provide independent probe of the cosmological principle (Singal 2019b). As previously discussed there is an expected anisotropy related to the CMB dipole anisotropy (about 1 part in 1000) due to our motion with respect to the CMB rest frame.

Secrest *et al.* (2021) used mid-infrared data from the Wide-field Infrared Survey Explorer (WISE) (Wright *et al.* 2010) to create reliable AGN/quasar catalogs and a custom quasar sample from the new CatWISE2020 data release (Eisenhardt *et al.* 2020). It was shown that there is a statistically significant dipole in the density of distant quasars with direction  $(l, b) = (238.2^{\circ}, 28.8^{\circ})$  which is  $27.8^{\circ}$  away from the direction of the CMB dipole. Its amplitude was found to be 0.01554,  $\sim 2$  times larger than predicted, with statistical significance at the  $4.9\sigma$  level (or with a p-value of  $5 \times 10^{-7}$ ) for a normal distribution. This result is in conflict with the cosmological principle.

### III.3.3 Fine structure constant $\alpha$ dipole

In the past 20 years there has been interest in the possibility of the variation of the fine structure constant

$\alpha \equiv e^2/(4\pi\epsilon_0\hbar c)$  (where  $e$ ,  $\epsilon_0$ ,  $\hbar$ , and  $c$  are the electron charge, the vacuum permittivity, the reduced Planck's constant, and the speed of light) (Bainbridge *et al.* 2017; Berengut *et al.* 2013; Evans *et al.* 2014; Hu *et al.* 2019; Lee *et al.* 2021; Milaković *et al.* 2020; Molaro *et al.* 2013; Murphy *et al.* 2003; Webb *et al.* 2011, 2014) (for review of varying fine structure constant, see Martins 2017).

The analysis by King *et al.* (2012); Webb *et al.* (2011) uses the “many multiplet” (MM) method (Dzuba *et al.* 1999; Murphy *et al.* 2001; Webb *et al.* 1999, 2001) to analyze quasar absorption line spectra obtained using the Ultraviolet and Visual Echelle Spectrograph (UVES) (Dekker *et al.* 2000) on the Very Large Telescope (VLT). It indicates both the violation of the cosmological principle and the spatial variation of the fine structure constant  $\alpha$  which is approximated as a spatial dipole with direction  $(l, b) = (330^\circ \pm 15^\circ, -13^\circ \pm 10^\circ)$  and amplitude  $0.97^{+0.22}_{-0.20} \times 10^{-5}$ , preferred over a simple monopole model with significance at the  $4.2\sigma$  (for possible systematics in this analysis, see Cameron and Pettitt 2012; Kanekar *et al.* 2012; Levshakov *et al.* 2012).

The variation of  $\alpha$  across the sky was shown to be well fit by an angular dipole model of the form (King *et al.* 2012)

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha} = C_A \cos\theta + C_B \quad (3.36)$$

where  $\alpha_0$  is the present local value,  $\theta$  is the angle with respect to the dipole direction,  $C_A$  is the angular amplitude of the dipole term and  $C_B$  is a monopole term.

It is worth to note that the analysis by Martins and Pinho (2017) suggests that there are no robust indications of time or space variations of  $\alpha$ . However recent measurements of quasar absorption-line spectra indicate a spatially dependent value of fine structure constant  $\alpha$  at a  $\sim 4\sigma$  significance level over a simple monopole (no-variation) model (Dumont and Webb 2017; Wilczynska *et al.* 2020). In addition, it is found that the fine structure constant  $\alpha$  dipole is anomalously aligned with other dipoles and the preferred direction in  $\Delta\alpha/\alpha$  is correlated with the one in the distribution of SNIa (Mariano and Perivolaropoulos 2012, 2013).

### III.4 BAO curiosities

As mentioned above (see Subsection II.1.2) the BAO measurements can be classified in two classes: galaxy BAO and Ly $\alpha$  BAO (with Ly $\alpha$  auto-correlation function and Ly $\alpha$ -quasar cross-correlation function). A  $2.5 - 3\sigma$  discrepancy between the BAO peak position in the Ly $\alpha$  at an effective redshift of  $z \sim 2.34$  and the CMB predictions from Planck/ $\Lambda$ CDM cosmological model has been found (Aubourg *et al.* 2015; du Mas des Bourboux *et al.* 2017; Font-Ribera *et al.* 2014).

For example, Aubourg *et al.* (2015) use the Ly $\alpha$  auto-correlation function and the Ly $\alpha$ -quasar cross-correlation function to report the measurements of the BAO scale in

the line-of-sight direction

$$D_H(z = 2.40)/r_s = 8.94 \pm 0.22 \quad (3.37)$$

and in the transverse direction

$$D_M(z = 2.40)/r_s = 36.6 \pm 1.2 \quad (3.38)$$

where  $D_H(z) \equiv \frac{c}{H(z)}$  is the Hubble distance and  $D_M(z) \equiv (1+z)D_A(z) = d_A(z)$  is the comoving angular diameter distance. These values are in  $\sim 2.3\sigma$  tension with CMB predictions  $D_H(z = 2.40)/r_s = 8.586 \pm 0.021$  and  $D_M(z = 2.40)/r_s = 39.77 \pm 0.09$  by Planck 2015 flat  $\Lambda$ CDM cosmology (Ade *et al.* 2016a).

The galaxy BAO peak position in the matter correlation function  $\xi(s)$  (see Eq. (2.19) and Fig. 7) and the measurements  $D_H(z = 2.40)/r_s$  and  $D_M(z = 2.40)/r_s$  were found to be consistent with CMB predictions. This discrepancy between galaxy and Ly $\alpha$  BAO constitutes the BAO anomaly which has been investigated in Addison *et al.* (2018); Cuceu *et al.* (2019); Evslin (2017).

Using new Ly $\alpha$  BAO measurements from the BOSS survey and from its extended version eBOSS in the SDSS DR14 the tension with CMB predictions was reduced to  $\sim 1.7\sigma$  (Blomqvist *et al.* 2019; de Sainte Agathe *et al.* 2019) and from eBOSS in the SDSS DR16 to only  $\sim 1.5\sigma$  (du Mas des Bourboux *et al.* 2020).

Evslin (2017) argues that this anomaly arises by cosmological effects at  $z < 2.34$  and the tension is caused by evolution of dark energy equation of state  $w(z)$  for redshift range  $0.57 < z < 2.34$ .

### III.5 Parity violating rotation of CMB linear polarization (Cosmic Birefringence)

In the standard model of elementary particles and fields, parity violation is observed only in the weak interaction sector (Lee and Yang 1956; Wu *et al.* 1957). A certain class of quintessence models should generically generate such parity asymmetric physics (Carroll 1998; Carroll *et al.* 1990). In particular a parity violating (nearly) massless axionlike scalar field  $\phi$  (dark matter or dark energy) would rotate CMB polarisation angles of CMB photons as they travel from the last scattering surface ( $z \approx 1000$ ) to the present by a non-zero angle  $\beta_a$  (cosmic birefringence).

A Chern-Simons coupling between a time-dependent axionlike field  $\phi(t)$  and the electromagnetic tensor and its dual in the Lagrangian density (e.g. Carroll *et al.* 1990; Ni 1977; Turner and Widrow 1988)

$$\mathcal{L} = \frac{1}{4}g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \quad (3.39)$$

induces a cosmic isotropic birefringence angle (e.g. Fujita *et al.* 2021a; Harari and Sikivie 1992)

$$\beta_a = \frac{1}{2}g_{\phi\gamma}\int_{t_s}^{t_0}\dot{\phi}dt \quad (3.40)$$

and produces a non-zero observed  $EB$  spectrum (Lue et al. 1999)

$$C_l^{EB} = \frac{1}{2} \sin(4\beta_a) (C_l^{EE} - C_l^{BB}) \quad (3.41)$$

where  $g_{\phi\gamma}$  is a Chern-Simons coupling constant which has mass-dimension  $-1$ ,  $\tilde{F}^{\mu\nu}$  is the dual of the electromagnetic tensor of  $F_{\mu\nu}$ , and  $t_0$  and  $t_s$  are the times at present and last scattering surface, respectively.

Using a novel method developed in Minami (2020); Minami and Komatsu (2020b); Minami et al. (2019), a non-zero value of the isotropic cosmic birefringence  $\beta_a = 0.35 \pm 0.14$  deg (68% C.L) was recently detected in the Planck18 polarization data at a  $2.4\sigma$  statistical significance level by Minami and Komatsu (2020a). More recently, using the latest Planck public data release 4 (Akrami et al. 2020e) a birefringence angle of  $\beta_a = 0.30 \pm 0.11$  deg (68% C.L) was reported by Diego-Palazuelos et al. (2022).

These recent evidences of the non zero value of birefringence poses a problem for standard  $\Lambda$ CDM cosmology and indicates a hint of a new ingredient beyond this model.

An axion or an axion-like particle with a weak coupling to photon as a possible source of the cosmic birefringence was investigated by Fujita et al. (2021b) and a two-axion alignment model with periodic potential was investigated by Obata (2021). Takahashi and Yin (2021b) showed that if an ultralight axion coupled to photons forms domain walls due to inflationary fluctuations, the domain-wall network can explain the hint for isotropic cosmic birefringence found by Minami and Komatsu (2020a). This model predicts a testable peculiar anisotropic cosmic birefringence as well. In contrast to the approach of Fujita et al. (2021b), this scenario explains the birefringence with the photon anomalous coefficient of the axion-like particle  $\sim O(1)$ . Furthermore, birefringence inducing axion-like particles could be candidates for an early dark energy resolution to the Hubble tension (Fujita et al. 2021b). Bianchini et al. (2020); Namikawa et al. (2020) study the anisotropic birefringence and constraints are derived. The axion field fluctuations over space and time generate anisotropic birefringence.

