Chapter I

Formation of the Viewpoint, Alpha-Like Four-Body Correlations and Molecular Aspects in Nuclei

Kiyomi IKEDA, Toshio MARUMORI,*
Ryozo TAMAGAKI** and Hajime TANAKA***

Department of Physics, Niigata University, Niigata
**Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo*
and Department of Physics, Kyushu University, Fukuoka
***Research Institute for Fundamental Physics*
Kyoto University, Kyoto
****Department of Physics, Hokkaido University, Sapporo*

(Received December 15, 1972)

Contents

§1. Introduction

The nucleus is a many-body quantal system self-sustained by the strong interaction. Usually we can regard this system as an aggregate of nucleons by absorbing main effects of the meson fields in determining nuclear structure into nuclear forces. Nuclear forces possess many characteristic features such as the one-pion-exchange tail, a repulsive core with surrounding attraction, strong noncentral forces and non-simple exchange property. The peculiarity of nuclear forces among various strong interactions is realized in such an aspect that the two-nucleon system has only one loosely bound system, the deuteron and no resonant state. Nuclear forces are strong enough to give strong binding effect only when an aggregate of nucleons is formed, while they are not so strong as to make the neutron (or proton) matter bound. The existence of two kinds of fermions is indispensable for nuclear binding, as shown up by the strong binding of the α-particle. The combined effects of such forces and the Pauli principle bring about the saturation property, the most basic average property of nuclei, which always persists in all the nuclear properties.

In the ground states of nuclei except for few cases, the independent particle aspect holds owing to the healing effect, which is one of the important roles of the Pauli principle. In the past two decades since the brilliant success of the shell model proposed by Mayer and Jensen,[1] main development of nuclear structure theory has been made in the framework of the single-particle field with the effective two-body correlations, under the assumption that the average effects of nuclear forces can be well replaced by the single-particle field and the effects not absorbed in this field are described by the effective two-body correlations.

The scheme, "single-particle field plus two-body correlations," provides the important basis for unified understanding of abundant nuclear phenomena. However, this does not necessarily mean that the single-particle field (regardless of whether deformed or not) is always stable and essential correlations are limited to the two-body type, because the scheme has been established only through the information about especially low-lying states. It seems quite natural to suppose complex actions of nuclear forces in the nuclear system, and we would expect to find rich phenomena which could not be understood only by the single scheme mentioned above. The viewpoint entitled "Alpha-Like Four-Body Correlations and Molecular Aspects in Nuclei" presented in this article provides another scheme for deeper understanding and wider prospect about nuclear properties, in addition to the scheme already established.

The contents of this article are presented on the following conceptional basis which has been formed on the way of investigations:[*]

(1) *The molecular aspects, including the aspects dominated by the alpha-like four-body correlations as a typical case,*[**] *constitute one of the essential facets of nuclear structure*, which may be called "molecular phase" in sharp

---

[*] As schematic illustration, Fig. 3 in Chapter II and Figs. 1·1 and 1·2 in Chapter III are useful.

[**] We often abbreviate this as "alpha-correlations".

contrast to the aspects based on the shell model, "shell phase." The correlations are realized as spatial localization (real substance) in the "molecular phase," and as predisposition or function of motion in the "shell phase." There exist *many series of quasimolecular states* accompanying systematic structure-change between the two basic aspects ("phases").

(2) The alpha-correlations and the molecular aspects are characterized by such property that the internal correlation in each cluster is very strong but the external (relative) correlation between clusters become weak once the saturating clusters are formed. We call this property of the correlations the *internally strong but externally weak* character.

(3) The Pauli principle brings about the independent particle aspect through the healing function in the "shell phase", while the Pauli principle plays a role to grow the molecular aspects through the effective exchange repulsion in the "molecular phase." We call this the *dual roles of the Pauli principle.*

The molecular point of view is not completely new, because this has been adopted often since the proposal of the α-particle model by Wefelmeier in 1937[2], the works with the resonating group method by Wheeler in 1973[3] and so forth at the early stage of nuclear physics. However development of this viewpoint was stagnant until quite recently. This is in sharp contrast to the early revival of the liquid drop model with the nuclear collective model presented by A. Bohr in 1952.[4] In the establishment of the collective model, the existence of the low-lying rotational levels which could not be explained by any other models was decisively important, although the foundation of this model was postponed to later investigation.

The proposal of the α-particle model was made on the binding energy consideration. This was nothing but an attempt to study the saturation problem in light nuclei, which had been a big puzzle for nuclear physicists until the nuclear matter theory initiated by Brueckner[5] gave a way of its answer. From our present knowledge, it could be a natural way for the α-particle model to be verified first of all by the fact that ^{8}Be can be described by the potential picture with the molecular type. Even such basic point was in confused situation due to the poor experimental information at that time. Then the criticism to the α-particle model was overshouting, although it was correct in pointing out that a simple analogy with the molecules does not hold in nuclei. Such a situation around the beginning of the 1950's is stated in the text book of Blatt and Weisskopf.[6] Despite of the criticism, efforts to extract the positive meaning implied in the α-particle model were started on the basis of the improved α-α scattering data.[7] In Japan, an attempt on the α-particle model was made in 1956 by the group of Rikkyo University, who pointed out the validity of the α-α potential concept for ^{8}Be.[8] Unfortunately the succeeding development did not follow. There was a general trend at that time to object this approach from the reason that nuclear structure theory should be made on the basis of the shell model.

Rather, stimulated by the remarkable success of the shell model, the thought to dissolve the function of the α-particle model into the representation with the maximum spatial symmetry in the shell model scheme became prevailing. For example, Perring and Skyrme[9)] showed that the harmonic oscillator shell model provides the alpha-model-like wave functions by grouping nucleons under the antisymmetrization. The cluster model was proposed as another aspect of nuclei by Wildermuth and others[10)] in 1958, who noticed the importance of relative excitation modes of clusters. Their viewpoint, as the consequence of emphasizing a role of the Pauli principle to make the cluster model similar to the shell model, tended to putting primary significance on "cluster representation," a useful alternative of the shell model,[11)] which conforms to the statement of Bayman and Bohr[12)] that the relative excitation mode in the cluster model can be comprised into the $SU(3)$ scheme.[13)] The cluster-coupling shell model was attempted by Horie with a similar idea at that time.[14)]

In the 1960's, however, the verification of cluster structure was pursued in the $1p$-shell nuclei through variational calculations and by use of various data.[11)] Especially the works made by the group of Neudatchin and Smirnov[15)] showed the direct evidences on the cluster structure in light nuclei such as ^{6}Li and ^{9}Be and estimated the degree of clusterization of some typical $1p$-shell nuclei between the shell model limit and the α-particle model one.

Although the ground and low-lying states in most of the $4n$-nuclei have been fairly well understood in the shell model base by taking into account the coupling scheme with the maximum spatial symmetry, it is doubtful that this scheme can describe most of the higher excited states of $4n$-nuclei. In fact we need to take the $4p$-$4h$ or even more particle-hole configurations for some excited states with collective nature, for example the excited rotational bands of the double-closed shell nuclei. Such picture becomes not so transparent, because the increase of the number of excited particles means inadequateness of the starting point. Brink[16)] developed the α-particle model with a microscopic treatment as another way of specifying nuclear states which is different from the particle-hole description. The calculation procedure is the application of the atomic-orbital method adopted by Margenau for the α-α interaction problem,[17)] in which the aggregate of the alpha-clusters around some geometrical centers (an α-cluster intrinsic state) is considered and no average field is introduced from the outset.

