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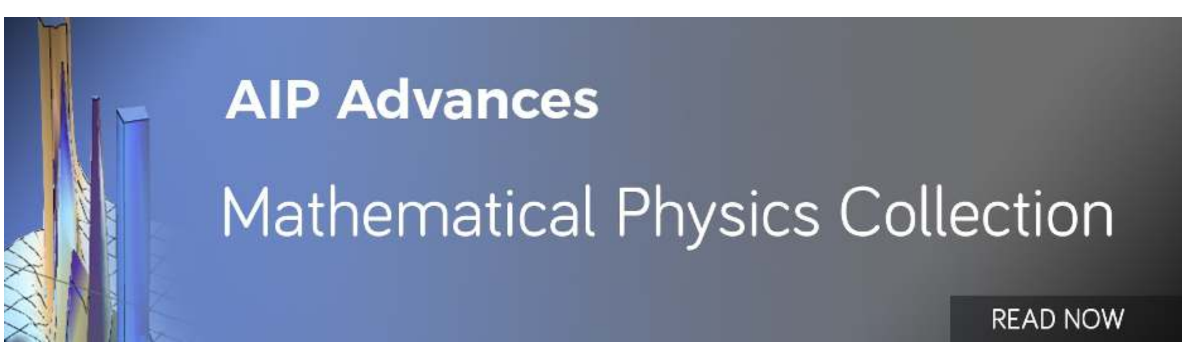

Channel flow of Ellis fluid due to peristalsis

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An analysis is carried out to investigate the peristaltic pumping of a non-Newtonian Ellis fluid in a planar channel. The coupled nonlinear partial differential equations governing the problem are simplified under the widely used assumption of long wavelength and low Reynolds number. A semi- analytical approach is adopted to obtain the expressions for stream function, longitudinal velocity, pressure gradient and pressure rise per wavelength. The important characteristics of the peristaltic motion are explained graphically for several values of the material parameter of the Ellis fluid. [http://dx.doi.org/10.1063/1.4932042]

I. INTRODUCTION

In recent years the peristaltic flow has been widely discussed because of its relevance in industry and physiology. In physiology, peristalsis is involved in the urine transport from kidney to the bladder, vasomotion of small blood vessels, movement of chyme and in many processes of reproduction. In industry the roller and finger pumps operate according to the peristaltic mechanism. The rheology of fluid material driven by peristaltic mechanism in the many flow cases can be characterized by the Newtonian constitutive equation. For instance the rheology of urine can be well described by assuming it a Newtonian fluid. However, there are numerous examples where the choice of Newtonian fluid is not appropriate. Many biological fluids such as blood, chyme, and spermatic fluid are some examples of non-Newtonian fluids. Undoubtedly the mechanics of non-Newtonian fluids presents special challenges to engineers, physicists, modellers, numerical analyst and mathematicians. The flows of non-Newtonian fluids are not only important because of their technological significance but also due to the interesting mathematical features presented by their governing equations. The rheological behavior of non-Newtonian fluids is very complex and it is not possible to find a universal constitutive relation valid for all non-Newtonian fluids. After the first investigation of Latham[1] on peristaltic motion of a Newtonian fluid, numerous attempts have been made to analyze peristaltic motion of non-Newtonian fluids theoretically. Some interesting studies on these flows have been carried out by Siddiqui and Schwarz,[2,3] Mekheimer, Mekheimer et al.[4] Mekheimer and Elmaboud[5–9] Hayat et al.[10–13] Hayat and Ali,[14–16] Wang et al.,[17] Ali et al.,[18,19] Srinivas and Kothandapani,[20] Tripathi et al.[21] and Abbasi et al.[22] All the above cited investigations cover the peristaltic flows of different non-Newtonian fluids. In some of these articles the magnetic field and heat transfer effects have also been discussed. However, there is no attempt is available which describe peristaltic motion of Ellis fluid. The Ellis model fall in the category generalized Newtonian fluid (GNF) models. The well-known GNF models are power law model, Carreau model, Herschel-Bulkley etc. The peristaltic motion of these models can be found in Refs. 23–25. The main advantage of Ellis equation is that it predicts the Newtonian behavior at small shear stresses and the power law behavior at large shear stresses. This advantage of Ellis model enables

