Chaotic Dynamics of Falling Disks

Stuart B. Field
Colorado State University

Franco Nori
Melody Klaus
Mitchell Moore*
University of Michigan

*Present address: University of Texas, Austin

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Introduction

Why study the motion of falling objects?

- A problem of great practical importance:

Atmospheric science/meteorology:

Fall and growth of snowflakes, hailstones, & raindrops; balloon wind sensors

Chemical Engineering:

Centrifuges, dust collectors, pneumatic conveyors, fiber suspensions, catalytic reactors, etc.

Biological Sciences

Spread of seeds from trees as they fall; “Experiments on the settling, overturning and entrainment of bivalve shells and related models”

Sedimentology:

Sedimentation of silt in river banks

- An interesting historical perspective

Problem dates back to Newton; studied by Maxwell, Kelvin, Stokes, Eiffel, Reynolds...

- An interesting everyday phenomenon
  - Falling leaves & paper
  - Coins in fountains
  - Snowflakes
  - Bar tricks...

Historical Notes

After Viets and Lee, AIAAJ. **9**, 2038 (1971)

Many workers over the centuries have been fascinated by the dynamics of falling objects. Many of their descriptions include references to wavering and unpredictable motion.

Isaac Newton, *Principia*, (1726)

"Exper. 7 ... I procured a square wooden vessel, whose breadth on the inside was 9 inches *English* measure, and its depth 9 $^1/_2$ feet; this I filled with rain water; and having provided globes made up of wax, and lead included therein, I noted the times of the descents of these globes..."

"the globes ... oscillated about their centers; that side which chanced to be the heavier descending first, and producing an oscillating motion."

"... the globe always recedes from that side of itself which is descending in the oscillation, and by so receding comes nearer to the sides of the vessel, so as even to strike them sometimes.

"Exper. 14. In the year 1719 ... Dr. *Desaguliers* made some experiments of this kind again, by forming hog's bladders into spherical orbs...These were let fall from the lantern on the top of the cupola of the same church [St. Paul's in London]... But the bladders did not always fall straight down, but sometimes fluttered a little in the air, and waved to and fro as they were descending."

Maxwell (1853)

"Every one must have noticed that when a slip of paper falls through the air, its motion, although undecided and wavering at first, sometimes becomes regular."

G. Eiffel (1912)

Dropped spheres off the Eiffel tower to obtain drag coefficients.

L. F. Richardson (1923)

Fired cannon balls vertically from a cannon.

R. G. Lunnon (1926)

Dropped spheres in mine shafts of Tynesdale, England, coal district. Also dropped spheres in water: "...the falls were not straight. There was always some swerving in the path of falling spheres."

W. W. Willmarth *et al.* (1963)
G. E. Stringham *et al.*, (1969)

Dropped spheres, cylinders, disks, spheroids in liquids. Found steady-falling, periodic oscillating, continuous tumbling, and "glide-tumbling" behaviors.

For this last type of motion, a disk would tumble several times "in an apparently random manner," and when the disks finally hit the bottom, "their ultimate location...could never be predicted."

All these statements were made before the development of the ideas of deterministic chaos, and yet they may contain at least the seeds of these ideas.

Possible Theoretical Approaches

Full Navier-Stokes Approach

- Computationally very expensive
- Difficult to probe a wide range of parameter space
- Dificult to extract relevant physics

Exact Models with Simpified Assumptions

- Can get analytic results in restricted models
- Leave out terms like viscosity, gravity

Phenomenological Approaches

- Model lift, drag, etc. phenomenologically
- Can include in this way all relevant forces
- Fluid degrees of freedom difficult to treat

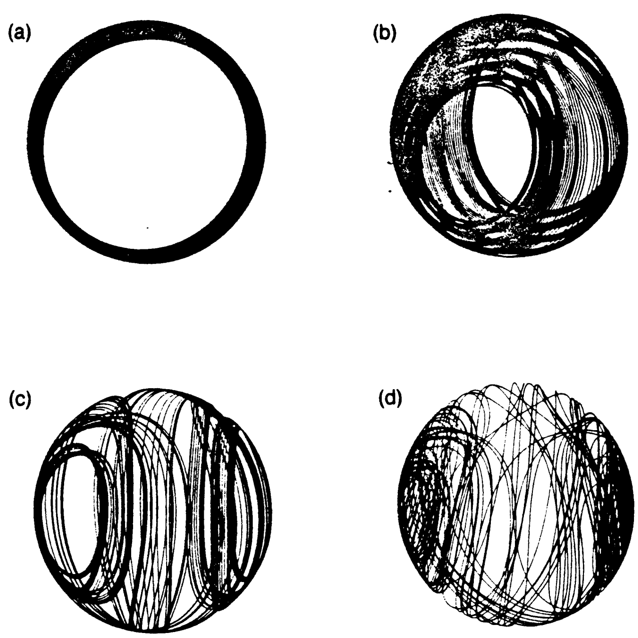
Recent Theoretical Approaches

Solutions of Kirchoff's equations for an Ideal Fluid
Aref & Jones (1993)

Study the motion of a body through an *ideal fluid* – incompressible, inviscid, irrotational.

Kirchoff's derived a set of ordinary DEQs which govern the motion of a body under such conditions.

Treatment includes important effects of fluid inertia by way of added mass tensor.



Leaves out: viscosity, vorticity, gravity.

Dynamics is not driven, dissipative chaos as in falling card problem.

Phenomenological Models of Falling Objects

Tanabe and Kaneko (1994)

Falling of a 1D “stick” in a 2D fluid.

