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Channel and Noise Variance Estimation and Tracking Algorithms for Unique-Word Based Single-Carrier Systems

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Abstract— Single-carrier (SC) wireless communication systems generally require knowledge of the channel and the variance of the additive noise process to equalize a received message. Obtaining this information can be straightforward in stationary environments; however, these parameters constantly change in mobile environments. In this paper, we propose novel algorithms for estimating and tracking the channel and noise variance in SC systems by exploiting a unique word (UW) extension. These UW-based algorithms benefit from low complexity and lend themselves to SC systems employing frequency-domain equalization at the receiver.

Index Terms— Frequency-domain equalization (FDE), channel estimation and tracking, unique word (UW).

I. INTRODUCTION

A fundamental problem in high data rate wireless communication systems employing single-carrier (SC) transmission is the equalization of the received message. In the past, researchers have mainly focused on equalization techniques that can be implemented in the time domain. Recently, however, the growing popularity of low complexity multi-carrier (MC) modulation techniques, such as orthogonal frequency division multiplexing (OFDM) [1]–[4], has led researchers to consider equalization of SC transmissions in the frequency domain [5]–[8]. Typically, systems employing SC transmission with frequency-domain equalization (SC-FDE) require knowledge of the channel and the variance of the additive noise process to perform equalization on the received symbols. These parameters can be estimated relatively easily prior to data transmission; however, in non-stationary channels, they vary with time and must be tracked by some means. In this paper, we present channel and noise variance estimation and tracking algorithms for SC-FDE systems employing a unique word (UW) extension.

The concept of using the UW in SC block transmissions as an alternative to the well-known cyclic prefix extension was presented in [9]. The topics of synchronization and phase tracking using the UW were addressed in [9]–[11] and

methods for exploiting the UW for equalizer training were described in [7], [8], [12]. Channel estimation and tracking techniques based on the UW were also briefly discussed in [7]–[10], [12], [13]. In this paper, we provide a rigorous explanation and analysis of a method of performing least squares (LS) channel estimation that is related to the equalizer training technique discussed in [7], [8]. We also derive bounds on the performance of this channel estimation technique. Furthermore, we propose and analyze a novel, low-complexity technique that uses the UW and the recursive least squares (RLS) algorithm to track a temporally fading channel and estimate the variance of the additive noise process.

The paper is arranged as follows. The channel and noise variance estimation and tracking algorithms are discussed in section II. Performance bounds are derived in section III. Finally, simulation results are illustrated in section IV, and conclusions are presented in section V.

Notation: We use a bold uppercase (lowercase) font to denote matrices (column vectors); frequency-domain variables are denoted by a tilde (e.g. $\tilde{a}$); $\mathbf{F}$ is the normalized $K \times K$ DFT matrix where its (k, i)th element is given by $F_{k,i} \triangleq K^{-1/2} \exp(-j2\pi ki/K)$ for $k, i = 0, \ldots, K-1$; $\mathbf{F}_m$ denotes the first m columns of $\mathbf{F}$ and $\mathbf{F}'_m$ denotes the last m columns of $\mathbf{F}$; $\mathbf{I}_m$ is the $m \times m$ identity matrix; $\mathbf{0}_{m \times n}$ is an $m \times n$ all-zero matrix; $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)_m$, and $|\cdot|$ denote the complex conjugate, transpose, conjugate transpose, modulo-m, and absolute value operations, respectively; $\otimes$ is the Kronecker product operator; $E\{\cdot\}$ is the expectation operator; $\text{tr}\{\cdot\}$ is the trace operator; $\mathcal{D}\{\mathbf{x}\}$ denotes a diagonal matrix with the elements of $\mathbf{x}$ on the diagonal.

II. CHANNEL AND NOISE VARIANCE ESTIMATION AND TRACKING

In this section, we describe two methods by which the channel can be estimated and tracked in an SC system. A simple procedure for estimating the variance of the additive noise process is also proposed. The first of the channel estimation methods is deterministic, utilizing a double-length UW to perform LS channel estimation. A similar technique was used to train a decision-feedback equalizer in [7], [8]. The second method is novel and relies on the recursive least squares (RLS) algorithm along with a separate feedback step to estimate and track the channel state information (CSI). Although any time-domain or frequency-domain detection technique can be employed once the channel has been estimated, these
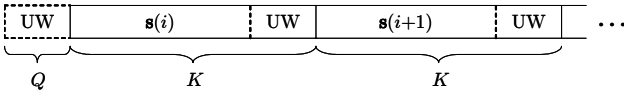
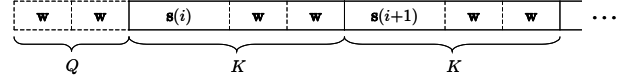Fig. 1.   Example of the basic UW block structure.

Fig. 2.   Example of the UW block structure used to perform deterministic LS channel estimation and tracking.

algorithms lend themselves to FDE systems since the use of the UW gives the system a cyclic nature.

Consider a system employing SC block transmissions with a UW extension. The ith length-K block of transmitted symbols, denoted by $\mathbf{x}(i)$, can be partitioned into a length-P vector $\mathbf{s}(i)$ of data symbols and a length-Q vector representing the UW. An illustration of this block structure is depicted in Fig. 1.

In order to mitigate inter-block interference (IBI), we assume that $Q \geq L$ where L is the memory order of the channel impulse response (CIR). This condition also induces circularity in the system, allowing us to express the ith length-K block of received symbols by

$$\mathbf{y}(i) = \mathbf{H}(i)\mathbf{x}(i) + \mathbf{n}(i) \qquad (1)$$

where $\mathbf{H}(i)$ is a $K \times K$ circulant matrix representing the channel at time i and $\mathbf{n}(i)$ is a length-K vector of uncorrelated, zero-mean, complex Gaussian noise samples, each with a variance of $\sigma_n^2/2$ per dimension. Specifically,

$$\mathbf{H}(i) = \begin{pmatrix} h_0(i) & 0 & \cdots & h_L(i) & \cdots & h_1(i) \\ \vdots & h_0(i) & \ddots & 0 & \ddots & \vdots \\ h_L(i) & \vdots & \ddots & \vdots & \ddots & h_L(i) \\ 0 & h_L(i) & & \ddots & & 0 \\ \vdots & & \ddots & & \ddots & \vdots \\ 0 & \cdots & 0 & h_L(i) & \cdots & h_0(i) \end{pmatrix}$$

where $h_m(i)$ is the mth complex tap coefficient of the CIR at time i. This system model will be used in the description of both of the aforementioned channel estimation and tracking techniques.

