Channel Estimation for STAR-RIS-aided Wireless Communication

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Abstract-In this letter, we study efficient uplink channel estimation design for a simultaneously transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) assisted two-user communication systems. We first consider the time switching (TS) protocol for STAR-RIS and propose an efficient scheme to separately estimate the channels of the two users with optimized training (transmission/reflection) pattern. Next, we consider the energy splitting (ES) protocol for STAR-RIS under the practical coupled phase-shift model and devise a customized scheme to simultaneously estimate the channels of both users. Although the problem of minimizing the resultant channel estimation error for the ES protocol is difficult to solve, we propose an efficient algorithm to obtain a high-quality solution by jointly designing the pilot sequences, power-splitting ratio, and training patterns. Numerical results show the effectiveness of the proposed channel estimation designs and reveal that the STAR-RIS under the TS protocol achieves a smaller channel estimation error than the ES case.

Index Terms—Channel estimation, reconfigurable intelligent surface, simultaneous transmission and reflection.

I. INTRODUCTION

Recently, reconfigurable intelligent surface (RIS) has emerged as a promising technology to improve the spectrum and energy efficiency of future wireless systems [1]. Specifically, RIS enables reconfigurable radio environment by smartly tuning the signal propagation via a large number of lowcost elements. However, in most existing works, RISs can only reflect signals within its front half-space [2], thus can only serve users on one side. To overcome this limitation, a novel concept of simultaneously transmitting and reflecting RISs (STAR-RISs) has been recently proposed [3], which can transmit and/or reflect the incident signals, thus enabling serving users in both sides of RIS to achieve full-space smart radio environment. This has motivated growing research interests to investigate the benefits of deploying STAR-RISs in wireless networks, such as extending network coverage [4], reducing power consumption [5], and enhancing system throughput [6].

To reap the passive beamforming gain of RISs/STAR-RISs, channel state information (CSI) is indispensable, which, however, is practically challenging to acquire due to the lack of

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Y. Cai is with the College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China (email: ylcai@zju.edu.cn). signal transmission/processing capabilities for passive surface. To address this issue, various RIS channel estimation schemes have been proposed in the literature to attain the CSI by efficiently turning the reflections of RIS over time based on e.g., the ON/OFF-based [7] and discrete Fourier transform (DFT)-based [8] training reflection pattern. However, these schemes [7], [8] cannot be directly applied to STAR-RIS due to the following reasons. First, the channel estimation design for STAR-RIS hinges on its hardware design and operation protocol, e.g., time switching (TS) and energy splitting (ES) [3], [5]; Though ES is shown to be preferable for downlink data transmission [5], it is unknown whether ES maintains the superiority over TS in terms of channel estimation. Thus, new channel estimation schemes tailed to STAR-RIS need to be devised. Second, in contrast to the conventional RIS that requires designing a single reflection pattern only, both the transmission and reflection patterns of the STAR-RIS need to be jointly designed with the pilot sequences. In particular, for ES, the STAR-RIS phase-shifts for transmission and reflection are intricately coupled in practice, which makes the channel estimation design more complicated.

To address the above issues, we study in this letter efficient channel estimation schemes for a STAR-RIS-assisted twouser communication system based on two practical protocols of TS and ES. For TS, we design the optimal training (transmission/reflection) pattern to separately estimate the concatenated channels of the two users. For ES, we consider a practical coupled phase-shift model for STAR-RIS and propose an efficient scheme to simultaneously estimate the channels of both users. Specifically, to minimize the mean square error (MSE) of channel estimation for the ES case, a joint optimization problem of the power-splitting ratio, pilot sequences, and training patterns is formulated. To solve this challenging problem, we first obtain the optimal solution under the ideal case where the transmission-and-reflection patterns are independently controlled, based on which a near-optimal solution under the practical phase-shift model is then developed. Interestingly, it is shown that the channel estimation error for ES is generally larger than the TS case as ES results in power leakage in the uplink channel estimation. Simulation results corroborate the effectiveness of our proposed designs and theoretical analysis.