This approach, a microscopic α-particle model, and the investigations following the original version of the alpha-cluster model were the attempts to extract positive meaning of the α-particle model not dissolved into the shell model scheme. In other words, the investigations were performed to look for the dynamical consequences of the alpha-correlations, which make the [4] symmetry predominate, beyond the group theoretic scheme, although in any model this symmetry has been considered as of important persistence in light nuclei. However the scope of considerations was still limited mostly to supplementing defects of the shell model.

It is a big problem in nuclear structure physics whether or not the nucleus possesses some essential and widely existent aspects not to be dissolved into the scheme, the single-particle field with the effective two-body correlations. What we want to emphasize in this article is that we have recognized the real existence of such aspects through the alpha-like four-body correlations and the molecule-like structures in light nuclei.

It is worth while mentioning that this viewpoint is related to the consideration of the saturation property as follows:

i) The alpha-like four-body correlations arise from the strong attractive correlation existent among four nucleons coupled each other in the relative S-states. This is the reason why the α-particle is the lightest saturating nucleus with the large binding energy and the extreme stiffness. Therefore the nucleus has the predisposition to dissociate the α-particle with a small energy input.

ii) Molecular aspects, visualized typically as the formation of a "Nuclear-Molecule," are realized only when the nucleus can be divided into saturating subunits without a great loss of binding energy.

Because these correlations are intimately related to the saturation property, their realization can be expected in a wide region of nuclei and of excitation energies. In fact, strong interest in the molecular aspects has been recently stimulated by new experimental information afforded in heavy ion reactions, α-nucleus scattering and so forth.

In Japan a particular background in theoretical side was indispensable for our recognition of the importance of these correlations, which was gradually built up in the 1960's, when the shell model and the collective model were prevailing.

In order to reveal physical implications of the α-particle model, the group of Hokkaido University (Hiura, Shimodaya, Tamagaki and Tanaka) initiated the approach in Beryllium region (^{8}Be and ^{9}Be) at the beginning of the 1960's. The important points obtained by them are to characterize *the alpha-correlations by the internally strong but externally weak property*, to recognize *the important role of the Pauli principle to grow the alpha-structure* of ^{8}Be and to establish *the validity of the α-particle model in the Be-region.*

In the joint discussions of the Hokkaido group, Arima and Marumori, which were performed in 1966 as a part of the project "Alpha-Cluster in Light Nuclei" organized by the Research Institute for Fundamental Physics (RIFP), the following prospect was obtained: *The alpha-like four-body cor-*

relations are indispensable to understand the sd-shell nuclei as well as the Be-region. The weak coupling model played an important role to develope our approach at this stage. Soon after that, in 1967, Ikeda proposed the viewpoint, the *molecule-like structure in light nuclei as the real existence in the excited energy region.* This provided us with a clearer concept on the structure-change of nuclear systems, which takes place primarily as a consequence of the dominance of the alpha-correlations in light nuclei, and with a wider prospect about the variety of existence form of nuclei.

Taking into account these viewpoints in a unified way we reached the conceptional basis in our approach, which is summarized at the beginning, and our collaboration has developed widely since 1968. The later stage of our approach from 1968 to 1971 proceeded along the unified viewpoint, under the project "Alpha-Like Four-Body Correlations and Molecular Aspects in Nuclei" organized by RIFP, with the collaboration of the groups in Hokkaido, Kyoto, Kyushu, Niigata and Tokyo. This article is devoted to presenting our viewpoint, reviewing the main results obtained in various model representations and giving theoretical considerations for the foundation of the alpha-correlations and molecule-like structures from the sides of many-body theory and realistic nuclear forces. In the following, the process of formation of our viewpoint is also reviewed.

§2. Verification of realization of the alpha-correlations in Beryllium region

Why did we initiate the approach to light nuclei on the standpoint of the α-particle model in the 1960's when nuclear structure theory based on the shell model picture was prevailing? Most of the attempts taking account of the alpha-like structure were intended how to find another representation of the shell model calculations, starting with the equivalence under the antisymmetrization between the cluster model wave functions and the shell model ones in the harmonic oscillator basis. It seemed difficult to extract positive meaning of the α-particle model in such a line of thought. At that time the application of the Brueckner theory to finite nuclei began to be attempted, but its applicability was considered as limited to the nuclei larger than about ^{16}O. As often emphasized by H. Tanaka,[18] each light nucleus has strong individual character, and the investigations on light nuclei reveal various important aspects of nuclei yet unknown. Following such consideration, the alpha-like structure strongly indicated in very light nuclei was regarded as an important aspect which cannot be absorbed into the shell model picture and is inherent in nuclei composed of two kinds of fermions interacting mainly with the relative S-state attraction in average. As the most typical one of the alpha-like structure, the group of Hokkaido University selected the problem in the Be-region and initiated their approach in 1960, because ^{8}Be decays into two α-particles despite of its large binding energy (instability of the single-particle field in the ground state), and therefore the loosely bound ^{9}Be with weak binding forces for three pairs was expected to have the $(\alpha+\alpha+n)$ structure. It should be emphasized that this viewpoint to stress heterogeneous character of nuclei outside of the scope of the shell model scheme was seen as singular at that time.

Our main standpoint was to regard the essential ingredient of the alpha-like structure as to be attributed to such characteristics that *internal correlation is strong but external* (*relative*) *correlation is weak.* It was advantageous that the old misleading experimental information of ^{8}Be, especially the existence of a low excited 0^+ state incompatible with the inter-α potential picture, had been removed at that time. Consequently it was clear that the phenomenological potentials to reproduce the α-α scattering data should have a repulsive core with a radius about 2 fm and a surrounding attractive potential, similar to the molecular potential. Therefore our investigation aimed to obtain the theoretical foundation for the internally strong but externally weak correlation, explaining the main features of the phenomenological α-α potentials as the interplay of realistic nuclear forces and the Pauli principle. The resonating group method[3)] was available as the mathematical tool to be used. Also the following backgrounds were helpful at the start of the project; the previous work to derive an effective nucleon-α interaction from realistic nuclear forces[19)] and the cooperative discussions in the project for "the Nuclear Force and Nuclear Structure" organized by RIFP for several years from 1960.

The strong internal binding of the α-particle was explained by the even-state attraction including the large effect of the triplet-even tensor force. Contrary to this the relative α-α potential has no long-range direct (local) attraction, because not only tensor forces (except higher order effects) but also the OPE-central potential do not contribute and the odd-state central forces are weak. These properties and the inside exchange repulsion originating from the Pauli principle lead to the weak relative binding.[20)] Later, to the origin of the repulsive core, a deeper understanding was given by Tamagaki and Tanaka[21),22)] through noticing the existence of the almost energy-independent inner oscillation in the S- and D-waves. Its outermost node around 2 fm plays an equivalent role to the phenomenological repulsive core. Also Saito[23)] showed that this inner oscillation is well reproduced by the orthogonality condition with the relative states forbidden by the Pauli principle (redundant solutions). The Pauli principle has another important role to grow the alpha-structure through the inner oscillation, beside the healing role responsible for the independent particle aspect, as emphasized by Tamagaki.

The implications of the internally strong but externally weak correlation can be stated as follows:

(1) The effective inner repulsion due to the Pauli principle and the short-tailed property of the outside attraction are responsible for the 2α-structure in ^{8}Be.

(2) The relative wave functions outside the outermost node of the inner oscillation spread much wider with much larger amplitude than the shell model predicts, and constitute main parts. Thus we can take the α-particle model for ^{8}Be.