A. Deterministic LS Channel Estimation

We briefly describe the deterministic approach to channel estimation. Although this method is very similar to an equalizer training technique presented in [7], [8], the mathematical framework defined in this section is a precursor to the analysis included in section III. Furthermore, the technique discussed will be used as a benchmark in section IV where the performances of several channel estimation/tracking algorithms are examined.

The UW employed with this technique is simply two identical length-$Q/2$ UWs concatenated to form one double-length UW as illustrated in Fig. 2 where $\mathbf{w}$ is the length-$Q/2$ UW.

It will be shown that the double-length UW can be exploited to perform channel estimation with each received block, thus

facilitating channel tracking in mobile systems. In contrast to this method, conventional LS channel estimation techniques rely on a preamble to estimate the channel and this estimate is used for all signal processing operations until the channel can be re-estimated with another preamble.

We will see that a sufficient condition for channel identifiability is

$$\frac{Q}{2} \geq L + 1. \qquad (2)$$

If the length of each UW $\mathbf{w}$ satisfies this condition, we may omit the time index i for brevity and partition (1) into an upper and lower part, giving

$$\begin{bmatrix} \mathbf{y}_a \\ \mathbf{y}_b \end{bmatrix} = \begin{bmatrix} \mathbf{H}_a \\ \mathbf{H}_b \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_a \\ \mathbf{n}_b \end{bmatrix} \qquad (3)$$

where $\mathbf{H}_a$ and $\mathbf{H}_b$ denote the first $K - Q/2$ rows and last $Q/2$ rows of $\mathbf{H}$, respectively, and $\mathbf{y}_a$, $\mathbf{y}_b$, $\mathbf{n}_a$, and $\mathbf{n}_b$ are each defined in a similar manner. The Toeplitz matrix $\mathbf{H}_b$ is defined by its first row $(\mathbf{0}_{1 \times K-L-Q/2}, h_L, \ldots, h_0, \mathbf{0}_{1 \times Q/2-1})$ and its first column $\mathbf{0}_{Q/2 \times 1}$, and can be partitioned further to yield

$$\mathbf{H}_b = \begin{bmatrix} \mathbf{0}_{Q/2 \times K-Q} & \mathbf{H}_1 & \mathbf{H}_0 \end{bmatrix} \qquad (4)$$

where $\mathbf{H}_0$ and $\mathbf{H}_1$ are $Q/2 \times Q/2$ Toeplitz matrices. The first row and column of $\mathbf{H}_0$ are $(h_0, \mathbf{0}_{1 \times Q/2-1})$ and $(h_0, \ldots, h_L, \mathbf{0}_{1 \times Q/2-L-1})^{\mathrm{T}}$, respectively, and the first row and column of $\mathbf{H}_1$ are $(\mathbf{0}_{1 \times Q/2-L}, h_L, \ldots, h_1)$ and $\mathbf{0}_{Q/2 \times 1}$, respectively. Thus, noting that

$$\mathbf{x} = \begin{bmatrix} \mathbf{s} \\ \mathbf{w} \\ \mathbf{w} \end{bmatrix} \qquad (5)$$

$\mathbf{y}_b$ can be rewritten as

$$\begin{aligned} \mathbf{y}_b &= \mathbf{0}_{Q/2 \times K-Q}\mathbf{s} + \mathbf{H}_1\mathbf{w} + \mathbf{H}_0\mathbf{w} + \mathbf{n}_b \\ &= \underbrace{(\mathbf{H}_1 + \mathbf{H}_0)}_{\mathcal{H}}\mathbf{w} + \mathbf{n}_b \end{aligned} \qquad (6)$$

where $\mathcal{H}$ is a circulant matrix. As long as (2) is true, each element of $\mathcal{H}$ is either a unique CIR coefficient or zero. If (2) is not true, the positions of some of the CIR coefficients in $\mathbf{H}_0$ and $\mathbf{H}_1$ overlap, which results in a superposition of CIR coefficients in $\mathcal{H}$. It should be noted that since the first column of $\mathbf{H}_1$ is composed of zeros, the first symbol of the first constituent UW can be replaced with a data symbol, thus relaxing the restriction given in (2). Consequently, a necessary and sufficient condition for channel identifiability is $Q \geq 2L + 1$. This is a minor point since only one additional data symbol can be transmitted in each block when this approach is taken.

Assuming the UW is designed such that (2) holds, we may rewrite (6) as

$$\begin{aligned} \mathbf{y}_b &= \mathcal{H}\mathbf{w} + \mathbf{n}_b \\ &= \mathcal{W}\mathbf{h} + \mathbf{n}_b \end{aligned} \qquad (7)$$

where $\mathbf{h} \triangleq (h_0, h_1, \ldots, h_L, \mathbf{0}_{1 \times Q/2-L-1})^{\mathrm{T}}$ and $\mathcal{W}$ is a circulant matrix composed of the elements of $\mathbf{w}$. Thus, as long as $\mathcal{W}$ has full column rank, the LS channel estimate $\hat{\mathbf{h}}_{LS}$ is given by

$$\hat{\mathbf{h}}_{LS} = \mathcal{W}^{\dagger}\mathbf{y}_b \qquad (8)$$

where $\mathcal{W}^{\dagger} = \left(\mathcal{W}^{H}\mathcal{W}\right)^{-1}\mathcal{W}^{H}$ is the pseudoinverse of $\mathcal{W}$.

Using training sequences interspersed throughout data packets to perform channel estimation is not a new concept. Indeed, one example of an application of this technique can be found in GSM/EDGE [14]. The method presented here, however, provides a solution to the channel estimation problem in the context of UW-based SC systems that employ FDE.