### III.6 Small-scale curiosities

On small scales (on scales of hundreds of  $kpc$  and below) the predictions of  $\Lambda$ CDM model are in many cases inconsistent with observations (Kroupa et al. 2010; Nakama et al. 2017; Weinberg et al. 2015). In particular observations on galaxy scales indicate that the  $\Lambda$ CDM model faces several problems in describing structures at small scales ( $\lesssim 1Mpc$ ) (see Bullock and Boylan-Kolchin 2017; Del Popolo and Le Delliou 2017; Kroupa 2012, 2015; Salucci 2019, for a review). Alternative models that modify the nature of dark matter have been used to solve these problems e.g. warm (Abazajian 2006; Bode et al. 2001; Schneider et al. 2014; Viel et al. 2013, 2005), fuzzy

(Hu et al. 2000; Hui et al. 2017; Iršič et al. 2017; Schive et al. 2014), self-interacting (Blennow et al. 2017; Carlson et al. 1992; Garcia-Cely and Chu 2017; Tulin and Yu 2018) and meta-cold dark matter (Strigari et al. 2007) (see also de Martino et al. 2020, for a review). Other models which have the potential to provide a solution to these problems have been proposed by Archidiacono et al. (2017); Foot and Vagnozzi (2015, 2016); Sawala et al. (2016); Schewtschenko et al. (2016); Vogelsberger et al. (2016). In particular Foot and Vagnozzi (2015, 2016) argued that the existence of a dissipative hidden dark matter sector (dark matter coupled to a massless dark photon) can solve some of these problems (core-cusp, missing satellites, and plane of satellites problem).

These small scale signals include the following:

#### III.6.1 The core-cusp curiosity

The core-cusp curiosity (de Blok 2010; Flores and Primack 1994; Moore 1994) (see also Cooke et al. 2022; Lelli 2022) refers to a discrepancy between the density of a dark matter halo profile of low-mass galaxies  $\rho(r) \propto r^{-x}$  in N-body simulations (an important tool for evaluating the predictions of the  $\Lambda$ CDM model) with  $1 \lesssim x \lesssim 1.5$  (cusp profile)<sup>54</sup> (Ferrero et al. 2012; Moore et al. 1999b; Navarro et al. 1996a, 1997) and the astronomical observed profile with  $x \sim 0$  (core profile) (Amorisco and Evans 2012; Battaglia et al. 2008; Davis et al. 1985; Flores and Primack 1994; Genina et al. 2017; Moore 1994; Navarro et al. 1996a, 1997; Walker and Penarrubia 2011). McGaugh and de Blok (1998) probe this problem in low surface brightness galaxies.

#### III.6.2 The missing satellites problem (or dwarf galaxy problem)

The missing satellites problem (or dwarf galaxy problem) (Bullock 2010; Kauffmann et al. 1993; Klypin et al. 1999; Moore et al. 1999a) refers to an over-abundance of the predicted number of halo substructures in detailed collisionless N-body simulations compared to the observed number of satellite galaxies in the Local Group. In particular the  $\Lambda$ CDM model predicts orders of magnitude larger number of satellites ( $\sim 1000$ ) than the observed number of dwarf galaxies ( $\sim 50$ ) (Mateo 1998; Moore et al. 1999a).

#### III.6.3 The Too Big To Fail (TBTF) problem

The Too Big To Fail (TBTF) problem (Boylan-Kolchin et al. 2011, 2012; Garrison-Kimmel et al. 2014; Kaplinghat et al. 2019; Papastergis et al. 2015; Tollerud et al. 2014) refers to an inconsistency between the predicted mass of dark matter subhaloes in  $\Lambda$ CDM theory and the

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<sup>54</sup> The well known Navarro–Frenk–White profile (Navarro et al. 1996b, 1997) is cusped with  $\rho(r \rightarrow 0) \sim r^{-1}$ .

observed central mass of brightest satellite galaxies in the Local Group (Garrison-Kimmel *et al.* 2014; Papastergis *et al.* 2015) (also in the MW Boylan-Kolchin *et al.* 2011, 2012 or in the Andromeda (M31) Tollerud *et al.* 2014).

In particular the  $\Lambda$ CDM predicted central densities of the most massive dark matter subhalos are systematically larger than the inferred from kinematics of the brightest Local Group satellites (Boylan-Kolchin *et al.* 2012; Garrison-Kimmel *et al.* 2014; Read *et al.* 2006). An observed bright satellite is more likely to reside in subhalos with lower mass than is expected in a  $\Lambda$ CDM model. The simulated massive dark matter subhalos ‘failed’ to form a comparatively bright satellite galaxy.

This problem is possibly related to the missing satellites problem but it is a distinct problem which dependents on the internal structure of subhalos or the central shapes of density profiles of satellite halos (Garrison-Kimmel *et al.* 2014).

Alternative models that modify the nature of dark matter have been investigated to solve this problem: non-trivial dark matter physics (Bozek *et al.* 2019; Lovell *et al.* 2017), interaction between the dark matter and dark radiation components (Schewtschenko *et al.* 2016; Vogelsberger *et al.* 2016), self-interacting dark matter (Vogelsberger *et al.* 2012; Zavala *et al.* 2013) and fuzzy dark matter (Schive *et al.* 2014)

### III.6.4 The problem of satellite planes

In the problem of satellite planes (Conn *et al.* 2013; Ibata *et al.* 2014; Kroupa *et al.* 2005; Pawłowski 2018; Pawłowski *et al.* 2014) several satellite galaxies of the MW, of neighboring Andromeda galaxy (M31), and of Centaurus A (CenA) are part of thin plane that is approximately perpendicular to the Galactic disk. Moreover measurement of the motions of satellite galaxies has shown that their orbits appear to be correlated (Ibata *et al.* 2013; Pawłowski *et al.* 2012; Sohn *et al.* 2020). This flattened structure and coherent motions of satellite galaxy systems is in inconsistency with the prediction of the  $\Lambda$ CDM model as inferred from simulations (Pawłowski 2018). The simulations based on  $\Lambda$ CDM cosmology indicate uncorrelated and close to isotropic satellite structures (Libeskind *et al.* 2005; Zentner *et al.* 2005). In these simulations the observed structure formations with spatial and kinematic coherence distribution are very rare with a probability  $\sim 10^{-3}$  (Pawłowski 2018; Pawłowski *et al.* 2014).

### III.6.5 The angular momentum catastrophe

The angular momentum catastrophe (van den Bosch *et al.* 2001) concerns a catastrophic angular momentum loss of gas during disk galaxies formation in Smooth Particle Hydrodynamics (SPH) (Monaghan 1992) simulations. The formed disks in simulations according to the predictions of  $\Lambda$ CDM have smaller scale lengths by a factor of 2 – 3 compared with observed ones (Bullock *et al.* 2001). An axion dark matter model may resolve

this discrepancy between the observed and predicted angular momentum distributions of baryons (ordinary cold dark matter) in the dwarf galaxies (Banik and Sikivie 2013).

### III.6.6 Baryonic Tully-Fisher Relation (BTFR)

Baryonic Tully-Fisher Relation (BTFR) (Lelli *et al.* 2016b; McGaugh 2011). As mentioned above, the well known Tully-Fisher (TF) (Tully and Fisher 1977) empirical relation connects the velocity of rotation of a spiral galaxy with its intrinsic luminosity while the Baryonic Tully-Fisher Relation (BTFR) (Gurovich *et al.* 2004; McGaugh *et al.* 2000; Verheijen 2001) Eq. (2.41) is a scaling relation between the observed total baryonic mass  $M_b$  (stars plus gas) of a spiral galaxy and its rotation velocity  $V_c$  (see Subsection II.1.6). The problem for  $\Lambda$ CDM model as inferred from simulations (e.g. Dutton 2012) is that the BTFR leads to existence of a higher intrinsic scatter ( $\sim 0.15$  dex) and a lower slope ( $s = 3$ ) compared to the observed ( $\sim 0.10$  dex and  $s \sim 4$ ) (Lelli *et al.* 2016b). McGaugh (2012) suggests the Modified Newtonian Dynamics (MOND) (Milgrom 1983) as a possible solution to this problem. However some simulations or semi-analytic approaches of galaxy formation within a  $\Lambda$ CDM cosmological context can reproduce a realistic BTFR slope but not its small scatter (e.g. Di Cintio and Lelli 2016; Geha *et al.* 2006; Sales *et al.* 2017; Santos-Santos *et al.* 2016).

### III.6.7 The void phenomenon

The void phenomenon (Peebles 2001) refers to the emptiness of voids (the number of small galaxies in the void). Cosmological N-body simulations in the context of  $\Lambda$ CDM have established a clear prediction (Gottlöber *et al.* 2003) that many small dark matter haloes should reside in voids (Peebles 2007, 2005). This is consistent with observations on large scales but is inconsistent with observations on small scales. In particular the local void contains much fewer galaxies than expected from  $\Lambda$ CDM theory (Tikhonov and Klypin 2009).

## III.7 Age of the Universe

A lower limit can be set on the age of the Universe by the ages of the oldest stars (or oldest astrophysical objects) because on cosmological timescales they form shortly after the Big Bang. In the context of  $\Lambda$ CDM cosmology, the standard theory (Abel *et al.* 2002; Bromm 2013; Bromm *et al.* 2002; Bromm and Larson 2004; Haiman *et al.* 1996) and cosmological numerical simulations (Hirano *et al.* 2015; Xu *et al.* 2016; Yoshida *et al.* 2003) predict that the first stars, the so-called population III (Pop III), formed in dark matter minihaloes of typical mass  $M \sim 10^5 - 10^6 M_\odot$  at redshifts  $z \sim 20 - 30$  (about 100 million years after the Big Bang i.e. about around the end of the cosmic dark ages) (for models indicating

late,  $z \sim 2 - 7$ , Pop III star formation, see [Johnson 2010; Tornatore et al. 2007](#)).

