CHANNEL REUSE MULTIPLE ACCESS PROTOCOL
FOR BIDIRECTIONAL BUS NETWORKS

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Abstract

We propose a novel contention-free multiple access protocol for bidirectional passive bus packet networks. One version of the protocol has a simple fixed assignment transmission scheduling similar to that of the Time Division Multiple Access protocol, except that left-going and right-going packets may only be transmitted in alternate rounds. Another version of the protocol employs a simple reservation scheme to provide a demand assignment transmission scheduling. In either case, the protocol may offer a maximum throughput exceeding one packet per packet transmission time under appropriate conditions, where throughput is defined to be the number of distinct packets successfully transmitted per packet transmission time. This is possible through channel reuse as explained in this paper. For this reason, we call this protocol *Channel Reuse Multiple Access* protocol [1].

1 Introduction

Multiple access protocols for different types of bus packet networks have been widely reported [2]. The broadcast capability of bidirectional passive bus networks is often exploited to simplify transmission scheduling. For point-to-point communications, each transmission is designated for only one station, and the success of a transmission depends on its being free of collision only where and when the reception is intended. Due to propagation delays, packets may overlap non-destructively. When this occurs. we say that there is *channel reuse*. An example of channel reuse is shown in the space-time diagram in Figure 1. Space-time diagrams are useful for the analysis of multiple access protocols on bus networks; e.g. [3].

Let p be the average packet transmission time. and $a.p$ be the propagation delay from one end of the bus to the other end. In many multiple access protocols for bus networks. maximum throughput is usually a decreasing function of a. and is strictly no greater than one packet per packet transmission time [2]. For contention-free protocols. the inefficiency is primarily due to the time overhead in passing some kind of control token from one station to another. In the *Channel Reuse Multiple Access* (CRMA) protocol. channel reuse is exploited for improved throughput performance.

Levy and Kleinrock have investigated the behavior of a bidirectional bus network with N regularly spaced stations. where $a \geq (N-1)$ [4][5]. They have proposed some transmission patterns. with which throughputs arbitrarily close to, but no greater than, two packets per packet transmission time are attainable. In [1]. the concept of channel reuse is introduced to gain insights on resource allocation in bidirectional bus networks. Among other results, the author has derived the same upper bound on the maximum throughput for bidirectional bus networks with any linear topology for any normalized end-to-end propagation delay. The author has also presented the CRMA protocol which is applicable to any bidirectional passive bus network. without any restriction on the normalized propagation delay. In addition, the maximum throughput of the protocol may exceed one packet per packet transmission time under appropriate conditions.

The performance of the CRMA protocol depends on the distribution of the stations. as it does in Ethernet-type bus networks [6]. Let the N stations on the bus be labeled s_1 through s_N from, say. left to right. The bus, of length D. is assumed to have perfectly non-reflecting terminations at both ends. For $k \in \{1,2,\ldots,N-1\}$. let the distance between station s_k and station s_{k+1} be d_k. where $0 \leq d_k \leq D$. *Station configuration* refers to how stations are located along the bus. It may be specified by the spatial intervals. d_1 through d_{N-1}. We consider both deterministic and probabilistic topologies. A special case of the deterministic topology is the regular bus network where $d_k = D/(N-1), \forall k \in \{1,2,\ldots,N-1\}$. In the probabilistic topology of interest. each station. except the end-stations. is independently located at a uniformly distributed point on the bus. It can be shown that the spatial intervals have the following common probability density function [7]

$$f_{d_k}(d) = \left(\frac{N-2}{D}\right)\left(1-\frac{d}{D}\right)^{N-3}, \quad 0 \leq d \leq D. \quad (1)$$

Traffic configuration refers to the distribution of traffic flows among the stations. We distinguish between *symmetric traffic*. where a packet transmitted by a station is equally likely to be designated for any one of the other stations. and *balanced traffic*. where a packet transmitted by a station. except for the end-stations. is equally likely to be designated for a station on the left as one on the right. Although balanced traffic is not a very realistic assumption. it is used merely to demonstrate the potential of the CRMA protocol.