B. RLS Channel Estimation and Tracking

The deterministic nature of the UW can be used in another way to perform channel estimation and channel tracking. For this method, the only restriction we initially place on the length-Q UW is $Q \geq L$ to account for IBI. Referring to (1), we consider the transformation of the received symbol vector $\mathbf{y}(i)$ into the frequency domain, which is given by

$$\tilde{\mathbf{y}}(i) = \widetilde{\mathbf{H}}(i)\tilde{\mathbf{x}}(i) + \tilde{\mathbf{n}}(i) \tag{9}$$

where $\tilde{\mathbf{y}}(i) = \mathbf{F}\mathbf{y}(i)$, $\tilde{\mathbf{n}}(i) = \mathbf{F}\mathbf{n}(i)$, $\tilde{\mathbf{x}}(i) = \mathbf{F}\mathbf{x}(i)$, and $\widetilde{\mathbf{H}}(i) = \mathbf{F}\mathbf{H}(i)\mathbf{F}^{H}$ is a diagonal matrix with the channel frequency response coefficients on the diagonal. The transmitted vector $\tilde{\mathbf{x}}(i)$ can be partitioned into a data part and a UW part as given by

$$\begin{aligned}\tilde{\mathbf{x}}(i) &= \begin{bmatrix} \mathbf{F}_{P} & \mathbf{F}'_{Q} \end{bmatrix} \begin{bmatrix} \mathbf{s}(i) \\ \mathbf{u} \end{bmatrix} \\ &= \tilde{\mathbf{s}}(i) + \tilde{\mathbf{u}} \end{aligned} \tag{10}$$

where $\mathbf{u}$ is the length-Q UW, $\tilde{\mathbf{s}}(i) = \mathbf{F}_{P}\mathbf{s}(i)$, and $\tilde{\mathbf{u}} = \mathbf{F}'_{Q}\mathbf{u}$. Therefore,

$$\tilde{\mathbf{y}}(i) = \widetilde{\mathbf{H}}(i)\tilde{\mathbf{s}}(i) + \widetilde{\mathbf{H}}(i)\tilde{\mathbf{u}} + \tilde{\mathbf{n}}(i). \tag{11}$$

If the channel varies with time, as is the case in mobile environments, the channel state must be tracked by some means. Of course, (11) shows that each received block is dependent upon random signals (the data and noise) as well as the deterministic UW. Consequently, the UW can be exploited to track channel variations by applying the RLS algorithm and treating the UW as the desired signal and the data signal as interference. In this approach, it is beneficial to remove as much of the interference from the received signal as possible prior to channel updating. With this aim, the interference caused by the data can be partially removed from each received block by first equalizing and detecting (quantizing) the data using a previous channel estimate and then subtracting the estimated interference signal from each received block in the frequency domain, which gives

$$\tilde{\mathbf{y}}_{u}(i) = \tilde{\mathbf{y}}(i) - \mathcal{D}\left\{\mathbf{F}_{P}\hat{\mathbf{s}}(i)\right\}\hat{\tilde{\mathbf{h}}}(i-1) \tag{12}$$

where $\hat{\mathbf{s}}(i)$ is the length-P vector of data symbols detected at time i and $\hat{\tilde{\mathbf{h}}}(i)$ is a length-K vector of the ith estimated channel frequency response coefficients. This feedback step is not used in existing UW-based channel estimation techniques, where data is treated as zero-mean colored noise and either removed from the received signal through the averaging of multiple received blocks or suppressed through an iterative weighted LS algorithm [15]. In contrast, data cancellation as given by (12) aids the performance of the proposed algorithm and allows us to design optimal UW structures for use with this algorithm as discussed in section III-B.

Using the modified received signal given by (12), the RLS algorithm can be employed with the cost function

$$\varphi(i) = \sum_{k=1}^{i} \rho^{i-k} |\mathbf{e}(k,i)|^{2} \tag{13}$$

where ρ is the standard RLS forgetting factor that is usually close to, but less than, one [16]. The error term $\mathbf{e}(k,i)$ in (13) is defined as

$$\mathbf{e}(k,i) = \tilde{\mathbf{y}}_{u}(k) - \widetilde{\mathbf{U}}\hat{\tilde{\mathbf{h}}}(i). \tag{14}$$

Taking the gradient of (13) with respect to $\hat{\tilde{\mathbf{h}}}(i)$, setting the result equal to zero, and performing some algebraic manipulations results in an expression for the channel estimate vector

$$\hat{\tilde{\mathbf{h}}}(i) = \underbrace{\frac{1}{\sum_{k=1}^{i}\rho^{i-k}}\widetilde{\mathbf{U}}^{-1}}_{\mathbf{P}^{-1}(i)}\underbrace{\sum_{k=1}^{i}\rho^{i-k}\tilde{\mathbf{y}}_{u}(k)}_{\mathbf{r}(i)}. \tag{15}$$

The channel estimate may be updated with the ith received block by noting that

$$\mathbf{P}(i) = \rho\mathbf{P}(i-1) + \widetilde{\mathbf{U}} \tag{16}$$

and

$$\mathbf{r}(i) = \rho\mathbf{r}(i-1) + \tilde{\mathbf{y}}_{u}(i). \tag{17}$$

Note that (15) requires the inverse of $\mathbf{P}(i)$ to compute the updated channel estimate. Since $\mathbf{P}(i)$ is a diagonal matrix, however, this inversion does not pose a problem. Consequently, this method of channel tracking benefits from very low complexity since only three complex multiplications are required to update the channel estimate on a given frequency tone.

For a better estimate, the estimate of the channel frequency response can be filtered to remove noise. Various frequency-domain filters can be used; however, the most common technique of removing noise from the channel estimate amounts to transforming the channel frequency response estimate into the time domain with an IDFT where it is windowed according to the CIR length, then transformed back into the frequency domain with a DFT. This procedure, which is commonly used in OFDM systems [17]–[19], effectively separates the channel subspace from the noise-only subspace, thereby removing unwanted noise from the channel estimate. By combining the DFT and windowing steps into partial fast Fourier transform (FFT) operations where some of the butterfly operations are culled in accordance with the window size, the complexity of this windowing technique is minimized.

C. Initialization of the RLS Method

For RLS channel estimation and tracking, $\mathbf{r}$ can be initialized to zero. Alternatively, if a reliable initial channel estimate $\hat{\tilde{\mathbf{h}}}(0)$ is available, $\mathbf{r}$ and $\mathbf{P}$ can be initialized to

$$\mathbf{r}(0) = \beta\widetilde{\mathbf{U}}\hat{\tilde{\mathbf{h}}}(0) \tag{18}$$

and

$$\mathbf{P}(0) = \beta\widetilde{\mathbf{U}} \tag{19}$$

where β is a positive real number. From (15), we note that

$$f(i) = \sum_{k=1}^{i} \rho^{i-k} \tag{20}$$

is a geometric series, and therefore

$$f(i) = \frac{1 - \rho^i}{1 - \rho}. \tag{21}$$

Consequently, we may intelligently choose β to be

$$\begin{aligned} \beta &= \lim_{i \to \infty} f(i) \\ &= \frac{1}{1 - \rho}. \end{aligned} \tag{22}$$

Defining β as above and initializing $\mathbf{r}$ and $\mathbf{P}$ as in (18) and (19) is equivalent to initializing the channel estimator by transmitting an infinite number of blocks containing only the UW over a static channel, which is denoted here by $\hat{\mathbf{h}}(0)$, and computing $\mathbf{r}(\infty)$ and $\mathbf{P}(\infty)$. This definition of β produces results as shown in section IV.