The age of the Universe  $t_*$  as obtained from local measurements using the ages of oldest observed stars (the so-called population II (Pop II)) in the MW appears to

$$t(z) = \int_0^{z_t} \frac{dz'}{(1+z')H(z')} = \frac{1}{H_0} \int_0^{z_t} \frac{dz'}{(1+z') [\Omega_{0m}(1+z')^3 + \Omega_{0r}(1+z')^4 + (1-\Omega_{0m})]^{1/2}} \quad (3.42)$$

where  $t$  is the cosmic time corresponding to redshift  $z_t$ . Thus the age of the Universe is  $t_U = t(z_t = \infty)$ .

For example the age of the MW Population II halo, metal deficient, high velocity subgiant HD-140283 (also known as Methuselah star) is estimated to be  $t_* = 14.46 \pm 0.31 \text{ Gyr}$  by [Bond et al. \(2013\)](#) and using new sets of stellar models is estimated to be  $t_* = 14.27 \pm 0.80 \text{ Gyr}$  by [VandenBerg et al. \(2014\)](#). These estimates of the age of this star are slightly higher ( $\sim 2\sigma$ ) than the age of Universe  $t_U = 13.800 \pm 0.024 \text{ Gyr}$  inferred by CMB Planck18 data ([Aghanim et al. 2020e](#)) but within the errors it does not conflict with this age.

Despite of the above indications the analysis by [Jimenez et al. \(2019\)](#) using new parallaxes from the Gaia space mission ([Perryman et al. 2001; Prusti et al. 2016](#)) in place of the older HST, reports a revision of the age of HD-140283 to  $t_* = 13.5 \pm 0.7 \text{ Gyr}$  which is more compatible with the age  $t_U$  inferred by Planck data. Also the analysis by [Valcin et al. \(2020\)](#) using populations of stars in globular clusters (very-low-metallicity stars) reports age of the Universe constrained to be larger than  $t_* = 13.5^{+0.16}_{-0.14} \text{ Gyr}$ .

Clearly, Eq. (3.42) indicates that in a  $\Lambda$ CDM Universe the quantities  $H_0$ ,  $t_U$  and  $\Omega_{0m}$  are related. Therefore the determination of the age of older objects based on local Universe observations provides a test of the current cosmological model and plays an important role in the studies of Hubble and spatial curvature tensions ([Di Valentino et al. 2019b, 2021h; Jimenez et al. 2019; Wei and Melia 2022](#)).

### III.8 The Lithium problem

It has long been known (since the early 80's) that absorption lines in the photospheres of old, metal-poor (Population II) halo stars in the Milky Way's halo indicate  $\sim 3.5$  times less primordial abundance of lithium isotope  ${}^7\text{Li}$  compared to the prediction of the standard BBN theory ([Asplund et al. 2006; Cyburt et al. 2003, 2008](#)). The observed value of the lithium abundance<sup>55</sup>  ${}^7\text{Li}/H = (1.6 \pm$

be larger and in some tension with the corresponding age of the Universe  $t_U$  obtained using the CMB Planck data in the context of  $\Lambda$ CDM ([Verde et al. 2013](#)).

The age of the Universe in the flat  $\Lambda$ CDM model is an observable determined by the integral

$0.3) \times 10^{-10}$  ([Zyla et al. 2020](#)) is smaller than the theoretically expected value  ${}^7\text{Li}/H = (5.62 \pm 0.25) \times 10^{-10}$  ([Pitrou et al. 2018](#)) at a level  $\sim 5\sigma$ . This constitutes the lithium problem ([Fields 2011](#)). No such problem exists for the observed abundances of other light elements  ${}^2\text{H}$  (or  $D$ ),  ${}^3\text{He}$ , and  ${}^4\text{He}$  that are in broad quantitative agreement with BBN predictions + WMAP/Planck cosmic baryon density  $\Omega_b$  which is deduced by the CMB ([Cyburt et al. 2016; Tanabashi et al. 2018](#)).

A number of theoretical or experimental studies in the literature have attempted to address the lithium problem (e.g. [Coc and Vangioni 2017; Goudelis et al. 2016; Hammache et al. 2013; Hayakawa et al. 2020; Hou et al. 2017; Iliadis and Coc 2020; Ishikawa et al. 2020; Kusakabe et al. 2014; Mori and Kusakabe 2019; Pizzzone et al. 2014; Poulin and Serpico 2015; Salvati et al. 2016; Sato et al. 2017a; Yamazaki et al. 2014](#)). For example the analysis by [Clara and Martins \(2020\)](#) shows that the variations in Nature's fundamental constants on primordial nucleosynthesis provide a possible solution to the lithium problem. Specifically, they determined that if the value of the fine-structure constant  $\alpha$  at the primordial nucleosynthesis epoch was larger than the present one by ten parts per million of relative variation, the lithium problem could be resolved.

It was also proposed by [Di Bari et al. \(2013\)](#) that decaying dark matter into dark radiation in the early Universe can solve the long-standing lithium problem, leaving completely unaffected the abundance of other light elements. This mechanism was also proposed to alleviate the  $H_0$  tension (see Subsection II.2.1) but is severely constrained by the Planck data ([Anchordoqui 2021](#)).

Measurements of lithium (e.g. [Melendez et al. 2010; Sbordone et al. 2010](#)) may not be representative of the cosmological production mechanism ([Iocco 2012; Spite et al. 2012](#)). It is thus possible that the solution to the lithium problem lies in the effects of stars in the lithium abundance. Therefore a precise knowledge of the stellar formation process and physics of stellar atmosphere is necessary to provide a fully satisfactory solution. Thus, possible solutions to this persistent problem can be classified into four categories (see [Cyburt et al. 2016; Fields 2011; Mathews et al. 2020](#), for a review):

- Cosmological solutions (e.g. new theory beyond the standard BBN including variations of funda-

<sup>55</sup> Usually in the literature the abundance of lithium is expressed by  $A({}^7\text{Li}) = 12 + \log_{10}[n({}^7\text{Li})/n(H)]$  where  $n$  is the number density of atoms and 12 is the solar hydrogen abundance.

mental constants) (Berengut *et al.* 2010; Cheoun *et al.* 2011; Clara and Martins 2020; Coc *et al.* 2007; Franchino-Viñas and Mosquera 2021; Hou *et al.* 2017; Kawasaki and Kusakabe 2011; Kohri and Takayama 2007; Larena *et al.* 2007; Luo *et al.* 2019)

- Nuclear Physics solutions (e.g. reactions destroy lithium during or after BBN) (Boyd *et al.* 2010; Broggini *et al.* 2012; Chakraborty *et al.* 2011; Coc *et al.* 2012; Cyburt *et al.* 2004; Hayakawa *et al.* 2020; Ishikawa *et al.* 2020; Mori and Kusakabe 2019)
- Astrophysical solutions (e.g. stars destroy lithium after BBN) (Fu *et al.* 2015; Korn *et al.* 2006; Pinsonneault *et al.* 2002; Richard *et al.* 2005)
- Extensions of the standard model (e.g. simultaneous imposition of photon cooling after BBN, X-particle decay and a primordial magnetic field (Yamazaki *et al.* 2014, 2017), destruction of  $^7\text{Be}$  due to the decay of a sterile neutrino (Salvati *et al.* 2016) and including new particles or interactions (Goudelis *et al.* 2016).

### III.9 Quasars Hubble diagram

The quasar distances can be estimated from their X-ray (coronal) emission generated by a plasma of hot relativistic electrons around the accretion disk. The emission is induced through inverse-Compton scattering processes and UV emission generated by the accretion disk where the gravitational energy of the infalling material is partially converted to radiation (Lusso *et al.* 2019; Risaliti and Lusso 2019).

In recent years model independent derivation<sup>56</sup> of the distance modulus-redshift relation using high- $z$  quasars ( $z \lesssim 7$ ) as distance indicators (quasars Hubble diagram) provides a new bright standard candle in the higher redshifts and earlier times beyond SnIa. The method used is based on a non-linear relation between the X-ray and the UV emissions at low redshift which is of the form (Avni and Tananbaum 1986; Lusso *et al.* 2010; Risaliti and Lusso 2015, 2019; Steffen *et al.* 2006)

$$\log_{10} L_X = \gamma_q \log_{10} L_{UV} + \beta_q \quad (3.43)$$

where  $L_X$  and  $L_{UV}$  are the rest-frame monochromatic luminosities at 2 keV and at 2500 Å, respectively (Avni and Tananbaum 1986). Also  $\gamma_q \sim 0.6$  (Bisogni *et al.* 2021; Lusso *et al.* 2010; Risaliti and Lusso 2019; Salvestri *et al.* 2019; Steffen *et al.* 2006; Vagetti *et al.* 2010; Vignali *et al.* 2003) and  $\beta_q$  are fitting parameters of the luminosities.

<sup>56</sup> Yang *et al.* (2020a) argued that even though the data used in this approach are valid, their analysis involves significant uncertainties as it may lead to spurious artificial tensions.

Extending a Hubble diagram up to redshift  $z = 5.5$  shows hints for phantom dark energy (Banerjee *et al.* 2021b; Lusso *et al.* 2019; Risaliti and Lusso 2019). In particular the distance modulus-redshift relation for a sample of 1598 quasars at higher redshift ( $0.5 < z < 5.5$ ) is in disagreement with the concordance model at a  $\sim 4\sigma$  significance level<sup>57</sup> (Risaliti and Lusso 2019). Moreover, the analysis by Lusso *et al.* (2019) building a Hubble diagram by combining three samples of Pantheon, quasars, and gamma-ray bursts (GRBs) reported tension at more than the  $\sim 4\sigma$  statistical level with the flat  $\Lambda\text{CDM}$  model.