2 Fixed Assignment CRMA

The fixed assignment CRMA protocol is a contention-free multiple access protocol designed for a bidirectional bus network with steady traffic. Each station on the network is scheduled for a potential transmission following a periodic pattern. Packets are allowed to overlap one another while avoiding destructive collisions. For simplicity, we assume that all packets require the same transmission time, p. With increasing availability of high speed networks, applications with fixed packet transmission times are becoming popular. An example is broadband ISDN.

Suppose that all stations know p, D, a, and d_1 through d_N. Let the stations schedule potential transmissions in a sequential order, consistent with their physical ordering, alternating between an increasing order for left-going packets and a decreasing order for right-going packets. In the left-going round, reserved for left-going packets, an implicit token is passed from the left to the right. In the right-going round, reserved for right-going packets, an implicit token is passed from the right to the left. The two symmetric rounds together constitute a transmission period.

Consider the mth transmission period, for $m \in \{1, 2, \ldots\}$. For $k \in \{1, 2, \ldots, N-1\}$, let $t_k^L(m)$ and $t_k^R(m)$ denote the points in time, respectively, during the left-going and right-going rounds, at which station s_k is allowed to transmit a packet. Then, the transmission scheduling for the fixed assignment CRMA protocol is given below, with $t_1^R(0) = 0$.

$$t_2^L(m) = t_1^R(m-1) + \left(1 + \frac{ad_1}{D}\right)p; \qquad (2)$$

$$t_{k+1}^L(m) = t_k^L(m) + \max\left\{\frac{ad_k}{D}, \left(1 - \frac{ad_k}{D}\right)\right\}p, \qquad (3)$$
$$\forall k \in \{2, 3, \ldots, N-1\};$$

$$t_{N-1}^R(m) = t_N^L(m) + \left(1 + \frac{ad_{N-1}}{D}\right)p; \qquad (4)$$

$$t_k^R(m) = t_{k+1}^R(m) + \max\left\{\frac{ad_k}{D}, \left(1 - \frac{ad_k}{D}\right)\right\}p, \qquad (5)$$
$$\forall k \in \{1, 2, \ldots, N-2\}.$$

The CRMA transmission pattern for a typical topology is shown in Figure 2. Suppose that in a given transmission period, all stations are active. Then, in the left-going round, packets may be successfully received by stations to the left of a transmitting station only; and in the right-going round, packets may be successfully received by stations to the right of a transmitting station only. By transmitting the same packet in both directions, broadcast may be achieved within a transmission cycle. Note that there are gaps in the transmission pattern. It turns out that avoiding these gaps is not very fruitful. It complicates the transmission pattern while the increase in throughput is not very significant.

2.1 Transmission Period

For a fixed assignment protocol, the transmission period is independent of traffic. It depends, however, on the normalized end-to-end propagation delay and the station configuration. Let $\mathcal{T}_a(N)$ be the CRMA transmission period normalized by the packet transmission time. Then, from (2) through (5), we obtain the following equivalent expressions for $\mathcal{T}_a(N)$ [1].

$$\mathcal{T}_a(N) = \{2(N-1) - 2a\} + \left\{\frac{2ad_1}{D} + \left(\frac{2ad_1}{D} - 1\right)^+\right\}$$

$$+ \left\{\frac{2ad_{N-1}}{D} + \left(\frac{2ad_{N-1}}{D} - 1\right)^+\right\}$$
$$+ 2\sum_{k=2}^{N-2}\left(\frac{2ad_k}{D} - 1\right)^+; \qquad (6)$$

$$\mathcal{T}_a(N) = (N+1) + \left(\frac{2ad_1}{D} - 1\right)^+ + \left(\frac{2ad_{N-1}}{D} - 1\right)^+$$
$$+ \sum_{k=2}^{N-2}\left\{\left(\frac{2ad_k}{D} - 1\right)^+\right\}$$
$$+ \sum_{k=2}^{N-2}\left\{\left(1 - \frac{2ad_k}{D}\right)^+\right\}; \qquad (7)$$

$$\mathcal{T}_a(N) = (2+2a) + \left(1 - \frac{2ad_1}{D}\right)^+ + \left(1 - \frac{2ad_{N-1}}{D}\right)^+$$
$$+ 2\sum_{k=2}^{N-2}\left(1 - \frac{2ad_k}{D}\right)^+, \qquad (8)$$

where $(*)^+ = \max(*, 0)$. From (8), it is easy to verify that $\mathcal{T}_a(N) \leq 2(N-1) + 2a$. Note that $\mathcal{T}_a(N)$ is a convex $\cup$ function of a since it is the sum of convex $\cup$ functions of a.

