



# Changes in Cross-Correlations as an Indicator for Systemic Risk

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The 2008–2012 global financial crisis began with the global recession in December 2007 and exacerbated in September 2008, during which the U.S. stock markets lost 20% of value from its October 11 2007 peak. Various studies reported that financial crisis are associated with increase in both cross-correlations among stocks and stock indices and the level of systemic risk. In this paper, we study 10 different Dow Jones economic sector indexes, and applying principle component analysis (PCA) we demonstrate that the rate of increase in principle components with short 12-month time windows can be effectively used as an indicator of systemic risk—the larger the change of PC1, the higher the increase of systemic risk. Clearly, the higher the level of systemic risk, the more likely a financial crisis would occur in the near future.

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In finance, systemic risk is the risk associated with the whole financial system, as opposed to any individual entity or component<sup>1</sup>. It can be defined as any set of circumstances that threatens the stability of the financial system, and so potentially initiates financial crisis<sup>2</sup>. It generally holds that the larger systemic risk, the larger are the threats to financial stability. An example is a bank runs associated with a large group of clients deciding to withdraw their deposits immediately, creating shortage of cash that might lead to multiple bank failures and cascades into a global financial crisis<sup>3</sup>. Systemic risk is commonly defined as the probability of a series of correlated defaults among financial institutions, occurring over a short time span, which in turn trigger a widespread liquidity and loss of confidence in the financial system as a whole<sup>2,4</sup>. The 2008–2012 global financial crisis makes researchers re-cognize the importance of the measurement and forecast of systemic risk.

The empirical studies on systemic risk are loosely divided into three categories<sup>2,5</sup>. Two of them are directly related to the performance of banks. The first one involves bank contagion, and is mostly based on the auto-correlation of the number of bank defaults, bank returns and fund withdrawals<sup>6–14</sup>. The other one is focused on bank capital ratios and bank liabilities, and show that aggregate variables such as macroeconomic fundamentals, which provide evidence in favor of the macro perspective on systemic risk on the banking sector<sup>15–18</sup>.

The third group of empirical studies on the systemic risk put emphasis on contagion, spillover effects and comovement in financial markets<sup>3,19,20,22–29</sup>. These studies are based on cross-correlations and causality relationships among securities and currency price time series. Most of the studies are carried out on financial sectors such as banks, brokers, insurance companies and hedge funds<sup>22–25</sup>. Many measures of the systemic risks are based on principal components analysis (PCA) or Granger-causality test<sup>24–26</sup>. For example, Kritzman *et al.*<sup>25</sup> proposed a systemic risk measurement called the absorption ratio based on PCA, Billio *et al.*<sup>2</sup> used the first two eigenvalues to detect the systemic risk from banks, brokers, insurers and hedge funds. They also proposed an indicator named dynamic causality index (DCI) calculated from Granger-causality test to measure the degree of systemic risk. Kaminsky and Reinhart<sup>24</sup> used a simple vector auto-regression model to run Granger-causality test between the interest and exchange rates of five Asian economies before and after the Asian crisis. Previous work has shown that the first PC is closely related to the average correlation. Billio *et al.*<sup>2</sup> reported that four financial sectors including hedge funds, banks, brokers, and insurance companies become highly interrelated and less liquid prior to crisis periods. Recently, some of the security cross-correlation studies report that the first principal component substantially increases during financial crisis<sup>19,20</sup>. The same studies reported that the volatility cross-correlations exhibit long memory<sup>21</sup>, implying that once high volatility (risk)<sup>30,31</sup> is spread across the entire market, it could last for a long time.



On top of what was proposed by previous studies<sup>24,25</sup> that use principle components, we develop an alternative method to use change in principle component to capture the systemic risk dynamics. Here based on 10 major economic sectors of US economy and 10 major economic sectors of European economy, each represented by the Dow Jones sector index, we apply principal component analysis with 12-month moving windows and demonstrate that the steepest increase in the first principal component PC1 may serve as an indicator of systemic risk—the larger the peak in the change of PC1, the higher is the systemic risk. Since larger systemic risk means potentially unstable financial market, a large change of PC1 implies a crisis is likely to occur in the near future.