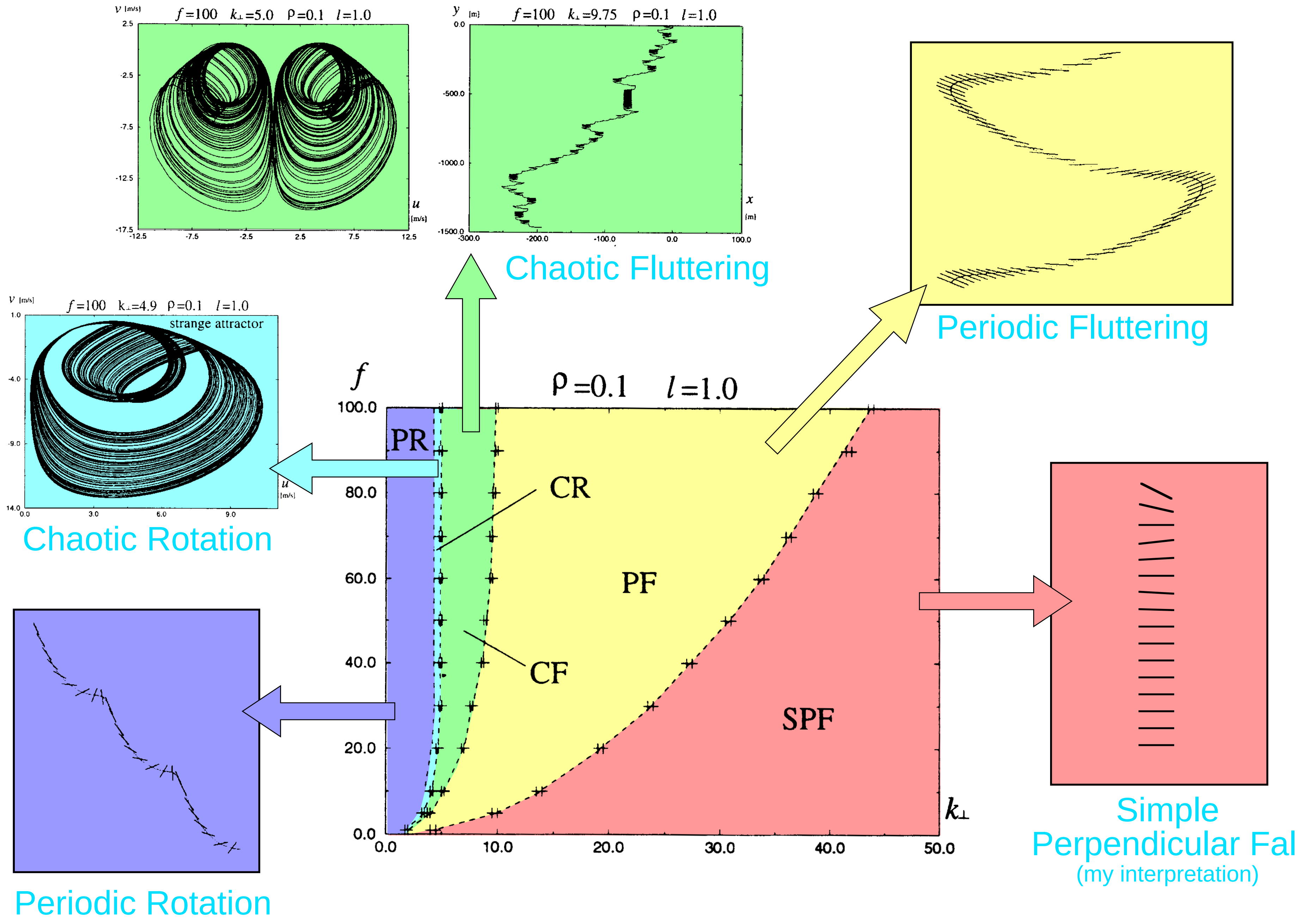
Phenomenologically model lift, drag, and gravitational forces.

Find several dynamical phases as system parameters are varied. Two of these are *chaotic* in nature.

The *fluid* degrees of freedom are ignored.

“Falling Paper” Behaviors Observed by Tanabe and Kaneko

PRL 73, 1372 (1994)



Phenomenological Models of Falling Objects
Mahadevan (1996)

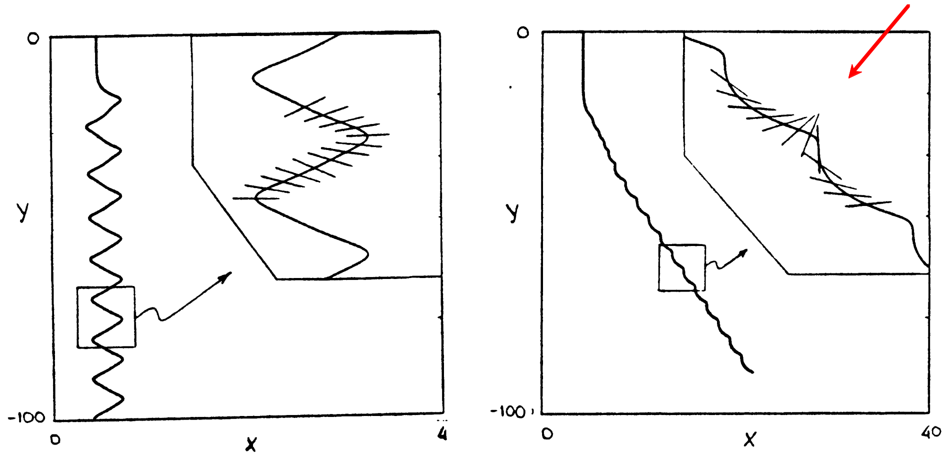
More rigorous treatment of previous theory.

Falling of an infinite cylinder in an ideal 2D fluid with a phenomenological dissipative term.

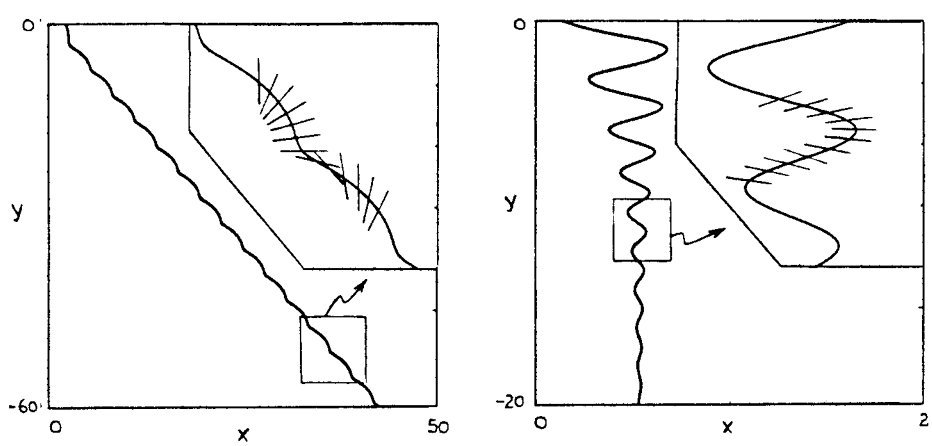
Characterization of motion based on fundamental system parameters–shape, mass anisotropy, densities, etc.

Numerics show periodic oscillations, tumbling, steady fall, and chaotic regimes.