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it to correctly reflect the rheological behavior of typical polymeric fluids.[26] Motivated by above facts the main objective of this paper is to analyze the peristaltic motion of Ellis fluid in a planner channel under long wavelength assumption. The organization of the paper is as follow. The basic governing laws are described in section II. The problem is formulated in section III. A semi- analytical solution of the problem formulated in previous section is reported under the long wavelength assumption in section IV. The effects of pertinent parameters on various characteristic of peristaltic motion are discussed in detail in section V. The main conclusions are reported in section VI.

II. GOVERNING EQUATIONS

For the flow under consideration the balances of mass and linear momentum in the absence of body forces are

$$div\boldsymbol{V} = 0, \tag{1}$$

$$\rho\frac{d\boldsymbol{V}}{dt} = -\nabla p + div\boldsymbol{S}, \tag{2}$$

where $\boldsymbol{V}$ is the velocity, ρ is the density d/dt is the material derivative, p is the hydrostatic pressure and $\boldsymbol{S}$ is the extra stress tensor. For an Ellis fluid the extra stress $\boldsymbol{S}$ is given by[27]

$$\boldsymbol{S} = \frac{\mu}{1+\left(\frac{\prod_s}{\tau_0^2}\right)^{\alpha-1}}\boldsymbol{A_1} \tag{3}$$

In the above equation μ is the dynamic viscosity, τ_0 and α material constants, $\prod_s$ is second order invariant of stress tensor and $\boldsymbol{A_1}$ is the first Rivilin-Ericksen tensor. The constant τ_0 is commonly defined as the shear stress corresponding to the half dynamic viscosity. The model (4) reduces to Newtonian Model for $\alpha = 1$ and $1/\tau_0^2 \to 0$.

III. MATHEMATICAL FORMULATION

Let us consider the flow of an incompressible Ellis fluid due to the propagation of infinite wave train traveling along the walls of the channel. We introduce a Cartesian Coordinates system (X,Y) in which X-axis is along the direction of flow and Y-axis normal to it. The wall of the channel is mathematically defined by the following equation

$$h(X,t) = a + b\sin\left[\frac{2\pi}{\lambda}(X-ct)\right], \tag{4}$$

where a is half width of the channel, b, λ and c are amplitude, wavelength, and wave speed, respectively and t is the time. The flow under consideration can be modeled unsteady and two-dimensional. Therefore, we define

$$\boldsymbol{V} = [U(X,Y,t), V(X,Y,t), 0], \tag{5}$$

with U and V as velocity components in X and Y directions, respectively. In view of Eq. (5), Eqs. (1)-(3) yield the following scalar equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{6}$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)U = -\frac{\partial p}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y}, \tag{7}$$

$$\rho\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)V = -\frac{\partial p}{\partial Y} + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y}. \tag{8}$$

Inherently the flow is unsteady in the laboratory frame (X,Y). However, it can be treated as steady in a frame moving with the speed of wave. Such a frame is known as wave frame. The transformations between the two frames are:

$$x = X - ct, \quad y = Y, u = U - c, v = V, \tag{9}$$

where u and v are components of velocity in x and y directions, respectively. Employing the transformation defined by Eq.(9) and introducing the dimensionless variables given by

$$x = \frac{\lambda x^*}{2\pi}, y = ay^*, \quad u = cu^*, \quad v = cv^*, \quad \boldsymbol{S} = \frac{\mu c}{a}\boldsymbol{S}^* \, p = \frac{\lambda \mu c p^*}{2\pi a^2} h = ah, \tag{10}$$

