Changes of Chaoticness in Spontaneous EEG/MEG

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Abstract—Depending on the task being investigated in EEG/MEG experiments, the corresponding signal is more or less ordered. The question still open is how can one detect the changes of this order while the tasks performed by the brain vary continuously. By applying a static measurement of the fractal dimension or Lyapunov exponent, different brain states could be characterized. However, transitions between different states may not be detected, especially if the moments of transitions are not strictly defined. Here we show how the dynamical measure based on the largest local Lyapunov exponent can be applied for the detection of the changes of the chaoticity of the brain processes measured in EEG and MEG experiments. In this article, we demonstrate an algorithm for computation of chaoticity that is especially useful for nonstationary signals. Moreover, we introduce the idea that chaoticity is able to detect, locally in time, critical jumps (phase-transition-like phenomena) in the human brain, as well as the information flow through the cortex.

Introduction

THE ESSENTIAL CHARACTERISTICS of nonlinear systems are accessible only through dynamical measures. Such measurement, however, is limited by noise (environmental and instrumental noise and intrinsic noise of the system under investigation) and by the adequacy of the A/D conversion (sampling rate and signal resolution). Through nonlinear dynamical studies of brain function (Babloyantz, 1985; Mayer-Kress, 1987) it has become possible in some cases to quantify the physiological state of the system generating EEG (electroencephalogram) and MEG (magnetoencephalogram). This quantification usually focuses on the dimension of the attractor (see Schuster, 1989 and references therein), the deterministicity measure (Glass & Kaplan, 1992 and Mühlnickel, et al., in this volume) and, as we will demonstrate, the predictability of EEG/MEG time series. From the point of view of clinical applications, an interesting measure is the predictability, a measure that results from the solution of the stability problem, i.e., the spectrum of Lyapunov exponents (Farmer et al., 1983). Although all mentioned quantities are linked to each other in the idealistic system, they provide independent information when extracted from experimental data bases. In this article, we discuss different ways to implement measures from chaos theory for the case of nonstationary or not strictly stationary systems, such as spontaneous EEG and MEG or other physiological times series. One important question concerns the predictability and the deterministicity of the system state as these characteristics are required to justify the analytical implementations derived from the theory of nonlinear dissipative systems. The predictability in low-dimensional systems is only inversely proportional to the Largest Lyapunov Exponent (LLE) in cases were stationarity can be assumed. Unfortunately, for

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neural mass activity we cannot assume a stable attractor and, therefore, we cannot come to straightforward conclusions concerning dynamics. Consequently, we must determine a measure characterizing the systems momentary stability or the variation in its information capacity. The chaoticity measure derived in this article fulfills these conditions and, hence, may prove useful for the analysis of biomedical time series.

Lyapunov Exponents as Chaoticity Measure

We will define (after Farmer et al., 1983) the deterministic system as chaotic if it possesses at least one positive LLE. Thus, in study nonlinear dynamics, the first step is to verify the deterministism of the signal under consideration (e.g., according to the procedure suggested by Glass and Kaplan and implemented by Mühlnickel et al. for the analysis of brain dynamics). If the system proves to be deterministic, then an attractor exists and its properties can be evaluated.

One method of determining the stability of a system is to disturb it by altering one of its variables and to then follow its subsequent response. A given system in one of its stable states should return to its original state after a small pertubation. This process is one of relaxation with a convergence that follows an exponential function. For unstable conditions, the system diverges exponentially from the original state. The evolution of the disturbance δ in the consecutive time points is described then by: $\delta_n = \delta_0 \exp(\lambda n)$. It is easy to see that δ approaches zero with an increase in time (n) for negative λ, and δ approaches infinity if λ is positive. This formulation of the stability shows us that an N-dimensional dynamical system consists of multi-directional N-values of the Lyapunov exponent (LE). However, this set of values, known as the spectrum of Lyapunov Exponents, does not consist of all possible combinations of LE values. Because the system is a priori dissipative, the sum of all LE must be negative (the phase space volume must be shrinking), i.e., there exists at least one LE that is negative. Most interesting is the value of the largest LE (LLE): If an LLE is positive the system is expanding in one direction, neighboring trajectores are diverging and, consequently, the system is unpredictable, meaning that it is chaotic. Moreover, the greater the value of the LE the shorter the predictability across time and the larger the chaoticity. Thus, for the strictly defined ergodic, stationary and deterministic systems LLE is the measure of the chaoticity of the state.

In practice, we do not disturb the trajectory in order to measure the LE, rather we search for nearby trajectories in the reconstructed phase space and observe the time evolution of distances between them. Repeating this for many pairs of trajectories, we can thus estimate an average logarithmic ratio of the developed distances, which characterize the stability of the system locally in time and phase space. This number is negative for periodic signals, zero for quasiperiodic or intermittent, and positive for a chaotic motion. In N-dimensional space LEs are equal to the length of axes of the N-hyperellipsoid (in 1-D this is an interval, in 2-D this is an ellipse, in 3-D ellipsoid, etc.). The LLE, coupled to the longest axis, determines the direction in space where maximal volume changes occur as time progresses, i.e., the direction of maximal instability.

There are many methods for reconstruction of an attractor from the experimental time-series. The most popular is based on Takens theorem (Takens, 1981) allowing the reconstruction of an attractor, which has the same topological features as the original one. Let X stand for the measured time series: $X=\{x_1,x_2,x_3, \ldots\ldots, x_N\}$, where the indices at x indicate discretized time.

