Changing Attitudes to University Mathematics through Problem Solving

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University mathematics is often presented in a formal way that causes many students to cope by memorising what they perceive as a fixed body of knowledge rather than learning to think for themselves. This research studies the effects on students' attitudes of a course encouraging co-operative problem-solving and reflection on the thinking activities involved. The attitudinal questionnaire was shown to the students' teachers who were asked to specify the attitudes they ***expect*** *from their students and the attitudes they* ***prefer****. This was used to give a "desired direction of change" from expected to preferred. Before the course, half the students responded that university mathematics did not make sense. A majority declared negative attitudes such as anxiety, fear of new problems and lack of confidence. During the problem-solving course the changes were almost all in the* ***desired*** *direction. During the following six months of standard mathematics lecturing, almost all changes were in the* ***opposite*** *direction.*

Introduction

> Maths education at university level, as it stands, is based like many subjects on the system of lectures. The huge quantities of work covered by each course, in such a short space of time, make it extremely difficult to take it in and understand. The pressure of time seems to take away the essence of mathematics and does not create any true understanding of the subject. From personal experience I know that most courses do not have any lasting impression and are usually forgotten directly after the examination. This is surely not an ideal situation, where a maths student can learn and pass and do well, but not have an understanding of his or her subject.
>
> *Third Year U.K. Mathematics Student, 1991*

The traditional methods of teaching mathematics at university, which are intended to inculcate rigorous standards of mathematics proof, often lead to a "deficit model" of rote-learning material to pass examinations. The resulting procedural forms of thinking and working often prove resistant to change (Sierpinska, 1987; Schoenfeld, 1989; Williams, 1991). The knowledge gained may be appropriate for solving routine problems but it can fail in contexts requiring more conceptual insight (Selden, Mason & Selden, 1994). In the memorable phrases of Skemp (1971), students learn the "product of mathematical thought" rather than the "process of mathematical thinking".

There is evidence that a supportive problem-solving environment can be of help in changing students' attitudes (Schoenfeld, 1985, 1987; Davis & Mason, 1987, 1988; Rogers, 1988). A course based on Mason, Burton & Stacey (1982) has been given for over a decade by the second-named author which also focuses on the emotional effects of cognitive success and failure. The students are acquainted with the theory of Skemp (1979) that distinguishes between a goal to be achieved and an anti-goal to be avoided. He theorised that a goal is associated with positive attitudes, both in pleasure of success and positive response to difficulties by seeking alternative methods of attack. But an anti-goal is associated with negative feelings of fear and anxiety. Long term lack of success is likely to cause a change in attitude from positive goals to the anti-goal of avoiding failure.

This theoretical position suggests that students may be helped to gain control over negative feelings by identifying cognitive difficulties and focusing on positive activities to solve the problem.

In Malaysia, similar difficulties occur (Mohd Yusof & Abd Hamid, 1990). It is the tradition that learning is based on discipline and obedience to work hard and children learn from an early age that it is best to follow the rules. At university students were keen to succeed by learning procedures to pass examinations. As the problems involve more complex procedures, a Catch-22 situation occurs. Students who are failing seek the security of learned procedures but, because these procedures are becoming more burdensome, the students continue to fail.

It was considered that

> … a plausible way in which students may become more successful is to become consciously aware of more successful thinking strategies and this must be done in a context designed to impose less cognitive strain.
>
> Razali & Tall, 1993, p. 219

To test this hypothesis, the first-named author translated the course at Warwick University into Bahasa Malaysia and used it to attempt to develop positive attitudes to mathematical thinking. Universities in Malaysia include a wide range of ability (from the 50th to 90th percentile, with the top 10% going abroad for their education). However, it was considered that the same range of problems would prove effective in improving students' attitudes.

The Method

The first-named author taught a ten week, thirty hour course, with a two hour group problem-solving session and a one hour meeting in smaller groups every week. The two hour session began with the instructor focusing on a specific aspect of problem-solving, followed by students working on problems illustrating this aspect in self-selected groups of three or four. The instructor reviewed the situation after half an hour or so, to see how things were progressing, ensuring that everyone was focused on the same problem and considering ideas generated by the students. She gave no clues to lead students towards a possible solution. They were encouraged to experience all aspects of mathematical thinking—formulating, modifying, refining, reviewing problems and solutions, specialising to special cases, generalising through systematic specialisation, seeking patterns, conjecturing, testing and justifying.