Eqs. (6) – (8) after dropping the asterisks take the following form

$$\delta \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

$$\text{Re}\left[\left(\delta u \frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u\right] = -\frac{\partial p}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \tag{12}$$

$$\delta\text{Re}\left[\left(\delta u \frac{\partial}{\partial x} + v\frac{\partial}{\partial x}\right)v\right] = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial S_{xy}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y}, \tag{13}$$

$$S_{xx} = \frac{2\delta \frac{\partial u}{\partial x}}{1 + (\beta\chi)^{\alpha-1}}, \tag{14}$$

$$S_{xy} = \frac{\left(\frac{\partial u}{\partial y} + \delta \frac{\partial v}{\partial x}\right)}{1 + (\beta\chi)^{\alpha-1}}, \tag{15}$$

$$S_{yy} = \frac{2\frac{\partial v}{\partial y}}{1 + (\beta\chi)^{\alpha-1}}, \tag{16}$$

where

$$\chi = \left(\frac{1}{2}\left((S_{xx})^2 + 2(S_{xy})^2 + (S_{yy})^2\right)\right)^{\frac{1}{2}}, \tag{17}$$

$\text{Re} = \rho ca/\mu$ is the Reynolds number, $\beta = c/a\tau_0^2$ is the dimensionless material parameter and $\delta = 2\pi a/\lambda$ is the wave number. Defining the stream function $\psi(x, y)$ by the relations $u = \partial\psi/\partial y$, $v = -\delta\partial\psi/\partial x$ the continuity equation (11) is satisfied identically and Eqs. (12)–(16) become

$$\text{Re}\delta\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right] = -\frac{\partial p}{\partial x} + \delta\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \tag{18}$$

$$\delta^3\text{Re}\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial x}\right] = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial S_{xy}}{\partial x} + \delta\frac{\partial S_{yy}}{\partial y}, \tag{19}$$

$$S_{xx} = \frac{2\delta\frac{\partial^2\psi}{\partial y^2}}{1 + (\beta\chi)^{\alpha-1}}, \tag{20}$$

$$S_{xy} = \frac{\left(\frac{\partial^2\psi}{\partial y^2} - \delta^2\frac{\partial^2\psi}{\partial x^2}\right)}{1 + (\beta\chi)^{\alpha-1}}, \tag{21}$$

$$S_{yy} = \frac{-\delta\frac{\partial^2\psi}{\partial y\partial x}}{1 + (\beta\chi)^{\alpha-1}}. \tag{22}$$

Eqs. (18) and (19) are subject to the symmetry condition at the centerline and no slip condition at the channel wall. These conditions are mathematically expressed in term of stream function as

$$\frac{\partial^2\psi}{\partial y^2} == 0 \text{ at } y = 0 \text{ and } \frac{\partial\psi}{\partial y} = -1, \text{ at } \quad \text{at } h = 1 + \phi\, Cosx. \tag{23}$$

where $\phi = b/a$ is the amplitude ratio.