In order to become a state vector instead of one scalar value at a fixed moment of time t, the time-lag technique is employed. The e-dimensional state vector is then:

$$\{x(t), x(t-\tau), \ldots\ldots, x(t-(e-l)^* \tau)\} \tag{1a}$$

where delay-time $\tau = p^*dt$, with dt sampling interval, $t \in [dt, N^*dt]$, p, e are integer, and e is large enough to embed the phase space (embedding dimension). To obtain the LLE, we must now estimate the average rate of the temporary changes of a chosen small volume element (multi-dimensional ellipsoid). LLE is then defined as:

$$\lambda_1 = \lim_{t \to \infty} ({}^1/t)^* \log[\mathrm{dist}(t)/\mathrm{dist}(t_0)] \tag{2}$$

where dist(t) denotes the length of the longest (principal) axes of the elipsoid at time t and dist(t_0) denotes the initial disturbance, respectively. The practical limitation of this definition is that we cannot fulfill the requirements of an infinite time series: $t=N \cdot dt$ and both N and δt are finite. Nevertheless, if the system conserves its total energy and if the analysed time series is long enough (to reconstruct more or less the attractor), the estimated value will be sufficiently close to the theoretical one. Practical methods for the estimation of the LE from experimental data sets have been amply described in the literature (Wolf et al., 1985; Nychka et al., 1992; Sano & Sawada, 1985; Rosenstein, Collins, DeLuca, 1993). The method we used is based on the definition of Lyapunov exponents in the sense of the local Lyapunov exponent presented by Wolff (1992) (see also the interpretations by Mayer-Kress, 1985; Eckmann, et al., 1986; and Wolf et al., 1985). To estimate the LLE one must first estimate partial Lyapunov exponents for a given pair of points in space. Namely, for points k and m:

$$\lambda_{km} = (1/2 \cdot \delta t \cdot \log 2) \log[\mathrm{dist}(x_k(t+\delta t), x_m(t'+\delta t) / \mathrm{dist}(x_k(t), x_m(t')] \tag{3}$$

where $\mathrm{dist}(x_k(t), x_m(t')) = \Sigma\, [x_k(t) - x_m(t')]^2$ means the square of the distance between trajectories k and m at a moment t.

Repeating this procedure for N pairs and averaging this, i.e.,

$$L = (1/N) \sum_{s=1}^{N} \lambda_{km}(s) \tag{4}$$

should yield the Largest Lyapunov Exponent if N is a large number and if the system dynamics do not change.

Nonstationary Data Sets

Spontaneous EEG and MEG signals measured during an uncontrolled brain task are not able to remain stationary over intervals long enough to provide sufficient data for reliable estimation. Some investigators (Mayer-Kress & Holzfuss, 1987; Ellner, 1988; Freeman & DiPrisco, 1986) have suggested that these mutually exclusive requirements represent a fundamental limitation in the application of chaos theory to biological systems. Theoretically, there are two possible treatments of the nonstationarity problem: One is to use very short-time intervals such that the dynamics remain unchanged and the other is to use very long-time intervals where the average energy remains constant. Unfortunately, the latter solution assumes, when applied to EEG/MEG, a constant brain state for the duration of the measurement period, an assumption that is generally unrealistic. Therefore, the only solution is to use short epochs (i.e., epochs that must still be presumed to be stationary). Large errors in estimation, arising because of small data samples will be systematic, however, thus enabling control and experimental comparisons to be made.

Skinner and associates (1992), using data recorded from the olfactory bulb of the rabbit found that epochs as short as 500 msec were necessary to achieve statistical stationarity. Following a novel odor, the 500 msec epochs still appeared to be stationary within this condition, but now the fractal dimension estimates were observed to be significantly increased compared to control. Rapp and associates (1990) proposed that human EEG epochs of 1 s remain stationary, demonstrating that following a target stimulus (active reaction on driver) the D2-dimension values were significantly lower than those following a nontarget stimulus (passive receiving).

The problem with these brief-interval studies is that the post-stimulus dimensions may appear stationary, although in reality they are not. That is, the dimensional shift may not change to a new stable state, but may actually undergo a variety of rapid nonstationary changes.

Another solution of this problem of data nonstationarity could be to apply a method of continuous estimation of the dynamical variable (dimension, entropy, Lyapunov exponent). In the case of a fractal dimension one expects that the reference vector (i.e., the "point"), which spans only a short interval, would remain stationary and would dominate the calculations, making the overall estimate less sensitive to nonstationarities (Farmer, et al., 1983; Skinner, et al., 1990; Soong & Stuart, 1989). When the pointwise scaling dimension (referred to as D2) was applied to biological data (EEG, EMG, heartbeat intervals), it was found to have a considerable error of estimation. This error could have arisen because the method still required data stationarity over the whole epoch. Another method, called the point-D2 (PD2), was developed to circumvent the requirement for data stationarity that use of an algorithm, in which each reference vector sought only its own stationary subepochs in which to make the vector-difference lengths (that are used to determine the dimensional estimate) at that point in the data (Skinner et al., 1990 and 1992). It has been demonstrated by James Skinner that this deterministic measure (i.e., the PD2) is inherently more sensitive to the output of the underlying biological system than is a stochastic measure, e.g., the signal-averaged mean (Freeman & Skarda, 1985) or the standard deviation (Freeman, 1979) applied to the same data.