D. Estimation of Noise Variance

The UW can also be exploited to estimate the variance of the additive noise process at the receiver. Here, we assume linear FDE is employed where the received message is converted into the frequency domain, equalized with a linear minimum mean-square error (LMMSE) equalizer, then converted back into the time domain prior to further processing such as decoding and detection. It should be noted that this approach to noise variance estimation can be employed with other FDE techniques as well.

Using an estimate of the channel, the contribution of the UW to the received message is first removed from the original received vector in the frequency domain, giving

$$\tilde{\mathbf{y}}_s = \tilde{\mathbf{y}} - \widetilde{\mathbf{U}}\hat{\mathbf{h}} \tag{23}$$

where again we have omitted the time index i for brevity. We can write the length-K vector $\mathbf{z}_s$ of time-domain symbols that are output from the conventional LMMSE equalizer as [20]

$$\mathbf{z}_s = \mathbf{F}^{\mathrm{H}} \left(\widehat{\widehat{\mathbf{H}}}^{\mathrm{H}} \widehat{\widehat{\mathbf{H}}} + \frac{\hat{\sigma}_n^2}{\sigma_s^2} \mathbf{I}_K \right)^{-1} \widehat{\widehat{\mathbf{H}}}^{\mathrm{H}} \tilde{\mathbf{y}}_s \tag{24}$$

where $\widehat{\widehat{\mathbf{H}}} = \mathcal{D}\{\hat{\hat{\mathbf{h}}}\}$, σ_s^2 is the variance of the transmitted data, $\hat{\sigma}_n^2$ is the current estimate of the noise variance, and it is assumed that $\mathrm{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\} = \sigma_s^2 \mathbf{I}_K$. Although this assumption is clearly untrue since a UW is employed, it is necessary in order to estimate the noise variance.

Letting $\mathbf{x}_s \triangleq (\mathbf{s}^{\mathrm{T}}, \mathbf{0}_{1 \times Q})^{\mathrm{T}}$ and assuming $\widehat{\widehat{\mathbf{H}}}$ is a reliable channel estimate, we can define the error covariance matrix $\mathbf{C}_e$ of the detected symbols as

$$\mathbf{C}_e \triangleq \mathrm{E}\left\{ (\mathbf{z}_s - \mathbf{x}_s)(\mathbf{z}_s - \mathbf{x}_s)^{\mathrm{H}} \right\}. \tag{25}$$

Evaluating (25) and performing some algebraic manipulations yields

$$\mathbf{C}_e = \sigma_n^2 \mathbf{F}^{\mathrm{H}} \mathbf{B} \left(\widehat{\widehat{\mathbf{H}}}^{\mathrm{H}} \widehat{\widehat{\mathbf{H}}} + \frac{\hat{\sigma}_n^2}{\sigma_s^2} \mathbf{F}_P \mathbf{F}_P^{\mathrm{H}} \right) \mathbf{B} \mathbf{F} \tag{26}$$

where

$$\mathbf{B} = \left(\widehat{\widehat{\mathbf{H}}}^{\mathrm{H}} \widehat{\widehat{\mathbf{H}}} + \frac{\hat{\sigma}_n^2}{\sigma_s^2} \mathbf{I}_K \right)^{-1}. \tag{27}$$

Since we removed the UW contribution from the received vector in (23), we expect the last Q terms of $\mathbf{z}_s$ to be zero[1]; however, due to the additive noise, these terms are in general not zero. Denoting the last Q terms in $\mathbf{z}_s$ by $\mathbf{z}_s^Q \triangleq (z_{s;0}^Q, \ldots, z_{s;Q-1}^Q)^{\mathrm{T}}$ and the last Q elements on the diagonal of $\mathbf{D}_e = \mathbf{C}_e / \sigma_n^2$ by the vector $\mathbf{d}_e^Q \triangleq (d_{e;0}^Q, \ldots, d_{e;Q-1}^Q)^{\mathrm{T}}$, the new estimate $\hat{\sigma}_n^2$ of the noise variance can be computed as follows:

$$\hat{\sigma}_n^2 = \frac{1}{Q} \sum_{m=0}^{Q-1} \frac{|z_{s;m}^Q|^2}{d_{e;m}^Q}. \tag{28}$$

III. PERFORMANCE BOUNDS

In this section, lower bounds on the mean-square error (MSE) of the channel estimates given in sections II-A and II-B are derived. Furthermore, the issue of UW design is addressed where optimal UW structures are given for each channel estimation/tracking technique. We begin by addressing the LS method discussed in section II-A.

A. MSE of Deterministic LS Channel Estimate

Define the LS channel estimation error vector by

$$\boldsymbol{\epsilon}_{LS} = \hat{\mathbf{h}}_{LS} - \mathbf{h}. \tag{29}$$

Assuming, in this case, that $Q = 2(L+1)$ and the UW is normalized, the MSE of the LS channel estimate shown in (8) is given by

$$\begin{aligned} \mathcal{E}_{LS} &= \frac{1}{L+1} \mathrm{tr}\left\{ \mathrm{E}\left\{ \boldsymbol{\epsilon}_{LS} \boldsymbol{\epsilon}_{LS}^{\mathrm{H}} \right\} \right\} \\ &= \frac{\sigma_n^2}{L+1} \mathrm{tr}\left\{ \left(\boldsymbol{\mathcal{W}}^{\mathrm{H}} \boldsymbol{\mathcal{W}} \right)^{-1} \right\}. \end{aligned} \tag{30}$$

Using a similar argument as was used in [21], we arrive at a lower bound on the channel estimation MSE. This bound is given by

$$\mathcal{E}_{LS} \geq \frac{\sigma_n^2}{L+1} \tag{31}$$

where the equality is met if and only if

$$\left(\boldsymbol{\mathcal{W}}^{\mathrm{H}} \boldsymbol{\mathcal{W}} \right)^{-1} = \frac{1}{L+1} \mathbf{I}_{L+1}. \tag{32}$$

Equation (32) suggests that the UW must have perfect periodic correlation properties. Of course, one sequence that meets this criterion is the Kronecker delta function. However, this sequence suffers from a high peak-to-average power ratio (PAPR). Another choice of UW that meets the criterion stated in (32) without compromising the PAPR is a Chu sequence [7], [22]. A Chu sequence benefits from having perfect periodic correlation properties, which satisfies (32), as well as a constant envelope in the time-domain, thus precluding PAPR problems. If a Chu sequence is used as a UW, (31) is met with equality and (8) reduces to a scaled circular correlation between the UW and the received vector $\mathbf{y}_b$.