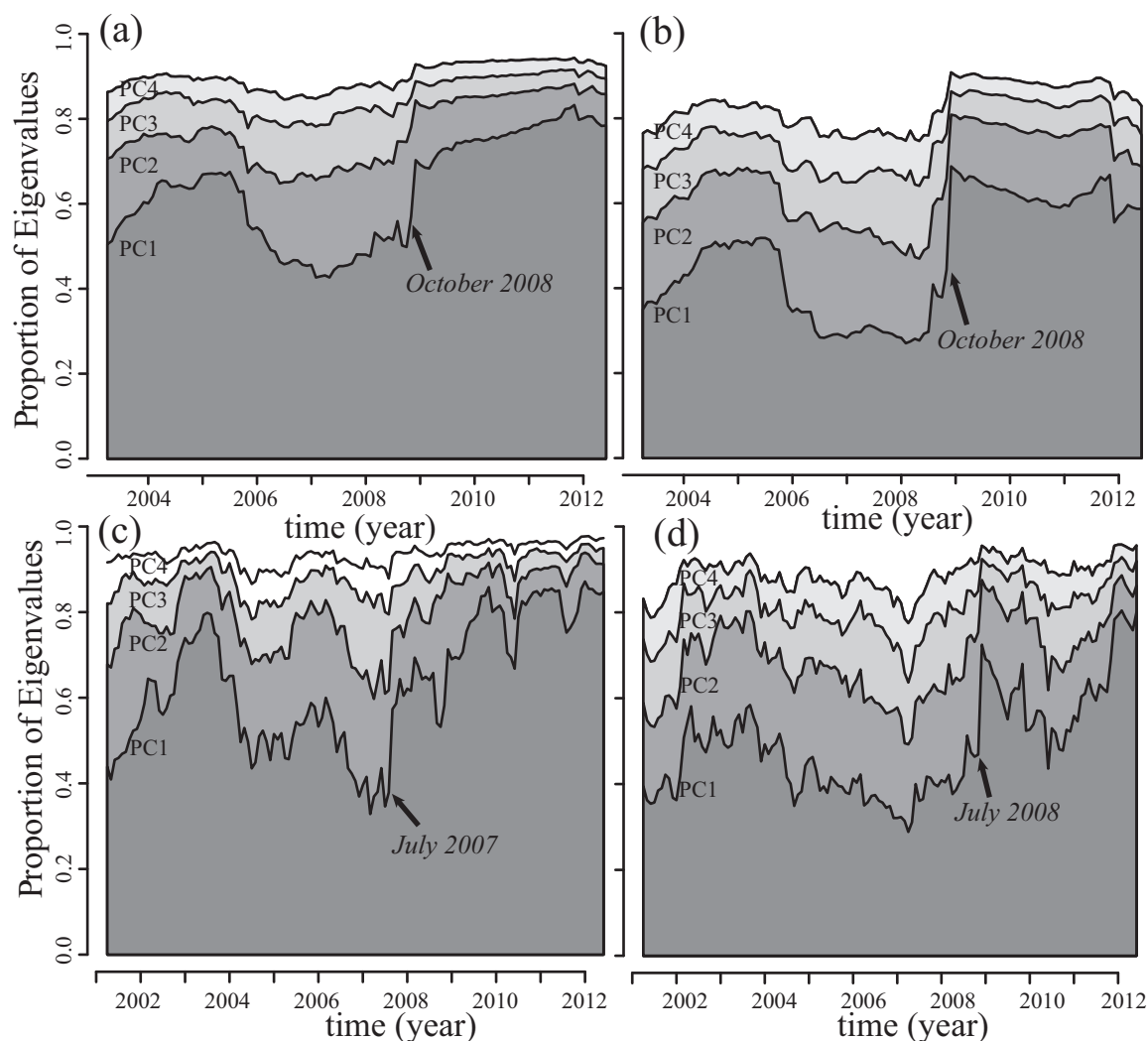
## Results

**Dynamics of principal components.** Billio *et al.*<sup>2</sup> suggested several econometric measures of systemic risk to quantify the interrelationship between the monthly returns of hedge funds, banks, brokers, and insurance companies based on principal components analysis (PCA) and Granger-causality tests. They reported that all four sectors have become highly intercorrelated over the past decade, increasing the level of systemic risk in the entire financial sector. The measures contain predictive power for the 2008–2012 global

financial crisis. When applying PCA, the basic idea is that systemic risk is getting higher when the largest eigenvalue increases explaining most of variation of the data. When applying Granger-causality test, the dynamic causality index (DCI) is defined in the way that an increase in the DCI indicates a higher level of system interconnectedness.

Here we study 10 major sectors of the US economy each quantified by the corresponding Dow Jones sector index and the Dow Jones Industrial Average. There are 11 indexes in total. Based on principal component analysis (PCA), using moving windows of size  $n$ , first we calculate the covariance at each time  $t$  based on previous  $n$  either (a) returns of Eq. (2) or (b) absolute returns of Eq. (3) (volatilities).

Fig. 1 shows the time series of the first four principal components (PCs), each quantifying the variability in (a) returns and (b) absolute returns. Note that the principal components are first normalized so that they sum up to one (see section Data). In (a) and (b) we use 36-month moving windows as commonly applied in financial literature<sup>2,4</sup>. For the time series of returns (absolute returns), we find that the first principal component captures the major variation in return (volatility) series. We also find that the eigenvalues from returns cover more variation than the eigenvalues of absolute returns. For example, the first eigenvalue of returns approximately captures the variation ranging from  $\approx 35\%$  to  $\approx 85\%$ . This is larger than the first



**Figure 1** | Principal components analysis of the monthly (a) return and (b) volatility of 10 Dow Jones Supersector indexes during March 2000 to June 2012 over 36-month rolling-window proportion of variability (eigenvalues) for principal components 1–4. Note that PC2 actually represents PC1+PC2, and accordingly PC4 represents PC1+PC2+PC3+PC4. (c) and (d) show the principal components 1–4 where 36-month moving time windows are replaced by 12-month moving time windows. Highest peaks locate at earlier months for 12-month windows.



eigenvalue of absolute returns which captures the variation ranging from  $\approx 30\%$  to  $\approx 70\%$ . For return time series, we find that the four PCs shows a rich dynamics over the entire period from April 2003 to June 2012. PC1 eigenvalue fluctuates from  $\approx 41\%$  at the beginning of 2007 to  $\approx 86\%$  in 2011. We find that PC1 captures  $\approx 60\%$  of variability among 10 major US sectors, and that the first and second PCs together (PC2 in figure represents PC1+PC2) explain  $\approx 77\%$  of the return variation. We note that roughly at the beginning of 2007 the proportion of variance explained by the first eigenvalue PC1 starts to increase almost monotonically. We find that the steepest increase of PC1 occurred in (a) only couple of months before 2009.

We note that the choice for window size  $n$  clearly affects the time coordinate which one can expect the steepest increase in cross-correlations. To this end, in Fig 1(c) we apply shorter 12-month moving windows instead of 36-month. The steepest increase in PC1 occurred in August 2007 before the global recession and before the markets started to decline.