During the one hour meeting, the students were encouraged to reflect on their mathematical experience and talk about their attempts to solve problems. The instructor encouraged them to consider the effectiveness of their solutions—where things may have gone wrong, where they may have failed to take advantage of certain things—ending by summarising their progress.

The students' performance and attitudes were monitored by:

- an attitudinal questionnaire administered before, just after, and six months following the course,
- classroom observation,
- semi-structured interviews with selected students and staff.

The attitudinal questionnaire was circulated to all staff teaching mathematics to the students, inviting them first to specify attitudes they *expect* students to have and then what they *preferred*. For each question the desired direction of change from what staff expect to what they prefer is called the "desired direction of attitudinal change". It was hypothesised that problem-solving would cause a change in students attitudes in the desired direction, but was suspected that these changes may be reversed when the students returned to standard mathematics.

The Students

The 24 males and 20 females taking part in the research were a mixture of third, fourth and fifth year undergraduates aged 18 to 21 in Industrial Science, majoring in Mathematics (SSI) and Computer Education (SPK), covering the full honours degree range (Table 1).

	Degree classification					
Students	I	II-1	II-2	III	P	F
SPK year 5	2	8	5	1	0	0
SPK year 4	3	11	7	1	0	0
SSI year 3	1	5	0	0	0	0
Total	**6**	**24**	**12**	**2**	**0**	**0**

Table 1 : Degree classification of students in the experiment

Classroom Observation

Initially, the students were very confused. They kept asking questions like "What shall I do now?", "Is this the right way of doing it?" when they became stuck after a frantic attack on the given problem. They showed enormous resistance which began to be worn away, little by little until, after four weeks, they were beginning to make decisions and think for themselves.

By this time they began to write a "rubric" commentary outlining their problem-solving activity.

At first they were set simple problems designed to promote a sense of success to help build self-confidence. As a policy the instructor did not work out all the problems beforehand and was willing to tackle a problem in front of the class to show that even mathematicians do not produce neat solutions at first. This encouraged students to feel less reluctant to make conjectures which might prove to be wrong on the possible route to success. Their discussion became livelier as they found they could explain things to their friends, rather than simply satisfying the course requirements or pleasing the their friends rather than simply satisfying the course requirements or pleasing the instructor. Their problem-solving became "a more creative activity, which includes the formulation of a likely conjecture, a sequence of activities testing, modifying and refining," (Tall, 1991).

The Questionnaire

An attitudinal questionnaire on mathematics and problem-solving was designed, based on common responses given in a pilot study at Warwick University. The participants were requested to respond to each item on a five point scale:

Y, y, –, n, N (definitely yes, yes, no opinion, no, definitely no).

Section A: Attitudes to Mathematics

1. Mathematics is a collection of facts and procedures to be remembered.
2. Mathematics is about solving problems.
3. Mathematics is about inventing new ideas.
4. Mathematics at the University is very abstract.
5. I usually understand a new idea in mathematics quickly.
6. The mathematical topics we study at University make sense to me.
7. I have to work very hard to understand mathematics.
8. I learn my mathematics through memory.
9. I am able to relate mathematical ideas learned.

Section B : Attitudes to Problem-Solving

1. I feel confident in my ability to solve mathematics problems.
2. Solving mathematics problem is a great pleasure for me.
3. I only solve mathematics problems to get through the course.
4. I feel anxious when I am asked to solve mathematics problems.
5. I often fear unexpected mathematics problems.
6. I feel the most important thing in mathematics is to get correct answers.
7. I am willing to try a different approach when my attempt fails.
8. I give up fairly easily when the problem is difficult.

Students were also asked to respond to the following question:

In a few sentences describe your feelings about mathematics.

The “desired direction of attitudinal change” perceived by mathematics staff

Table 2 shows the responses of 22 mathematics lecturers and the desired direction of change from preferred to expected student attitudes. To highlight the total “Yes” responses (Y+y), these are added together and displayed in the “Yes” column, with the subset of “definite yes” responses (Y) given in brackets in the same column. A similar convention is used for “No (N)”.