(3) The damped inner oscillation appears just in accordance with the cluster representation of the harmonic oscillator shell model, and thus means the remnant of the shell model aspect in such a typical molecule-like structure.

(4) Cluster-interactive forces are quite different from cluster-constructive forces.

Since the α-particle model was established in ^{8}Be, Hiura and Shimodaya[24] proceeded to the study of ^{9}Be in this model with properly taking into account the Pauli principle and the weak binding property of the effective potentials between α-α and n-α. They extracted the features of coupling scheme for the odd-A system including well-localized alpha-clusters, from structure calculations. This work and the analysis of the $(\alpha, 2\alpha)$ reaction showed the validity of the α-particle model in ^{9}Be, in which the exchange effect is properly included. At this stage there were the cooperative discussions with the experimental group in Kyoto who performed $(\alpha, 2\alpha)$ experiments.[25]

Thus the investigations in the Be-region played a role to generate the strategy in our approach. Also up to the present it provides an important proto-type that we start with and return back to, when we intend to make some new approach: in the approach to link the resonating group method with the generator coordinate method by Horiuchi;[26] in the approach to study the alpha-correlations in terms of LCCO (linear combination of cluster orbitals) by Abe, Hiura and Tanaka;[27] in the approach to study the breathing mode by Saito and Yasuno[28] and so forth. All the approaches hitherto performed have confirmed the typical realization of the α-particle model aspect in the Be-region.

Chapter II is devoted to presenting our approach in the Be-region. The interrelation of the α-particle model picture to the independent particle picture is discussed in Chapter V.

§3. Indispensable role of alpha-like four-body correlations in the *sd*-shell region

3.1 Our viewpoints in the joint discussions in 1966

The questions we took care of in the way of studying the Be-region were as follows: In what manner do the alpha-correlations realized in the Be-region maintain general character as an indispensable aspect of nuclei? How do they show up in the nuclei beyond the Be-region? At first glance, ^{12}C seemed to be the next subject to be attacked. However, one inevitable drawback of the α-particle model was recognized; the 3^- state of ^{12}C arising from the rotation about the axis perpendicular to the 3α-plane becomes too low[29)] although the α-particle model gives a plausible explanation for the first 0^+ state at 7.66 MeV, which seemed difficult to be explained by the shell model picture.[30)] Furthermore we had no positive reason to adopt the α-particle model for the ground state of ^{16}O as in its old version in place of the shell model, although the α-particle model showed no serious contradiction with the experimental data about the excited states of ^{16}O and even seemed successful.[31)]

In such a situation the important motivations to proceed to the next step beyond the Be-region were afforded in the joint discussions of the Hokkaido group, Arima and Marumori, performed at Sapporo in autumn of 1966 as a part of the project "Alpha Cluster in Light Nuclei" organized by RIFP.[32)]

One of the motivations was brought from the weak coupling model proposed by Arima, Horiuchi and Sebe,[33)] which indicated the importance of the alpha-correlations in the *sd*-shell nuclei. The extensive shell model calculations performed in this region by the shell model group of Arima and others[34)] led to the conclusion that, for the understanding of the anomalously low-lying levels, 1.7 MeV 1^+ level of ^{18}F and 0.11 MeV $1/2^-$ level of ^{19}F, it is indispensable to assume the alpha-like correlations of four particles in the *sd*-orbits including particles excited from the ^{16}O core and the weak coupling between the alpha-like four particles with holes.[*)] The excited levels of ^{16}O were explained well by the direct product of the ground rotational band of ^{12}C and that of ^{20}Ne generated from the alpha-like four particles in the *sd*-shell, named by the $4p$-$4h$ mode.[**)] It is to be noted that as a background of the weak coupling model there was the recognition of the internally strong but externally weak character peculiar to the alpha-correlations which had been found in the Be-region.

Another important viewpoint obtained through the discussions was to regard the problem of the excited rotational bands in ^{16}O as a typical realization of the alpha-correlations in the excited states of light nuclei.

[*)] The very low-lying $1/2^-$ state in $^{19}F(^{19}Ne)$ had been pointed out as a positive evidence for the $^{15}N+\alpha(^{15}O+\alpha)$ structure from the cluster model.[35),11)] The important point lies in the recognition resulting inevitably from the shell model side, rather than as a possible explanation from the cluster model.

[**)] As a development of the weak coupling model in another direction, quartet scheme has been proposed.[36)]

The Copenhagen school had put the particular significance on the excited 0^+ states in some nearly-closed-shell nuclei such as ^{70}Ge, ^{42}Ca and ^{18}O as implying a new mode, calling them "mysterious second zero", Then, by noticing the appearance of the rotational bands upon such second 0^+ states in the closed-shell nuclei (^{16}O and ^{40}Ca), Brown proposed the viewpoint of the deformed excited states;[37] the deformation is regarded as the main reason for the low excitation energy of the first excited 0^+ state. Following this viewpoint, many works to explain the observed strong electromagnetic transition between the rotational band and the ground state were performed, by introducing a considerable mixing between a few specific $2p$-$2h$ or $4p$-$4h$ deformed excited states and the spherical ground state, where the ground state was thus considered to be of coexistence of spherical and "deformed" shapes (the coexistence model).[38] Eichler and Marumori[39] tried to understand the particularly low excitation energy of the "mysterious zero" state from the viewpoint of the stability of the spherical (normal) shell model ground state of the closed-shell nucleus. Although, in their formulation, they limited the excitation modes to the "dressed" $2p$-$2h$ type (in the new Tamm-Dancoff sense), they arrived at a qualitative conclusion that the spherical shell model ground state is still stable but is very near to the critical point for its instability, because a spherical ground-state correlation —the collective predisposition to produce the "deformed" excited state—is especially so strong to lower the excitation energy of the second 0^+ state.

In such a situation it was emphasized by Marumori in this joint discussions that (i) from the viewpoint of the deformed excited states, there is no clear justification for the preference of the $2p$-$2h$ mode than $4p$-$4h$ one because the calculated relative positions of the excited states resulting from two modes depend critically on details of the assumed effective interactions; (ii) we should consider the dominant $4p$-$4h$ correlation at first, based on the importance of the alpha-like four-body correlations in the *sd*-shell region as well as the Be-region; and (iii) Eichler and Marumori's idea and formulation can be extended to the $4p$-$4h$ mode.

The viewpoints we reached in 1966 are summarized as follows:

(1) To study the alpha-like four-body correlations, ^{20}Ne situates in such similar position in the *sd*-shell region as ^{8}Be does in the $1p$-shell region. In order to see to what extent the alpha-correlations reveal themselves in the form of the "α-particle," we need to investigate ^{20}Ne by presupposing an alpha-cluster outside the ^{16}O core.

(2) The excited rotational bands in ^{16}O should be regarded primarily as a consequence of the strong alpha-correlations, in other words as a consequence of the dominant $4p$-$4h$ mode (alpha-mode).

(3) As the alpha-correlations become stronger, the levels due to the alpha-mode go lower. The deformation of the excited states grows as a result of

the strong alpha-correlations. At the extreme the spatial localization appears as in ^{8}Be, and the α-particle model picture becomes valid.

