Channel Estimation Algorithms for OFDM Systems

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***Abstract*—In this work we have compared different type of channel estimation algorithm for Orthogonal Frequency Division Multiplexing (OFDM) systems. The result of the Adaptive Boosting (AdaBoost) algorithm was compared with other algorithms such Least Square (LS), Best Linear Unbiased Estimator (BLUE) and Minimum Mean Square Error (MMSE).**

***Index Terms*—OFDM, Channel Estimation, LS, MMSE.**

I. INTRODUCTION

In communication system many techniques, like Frequency Division Multiplexing Access (FDMA), Time Division multiplexing Access (TDMA) and Code Division Multiplexing Access (CDMA), are used for transmission of signal. Where FDMA has very bad spectrum usage and TDMA performance degrades by multipath delay spread causing Inter Symbol Interference (ISI). In contrast OFDM enables high bit-rate wireless applications in a multipath radio environment the need for complex receivers. OFDM is a multi-channel modulation system employing Frequency Division Multiplexing (FDM) of orthogonal sub-carriers, each modulating a low bit-rate digital stream. OFDM uses N overlapping (but orthogonal) sub bands, each carrying a baud rate of $1/T$ and they are spaced $1/T$ apart. Because of the selected frequency spacing, the sub-carriers are all mathematically orthogonal to each other. This permits proper demodulation of symbol streams without the requirement of non overlapping spectra.

Currently, there is increasing interest in OFDM as it combines the advantages of high data rates and easy implementation. This is reflected by the many standards that considered and adopted OFDM those for Digital Audio Broadcast (DAB) and Digital video Broadcast (DVB), high speed modems over digital subscriber lines, and Wireless Local Area Network (WLAN) broadband systems as of ieee 802.11a, 802.11b and 802.11g.

The accuracy of channel state information greatly influences the overall system performance [1]. The main challenges associated with OFDM systems today are- channel identification and tracking, channel coding and equalization. In wideband mobile channels, pilot-based signal correction schemes are feasible method for OFDM systems [2]. Most channel estimation methods for OFDM transmission systems have been developed under the assumption of a slow fading channel, where the channel transfer function is assumed stationary within one OFDM data block. In addition, the channel transfer function for the previous OFDM data block is used as the transfer function for the present data block. In practice, the channel transfer function of a wideband radio channel may have significant changes even within one OFDM data block. Therefore, it is preferable to estimate channel characteristic based on the pilot signals in each individual OFDM data block [3].

Recently, an elegant channel estimation method for OFDM mobile communication systems has been proposed by [4] in which a semi-blind low complexity frequency domain based channel estimation algorithm for multi-access OFDM systems was proposed [4]. Many researchers have pursued channel estimation in the time domain. A joint carrier frequency synchronization and channel estimation scheme, using the expectation-maximization (EM) approach, is presented in [5] while [6] used subspace tracking. In [7], a joint and data estimation algorithm is presented which makes a collective use of data and channel constraints. A joint frequency-offset and channel estimation technique for multi-symbol encapsulated (MSE) OFDM system is proposed in [8], while the authors of [9] presented a sequential method based on carrier frequency offset and symbol timing estimation. The authors of [10] estimated the channel based on power spectral density (PSD) and least squares (LS) estimation for OFDM systems with timing offsets. A pilot aided channel estimation algorithm in the presence of synchronous noise by exploiting the a priori available information about the interference structure was presented in [11] while [12] used implicit pilots for joint detection and channel estimation. A joint time domain tracking of channel frequency offset for OFDM systems is suggested in [13] while a time domain carrier frequency offset (CFO) tracking method based on Particle filtering is presented in [14]. Radial Basis Function (RBF) network based channel estimation has been investigated in [15]. This is based on using the radial basis function (RBF) network to model the dynamics of the fading process. In one-dimensional channel estimation, only the time-correlation of the fading channel is exploited, whereas in two-dimensional channel estimation, both the time and the frequency correlations of the fading channels are exploited. RBF networks are essentially nonlinear interpolators of the pilot channels. AdaBoost based channel estimation is proposed in [16]. AdaBoost not only increases the performance of the OFDM systems it also renders the

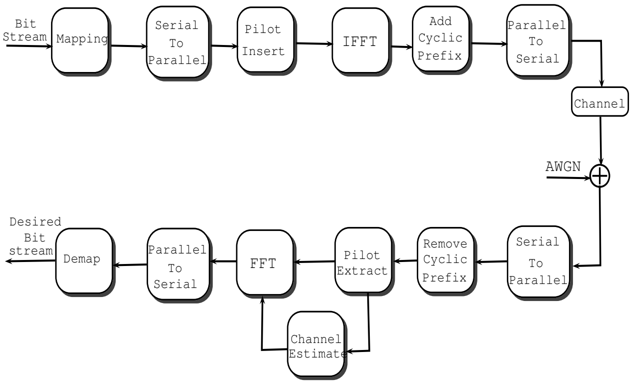


Fig. 1. Block Diagram of the OFDM systems

QAM mapping obsolete and thereby reducing the complexity of receiver designs.