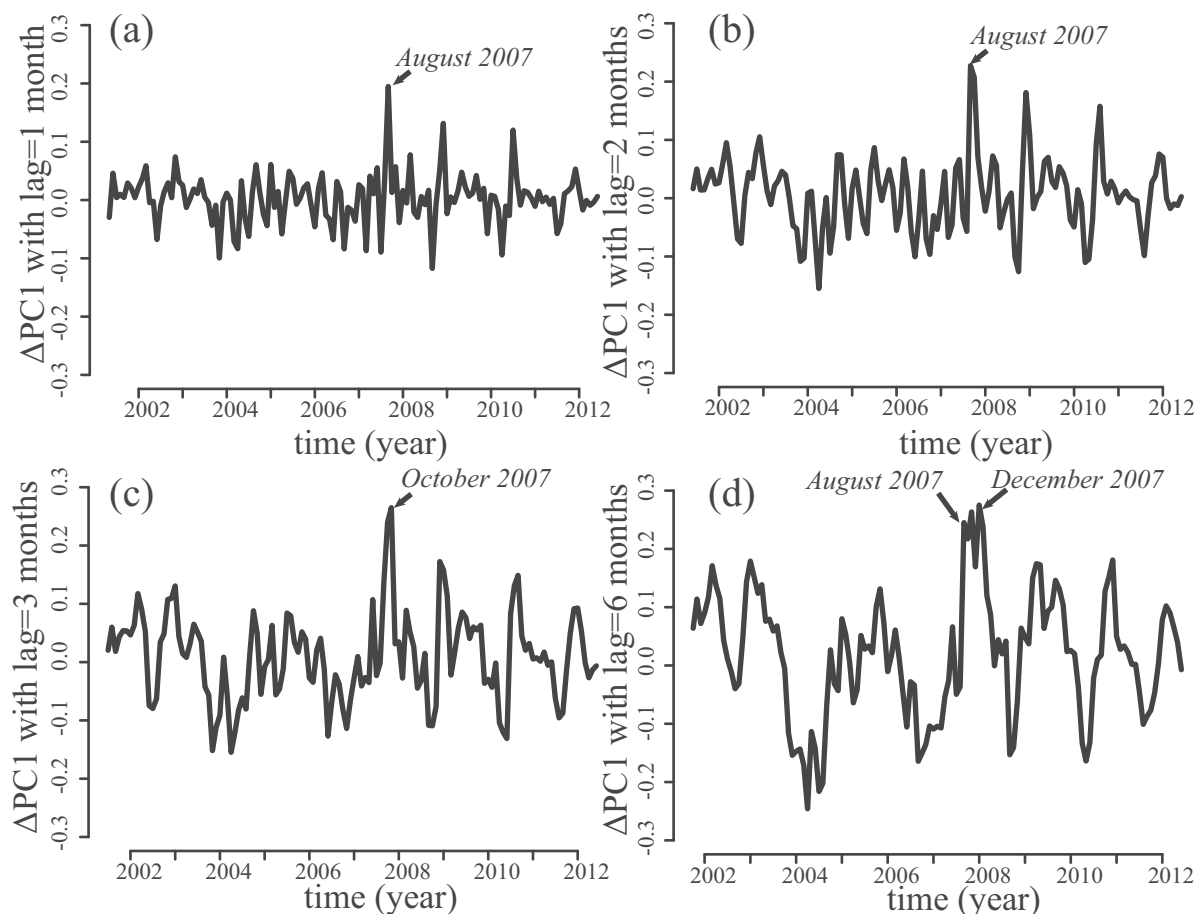
In Fig. 1(a) the steepest increase in PC1 occurred in late 2008, long after the crisis started. In Fig. 1(c), for 12-month moving windows—this has shorter window size than the one used in Fig. 1(a)—we show that the steepest increase of PC1 occurred before the 2007 global recession started in December 2007. We propose the following explanation: market crashes are always associated with large shocks, but if window size  $n$  is too large, large shocks are overridden by all other signals. Next we study the dynamics of PC1 in terms of the rate of change in PC1 values.

We define the temporal dependence of the change of the first component PC1 as

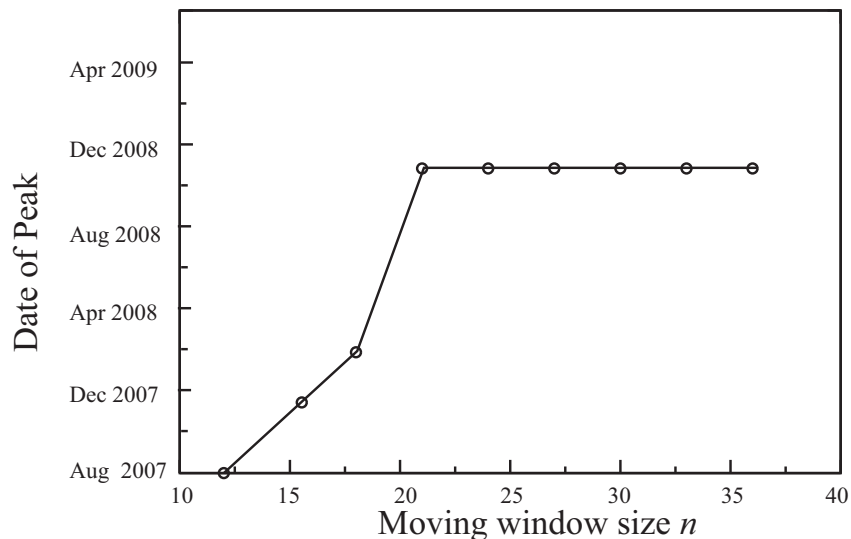
$$\Delta PC1(t) = PC1(t) - PC1(t - m). \quad (1)$$