It is important to note that this lower bound on MSE is greater than that presented for OFDM systems in [21] and for

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[1]In fact, these terms will be affected by residual ISI resulting from the MMSE equalizer, but we assume here that this ISI is small enough to ignore.

SC systems employing a cyclic prefix in [23]. This difference can be attributed to the fact that the UW only occupies a portion of each transmitted block, and therefore less energy is used for training in the UW based method than in the full-training-block methods described in [21], [23].

B. MSE of RLS Channel Tracking

The MSE of the channel tracking algorithm presented in section II-B was studied. In order to make the MSE derivation tractable, we assume the interference caused by the data is perfectly removed from $\tilde{\mathbf{y}}(i)$ in (12), which gives

$$\tilde{\mathbf{y}}_u(i) = \widetilde{\mathbf{U}}\tilde{\mathbf{h}}(i) + \tilde{\mathbf{n}}(i). \tag{33}$$

Furthermore, it is assumed that temporal fading follows Jakes' model [24] and the length of the UW is set equal to the memory order of the CIR (i.e. $Q = L$).

Define the MSE of the CIR estimate as

$$\mathcal{E}_{RLS}(i) = \frac{1}{L+1}\text{tr}\left\{\text{E}\left\{\left(\hat{\mathbf{h}}(i) - \mathbf{h}(i)\right)\left(\hat{\mathbf{h}}(i) - \mathbf{h}(i)\right)^{\text{H}}\right\}\right\} \tag{34}$$

where[2]

$$\hat{\mathbf{h}}(i) = \frac{1}{\sqrt{K}}\mathbf{F}_{L+1}^{\text{H}}\hat{\tilde{\mathbf{h}}}(i) \tag{35}$$

and $\mathbf{h}(i) \triangleq (h_0(i), h_1(i), \ldots, h_L(i))^{\text{T}}$. Under the assumptions stated above, it can be shown that the MSE of the CIR estimate at time i is given in (36) where the vectors $\boldsymbol{\rho}_i$ and $\mathbf{j}_i$ and the matrix $\mathbf{J}_i$ are given in the Appendix along with a detailed derivation of (36). In (36), $\tilde{u}_k$ denotes the kth element of $\tilde{\mathbf{u}}$. The first error term in (36) is due to the additive white Gaussian noise process at the receiver. The second term represents the error in the channel estimate due to temporal fading.

$$\mathcal{E}_{RLS}(i) = \frac{\sigma_n^2\,(1-\rho)\,(1+\rho^i)}{K^2\,(1+\rho)\,(1-\rho^i)}\sum_{k=0}^{K-1}\frac{1}{|\tilde{u}_k|^2} \tag{36}$$
$$+ \frac{1}{L+1}\left(1 - \frac{2\,(1-\rho)}{1-\rho^i}\boldsymbol{\rho}_i^{\text{H}}\mathbf{j}_i + \left(\frac{1-\rho}{1-\rho^i}\right)^2\boldsymbol{\rho}_i^{\text{H}}\mathbf{J}_i\boldsymbol{\rho}_i\right)$$

Equation (36) can be minimized with respect to two parameters: the UW and the forgetting factor ρ. First, consider the optimization of the UW for a given value of ρ. Applying the arithmetic-geometric mean inequality, we arrive at a lower bound on $\mathcal{E}_{RLS}(i)$, which is given by

$$\mathcal{E}_{RLS}(i) \geq \frac{\sigma_n^2\,(1-\rho)\,(1+\rho^i)}{K\,(1+\rho)\,(1-\rho^i)}\left(\prod_{k=0}^{K-1}\frac{1}{|\tilde{u}_k|^2}\right)^{1/K} + \mathcal{C}(i) \tag{37}$$

where

$$\mathcal{C}(i) \triangleq \frac{1}{L+1}\left(1 - \frac{2\,(1-\rho)}{1-\rho^i}\boldsymbol{\rho}_i^{\text{H}}\mathbf{j}_i + \left(\frac{1-\rho}{1-\rho^i}\right)^2\boldsymbol{\rho}_i^{\text{H}}\mathbf{J}_i\boldsymbol{\rho}_i\right)$$

is just the second term in (36). Equation (37) is met with equality if and only if $|\tilde{u}_0|^2 = |\tilde{u}_1|^2 = \cdots = |\tilde{u}_{K-1}|^2$.

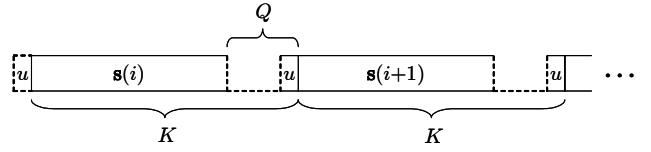[2]The time-domain windowing operation is employed here to remove noise from the channel estimate [17], [18].

Fig. 3. Example of a UW block structure used to perform RLS channel estimation and tracking.

Note that by optimizing the UW in this manor, the MSE is minimized for all values of i (i.e. during both the transient state and the steady state) for the given value of ρ.

One UW that meets this criterion is the Kronecker delta function. This is easily verified by recognizing that the DFT of a delta function has a constant envelope. As previously mentioned, however, employing a delta function as a UW leads to a PAPR problem. If the energy in the impulse is equal to the average energy of each transmitted data symbol, this PAPR problem is equivalent to that encountered in zero-padded (ZP) SC systems. It is important to note, however, that the implementation of a delta function as a UW does not reduce the UW-SC system to a ZP-SC system. This fact is illustrated in Figure 3 where the UW that is employed is simply $\mathbf{u} = (0, 0, \ldots, u)^{\text{T}}$.

Recall from the previous section that Chu sequences are optimal for the deterministic channel estimation method discussed in section II-A. However, Chu sequences do not provide optimal performance in the MSE sense when they are applied to the RLS method. This result arises from the fact that the optimal sequence (in the MSE sense) must be composed of $Q < K$ symbols but have a constant-modulus K-point DFT. Since Chu sequences have perfect periodic correlation properties, they benefit from having constant-modulus symbols in the frequency domain, but only when the DFT is taken over the complete sequence rather than a padded sequence [22]. Thus, the lower bound in (37) is not met with a Chu sequence unless it is a length-K sequence, in which case $\tilde{\mathbf{u}} = \mathbf{Fu}$. The UW is a full-length training block in this case and, therefore, can only be used as a preamble.