Notations: $\mathbb{C}^{M \times 1}$ denotes the space of $M \times 1$ complexvalued vectors. \mathbf{a}^T and \mathbf{a}^H denote the transpose and the conjugate transpose of vector \mathbf{a} , respectively. diag(\mathbf{a}) denotes a diagonal matrix with the diagonal elements given by \mathbf{a}^T . For a square matrix \mathbf{A} , $\text{Tr}(\mathbf{A})$, and \mathbf{A}^{-1} represent its trace and inverse, respectively. For any matrix \mathbf{B} , $\text{rank}(\mathbf{B})$ and $[\mathbf{B}]_{m,n}$ denote its rank and (m, n)th element. γ denotes the imaginary

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(a) A STAR-RIS aided two-user uplink (b) Operation protocols for STAR-RIS communication systems

Fig. 1: Illustration of system model.

unit, i.e., $j^2 = -1$. The distribution of a circularly symmetric complex Gaussian variable with mean μ and variance σ^2 is denoted by $\mathcal{CN} \sim (\mu, \sigma^2)$.

II. SYSTEM MODEL

As illustrated in Fig. 1(a), we consider a narrow-band wireless communication system, where a STAR-RIS with M_0 elements is deployed to assist the data transmission from two single-antenna users¹ located on its both sides to a singleantenna base station² (BS). We recall the elements-grouping strategy in [9], [10] to reduce the channel estimation overhead, where the STAR-RIS elements are divided into M subsurfaces, each of which consists of M_0/M adjacent elements that share a common transmission/reflection coefficient. The uplink signals of the two users can be either transmitted or reflected by the STAR-RIS to the BS, thus referred to as T user and R user, respectively. Let h_k and $\mathbf{r}_k \in \mathbb{C}^{M \times 1}$ denote the baseband equivalent channel from user $k, k \in \mathcal{K} \triangleq \{t, r\}$ to the BS and STAR-RIS, respectively. Further, we denote $\mathbf{g}^{H} \in \mathbb{C}^{1 \times M}$ as the channel from the STAR-RIS to the BS; and $\mathbf{q}_{k}^{H} \triangleq \operatorname{diag}(\mathbf{g})\mathbf{r}_{k} \in \mathbb{C}^{M \times 1}$ as the cascaded channels from user k to the BS. Moreover, we assume quasi-static block-fading channels and focus on the uplink communication in one typical fading block such that h_k , \mathbf{r}_k , and \mathbf{g} remain approximately constant.

We consider two practical operation protocols for STAR-RIS, namely, time switching and energy splitting [3], [5], as shown in Fig. 1(b). In the following sections, we will elaborate the STAR-RIS channel estimation scheme for TS and ES, respectively.

III. STAR-RIS CHANNEL ESTIMATION: TIME SWITCHING

In this section, we propose an efficient scheme to separately estimate the concatenated channels of STAR-RIS under the TS protocol.

Transmission and Reflection Model: For TS, the STAR-RIS switches all elements between the transmission and reflection modes in two separate time intervals (referred to as T period and R period). We denote τ_t and τ_r as the number of time slots allocated to the T period and R period, respectively. Moreover, we define $\boldsymbol{\theta}_i^{\text{TS}} \triangleq [e^{j\theta_{1,i}}, \dots, \dots, e^{j\theta_{M,i}}], \boldsymbol{\phi}_i^{\text{TS}} \triangleq [e^{j\phi_{1,i}}, \dots, \dots, e^{j\phi_{M,i}}]$ as the transmission and reflection coefficient vectors, respectively, where $\theta_{m,i}, \phi_{m,i} \in [0, 2\pi), m \in$

²The design in our work is readily extendable to the BS with multiple antennas by estimating their channels in parallel.

time slot i. <u>Signal Model</u>: In the following, we elaborate the channel estimation design for the T user, while the scheme can be

estimation design for the 1 user, while the scheme can be directly extended to the case of R user. Specifically, during the T period, the T user consecutively sends pilot symbols to the BS. Denote p as the maximum transmit power. Then, the baseband received signal at the BS in time slot $i, i = 1, ..., \tau_t$, is

$$y_{i} = [h_{t} + \boldsymbol{g}^{H} \operatorname{diag}(\boldsymbol{\theta}_{i}^{\mathrm{TS}})\mathbf{r}_{t}]\sqrt{p}s_{t,i} + n_{i}$$