Since the data are monthly recorded,  $m = 1$  denotes the change in PC1 within one month, and therefore, the choice  $m$  denotes the change in PC1 within  $m$  months. The change in PC1 for fixed 12-month moving windows and varying values of  $m$  are shown in Fig. 2. In Fig. 2(a) when  $m = 1$  we find that the largest increase in PC1 coincide as expected with the steepest increase in PC1 in Fig. 1(c) occurred in August 2007, that was a very special month for the financial system. It was the month when the Interbank market froze completely<sup>32</sup>. In Fig. 2(a)–(d) we find that with increasing  $m$ , in the estimator defined in Eq. (1) the change in PC1 becomes delayed, and that is the same mechanism we found when varying  $n$  in Fig. 1. To have a close examination of the effect of the size of window  $n$  on the forecasting power of our estimator, in Fig. 3 we show how the peak representing the largest increase in PC1 depends on  $n$ . We see that after  $n \approx 20$  the date where the peak occurs virtually saturates. Smaller time windows evidently have earlier peaks than larger ones.

Next in Fig. 4 for fixed 12-month moving windows we find that the probability distribution function (pdf) of all past changes in PC1 exhibits an asymmetric functional form (skewness = 0.29) where we hypothesize that the right-wing rare events are associated with highest increase in systemic risk. We also show the Gaussian pdf with the standard deviation equivalent to the one we found in PC1 data. Having information about the pdf of past changes in PC1, based on any future change in PC1 we can test how large the increase of systemic risk is.



**Figure 2** | For monthly recorded data we show changes of PC1 of returns for lag (a)  $m = 1$  month ( $m$  is defined in Eq. 1), (b)  $m = 2$  months, (c)  $m = 3$  months and (d)  $m = 6$  months calculated for 10 Dow Jones Supersector indexes during March 2000 to June 2012. In (a) the largest peak occurred in August 2007 before the global recession. With increasing  $m$  the highest peaks are becoming delayed.

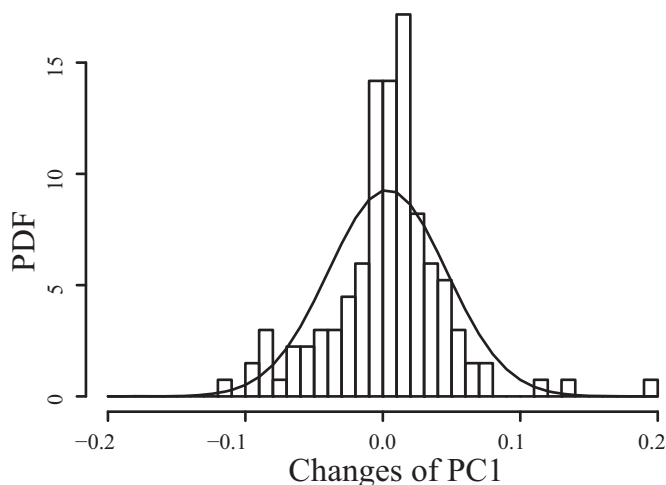


**Figure 3 | Size of the largest peak in PC1 as a function of window size  $n$ .** With increasing  $n$ , the largest peak increases first and saturates after approximately 20-month time window.

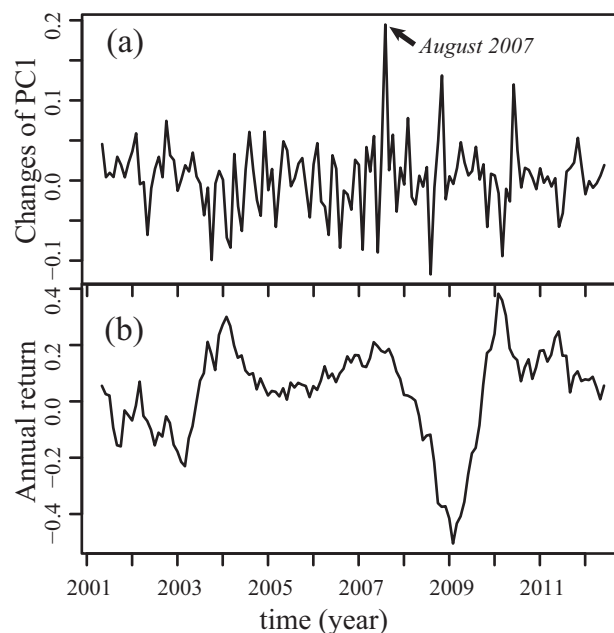
We further confirm our results by comparing in Fig. 5 the monthly change in PC1 values ( $m = 1$  in Eq. (1)) and 12-month (annual) returns of the Dow Jones Industrial Average. At each month  $t$ , using moving time window approach, a time series of changes in PC1 based on past 12-month returns—calculated between  $t$  and  $t - 12$ —and a time series of future 12-month returns of the Dow Jones Industrial Average—calculated between  $t$  and  $t + 12$ —are constructed. In Fig. 5 we find after the largest PC1 peak in August 2007 there is a rapid drop in index return. Even after the second largest peak in PC1 the annual return calculated for moving time window continues to decrease, suggesting that PCA with short-time windows can serve as a measure for systemic risk. Clearly, the higher the level of systemic risk, the more likely a financial crisis would occur in the near future.