The rest of the article is arranged as follows: Section. 2 describes the mathematical model of OFDM systems and its channel model along with two types of pilot arrangement. The implementation of AdaBoost and other algorithms (like LS, MMSE, and BLUE) is discussed in Section. 3 . Section. 4 describes the performance analysis of different algorithms and finally Section. 5 draws up the conclusions.

II. SYSTEM DESCRIPTION

The block diagram of an OFDM system is given in Figure.1. The binary information data are grouped and mapped into multi-amplitude-multi-phase signals. In this paper, we consider the 16-QAM modulation. After pilot insertion, the modulated data $X(k)$ are sent to an IDFT and are transformed and multiplexed into $x(n)$ as

$$x(n) = IFFT\{X(k)\} = \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N} \quad for\ \ n=0,1,...,N-1 \tag{1}$$

where N is the number of subcarriers. The guard interval N_g is inserted to prevent possible inter-symbol interference in OFDM systems, and the resultant samples $x_g(n)$ are

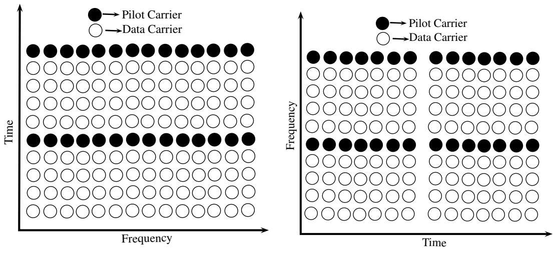
$$x_g(n) = \begin{cases} x(N+n) & n = N_g, N_g-1, \ldots, -1 \\ x(n) & n = 0,1,\ldots,N-1 \end{cases} \tag{2}$$

where N_g is the number of samples in the guard interval. The transmitted signal is then sent to a frequency selective multi-path fading channel. The received signal can be represented by

$$y_g(n) = x_g(n) \otimes h(n) \ + \ w(n) \tag{3}$$

Where $h(n)$ is the channel impulse response (CIR) and $w(n)$ is the Additive White Gaussian Noise (AWGN) and $\otimes$ is the circular convolution. The channel impulse response $h(n)$ can be expressed as [17]

$$h(n) = \sum_{i=0}^{r-1} h_i \ e^{j2\pi f_{Di}T\frac{n}{N}} \delta(\lambda - \tau_i) \tag{4}$$



(a) Block Type (b) Comb Type

Fig. 2. Pilot Arrangements

Where r the total number of propagation paths, h_i is the complex impulse response of the ith path, f_{Di} is the ith path's Doppler frequency shift which causes Inter channel Interference (ICI) of the received signals, λ is the delay spread index, and τ_i is the ith path delay time normalized by the sampling time. After removing the guard interval from $y_g(n)$, the received samples $y(n)$ are sent to a Fast Fourier Transform (FFT) block to demultiplex the multi-carrier signals.

$$Y(k) = FFT\{y(n)\} = \frac{1}{N}\sum_{n=0}^{N-1} y(n)e^{-j2\pi kn/N} \quad for\ k = 0,1,\ldots,N-1 \tag{5}$$

If we assume that the guard interval is longer than the length of channe1 impulse response- there is no inter-symbol interference between OFDM symbols- the demultiplexed samples $Y(k)$ can be represented by

$$Y(k) = X(k)H(k) + W(k), \quad k = 0,1,\ldots,N-1 \tag{6}$$

Where $H(k) = h_i \ e^{j2\pi f_{Di}T\frac{sin(\pi f_{Di}T)}{\pi f_{Di}T}} e^{-j\frac{2\pi\tau_i}{N}k}$ and $W(k)$ is the Fourier transform of the AWGN $w(k)$.

After that, the received pilot signals $\{Y_p(k)\}$ are extracted from $\{Y(k)\}$, the channel transfer function $\{H(k)\}$ can be obtained from the information carried by $\{H_p(k)\}$. With the knowledge of the channel responses $\{H(k)\}$, the transmitted data samples $\{X(k)\}$ can be recovered by simply dividing the received signal by the channel response:

$$\hat{x}(k) = \frac{y(k)}{\hat{H}(k)} \tag{7}$$

where $\hat{H}(k)$ is an estimate of $H(k)$. After signal demapping, the source binary information data are reconstructed at the receiver output.