"Falling card"

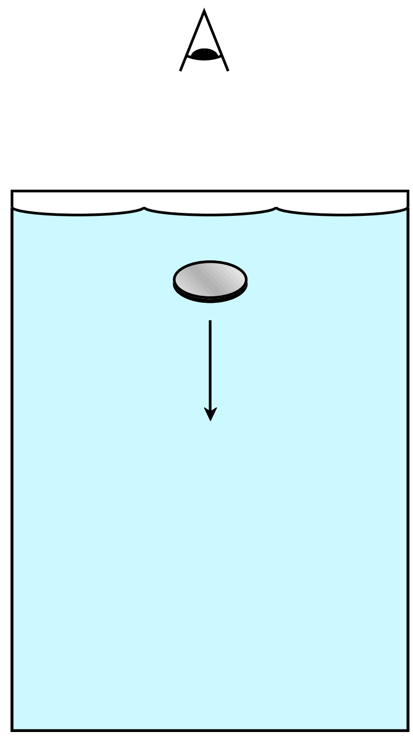


(*a*) (*b*)



(*a*) (*b*)

Direct Observation of Falling Disk Motion



Disk Parameters

Diameters: $d = 5.1$–18.0 mm
Thicknesses: $t = 0.076$–1.63 mm
Materials: ρ: Steel, Lead, Paper (in air)