To validate our approach, we study the U.S. financial sector that is a subset of the previous dataset, i.e. five U.S. financial indexes

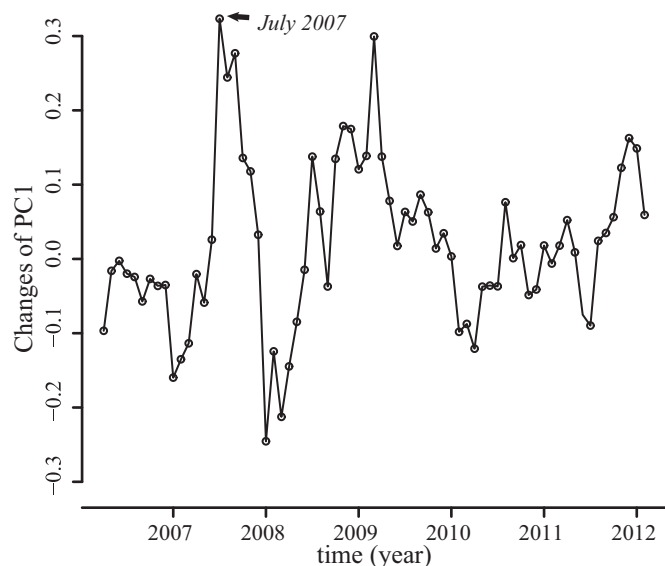
representing banking (KBW Bank Index), broker (NYSE Arca Securities Broker/Dealer Index), insurance companies (S&P 500 Insurance Index), hedge funds (Eurekahedge North American Hedge Fund Index), and real estate industry (Vanguard REIT Index Fund). Fig. 6 shows that financial sector also has the largest increase in PC1 occurred prior to financial crisis, in July 2007, which is prior to the global recession and global financial crisis. This implies that PCA formalism with the rate of increase in principle components and short-time windows may serve as an indicator for systemic risk.



**Figure 4 | Pdf of changes in the 1st principal component calculated for 10 Dow Jones Supersector indexes during March 2000 to June 2012 over 12-month moving windows.** The pdf is highly asymmetric and characterized by outliers. The density at extreme right is the highest peak in the change of PC1. It falls far from the distribution thus can be described as anomaly, since it is associated with the highest increase of the level of systemic risk. Theoretical pdf of Gaussian distribution (with equivalent average and standard deviation) is shown by the solid line. Here skewness and kurtosis are 0.29 and 5.49 respectively.

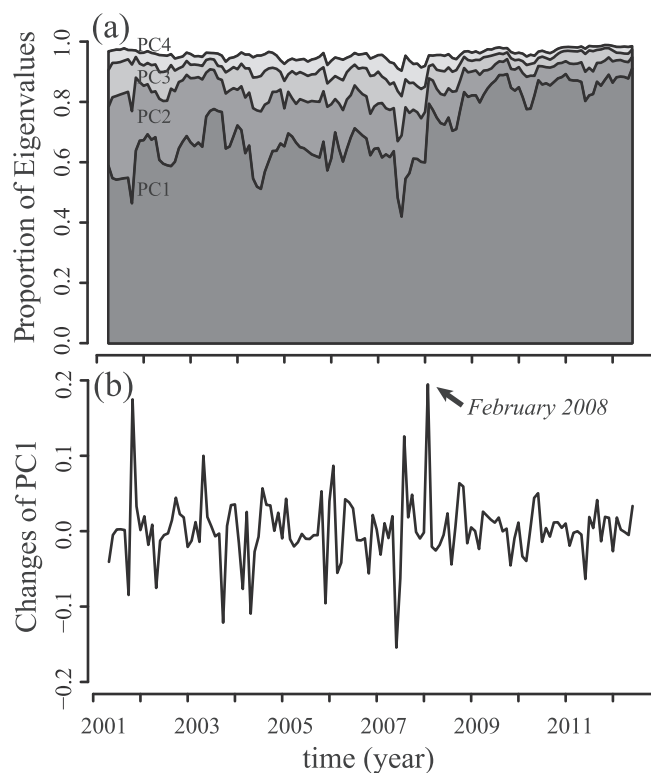


**Figure 5 | Past versus Future—comparison of monthly changes in PC1 values and future annual returns of the Dow Jones Industrial Average.** Both PC1 and returns are evaluated on 12-month time windows. However, PC1 values are calculated between  $t$  and  $t - 12$ , while returns are calculated between  $t$  and  $t + 12$ . It is evident that the largest change in PC1 value is followed by a large drop in 12-month returns, indicating a market crash. For the changes in PC1 values, we take  $m=1$  such that changes in one month are considered.



**Figure 6** | PC1 with short time windows. Changes for lag  $m = 1$  month of PC1 of returns calculated for five financial sector indexes representing banking, broker, insurance companies, hedge funds and real-estate industry over 12-month moving windows. The peak at July 2007 coincides with our finding for the Dow Jones indexes.  $m$  is defined in Eq. (1).