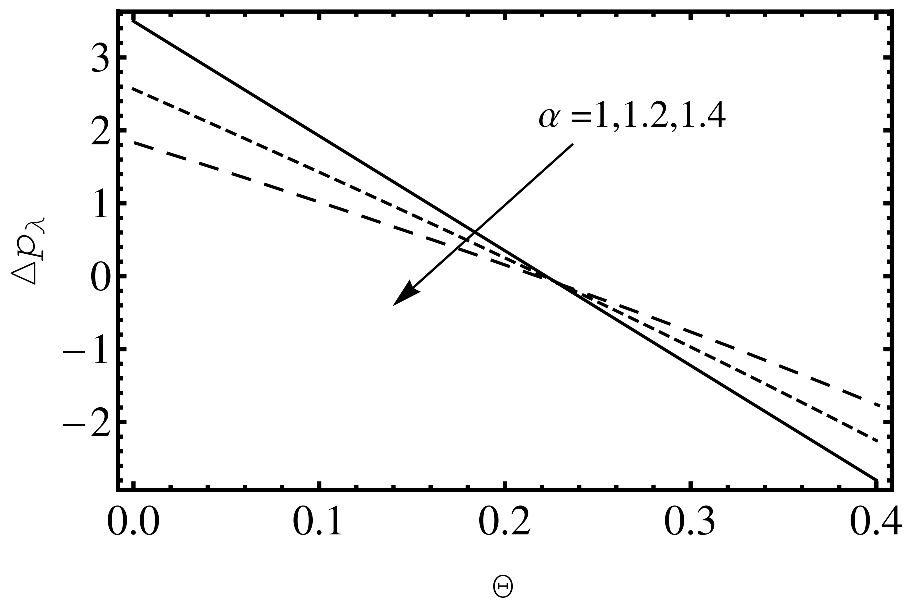

FIG. 1. Plots of pressure rise per wavelength (ΔP_λ) versus flow rate Θ for various values of material parameter α when $F = -0.8$, $\beta = 20$ and $\phi = 0.4$.

IV. SOLUTION OF THE PROBLEM

The set of Eqs. (18)-(22) comprises of higher-order nonlinear partial differential equations. It is difficult to obtain a closed from solution of these equations. However, in many practical physical problems related to peristalsis the wavelength is large as compare to the width (radius) of the channel.[28] The parameter δ in our problem represents the ratio of the channel width to the wavelength of the wave. Thus assuming δ to be small, Eqs. (18)-(22) reduce to

$$\frac{\partial p}{\partial x} = \frac{\partial S_{xy}}{\partial y}, \tag{24}$$

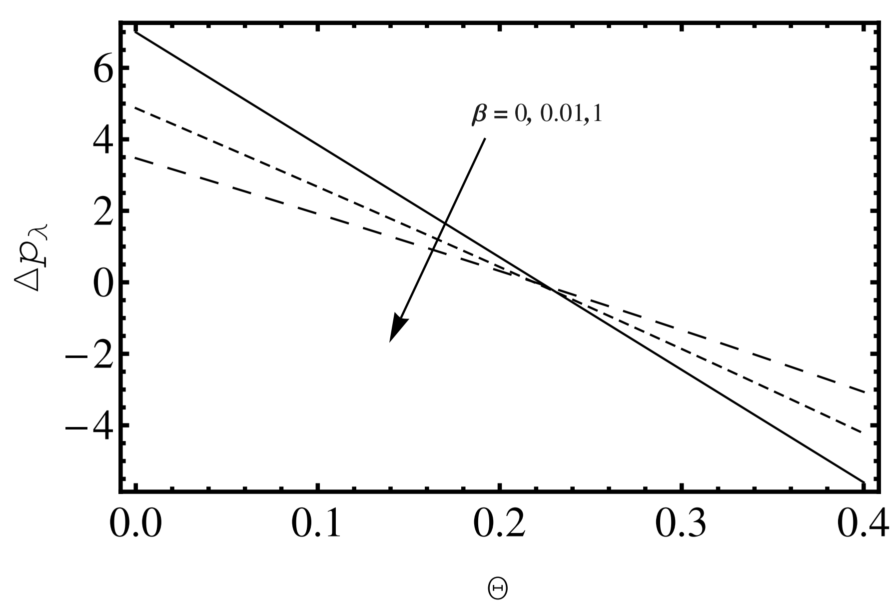

FIG. 2. Plots of pressure rise per wavelength (ΔP_λ) versus flow rate Θ for various values β of when $F = -0.8$, $\alpha = 1.5$ and $\phi = 0.4$.