				Expect					Prefer				
	Attitude	desired change		Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	facts and procedures	↓ +++ / +	<1%	20	(8)	2	(0)	0	13	(4)	9	(2)	0
	solving problems	↑ +++ / +++	n.s.	19	(9)	3	(0)	0	22	(9)	0	(0)	0
	inventing new ideas	↑ + / –	n.s.	8	(2)	14	(1)	0	11	(3)	11	(1)	0
	abstract	↓ +++ / ––	<1%	20	(6)	2	(0)	0	7	(0)	15	(4)	0
	understand quickly	↑ + / –––	<1%	3	(0)	19	(6)	0	15	(1)	7	(1)	0
	make sense	↑ ++ / –	<1%	8	(0)	14	(2)	0	19	(3)	3	(0)	0
	work hard	↓ +++ / ++	<1%	21	(13)	1	(0)	0	18	(4)	4	(0)	0
	learn by memory	↓ ++ / –––	<1%	15	(5)	7	(1)	0	2	(1)	20	(6)	0
	able to relate ideas	↑ +++ / ––	<1%	5	(0)	17	(5)	0	22	(5)	0	(0)	0
Problem Solving	confidence	↑ +++ / –	<1%	10	(1)	12	(0)	0	22	(3)	0	(0)	0
	pleasure	↑ +++ / +	<5%	15	(0)	7	(2)	0	21	(4)	1	(0)	0
	only to get through	↓ +++ / –	<1%	21	(9)	1	(0)	0	7	(2)	15	(3)	0
	anxiety	↓ ++ / –––	<1%	16	(5)	6	(0)	0	2	(0)	20	(5)	0
	fear unexpected	↓ ++ / ––	<1%	15	(7)	7	(0)	0	3	(0)	19	(5)	0
	correct answers	↓ ++ / –	<1%	19	(3)	3	(0)	0	6	(2)	16	(2)	0
	try different approach	↑ +++ / +	<1%	12	(1)	10	(0)	0	22	(4)	0	(0)	0
	give up	↓ ++ / ––	<1%	16	(2)	6	(0)	0	2	(0)	20	(2)	0

Table 2 : Lecturers’ perceptions of students preferred and expected attitudes

The arrows in the second column indicate the direction of movement, with the number of plus or minus signs indicating the average weighted strength of response. Each response is weighted with Y as 2, y as 1, n as –1 and N as –2. An average response of 1 or more is considered “strong” and denoted “+++” or “---”, from 0.5 and less than 1 is denoted by “++” or “--”, and less than 0.5 is considered “weak”, denoted “+” or “-”. For instance, in line 1, “facts and procedures” changes down from an expected strong agreement (+++) to a preferred weak agreement (+). In line 4, “being abstract” diminishes from an expected strong agreement (+++) to a preferred disagreement (--). The significance of the change is computed using the Wilcoxon Matched-pairs Signed-rank Test (as in Siegel, 1956) (with correction factor applied in the event of tied ranks) and is given as significant (<5%), highly significant (<1%) or not significant (n.s.).

Of the seventeen items, five do not change direction from "expect" to "prefer": *facts and procedures* $\left(\downarrow^{+++}_{+}\right)$, *solving problems* $\left(\upharpoonleft^{+++}_{+++}\right)$, *working hard* $\left(\downarrow^{+++}_{++}\right)$, *pleasure* $\left(\upharpoonleft^{+++}_{+}\right)$, *try a new approach* $\left(\upharpoonleft^{+++}_{+}\right)$. In these items the staff tend to expect the students to share their preferences. *Solving problems* is high on both counts and does not change significantly. The students are expected to get *pleasure* but the preference is more. The other three items are significant at the 1% level. The staff prefer students to think that mathematics consists of *procedures to be remembered* and to *work hard* but would prefer slightly less emphasis on both items. They expect students to be fairly willing to *try a new approach* but would prefer this to be greater.

Twelve items reverse direction. Only one is statistically insignificant—that mathematics involves *inventing new ideas* changes marginally from weakly expecting students to disagree to weakly preferring their agreement.

The remaining eleven items—the majority of items on the questionnaire—both reverse direction and are statistically significant at the 1% level. The lecturers *expect* the typical student to think mathematics is very *abstract*, will *not understand quickly*, will consider that mathematics does *not make sense*, and will *not relate mathematical ideas*. The students are expected only to *learn through memory*, will only solve problems *to get through the course* and consider getting *correct answers* is most important. They are expected *not to have confidence*, to feel *anxious*, to *fear* the unexpected, and to *give up* in the face of difficulty. In every case the lecturers *prefer* the student to have the opposite attitude.