However Khadka and Ratra (2021) using an updated, larger QSO dataset (Lusso *et al.* 2020) containing 2421 QSO measurements with redshifts up to  $z \sim 7.5$  have demonstrated that the  $L_X$ - $L_{UV}$  relation parameter values depend on the cosmological model thus cannot be used to constrain cosmological parameters. Recently Khadka and Ratra (2022) demonstrated that the parameter values of the largest of seven sub-samples in this QSO dataset depend on the cosmological model while the second and third biggest sub-samples appear standardizable.

### III.10 Oscillating signals in short range gravity experiments

The most constraining test of gravity at very short distance (sub-millimeter) scales looking for departures from Newtonian gravity is implemented via torsion balance experiments. A reanalysis of short range gravity experiments has indicated the presence of an oscillating force signal with sub-millimeter wavelength (Antoniou and Perivolaropoulos 2017; Perivolaropoulos 2017b). In particular Perivolaropoulos (2017b) has indicated the presence of a signal at  $2\sigma$  level of spatially oscillating new force residuals in the torsion balance data of the Washington experiment (Kapner *et al.* 2007). As an extension of the previous analysis the study by Antoniou and Perivolaropoulos (2017) using Monte Carlo simulation and analysing the data of the Stanford Optically Levitated Microsphere Experiment (SOLME) which involves force measurements an optically levitated microsphere as a function of its distance  $z$  from a gold coated silicon cantilever (Rider *et al.* 2016) reports a oscillating signal at about  $2\sigma$  level.

The sub-millimeter scale of the quantum nature of dark energy may be written as

$$\lambda_{de} \equiv \sqrt[4]{\frac{\hbar c}{\rho_{de}}} \approx 0.085 \text{ mm} \quad (3.44)$$

<sup>57</sup> The analyses of the high- $z$  quasar data has lead to a wide range of conclusions (Khadka and Ratra 2020b; Li *et al.* 2021b; Melia 2019; Velten and Gomes 2020; Yang *et al.* 2020a). For example Yang *et al.* (2020a) concludes that the log polynomial expansion generically fails to recover flat  $\Lambda\text{CDM}$  beyond  $z \sim 2$ , thus implying that the previously derived  $\sim 4\sigma$  tension may be artificial.

where it is assumed that  $\Omega_{0m} = 0.3$  and  $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$ .

Thus, if the accelerating expansion of the Universe is connected with effects of modified gravity due to quantum gravity (Addazi *et al.* 2021) it would be natural to expect some modification of Newton's law at the submillimeter scale.

The deviations from Newton's law of gravitation is usually described in the context of scalar-tensor (Esposito-Farese and Polarski 2001; Gannouji *et al.* 2006) and flat extra dimension theories (Antoniadis *et al.* 1998; Arkani-Hamed *et al.* 1998, 1999; Floratos and Leontaris 1999; Kehagias and Sfetsos 2000; Perivolaropoulos 2003; Perivolaropoulos and Sوردис 2002) by a short range Yukawa type potential of the form

$$V_{eff} = -G \frac{M}{r} (1 + \alpha_Y e^{-mr}) \quad (3.45)$$

where  $\alpha_Y$  and  $m$  are parameters to be constrained by the data.

Alternatively, a power law ansatz may also generalize the gravitational potential to the form

$$V_{eff} = -G \frac{M}{r} \left[ 1 + \bar{\beta}^k \left( \frac{1}{mr} \right)^{k-1} \right] \quad (3.46)$$

This power law parametrization is motivated by some brane world models (Benichou and Estes 2012; Bronnikov *et al.* 2006; Donini and Marimón 2016; Nojiri and Odintsov 2002).

For  $m^2 < 0$  the Yukawa gravitational potential becomes oscillating and takes the form

$$V_{eff} = -G \frac{M}{r} [1 + \alpha_Y \cos(mr + \theta)] \quad (3.47)$$

where  $\theta$  is a parameter.

Recently a reanalysis of the data of the Washington experiment searching for modifications of Newton's Law on sub-millimeter scales by Perivolaropoulos and Kazantzidis (2019) has indicated that a spatially oscillating signal is hidden in this dataset. In addition it is shown that even though this signal cannot be explained in the context of standard modified gravity theories<sup>58</sup> (viable scalar tensor and  $f(R)$  theories), it occurs naturally in nonlocal (infinite derivative) gravity theories (Edholm *et al.* 2016; Frolov and Zelnikov 2016; Kehagias and Maggiore 2014) that predict such spatial oscillations without the presence of ghosts (instabilities) and has a well-defined Newtonian limit.

The origin of oscillating signals could be due to three possible effects:

- A statistical fluctuation of the data.
- A periodic distance-dependent systematic feature in the data.
- A signal for a short distance modification of GR (e.g. non-local modified theory of gravity).

In the later case, it is important to identify modified theories that are consistent with such an oscillating signal and are not associated with instabilities (e.g. Biswas *et al.* 2014; Tomboulis 1997).

### III.11 Anomalously low baryon temperature

The Experiment to Detect the Global Epoch of Reionization Signature (EDGES) collaboration (Bowman *et al.* 2018) report anomalously low baryon temperature  $T_b \approx 4K$  at  $z \approx 17$  (half of its expected value). This temperature was inferred from the detection of global (sky-averaged) 21-cm absorption signal which is centred at a frequency of  $\sim 78 \text{ MHz}$ . The absorption depth of cosmic CMB photons at redshifts range  $15 \lesssim z \lesssim 20$  estimated by EDGES is more than twice the maximal value expected in the  $\Lambda\text{CDM}$  model, at  $\sim 3.8\sigma$  significance.

Possible explanations of this discrepancy were investigated and various models were proposed (e.g. Boyarsky *et al.* 2019; Fraser *et al.* 2018; Hill and Baxter 2018; Muñoz and Loeb 2018). For example Hill and Baxter (2018) argue that EDE can explain this anomaly.

The EDGES observation has been used to constrain various cosmological models of dark matter and dark energy (e.g. Barkana *et al.* 2018; Hill and Baxter 2018; Kovetz *et al.* 2018; Yang *et al.* 2019f).

### III.12 Colliding clusters with high velocity

Observed galaxy clusters like the massive ( $\sim 10^{15} M_\odot$ ) high-redshift ( $z = 0.87$ ) interacting pair known as El Gordo (ACT-CL J0102-4915) (Menanteau *et al.* 2012) have a very high relative velocity. This implies that formation of large structures may have taken place earlier than expected in  $\Lambda\text{CDM}$  cosmology. Asencio *et al.* (2020) based on light cone tomography estimated that the too-early formation of El Gordo rules out  $\Lambda\text{CDM}$  cosmology at  $6.16\sigma$  confidence. The early and rapid formation of clusters which consist of two colliding massive galaxy clusters at a high redshift may constitute a problem of the  $\Lambda\text{CDM}$  model. Asencio *et al.* (2020) argue that MOND with light sterile neutrinos model as suggested by Haslbauer *et al.* (2020) can resolve this issue.

<sup>58</sup> For a free massive scalar Perivolaropoulos and Skara (2020) investigate the physical conditions that can eliminate the tachyonic instabilities or at least drastically change their lifetime.

TABLE IV. Some existing and upcoming large-scale structure missions/experiments.

Experiments	Type	Probes	Redshift	Wavelengths	Operator	Duration	Refs.
Euclid	Space	WL, BAO	$z \lesssim 6$	$550\text{ nm} - 2\mu\text{m}$	ESA	> 2022	(Laureijs <i>et al.</i> 2011)
Vera C. Rubin Obs.	Ground	WL, BAO	$z \lesssim 7.5$	$320 - 1060\text{ nm}$	LSST	> 2022	(Abell <i>et al.</i> 2009)
Gaia	Space	Astrometry	$z \approx 0$	$320 - 1000\text{ nm}$	ESA	> 2013	(Prusti <i>et al.</i> 2016)
JWST	Space	WL	$z \lesssim 15$	$0.6 - 28.3\mu\text{m}$	NASA-ESA-CSA	> 2021	(Gardner <i>et al.</i> 2006)
GAUSS	Space	WL $3 \times 2\text{pt}$	$z \lesssim 5$	$0.5 - 5\text{ nm}$		> 2035	(Blanchard <i>et al.</i> 2021)

TABLE V. Some existing and upcoming CMB missions/experiments.

Experiments	Type	Detectors	Frequencies <sup>a</sup> (GHz)	Resolution <sup>b</sup> (arcmin)	Sensitivity <sup>c</sup> ( $\mu\text{K arcmin}$ )	Sky Cover	Duration	Refs.
Planck	Space	74	25 – 1000	5 – 33	$\sim 30$	All	2009-2013	(Aghanim <i>et al.</i> 2020b)
CMB S4	Ground	$500 \cdot 10^3$	30 – 270	0.8 – 11	$\sim 1$	70%	> 2027	(Abazajian <i>et al.</i> 2016)
SO LAT	Ground	$30 \cdot 10^3$	27 – 280	0.1	$\sim 6$	40%	> 2021	(Ade <i>et al.</i> 2019)
SO SATs	Ground	$30 \cdot 10^3$	90 – 280	0.5	$\sim 2$	10%	> 2021	(Ade <i>et al.</i> 2019)
SPT-3G:	Ground	$16 \cdot 10^3$	90 – 280	1	$\sim 3.5, 6$	10%	> 2017	(Benson <i>et al.</i> 2014)

<sup>a</sup> The main CMB channels have frequencies  $70 - 217\text{ GHz}$  with spectrum peak at a frequency of  $\sim 160\text{ GHz}$ .<sup>b</sup> The angular resolution is a function of frequency, with lower frequencies having a worse angular resolution.<sup>c</sup> The sensitivity is a function of frequency and its estimates scaled to  $1^0$  assuming that the noise is white.