It is to be noted that this viewpoint is much closer to Morinaga's original idea that in light nuclei there are excited rotational bands generated from the strongly deformed intrinsic states with the α-chain shape.$^{40)}$

3.2 Alpha-cluster plus ^{16}O-core model

The approach following this viewpoint started with the investigation of ^{20}Ne in 1967 by the group of Hokkaido University adopting *alpha-cluster plus* ^{16}*O-core model*. The positive role of this model lies in the following point. Owing to the alpha-correlations with the internally strong but externally weak character, the independent particle aspect holds for the nucleons in the closed shell, while effective forces among the outer four nucleons are attractive enough to form an alpha-cluster beyond the coupling given by the spatial [4] symmetry in the *sd*-shell orbits. Hiura, Abe, Saito and Endo$^{41)}$ obtained the following successful results: The ground $K^\pi=0^+$ rotational band is plausibly explained with an α-core effective potential with a repulsive core and state-dependent attractive well; owing to the particular state-dependence of an effective nucleon-α interaction and the effect of the Pauli principle, the weak coupling property of α-hole interaction holds in the low-lying anomalous-parity levels of ^{19}F, while the strong coupling scheme holds in the ground rotational band in ^{21}Ne; the $K^\pi=2^-$ band of ^{20}Ne* is well interpreted in the strong coupling scheme for the *p*-*h* pair. This work provided the important starting point for later works in the region ^{16}O*$\sim$Ne. Chapter IV is devoted to describing these works including the microscopic model treatments performed at a later time.

3.3 Approach based on the dominance of the alpha-correlation mode

In parallel with the project of the Ne-region in 1967 Marumori and Suzuki$^{42)}$ developed a microscopic theory to describe the alpha-like four-body correlations in the new Tamm-Dancoff sense. As this correlations go strong, the 4*p*-4*h* excited state dressed by the alpha-like mode appears low, and the ground state in the shell model sense becomes unstable when the excitation energy due to this mode tends to zero. If the alpha-correlations become still stronger, the "alpha-super state" similar to the α-particle model (α-boson-like) picture appears as the ground state, just in the same way as the "phase transition" from the pairing-vibrational state to the B.C.S. state for the pairing correlation. In this framework, the realization of the low-lying 0^+ excited states as the dressed 4*p*-4*h* states reflects that the closed-shell nuclei such as ^{16}O and ^{40}Ca have the predisposition to grow the strong alpha-correlations.

Investigations following this formulation have been developed by the group of Kyushu University, on the standpoint to take into account both the persistence of the shell model character and the strong alpha-correlations in the low-lying states of the *sd*-shell nuclei. Its background is the simultaneous success in ^{20}Ne of both the shell model calculations$^{34)}$ and the alpha-cluster plus ^{16}O-core model.$^{41)}$ A possibility to give the weak coupling feature between the 4*p*-mode and the single-hole mode was discussed by taking into account the ground-state correlations.$^{43)}$ The investigation by use of a schematic model composed of one single-particle and one single-hole level was attempted to see the effect of the ground-state correlations in ^{20}Ne as an example.$^{44)}$ The results of this work showed the importance of full use of the available higher shells to describe the alpha-correlations. In the recent work by Kamimura, Matsuse and Takada,$^{45)}$ the alpha-like spatial correlations of the outer four particles are described in a subspace in which the single-particle states are vertically truncated with the [4] symmetry. The residual interaction responsible for reproducing well the ground state rotational band in this vertically truncated subspace becomes much weaker than that used in the usual shell model calculations. Works are now in progress to study roles of the ground-state correlations and physical meaning of "phase transition" in nuclei due to the alpha-correlations.

Chapter VI is devoted to presenting the formulation of the alpha-correlation mode, showing the main results obtained in this framework and giving future prospects concerning the alpha-like four-body correlations in nuclear many-body theory.

3.4 Connection with the viewpoint of molecule-like structure

From the studies on low-lying states of ^{20}Ne it seemed not so clear-cut to discriminate which viewpoint is real; the dominance of the deformation with the strong coupling to the core or the dominance of the alpha-correlated structure with the weak coupling to the core. Various attempts were performed to show the reality of the latter possibility. It was necessary to find evidences that the alpha-correlations are dominantly realized as in ^{8}Be, in the nuclear states including highly excited ones. In connection with this point, the viewpoint of molecule-like structure was proposed (see §4): (i) The $K^{\pi}=0^{-}$ rotational bands of ^{16}O* and ^{20}Ne have the dominantly alpha-correlated character, and the $K^{\pi}=0^{+}$ (ground) rotational band of ^{20}Ne can be regarded as the partner of the inversion doublet. (ii) The typical alpha-correlated states including the α-chain structure have the common important feature to be represented by the threshold rule.

On the way of developments of the viewpoint of the molecule-like structure, the problems of the $K^{\pi}=0^{+}$ bands of ^{16}O* and ^{20}Ne have been investigated, and their position in nuclear states has been clearly understood as in "critical" (intermediate) situation, as mentioned in §§4 and 6.

§4. Viewpoint of molecule-like structure in nuclei

Molecule-like structure, to be defined primarily as the states well localized into subunits, appears as a consequence of the tight binding of constituent units and of the mutual interactions not so strong as to destroy the stability of these subunits. The correlation characteristic, being internally strong but externally weak in ^{8}Be, is nothing but the condition to produce the molecule-like structure in nuclei. The resulting effective α-α potential with the effective inside repulsion originating from the Pauli principle and the surrounding attraction is very similar to the molecular potential. Contrary to the actual molecules, this potential gives such weak binding effect as to bring about a quasibound state in the ground state, otherwise two α-particles dissolve into a shell model state. The molecular aspect of ^{8}Be shows up in the phenomenon; 1) the appearance of the rotational band ($J^\pi=0^+, 2^+, 4^+$) and its large moment of inertia corresponding to the large α-α separation about 4 fm, and 2) the large α-reduced width near the Wigner limit.

If such feature as in ^{8}Be holds between alpha-clusters in some excited states, we can expect the molecule-like states composed of several α-particles. Such states may be strongly deformed, but the resulting deformation, where the higher order ones equally contribute as the quadrupole order, should be regarded as the effect induced by the dominance of the alpha-correlations. In 1967 Ikeda made Hartree-Fock calculations[46] to allow well-localized alpha-structure in a schematic model, in order to see which is dominant in $4n$-nuclei, the alpha-correlations or the conventional quadrupole deformation. His results showed that the H-F solution localized about two centers is stable in ^{8}Be and the linear α-chain states proposed by Morinaga[40] are probably realized in larger $4n$-nuclei.

An important step in getting a key point to verify the molecule-like structure was taken forward by Horiuchi and Ikeda in 1967:[47] They showed that the two rotational bands with $K^\pi=0^+$ and $K^\pi=0^-$ in ^{16}O* (with the 6.06 and 9.58 MeV band head, respectively) and in ^{20}Ne (building on the ground state and 5.8 MeV level, respectively) can be unified as the inversion-doublet with the same molecular structure of an "α-particle" plus a residual nucleus. They noticed the following quantities: The growth of the molecular structure is more remarkable according as; (i) the gap energy between two band heads of the inversion-doublet becomes smaller, (ii) the band head is more proximate to the threshold energy for the α-decay (threshold rule) and (iii) the α-reduced width becomes larger. According to this measure the inversion-doublet bands in ^{16}O* show clearer molecular aspects than those in ^{20}Ne, and similarly the negative parity bands have more grown-up molecule-like structure than the positive parity partners in ^{20}Ne and ^{16}O*.