This reveals that the lecturers expect the students to have profoundly different attitudes from those that they desire. Their expectation of such emotions as anxiety and fear of the unexpected intimates an anti-goal of avoiding failure rather than the goal of achieving understanding. The expectation of pleasure from being able to solve problems coupled with a low expectation of linking ideas together suggests that the students are expected to settle for the lesser goal of being able to carry out procedures to solve routine problems.

Changes in student attitudes in problem solving and mathematics lectures

The attitudinal questionnaire was given before and after the problem-solving course, then six months later after a semester of standard mathematics lectures (Table 3).

		Before P S					After P S					After Math				
		Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–	Yes	(Y)	No	(N)	–
Mathematics	facts and procedures	34	(18)	8	(2)	2	11	(3)	32	(8)	1	30	(9)	14	(1)	0
	solving problems	27	(10)	16	(4)	1	42	(21)	0	(0)	2	32	(22)	12	(0)	0
	inventing new ideas	21	(4)	21	(6)	2	37	(15)	5	(0)	2	24	(4)	18	(1)	2
	abstract	25	(13)	17	(0)	2	15	(8)	27	(3)	2	22	(7)	21	(0)	1
	understand quickly	9	(0)	30	(5)	5	20	(3)	21	(2)	3	16	(2)	26	(1)	2
	make sense	22	(4)	22	(5)	0	35	(5)	7	(0)	2	29	(4)	14	(0)	1
	work hard	37	(15)	5	(1)	2	28	(8)	13	(0)	3	32	(8)	12	(1)	0
	learn by memory	30	(1)	12	(2)	2	11	(0)	31	(7)	2	20	(2)	22	(1)	2
	able to relate ideas	24	(8)	18	(2)	2	35	(11)	8	(0)	1	31	(5)	10	(0)	3
Problem Solving	confidence	26	(7)	17	(2)	1	36	(12)	6	(0)	2	34	(7)	10	(0)	0
	pleasure	43	(25)	1	(1)	0	42	(21)	0	(0)	2	42	(21)	1	(0)	1
	only to get through	16	(4)	27	(8)	1	4	(0)	37	(10)	3	14	(1)	30	(5)	0
	anxiety	17	(1)	24	(4)	3	6	(0)	36	(9)	2	9	(0)	33	(4)	2
	fear unexpected	30	(10)	12	(3)	2	10	(3)	31	(9)	3	16	(3)	28	(2)	0
	correct answers	21	(4)	21	(3)	2	5	(1)	36	(11)	3	17	(0)	25	(7)	2
	try different approach	42	(17)	0	(0)	2	43	(20)	0	(0)	1	43	(16)	1	(0)	0
	give up	19	(3)	24	(9)	1	5	(0)	37	(20)	2	9	(0)	33	(12)	2

Table 3 : The changing attitudes of students before and after problem-solving and "after math"

Calculating the significance in the change of the responses and using a weighted average response as in Table 2, we find the changes given in Table 4. These show a remarkable story of two opposing changes.

During the problem-solving course, the attitudinal changes are in the direction desired by the lecturers, except for "pleasure" which remains highly rated, changing only marginally down from 43 to 42 (out of 44).