Adding a tight constraint to the PAPR of the UW, such as requiring the UW to be constant-modulus, results in neither Chu sequences nor the Kronecker delta function being optimal. The objective in this case is to find a length-Q constant-modulus sequence that also has a constant-modulus K-point DFT. Although this task is beyond the scope of this paper, it should be noted that some methods that can be used to find sequences that closely match these criteria have been documented in the literature [25], [26].

As previously mentioned, the MSE of the RLS channel estimate can also be minimized with respect to the forgetting factor ρ. This minimization is most useful when computed for the steady-state MSE (i.e. as $i \to \infty$), which must be performed numerically since taking the derivative of (36) with respect to ρ as $i \to \infty$ does not yield a closed form solution. To illustrate the feasibility of minimizing the MSE with respect to ρ, the MSE of the channel estimate as given by (36) was calculated as a function of ρ for an SNR of 15 dB and normalized Doppler spreads of $f_n = 10^{-3}$, $f_n = 10^{-4}$, and $f_n = 10^{-5}$. A Kronecker delta function was used for the
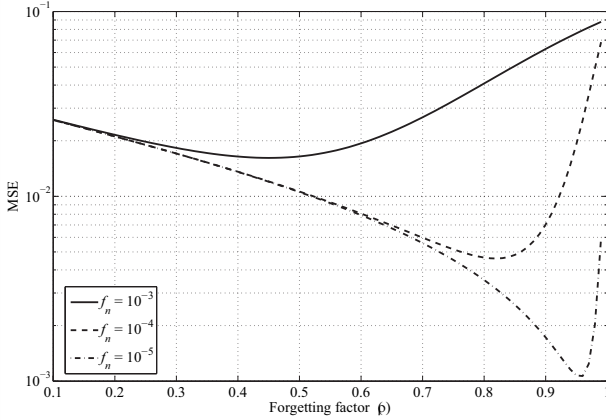
Fig. 4. MSE of RLS channel estimate as a function of ρ for various normalized Doppler spreads and an SNR of 15 dB.

TABLE I
SUMMARY OF SIMULATED SC-FDE SYSTEMS.

System	Channel knowledge	σ_n^2 knowledge
1	Perfect	Perfect
2	Preamble only	Estimated
3	Double-length UW w/ LS estimation (every block)	Perfect
4	Preamble w/ RLS update	Estimated

UW. The results of these calculations are depicted in Fig. 4, where clear minima are observed for the normalized Doppler values that were studied.

As observed in Fig. 4, the proposed RLS algorithm does not track well for high Doppler spreads and should, therefore, only be implemented in low-mobility environments as will be shown in the next section.

IV. RESULTS AND DISCUSSION

The algorithms described in section II were implemented in computer simulations in order to observe their performance relative to other techniques. In total, four uncoded systems were simulated. In each of these systems, UWs were appended to sets of QPSK data symbols to form blocks of $K = 64$ symbols. These blocks were transmitted over an 11-tap, exponentially decaying channel in bursts of 500 blocks per channel use. The channel realizations were generated with a Rayleigh fading profile from burst to burst, and Jakes' model was used to simulate temporal fading within each burst [24]. At the receiver, each system utilized its own knowledge of the channel and the noise variance to equalize the received message with an LMMSE FDE [20]. The equalized symbols were then mapped to QPSK symbols.

The first system was assumed to have perfect knowledge of the channel and the noise variance. The second system used an initial channel estimate, which was gleaned from a preamble, to construct an LMMSE FDE. Only one channel estimate was obtained for each burst. This system also used the method presented in section II-D to estimate the noise variance. Each block transmitted by the third system included a double-length UW, which was used at the receiver to perform LS channel estimation as described in section II-A. It was assumed that this system had perfect knowledge of the noise variance. Finally,
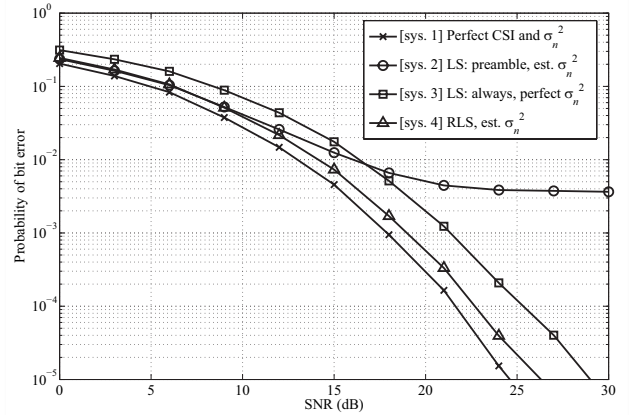

Fig. 5. Probability of bit error for SC-FDE systems employing various channel estimation/tracking techniques in a channel with a normalized Doppler spread of $f_n = 1.5 \times 10^{-6}$.
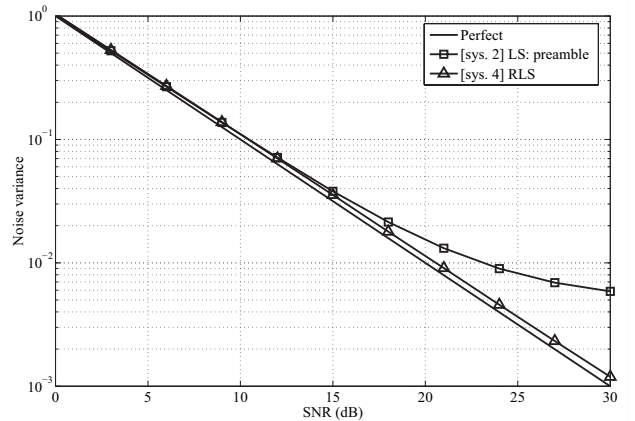

Fig. 6. Estimated noise variance over a range of SNR values.

the fourth system employed the RLS channel tracking method detailed in section II-B and the noise variance estimation algorithm given in section II-D. This system initialized the metrics $\mathbf{r}$ and $\mathbf{P}$ in accordance with (18) and (19) where the initial channel estimate was obtained through a preamble. The windowing/filtering technique described in section II-B and [17]–[19] was employed to improve the quality of the channel estimate. A forgetting factor of 0.96 was used. Each of these systems is summarized in Table I.

Fig. 5 depicts the probability of bit error of each of the systems described above. In this example, the normalized Doppler spread that was used with Jakes' model is $f_n = 1.5 \times 10^{-6}$. The system that employs RLS channel tracking and noise variance estimation performs better than the two systems that use deterministic LS channel estimation in this slow-fading environment. Indeed, the system employing a double-length UW with constant LS channel updating performs approximately 2 dB worse than the RLS method while providing only 75% of the raw throughput. Furthermore, it should be noted that the difference in performance between the RLS method and the ideal case is only 1 dB.