Fluid Parameters

Fluid Used: ρ_f: Water/Glycerol Mixtures
Viscosities: $\nu = 0.01$–0.12 cm^2/s

Other Parameters

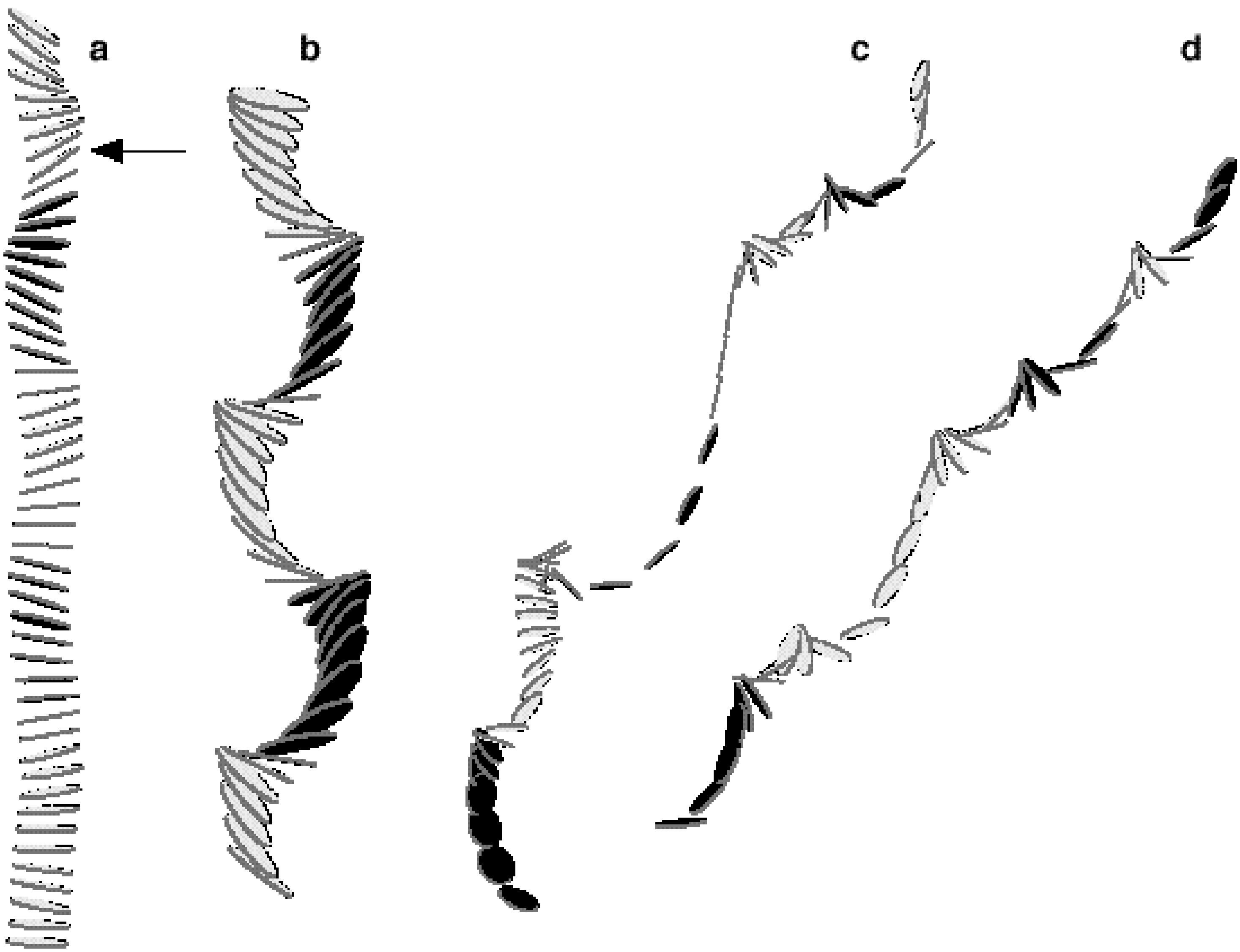
Drop Heights: 0.3–1.0 m

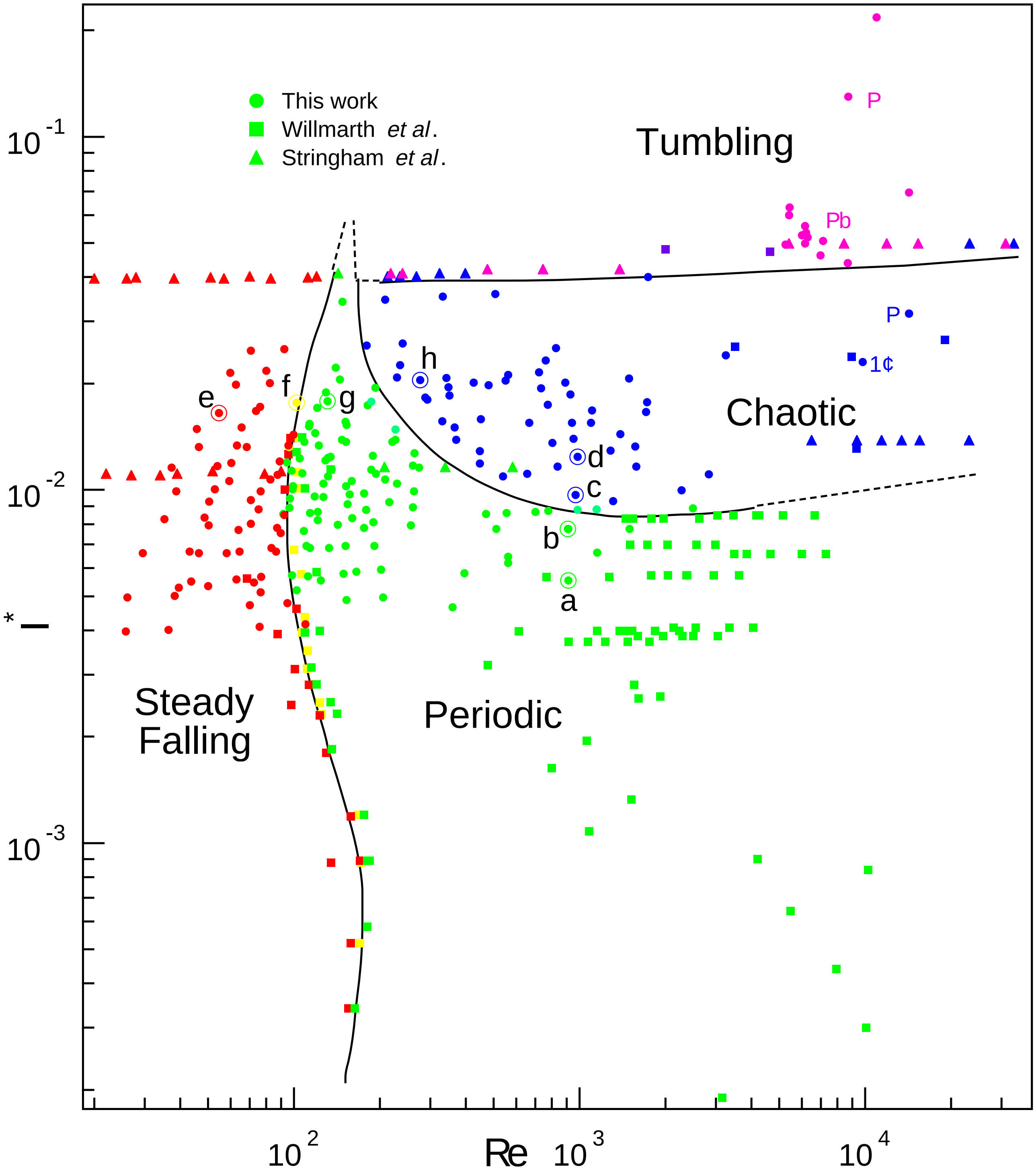
These five parameters d, t, ρ, ρ_f, and ν imply a very large parameter space.

Can reduce these five to only three independent *dimensionless ratios*:

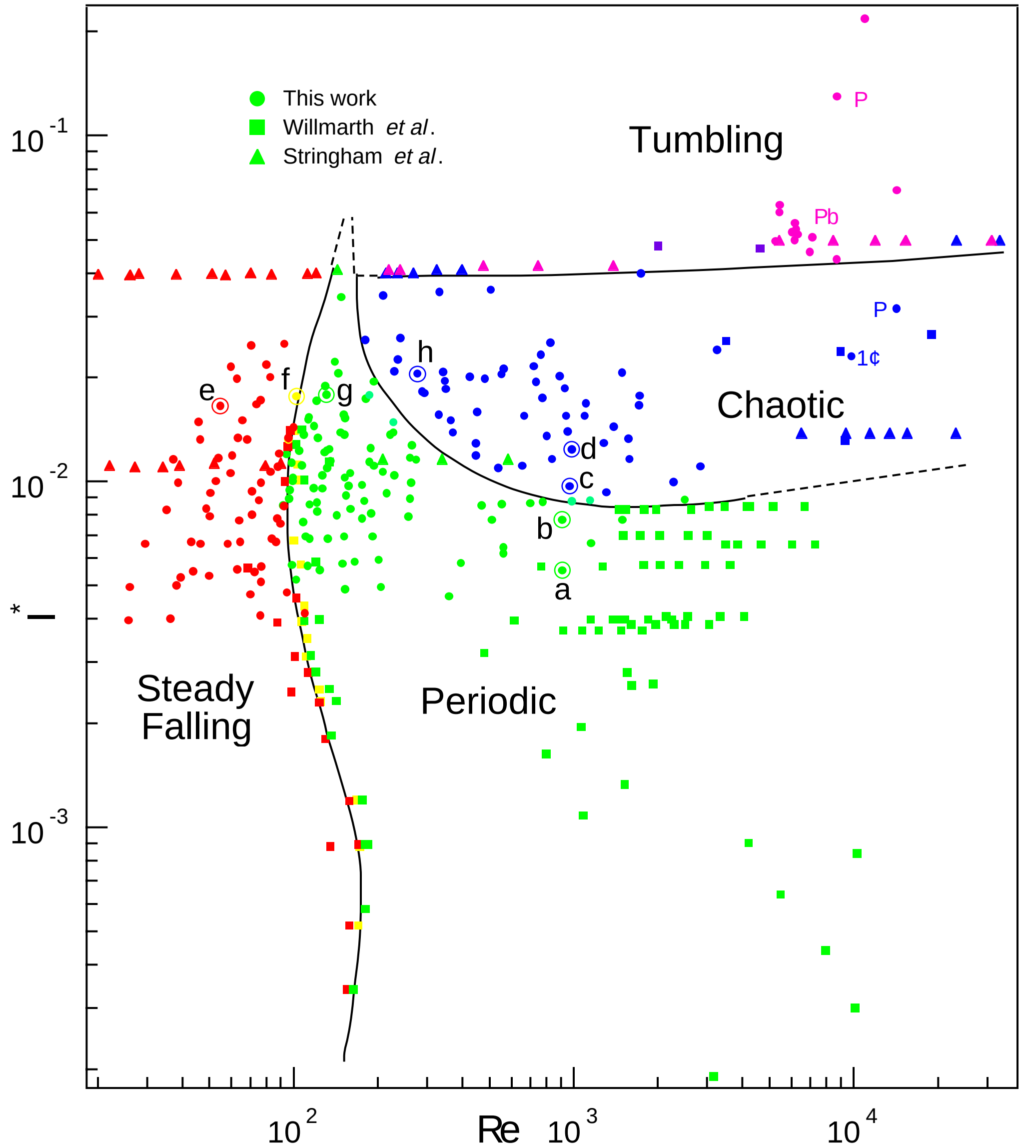
- $I^* = \pi\rho\, t / 64\rho_f d \propto \dfrac{I_{\text{disk}}}{I_{\text{sphere of fluid}}}$
- $\mathrm{Re} = Ud/\nu$
- $t/d \ll 1$ (ignore)