TABLE VI. Some existing and upcoming GW experiments/observatories

Experiments	Type/Detectors	Arms	Frequencies <sup>a</sup> (Hz)	Location	Duration	Refs.
Adv. LIGO	Ground/Laser interf.	$2 \times 4\text{ km}$	$10 - 10^3$	Hanford, USA	> 2015	(Aasi <i>et al.</i> 2015)
Adv. LIGO	Ground/Laser interf.	$2 \times 4\text{ km}$	$10 - 10^3$	Livingston, USA	> 2015	(Aasi <i>et al.</i> 2015)
Adv. Virgo	Ground/Laser interf.	$2 \times 3\text{ km}$	$10 - 10^3$	Pisa, Italy	> 2016	(Acernese <i>et al.</i> 2015)
KAGRA	Undergr./Laser interf.	$2 \times 3\text{ km}$	$10 - 10^3$	Kamioka, Japan	> 2020	(Somiya 2012)
CE	Ground/Laser interf.	$2 \times 40\text{ km}$	$5 - 4 \cdot 10^3$	USA	> 2030	(Abbott <i>et al.</i> 2017b)
LISA	Space/Laser interf.	$3 \times 2.5\text{ Gm}$	$10^{-4} - 10^{-1}$	Heliocentric orbit	> 2034	(Amaro-Seoane <i>et al.</i> 2017)
Taiji	Space/Laser interf.	$3 \times 2\text{ Gm}$	$10^{-4} - 10^{-1}$	Heliocentric orbit	> 2033	(Hu and Wu 2017)
TianQin	Space/Laser interf.	$3 \times 0.1\text{ Gm}$	$10^{-4} - 1$	Geocentric orbit	> 2035	(Luo <i>et al.</i> 2016)
DECIGO	Space/Laser interf.	$4 \times 3 \times 1\text{ Mm}$	$1 - 10$	Heliocentric orbit	> 2027	(Kawamura <i>et al.</i> 2006, 2011)
ET	Undergr./Laser interf.	$3 \times 2 \times 10\text{ km}$	$1 - 10^4$		> 2035	(Punturo <i>et al.</i> 2010)

<sup>a</sup> GW spectrum could span a wide range of frequencies, thus there are numerous proposed detectors, including pulsar timing arrays (PTAs) (Detweiler 1979; Foster and Backer 1990; Hellings and Downs 1983) ( $10^{-9}$  to  $10^{-6}\text{ Hz}$ ) such as Square Kilometre Array (SKA) (Janssen *et al.* 2015) (with the three collaborations, North American Nanohertz Observatory for Gravitational-waves (NANOGrav) (McLaughlin 2013), European Pulsar Timing Array (EPTA) (Desvignes *et al.* 2016) and Parkes Pulsar Timing Array (PPTA) (Hobbs 2013) members of the International Pulsar Timing Array (IPTA) (Verbiest *et al.* 2016)) and atom interferometry such as Atomic Experiments for Dark Matter and Gravity Exploration (AEDGE) (El-Neaj *et al.* 2020).

## IV Conclusions-Discussion- Outlook

In the present review, we discussed in a unified manner many existing curiosities in cosmological and astrophysical data that appear to be in some tension ( $2\sigma$  or larger) with the standard  $\Lambda$ CDM model as specified by the Planck18 parameter values. The Hubble tension is the most significant observational indication that the current standard model  $\Lambda$ CDM may need to be modified after more than 20 years since its establishment.

In addition to the well known tensions ( $H_0$  tension,  $S_8$  tension and  $A_L$  anomaly), we provided a list of the non-standard cosmological signals in cosmological data. We presented the current status of these signals and their level of significance and also referred to recent resources where more details can be found for each signal. These signals have a lower statistical significance level than the  $H_0$  tension but may also constitute hints towards new physics. We also briefly discussed possible theoretical approaches that have been considered in order to explain the non-standard nature of these signals.

We also discussed the possible generic extensions of  $\Lambda$ CDM model. Generic extensions of  $\Lambda$ CDM may allow for a redshift dependence of the parameters  $w$ ,  $\mu_G$ ,  $\Sigma$  and  $\alpha$  as well as a possible large scale spatial dependence of these parameters which could violate the cosmological principle. *Varying fundamental constants* can potentially address the Hubble tension, the fine structure constant  $\alpha$  dipole, the lithium problem, the growth tension, the curious SnIa  $M$  signals (variation of the SnIa absolute magnitude  $M$ ), quasar signals and the ISW CMB signal.

The strategic approach required for the identification of new physics may include the following three steps:

- Tune current missions towards verification or rejection of non-standard signals.
- Identify favored parametrizations of  $H(z, w(z), r)$ ,  $\mu_G(z, r)$ ,  $\Sigma(z, r)$ ,  $\alpha(z, r)$  assuming that at least some of the non-standard signals are physical.
- Identify theoretical models (field Lagrangians) that are consistent with these parametrizations that can address simultaneously more than one of these tensions. Interestingly, for example only a small subset of modified gravity models is consistent with the weak gravity +  $\Lambda$ CDM background (Gannouji *et al.* 2018, 2021; Pizzuti *et al.* 2019; Wittner *et al.* 2020) suggested in the context of the  $S_8$  tension.

In the next decades new observational data from existing and upcoming missions/experiments (see Tables IV, V and VI) will improve measurements and open up a wide range of new directions in the explanation of the curiosities of  $\Lambda$ CDM cosmology and understanding of cosmological physics. Here we provide an incomplete list of these missions:

- **Euclid:** The European Space Agency (ESA) Euclid mission (Laureijs *et al.* 2011) is planned for launch in July-December 2022. The goals of Euclid are to investigate the nature of dark matter, dark energy and gravity and thus to provide a better knowledge of the origin of the accelerated expansion of the Universe (Amendola *et al.* 2013; Cimatti *et al.* 2009; Laureijs *et al.* 2011; Refregier 2009; Sartoris *et al.* 2016). The optical and NIR Euclid survey using the cosmological WL and BAO probes will detect a high number of galaxy clusters up to redshift  $z \sim 2$  (and possibly higher) in a redshift range that is sensitive to dark energy (Adam *et al.* 2019) and will provide consistent growth rate data in both the low- $z$  and high- $z$  regimes. Therefore, Euclid will improve significantly the constraints on cosmological parameters such as  $\sigma_8$  and the mass density parameter  $\Omega_{0m}$  with a precision of  $\sim 10^{-3}$  for  $\Lambda$ CDM (Sartoris *et al.* 2016). The Euclid survey will also measure the equation of state parameter of dark energy  $w_{DE}$  with higher precision ( $\sim 1\%$ ) than precursor surveys. Stochastic inhomogeneities are expected to lead to an intrinsic uncertainty in the values of cosmological parameters obtained with such high redshift surveys. The corresponding cosmic variance in the context of Euclid for the measurement of  $H_0$  has been shown to be limited to about 0.1% (Fanizza *et al.* 2021). Thus Euclid and other deep surveys ( $z \gtrsim 0.15$ ) will provide an estimation of the  $H_0$  which will be more precise than the low redshift surveys ( $z \lesssim 0.15$ ). Such improved constraints from Euclid in combination with contemporary surveys (Martinelli *et al.* 2020, 2021; Nesseris *et al.* 2021) will allow the verification or rejection of many of the non-standard signals discussed in this review and will also help distinguish among the favored theoretical models that have been proposed for the explanation of these signals.

- **Vera C. Rubin Observatory Legacy Survey of Space and Time:** The Large Synoptic Survey Telescope (LSST), recently renamed the Vera C. Rubin Observatory LSST (Abell *et al.* 2009; Ivezić *et al.* 2019) is a future survey of the southern sky planned for the beginning in 2022. The Vera C. Rubin Observatory based in Chile with an 8.4 m (6.5 m effective) telescope in six bands, targeting at least  $18,000 \text{ deg}^2$  of high galactic latitude sky, will provide databases including 25 billion galaxies with  $\gtrsim 0.2$  arcsecond pixel sampling (Marshall *et al.* 2017). The main cosmological goals of the Vera C. Rubin Observatory ground-based project are to investigate the nature of dark matter and the dynamical behavior of dark energy by measuring WL and BAO (Marshall *et al.* 2017). Vera C. Rubin observatory will detect enormous number of galaxies and in combination with the Euclid BAO survey will probe an unprecedented range of

redshifts. These surveys can determine  $w_{DE}(z)$  in bins of redshift and their dark energy constraining power could be orders of magnitude greater than that of precursor surveys (Abate *et al.* 2012). The provided improved constraints on cosmological parameters allow to address potential systematics and to ensure that any measured tension is robust. In addition, the Vera C. Rubin project would be a useful tool in testing the models which have been used to explain these tensions.