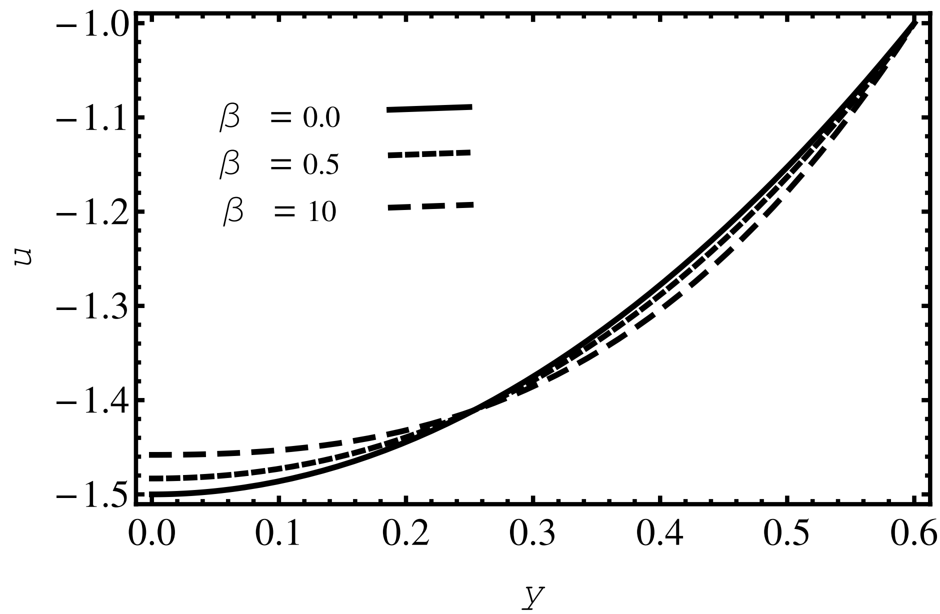

FIG. 3. Plots of longitudinal velocity (u) for various values of β when $F = -0.8$, $\alpha = 2$ and $\phi = 0.4$.

$$\frac{\partial p}{\partial y} = 0, \tag{25}$$

$$S_{xx} = S_{yy} = 0, \tag{26}$$

$$S_{xy} = \frac{\frac{\partial^2 \psi}{\partial y^2}}{1 + (\beta S_{xy})^{\alpha - 1}}. \tag{27}$$

Eq. (25) dictates that p is not a function of y. Thus the only possibility is that p is a function of x.This implies that dp/dx is only a function of x and therefore can be treated as a constant while integrating Eq. (24). Integrating Eq. (24) with respect to y and using the boundary condition $\partial^2\psi/\partial y^2 = 0$, we get

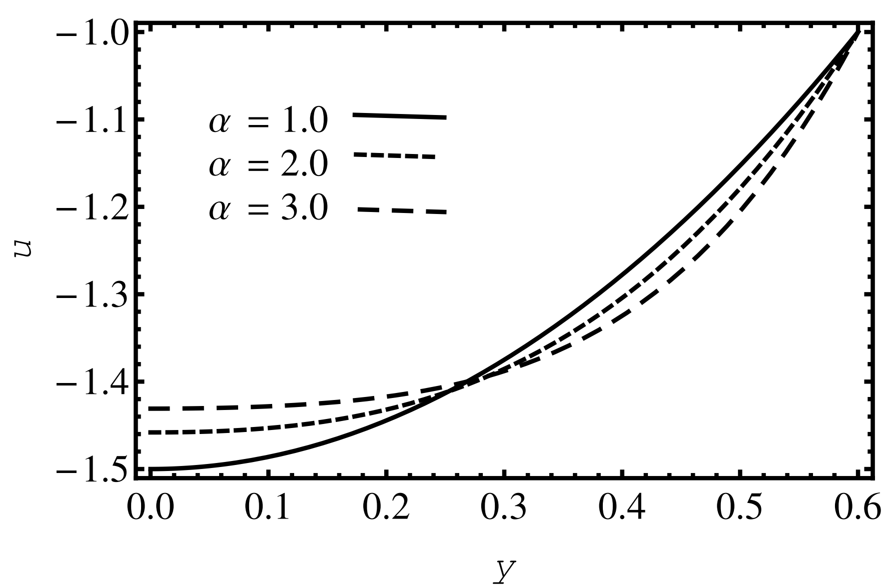

FIG. 4. Plots of longitudinal velocity (u) for various values of α when $F = -0.8$, $\phi = 0.4$ and $\beta = 2$.