The previously mentioned considerations for dimensional estimates explain principally why the values of the nonlinear characteristics, which are estimated in case of biological signals, seem to be relative and why they cannot mirror the properties of the hypothetical models. What length an analysed time interval should have is in our opinion very individual and strongly dependent on the kind of brain processing being under consideration. For spontaneous brain activity we find the range between 0.5 s and 40 s as a (quasi) stationary one.

Chaoticness of Nonstationary Signals

It is very difficult to characterize the chaoticity of the nonstationary signals where the total system energy is not constant across time and, therefore, the energy and entropy for two subsequent, non-overlapping time segments differ. Nevertheless, scanning nonlinear quantities by means of LLE or K-S entropy (corresponding to measured signal) will indicate the stationarity of the measured signal. In general, there are time series in which the average change of the LLE (or entropy) is rapid and other time series for which the average change is gradual. We will suppose that a rapid change indicates phase-transition like behavior (Badii, 1988; Fuchs & Kelso, 1992). A good example for these considerations is provided by the EEG signal recorded under resting conditions. It is both nonstationary and high-dimensional. For every subsequent process uncorrelated to the other ongoing processes in the brain an additional degree of freedom will be added. Imagine, for instance, a model of brain activity such that an external event could trap the system for a brief period in any one particular attractor. There will be attracting transient trajectories. The appearance of the attractor, however, will be too short such that only a fuzzy image can be reconstructed. There will not be enough orbits to search in many dimensions; consequently, it will not be possible to estimate the traditional chaotic features. The scanning procedure to be presented further provides a possible solution to this problem.

It is strongly important to emphasize that the change in attractor properties can occur even in well-defined stationary systems such as in the case of an interior crisis (Grebogi & Yorke, 1983; Lai et al., 1992). We have experimentally observed such critical jumps in the bouncing ball. There the attractor consists of two topologically different parts, a periodic or quasiperiodic (region of small jumps) and another randomly reanimating, "strange" part (Kowalik et al., 1988). The ubiquitous noise caused alterations in the system dynamics. There is no way to differentiate between the following two cases: (a) transitions occur between coexisting attractors within one set of equations, and (b) transitions from one system dynamic to the next one (i.e., a change in system equations). The situation for brain signals may be comparable, i.e., the informational external noise could evoke the change of the brain-attractor but also the change of the governed dynamics.

As already mentioned, the LLE may be useful in evaluating chaoticity if the dynamic behavior is stationary and deterministic. Our goal here was to develop a chaoticity measure based on the definition of the Lyapunov exponent, which allows conclusions even when constraints regarding stationarity cannot be met and which, consequently, can be utilized for the analysis of most experimental data sets. Moreover, it should be applicable for the system with an unknown number of degrees of freedom. The greatest problem when estimating the LLE from experimental data is an insufficient amount of data and the sensitivity of the estimation to noise. The first problem is simply a problem of nonstationarity due to the dependence of energy on time (number of points). Because the value in Eq. 4 has to be time-dependent in such a case, our proposal in this place is to use averaging across time (in addition to averaging across a small ensemble as in Eq. 4.) Thus, the average scanned LLE, which is denoted as $\Lambda^{\sim}$ is given by:

$$\Lambda^{\sim} = \langle L(\alpha,t) \rangle_{\Delta t} \tag{6a}$$

with

$$L = L(\alpha,t) \tag{6b}$$

where Δt denotes averaging across a time-interval (scanning window), α means a set of all parameter used for the reconstruction of the attractor, and L is estimated from Eq. 4, for fixed k (only one arbitrary reference trajectory for each time slice).

One other known problem of estimating the LLE is the resolution of the digitizer. A change in even the least significant bit in the digital representation of data can result in a different L-estimate. In order to avoid this problem, we propose basing the computations on a normalized data set or directly using the integer data produced by A/D-conversion. A simple reasoning for this choice is that the thickness of the trajectory, in the best case, will be equal to 1 bit and numerical noise will be at least equal to 2 (± 1) bit.

Let us assume a time series generated by a multidimensional system that is not long enough to obtain a satisfying estimate of the correlation dimension (no saturation with growing embedding dimension). Consequently, it will also be more difficult to estimate the correct value of the LLE as the reconstruction of the attractor cannot be sufficiently detailed (Eckmann & Ruelle, 1992), and, hence, the LLE will not converge with an increasing number of points (compare Wolf et al., 1985). Nevertheless, we may obtain convergence with the number of iterations. In this case, the value reached will obviously not be the correct LLE. However, it will be closer to the true value for a larger number of data points. This statement is valid on a statistical basis only. This is particularly true as the values may oscillate with a number of iterations before saturation is reached. We propose the following procedure. Let us first assume we have a deterministic signal measured or estimated with a defined accuracy, for instance, B-bits. Before we reconstruct the phase space and start tracing two close trajectories, we must scale a whole data set to the integer range [O...NMAX=2^B–1]. For a fixed parameter set (delay, embedding dimension, etc.), we then follow nearby trajectories and estimate the rate of their average exponential divergence according to the definition (4). Following this step, we shift the chosen time sequence about the few points and repeat the procedure. After N scan steps we obtain N different values of quasi-LLE which allows to characterize the momentary stability of the investigated systems.