- **CMB-S4:** The fourth generation<sup>59</sup> (Stage-4) ground-based CMB experiment (CMB-S4) (Abazajian *et al.* 2019, 2022, 2016), is planned to start observations in 2027. It is anticipated to be the definitive CMB polarization experiment. The goals of CMB-S4 are to detect the signature of primordial gravitational waves in order to shed light on models of inflation, to search for previously undiscovered light relic particles in order to study the dark Universe, to map normal and dark matter in the cosmos separately and to explore the time-variable millimeter-wave sky (Abazajian *et al.* 2016). The CMB-S4 survey in combination with external cluster surveys which are sensitive to different redshift ranges such as the Vera C. Rubin will provide detailed cluster data (Abazajian *et al.* 2016). These data will be used to study the growth of cluster scale perturbations, to improve constraints on cosmological parameters and to test the alternative models or extensions of  $\Lambda$ CDM which can be used to clarify the origin of many of the tensions and non-standard signals referred in the present review. The CMB-S4 will also contribute to neutrino cosmology providing compelling sensitivity in the constraint of the effective number of relativistic species,  $N_{eff}$  and of the sum of the neutrino masses  $\sum m_\nu$  (see Aghanim *et al.* 2020e; Di Valentino and Melchiorri 2021, for cosmological constraints and Capozzi *et al.* 2021, for three-neutrino analysis). This project has also the potential to constrain  $\Delta N_{eff} \equiv N_{eff} - N_{eff}^{SM} \simeq 0.060$  at 95% confidence level (Abazajian *et al.* 2019). Planck has provided a constraint  $\Delta N_{eff} \simeq 0.126$  at 95% confidence level using temperature and polarization TT, TE, EE + lowE data (Aghanim *et al.* 2020e). The improved constraints on  $N_{eff}$  will enable us to test the scenarios with modifications of  $\Lambda$ CDM model in the light relic sector.
- **Gaia:** The Gaia satellite was launched at the end of 2013 (Perryman *et al.* 2001; Prusti *et al.* 2016). This European Space Agency (ESA) mission Gaia provides data that allow us to determine with high

accuracy positions, parallaxes and proper motions for more than 1 billion sources. There have been two data releases GDR1 (Brown *et al.* 2016) and GDR2 (Brown *et al.* 2018) of Gaia results. Using quasars Lindegren *et al.* (2018) found that the GDR2 suffer from the parallax zero point (ZP) error. Riess *et al.* (2021a) refer to this additional error as parallax offset because it is not a single value but depends on the color or/and magnitude of the source and its position on the sky. Riess *et al.* (2018a) found that the parallax offset can be measured directly from the Cepheids, but with a reduced precision of the distance scale from GDR2. This reduction leads to a increased uncertainty of the determination of  $H_0$  value by a factor of 2.5.

Recently the Gaia team presented the Gaia Early Data Release 3 (EDR3) (the full Gaia DR3 release is expected in 2022) (Gaia Collaboration *et al.* 2020) with improved parallaxes since GDR2. Using the EDR3 parallaxes and Cepheid PL relation the latest analysis of the SH0ES Team (Riess *et al.* 2021a) achieved a precision of 1.0% in the geometric calibration of Cepheid luminosities. The precision of the geometric calibration of Cepheids will approach 0.5% by Gaia DR4 (Riess *et al.* 2021a). This higher precision will be sufficient to confirm the present  $H_0$  tension.

- **James Webb Space Telescope:** The James Webb Space Telescope (JWST or 'Webb') (Gardner *et al.* 2006; Sabelhaus and Decker 2004) is a joint NASA-ESA-CSA (National Aeronautics and Space Administration -European Space Agency-Canadian Space Agency) large, cold (under 50 K), infrared optimized ( $0.6 < \lambda < 28.30 \mu\text{m}$ ), space telescope and its launch is currently planned for 31 October 2021. JWST is a scientific successor to HST and will extend its discoveries to higher redshifts. It is nearly twice as big as HST with 6.6 m gold-plated primary mirror much larger than 2.4 m of HST.

The two main goals of this upcoming, next-generation telescope are to look much closer to the Big Bang and to investigate the light from the first stars and galaxies that formed in the Universe (see Gardner *et al.* 2006, for other goals). The observational data of this mission will essentially enhance our understanding of the formation and evolution of galaxies, stars, and planetary systems. The JWST will detect galaxies out to a redshift of  $z \geq 15$ . It will probably be able to detect Pop III stars (see Subsection III.7) in the high-redshift galaxies (Rydberg *et al.* 2013; Zackrisson *et al.* 2011) in a mass range  $140 - 260 M_\odot$  as pair-instability supernovae (Hartwig *et al.* 2018; Whalen *et al.* 2013). Various projects using the JWST observations will provide stronger nucleosynthesis constraints inside the first supernova (Bromm 2013) and constraints on the nature of dark matter (Maio and Viel 2015; Schultz

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<sup>59</sup> Planck was the third generation space mission which mapped the anisotropies of CMB.

*et al.* 2014). These constraints and other unprecedented information from JWST could potentially help address the lithium problem, explain small-scale curiosities and improve constraints on the age of Universe.

- **Simons Observatory:** The Simons Observatory (SO) (Abitbol *et al.* 2019; Ade *et al.* 2019; Galitzki *et al.* 2018) is a next generation CMB ground-based experiment. SO consists of one  $6\text{ m}$  Large Aperture Telescope (LAT) and three  $0.42\text{ m}$  Small Aperture Telescopes (SATs) at the Atacama Desert, Chile. It will provide more accurate measurements of the primary CMB temperature and polarization signals. The main targets of SO as described by Ade *et al.* (2019) are: primordial perturbations, effective number of relativistic species, neutrino mass, deviations from  $\Lambda\text{CDM}$ , galaxy evolution (feedback efficiency and non-thermal pressure in massive halos) and reionization (measurement of duration). Also a goal of SO survey is to provide a catalog of 16.000 galaxy clusters and more than 20.000 extragalactic sources. The sky region from SO survey overlaps with many surveys such as LSST, DES, DESI and Euclid at different wavelengths (Ade *et al.* 2019). This overlap is extremely beneficial as it will allow data cross correlation tests (for a detailed discussion, see Dodelson *et al.* 2016). Like CMB-S4, SO will provide improved constraints on the effective number of relativistic species  $N_{\text{eff}}$ , the sum of the neutrino masses  $\sum m_\nu$  and the dark energy equation of state  $w_{DE}$  (see Ade *et al.* 2019, for the forecast constraints on cosmological parameters). Also the SO and CMB-S4 experiments will measure the primordial tensor-to-scalar ratio  $r$  to a target sensitivity of  $\sigma_r \sim 0.002$  (for an  $r = 0$  model). This will be an improvement by a factor of approximately 5 compared to Planck sensitivity. In addition the uncertainty of the determinations of  $H_0$  from SO will be two and five times better than that inferred from Planck and local direct measurement respectively. Therefore, the SO data will enable us to improve constraints on extensions of  $\Lambda\text{CDM}$  which alleviate its tensions and curiosities. In addition the improved quality of lensing data from SO as well as CMB-S4 will improve our understanding of the CMB anisotropy anomalies.

- **SPT-3G:** This is a third generation CMB experiment (Benson *et al.* 2014; Dutcher *et al.* 2021). It uses the third survey camera SPT-3G which was installed on the South Pole Telescope (SPT) in 2017. The SPT-3G with the 10-meter diameter telescope targets at least  $1,500\text{ deg}^2$  region of low-foreground sky in three spectral bands centered at 95, 150, and  $220\text{ GHz}$  with  $\sim 16,000$  detectors (10 times more than its predecessor SPTpol Austermann *et al.* 2012; Keisler *et al.* 2015).

Its scientific goals aim to constrain the physics of

the cosmic inflation, to explore the neutrino sector, and to constrain the relativistic energy density of the Universe (Benson *et al.* 2014; Dutcher *et al.* 2021). The SPT-3G survey in combination with the deep and wide optical survey DES, will provide detailed data on  $\sim 200\text{ Mpc}$  scales which may be used to test General Relativity. The SPT-3G will also provide stringent and improved constraints on the effective number of relativistic species,  $N_{\text{eff}}$  and on the sum of the neutrino masses  $\sum m_\nu$  by synergy with Planck.

- **GAUSS:** This space mission concept combines the WL and galaxy clustering probes using three two-point correlation functions ( $3 \times 2\text{pt}$  analysis) of gravitational lensing and galaxy positions: the cosmic shear, the galaxy clustering and galaxy-galaxy lensing. GAUSS aims to fully map the cosmic web up to redshift  $z \sim 5$  and to provide a catalog with the spectroscopic redshifts and the shapes of 10 billions of galaxies (Blanchard *et al.* 2021) increased by a factor of approximately  $10^3$  compared to DESI which measures the spectra of 35 million galaxies and quasars (Aghamousa *et al.* 2016a). The very large sky coverage and the high galaxy density provided by the GAUSS will facilitate the construction of the 3D matter power spectrum of all scales (large and small) in detail. The  $3 \times 2\text{pt}$  correlation functions in combination with 3D matter power spectrum will provide stronger constraints and break parameter degeneracies (Blanchard *et al.* 2021). The constraining power of the GAUSS will be an order of magnitude larger than that of any currently planned projects such as Euclid and Vera C. Rubin Observatory.

- **Laser Interferometer Gravitational-Wave Observatory:** The Laser Interferometer Gravitational-Wave Observatory (LIGO) proposed by Abramovici *et al.* (1992) is a large-scale experiment that uses ground-based laser interferometers with  $L = L_x = L_y = 4\text{ km}$  long orthogonal arms to detect GWs<sup>60</sup>.

There are two identical LIGO instruments, one in Hanford (LHO) and one in Livingston (LLO) separated by roughly  $3000\text{ km}$ . The principle of op-

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<sup>60</sup> Another class of GW detectors are the resonant mass antennas (Aguiar 2011; Astone *et al.* 2010; Maggiore 2000; Pizzella 1997) in the frequency range from  $15\text{ Hz}$  to few  $k\text{Hz}$ . The principle of operation of mass resonance detectors is related to the periodic dimensional changes caused by the ripple effect of GWs on solid bodies. The Weber bar (Weber 1960) is a first generation resonant mass detector, the ALLEGRO (Mauceli *et al.* 1996), NAUTILUS (Amaldi *et al.* 1990; Astone *et al.* 1996, 1997), EXPLORER (Ciufolini and Fidecaro 1997), AURIGA (Cerdonio *et al.* 1997), NIOBE (Blair *et al.* 1995; Heng *et al.* 1996) are the second generation and Mario Schenberg (Aguiar *et al.* 2002), MiniGRAIL (de Waard *et al.* 2003) are the third generation.

eration of laser interferometers (Gertsenshtein and Pustovoit 1962; Moss *et al.* 1971) is similar to that of a simple interferometer, such as that used by Michelson and Morley. Detection of GWs with strain amplitude  $h \sim 10^{-21}$  by a ground detector with arms of length  $L = 4\text{ km}$  requires length change measurement (Gertsenshtein and Pustovoit 1962)

$$\Delta L = \delta L_x - \delta L_y \sim hL \sim 4 \cdot 10^{-18} \text{ m} \quad (4.1)$$

For the period between 2002 and 2010, the two LIGO observatories were unable to detect GWs. The detectors were later replaced by much improved Advanced LIGO versions (Aasi *et al.* 2015; Harry 2010). The improved detectors that officially went into operation in 2015 have about ten times the sensitivity to detect GWs in the frequency range around  $\sim 100\text{ Hz}$  compared to the initial LIGO interferometers (Aasi *et al.* 2015). In addition Advanced LIGO extends the low frequency end from  $40\text{ Hz}$  down to  $10\text{ Hz}$ . Much of the research and development work for LIGO/Advanced LIGO projects was based on the groundbreaking work of the GEO 600 detector (Affeldt *et al.* 2014; Willke *et al.* 2002) which is a  $600\text{ m}$  interferometer in Hanover, Germany.

