

Challenges in teaching mathematical reasoning and proof – Introduction

Christine Knipping, Oldenburg (Germany)

This special issue presents recent work in the research area of proof. The ideas discussed in the articles open new perspectives from very different points of view. Patricio Herbst in the first article discusses the role of diagrams in proving from a socio-mathematical standpoint. David Reid & Rosemary Roberts in the second article focus on the learners' perspective and specifically their criteria for explanations. In the third article Aiso Heinze presents results of interviews with grade 8 students who participated in a study investigating students' abilities to perform geometrical proofs. Finally, Aiso Heinze, Ying-Hao Cheng & Kai-Lin Yang take an international comparative approach, comparing students' performance in reasoning and proof in Taiwan and Germany. The articles readdress important questions that are fundamental to the field of proof and have not yet been solved: How could teaching encourage students to take more responsibility in proving? What are conditions for teaching that enable students and teachers to share criteria for mathematically acceptable explanations? How can we overcome our cultural biases of teaching proof?

Herbst addresses the question of how teaching could foster students' taking more responsibility, from a point of view that investigates the didactical and social constraints in the classroom. Concretely he analyses the role of interactions with diagrams in proving activities. He makes visible how certain forms of interactions with diagrams help the teacher and the students to come up with a proof. His socio-mathematical as well as semiotic perspective on proving activities in the mathematics classroom provides intriguing insights into the challenge of using diagrams in proving in geometry. From a cognitive, epistemological and psychological perspective a wide range of research has already explored the dialectic role of diagrams in proving: the visual force of representations, on the one side, and the constraints imposed by the same representations on the other side. Recently, the educational research in the field of new technologies (e.g. Dynamic Geometry Software) has explored the implications of this dialectic for students' learning of mathematical proof.

Herbst offers here a different approach to exploring the role of diagrams in the teaching of proof. He provides theoretical explanations that help us to understand the problematique of teaching proof based on diagrams. Looking closer at the didactical constraints of the situation, he not only reconstructs, but also explains the different ways diagrams are used in proving tasks. He examines closely the work of the teacher and the work of the students. The complex interplay of teachers' constructions of diagrams and students' use of these diagrams is discussed. The importance of the didactical contract between teacher and students becomes obvious. Herbst also gives historical reasons, related to teaching

goals and principles, that are part of the didactical constraints in the situation. He shows how these principles still influence the teaching and learning of proof. Finally, he investigates attempts in a current research project to use diagrams in a way that fosters students own mathematical reasoning. But, he also gives examples of the challenges teachers face to use such diagrams within the constraints of the existing didactical contract. All this illustrates the complexity of the goal of students taking more responsibility in proving.

Reid & Roberts investigate the question of what conditions might enable students and teachers to share criteria for mathematically acceptable explanations. They first revisit large scale and longitudinal studies on students' preferences for proofs. Results of these studies show that many students for various reasons do not share mathematical criteria for explanations. Students, for example, declare arguments with logical flaws to be acceptable relying only on the form of the argument. They also produce proofs without sound underlying reasoning, trying to achieve certain formal characteristics of a proof. These results suggest that part of students' difficulties with proof is students' focus on the formulation of proofs, instead of the reasoning involved. Facing these results Reid & Roberts ask if these two aspects, reasoning and form, on which research focuses, are not due to a research bias. Because of our mathematically biased understanding of acceptable explanations we might tend in our research to focus too much on characteristics of an argument that are important to mathematics rather than students' thinking. Focussing on learners' perspectives Reid & Roberts find in their investigative small scale study that adult learners hold criteria for acceptable explanations that are different from the ones discussed above. These findings call for further research. Do other learners as well share the criteria of the adult learners investigated here? What further criteria might be important for acceptable arguments, according to students? What does this imply for teaching? Reid & Roberts raise these important questions and make us aware of a dilemma. If we respect students' criteria for acceptable arguments we risk being in conflict with mathematically acceptable arguments. If we force students to come up with mathematically sound arguments in class we might create a "teaching illusion" that these arguments are convincing, whereas students' might already react on the level of the didactical contract, instead of following the underlying reasoning. Herbst illustrates in his article how this is possible in the context of interactions with diagrams.

Heinze's interview study is an extension of a previous research project. In that project grade 7 and grade 8 students were given written tests on reasoning and proof. The analysis of their performance revealed that the students' had limited content and conceptual knowledge to support their reasoning as well as weaknesses in the structuring of their reasoning. Heinze interviewed a small number of these students in order to get further insights into their difficulties. The interviews show that students in interaction with an interviewer are able to demonstrate more knowledge, but still not enough to come up with a sound argument or even a plausible conjecture to be

proven. Most of the interviewed students had serious difficulties in structuring their reasoning in a way that the interviewer would accept as mathematical. The overall impression one gets is that these students have never before been asked as individuals to come up with a coherent argument or a plausible conjecture. As Herbst's article suggests the didactical contract framing their past school experiences may have deprived them of any responsibility for producing reasoned arguments. If this is the case it is hardly surprising that the interviewed students have these difficulties. This reveals the importance of teaching, but it is possible that this is not the whole story as the following article indicates.

Heinze, Cheng & Yang in their article discuss German and Taiwanese teaching styles as two examples for cultural biases in teaching proof in East Asia and Western Europe. But, the authors also illustrate that these teaching styles only partly explain why Taiwanese students perform significantly better in international surveys, and particularly well in national surveys on proving competencies. Teaching styles seem to be more explanatory for German students who only show very moderate or low mathematical performance in international studies, as well as in national surveys on proof. Video Studies on German classroom interactions demonstrate that students are generally not involved in cognitively demanding activities. The analyses of the videos also illustrate that students in general are not encouraged to take responsibility in proving, although the discursive interaction format in the classroom might suggest so on a surface level. Research describes a different teaching style in Asian countries. A recent video study in Taiwan as well as video studies of other Asian countries, report a teaching style in these countries that is based on a lecture by the teacher, dominated by demonstrating procedures, and exercise phases focused on procedural skills. Teachers rarely require reasoning activities. Heinze, Cheng & Yang see it as a paradox that Asian students nevertheless manage to belong to high achieving samples in comparative studies, even in complex problem solving and proving.

One hypothesis the authors come up with to explain this paradox is the familiarity of Asian students with tests and test questions. This might include familiarity with certain proving tasks in the tests. This relates to results in the article by Reid & Roberts in this issue who suggest that from a learners' perspective familiarity with an argument is more important than other characteristics of a proof. This could also explain why successfully producing a proof does not necessarily mean that the proof is understood on the level of reasoning. Future research needs to give more attention to this hypothesis.

Another important idea Heinze, Cheng & Yang come up with is to have a closer look at learning and learning strategies. Teaching styles are only one side of the coin, learning styles are the important other side. The authors develop initial conjectures about different culturally embedded learning styles. A closer view on students' learning strategies and on Asian mathematics classrooms from a socio-mathematical point of view, promises to provide new elements for understanding the Asian paradox.

Comparative studies allow us to learn from each other in many respects. One is to question our own implicit assumptions about teaching and learning. The article by Heinze, Cheng & Yang suggests that this might challenge us more than we would like.

Author

Knipping, Christine, Carl-von-Ossietzky-Universität, Institut für Mathematik, Fakultät 5, D-26111 Oldenburg
E-Mail: christine.knipping@uni-oldenburg.de