The OFDM transmission scheme makes it easy to assign pilots in both time and frequency domain. Figure.2 shows two major types of pilot arrangement. The first kind of pilot arrangement shown in Figure.2a is denoted as block-type pilot arrangement. The pilot signal is assigned to a particular OFDM block, which is sent periodically in time domain. This type of pilot arrangement is especially suitable for slow-fading radio channels. The estimation of channel response is usually obtained by either LS or MMSE estimates of training pilots [5].

The second kind of pilot arrangement, shown in Figure. 2b, is denoted as comb-type pilot arrangement. The pilot signals are uniformly distributed within each OFDM block. Assuming that the payloads of pilot signals of the two arrangements are the same, the comb-type pilot assignment has a higher retransmission rate. Thus, the comb-type pilot arrangement system is provides better resistance to fast-fading channels. Since only some sub-carriers contain the pilot signal, the channel response of nonpilot subcarriers will be estimated by interpolating neighboring pilot sub-channels. Thus, the comb-type pilot arrangement is sensitive to frequency selectivity when comparing to the block-type pilot arrangement system. That is, the pilot spacing $(\Delta f)_p$, must be much smaller than the coherence bandwidth of the channel $(\Delta f)_c$ [3].

III. CHANNEL ESTIMATION

For comb-type pilot sub-carrier arrangement, the N_p pilot signals $X_p(m)\ \ , m = 0, 1, ..., N_p - 1$, are uniformly inserted into $X(k)$. That is, the total N sub-carriers are divided into N_p groups, each with $L = N/N_p$ adjacent sub-carriers. In each group, the first sub-carrier is used to transmit pilot signal. The OFDM signal modulated on the kth sub-carrier can be expressed as

$$\begin{aligned} X(k) &= X(mL + l) \\ &= \begin{cases} X_p(m), & l = 0, \\ information\ data, & l = 1, 2, \ldots, L-1 \end{cases} \end{aligned} \tag{8}$$

Let

$$\begin{aligned} H_P &= [H_p(0)\ H_p(1)\ \ldots H_p(N_p - 1)]^T \\ &= [H(0)\ H(L-1)\ \ldots H((N_p - 1)(L-1))]^T \end{aligned} \tag{9}$$

be the channel response of pilot carriers, and

$$Y_p\ =\ [Y_p(0)\ Y_p(1) \ldots Y_p(N_p - 1)] \tag{10}$$

be a vector of received pilot signals. The received pilot signal vector Y_p can be expressed as

$$Y_p\ =\ X_p H_p\ + W_p \tag{11}$$

where

$$X_p\ =\ \begin{bmatrix} X_p(0) & 0 & \ldots & 0 \\ \vdots & X_p(1) & \ldots & \vdots \\ 0 & 0 & \ldots & X_p(N_p - 1) \end{bmatrix}$$

where W_p is the vector of Gaussian noise in pilot sub-carriers.

A. LS Estimation

In conventional comb-type pilot based channel estimation methods, the estimation of pilot signals, is based on the LS method is given by

$$\begin{aligned} \hat{H}_{p,ls} &= [H_{p,ls}(0)\ H_{p,ls}(1)\ \ldots\ H_{p,ls}(N_p - 1)] \\ &= X_p^{-1} Y_p \\ &= [\tfrac{Y_p(0)}{X_p(0)}\ \tfrac{Y_p(1)}{X_p(1)}\ \ldots\ \tfrac{Y_p(N_p-1)}{X_p(N_p-1)}] \end{aligned} \tag{12}$$

The LS estimate of H_p is susceptible to AWGN and Inter-Carrier Interference (ICI). Because the channel responses of data subcarriers are obtained by interpolating the channel characterstics of pilot subcarriers, the performance of OFDM systems which are based on comb-type pilot arrangement is highly dependent on the rigorousness of estimate of pilot signals. Thus, a estimate better than the LS estimate is required. The MMSE estimate has been shown to be better than the LS estimate for channel estimation in OFDM systems based on block-type pilot arrangement. The Mean Square Error (MSE) estimation, given in [17], shows that MMSE estimate has about 10-15 dB gain in SNR over the LS estimate for the same MSE values. The major drawback of the MMSE estimate is its high complexity, which grows exponentially with the observation samples.