When the stations are regularly spaced, the normalized CRMA transmission period is

$$\mathcal{T}_a^R(N) = \begin{cases} 2(N-1) - 2a\left(\frac{N-3}{N-1}\right) \\ \qquad\qquad \text{if } 0 \leq a \leq \frac{1}{2}(N-1); \\ (2+2a) \quad \text{if } a \geq \frac{1}{2}(N-1). \end{cases} \qquad (9)$$

Note that $\min_{a \geq 0}\{\mathcal{T}_a^R(N)\} = (N+1)$, where the minimum is achieved with $a = (N-1)/2$. The transmission pattern allows up to $2(N-1)$ distinct packets to be transmitted in each period. Hence, in a regular bidirectional bus network with $N \approx (2a+1)$ stations, it is possible to achieve throughput of $2(N-1)/(N+1)$, which approaches 2 for large N.

For the probabilistic topology, we have the following normalized mean transmission period.

$$E\{\mathcal{T}_a(N)\} = \begin{cases} 2(N-1) - 2a\left(\frac{N-3}{N-1}\right) \qquad \text{if } 0 \leq a \leq \frac{1}{2}; \\ 2(N-1) - 2a\left(\frac{N-3}{N-1}\right) \\ \qquad + 4a\left(\frac{N-2}{N-1}\right)\left(1 - \frac{1}{2a}\right)^{N-1} \quad \text{if } a \geq \frac{1}{2}, \end{cases} \qquad (10)$$

where the expectation is taken over the joint distribution of the intervals, d_1 through d_{N-1} [1]. Note that $E\{\mathcal{T}_a(N)\}$ decreases with a for small a, but increases with a for large a.

The graph of $\mathcal{T}_a^R(N)$ in (9) has a V shape. It is clear from (9) and (10) that $E\{\mathcal{T}_a(N)\}$ coincides with $\mathcal{T}_a^R(N)$ both for very small and very large a. Since $\mathcal{T}_a(N)$ is a convex $\cup$ function of a, $E\{\mathcal{T}_a(N)\}$ is also a convex $\cup$ function of a. It follows that $E\{\mathcal{T}_a(N)\}$ must be bounded from below by $\mathcal{T}_a^R(N)$ for all $a \geq 0$.

2.2 Network Utilization

Network utilization is the sum of individual station's channel utilization, which is the average number of packets successfully transmitted by each station during a transmission period. Let ρ_k be the channel utilization for station s_k, in number of packets per transmission period. Let P_k^L and P_k^R be the probabilities that a packet, transmitted from station s_k, is to be transmitted to the left and to the right respectively. Then, the channel utilizations associated with the left-going and the right-going channels are

respectively given below for all $k \in \{1, 2, \ldots, N\}$.

$$\rho_k^L = P_k^L \rho_k; \quad \text{and} \quad \rho_k^R = P_k^R \rho_k. \tag{11}$$

Since $0 \le \rho_k^L \le 1$ and $0 \le \rho_k^R \le 1$. we have

$$0 \le \rho_k \le \rho_k^* = \min\left(\frac{1}{P_k^L}, \frac{1}{P_k^R}\right). \tag{12}$$

Note that. for any traffic configuration. we have

$$\rho_1^* = \rho_N^* = 1; \quad 1 \le \rho_k^* \le 2, \quad \forall k \in \{2, 3, \ldots, N-1\}. \tag{13}$$

The network utilization of the CRMA protocol and its maximum are respectively

$$\rho(N) = \sum_{k=1}^{N} \rho_k; \quad \text{and} \quad \rho^*(N) = \sum_{k=1}^{N} \rho_k^*. \tag{14}$$

Note from (13) and (14) that. for any traffic,

$$N \le \rho^*(N) \le 2(N-1). \tag{15}$$

In Lemma 1. we show tighter bounds on $\rho^*(N)$ for symmetric traffic. and an exact expression of $\rho^*(N)$ for balanced traffic.