= $(h_{t} + \boldsymbol{\theta}_{i}^{\mathrm{TS}}\mathbf{q}_{t}^{H})\sqrt{p}s_{t,i} + n_{i},$ (1)

where $n_i \sim C\mathcal{N}(0, \sigma^2)$ is the additive white Gaussian noise at the BS; $|s_{t,i}|$ is the pilot symbol in time slot *i*, which can be set as $s_{t,i} = 1$ for simplicity. As such, the overall received signal during the T period at the BS can be written as

$$\mathbf{y}^{t} = [y_1, \dots, y_{\tau_t}]^T = \sqrt{p} \mathbf{\Theta} \mathbf{x}^t + \mathbf{n},$$
(2)

where $\boldsymbol{\Theta}$ denotes the transmission pattern matrix, the i-th row of which is given by $[1, \boldsymbol{\theta}_i^{\text{TS}}]$. $\mathbf{x}^t = [h_t, \mathbf{q}_t]^T$ denotes the composite channel vector associated with the T user, and $\mathbf{n} = [n_1, ..., n_{\tau_t}]^T$.

A. Problem Formulation

According to (2), if Θ is of full column rank, the leastsquare (LS) estimate of \mathbf{x}^t is given by $\hat{\mathbf{x}} = \frac{1}{\sqrt{p}} \Theta^{\dagger} \mathbf{y}^t$, where $\Theta^{\dagger} = (\Theta^H \Theta)^{-1} \Theta^H$ is the pseudo-inverse of Θ . The MSE of the above LS estimation is [8]

$$MSE^{T} = \mathbb{E}\left[\|\hat{\mathbf{x}} - \mathbf{x}\|_{2}^{2}\right] = \frac{\sigma^{2}}{p} Tr[(\boldsymbol{\Theta}^{H}\boldsymbol{\Theta})^{-1}].$$
(3)

The optimization problem for minimizing the MSE can be thus formulated as

(P1):
$$\min_{\boldsymbol{\Theta}} \frac{\sigma^2}{p} \operatorname{Tr}[(\boldsymbol{\Theta}^H \boldsymbol{\Theta})^{-1}]$$
 (4a)

s.t.
$$\theta_{m,i}, \phi_{m,i} \in [0, 2\pi), m \in \mathcal{M}, i = 1, ..., \tau_t$$
, (4b)

$$\operatorname{rank}(\mathbf{\Theta}) = M + 1. \tag{4c}$$

B. Proposed Solution

According to [8], an optimal solution to problem (P1) is an $(M+1) \times (M+1)$ DFT matrix \mathbf{D}_{M+1} , whose entries are given by

$$[\mathbf{D}_{M+1}]_{m,n} = e^{-j\frac{2\pi(m-1)(n-1)}{M+1}}, 1 \le m, n \le M+1.$$
 (5)

Following the same procedure in the R period, the CSI of the R user can be acquired. Then, it can be easily shown that under the minimum required overhead $\tau_t + \tau_r = 2M + 2$, the sum MSE of channel estimation by TS is

$$MSE^{TS} = \frac{2\sigma^2}{p}.$$
 (6)

IV. STAR-RIS CHANNEL ESTIMATION: ENERGY SPLITTING

In this section, we introduce the channel estimation scheme for ES, where the channels of both users are estimated simultaneously.

¹Our design can be extended to the multi-user case by dividing users into multiple groups, each consisting of two users located on the two sides of RIS, and allocating orthogonal time/frequency resources to each group.

<u>Transmission and Reflection Model</u>: For the ES protocol, the signals impinged on each sub-surface is split into transmitted ones and reflected ones with an energy splitting ratio of β_m^t, β_m^r . Denote τ as the number of time slots for channel estimation. Accordingly, the transmission and reflection vectors at time slot *i* is defined as $\boldsymbol{\theta}_i^{\text{ES}} \triangleq [\beta_1^t e^{j\theta_{1,i}}, \dots, \beta_M^t e^{j\theta_{M,i}}]$ and $\boldsymbol{\phi}_i^{\text{ES}} \triangleq [\beta_1^r e^{j\phi_{1,i}}, \dots, \beta_M^r e^{j\phi_{M,i}}]$, respectively. Note that according to the law of energy conservation, we have $\beta_m^t + \beta_m^r \leq 1$ [3].