Thus from dimensional considerations alone expect behavior of a (thin) disk to depend only on two parameters I^* and Re


a
b
c
d



This work
Willmarth et al.
Stringham et al.
Tumbling
Chaotic
Periodic
Steady
Falling
P
Pb
1¢
a
b
c
d
e
f
g
h
10^{-1}
10^{-2}
10^{-3}
I^*
10^2
10^3
10^4
Re



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Tumbling
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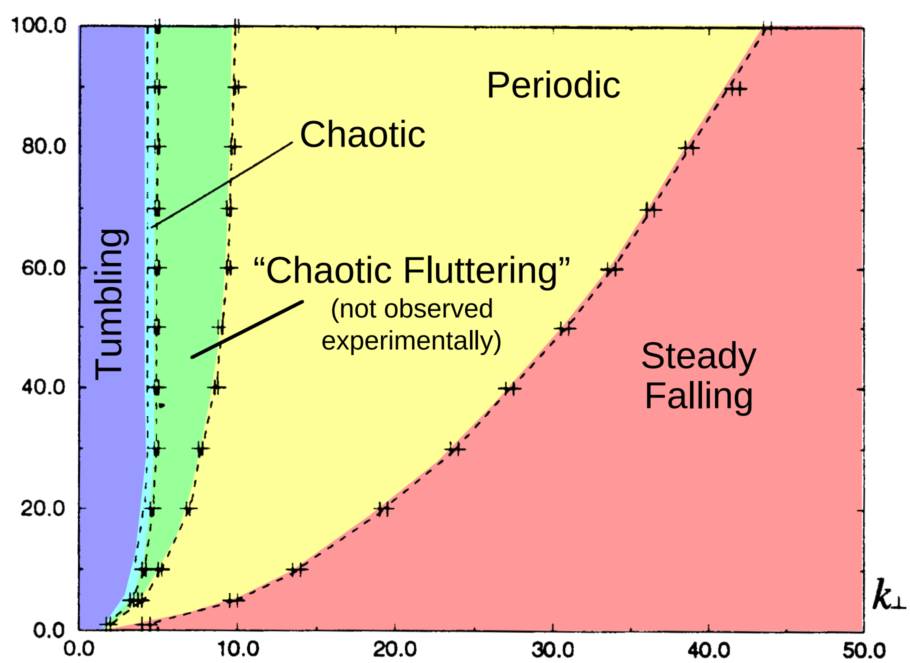
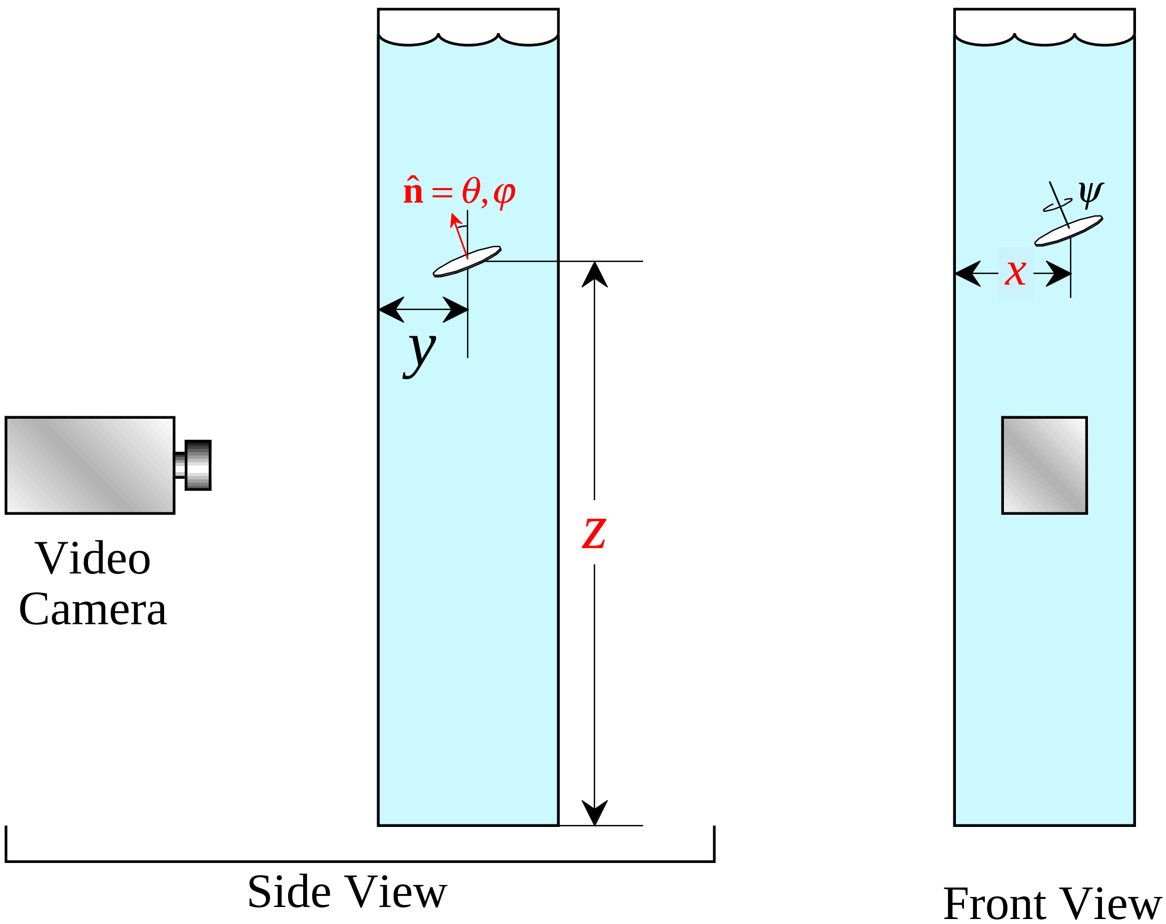
h
I*
Re
10^-1
10^-2
10^-3
10^2
10^3
10^4



Tumbling
Chaotic
Periodic
"Chaotic Fluttering"
(not observed
experimentally)
Steady
Falling
k⊥
100.0
80.0
60.0
40.0
20.0
0.0
10.0
30.0
50.0

Experimental Determination of Iteration Maps

(I^*, Re) Phase diagram provides compact overview of disks' behavior
Can we find a more quantitative measure of their motion?