Lemma 1

- *Symmetric Traffic*

$$\rho^*(N) \le \begin{cases} \gamma(N) + 2(N-1)\ln\left(\frac{N-1}{N-2}\right) \\ \qquad \text{for even } N; \\ \gamma(N) + 2 \quad \text{for odd } N, \end{cases} \tag{16}$$

$$\rho^*(N) \ge \begin{cases} \gamma(N) \quad \text{for even } N; \\ \gamma(N) + 2 - 2(N-1)\ln\left(\frac{N+1}{N}\right) \\ \qquad \text{for odd } N, \end{cases} \tag{17}$$

where $\qquad \gamma(N) = 2(N-1)\ln 2. \tag{18}$

And, for both even and odd values of N, we have

$$0 \le \rho^*(N) - \gamma(N) \le 2\left(\frac{N-1}{N-2}\right). \tag{19}$$

- *Balanced Traffic*

$$\rho^*(N) = 2(N-1). \tag{20}$$

∎

Proof of Lemma 1

With symmetric traffic. we have. for all $k \in \{1, 2, \ldots, N\}$.

$$P_k^L = \left(\frac{k-1}{N-1}\right) \quad \text{and} \quad P_k^R = \left(\frac{N-k}{N-1}\right). \tag{21}$$

To derive the bounds in (16) and (17). we use (12). (14). (21). and the following bounds.

$$\ln\left(\frac{n+1}{m}\right) \le \left\{\sum_{k=m}^{n} \frac{1}{k}\right\} \le \ln\left(\frac{n}{m-1}\right), \quad 0 < m < n. \tag{22}$$

The bounds in (19) follow from (16) and (17) because $\ln(u) \le (u-1)$ for all $u \in \mathcal{R}$.

With balanced traffic. we have

$$P_1^L = P_N^R = 0, \qquad P_1^R = P_N^L = 1, \tag{23}$$

and $\qquad P_k^L = P_k^R = \frac{1}{2}, \quad \forall k \in \{2, 3, \ldots, N-1\}. \tag{24}$

It then follows that

$$\rho_k^* = \begin{cases} 1 & \text{if } k \in \{1, N\}; \\ 2 & \text{if } k \in \{2, \ldots, N-1\}. \end{cases} \tag{25}$$

Then. from (14) and (25). we obtain (20).

Q.E.D.

2.3 Throughput Performance

The throughput of the CRMA protocol for a bidirectional bus network with deterministic topology is the ratio of the network utilization to the transmission period. For a probabilistic bus network. we consider the expected throughput. where the expectation is taken over the distribution of the topology. An exact analysis of the expected throughput is complicated. We thus derive lower bounds on the maximum expected throughput.

Lemma 1 permits us to assume. without sacrificing much accuracy. the following maximum network utilization for large N.

$$\rho^*(N) = 2(N-1)\nu, \tag{26}$$

where $\qquad \nu \begin{cases} \approx \ln 2 & \text{for symmetric traffic;} \\ = 1 & \text{for balanced traffic.} \end{cases} \tag{27}$

Let a_N be the normalized average propagation delay between adjacent stations. Thus. $a_N = a/(N-1)$. When N is large. $S^R(a_N)$. the CRMA maximum throughput for the regular bus network is as follows.

$$S^R(a_N) = \frac{\rho^*(N)}{T_a^R(N)} = \begin{cases} \frac{\nu}{1-a_N} & \text{if } 0 \le a_N \le \frac{1}{2}; \\ \frac{\nu}{a_N} & \text{if } a_N \ge \frac{1}{2}, \end{cases} \tag{28}$$

where $T_a^R(N)$ is given in (9). Minimizing (28) over $a_N \ge 0$. we have $\min_{a_N}\{S^R(a_N)\} = 2\nu$. where the minimum is obtained with $a_N = 0.5$. The maximum throughput for the regular bus network is shown in Figure 3.