The experiment by Chevallier et al.[48)] indicating a possibility of the rotational band with the linear 4α-chain structure was very encourageous. Prior to this Morinaga[49)] pointed out, as a verification of his original idea of the α-linear chain states,[40)] the possibility to regard the 7.66 MeV 0^+ state and a broad 2^+ state around 10 MeV in ^{12}C as the rotational band generated on the linear 3α-chain state. Based on the establishment on the 2α-structure of ^{8}Be, theoretical suggestion of general linear $n\alpha$-chain states and these experimental information on the 3α- and 4α-chain states, Ikeda[50)] proposed in 1968 the viewpoint that the linear α-chain states constitute the other basic series of nuclear states with sharp contrast to the ground-state series with the shell model character. There are manifold series of states with intermediate structures between the two basic series; the series with several dissociated alpha-clusters (α-core, 2α-core, etc.), the dimolecular states composed of two saturating nuclei larger than α (^{12}C-^{12}C, ^{12}C-^{16}O, ^{16}O-^{16}O, etc.), the states accompanying several particles and holes coupled with these multi-center states. This viewpoint of molecule-like structure has been stated later in comprehensive way by Ikeda, Takigawa and Horiuchi.[51)] It is pictorially illustrated by a diagram shown in Figs. 1·1 and 1·2 in Chapter III, which we call the Ikeda diagram. Now the physical significance of this viewpoint shown by the Ikeda diagram is being confirmed by rich supply of experimental information provided with heavy ion reactions, such as selective excitation in α-transfer reactions, quasimolecular resonances and so forth.

At first glance molecule-like structure in nuclei seems analogous to the actual molecules. Partly it may be true, but is completely different in the following point: A composite system, an actual molecule, is a completely different substance from the subunits (atoms), because of the real existence of the central bodies, while there is no such substance in the nucleus, and a composite system with the "molecular" character is also one nucleus composed of the same nucleons as in subunit-nuclei. Therefore the concept, "molecular aspects" or "molecule-like structures" has a limited meaning to be considered in comparison with the shell model character. We should look for the criteria of realization of the molecular aspects of nuclei in the dynamical properties of the system such as threshold energies, gap energies, reduced widths and moments of inertia. Developments in theoretical studies on reduced widths have made it possible to determine the properties of the states important for the molecular viewpoint.[52)~54)]

Chapter III is devoted to developing this viewpoint and to showing the fruitful results obtained along this line.

In establishing our viewpoint, the period from 1966 to 1967 was important, as can be seen from the contexts in §§2~4. The unified viewpoint comprising the key points found until 1967 was proposed in the beginning of 1968 as a project in RIFP entitled "Alpha-Like Four-Body Correlations and Molecular Aspects in Nuclei."

§5. Dual roles of the Pauli principle

Since the success of the shell model, it had been a serious question why the independent particle aspect persists even in the presence of strong nuclear forces, until it was answered by the nuclear matter theory. The nucleus can be regarded as a low-density fermion system with respect to the short-range singular part of nuclear forces such as a repulsive core, while it can be regarded as a high-density fermion system with respect to the tail part of nuclear forces. The attractive forces just outside a repulsive core pull in the relative S-wave pushed out by the repulsive core, and the resultant shift of the wave function is small at relative distance about 1 fm, the separation distance,[55)] beyond which two-nucleon potentials are not so strong. Therefore, the Pauli principle acts very efficiently to recover the relative wave function in nuclear matter into the free wave outside the *healing distance*[56)] much less than the mean two-nucleon distance at the normal density (~1.9 fm). This healing effect of the Pauli principle provides the foundation of the independent particle aspect,[56),57)] and we can say that the Pauli principle plays a dominant role to realize the shell model character in the ground-state series.

The above argument, however, does not necessarily mean that the Pauli principle always acts to bring about the independent particle aspect against the growth of correlations. We have recognized that there are manifold molecular states covering a wide region of the excited states of light nuclei, as illustrated in the Ikeda diagram. In order that these molecular states exist without melting into the one-center field, there must be an effective inside repulsion similar to that of the effective α-α potential. If we consider the linear α-chain series starting from ^{8}Be, all the relative distances between α-clusters are almost the same as the α-α separation distance in ^{8}Be, about 4.5 fm (adjusted to the moment of inertia). In other words, main features of the linear α-chain series are well described by imposing a particlular geometrical restriction on the α-particle model with the α-α interaction established in ^{8}Be. As mentioned in §2, the α-α inside repulsion is understood in terms of the damped inner oscillation originating from the Pauli principle. We can say that in ^{8}Be the Pauli principle has a decisive role to grow the alpha-molecular aspect. When this role of the Pauli principle is not spoiled by the presence of the other nucleons or clusters in nuclei larger than ^{8}Be, the alpha-molecular aspect persists. In fact Tamagaki[58)] showed that in the linear α-chain configuration in ^{12}C* and ^{16}O* the overall-exchange effect (the effect of the antisymmetrization intervening among all the clusters except that restricted to two-α relative parts) does not alter the essential feature of the

α-α interaction in ^{8}Be, and even strengthens the inside repulsion. On the other hand, in the ground states of ^{12}C and ^{16}O, even if the inside exchange repulsion is introduced at the outset, it is entirely weakened by the overall-exchange effect so as to form the one-center field (triangular deformed in ^{12}C and spherical in ^{16}O).

Based on the above arguments, we can say that the Pauli principle has the dual roles; the one to produce the independent particle aspect in the average field through the healing function and the other to grow the molecular aspect through the inside exchange repulsion. The dual roles of the Pauli principle are responsible for the existence of the molecular series and the ground-state series.

Such duality character is of the interesting cases where a fundamental factor inherent in a physical system causes opposite aspects in the same system. Recently much attention has been attracted to the molecular-aspect-producing role of the Pauli principle in connection with phenomena of heavy ion scattering such as quasi-molecular resonances.

§6. "Critical" region

——*Coexistence of "molecular phase" with "shell phase" and its growth*——

There are many nuclear states between the two basic series, the ground-state series and the linear α-chain series, which we are interested in. Some of them have simultaneously both the shell model aspects and the molecular aspects. It seems suitable to call such nuclear states with coexistent feature as being in "critical" region, in the sense of phase transition that the different aspects appear in the same domain. The underlying reason of coexistence is primarily attributed to the saturation property.

For nuclear states in "critical" region we can approach problems fairly equally well from the shell model picture and from the molecular picture, but the approach from the one side only cannot give full description for them. In fact the ground-state band with $K^{\pi}=0^{+}$ of ^{20}Ne has been considerably well described by the shell model if a large effective charge ($e^{p}\cong1.5e$ and $e^{n}\cong0.5e$) is introduced,[34] while it has been plausibly explained by the alpha-cluster plus ^{16}O-core model.[41] On the contrary, the negative parity band with $K^{\pi}=0^{-}$ built on the 5.80 MeV level can be regarded as the typical molecular states because of the large α-reduced width. In the inversion-doublet rotational bands in ^{20}Ne, there exists clear difference in the degree of dissociation of the alpha-cluster. Also, observing the change of the alpha-core structure from ^{20}Ne to ^{16}O*, we can see the growth of the molecular aspect because of the smaller energy shifts from the α-threshold and the larger strength (or α-reduced width) for the α-transfer reaction.[59]

In order to treat the nuclear states in such transient situation, we need a formulation to include both limits, namely the shell model states at one extreme and the molecular states at the other extreme. As an approximate microscopic treatment to describe such transient situation, we can adopt an approach following a microscopic α-particle model by Bloch and Brink.[16] The wave functions are constructed by taking the antisymmetrized product of the alpha-cluster wave functions with some geometrical configuration. Such alpha-cluster intrinsic states are chosen to generate rotational bands under consideration as suitably as possible, because most of the noted states of light nuclei are classified into some rotational bands. If we treat the position parameters as generator coordinates, we have a better description taking into account vibrational motion, although the generator coordinate method has been actually applied only to several di-cluster states.