On February 11, 2016 the LIGO Scientific Collaboration (Abbott *et al.* 2009) and the Virgo Collaboration (Acernese *et al.* 2008; Giazotto 1990) announced the first directly observed GWs from a signal detected on September 14, 2015 by the Advanced LIGO devices (the Virgo was not working at the time due to an upgrade). The detected signal was named GW150914 and its source was the merger of two stellar-mass BHs (Abbott *et al.* 2016).

The Advanced Virgo (Acernese *et al.* 2015) with  $3\text{ km}$  arm length interferometer contributes to the reliability of Advanced LIGO experimental device detections allowing for greater accuracy in locating the source in the sky (triangulation i.e 3-detector localization) (Singer and Price 2016) and more accurate reconstruction of the signal waveform (for the relevant method, see Cornish and Littenberg 2015; Cornish *et al.* 2021; Dálya *et al.* 2021; Ghonge *et al.* 2020; Littenberg and Cornish 2015). For example, in the case of the event GW170814 the three detectors improved the sky localization of the source, reducing the area of the 90% credible region from  $1160\text{ deg}^2$  using only the two LIGO detectors to  $60\text{ deg}^2$  using all three LIGO/Virgo detectors and reduced the luminosity distance uncertainty from  $570_{-230}^{+300}\text{ Mpc}$  to  $540_{-210}^{+130}\text{ Mpc}$  (Abbott *et al.* 2017c).

In 2019 the Advanced LIGO (Aasi *et al.* 2015), the Advanced Virgo (Acernese *et al.* 2015) and

the Japanese successor of the Tama300 (Ando *et al.* 2001; Arai 2008), Kamioka Gravitational (KAGRA) wave detector (Akutsu *et al.* 2020; Aso *et al.* 2013; Somiya 2012) (previously called LCGT Kuroda 2010) signed collaboration agreement to begin joint observation. The LIGO, Virgo and KAGRA collaboration will be probably complemented by other interferometers like the planned Indian LIGO by the Indian Initiative in Gravitational Wave Observations (IndIGO) consortium (Unnikrishnan 2013). In addition a future third-generation ground-based detector the Cosmic Explorer (CE) (Abbott *et al.* 2017b; Reitze *et al.* 2019a,b) is envisioned to begin operation in the 2030s in the USA. It will contribute to the GW Astronomy beyond LIGO. CE with ten times longer arms ( $40\text{ km}$ ) than Advanced LIGO's will amplify the amplitude of the observed signals (Chamberlain and Yunes 2017; Essick *et al.* 2017) and will significantly increase the sensitivity of the observations (Reitze *et al.* 2019a,b).

Many events ( $\sim 50$  compact binary coalescences) were observed by the Advanced LIGO/Virgo interferometers during three observing run periods (O1, O2 and O3)<sup>61</sup>. The full three-detector network provided data which enabled the standard siren measurement of the Hubble constant  $H_0$  (see Subsections II.1.4). These data are not yet sufficiently constraining the Hubble constant but in the future they are expected to improve significantly.

- **Laser Interferometer Space Antenna:** The Laser Interferometer Space Antenna (LISA) (Amaro-Seoane *et al.* 2017; Caprini *et al.* 2016) is a large-scale space mission proposed by ESA, planned for launch in 2034. It will consist of three spacecrafts placed in an equilateral triangle with arms 2.5 million kilometers long which will be placed near the Earth in a heliocentric orbit. In order to pave the way for the LISA mission ESA launched LISA Pathfinder in 2015 and it was operational from 2016 to 2017 (Armano *et al.* 2016, 2018). The results from scientific research show that LISA Pathfinder works exactly five times better than required, with a successful demonstration of the basic technologies for a large gravitational wave observatory.

LISA is designed to detect GWs in the frequency range from  $0.1\text{ mHz}$  to  $10^{-1}\text{ Hz}$  (Amaro-Seoane

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<sup>61</sup> For the first Gravitational Wave Transient Catalog (GWTC-1) during O1 and O2, see Abbott *et al.* (2019a), for second Gravitational Wave Transient Catalog (GWTC-2) from the first part of the third observing run (O3a), see Abbott *et al.* (2020b) and for third Gravitational Wave Transient Catalog (GWTC-3) from the second part of the third observing run (O3a), see Abbott *et al.* (2021c)

*et al.* 2012, 2013) targeting very different source populations from ground-based detectors such as LIGO, Virgo and KAGRA which operate in the frequency range<sup>62</sup> from  $10\text{ Hz}$  to  $10^3\text{ Hz}$  (Abbott *et al.* 2018a).

There are many different sources of GWs (see Cai *et al.* 2017a; Guzzetti *et al.* 2016; Sathyaprakash and Schutz 2009, for a review of GW physics). LIGO and Virgo can detect the merger events of binaries with masses  $\lesssim 100M_\odot$  while LISA will be able to detect the merger of massive BHs ( $10^5 - 10^7M_\odot$ ) with higher signal-to-noise ratio (SNR) and thus to perform precision tests in the strong gravity regime of  $\Lambda\text{CDM}$  model. LISA will detect events lasting weeks, months or years allowing us to observe a much larger volume of the Universe. It may improve our understanding of the early Universe. In addition the LISA mission will be able to detect sources like primordial BHs ( $\sim 10^{-12}M_\odot$ ) which correspond to the mHz frequency (Bartolo *et al.* 2019a,b; Cai *et al.* 2019). This possibility can help to test primordial BH dark matter scenario. Finally, LISA and the Big Bang Observer (BBO) (Crowder and Cornish 2005; Harry *et al.* 2006), which is a proposed LISA's successor will detect many other known or currently unknown exotic sources. Thus it will enable us to explore alternative gravity theories and to address the problems of the  $\Lambda\text{CDM}$  cosmology.

- **Taiji:** Taiji (Hu and Wu 2017; Ruan *et al.* 2020a) meaning ‘supreme ultimate’ is a Chinese large-scale space mission, planned for launch in 2033. Like LISA, Taiji is a laser interferometric GW detector which will consist of three spacecraft placed in an equilateral triangle with arms 2 million kilometers long in orbit around the Sun. Taiji will detect GWs in the frequency range from  $0.1\text{ mHz}$  to  $10^{-1}\text{ Hz}$ . Like LISA, Taiji can detect many possible GW sources such as a stochastic GW background generated in the early Universe and the merger of two super massive BHs.

A potential LISA-Taiji network was explored by Ruan *et al.* (2019b, 2020b). This network with a separation distance of about  $0.7\text{ AU}$  can accurately localize the sky position of a GW source and may completely identify the host galaxy.

- **TianQin :** TianQin (Luo *et al.* 2016; Mei *et al.* 2020) is a Chinese large-scale space mission. It aims to launch a laser interferometric GW detector around 2035. Like other space-base observatories, TianQin observatory consist of three space-crafts placed in an equilateral triangle with arms

$\sim 0.1\text{ Gm}$  long but in geocentric orbit with an orbital radius of about  $10^5\text{ km}$  (Luo *et al.* 2016; Mei *et al.* 2020). TianQin aims to detect GWs in the frequency range from  $10^{-4}\text{ Hz}$  to  $1\text{ Hz}$  (overlapping with that of LISA near  $10^{-4}\text{ Hz}$  and with that of DECIGO near  $1\text{ Hz}$ ). It will search for GW signals from various cosmological sources such as the inspiral of supermassive BBH (Feng *et al.* 2019), stellar-mass BBH (Liu *et al.* 2020b), the merger of massive BBHs (Wang *et al.* 2019) and stochastic GW background originating from primordial BHs (Di and Gong 2018) and/or cosmic strings (Olmez *et al.* 2010). As a precursor mission of TianQin, TianQin-1 experimental satellite has been launched on 20 December 2019. The results from scientific research shows that TianQin-1 satellite has exceeded all of its mission requirements.

- **Deci-hertz Interferometer Gravitational wave Observatory:** The DECi-hertz Interferometer Gravitational wave Observatory (DECIGO) is a Japanese large-scale space mission (Kawamura *et al.* 2006, 2008a,b, 2011) which was proposed by Seto *et al.* (2001) and is planned for launch in 2027. DECIGO consists of four clusters (with two of them at the same position) and each cluster consists of three spacecrafs placed in an equilateral triangle with  $1000\text{ km}$  arm lengths in heliocentric orbit (Kawamura *et al.* 2021). As a precursor mission of DECIGO, B-DECIGO (smaller version of DECIGO) will be launched before 2030 with  $100\text{ km}$  arm lengths orbiting around the earth at  $2000\text{ km}$  altitude above the surface of the earth (Kawamura *et al.* 2019, 2021; Sato *et al.* 2017b).