To further verify our findings, we study 10 major sectors of the European economy and separately 10 major sectors of the worldwide developed markets excluding the US and each sector is quantified by

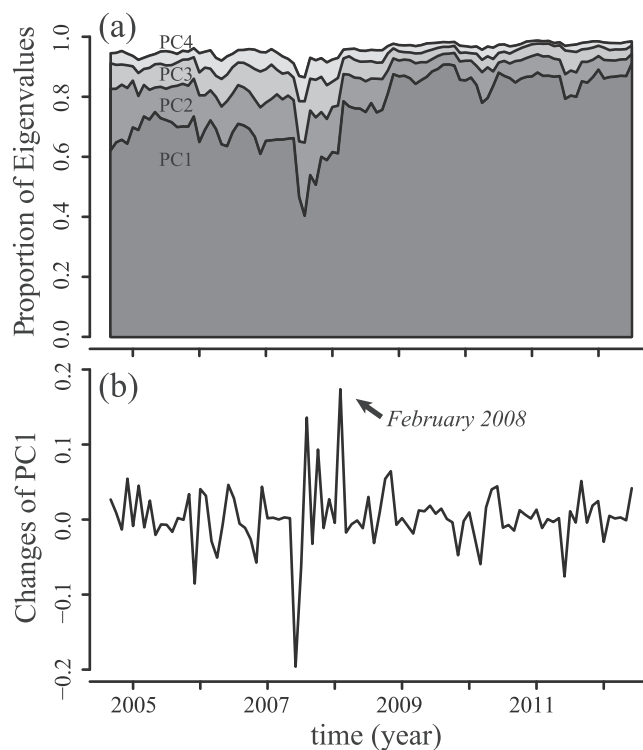


**Figure 7** | (a) PCA of the monthly return of 10 Dow Jones Supersector indexes of major sectors of the European economy during January 2000 to Jun 2012 over 12-month rolling-window proportion of variability (eigenvalues) for principal components 1–4. Note that PC2 actually represents PC1+PC2, and accordingly PC4 represents PC1+PC2+PC3+PC4. In (b) we show changes of PC1 of returns for lag  $m = 1$  month ( $m$  is defined in Eq. 1).

the corresponding Dow Jones sector index (Data section). For the result from European data in Fig. 7, by applying PCA we show again the time-series of first four principal components (PCs) for 12-month moving windows, each quantifying the variability in returns. We find that the PC1 started to increase almost monotonically in the mid 2007. In Fig. 7 we find that the largest change in PC1 occurred in February 2008. Note that very similar results we obtain in Fig. 8 by analyzing the developed markets excluding the US economy. These two dates are a few months later after the result from US market, and this is because it took time for the crisis to spread from US to Europe in 2008.

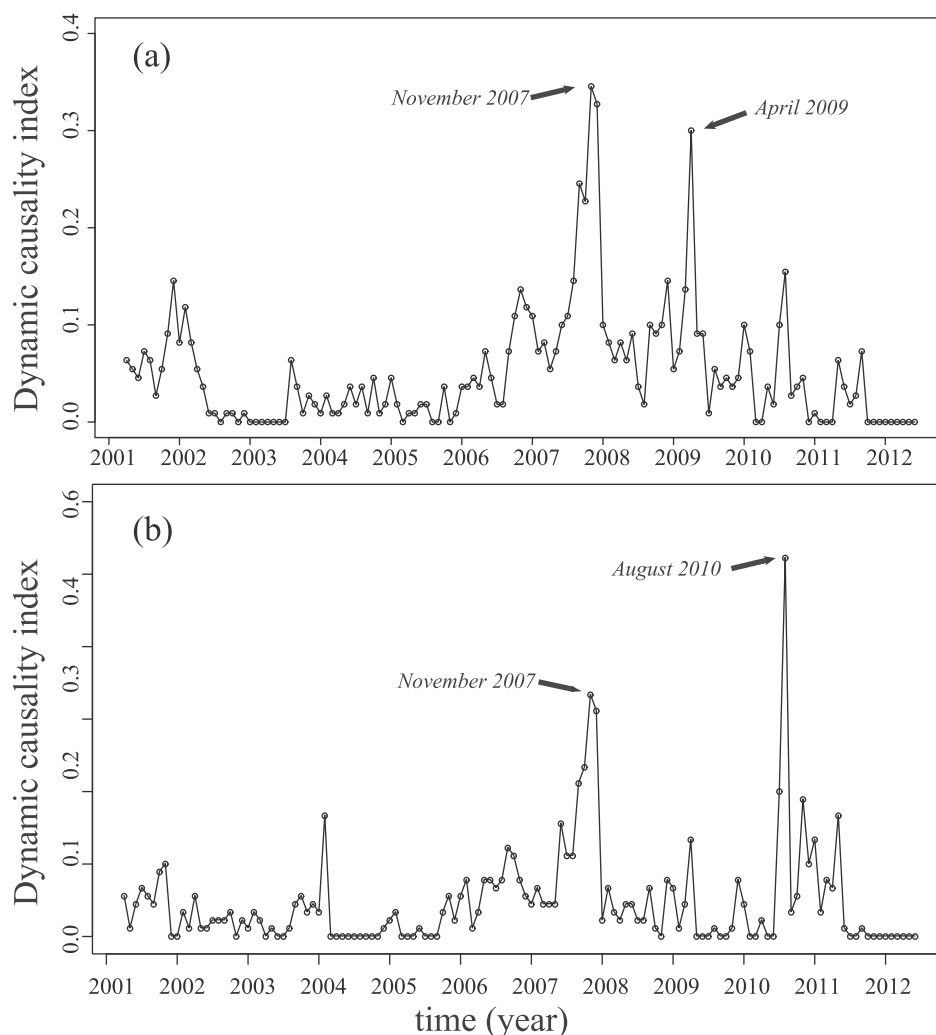
Billio *et al.*<sup>2</sup> reported that principal components estimates and Granger-causality tests yield an asymmetry in the connections between different financial industries: the returns of banks and insurers have more significant impact on the returns of hedge funds and brokers than vice versa. Besides, they found that this asymmetry became highly significant prior to the Financial Crisis of 2007–2009, indicating that the measures may be useful as early-warning indicators of systemic risk. In order to support our results reported in Figs. 1, 2, and 5, next instead of using 36-month time-window, we apply 12-month time window and calculate the dynamic causality index ( $L_{DCI}$ ) of Eq. (6) for monthly recorded time series of 10 Dow Jones indexes representing 10 super-sectors (see section Methods). The plot of  $L_{DCI}$  versus time in Fig. 9(a) shows that  $L_{DCI}(t)$  exhibits the sharp increase in the number of causality links just prior the financial crisis, which means that  $L_{DCI}(t)$  with shorter time window of 12 months captures the increase of systemic risk prior the financial crisis.