In order to clarify the alpha-core molecular aspect, a typical series running just above the ground-state series in the Ikeda diagram, Nemoto and Bando[60] investigated the region from ^{20}Ne to ^{16}O* by extending of the method mentioned above. They clarified the origin of the weak alpha-hole coupling scheme in terms of the alpha-clusterization and the overall-exchange effect. This indicates the reason of success of the alpha-cluster plus ^{16}O-core model,[41] where the weak alpha-hole coupling was explained in terms of the state-dependence of an effective nucleon-α interaction. They also explained the mechanism for the gradual growth of the alpha-core molecular aspect from ^{20}Ne to ^{16}O*. The enhancement of outer parts of the relative wave functions thus obtained is in favor of the α-transfer reaction data. The investigations on these subjects are described in Chapter IV.

The low-lying states of ^{12}C, except the second 0^+ state, can be said to be in such transient situation, because the ground state of ^{12}C is situated between ^{8}Be with the 2α-structure and the ground state of ^{16}O with the shell model character. Takigawa and Arima[61] investigated the low-lying states of ^{12}C in the Heitler-London approximation, in a hybrid way to take account of the mixing of the spatial symmetry [31] due to the $\boldsymbol{l}\cdot\boldsymbol{s}$ force into 3α-cluster structure. Their results show the importance of both the clustering feature (even though it is less pronounced than in ^{8}Be) and the shell model aspect. As a result of moderate dissolution of the alpha-clusters, the inter-cluster distances in the triangular configuration become small ($\cong$2.3 fm) but still significant to reproduce the enhanced $B(E2)$ between the ground 0^+ and 4.43 MeV 2^+ states without introducing effective charge. Also the possibility was shown that, as a result of the spin-orbit coupling effect, the excitation energies of the 2^+ and 4^+ states of the ground rotational band are raised to the positions near the experimental values. However, it seems premature to answer whether or not the triangular deformation really exists in ^{12}C, if we consider the results together with those of the Hartree-Fock approach.[27],[62] The problems concerning ^{12}C are described in Chapter V.

§7. Justification of density localization through the Hartree-Fock approach

The various approaches hitherto stated concerning $4n$-nuclei are more or less based on the presupposition that constituent clusters undergo the modification restricted at most to the change in spread from the isolated α-particle. Strictly speaking, the alpha-like molecular aspect remains still open until the stability of the constituent alpha-clusters against polarization is confirmed, even if the energy minimum is obtained at large α-α distance and for a suitable spread parameter of the alpha-clusters in the variational calculations.

Our task is to show the stability of the density-localized solution, starting with the one-particle basis functions. Ikeda's work[46] was the first to approach this problem. Then Kubodera and Ikeda[63] proceeded to the restricted Hartree-Fock calculation by use of the di-spherical potential model in ^{8}Be.

Abe, Hiura and Tanaka[27] investigated more extensively this problem in ^{8}Be and ^{12}C in the Hartree-Fock calculation, in order to test the degree of the polarization of the alpha-clusters. They took the basis state vectors with the linear combination of cluster orbitals (LCCO) including the higher orbitals at each center in accordance with the point group symmetries, corresponding to the dumbell geometry for ^{8}Be and the equilateral triangle one for the ground configuration of ^{12}C. The density distribution obtained in this molecular-orbital model clearly shows the spatial alpha-like localization, although the binding energy curves become almost flat over considerably wide region of d(α-α distance). The α-α distance minimizing the total intrinsic energy in Brink's microscopic α-particle model ($d=d_\alpha$), recovers its physical meaning in the sense that the polarization (mixing of the higher orbitals) is the smallest at d_α. In other words, if we construct the molecular-orbit Hartree-Fock wave functions at d far off d_α, a large mixing of the higher orbitals is needed to result in the spatial localization separated by d_α. Thus they concluded that the dumbbell alpha-cluster structure of ^{8}Be is very stable against the polarization, and the equilateral triangle alpha-cluster structure of ^{12}C has the same tendency.

Study of the alpha-like four-body correlations and molecular aspects from the one-particle basis is important, because (i) it gives an evidence for the realization of these aspects without presupposing a model to represent clusterization from the outset, (ii) it provides a deeper understanding of the relation between the approaches made by presuming clusters and those based on the shell model, and (iii) we can obtain the one-particle motion for the states with apparent localization, in particular for the corresponding states of neighbouring odd-A nuclei. These problems are described in Chapter V.

§8. Connection with saturation property and nuclear forces

One of the important characters of the molecular aspects lies in the realization closely related to the saturation property of nuclei, as indicated by the threshold rule (§4) and by the coexistence of the "molecular phase" and the "shell phase" at almost the same energy region (§6). However the models of nuclear structure except the α-particle model had been developed in a way almost independent of the saturation problem, until the important consequences achieved in the nuclear matter theory began to be applied to the problems of finite nuclei. Even in such applications, main efforts have been limited to giving the single-particle field and two-body effective interactions, starting with realistic nuclear forces.

We used the terminology, *realistic nuclear forces* in the following sense. Our knowledge on nucleon-nucleon interaction consists of parts which have been recognized in three different stages following the Taketani theory on nuclear forces;[64] the essentialistic one about the outermost part ($r \gtrsim 2$ fm, r being the inter-nucleon distance) which is dominated by the OPEP, the substantialistic one about the intermediate part ($r \cong 1 \sim 2$ fm) which is reasonably understood by the one-boson-exchange model with the necessary modification due to the uncorrelated two-pion exchange process, and the phenomenological one about the innermost part ($r \lesssim 1$ fm) to be determined by comparison with the data at present.[65] We consider these nuclear forces as *realistic* because these are constracted with proper account of the theoretical reliability and are known enough to study nuclear structure problems.

The α-particle model proposed to explain the systematics of the ground-state energies of $4n$-nuclei[2] was a model for the saturation property of light nuclei, and even now it remains the simplest way for this aim. From our viewpoint, however, the binding energy formula of the α-particle model can be restated as follows: Owing to the large binding energy of the α-particle and the predisposition of the alpha-correlations in the ground state, the α-chain states are realized with a small energy input (~ 2.5 MeV per bond) to the ground state. Thus we can naturally understand the fact that ^{8}Be (the starting base) remained as the only exception in the binding energy formula of the old α-particle model. Therefore, the saturation problem in light nuclei can be stated as follows: Main parts of the ground-state energies are stored in the form of the alpha-cluster internal energies, and the relatively small but significant energy gain takes place due to the dissolution of the alpha-clusters into the shell "phase" except the Be-region. The approach to the saturation property of nuclear matter along such thought is another possible way to explain the nuclear saturation which is different from the usual nuclear matter theory. Considerably good results for the binding enerigy of α-matter (without dissolution)[66a)] mean that the states composed of dissociated α-particles are expected to appearat rather low excitation energies or at low density such as at the nuclear surface.[66)]

The first step to approach the alpha-correlation starting with realistic nuclear forces is to explain the binding mechanism of the α-particle, the second to derive *modified* forces (in the sense of reaction matrix in the nucleus) describable for the ground-state energies of light nuclei, and the third to find the properties of *effective* interactions induced by and/or responsible for the strong spatial localization. These investigations are now in progress and very recently some successful results have been obtained.