The performance of the noise variance estimation technique is illustrated in Fig. 6 for the example discussed above. From this figure, it is observed that the estimation technique works well despite the SNR, although as the quality of the channel
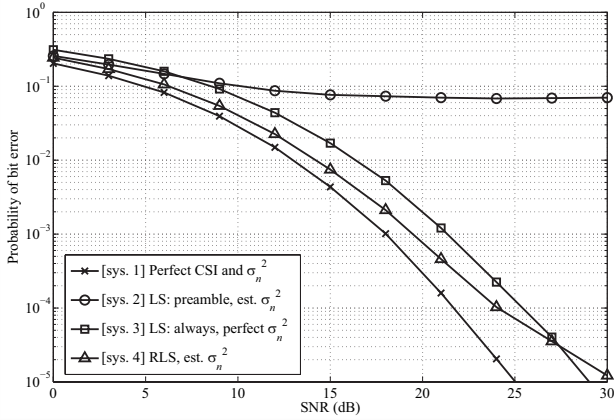
Fig. 7. Probability of bit error for SC-FDE systems employing various channel estimation/tracking techniques in a channel with a normalized Doppler spread of $f_n = 5 \times 10^{-6}$.

Fig. 9. MSE of RLS channel estimate as a function of ρ for various normalized Doppler spreads and an SNR of 27 dB.
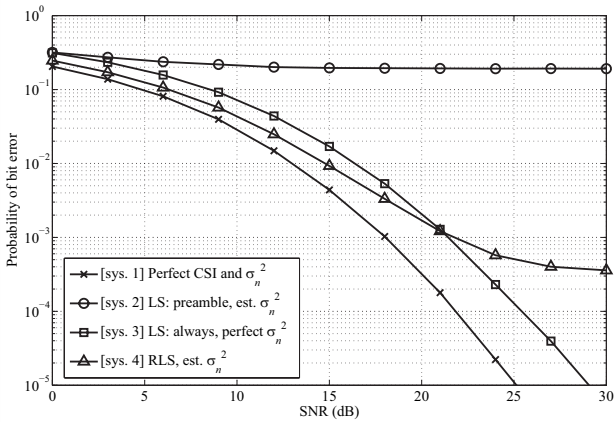

Fig. 8. Probability of bit error for SC-FDE systems employing various channel estimation/tracking techniques in a channel with a normalized Doppler spread of $f_n = 1 \times 10^{-5}$.

estimate degrades, the error in the noise variance estimate increases significantly as shown by the curve corresponding to the second system outlined in Table I. Although there is a slight error in the noise variance estimate for the system employing RLS channel tracking, this error obviously does not greatly degrade the performance of this system as shown in Fig. 5.

So far, we have only discussed the performance of the channel estimation/tracking algorithms in channels that change very slowly with time. It is beneficial to observe the performance of the algorithms in channels with higher normalized Doppler spreads. Fig. 7 and Fig. 8 depict the probability of bit error for each of the systems outlined in Table I operating in channels with a normalized Doppler spread of $f_n = 5 \times 10^{-6}$ and $f_n = 1 \times 10^{-5}$, respectively. All other system parameters in these examples are the same as those specified for the example detailed above. As observed in Fig. 7 and Fig. 8, an error floor begins to emerge for the RLS technique, which is obviously due to the increased rate of temporal variations in the channel. This remark is corroborated by Fig. 9, which depicts the MSE as a function of ρ for the three normalized Doppler spreads in question at an SNR of 27 dB. Observe that for a forgetting factor of $\rho = 0.96$, the MSE steadily increases
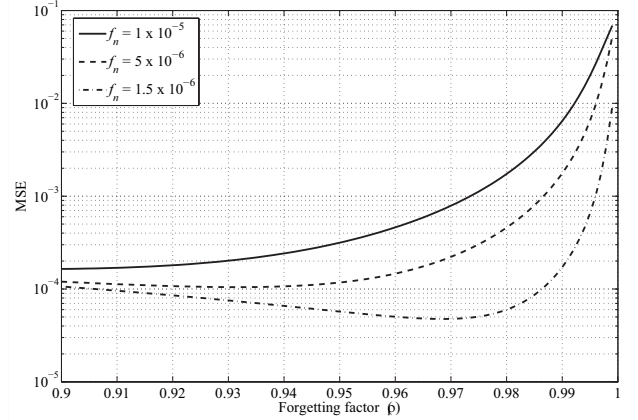
as the normalized Doppler spread increases.

It is interesting to note that the system employing a double-length UW and constant deterministic LS updating of the channel performs well in all of the examples presented here. Consequently, this technique is a good choice of transmission scheme for implementation in mobile environments. The only drawback is the increased overhead due to the double-length UW. Of course, this drawback is minimized when $K >> Q$.

V. CONCLUSIONS

In this paper, we presented algorithms for estimating and tracking the channel and noise variance in moderately mobile wireless communication systems. These algorithms utilize the constant nature of the UW extension to perform these estimation tasks. It was shown that a deterministic LS channel estimation technique that uses a double-length UW performs very well in mobile channels, but suffers from a low throughput due to the long UW. A novel technique based on the RLS algorithm was also presented. This method was shown to benefit from very low complexity and better throughput than the aforementioned technique, but its use is limited to slow-fading environments. In these environments, the performance of the RLS-based technique excels, providing a performance gain of 1-2 dB over the low-throughput deterministic LS method.

APPENDIX
MSE DERIVATION FOR THE RLS ALGORITHM

Consider the MSE at time i, which is defined by

$$\mathcal{E}_{RLS}(i) = \frac{1}{L+1} \text{tr} \left\{ \text{E} \left\{ \boldsymbol{\epsilon}(i) \boldsymbol{\epsilon}^{\text{H}}(i) \right\} \right\} \tag{38}$$

where $\boldsymbol{\epsilon}(i) = \hat{\mathbf{h}}(i) - \mathbf{h}(i)$. Assuming the interference is perfectly removed from the ith received block, the windowed CIR estimate at time i can be written as in (39) where we have used the fact that $\tilde{\mathbf{h}}(i) = \sqrt{K} \mathbf{F}_{L+1} \mathbf{h}(i)$ (cf. (15), (21), and (35)). Noting that the mean of each CIR tap is zero and the channel and noise are uncorrelated, the expectation in the expression for the MSE can be evaluated to give (40).