We also calculate the ( $L_{DCI}$ ) of Eq. (6) for monthly recorded time series of 10 Dow Jones indexes representing 10 super-sectors in Europe. The plot of  $L_{DCI}$  versus time in Fig. 9(b) exhibits not only



**Figure 8** | (a) PCA of the monthly return of 10 Dow Jones Supersector indexes of major sectors of the worldwide developed markets excluding the US economy during July 2003 to Jun 2012 over 12-month rolling-window proportion of variability (eigenvalues) for principal components 1–4. Note that PC2 actually represents PC1+PC2, and accordingly PC4 represents PC1+PC2+PC3+PC4. In (b) we show changes of PC1 of returns for lag  $m = 1$  month ( $m$  is defined in Eq. 1).





**Figure 9** | Early-warning indicator—dynamic causality index of Ref [2] defined as the proportion of significant causal connections at 5% level of statistical significance out of all possible causal connections based on 12-month rolling-windows. Linear Granger-causality relationships are calculated for the (a) US economy over the period from March 2000 to June 2012 for the monthly returns of 10 Dow Jones Supersector indexes and for the (b) European economy over the period from January 2000 to June 2012 for the monthly returns of 10 Dow Jones Supersector indexes.

the sharp increase in the number of causality links just prior the financial crisis, but also even larger increase in the number of causality links in the mid of 2010 that is most probably related to the European sovereign debt crisis when some countries in the euro area were not capable to repay or re-finance their government debt.

## Discussion

Over the past two decades of financial innovations and deregulation, the global financial system has become considerably more complex and interrelated. The increase in cross-correlations among securities<sup>30,33,34</sup> and industry sectors has great impact in the risks at the systemic level that has never been present before.

In this paper, we proposed a systemic risk estimator which is built upon principal components analysis (PCA). When PCA is applied, as proposed in Refs. 2,5, systemic risk is higher when the largest eigenvalue explains most of variation of the data, or alternatively, with increasing the largest eigenvalue. The more PC1 explains the variation of the data, the larger the systemic risk, implying that the larger the change of PC1, the higher the increase of systemic risk. The larger increase of systemic risk, the larger are the threats to financial stability, implying it is more probable the crisis will occur in the near future.

By studying 10 major US economic sectors and their indexes, we found that within PCA framework and short time windows of 12

months, the change in the first principal component PC1 can be used as an indicator of systemic risk, where the larger the peak in the change of PC1, the higher the increase of systemic risk. We obtained that the largest peak in the PCA approach coincide with the largest peak obtained by using the dynamic causality index ( $L_{DCI}$ ) with 12-month time window.  $L_{DCI}$  versus time exhibits the sharp increase in the number of causality links just prior the financial crisis, which means that  $L_{DCI}(t)$  with shorter time window of 12 months captures the increase of systemic risk during the financial crisis.

Similar results were obtained on the 10 major European economic sectors and their indexes. By applying the dynamic causality index ( $L_{DCI}$ ), besides the sharp increase in the number of causality links just prior the financial crisis, we also obtained even larger increase in the number of causality links in the mid of 2010 that is most probably related to the ongoing European sovereign debt crisis.

## Methods

**Data.** We analyze monthly recorded time series of 10 Dow Jones indexes representing 10 super-sectors. The super-sectors are categorized following classification sector rules of Dow Jones Indexes<sup>1</sup>. Each Dow Jones sector Index captures 95% market capitalization of the U. S.-traded stocks of the sector. In total, ten sectors are represented by the following indexes: Dow Jones U.S. Oil & Gas Index, Dow Jones U.S. Basic Materials Index, Dow Jones U.S. Industrials Index, Dow Jones U.S. Consumer Goods Index, Dow Jones U.S. Health Care Index, Dow Jones U.S.



Consumer Services Index, Dow Jones U.S. Telecommunications Index, Dow Jones U.S. Utilities Index, Dow Jones U.S. Technology Index, Dow Jones U.S. Financials Index. We obtain all the data from finance.yahoo.com and analyze these database for the period starting at March 2000 and ending at June 2012. We also study financial subsector using the following five indexes: S&P Insurance Index, KBW Bank Index, Vanguard REIT Index ETF, NYSE Arca Securities Broker/Dealer Index, Eurekahedge North American Hedge Fund Index.

We also study monthly recorded time series of 10 Dow Jones indexes representing 10 super-sectors in Europe. In total, ten European sectors are represented by the following indexes: Dow Jones Europe Oil & Gas Index, Dow Jones Europe Basic Materials Index, Dow Jones Europe Industrials Index, Dow Jones Europe Consumer Goods Index, Dow Jones Europe Health Care Index, Dow Jones Europe Consumer Services Index, Dow Jones Europe Telecommunications Index, Dow Jones Europe Utilities Index, Dow Jones Europe Technology Index, Dow Jones Europe Financials Index, For European data the range is from Jan 2000 to Jun. 2012.