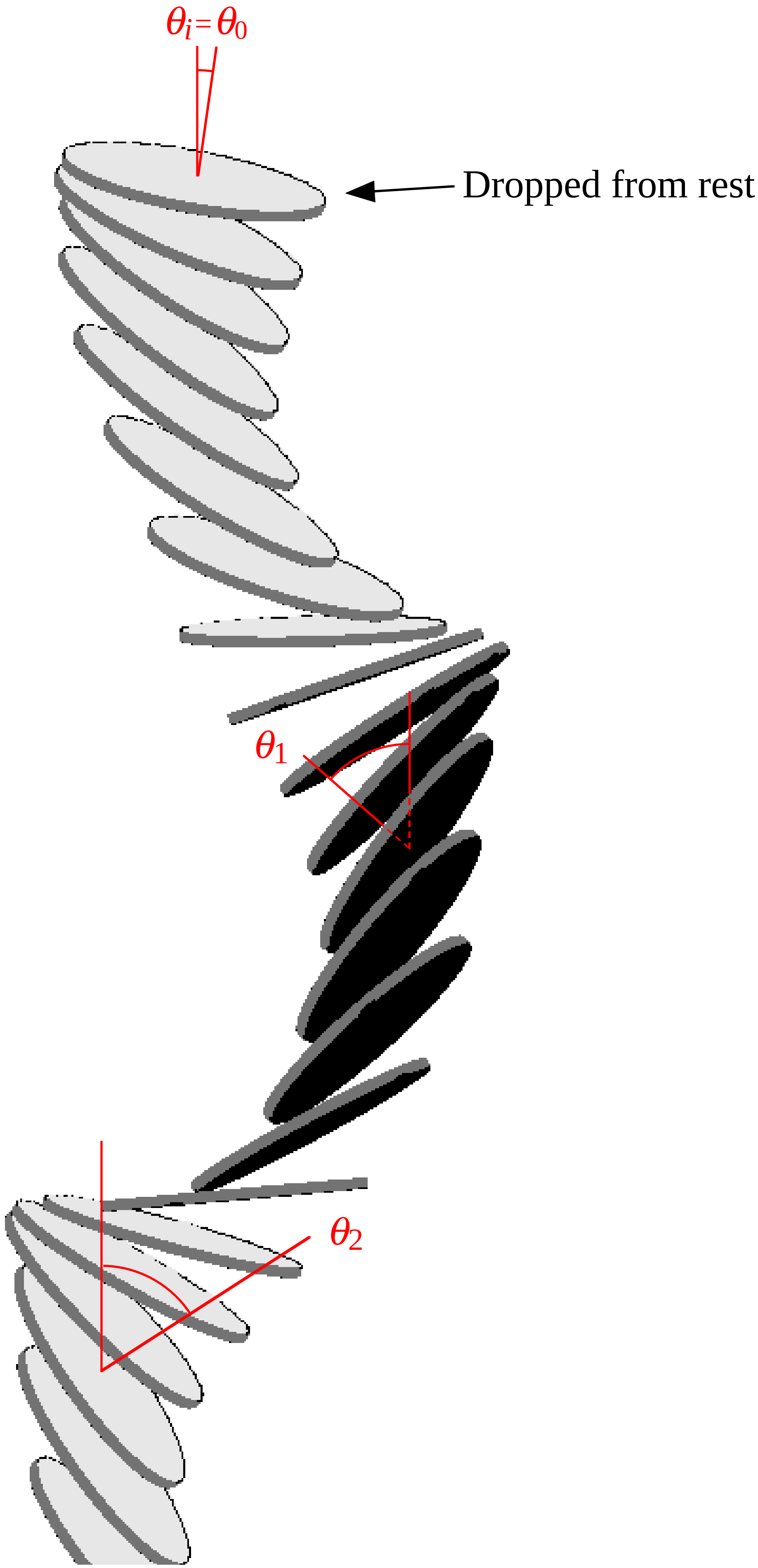


Six degrees of freedom;
we record *four*:

- x, z
- $\hat{\mathbf{n}} = \theta, \varphi$

Not recorded:

- y
- ψ



$\theta_i = \theta_0$
Dropped from rest
θ_1
θ_2

Why might θ_1 determine θ_2 ?

Label extrema of motion θ_0, θ_1, θ_2...

Deterministic motion: Since disk dropped from rest at θ_0, $\theta_1 = f(\theta_0)$.

Question: does $\theta_2 = f(\theta_1)$, with the same mapping f?

Yes, if θ alone completely specifies the initial conditions.

- At extrema, other angular velocities & horizontal velocities ≈ 0.
- Vertical velocity ≈ constant.
- Fluid degrees of freedom: vortex shedding, turbulence, etc.
- But for repetitive oscillations of disk, motion of fluid near disk also repeats–fluid motion included in renormalized mass of disk (I*).

Experiment will show if these ideas hold.