Most of the calculations in various models reported in this article have been done by use of simple phenomenological effective forces. Of course, unsatisfactory features are present in these forces. For example, in order to explain the binding energy of the α-particle and those of the larger $4n$-nuclei, we are obliged to use a stronger odd-state repulsion as the mass number A increases. The saturation property is assured grossly by the repulsive core contribution mainly effective in the relative S-state and the overall saturation (from the α-particle to the nuclear matter) can be described by the decrease of the relative weight of the tensor force contributions when A goes larger, as pointed out by Akaishi and Nagata.[67)] Although the odd-state repulsion leads to the enhancement of the alpha-clusterization, this feature should not be taken literally but regarded as a simulated one in some sense. Validity of the results obtained in model calculations by use of such effective forces should be confirmed by use of realistic nuclear forces, in principle.

Chapter VII is devoted to reviewing the present status concerning the approaches starting with realistic nuclear forces. The particular role of strong tensor forces is emphasized, in reproducing the binding energy of ^{3}H (^{3}He) and ^{4}He and with respect to the observation that the induced effective interaction as a consequence of strong alpha-clustering plays a role to help the stability of density localization, as shown for ^{8}Be and ^{12}C by Bando, Nagata and Yamamoto.[68)] These results throw light on the origin of the internally strong but externally weak correlation and the physical meaning of phenomenological effective forces.

It is to be noted that the present stage is still far from the final goal, because in any way a model is assumed for the basis states on which problems (such as the reaction matrix equation) are solved. At the same time, it is to be noted that the binding mechanism of light nuclei has been clarified in comparison with that of the interior of nuclei represented by nuclear matter, and *vice versa*. Especially, in Japan, this point of view to consider simultaneously both the limits of nuclei has been taken continuously. From the recent development in the study of very light nuclei, we may say that now we have some materials necessary for studying the pair scattering correlations and the many-particle molecular correlations (including alpha-ones) simultaneously.

§9. Remarks on present status and future problems

We have recognized that there exist many series of nuclear states with the molecular aspects in the wide domain spanned by the ground-state series with the shell-model character and the α-chain series with the complete alpha-cluster dissociation. Our investigations have been performed about the structure problems of the nuclei important for establishing our viewpoint entitled "Alpha-Like Four-Body Correlations and Molecular Aspects in Nuclei". Various models have been proposed and the conclusions obtained by a particular model have been examined from other sides, by adopting a different starting point and/or a different method. Based on the results of these investigations, we can say that our viewpoint is certainly valid for comprehensive understanding of main features of the nuclear states in light nuclei. Thus the importance of the molecular aspects starting with the most basic ones dominated by the alpha-correlations has been verified.

Our main aims are to pursue the possible basic aspects of nuclei and the manifold structure-changes derived from them and to clarify the interrelation between the realization of each basic aspect and the particular role of nuclear forces responsible for the aspect. With respect to the molecular aspects in light nuclei, the results described in this article have been obtained by use of the experimental data still limited, and more implications of the molecular aspects will be revealed on the basis of new systematic data. From the viewpoint of nuclear many-body theory and in relation to realistic nuclear forces, the foundation of the alpha-like four-body correlations and the molecular structures of nuclei should be investigated further. Future developments will follow mainly in the above-mentioned two directions. Of course, the studies on both directions are related and the developments in the one side will accelerated those in the other side.

(1) The viewpoint presented in this article will provide an important footing in understanding rich experimental information which will be supplied mainly through heavy ion reactions. Phenomenological and model approaches will be made to show and to verify various kinds of structure-changes, in particular, the multi-cluster states in light nuclei and the states dominated by the alpha-like four-body correlations in the heavier region. In order to extract reliable structure information from the phenomenon such as quasi-molecular resonances and cluster-transfer reactions, it is needed to develope

theoretical study on the characteristics of the interactions between two nuclei not less than the α-particle and on the mechanism of heavy ion reactions.

(2) For the deeper understanding of the molecular aspects, further study on the many-body correlations including the alpha-like four-body ones is needed, to clarify the mechanism of structure-charge, in other words, the mechanism of "phase transition" due to these many-body correlations, taking into account the importance of spatial correlations often stressed in this article. With regard to nuclear forces, it is wanted to construct effective interactions applicable in both the "shell-phase" and the "molecular phase." We have noticed the important role of strong tensor forces, which are indispensable for the overall saturation, the internally strong but externally weak correlation and the interdependent effects between effective interactions and clustering. This means that the molecular aspects cannot be understood by some simply averaged attraction. Special attention should be paid to the particular actions of realistic nuclear forces under the strong influence of the Pauli principle. This problem will be developed to a more extensive subject to construct the formulation to describe simultaneously the pair-scattering correlation and the molecular (or clustering) correlation. In connection with this, it may be said that our viewpoint will provide a prospective way in order that nuclear theory will reach the stage of microscopic understanding based on realistic nuclear forces.

One of the significances of the viewpoint thus formed in the investigations of light nuclei is the confidence that new characteristic features of the nucleus appear just when it is divided into saturating subsystems. In light nuclei we have expressed them concisely by the threshold rule which indicates the realization of the typical molecular states as loosely bound or quasibound states. When we consider this characteristic in light nuclei together with the appearance of the intermediate structure in fission process, it may be said that nuclear structure physics has entered into a new stage to comprehend the molecular aspects, which are regarded as dynamical change as a whole in close connection with the saturation property, in addition to the aspects described in the framework "single-particle field plus effective two-body correlations," where possible dynamical changes are considered under the premise of the stability of the nucleus against drastic dissociation.

References

1) M. G. Mayer, Phys. Rev. **75** (1949), 1969.
O. Haxel, J. H. D. Jensen and H. S. Suess, Z. Phys. **128** (1950), 295.
2) W. Wefelmeier, Z. Phys. **107** (1937), 332.
3) J. A. Wheeler, Phys. Rev. **52** (1937), 1083, 1107.
4) A. Bohr, Kgl. Danske Videnskab, Selskab, Mat. -fys. Medd. **26** (1952), No. 14.