$$S_{xy} = \frac{dp}{dx} y. \tag{28}$$

Substituting Eq. (28) into Eq. (27) and integrating twice one finds

$$\psi = \frac{y^3}{6}\frac{dp}{dx} + \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)(\alpha+2)} y^{\alpha+2} + C_3 y + C_4, \tag{29}$$

where C_3 and C_4 are integration constants. Since dp/dx is unknown, therefore the expression (29) involves three unknowns. The boundary condition $\partial\psi/\partial y = -1$ at $y = h$ is not sufficient to calculate uniquely the value of all these unknowns. The additional boundary condition on stream function can be imposed by prescribing the flow rate at each cross-section as a constant. This assumption yields the following additional boundary conditions[2,7]

$$\psi = 0 \text{ at } y = 0 \text{ and } \psi = F \text{ at } y = h, \tag{30}$$

where F is the prescribed flow rate in the wave frame. In view of no-slip condition and first condition in (30) the following expression of the stream function can be obtained.

$$\psi = \frac{y^3}{6}\frac{dp}{dx} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)(\alpha+2)} y^{\alpha+2} + \left[-1 - \frac{h^2}{2}\frac{dp}{dx} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)} h^{\alpha+1}\right] y. \tag{31}$$

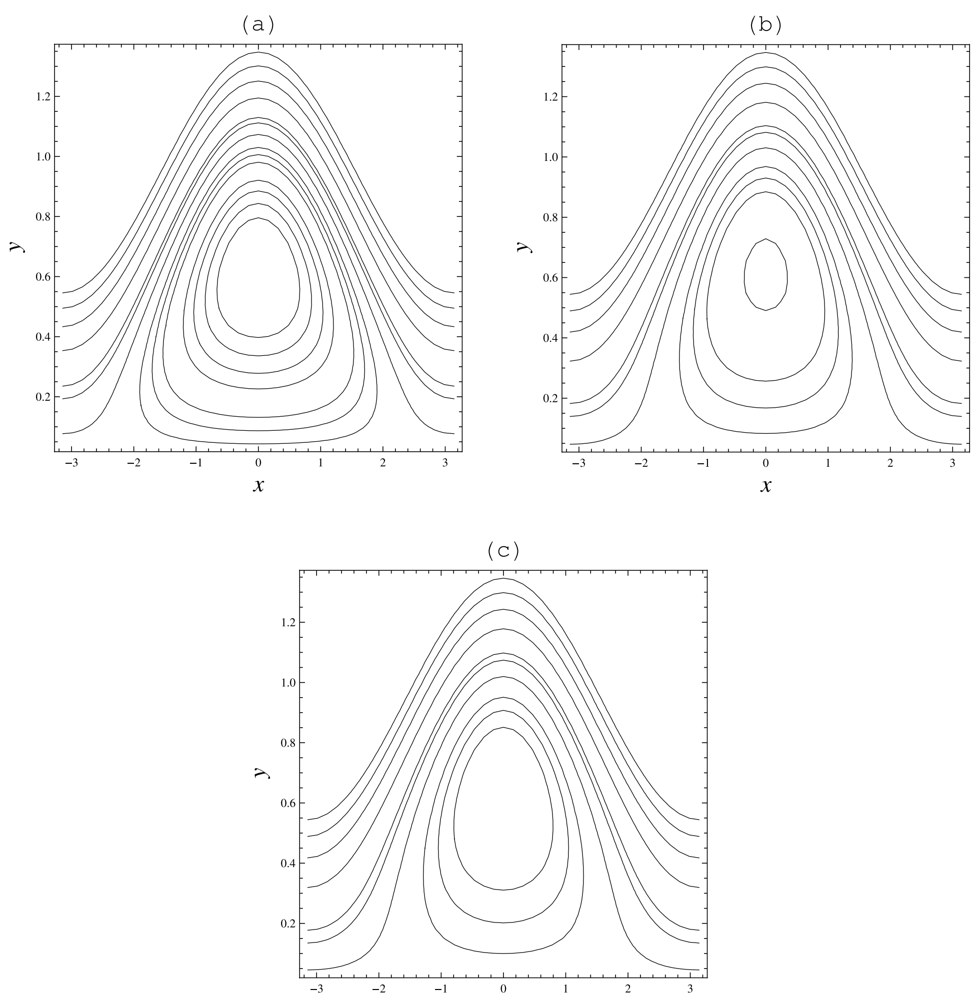

FIG. 5. Plots of streamlines for (a) $\beta = 0$ (b) $\beta = 2$ (c) $\beta = 10$ when $F = -0.25$, $\alpha = 2$ and $\phi = 0.4$.