Finally we study monthly recorded time series of 10 Dow Jones Developed Markets ex-U.S: Dow Jones Developed Markets ex-U.S. Oil & Gas Index, Dow Jones Developed Markets ex-U.S. Basic Materials Index, Dow Jones Developed Markets ex-U.S. Industrials Index, Dow Jones Developed Markets ex-U.S. Consumer Goods Index, Dow Jones Developed Markets ex-U.S. Health Care Index, Dow Jones Developed Markets ex-U.S. Consumer Services Index, Dow Jones Developed Markets ex-U.S. Telecommunications Index, Dow Jones Developed Markets ex-U.S. Utilities Index, Dow Jones Developed Markets ex-U.S. Technology Index, Dow Jones Developed Markets ex-U.S. Financials Index. For developed markets ex US, the range is from July 2003 to Jun. 2012.

For a given index  $i$ , its return series is defined as

$$R_i(t) = \ln P_i(t) - \ln P_i(t-1) \quad (2)$$

where the  $P(t)$  denote the price of a given index  $i$  in month  $t$ . The volatility is defined as the absolute of return series:

$$V_i(t) = |R_i(t)|. \quad (3)$$

With the time series of raw returns, we further normalize them into time series with zero mean and unit variance:

$$g_i(t) = \frac{R_i(t) - \langle R_i \rangle}{\sigma_R} \quad \text{or} \quad \bar{g}_i(t) = \frac{V_i(t) - \langle V \rangle}{\sigma_V} \quad (4)$$

where  $i$  refers to the  $i_{th}$  super-sector index;  $\sigma_R = \sqrt{\langle R^2 \rangle - \langle R \rangle^2}$  and  $\sigma_V = \sqrt{\langle V^2 \rangle - \langle V \rangle^2}$  are the standard deviations of  $R(t)$  and  $V(t)$  respectively;  $\langle \dots \rangle$  denotes a time average over the period studied.

**Principle component analysis.** In order to investigate the systemic risk among indexes of different economic sectors, we apply principal components analysis (PCA), which is a widely used method for capturing correlation dynamics<sup>35–37</sup>. PCA is a mathematical tool that converts a group of correlated variables into orthogonal linear combinations (called principle components) so that the new basis are linearly uncorrelated. The principle components are usually calculated from eigenvalue decomposition of a cross-correlation matrix.

If by  $C$  we denote the cross-correlation matrix, the squares of the non-zero singular values of  $C$  are equal to the non-zero eigenvalues of  $CC^+$ , where  $C^+$  denotes the transpose of  $C$ . In a singular value decomposition  $C = U\Sigma V^+$ , the diagonal elements of  $\Sigma$  are equal to the singular values of  $C$ . The columns of  $U$  and  $V$  are the left and the right singular vectors of the corresponding singular values. For the 11 times series in our analysis,  $C$  represents the  $11 \times 11$  correlation matrix of the 11 return time series.

In finance, we base our risk estimate on cross correlation matrices derived from asset and investment portfolios<sup>38–41</sup>. The larger the discrepancy between the correlation matrix  $C$  between empirical time series and the Wishart matrix  $W$  obtained between uncorrelated time series, the stronger are the cross correlations in empirical data. In Ref.<sup>38</sup> in the case of 406 time series of companies comprising the S&P 500, 94% of the total number of eigenvalues fall in the region where the theoretical formula that holds for uncorrelated time series applies. Hence, less than 6% of the eigenvectors, which are responsible for 26% of the total volatility, appear to carry some information.

**Dynamic causality index and its changes.** We study causality links based on the linear Granger-causality tests<sup>42–44</sup>. In this study, we analyze the correlation dynamics among return series. Precisely we study not the return time series of indexes but their normalized time series.

In order to calculate the dynamic causality index, that is an early-warning indicator for financial crisis, first we need to define a Granger causality for a pair of indexes:

$$\begin{aligned} X_t &= \sum_{j=1}^m a_j X_{t-j} + \sum_{j=1}^m b_j Y_{t-j} + \epsilon_t \\ Y_t &= \sum_{j=1}^m c_j X_{t-j} + \sum_{j=1}^m d_j Y_{t-j} + \eta_t \end{aligned} \quad (5)$$

where  $X_t$  and  $Y_t$  are two time series each defined with zero mean and unit variances. In this paper each series is normalized and so we use the normalized return and volatility time series of Eq. 4. In Eq. 5,  $\epsilon_t$  and  $\eta_t$  are two uncorrelated *i.i.d.* processes,  $m$  is the maximum lag considered, and  $a_j, b_j, c_j, d_j$  are coefficients of the model. The definition

of causality implies that  $Y$  caused  $X$  when  $b_j$  is statistically significant from zero. Likewise  $X$  cause  $Y$  when  $c_j$  is statistically significant from zero. When both  $b_j$  and  $d_j$  are statistically significant from zero, there is a feedback relationship between the two time series. In practice, the causality is based on the F-test where the null hypothesis is defined that coefficients  $b_j$  and  $c_j$  are equal to zero.

We analyze the pairwise Granger causality between the  $t-1$  and  $t$  monthly returns of the 10 indexes. We follow the definition of Billio *et al.*<sup>2</sup>,  $X$  Granger-causes  $Y$  if  $C_1$  has a p-value of less than 5%, similarly  $Y$  Granger-causes  $X$  if  $b_1$  has a p-value of less than 5%. Billio *et al.*<sup>5</sup> defines the dynamic causality index (DCI) series as

$$L_{DCI}(t) = \frac{\text{number of causal relationships over a given period}}{\text{total possible number of causal relationships}} \quad (6)$$

where the rolling-windows have length of 12 months,  $t$  indicate the month immediately following the rolling-window.

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## Author contributions

BP, ZZ, and LF performed analysis, and BP, ZZ, LF, and BL discussed the results, and contributed to the text of the manuscript.

## Additional information

**Competing financial interests:** The authors declare no competing financial interests.

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