We consider in this paper the *practical coupled phase-shift* model, where the phase-shifts of each element for transmission and reflection, i.e., $\theta_{m,i}$, $\phi_{m,i}$ are coupled with each other.³ This model is practically accurate for a fully-passive STAR-RIS under the hardware constraint [12]. Specifically, according to [12], [13], the coupled phase-shifts of each sub-surface m should satisfy

$$\cos(\theta_{m,i} - \phi_{m,i}) = 0. \tag{7}$$

Signal Model: During the channel estimation stage, both users keep sending pilot symbols to the BS, while the training (transmission and reflection) patterns of the STAR-RIS are properly designed to assist the channel estimation. For a fair comparison, we set the same total transmit power of the users in each time slot as in the TS case; thus, the transmit power of the T user and R user is given by $p_t = p_T = \frac{p}{2}$. As such, the baseband received signal at the BS in time slot *i* is

$$y_i = (h_t + \boldsymbol{\theta}_i^{\mathrm{ES}} \mathbf{q}_t^H) \sqrt{p/2} s_{t,i} + (h_r + \boldsymbol{\phi}_i^{\mathrm{ES}} \mathbf{q}_r^H) \sqrt{p/2} s_{r,i} + n_i,$$
(8)

where $s_{k,i}$ denotes the pilot symbol of user k in time instant i. Then, the overall received signal at the BS during the channel estimation stage is given by

$$\mathbf{y} = [y_1, \dots, y_\tau]^T = \sqrt{p/2} \mathbf{V} \mathbf{x} + \mathbf{n}, \tag{9}$$

where $\mathbf{x} = [h_t, \mathbf{q}_t, h_r, \mathbf{q}_r]^T$ is the composite channel coefficient vector, $\mathbf{n} = [n_1, ..., n_\tau]^T$, and \mathbf{V} is given at the bottom of this page. If \mathbf{V} is of full rank, the LS estimate of \mathbf{x} is given by $\hat{\mathbf{x}} = \sqrt{\frac{2}{p}} \mathbf{V}^{\dagger} \mathbf{y}$, with the minimum overhead $\tau \ge 2M + 2$. Then, the MSE of channel estimation for the ES protocol can be expressed as [8]

$$MSE^{ES} = \frac{2\sigma^2}{p} Tr[(\mathbf{V}^H \mathbf{V})^{-1}].$$
 (11)

A. Problem Formulation

Define $\bar{\Theta}$ and $\bar{\Phi}$ as the training pattern matrix that stacks the vector θ_i^{ES} and ϕ_i^{ES} of each time slot, respectively, i.e., $\bar{\Theta} \triangleq [\theta_1^{\text{ES}}; ...; \theta_{\tau}^{\text{ES}}], \bar{\Phi} \triangleq [\phi_1^{\text{ES}}; ...; \phi_{\tau}^{\text{ES}}]$. For the ES

³We assume continuous phase-shifts in this letter, while the results can be extended to a more general case with practical discrete phase-shift by proper quantization [11].

protocol, we aim to minimize the MSE of channel estimation by jointly designing the pilot sequences of the users $\mathbf{s}_k \triangleq \{s_{k,1}, ..., s_{k,\tau}\}^T, \forall k \in \mathcal{K}$, energy splitting ratio β_m^k , and training pattern matrices $\bar{\boldsymbol{\Theta}}, \bar{\boldsymbol{\Phi}}$, which can be formulated as

(P2):
$$\min_{\{\mathbf{s}_k, \beta_m^k, \bar{\mathbf{\Theta}}, \bar{\mathbf{\Phi}}\}} \frac{2\sigma^2}{p} \operatorname{Tr}[(\mathbf{V}^H \mathbf{V})^{-1}]$$
(12a)

s.t.
$$\theta_{m,i}, \phi_{m,i} \in [0, 2\pi), m \in \mathcal{M}, i = 1, ..., \tau,$$

$$\operatorname{rank}(\mathbf{V}) = 2M + 2, \tag{12c}$$

$$\beta_m^\iota + \beta_m^r \le 1,\tag{12d}$$

$$\beta_m^k > 0, k \in \mathcal{K},\tag{12e}$$

$$\cos(\theta_{m,i} - \phi_{m,i}) = 0. \tag{12f}$$

B. Proposed Solutions

To obtain useful insights, we first drop the constraints in (12f) and denote the relaxed problem as (P3). Note that (P3) corresponds to the ideal case, where the phase-shifts for transmission and reflection can be adjusted independently.