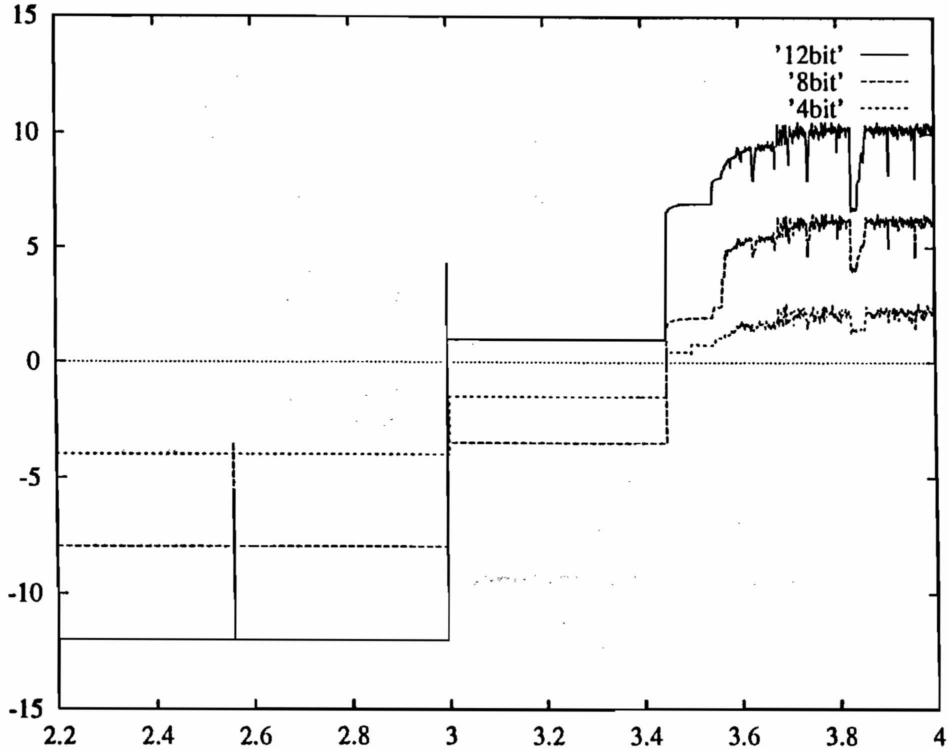
The single run of an estimation will then read as follows:

(i) Read the block of data (the scan width depends on sampling rate and frequency range—usually 518–8192 points).
(ii) Normalize the block (typically it is 12 bit A/D converter, i.e., the range [0, 4095]). This step is required because we ask about “pure” system dynamics changes and the amplitude relations for the successive scanned windows are not important.
(iii) Choose an initial point (one of the initial points larger than the delay) and the parameters for a phase space reconstruction (embedding dimension, delay, step).
(iv) Find the smallest distance (as in the algorithm of Wolf, et al., 1985) greater than the noise level. As a rule if the distance is equal to zero in estimations we exchange the zero with one. This has a strong implication in the behavior of quasi-LLE: no zero means no $-\infty$ of the exponent. Due to noise there is no zero possible to detect in the real data and the above treatment will make more sense in experiments.
(v) Follow the definition of the Lyapunov exponent given by Eq. 2. (output L).
(vi) Average within the scanning window (output Λ).
(vii) Estimate the standard deviation (output σ_Λ).

This method can be used both for the data generated by maps and for digitized experimental time series. In order to characterize the chaoticness of the time series we must examine all three values: L, Λ and σ_Λ, or, in the case of discrete maps, only L and Λ. Figure 1 presents an evolution of the static values of the chaoticity Λ for the logistic map as a function of the nonlinear parameter.

As shown in Figure 1A, the values of chaoticity are proportional to the number of bits, and, for instance, for 12-bit converting these can be from –12 to 12. In general, because of steps (ii) and (iv), for ζ-bit converters L reaches values from the region $[-\zeta,+\zeta]$. Because $\Lambda_{min}=-\zeta$, the evolution of chaoticity is more similar in its form to the form of evolution observed in entropy than to this in the evolution of the Lyapunov exponent.

In order to prove the applicability of this method in detecting changes in the system