DECIGO is designed to detect GWs in the frequency range from  $0.1\text{ Hz}$  to  $10\text{ Hz}$  which is located in a gap between the frequency band of the LISA/Taiji and ground-based detectors such as advanced LIGO, advanced Virgo, and KAGRA. It aims to observe the primordial gravitational waves i.e. the beginning of the universe ( $10^{-36} - 10^{-34}\text{ sec}$  right after the birth of the Universe), the formation of giant black holes in the center of galaxies and the compact binaries, such as white dwarf binaries (Farmer and Phinney 2003).

- **Einstein Telescope:** Einstein Telescope (ET) or Einstein Observatory is a European proposed underground laser interferometric GW detector (Punturo *et al.* 2010). It will be located underground at a depth of about  $100 - 300\text{ m}$  in order to reduce the seismic noises. ET will consist of three nested detectors placed in an equilateral triangle, each in turn composed of two interferometers with arms  $10\text{ km}$  long. Using two arms in each side of the triangle will enable the determination of the polarisation of GWs. As a third-generation observatory is targeting a sensitivity 10 times better than of current second-generation laser-interferometric detec-

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<sup>62</sup> The frequency range from  $0.1\text{ mHz}$  to  $10^{-1}\text{ Hz}$  is unobservable by any proposed ground based detectors, due to seismic noise.

tors such as advanced LIGO, advanced Virgo, and KAGRA (Hild *et al.* 2010). ET will reduce thermal noise compared to the first and second generations of GW detectors by operating the mirrors at cryogenic temperatures as low as  $10\text{ K}$  (Hild *et al.* 2011). It is planned to start observations in 2035 with two candidate sites: north of Lula in Sardinia (Italy) and in Meuse-Rhine Euroregion (the border area of Belgium, Germany, and the Netherlands) (Amann *et al.* 2020). ET will detect GWs in the frequency range from  $\sim 1\text{ Hz}$  to  $\sim 10\text{ kHz}$ . This will allow the detection of BNS up to a redshift of  $z \sim 2$ , stellar-mass BBH at  $z \sim 15$ , and intermediate-mass BBH ( $10^2 - 10^4 M_\odot$ ) at  $z \sim 5$ . The observations of these standard sirens will be useful to calibrate the cosmic distance ladder and will improve the estimation of the Hubble constant.

Cosmology is entering an even more exciting era! The combination of the existing puzzling observational signals discussed in this review, along with the upcoming revolutionary improvement in the quality and quantity of data creates anticipation for exciting new effects and new physics discoveries in the coming two decades.

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## A Appendix

In this appendix we present the list of used acronyms.

TABLE VII: List of used acronyms.

Acronym	Meaning	Acronym	Meaning
ACS	Advanced Camera for Surveys	KiDS	Kilo Degree Survey
ACT	Atacama Cosmology Telescope	LAT	Large Aperture Telescope
AEDGE	Atomic Experiments for Dark Matter and Gravity Exploration	LDE	Late Dark Energy
AGB	Asymptotic Giant Branch	LIGO	Laser Interferometer Gravitational-Wave Observatory
AGN	Active Galactic Nucleus	LISA	Laser Interferometer Space Antenna
AIC	Akaike Information Criterion	LMC	Large Magellanic Cloud
AO	Adaptive Optics	LOS	Line-Of-Sight
AvERA	Average Expansion Rate Approximation	LSS	Large Scale Structure
BAO	Baryon Acoustic Oscillations	LSST	Large Synoptic Survey Telescope
BBH	Binary Black Holes	LwMPT	Late $w - M$ Phantom Transition
BBN	Big Bang Nucleosynthesis	MCP	Megamaser Cosmology Project
BBO	Big Bang Observer	MCT	Multi-Cycle Treasury
BH	Black Hole	MEDE	Modified Emergent Dark Energy
BIC	Bayesian Information Criterion	MGS	Main Galaxy Sample
BNS	Binary Neutron Stars	MM	Many Multiplet
BOSS	Baryon Oscillation Spectroscopic Survey	MST	Mass Sheet Transformation
BTFR	Baryonic Tully Fisher Relation	MW	Milky Way
CC	Cluster Counts	NANOGrav	North American Nanohertz Observatory for Gravitational-waves
CCH	Cosmic Chronometric	NASA	National Aeronautics and Space Administration
CCHP	Carnegie–Chicago Hubble Program	NEDE	New Early Dark Energy
CDI	Cold Dark matter Isocurvature	NIR	Near InfraRed
CDM	Cold Dark Matter	NRAO	National Radio Astronomy Observatory
CE	Cosmic Explorer	NS	Neutron Star
CFHTLenS	Canada-France-Hawaii Telescope Lensing	NVSS	NRAO VLA Sky Survey
CHP	Carnegie Hubble Program		

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TABLE VII – continued from previous page

Acronym	Meaning	Acronym	Meaning
CL	Confidence Level	PEDE	Phenomenologically Emergent Dark Energy
CMB	Cosmic Microwave Backgroun	PL	Period–Luminosity
COBE	Cosmic Background Explorer	PTAs	Pulsar Timing Arrays
COSMOGRAIL	COSmological MOonitoring of GRAVItational Lenses	QFT	Quantum Field Theory
CP	Cosmological Principle	QSO	Quasi-Stellar Object (quasar)
CPL	Chevallier - Polarski - Linder	ROSAT	ROentgen SATellite
CSA	Canadian Space Agency	RSD	Redshift Space Distortions
CSP	Carnegie Supernova Project	RVM	Running Vacuum Model
DE	Dark Energy	SATs	Small Aperture Telescopes
DEBs	Detached Eclipsing Binary stars	SBF	Surface Brightness Fluctuations
DECIGO	DECi-hertz Interferometer wave Observatory	SDSS	Sloan Digital Sky Survey
DES	Dark Energy Survey	SH0ES	Supernovae $H_0$ for the Equation of State
DESI	Dark Energy Spectroscopic Instrument	SKA	Square Kilometre Array
DIC	Deviance Information Criterion	SLACS	Sloan Lens ACS Survey
DM	Dark Matter	SM	Standard Model
EBL	Extragalactic Background Light	SMBH	SuperMassive Black Hole
eBOSS	Extended Baryon Oscillation Spectroscopic Survey	SnIa	Supernova Type Ia
EDE	Early Dark Energy	SneII	Supernovae Type II
EDGES	Experiment to Detect the Global Epoch of Reionization Signature	SNR	Signal-to-Noise Ratio
EDR	Early Data Release	SO	Simons Observatory
EDS	Early Dark Sector	SOLME	Stanford Optically Levitated Microsphere Experiment
EFTofLSS	Effective Field Theory of Large-Scale Structure	SPH	Smooth Particle Hydrodynamics
EM	ElectroMagnetic	SPT	South Pole Telescope
EPTA	European Pulsar Timing Array	STRIDES	STRong-lensing Insights into Dark Energy Survey
eROSITA	extended ROentgen Survey with an Imaging Telescope Array	TBTF	Too Big To Fail
ESA	European Space Agency	TD	Time-Delay
ET	Einstein Telescope	TDCOSMO	Time-Delay COSMOgraphy
ETHOS	Effective THeory Of Structure formation	TDE	Transitional Dark Energy
FJ	Faber–Jackson	TFR	Tully-Fisher Relation
FLRW	Friedmann-Lemaître-Roberson-Walker	TGSS	TIFR GMRT Sky Survey
FMOS	Fiber Multi-Object Spectrograph	TIFR	Tata Institute of Fundamental Research
FP	Fundamental Plane	TPCF	Two-Point Correlation Functions
GAMA	Galaxy and Mass Assembly	TRGB	Tip of the Red Giant Branch
GAUSS	Gravitation And the Universe from large Scale-Structures	tSZ	thermal Sunyaev-Zel'dovich
GDR	Gaia Data Release	UV	Ultraviolet
GEDE	Generalised Emergent Dark Energy	UVES	Ultraviolet and Visual Echelle Spectrograph
GEHR	Giant Extragalactic HII Region	VCDM	Vacuum Cold Dark Matter
GGI	Galaxy-Galaxy Lensing	VHE	Very High Energy
GMRT	Giant Metrewave Radio Telescope	VIKING	VISTA Kilo-Degree Infrared Galaxy
GP	Gaussian Process	VIMOS	VIable MultiObject Spectrograph
GR	General Relativity	VIPERS	VIMOS Public Extra-galactic Redshift Survey
GRB	Gamma-Ray Burst	VISTA	Visible and Infrared Survey Telescope for Astronomy
GW	Gravitational Waves	VLT	Very Large Telescope
GWTC	Gravitational Wave Transient Catalog	VM	Vacuum Metamorphosis
HMF	Halo Mass Function		
HSC	Subaru Hyper Suprime-Cam lensing survey		
HST	Hubble Space Telescope		

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TABLE VII – continued from previous page

Acronym	Meaning	Acronym	Meaning
H0LiCOW	$H_0$ Lenses in COSMOGRAIL's Wellspring	VSF	Violent Star Formation
ICM	IntraCluster Medium	VVDS	VIMOS-VLT Deep Survey
IDE	Interacting Dark Energy	WISE	Wide-field Infrared Survey Explorer
IGM	InterGalactic Medium	WL	Weak Lensing
ISW	Integrated Sachs-Wolfe	WMAP	Wilkinson Microwave Anisotropy Probe
JWST	James Webb Space Telescope	WtG	Weighting the Giant
IndIGO	Indian Initiative in Gravitational wave Observations consortium	ZTF	Zwicky Transient Facility
IPTA	International Pulsar Timing Array	2dFGRS	2-degree Field Galaxy Redshift Survey
JLA	Joint Light-curve Analysis	2dFlenS	2-degree Field Lensing Survey
KAGRA	Kamioka Gravitational	6dFGS	6-degree Field Galaxy Survey

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