5) K. A. Brueckner, C. A. Levinson and H. M. Mahmoud, Phys. Rev. **95** (1954), 217; see also Ref. 57).
6) J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, New York and London, 1952), p. 292.
7) R. R. Haefner, Rev. Mod. Phys. **23** (1951), 228.
8) K. Sekine, M. Taketani, T. Toyoda and K. Inoue, Soryusiron Kenkyu (Mimeographed Circular in Japanese) **11** (1956), 195.
9) J. K. Perring and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A69** (1956), 600.
10) K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. **7** (1958), 150; **9** (1958/59), 449.
11) K. Wildermuth and W. McClure, *Springer Tracts in Modern Physics*, Vol. 41, "*Cluster Representation of Nuclei*" (Springer Verlag, Berlin, 1966).
12) B. F. Bayman and A. Bohr, Nucl. Phys. **9** (1958/59), 596.
13) J. P. Elliott, Proc. Roy. Soc. (London) **A245** (1958), 128; **A245** (1958), 562.
14) H. Horie, Report at the Symposium on Pairing Correlation and Cluster held at Japan Atomic Energy Research Institute (JAERI) in 1962.
15) V. G. Neudatchin and V. N. Orlin, JETP (USSR) **41** (1961), 874; S. Matthies, V. G. Neudatchin and Yu. F. Smirnov, Nucl. Phys. **49** (1963), 97; **66** (1965), 25; Yu. A. Kudeyarov, Z. Matties, V. G. Neudatchin and Yu. F. Smirnov, Nucl. Phys. **65** (1965), 529; V. G. Neudatchin and Yu. F. Smirnov, Prog. in Nucl. Phys. **10** (1969), 275.
16) D. Brink, *Proceedings of the International School of Physics*, Enrico Fermi **36** (Academic Press, New York and London, 1966), p. 247.
17) H. Margenau, Phys. Rev. **59** (1941), 37.
18) H. Tanaka, Soryusiron Kenkyu (Mimeographed Circular in Japanese) **31** (1965), 100.
19) S. Nagata, T. Sasakawa, T. Sawada and R. Tamagaki, Prog. Theor. Phys. **22** (1959), 274.
20) I. Shimodaya, R. Tamagaki and H. Tanaka, Prog. Theor. Phys. **27** (1962), 793.
21) R. Tamagaki and H. Tanaka, Prog. Theor. Phys. **34** (1965), 191. Independently S. Okai and S. C. Park, Phys. Rev. **145** (1966), 787.
22) R. Tamagaki, Prog. Theor. Phys. Suppl. Extra Number (1968), 242.
23) S. Saito, Prog. Theor. Phys. **41** (1969), 705.
24) J. Hiura and I. Shimodaya, Prog. Theor. Phys. **30** (1963), 585; **36** (1966), 977.
25) T. Yanabu, S. Yamashita, K. Takimoto and K. Ogino, J. Phys. Soc. Japan **20** (1965), 1303.
26) H. Horiuchi, Prog. Theor. Phys. **43** (1970), 375.
27) Y. Abe, J. Hiura and H. Tanaka, Prog. Theor. Phys. **46** (1971), 352; **49** (1972), 800.
H. Tanaka, *Proceedings of the International Conference on Clustering Phenomena in Nuclei at Bochum, 1969* (I.A.E.A., Vienna, 1969), p. 137.
28) S. Saito and M. Yasuno, private communication.
29) A. E. Glassgold and A. Galonsky. Phys. Rev. **103** (1956), 701.
30) S. Cohen and D. Kurath, Nucl. Phys. **73** (1965), 1.
31) D. M. Dennison, Phys. Rev. **96** (1954), 378.
S. L. Kameny, Phys. Rev. **103** (1956), 358.
32) Soryusiron Kenkyu (Mimeographed Circular in Japanese) **37** (1967), F1.
33) A. Arima, H. Horiuchi and T. Sebe, Phys. Letters **24B** (1967), 129.
34) T. Inoue, T. Sebe, H. Hagiwara and A. Arima, Nucl. Phys. **59** (1964), 1; T. Inoue, T. Sebe, K. K. Huang and A. Arima, Nucl. Phys. **A99** (1967), 305; Y. Akiyama, A. Arima and T. Sebe, Nucl. Phys. **A138** (1969), 273.
A. Arima, S. Cohen, R. Lawson and M. Macfarlane, Nucl. Phys. **A108** (1968), 94.
35) R. K. Sheline and K. Wildermuth, Nucl. Phys. **21** (1960), 196; K. Wildermuth and Y. Tang, Phys. Rev. Letters **6** (1961), 17.
36) A. Arima, V. Gillet and J. Ginocchio, Phys. Rev. Letters **25** (1970), 1043.
37) G. E. Brown, *Comptes Rendus du Congrès International de Physique Nucléaire* (Paris, 1964), p. 129.
38) G. E. Brown and A. M. Green, Nucl. Phys. **75** (1966), 401.
39) J. Eichler and T. Marumori, Kgl. Danske Videnskab. Selskab, Mat. -fys. Medd. **35** (1966),

No. 7.
40) H. Morinaga, Phys. Rev. **101** (1956), 254.
41) J. Hiura, Y. Abe, S. Saito and O. Endo, Prog. Theor. Phys. **42** (1969), 555.
42) T. Marumori and K. Suzuki, Nucl. Phys. **A106** (1968), 610.
43) J. Eichler, T. Marumori and K. Takada, Prog. Theor. Phys. **40** (1968), 60.
44) Y. Fukushima, M. Kamimura and K. Takada, Memoirs of the Faculty of Science, Kyushu University, Ser. B **4** (1969), 39.
45) M. Kamimura, T. Matsuse and K. Takada, Prog. Theor. Phys. **47** (1972), 1537.
46) K. Ikeda, Soryusiron Kenkyu (Mimeographed Circular in Japanese) **35** (1967), F. 67; *Proceedings of the International Conference on Clustering Phenomena in Nuclei at Bochum, 1969* (I. A. E. A., Vienna, 1969), p. 277.
47) H. Horiuchi and K. Ikeda, Prog. Theor. Phys. **40** (1967), 277.
48) P. Chevallier, F. Scheibling, G. Goldring, I. Plesser and M. W. Sachs, Phys. Rev. **160** (1967), 827.
49) H. Morinaga, Phys. Letters **21** (1966), 78.
50) K. Ikeda, Report at the Conference on Low-lying States in Medium- and Heavy-Mass Nuclei, JAERI **1158** (1968), 99; Report at the Symposium held at RIFP, Kyoto University in 1968, p. 101.
51) K. Ikeda, N. Takigawa and H. Horiuchi, Prog. Theor. Phys. Suppl. Extra Number (1968), 464.
52) Y. Suzuki, H. Horiuchi and K. Ikeda, Prog. Theor. Phys. **47** (1972), 1517.
53) H. Horiuchi, Prog. Theor. Phys. **47** (1972), 1058.
54) H. Horiuchi and Y. Suzuki, Prog. Theor. Phys. **50** (1973), No. 2.
55) S. A. Moszkowski and B. L. Scott, Ann. of Phys. **11** (1960), 65.
56) L. C. Gomes, J. D. Walecka and V. F. Weisskopf, Ann. of Phys. **3** (1958), 241.
57) K. A. Brueckner and J. L. Gammel, Phys. Rev. **109** (1958), 1023.
58) R. Tamagaki, Prog. Theor. Phys. **42** (1969), 748.
59) a) R. Middelton, *Proceedings of the International Conference on Nuclear Reaction Induced by Heavy Ions* (Heidelberg, 1969), p 263.
  b) A. A. Ogloblin, ibid., p. 231.
  c) K. Bethge, Ann. Rev. Nucl. Sci. **19** (1969), 255.
60) F. Nemoto and H. Bando, Prog. Theor. Phys. **47** (1972), 1210.
61) N. Takigawa and A. Arima, Nucl. Phys. **A168** (1971), 593.
62) J. Eichler and A. Feassler, Nucl. Phys. **A157** (1970), 166.
63) K. Kubodera and K. Ikeda, Prog. Theor. Phys. **42** (1969), 740.
64) M. Taketani, S. Nakamura, M. Sasaki, Prog. Theor. Phys. **6** (1951), 581. See also Prog. Theor. Phys. Suppl. No. 50. (1971), 53.
65) Prog. Theor. Phys. Suppl. No. 3 (1956), No. 39 (1967) and No. 42 (1968), in the series of this journal. As a recent review on the works made during the past decade, for example, see M. J. Moravcsik, Reports on Prog. in Phys. (to be published).
66) a) J. W. Clark and T. P. Wang, Ann. of Phys. **40** (1966), 127; G. P. Mueller and J. W. Clark, Nucl. Phys. **A155** (1970), 561.
  b) M. Harada, R. Tamagaki and H. Tanaka, Prog. Theor. Phys. **29** (1963), 933.
67) Y. Akaishi and S. Nagata, Prog. Theor. Phys. **48** (1972), 133.
68) H. Bando, S. Nagata and Y. Yamamoto, Prog. Theor. Phys. **44** (1970), 646; **45** (1971), 1515.