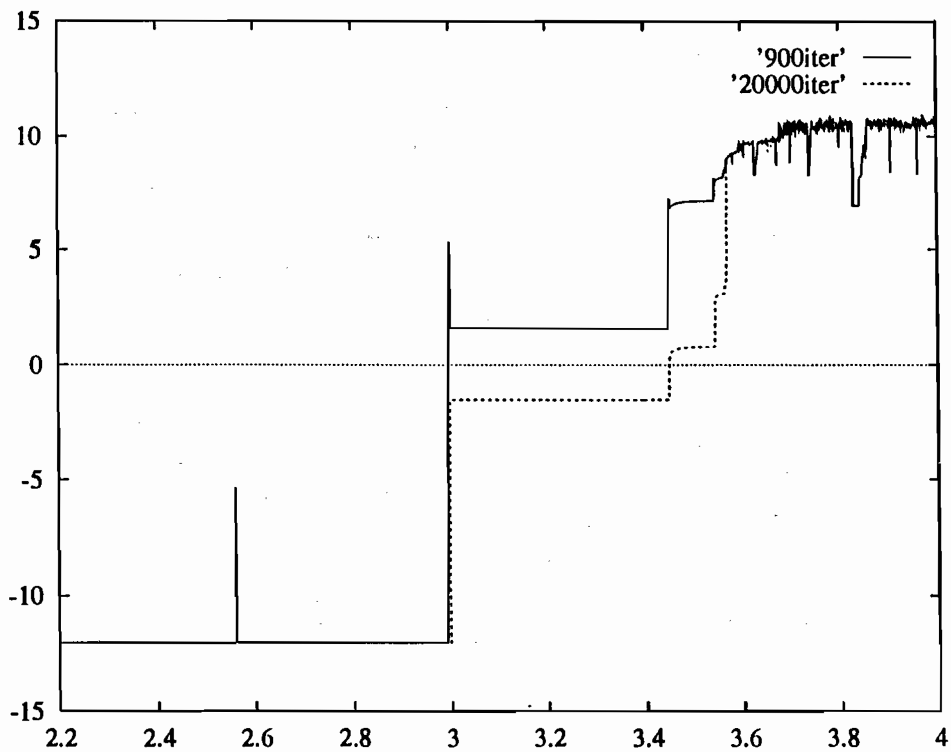


FIG. 1. Chaoticity of a logistic map. (A) Dependence of chaoticity on accuracy of digital presentation. (B) How sensitive is the chaoticity measure shows the evolution of the chaoticity as a function of number of initial iterations (note, there are almost no differences between both patterns in the chaotic region).

state, we created a synthetically assembled time series. We present an example formed by intervals of chaotic dynamics: Henon, Lorenz, Van-der-Pols-like oscillator (Kowalik, et al., 1994) as well as measured brain signals: EEG, MEG, with intervals of sine functions interspersed. Each of the chaotic attractors is interrupted by the sum of two incommensurate sine functions with a frequency ratio 23:11. Figure 2 shows the corresponding evolution of the chaoticity for this time series.

One can easily observe corresponding changes of chaoticity in moments where the dynamics transform. There is always in the plot of L-series a crossing over zero line at the beginning and at the end of the successive trial.

Since we assumed, in our estimations, that the reconstructed part of the attractor in each of the time slices results from varying dynamics, the respective data set of particular window was independently normalized. The advantage of such normalizing is a weak dependence of the measured value on the artefacts and a better description of the changes in system dynamics. The average values estimated in the way proposed above will henceforth be referred to as the chaoticity measure.

Chaoticity of the Spontaneous MEG/EEG Time Series

It is very important to point out that there is no temporal change in an L-series for nonlinear stationary oscillations if no rapid change of system dynamics occur (see Figure 1 and Figure 2).

In the case of spontaneous activity of the brain, the signal is generated without a specially defined task. Thus, we cannot control the changes of the brain state and it is supposed that such changes are rather rare in the case of normal (control) brain. In order to have a signal with possible large amount of such changes we used the time series resulting from two investigations: MEG was measured in tinnitus (ringing in the ear) patients and EEG time slices were collected from schizophrenic patients.

For the numerical analysis intervals containing more than 10,000 points (in the case of EEG it was at least 100 s for 100 Hz sampling rate, bandwidth DC-100 Hz, in the case of MEG: more than 10 s, sampling rate 208.3 Hz, bandwidth DC-50 Hz and free of movement artifacts were used). The (central) EEG referred to linked mastoids was recorded in cooperative schizophrenic patients during a 20-minute interview with their therapist. The EEG was simultaneously recorded from the interviewer.

The MEG-signals were measured using a 37 Channel magnetometer (BTi, San Diego, USA) consisting of 37 DC- SQUIDs operating at liquid helium temperature. The magnetic measurements were performed inside a magnetically shielded room to reduce environmental noise. The dewar containing magnetometer was centered over the temporal lobe of both hemispheres successively. The spontaneous activity was recorded for tinnitus patient under resting conditions. For the measured data set we have estimated the spatial distribution of the chaoticity. Figure 3 presents one representative plot of a chaoticity-map measured over the left hemisphere.

Distinct local variations suggest that under resting conditions, the brain dynamics vary considerably between regions, meaning that there is only limited or even no coupling among the different brain regions. On the other hand, nearby channels display often equal values in their dynamical variables. This cannot be explained on the basis of magnetic field propagation alone. The nonhomogeneous image of the chaoticity maps (the same observed for LLE by Kowalik, et al., 1992) indicates the task-dependent coupling of various brain regions.