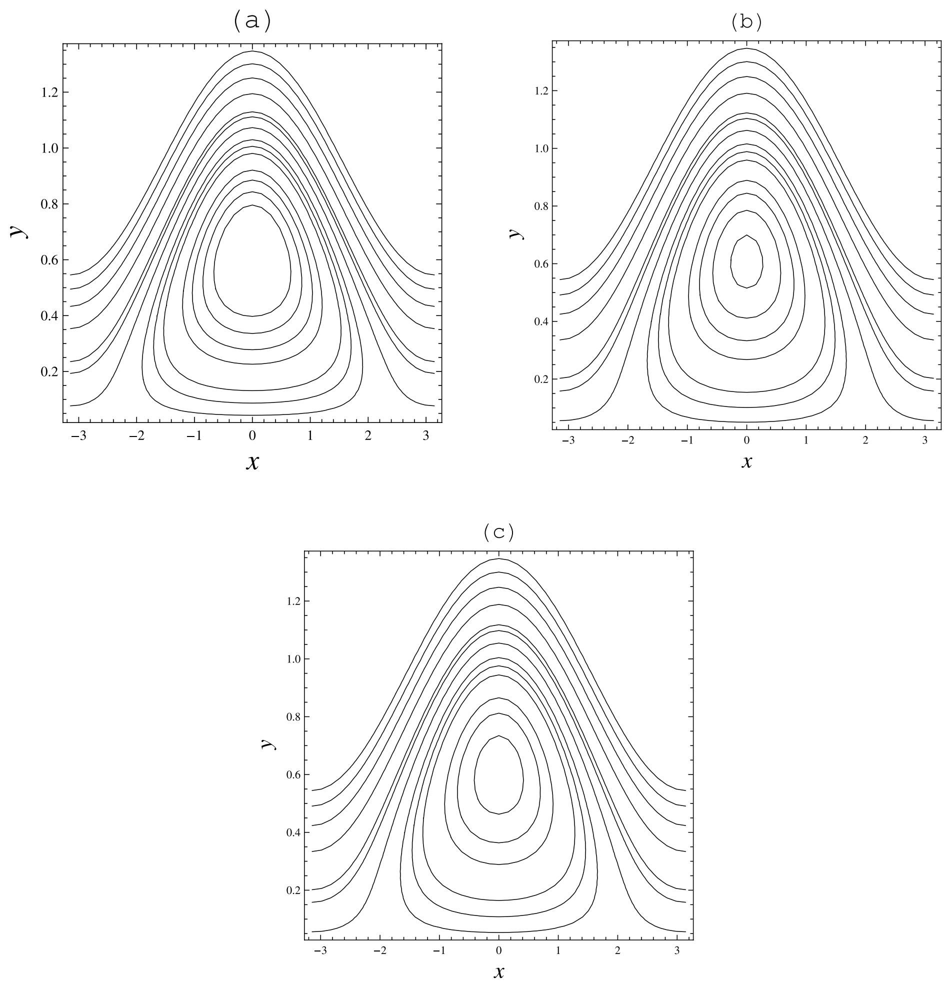

FIG. 6. Plots of streamlines for (a) $\alpha = 1$ (b) $\alpha = 2$ (c) $\alpha = 3$ when $F = -0.25$, $\beta = 0.5$ and $\phi = 0.4$.