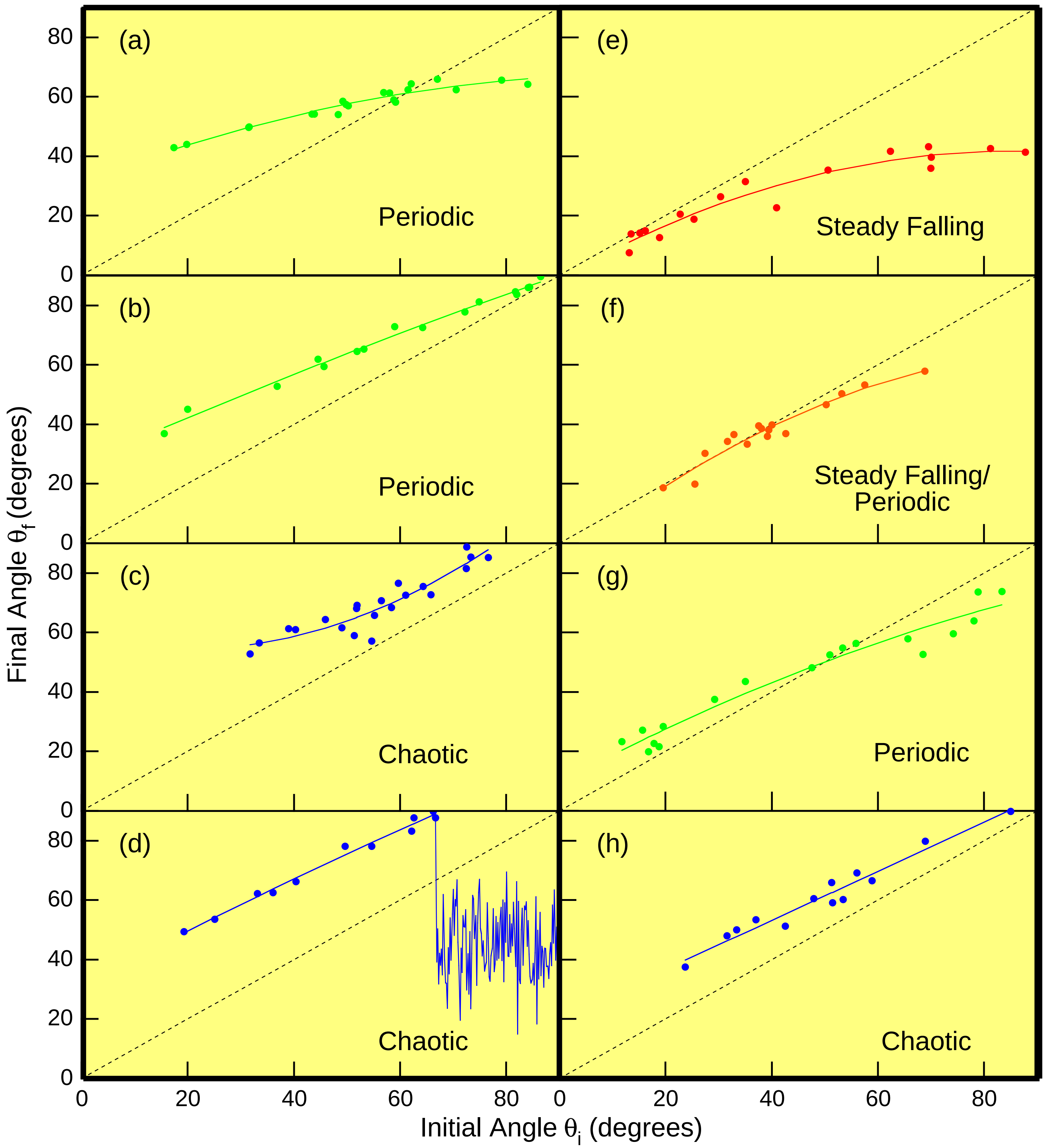
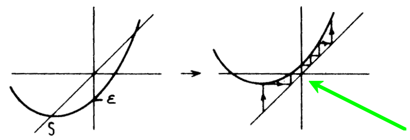
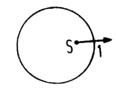
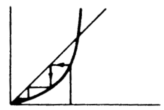
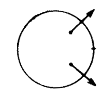
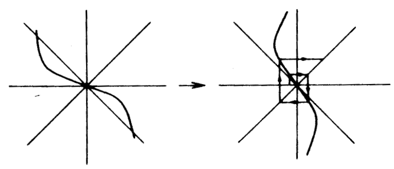
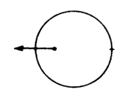


Fig. 3, Field et al.

Intermittency Route to Chaos

Table 5: Three types of intermittency.

Type	Charateristic behavior and maps	Typical map ($\varepsilon < 0 \rightarrow \varepsilon > 0$)	Eigenvalues
I	A real eigenvalue crosses the unit circle at + 1 $x_{n+1} = \varepsilon + x_n + ux_n^2$		
II	Two conjugate complex eigenvalues cross the unit circle simultaneously. $r_{n+1} = (1+\varepsilon)r_n + ur_n^3$ $\theta_{n+1} = \theta_n + \Omega$		
III	A real eigenvalue crosses the unit circle at − 1 $x_{n+1} = -(1+\varepsilon)x_n - ux_n^3$		

Schuster, *Deterministic Chaos*

In the intermittent transition, there is a "laminar" region where the trajectory moves through the narrow neck between the map and the diagonal.

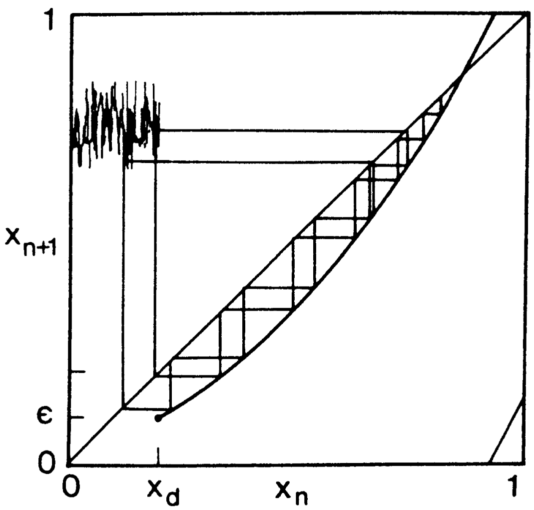
In our system, this neck is the narrow region near $\theta = 90°$.

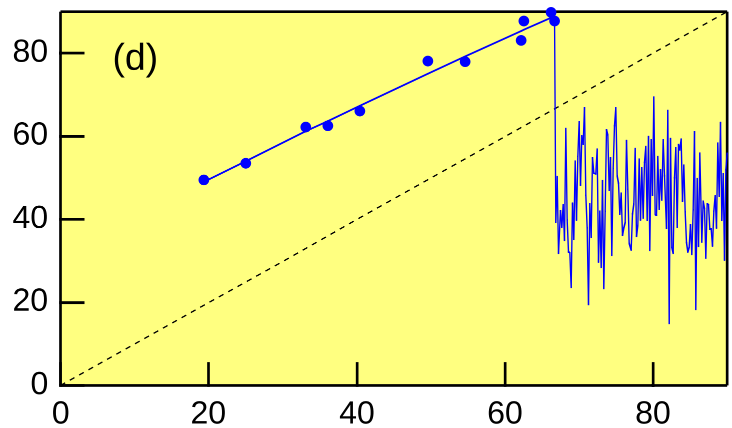
Thus the laminar region in our case is the oscillating regime building up to chaos.

Prediction: $\langle L \rangle \approx \varepsilon^{-1/2}$

Chaos with Discontinuous Maps

- In the above classification the maps are *differentiable* about the fixed point.
- Bauer *et al*. [PRL **68**, 1625 (1992)] discuss the case of *discontinuous* maps:





Trajectory moves along map until it passes point of discontinuity x_d. Then iterations are reinjected randomly into laminar region.