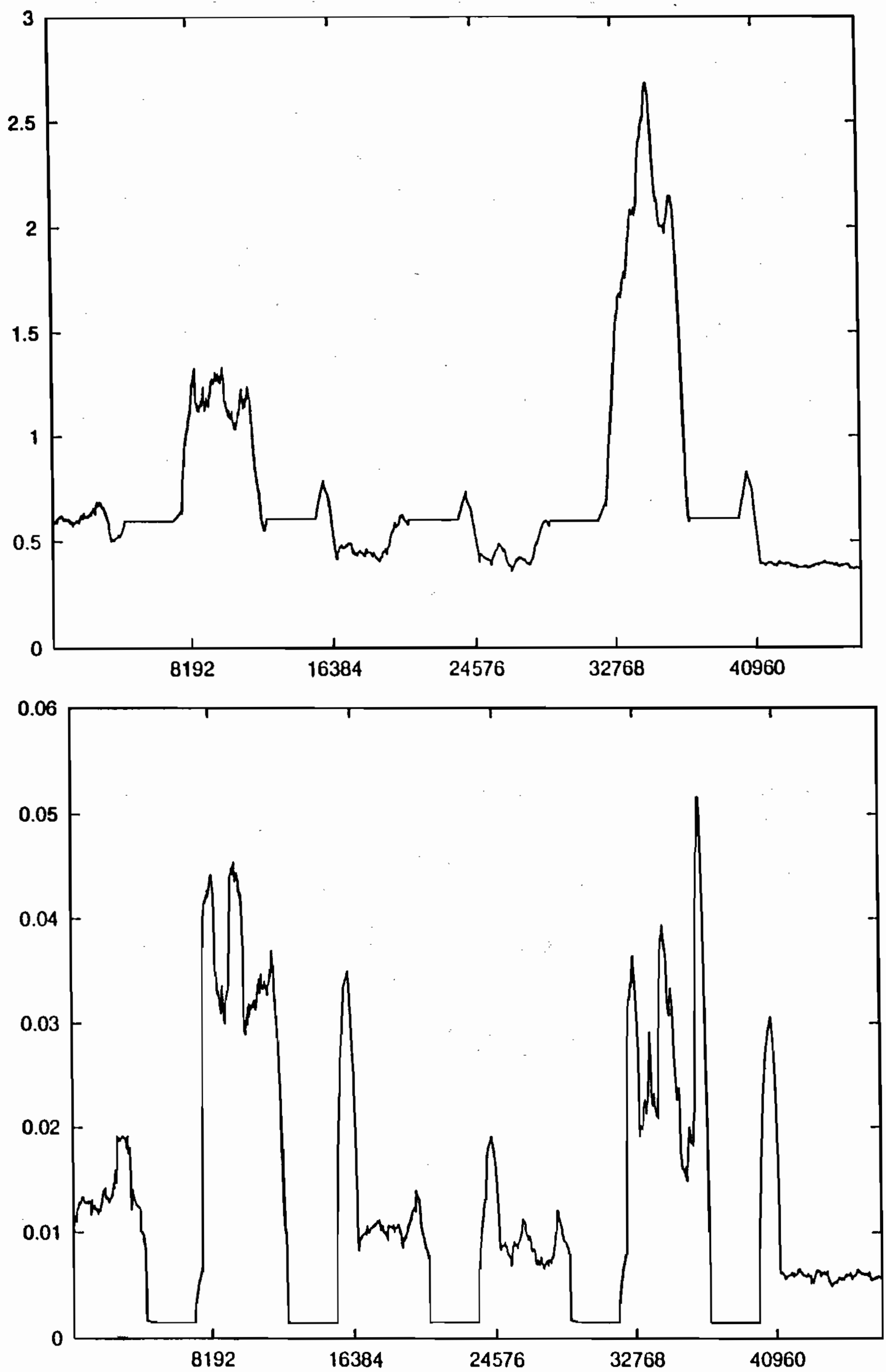


FIG. 2. Chaoticity Λ (A) ands σ_Λ (B) across time for the synthetized time series containing 11 intervals, each 4096 points. These are, successively: Henon attractor, sine combination: $\sin 2\pi f_1 t + \sin 2\pi f_2 t$ (with $f_1{:}f_2=23{:}11$), Lorenz attractor, sine again, MEG, sine, EEG, sine, Vander-Pol-like, sine, noise. Embedding dimension = 7, step=2 and delay=10 were used.

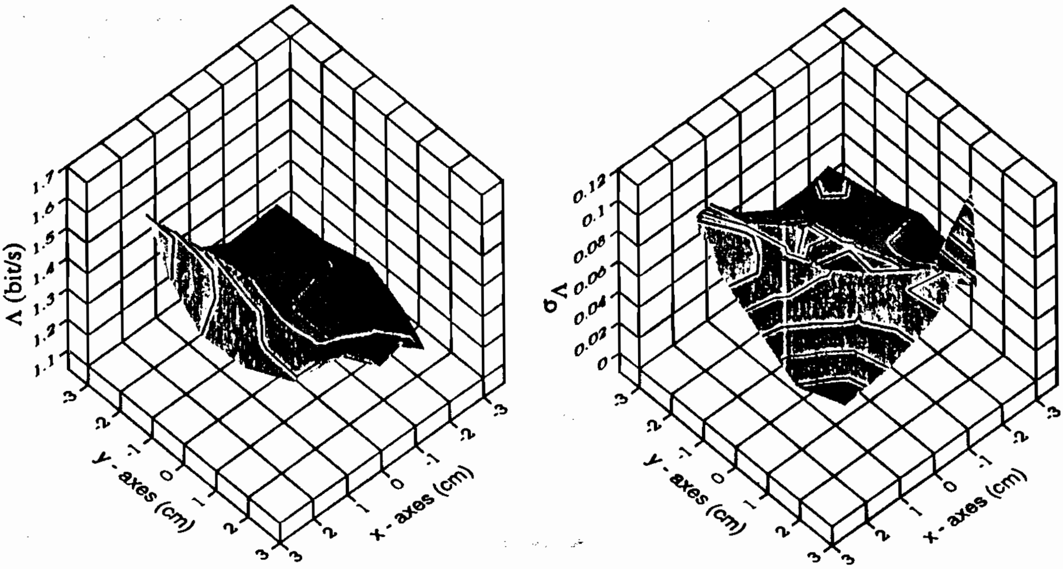


FIG. 3. 3D-plot of spatial distribution of chaoticity Λ (left) and σ_Λ (right) derived from the MEG measured from tinnitus patient.

The change of the average value is very rapid and the range of that change is significant. There is a convincing evidence that the use of the chaoticity measure can very well describe the delicate and invisible changes of the system dynamics across time.

However, we cannot expect that the dynamical states and the attractors which characterize such states remain constant across time. If the attention switches to another task we may expect variations in both the dynamical characteristics and the coupling between brain areas. The time evolution of the chaoticity suggests that this is indeed the case (Figure 4).