B. MMSE Estimation

The MMSE estimator employs the second order statistics of the channel conditions in order to minimizes Mean Square Error (MSE). Let $\underline{R}_{hh}$, $\underline{R}_{HH}$, and $\underline{R}_{YY}$ be the autocovariance matrix of h, H, and Y, respectively and $\underline{R}_{hY}$ be the cross covariance matrix between h and Y. Also σ_w^2 denotes the AWGN variance $E\{|w^2|\}$. Assume the channel vector h, and the AWGN w are uncorrelated, it can be derived that [18]

$$\underline{R}_{HH} = E\{HH^H\} = E\{(\underline{F}h)(\underline{F}h)^H\} = \underline{F}\ \underline{R}_{hh}\underline{F}^H \tag{13}$$

where

$$\underline{F} = \begin{bmatrix} TW_N^{00} & \ldots & TW_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ TW_N^{(N-1)0} & \ldots & TW_N^{(N-1)(N-1)} \end{bmatrix} \tag{14}$$

and $TW_N^{i,k} = \frac{1}{\sqrt{N}} e^{-j2\pi i \frac{k}{N}}$ is called as the twidle factor matrix. Assuming $\underline{R}_{hh}$ (thus $\underline{R}_{HH}$) and σ_w^2 are known at the receiver in advance, the MMSE estimator of the h is given by $\hat{h}_{MMSE} = \underline{R}_{hY}\underline{R}_{YY}YY^H$. At last it can be estimated that

$$\hat{H}_{MMSE} = \underline{F}\hat{h}_{MMSE} = \underline{R}_{HH}[\underline{R}_{HH} + \sigma_w^2 (XX^{Ht})^{-1}]^{-1}\hat{H}_{LS} \tag{15}$$

MMSE employs the second order statistics of the channel for estimation. Some times the channel statistics are not available, so it is difficult to estimate the channel in this situation. However, in OFDM systems the signal can be available at the receiver by means of pilot carriers.

C. BLUE

If we restrict the estimator to be linear in data and and find the linear estimator that is unbiased and minimum variance then the estimator is called Best Linear Unbiased Estimator (BLUE). BLUE can be determined with knowledge of only the first and second moment of the PDF. Since complete knowledge of the PDF is not necessary, the BLUE is more suitable for practical implementation [19].

Gauss-Markov Theorem 1. *If the data can be modeled in the following linear form*

$$Y\ = XH\ +\ W \tag{16}$$

where X is a known $N \times p$ matrix and H is a $p \times 1$ vector of parameters to be estimated, and W is a $N \times 1$ noise vector *with zero mean and covariance C (the PDF of W is otherwise arbitrary), then BLUE of H is given by*

$$H_{BLUE} = (H^{Ht}C^{-1}H)^{-1}H^{Ht}C^{-1}X \tag{17}$$

where $C = (X - E\{x\})(X - E\{x\})^{Ht}$ *and* Ht *is the conjugate transpose or Hermitian Transpose. and the minimum variance of* $\hat{H}_i$ *is*

$$var(\hat{H}_i) = [(H^{Ht}C^{-1}H)^{-1}]_{ii} \tag{18}$$

D. AdaBoost

Boosting is a general method for improving the accuracy of any given learning algorithm. AdaBoost was originally defined for two class problems but it can be extended boosting to multi-class and regression problems. The AdaBoost algorithm, introduced in 1995 by Freund and Schapire, solved many of the practical problems of the earlier boosting algorithms [20]. The AdaBoost algorithm for multiclass problem is described as below.

Suppose we are given a set of training data $(x_1, c_1),\ (x_2, c_2),\ \ldots,\ (x_n, c_n)$, where the input (prediction variable) $x_i \in \mathbb{R}^p$ and the output (response variable) c_i is quantitative values in afinite set, e.g. $1, 2, \ldots, K$. Where K is the number of classes. Usually it is assumed that the training data are composed from independently and identically distributed (iid) samples from a unknown probability distribution $Prob(X, C)$. The gaol is to find out a classification rule $C(x)$ from the training data, so that when given a new input x, we can assign a class label c from $1, 2, \ldots, K$. The misclassification error rate of a classifier $C(x)$ is given by $1 - \sum_{k=1}^{K} E_X[\mathbb{I}_{C(X)=k}\ Prob(C = k|X)]$. So from [20]