Theorem 1

For regular bus networks, the CRMA protocol achieves maximum throughput exceeding one packet per packet transmission time under the following sufficient conditions.

$$\begin{aligned} &\text{For symmetric traffic:} \quad (1 - \ln 2) \le a_N \le \ln 2; \\ &\text{For balanced traffic:} \quad 0 \le a_N \le 1. \end{aligned} \tag{29}$$

∎

Proof of Theorem 1

The bounds in (29) are obtained from (27) and (28) by considering the value of a_N that makes $S^R(a_N) = 1$.

Q.E.D.

For probabilistic bus networks. it is convenient to normalize $E\{T_a(N)\}$ by $2(N-1)$ since the maximum network utilization for large N is proportional to $2(N-1)$ for both symmetric and balanced traffic. Let the *adjusted mean transmission period*.

denoted $J(a_N)$, be defined as follows.

$$J(a_N) = \frac{E\{T_a(N)\}}{2(N-1)}, \qquad \forall a \geq 0. \qquad (30)$$

Then, from (10), we have

$$J(a_N) = \begin{cases} 1 - a_N \left(\frac{N-3}{N-1}\right) & \text{if } 0 \leq a_N \leq \frac{1}{2(N-1)}; \\ 1 - a_N \left(\frac{N-3}{N-1}\right) \\ \quad + 2a_N \left(\frac{N-2}{N-1}\right)\left(1 - \frac{1}{2(N-1)a_N}\right)^{N-1} \\ \qquad \text{if } a_N \geq \frac{1}{2(N-1)}. \end{cases} \qquad (31)$$

For large N, we have

$$J(a_N) \approx \begin{cases} (1 - a_N) & \text{if } 0 \leq a_N \leq \frac{1}{2(N-1)}; \\ (1 - a_N) + 2a_N e^{-\frac{1}{2a_N}} & \text{if } a_N \geq \frac{1}{2(N-1)}. \end{cases} \qquad (32)$$

Note that $\lim_{a_N \to \infty}\{J(a_N) - a_N\} = 1$. An even simpler approximation than (32) is

$$J(a_N) \approx 1 - a_N + 2a_N e^{-\frac{1}{2a_N}} \leq 1 + a_N, \qquad \forall a_N \geq 0. \quad (33)$$

A graph of $J(a_N)$ versus a_N is shown in Figure 4.

We now derive a lower bound on $\hat{S}_a(N)$, the maximum expected throughput for the probabilistic bus network. By definition, we have

$$\hat{S}_a(N) = E\left\{\frac{\rho^*(N)}{T_a(N)}\right\} = \rho^*(N) E\left\{\frac{1}{T_a(N)}\right\}, \qquad (34)$$

where the expectation is taken over the distribution of the station configuration. By Jensen's Inequality [8],

$$\hat{S}_a(N) \geq \frac{\rho^*(N)}{E\{T_a(N)\}} = \nu J(a_N)^{-1}, \qquad (35)$$

where ν has been defined in (27) and $J(a_N)$ may be approximated as in (33). For large N, we write $\hat{S}_a(N)$ as $\hat{S}(a_N)$.

Theorem 2

For probabilistic bus networks with balanced traffic, the maximum expected throughput can exceed one packet per packet transmission time if $0 \leq a_N \leq 0.72$. If $a_N \approx 0.30$, it can exceed 1.23, 0.86, and 0.61 for balanced traffic, symmetric traffic, and any other traffic respectively. ∎

Proof of Theorem 2

From (33), one can verify that $J(a_N)$ is no greater than unity for $0 \leq a_N \leq 0.72$. Considering this, and (27), in (35), one sees that $\hat{S}(a_N) \geq 1$ over the above range of a_N for balanced traffic. The mimimum of $J(a_N)$ in (33) is obtained with $a_N \approx 0.30$, and is equal to 0.81. Using this value, (15), (27), and (30), in (35), the remainder of the theorem may be easily verified.

Q.E.D.