Notice similarity to our map. At $\theta_d \approx 67°$, disk begins to tumble, yielding a very dense map for "random" reinjection.

For continuous maps, laminar length scales as

$\langle L \rangle \approx \varepsilon^{-1/2}$. For discontinuous maps, have $\langle L \rangle$

$\approx \log(\varepsilon)$.

Could not observe any such scaling - can't set ε precisely due to disk imperfections.

Future: Change viscosity via temperature?

Tumbling Regime

At very large values of I* > 0.04, the disks tumble continuously; there is no oscillating "laminar" regime.

Experimentally, this tumbling does not appear to be periodic. Is it *chaotic*?

Evidence for Periodic Tumbling: Two Tumbling Regimes?

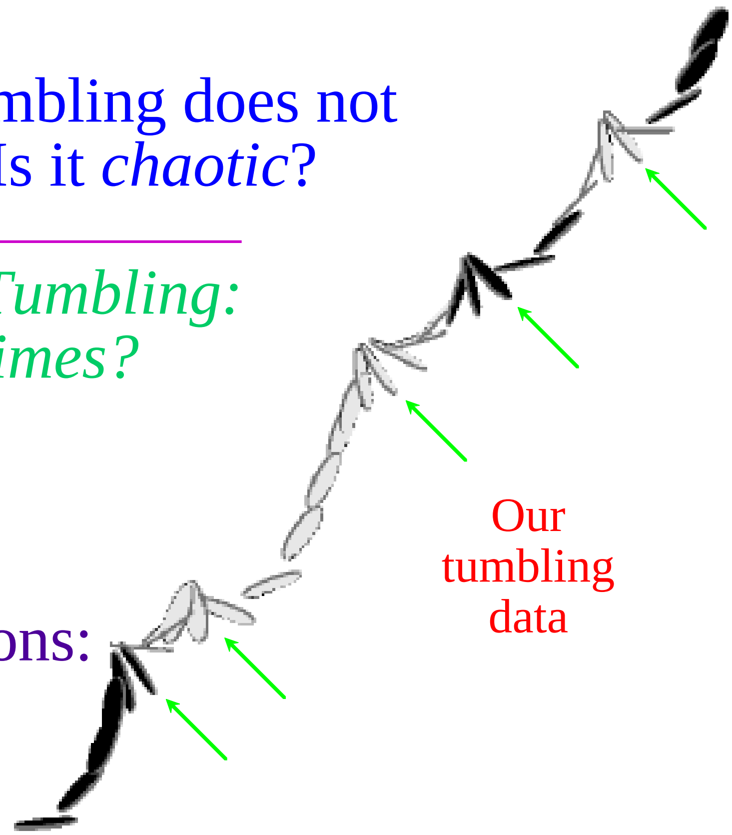
Theoretical Treatments:

Mahadevan (1996)

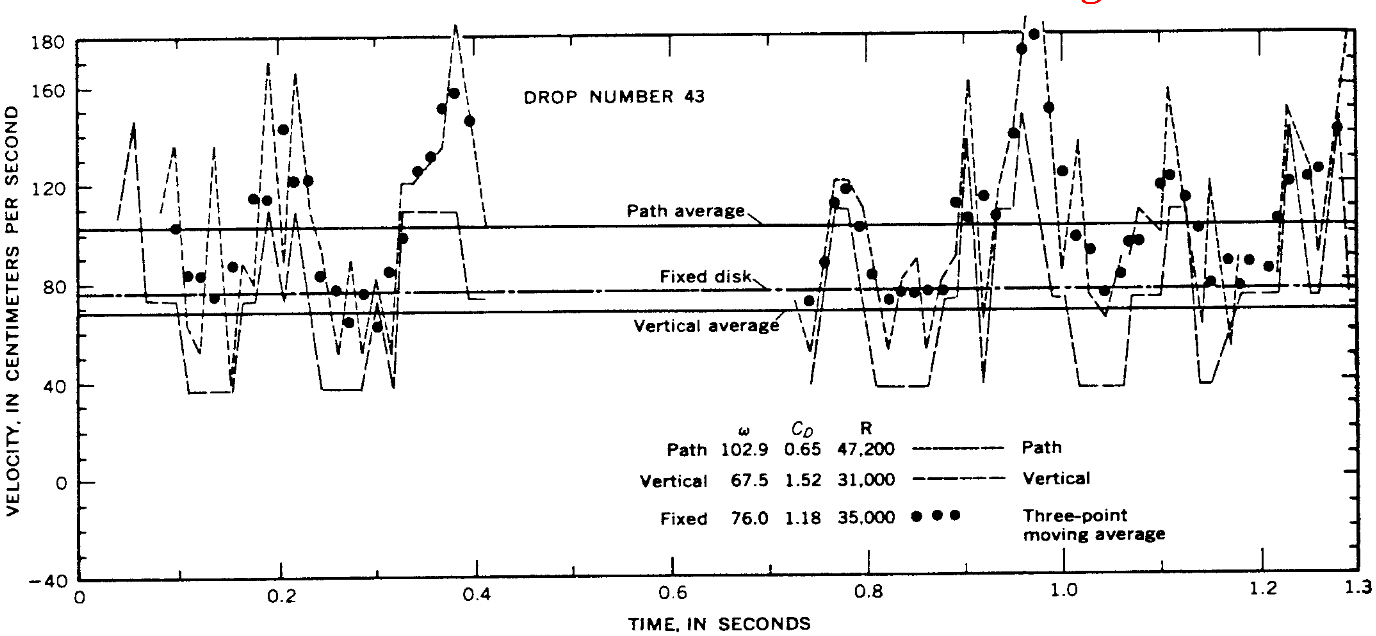
Maxwell(1853)

Experimental Observations:

Paper disks with *very* high I* appear to tumble periodically



Data of Stringham *et al.*



G. E. Stringham et al., USGS Prof. Paper 562-C (1969)

Conclusions

- Understanding motion of objects falling in a viscous medium of technological and scientific importance.
- Found four distinct dynamical phases (with perhaps two tumbling phases) with clear boundaries between them. These generally agree with theory, but not in detail.
- At least one chaotic phase clearly exists – an oscillating/tumbling phase.
- Complex behavior of disks can be reduced to a series of 1D maps.
- Chaos develops when fixed point collides with a discontinuity in the map, leading to a novel type of intermittency.

- Future directions:
  - Explore tumbling regime in greater detail
  - Look at scaling in laminar-to-chaotic transition
  - Better connections between hydrodynamic theory and chaos.