		desired change		After P S		After Math		Total change	
Mathematics	facts and procedures	$\downarrow^{+++}_{+}$	<1%	$\downarrow^{++}_{--}$	<1%	$\uparrow^{++}_{--}$	<1%	$\downarrow^{++}_{++}$	n.s.
	solving problems	$\uparrow^{+++}_{+++}$	n.s.	$\uparrow^{+++}_{+}$	<1%	$\downarrow^{+++}_{++}$	<1%	$\uparrow^{++}_{+}$	n.s.
	inventing new ideas	$\uparrow^{+}_{-}$	n.s.	$\uparrow^{+++}_{-}$	<1%	$\downarrow^{+++}_{+}$	<1%	$\uparrow^{+}_{-}$	n.s.
	very abstract	$\downarrow^{+++}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.*	$\uparrow^{+}_{-}$	n.s.	$\downarrow^{+}_{+}$	n.s.
	understand quickly	$\uparrow^{+}_{---}$	<1%	$\uparrow^{o}_{--}$	<1%	$\downarrow^{o}_{-}$	n.s.	$\uparrow^{-}_{--}$	n.s.
	make sense	$\uparrow^{++}_{-}$	<1%	$\uparrow^{++}_{-}$	<1%	$\downarrow^{++}_{+}$	n.s.	$\uparrow^{+}_{-}$	n.s.*
	work very hard	$\downarrow^{+++}_{++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.*	$\uparrow^{++}_{++}$	n.s.	$\downarrow^{+++}_{++}$	n.s.
	learn by memory	$\downarrow^{++}_{---}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	<5%	$\downarrow^{+}_{-}$	n.s.*
	able to relate ideas	$\uparrow^{+++}_{--}$	<1%	$\uparrow^{++}_{+}$	<5%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	n.s.
Problem Solving	confidence	$\uparrow^{+++}_{-}$	<1%	$\uparrow^{++}_{+}$	<5%	$\downarrow^{++}_{++}$	n.s.	$\uparrow^{++}_{+}$	<5%
	pleasure	$\uparrow^{+++}_{+}$	n.s.	$\boxed{\downarrow^{+++}_{+++}}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
	get through	$\downarrow^{+++}_{-}$	<1%	$\downarrow^{-}_{---}$	<1%	$\uparrow^{--}_{---}$	<1%	$\downarrow^{-}_{-}$	n.s.
	anxiety	$\downarrow^{++}_{---}$	<1%	$\downarrow^{-}_{--}$	<5%	$\boxed{\downarrow^{--}_{--}}$	n.s.	$\downarrow^{-}_{--}$	n.s.
	fear unexpected	$\downarrow^{++}_{--}$	<1%	$\downarrow^{++}_{--}$	<1%	$\uparrow^{-}_{--}$	n.s.	$\downarrow^{++}_{-}$	<1%
	correct answers	$\downarrow^{++}_{-}$	<1%	$\downarrow^{+}_{--}$	<1%	$\uparrow^{-}_{--}$	<1%	$\downarrow^{+}_{-}$	n.s.
	try new approach	$\uparrow^{+++}_{+}$	<1%	$\uparrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.	$\downarrow^{+++}_{+++}$	n.s.
	give up	$\downarrow^{++}_{--}$	<5%	$\downarrow^{-}_{---}$	<1%	$\downarrow^{--}_{---}$	n.s.	$\downarrow^{-}_{--}$	<5%

Table 4 : Desired changes compared with changes after problem-solving and after mathematics lectures

However, *during the return to mathematics lectures, the attitudinal changes are in the opposite direction from that desired by the lecturers,* with the exception of “anxiety” which incurs a small decrease. Even here, although the total “definitely not anxious” falls from 9 to 5, the total number feeling anxious actually increases from 6 to 9.

During the problem-solving course, only four changes are not statistically significant and, of these, *pleasure*, *willingness to work hard, willingness to try a new approach* remain highly rated, whilst the sense that *mathematics is abstract* improves slightly from positive to negative. Three items change significantly at the 5% level: *ability to relate ideas* and *confidence* both increase, whilst *anxiety* diminishes. All other items have highly significant changes in the desired direction. Some beliefs are reversed so that after problem-solving students now believe that *mathematics is more than facts and procedures,* it involves *inventing new ideas*, it *makes sense*, it is *not learnt just through memory*, there is *less fear of the unexpected*, it is *not just getting correct answers*. Others positive attitudes are greatly increased: *mathematics is more about solving problems*, it can be *understood more quickly*, and students are *less likely to give up* when encountering a difficulty.

However, six months later, after returning to the mathematics course many opinions have reverted back in the old direction. There is a significant *reduction in belief that mathematics is not just memorisation*, and highly significant *reversal in belief that mathematics is just facts and procedures*; it is *less about solving problems*, *less about inventing new ideas*, *less about doing the work for reasons other than to get through the course* and *less about things other than correct answers*.

Comparing the situation from before the problem-solving course with the status after six months back at regular mathematics lectures, many of the indicators revert back towards their old position. But three problem-solving attributes remain: *confidence* and *unwillingness to give up* remain significantly improved and *fear of the unexpected* is highly significantly reversed. Smaller changes are evident in the belief that mathematics *make sense* and that *it is not necessary just to learn by memory*. (These are improved by a factor that would be significant at the 10% level, marked “n.s.*” in Table 4.)

In addition, a number of items rated at least “++” or “--” carry over attitudes present in earlier mathematics learning. These are *mathematics is facts and procedures, is about solving problems*, students *work hard,* are *able to relate ideas*, take great *pleasure* in their work, have *low anxiety,* are *willing to try a new approach*. This is consistent with having success in procedural learning as a goal.