Proposition 1. *The optimal solution to problem (P3) should satisfy the following conditions:*

- V is an orthogonal matrix.
- The pilot symbols of both users are always non-zero.
- The energy splitting ratio of each sub-surface is set identically as β^t_m = β^r_m = 0.5, ∀m ∈ M.

Proof. The lower bound of MSE is attained when $\mathbf{V}^H \mathbf{V}$ is a diagonal matrix [8], or in other words, the columns in \mathbf{V} is orthogonal to each other. In this case, we have

$$\operatorname{Tr}[(\mathbf{V}^{H}\mathbf{V})^{-1}] \stackrel{(a)}{=} \sum_{q=1}^{2M+2} \frac{1}{\sum_{p=1}^{\tau} |[\mathbf{V}]_{p,q}|^{2}} \\ \stackrel{(b)}{\geq} \sum_{m=1}^{M} \frac{1}{\tau} (\frac{1}{\beta_{m}^{t}} + \frac{1}{\beta_{m}^{r}}) + \frac{2}{\tau}$$
(13)

where (a) is due to that the *q*-th diagonal element in $\mathbf{V}^{H}\mathbf{V}$ is $\sum_{p=1}^{\tau} |[\mathbf{V}]_{p,q}|^2$. If $\beta_m^k \neq 0$, the equality of (b) will hold when the pilot symbols are all non-zero, i.e., $|s_{k,i}| = 1, \forall k, i$. Finally, the MSE minimization is equivalent to minimizing $(\frac{1}{\beta_m^t} + \frac{1}{\beta_m^r})$, whose lower bound is achieved when $\beta_m^t = \beta_m^r = 0.5$. This completes the proof.

Next, we give an example of the optimal solution for problem (P3), which satisfies all the conditions in **Proposition 1**. First, we select any $(2M + 2) \times (2M + 2)$ orthogonal matrix \mathbf{D}_{2M+2} , e.g., the DFT/Hadamard matrix. The pilot sequences \mathbf{s}_t and \mathbf{s}_r are set as the first and (M+2)-th column of \mathbf{D}_{2M+2} , respectively. Then, the training patterns $\bar{\boldsymbol{\Theta}}, \bar{\boldsymbol{\Phi}}$ can be set as

$$\mathbf{V} = \begin{bmatrix} s_{t,1} & \sqrt{\beta_1^t} e^{j\theta_{1,1}} s_{t,1} & \dots & \sqrt{\beta_M^t} e^{j\theta_{M,1}} s_{t,1} & s_{r,1} & \sqrt{\beta_1^r} e^{j\phi_{1,1}} s_{r,1} & \dots & \sqrt{\beta_M^r} e^{j\phi_{M,1}} s_{r,1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ s_{t,\tau} & \sqrt{\beta_1^t} e^{j\theta_{1,\tau}} s_{t,\tau} & \dots & \sqrt{\beta_M^t} e^{j\theta_{M,\tau}} s_{t,\tau} & s_{r,\tau} & \sqrt{\beta_1^r} e^{j\phi_{1,\tau}} s_{r,\tau} & \dots & \sqrt{\beta_M^r} e^{j\phi_{M,\tau}} s_{r,\tau} \end{bmatrix}$$
(10)

$$[\bar{\mathbf{\Theta}}]_{m,n} = \frac{[\mathbf{D}_{2M+2}]_{m,n+1}}{s_{t,m}}, 1 \le m \le 2M+2, 1 \le n \le M$$
(14a)

$$[\bar{\Phi}]_{m,n} = \frac{[\mathbf{D}_{2M+2}]_{m,n+M+2}}{s_{r,m}}, 1 \le m \le 2M+2, 1 \le n \le M.$$
(14b)