The well visible jump in the middle part of the plot can be viewed as a phase transition-like phenomenon, i.e., a transition in the dynamical flow of the brain activity. Such critical transitions are exhibited more frequently for patients with a paranoid-hallucinatory disorder than the interviewer, from whom the EEG was recorded at the same time.

Conclusions

We can conclude our results as follows:

(1) The chaoticity Λ can be used for the mapping of brain functional activity similar to the LLE (Kowalik, et al., 1992),

(2) For the magnetic fields and electric potentials produced by the human brain we can observe rapid changes of the system dynamics (even very slight) by observing singularities in the L-pattern.

In order to differentiate spatially and temporally the functional activity of the human brain, it is probably not sufficient to simply analyze the amplitude, time and/or frequency domains of the measured MEG/EEG signals. It seems to be more natural and in agreement with the definition of the brain activity to estimate the temporal changes of the informational capacity or of the largest Lyapunov exponent (Iasemidis & Sackellares, 1991;

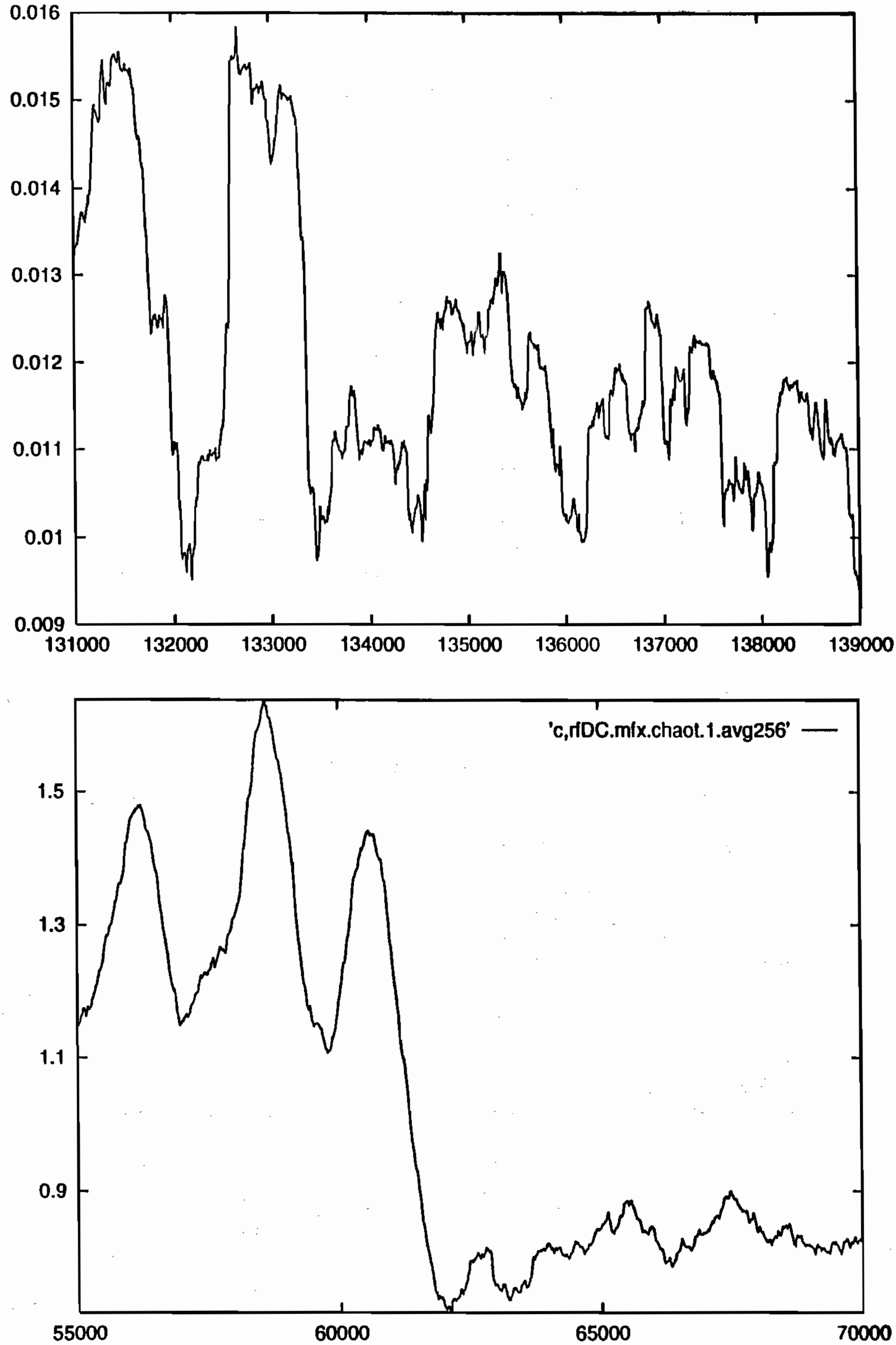


FIG. 4. Rapid changes of brain dynamics can be detected by the chaoticity measures. Cascade of transitions observed via σ_Λ estimated for EEG (90 seconds slice) of schizophrenics (A) and (B) single transition derived from MEG (72 seconds time slice) of tinnitus observed via Λ.

Kowalik, et al., 1992). The method proposed in this article involves the estimation of the signal chaoticity with the advantage that it supports each kind of experimental data, i.e., computer estimation is time efficient, can be applied for short time series and supplies the same information about system dynamics as the entropy rates and the LLE (for a review see Elbert, et al., 1994). Moreover, it enables a direct (on line) measurement of the chaoticity. Because the average change of chaoticity estimated from the EEG/MEG signals characterize the temporal information flow in a human brain there are numerous possible applications in studies of human brain.