The nature of the change in students' mathematical thinking

The data reveals the astonishing result that, when students are doing problem-solving, their attitudes change in the direction desired by the mathematics lecturers yet, when the students return to be taught by the mathematics lecturers, the students attitudes now move in the opposite direction by those who are teaching them. How can this be? There is certainly the possibility that the problems faced in the problem-solving course are less demanding than the mastery required of new ideas in the regular courses. It may be that the regular course involve difficulties in which students can no longer operate in any way other than rote-learning the material for reproduction in the examinations. Such comments were made in the original UK pilot experiment:

> Clearly it is better if students understand the mathematics they have studied. But in practice this is difficult to attain because of the nature of the subject and time constraint. Often many students find it is not always possible to understand the course so they may choose instead to memorise the syllabus and solutions to the example sheets. … It is perfectly possible to gain a good grade by rote-learning. *3rd Year U.K. Mathematics Student*

Student comments before and after problem-solving

In the questionnaire, the students were asked to write a few sentences describing their feelings about mathematics. These were grouped under three headings: the nature of mathematics, personal feelings (such as motivation, interest, pressure etc.) and teaching methods. For example, items classified under the "nature of mathematics" on the pre-test included negative comments such as 'too abstract', 'seems pointless', and 'theory more difficult than practice'. Negative "personal factors" included 'lack of motivation', 'put off by amount that needs to be done' and 'puzzled by what is going on'. Positive feelings were mostly about the course being 'enjoyable and challenging', 'great sense of satisfaction when able to understand new concepts and to solve problems' and 'effort put in is worthwhile'. All responses relating to teaching were all negative—such as 'difficult to follow' and 'delivered in a dull atmosphere'.

After the problem-solving, comments shifted in a positive direction (Table 5).

Student comments	Positive		Negative	
(*N* = 44)	Pre	Post	Pre	Post
nature of mathematics	5	12	7	2
personal	15	28	12	5
teaching	0	0	8	4
Total comments	20	40	27	11

Table 5: Classification of written responses

Comments written after the course include:

> I am beginning to think instead of just doing the tutorial questions. … I think I am learning more because I understand what is going on.
>
> *3rd Year Industrial Science Student, majoring in Mathematics*

> Mathematics has always given me a lot of problems because I don't have the ability for memorisation. …Now that I know about mathematical thinking, my interest and desire to learn maths have increased.
>
> *4th Year Computer Education Student*

> The course should have been introduced earlier. … After following the course I am more confident to solve any maths problem that is given.
>
> *5th Year Computer Education Student*

These are consistent with the classroom observations and the changes intimated by the questionnaire, supporting the hypothesis that the course in problem-solving changes the attitudes of students from mathematics as a body of procedures to be learned to mathematics as a process of thinking.

Student comments six months after returning to mathematics

The students comments after returning to standard mathematics learning suggest a number of factors that could explain their changes in attitudes. For instance, about a third (32%) reported that the regular mathematics did not allow them to think in a problem-solving manner:

> Since following the course I know mathematics is about solving problems. But whatever mathematics I am doing now doesn't allow me to do all those things. They are just more things to be remembered.
>
> *5th Year Computer Education Student*

> I believed mathematics is useful in that it helps me to think. Having said that it is hard to say how I can do this with the maths I am doing. Most of the questions given can be solved by applying directly the procedures we had just learned. There is nothing to think about.
>
> *3rd Year Industrial Science Student, majoring in Mathematics*

> There is little discussion and it provides no encouragement to do maths. The content is emphasised over everything else. We are crammed full of lots of bland mathematical abstract theory.
>
> *3rd Year Industrial Science Student, majoring in Mathematics*

Some emphasise the speed of presentation to complete the content:

> I did not enjoy most of the maths courses—too dependent on the lecturers. I don't find the way most of them teach particularly inspiring. We find ourselves hurrying through to keep up. There is no time to think about the mathematics we are doing.
>
> *3rd Year Industrial Science Student, majoring in Mathematics*

Some appreciate their knowledge in problem solving, suggesting it helps them to learn their mathematics and solve problems more effectively:

> The problem solving techniques help me come to terms with the abstract nature of the maths I am doing. I try to connect the ideas together and talk about them with my friends. It is not that easy though. But I feel all the effort worth it when I am able to do so.
>
> *3rd Year Industrial Science Student, majoring in Mathematics*

> I find the problem solving knowledge very useful in helping me understand the whys and the hows of advanced mathematics. It is much more