In above expression dp/dx is still unknown. On employing the second boundary condition in (30) the following non- linear algebraic equation in dp/dx can be obtained.

$$\frac{h^3}{6}\frac{dp}{dx} + \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)(\alpha+2)}h^{\alpha+2} + \left[-1 - \frac{h^2}{2} - \frac{\beta^{\alpha-1}\left(\frac{dp}{dx}\right)^{\alpha}}{(\alpha+1)}h^{\alpha+1}\right]h = F. \tag{32}$$

The above equation can be solved for dp/dx using any computational software at each cross-section x for a given set of parameter. Once dp/dx is known, the solution is complete. The integration of dp/dx over one wavelength gives the pressure rise across one wavelength. Employing the formula

$$\Delta p_{\lambda} = \int_{0}^{2\pi} \frac{dp}{dx} dx, \tag{33}$$

we have obtained the pressure rise per wavelength to illustrate the pumping characteristics in next section.

V. RESULTS AND DISCUSSION

This section displays the graphical illustrations of pressure rise per wavelength (ΔP_{λ}), longitudinal velocity and streamlines for various value of material constants of the Ellis model. Fig. 1 illustrates the effects of material constant α on pressure rise per wavelength. three distinct regions can be identified from this figure. The region where $\Theta > 0$ $\Delta P_\lambda > 0$ is known as peristaltic pumping region. In this region the peristalsis has to work against the pressure rise to propel the fluid. The region where $\Theta > 0$ $\Delta P_\lambda = 0$ is called free pumping region. The corresponding value of Θ for $\Delta P_\lambda = 0$ is called free pumping flux. Since $\Delta P_\lambda = 0$, the free pumping flux is solely due to peristaltic waves. The last region where $\Theta > 0$ $\Delta P_\lambda < 0$ is known as augmented pumping region. In this region the pressure assists the flow due to peristalsis. It is noted that for a fixed value of prescribed flow rate an increase in α decreases the pressure rise per wavelength in the peristaltic pumping region. The free pumping flux is nearly found to be independent of α. However, in augmented pumping region the assistance provided by the pressure decreases with increasing α for a fixed value of mean flow rate Θ. The second material constant which characterizes the Ellis model is β. It is found that the effects of β on the ΔP_λ are similar to the effects of α. Fig. 2 also shows that ΔP_λ decreases in going from Newtonian to Ellis fluid. The longitudinal velocity at a cross-section $x = -\pi$ is shown for different value of α and β in Figs. 3 and 4, respectively. These figures do illustrate that longitudinal velocity is significantly affected by the material constants of Ellis fluid. Figs. 3 and 4 indicate that the magnitude of longitudinal velocity near the channel center decreases with increasing either of α or β. On the contrary, near the vicinity of channel wall it follows converse trend. It is important to mention that a flattening trend in velocity is observed near channel center for larger values of α. This is perhaps due to enhanced shear-thinning in viscosity for larger values of α. It is further noted from Fig. 4 that magnitude of longitudinal velocity at the center of the channel decreases going to Newtonian to Ellis fluid.

The streamline patterns for different value of α and β are shown in Figs. 5 and 6. Again it is observed that the streamlines of the flow are affected in a similar manner by increasing either α or β. In fact it is observed that the strength of recirculating zone (trapped bolus) appearing in the wider part of the channel decreases by increasing α or β. However, it is observed that such a decrease is faster by increasing β.

VI. CONCLUSIONS

A semi-analytical approach is adopted to address the problem of peristaltic motion of an Ellis fluid under long wavelength assumption. The expressions of stream function reported here involve the unknown pressure gradient which is found by solving a nonlinear algebraic equation at each cross-section for given set of parameters. The flow velocity, pumping characteristics and trapping phenomena are analyzed for various values of material parameter of the Ellis fluid. The main observations of the present study are summarized as follows.

- An enhanced shear thinning in viscosity is observed for larger values of α and β. As a result of this shear-thinning in viscosity, a flattening trend in the longitudinal velocity is observed at the channel center.
- The pressure rise per wavelength in pumping region decreases with increasing either α or β for a fixed value of prescribed flow rate Θ.
- The value of prescribed flow rate Θ for which $\Delta P_\lambda = 0$ is unaffected by increasing either α or β.
- The strength of recirculating zone appearing in wider past of the channel decreases by increasing either α or β.

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