We compare the throughput performance of the CRMA protocol to that of the TDMA protocol whose scheduling is defined as follows. For $m \in \{1, 2, \ldots\}$, let $t_1(m)$ through $t_N(m)$ be the points in time, during the mth transmission period, at which stations s_1 through s_N are respectively allowed to transmit a packet, with $t_N(0) = 0$.

$$t_1(m) = t_N(m-1) + (1+a)p; \qquad (36)$$

$$t_{k+1}(m) = t_k(m) + \left(1 + \frac{a d_k}{D}\right)p, \qquad (37)$$

$$\forall k \in \{1, 2, \ldots, N-1\}.$$

We thus obtain the following maximum throughput of the TDMA protocol for large N.

$$S_{TDMA}(a_N) = \frac{N}{N + 2a} \approx \frac{1}{1 + 2a_N}, \qquad (38)$$

which is the same for balanced as well as symmetric traffic.

From (27), (28), (33), (35), and (38), we obtain the following results. For balanced traffic, the CRMA maximum throughput for the regular bus network and the CRMA maximum expected throughput for the probabilistic bus network are both no less than the TDMA maximum throughput, regardless of the value of a_N. For symmetric traffic, the CRMA maximum throughput for the regular bus network and the CRMA maximum expected throughput for the probabilistic bus network are both no less than the TDMA maximum throughput, provided that $a_N \geq a_N^*$, where $a_N^* \approx (1 - \ln 2)/(2\ln 2 + 1) \approx 0.13$.
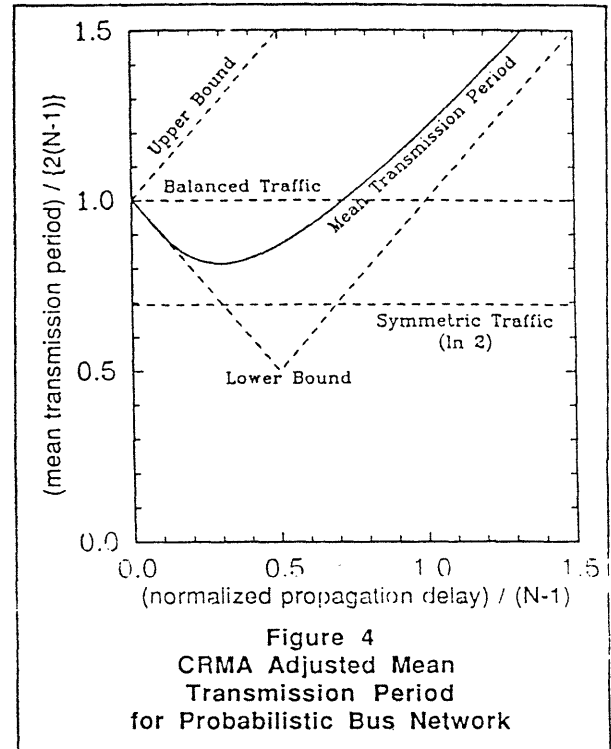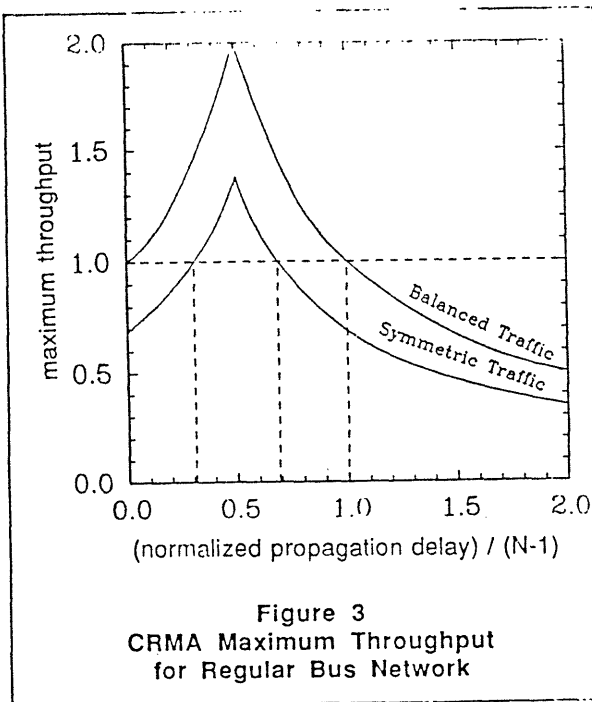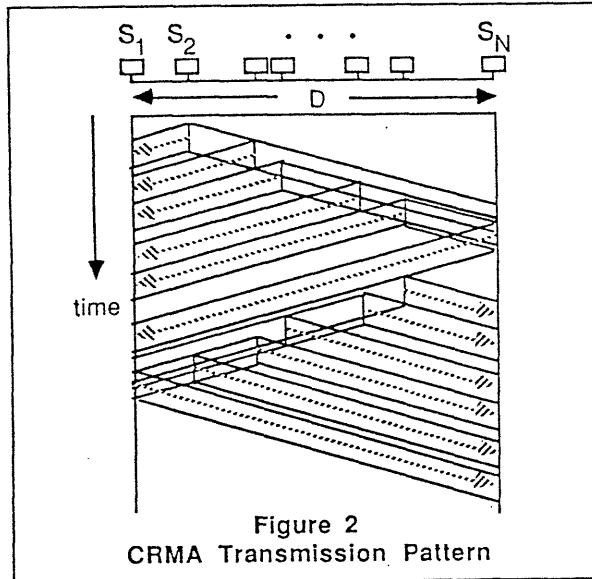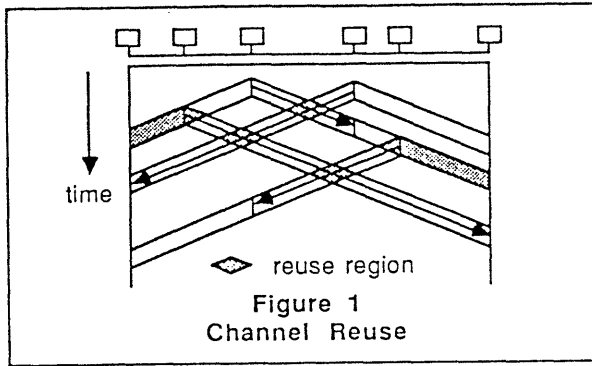
The relative efficiency of the CRMA protocol is due to non-exclusive sharing of channel resources. For small a_N, the potential for channel reuse is very limited. When there is symmetric traffic, the alternating left and right rounds of the CRMA protocol do not permit channel resources to be fully utilized. Hence, for small a_N, and with symmetric traffic, the CRMA protocol may have a poorer throughput performance than the TDMA protocol.