The optimality of the above design can be explained as follows: First, for ease of description, \mathbf{V} in (10) can be written as

$$\mathbf{V} = [\mathbf{s}_t, \frac{1}{\sqrt{2}} \operatorname{diag}(\mathbf{s}_t) \bar{\mathbf{\Theta}}, \mathbf{s}_r, \frac{1}{\sqrt{2}} \operatorname{diag}(\mathbf{s}_r) \bar{\mathbf{\Phi}}].$$
(15)

Since we divide the phase-shifts by the pilot symbols in (14), the *m*-th column ($\forall n \neq 1, M+2$) in V equals the *m*-th column in \mathbf{D}_{2M+2} times the energy splitting ratio $\frac{1}{\sqrt{2}}$. Meanwhile, the first and (M + 2)-th column of V is the same as that in \mathbf{D}_{2M+2} as introduced. Thus, V is an orthogonal matrix, which is an optimal solution to (P3). In this case, by substituting $\tau = 2M + 2$ and $\beta_m^k = 0.5$ into (13), the channel estimation MSE for ES is obtained as

$$MSE^{ES} = \left(\frac{4M}{2M+2} + \frac{2}{2M+2}\right)\frac{2\sigma^2}{p} = \frac{4M+2}{M+1}\frac{\sigma^2}{p}, \quad (16)$$

which serves as a performance upper bound when evaluating the impact of practical phase-shifts.

Proposed Solution to Problem (P2): Due to constraint (12f) introduced by the coupled phase-shifts, it is hard to find an optimal solution for problem (P2). Therefore, we aim to find a high-quality suboptimal solution by constructing a *nearly-orthogonal* matrix \mathbf{V} under the full-rank constraint.

Inspired by the transmission/reflection training design under the ideal phase-shift case, we target to retaining the orthogonality of $[\operatorname{diag}(\mathbf{s}_t)\overline{\mathbf{\Theta}}, \operatorname{diag}(\mathbf{s}_r)\overline{\mathbf{\Phi}}]$ in V. Interestingly, we find that constraint (12f) can be met if the pilot sequence of R user is changed to $\mathbf{s}_r = [j, -j, j, -j, ...]$ and the training pattern is designed as in (14). The reason is as follows: From (14), we can find that for $1 \leq \forall i \leq 2M + 2, 1 \leq \forall m \leq M$,

$$\frac{e^{j\theta_{m,i}}}{e^{j\phi_{m,i}}} = \frac{[\mathbf{D}_{2M+2}]_{i,m+1}}{[\mathbf{D}_{2M+2}]_{i,m+M+2}} \frac{s_{r,i}}{s_{t,i}}.$$
 (17)

If \mathbf{D}_{2M+2} is a DFT matrix, mathematically we have

$$[\mathbf{D}_{2M+2}]_{i,m+M+2} = (-1)^{i+1} [\mathbf{D}_{2M+2}]_{i,m+1}.$$
 (18)

Therefore, $\frac{e^{j\theta_{m,i}}}{e^{j\phi_{m,i}}} = j$, which satisfies (12f). If \mathbf{D}_{2M+2} is the Hadamard matrix, since its entries are either +1 or -1, $\frac{e^{j\theta_{m,i}}}{e^{j\phi_{m,i}}}$ is either j or -j according to (17), which also satisfies (12f). Based on the above, the columns of **V** are all orthogonal with each other except for one column, which is \mathbf{s}_r . Therefore, it is expected that the proposed STAR-RIS channel estimation scheme under the practical phase-shift model approaches the MSE performance of that under the ideal model.

C. Discussion

As introduced, the minimum overhead for channel estimation for TS and ES is the same, which is 2M+2. Nevertheless, from (6) and (16), we can observe that, the minimum channel



Fig. 2: NMSE performance under different energy splitting ratios β_t .

estimation error for the ES protocol is approximately twice as that for the TS protocol. This can be intuitively explained by the fact that after energy splitting, part of the uplink signals is transmitted or reflected towards the opposite side of the STAR-RIS from the BS, which leads to a reduction in the effective signal strength during channel estimation.