$$C^{\star}(x) = arg\ \max_k\ Prob(C = k|X = x) \tag{19}$$

will minimize this quantity with the misclassification error rate equal to $1 - E_X \max_k\ Prob(C = k|X)$. This classifier is known as *Bayes Classifier* and its rate is *Bayes Error Rate*. The multiclass AdaBoost algorithm tries to approximate the Bayes Classifier $C^{\star}(x)$ by combining many weak classifiers. Starting with an unweighted training sample, the AdaBoost builds a classifier that produces class labels. If a training data point is misclassified, the weight of that training data point is increased (boosted). A second classifier is built using these new weights, which are no longer equal. Again, misclassified training data have their weights boosted and the procedure is repeated. Typically, one may build 500 or 1000 classifiers this way. A score is assigned to each classifier, and the final classifier is defined as the score weighted linear combination of the classifiers from each stage. Specifically, let $T(x)$ denote a weak multi-class classifier that assigns a class label to x, then the AdaBoost algorithm proceeds as follows: [21]

Multiclass AdaBoost Algorithm: 1. *The Algorithm is as follows:*

1) *Initialize the observation weights* $w_i = 1/n,\ i = 1,\ 2,\ \ldots,\ n.$
2) *For m = 1 to M:*

TABLE I
OFDM PARAMETERS

No of Subcarriers	256
No. of Pilot Carriers	32
Guard Interval	64
Guard Type	Cyclic Extension

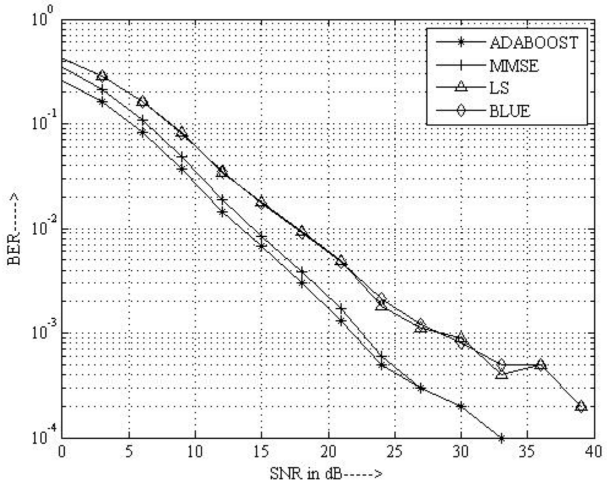


Fig. 3. Performance Comparison of Different Algorithms

   a) *Fit a classifier* $T^{(m)}(x)$ *to the training data using weights* w_i.
   b) *Compute*

$$err^{(m)} = \sum_{i=1}^{n} w_i\ \mathbb{I}(c_i \neq T^{(m)}(x_i)) / \sum_{i=1}^{n} w_i \tag{20}$$

   c) *Compute*

$$\alpha^{(m)} = \log \frac{1 - err^{(m)}}{err^{(m)}} + \log(K - 1) \tag{21}$$

   d) *Set*

$$w_i \leftarrow w_i\ exp(\alpha^{(m)}.\ \mathbb{I}(c_i \neq T^{(m)}(x_i))) \tag{22}$$

   for i=1, 2, . . . , n.
   e) *Re-normalize* w_i.

3) *Output*

$$C(x) = arg\ \max_k \sum_{m=1}^{M} \alpha^{(m)}\ \mathbb{I}\ (T^{(m)}\ (x) = k) \tag{23}$$

IV. SIMULATION RESULTS

The OFDM system channel estimation was simulated with LS, MMSE, BLUE and AdaBoost methods. In all simulations we have used QAM16 as the modulation scheme for the individual carriers. Other parameters of the simulation are given in the Table.I. Figure.3 shows comparison of Bit Error Rate (BER) for LS, MMSE, BLUE and AdaBoost. Figure.3 shows that the AdaBoost algorithm improves the performance specially in low SNR values. However, at high SNR both MMSE and AdaBoost show a similar performance. Furtheremore, AdaBoost gives better performance when compared to LS and BLUE. Nevertheless, BLUE and LS are performing same or we can tell BLUE gives very marginal improvement to LS. The reason for this performance increase is because of the covariance matrix used in the BLUE. As our noise is ASWGN and it has variance of 1 so the BLUE's performance is all most that of LS algorithm.

V. CONCLUSSION

The computational complexity of the MMSE increases exponentially as the number of carrier increases. Whereas the computatinal complexity is not exponential in the case of AdaBoost. Hence, AdaBoost can be employed when a high number of carriers is required. Furthermore, as it is a classification algorithm so in the receiver side we will require a separate demapper (or decoder) to get the desired data bits. AdaBoost not only increases the performance of the OFDM systems it also renders the QAM mapping obsolete and thereby reducing the complexity of receiver designs.

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