3 Demand Assignment CRMA

Just like other fixed assignment multiple access protocols, the fixed assignment CRMA protocol is inefficient under light traffic conditions. It is useful mainly for applications with steady demand for bandwidth. The protocol may be improved by adding a reservation capability. The demand assignment CRMA protocol employs basically the same transmission pattern as in the fixed assignment CRMA protocol. A left-going round and a right-going round alternate. Reservation for a transmission in any round must be made in the immediate previous round. Reservation transmissions are superimposed over data transmissions.

In the left-going round, an implicit token is passed from the left to the right. Each transmitting station must give way to those which are allowed to transmit before it. Hence, for the left-going round, reservation for transmission by any station must be made with every other station on its right. This may be achieved easily in the previous round since it happens to be a right-going round. By symmetry, reservations for the right-going traffic are similarly achieved. The separation between reservations to the left and the right has been proposed by Marsan and Gerla for TOKENET, in which there is channel reuse in the reservation phase that occupies a separate space-time window from the data transmission phase [9]. In the demand assignment CRMA protocol, the two phases are merged.

Under heavy traffic conditions, the performance of the demand assignment CRMA protocol is similar to that of the fixed assignment CRMA protocol, as long as the length of each reservation transmission is much smaller than p. Under light traffic conditions, the performance of the demand assignment CRMA protocol is expected to be better than that of the fixed assignment CRMA protocol.

Figure 1
Channel Reuse

Figure 2
CRMA Transmission Pattern

Figure 3
CRMA Maximum Throughput
for Regular Bus Network

Figure 4
CRMA Adjusted Mean
Transmission Period
for Probabilistic Bus Network

Acknowledgement

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