V. NUMERICAL RESULTS

In this section, we provide numerical results to verify the effectiveness of our proposed channel estimation schemes for TS and ES. In the simulation, the STAR-RIS consists of $M_0 = 80$ elements and is divided into M = 20 sub-surfaces. All involved channels are modeled as Rician fading with the Rician factor of 10 dB. The distance-dependent path losses are modeled as $l = \beta_0 (d/d_0)^{-\alpha}$, where $\beta_0 = -30$ dB denotes the path loss at the reference distance $d_0 = 1$ meter (m), d represents the individual link distance, and α is the path-loss exponent. We consider a two-dimensional coordinate system, where the BS is located at the origin and the reference center of the STAR-RIS is at (50m, 0). The T user and the R user are located at (54m, 3m) and (46m, -3m), respectively. The pathloss exponents of the user-BS, user-STAR-RIS, BS-STAR-RIS channels are set as 3.5, 2.8, and 2.2, respectively. We set the noise power as $\sigma^2 = -110$ dBm and the maximum transmit power of the user as p = 30 dBm, unless otherwise stated.

We compare the performance of the proposed channel estimation schemes with the following benchmarks:

- **ON/OFF scheme for TS:** Following the idea in [7], the direct links are estimated with all sub-surfaces turned off and the cascaded links are estimated with one sub-surface turned on at transmission/reflection mode sequentially.
- Two-phase channel estimation for ES: In this scheme, the direct links and cascaded links are estimated separately in two phases. Specifically, in the first phase, only the users send orthogonal pilot sequences with the subsurfaces off to estimate the direct links. In the second phase, the transmission and reflection patterns are set as (14) to estimate the cascaded channels.

In Fig. 2, we plot the normalized MSE (NMSE) for STAR-RIS channel estimation under different energy splitting ratios. It is observed that by varying β_t , there exists a trade-off



Fig. 3: NMSE performance versus the total transmit power under different channel estimation schemes.



Fig. 4: NMSE performance versus the number of sub-surfaces under different channel estimation schemes.

between the channel estimation accuracy of T and R users. Specifically, the minimum of the two users is obtained when $\beta_t = 0.5$, which verifies our analysis in **Proposition 1**. Besides, with the optimal energy splitting ratio, the NMSE using ES is approximately twice as that using TS. This is because ES results in power leakage in the uplink channel estimation (see, Section IV. C).

In Fig. 3 and Fig. 4, we compare the performance of our proposed channel estimation schemes against the benchmarks. In Fig. 3, we plot the NMSE versus total transmit power. The key observations are made as follows: First, the NMSE decreases with the increasing of transmit power for all schemes, and the TS protocol yields the smallest NMSE. Second, the channel estimation error of the ON/OFF scheme is much larger than that of the other schemes since the large aperture of the surface is not fully utilized in the channel estimation stage. Third, the two phase-based scheme for ES behaves worse than our proposed scheme due to the error propagation issue. Specifically, the channel estimation error in the first phase for estimating direct links will deteriorate the performance in the second phase. Finally, the proposed scheme for ES with practical phase-shifts achieves close NMSE performance as that with ideal phase-shifts, which verifies the effectiveness of our proposed near-orthogonal training pattern design.

In Fig. 4, we examine the channel estimation performance

of different schemes versus the number of sub-surfaces M. It is observed that the NMSE of the ON/OFF scheme increases with M since the impact of noise accumulates with longer channel estimation time. For other schemes, the NMSE keeps constant, which is consistent with the analysis in (6) and (16). Note that a larger M requires longer channel estimation overhead, which leads to a shorter time of data transmission. Therefore, there exists a trade-off between the achievable rate and channel estimation overhead [11], which is an interesting topic in the future.

VI. CONCLUSION

In this letter, we proposed efficient channel estimation schemes for a STAR-RIS assisted two-user communication system for the TS and ES protocols, respectively. We first presented the optimal training patterns for TS to separately estimate the channels of two users. Then, a novel scheme for ES under the coupled phase-shift model was developed by jointly optimizing the pilot sequences, energy splitting ratio, and the training patterns, which achieves near-optimal MSE performance. Numerical results demonstrated that TS is more cost-effective than ES in terms of uplink channel estimation.

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