> satisfying than rote-learning. Furthermore it is actually easier to remember something that you understand. *4th Year Computer Education Student*

There are some who have minor reservations on their problem solving experience. But they believe it is necessary to have a positive attitude:

> The main disadvantage is time. It would take several hours maybe days to understand each new concept. Under the current circumstances we are finding ourselves rapidly hurrying to keep up. Sometime we were too bogged down in the technical details and we end up purely taking down the notes without even concentrating. This really defeats the problem-solving techniques. … But I think with further support from good teaching as well as tailoring the courses to suit the needs of the students the situation can be improved. *5th Year Computer Education Student*

Individual interviews with lecturers

Interviews revealed substantial differences in meaning of ideas expressed in the questionnaire from the ideas of "mathematical thinking" in the problem-solving course. For instance, Kilpatrick & Stanic (1989) suggest three different perceptions of problem solving—as means to a focused end, as skill and as art. It soon became apparent that the lecturers see it more as a means to achieve a specific end or a skill to be learned rather than the art of thinking mathematically. "Inventing new ideas" was perceived as original research rather than just ideas new to the individual, as in the following quotation:

> To me mathematics is a tool for solving problems. One way of motivating the students is by showing them applications in the real world. In this way they get the knowledge and the skills for solving problems. … I do not think the students are capable of creating new ideas on their own.

Lecturers are not certain of the strategies used in the problem-solving course:

> … I am not sure of these [processes]. I have not thought about them and I don't know how to go about [teaching] them. I think I need to learn more about them before I can implement them. We developed certain abilities to look at problems but we are not sure how those abilities came to be with you.

Instead they show students how to do examples in the hope that they will develop their own techniques:

> The experience of making conjectures, generalising and the like, I think the students can get themselves on their own, from doing their project work. We do not have the time to teach them everything.
>
> We tell them how to do it – for example, what are the criteria that should be fulfilled in the formula before they can use it. Normally I explain only part of it then I think the students can complete it themselves. … I think that is sufficient for the students.

Some lecturers genuinely want to change the system but are not sure how:

> I would like students not only to see mathematics as a subject that they need to learn and pass in an exam but also as a discipline which enables them to think for themselves. My main aim is not in trying to finish the syllabus but rather in making the students learn the mathematics in a more meaningful way. … I am not really sure how but I am trying to do it.

> To me mathematics is a mental activity but I should say that at this level I presented it more as a formal system. Because we are confined by the syllabus and also depending on the students' background. … I would like it to change. How do I do that? I don't know.

> The system has been proven a failure. It has not been successful in producing good mathematicians, or engineers that can use mathematics effectively. They only know how to use procedures or computer packages without really understanding why they use them. …It's all down to the system. We are not training students to discover patterns, or how to prove a statement is true, for example. What we teach them is mainly how to use the procedures.

Summary

Although lecturers prefer students to have a range of positive attitudes to mathematics, they expect the reality to be different. They prefer students to see mathematics as solving problems, making sense, with students working hard, able to relate ideas without needing to learn through memory, having confidence, deriving pleasure, with low anxiety and fear, ready to try a new approach and unwilling to give up easily on difficult problems. On the other hand, they expect them to see mathematics as abstract, failing to understand it quickly, not making sense, working hard to learn facts and procedures through memory, unable to relate ideas, with less confidence, obtaining less pleasure, working only to get through the course, with anxiety, fear, seeking only correct answers, and ready to give up when things get difficult.

By assigning a "desired direction of change" in the direction from what lecturers expect to what they prefer, it transpires that when doing a problem-solving course almost all the changes are in the desired direction and when returning to mathematics lectures, almost all the changes are in the reverse direction.

The findings show that the lecturers have little confidence in the students' ability to think mathematically and teach them accordingly. The students acquiesce to this approach, and set their sights on the lower target of learning procedurally to be successful in routine tasks. In this there is a widespread sense of pleasure although, after the problem-solving course, opinions expressed suggest concern that the quantity and difficulty of the mathematics gives them little room for creative thinking.

Teaching problem-solving skills is not part of the lecturers' previous experience, consequently the lack of experience and the perceived difficulty of changing a formal system with so much content to be learned are severe deterrents to change. However, given the fact that problem-solving causes "positive changes in attitude" which are largely reversed in the standard course with its more difficult mathematical content, it is appropriate to pose the question:

> *Given this evidence, do professors wish to continue to get what they expect, or is it possible to change to attempt to get what they prefer?*

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