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References

Babloyantz, A. (1985). In H. Haken (Ed.), *Physics and Computers.* Berlin: Springer, 116–122.

Badii, R. (1988). *Il Nuovo Cimento,* 10, 819–840.

Eckmann, J.P., Kamphorst, S.O., Ruell, D., and Ciliberto, S. (1986). Lyapunov exponents from time series. *Phys. Rev.,* A34, 4971–4979.

Eckmann, J.-P., Ruelle, D. (1992). *Physica,* D56, 185–187.

Elbert, T., Ray, W.J., Kowalik, Z.J., Skeener, J., Graf, K.E., and Birbaumer N. (1994). Chaos and physiology. *Physiological Reviews,* 74, 1–47.

Ellner, S. (1988). *Physics Letters,* A133, 128–133.

Farmer, J.D., Ott, E., and York, J.A. (1983). *Physica,* 7D, 153–180.

Freeman, W.J. (1979). *Biol. Cybern.* 35, 221–234.

Freeman, W.J. and Skarda, C.A. (1985). *Brain Res. Rev.,* 10, 147–175.

Freeman, W.J. and DiPrisco, V. (1986). In G. Palm, Aertsen, A. (Eds.), *Brain theory,* Berlin, Heidelberg, New York: Springer-Verlag, 97–119.

Fuchs, A., Kelso, J.A.S. (1992). In B.H. Jansen, and M.E. Brandt (Eds.), Nonlinear dynamical analysis of the EEG, *World Scientific,* 269–284.

Glass, L., and Kaplan, D. (1992). *Phys. Rev. Lett.*

Grebogi, C., Ott, E., and Yorke, J.A. (1983). *Physica,* 7D, 181.

Iasemidis, L.D., and Sackellares, J.C. (1991). In D. Duke and W. Pritchard (Eds.), Measuring chaos in the human brain, Singapore, New Jersey, London, Hong Kong; *World Sci.,* 97–112.

Kowalik, Z.J., Franaszek, M., and Pieranski, P. (1988). *Phys. Rev.,* A37, 4016–4023.

Kowalik, Z.J., Goerke, N., Bode, M.T., and Purwins, H.-G. A driven Van-der-Pol-like oscillator in experiment, submitted to *Chaos,* 1994.

Kowalik, Z.J., Elbert, T., Hoke, M. (1992). In B.H. Jansen, M.E. Brandt (Eds.), Nonlinear dynamical analysis of the EEG, *World Scientific,* 156–164.

Lai, Y.-C., Grebogi, C., Yorke, J.A. (1992). In J.H. Kim and J. Stringer (Eds.), *Applied chaos,* John Wiley & Sons, Inc., 441–455.

Mayer-Kress, G. (Ed.) (1986). *Dimensions and entropies in chaotic systems.* Springer series in Synergetics, Vol. 39, Berlin, Heidelberg, New York, Tokyo: Springer-Verlag.

Mayer-Kress, G., Layne, S.P. (1987). In K.H. Koslov, A.J. Mandell, and M.F. Schlesinger (Eds.), Perspectives in Biological Dynamics and Theoretical Medicine. Ann. N.Y. Acad. Sci., 504, 62–87.

Mayer-Kress, G., Holzfuss, J. (1987). In Temporal Disorder in Human Oscillatory Systems. L. Rensing an der Heiden, U., Mackey, M.C. (Eds.), 57–68, Springer-Verlag Berlin, Heidelberg, New York, Tokyo.

Mühlnickel, W., Rendtorff, N., Kowalik, Z.J., Rockstroh, B., Miltner, W., and Elbert, T. Testing the determinism of EEG and MEG, this volume.

Nychka, D., Ellner, S., McCaffrey, D. and Gallant, A.R. (1992). *J.R. Statist. Soc.* B 54, 399–426.

Rapp, P.E., Bashore, T., Martinerie, J., Albano, A., Zimmermann, I., Mees, A., Dynamics of brain electrical activity. *Brain Topography,* 2, 99–118, 1990.

Rosenstein, M.T., Collins, J.J., De Luca, C.J. *Physica,* 65D, 117–134, 1993.
Sano, M., Sawada, Y., *Phys. Rev. Lett.,* 55, 1082, 1985.
Schuster, H.-G., Deterministic Chaos. An Introduction, Verlag Chemie, Weinheim 1989.
Skinner, J.E., J.L. Martin, C.E. Landisman, M.M. Mommer, K. Fulton, M. Mitra, W.D. Buron, and B. Saltzberg, in Brain Dynamics Progress and Perspectives, E. Basar and T.H. Bullock (Eds.), 119–134, Springer-Verlag, Berlin 1990.
Skinner, J.E., M. Molnar, T. Vybiral, and M. Mitra, Integrative Physiological and Behavioral Science, 27, 39–53, 1992.
Soong, A.C.K., and C.I.J.M., Stuart, Biol. Cybern., 62, 55–62, 1989
Takens, F., in "Lecture Notes in Mathematics," Vol. 898, p. 366, Dynamical systems and turbulence, D.A. Rand, and L.S. Young (Eds.), Springer-Verlag, Berlin, 1981.
Wolf, A., Swift, J.B., Swinney, H.L., Vastano, J.A., Physica 16D: 285–317, 1985.
Wolff, R.C.L., *J.R. Statist. Soc.* B54, 353–371, 1992.