Recall from section III-B that temporal fading is assumed to follow Jakes' model in this analysis. Consequently,

$$\text{E} \left\{ h_m(k) h_n^*(\ell) \right\} = \begin{cases} \sigma_{h_m}^2 J_0 \left(2\pi f_n K |k - \ell| \right) & , \quad m = n \\ 0 & , \quad \text{otherwise} \end{cases} \tag{41}$$

$$\hat{\mathbf{h}}\left(i\right) = \frac{1-\rho}{1-\rho^{i}} \sum_{k=1}^{i} \rho^{i-k} \mathbf{h}\left(k\right) + \frac{1-\rho}{\sqrt{K}\left(1-\rho^{i}\right)} \mathbf{F}_{L+1}^{\mathrm{H}} \widetilde{\mathbf{U}}^{-1} \sum_{k=1}^{i} \rho^{i-k} \tilde{\mathbf{n}}\left(k\right) \tag{39}$$

$$
\begin{aligned}
\mathrm{E}\left\{\boldsymbol{\epsilon}(i)\boldsymbol{\epsilon}^{\mathrm{H}}(i)\right\} \;=\; & \frac{\sigma_{n}^{2}\left(1-\rho\right)^{2}\left(1-\rho^{2i}\right)}{K\left(1-\rho^{i}\right)^{2}\left(1-\rho^{2}\right)} \mathbf{F}_{L+1}^{\mathrm{H}} \widetilde{\mathbf{U}}^{-1} \widetilde{\mathbf{U}}^{-\mathrm{H}} \mathbf{F}_{L+1} + \mathrm{E}\left\{\mathbf{h}\left(i\right)\mathbf{h}^{\mathrm{H}}\left(i\right)\right\} \\
& + \left(\frac{1-\rho}{1-\rho^{i}}\right)^{2} \sum_{k=1}^{i}\sum_{\ell=1}^{i} \rho^{2i-k-\ell} \mathrm{E}\left\{\mathbf{h}\left(k\right)\mathbf{h}^{\mathrm{H}}\left(\ell\right)\right\} \\
& - \frac{1-\rho}{1-\rho^{i}} \sum_{k=1}^{i} \rho^{i-k} \mathrm{E}\left\{\mathbf{h}\left(k\right)\mathbf{h}^{\mathrm{H}}\left(i\right)\right\} \\
& - \frac{1-\rho}{1-\rho^{i}} \sum_{k=1}^{i} \rho^{i-k} \mathrm{E}\left\{\mathbf{h}\left(i\right)\mathbf{h}^{\mathrm{H}}\left(k\right)\right\}
\end{aligned}
\tag{40}
$$

and

$$\mathrm{E}\left\{\mathbf{h}\left(k\right)\mathbf{h}^{\mathrm{H}}\left(\ell\right)\right\} = J_{0}\left(2\pi f_{n} K\left|k-\ell\right|\right)\boldsymbol{\Phi} \tag{42}$$

where $J_{0}(x)$ is the zeroth-order Bessel function of the first kind, $\boldsymbol{\Phi} \triangleq \mathcal{D}\{\sigma_{h_{0}}^{2}, \sigma_{h_{1}}^{2}, \dots, \sigma_{h_{L}}^{2}\}$, and $\sigma_{h_{m}}^{2} = \mathrm{E}\{|h_{m}|^{2}\}$. Substituting (42) into (40) and using (38), the MSE is found to be (43).

This expression can be simplified further by assuming the total power of the channel is normalized (i.e. $\sum_{m=0}^{L}\sigma_{h_{m}}^{2} = 1$). Finally, the expression for MSE given by (43) can be represented in matrix form as given by (44) through (47).

$$\mathcal{E}_{RLS}\left(i\right) = \frac{\sigma_{n}^{2}\left(1-\rho\right)\left(1+\rho^{i}\right)}{K^{2}\left(1+\rho\right)\left(1-\rho^{i}\right)} \sum_{k=0}^{K-1} \frac{1}{|\tilde{u}_{k}|^{2}} \tag{44}$$

$$+ \frac{1}{L+1}\left(1 - \frac{2\left(1-\rho\right)}{1-\rho^{i}}\boldsymbol{\rho}_{i}^{\mathrm{H}}\mathbf{j}_{i} + \left(\frac{1-\rho}{1-\rho^{i}}\right)^{2}\boldsymbol{\rho}_{i}^{\mathrm{H}}\mathbf{J}_{i}\boldsymbol{\rho}_{i}\right)$$

$$\mathbf{j}_{i} = \begin{pmatrix} J_{0}\left(2\pi f_{n} K\left(i-1\right)\right) \\ J_{0}\left(2\pi f_{n} K\left(i-2\right)\right) \\ \vdots \\ 1 \end{pmatrix} \tag{46}$$

$$\boldsymbol{\rho}_{i} = \begin{pmatrix} \rho^{i-1} \\ \rho^{i-2} \\ \vdots \\ 1 \end{pmatrix} \tag{47}$$

ACKNOWLEDGMENT

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$$
\begin{aligned}
\mathcal{E}_{RLS}(i) &= \frac{\sigma_n^2 \left(1-\rho\right)\left(1+\rho^i\right)}{K^2\left(1+\rho\right)\left(1-\rho^i\right)} \sum_{k=0}^{K-1} \frac{1}{\left|\tilde{u}_k\right|^2} + \frac{1}{L+1} \sum_{m=0}^{L} \sigma_{h_m}^2 \\
&+ \frac{\left(1-\rho\right)^2}{(L+1)\left(1-\rho^i\right)^2} \left(\sum_{k=1}^{i} \sum_{\ell=1}^{i} \rho^{2i-k-\ell} J_0\left(2\pi f_n K \left|k-\ell\right|\right) \right) \left(\sum_{m=0}^{L} \sigma_{h_m}^2 \right) \\
&- \frac{2\left(1-\rho\right)}{(L+1)\left(1-\rho^i\right)} \left(\sum_{k=1}^{i} \rho^{i-k} J_0\left(2\pi f_n K \left|k-i\right|\right) \right) \left(\sum_{m=0}^{L} \sigma_{h_m}^2 \right)
\end{aligned}
\tag{43}
$$

$$
\mathbf{J}_i = \begin{pmatrix}
1 & J_0\left(2\pi f_n K\left(1\right)\right) & \cdots & J_0\left(2\pi f_n K\left(i-1\right)\right) \\
J_0\left(2\pi f_n K\left(1\right)\right) & 1 & & \vdots \\
\vdots & & \ddots & \\
J_0\left(2\pi f_n K\left(i-1\right)\right) & \cdots & & 1
\end{pmatrix}
\tag{45}
$$

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