Chapter I

Introduction to Comprehensive Nuclear Structure Study Based on Cluster Correlations and Molecular Viewpoint

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Contents

§ 1. Development of the studies based on the viewpoint presented in P.T.P. Supplement No. 52

As a basic correlation of the nuclear many-body system there exist cluster correlations by which various kinds of molecule-like structure are realized in wide regions of mass number and excitation energy. Consequently the molecule-like structure constitutes a fundamental form of existence and motion of nuclei, together with the shell-model-like structure. This viewpoint was presented in the Supplement No. 52[1] of the Progress of Theoretical Physics (abbreviated to P.T.P.).

In the course of the progress of the successful studies by the nuclear shell model, the independent-particle picture was founded firmly, but at the same time it was recognized clearly that there exist in light nuclei groups of levels (ground or excited) with anomalous properties which were left outside the success of the shell model interpretations. These levels could not be

easily understood on the basis of the independent-particle picture. Another viewpoint different from the independent-particle one was needed for the understanding of these groups of levels. A basic viewpoint of the cluster correlation and molecule-like structure in nuclear system was formed through the accumulated studies aimed to understand the structure of these groups of levels. In P.T.P. Supplement No. 52, together with the presentation of the molecular viewpoint and with the historical background of investigations, there were reported the results of the theoretical studies which were necessary for the formation of the viewpoint and done under the research project of "Alpha-Like Four Body Correlations and Molecular Aspects in Nuclei" organized by the Research Institute for Fundamental Physics (abbreviated to RIFP) during 1968~1971.

The prospect[2] that the molecule-like structure can appear in wide region of nuclei both in mass number and in excitation energy has promoted an extensive development of study of cluster structure in light nuclei. After the presentation of the molecular viewpoint in P.T.P. Supplement No. 52, two research projects were organized by RIFP; one was on "Study of Composite Particle Scattering by Microscopic Model"[3] during 1972~1973 and the other was on "Study of ^{20}Ne Region Nuclei with Transient Character from Shell-Structure to Molecular One"[4] during 1973~1974. The former project aimed to develop the microscopic and semi-microscopic models which treat the relative motion and the interaction between clusters so as to make those models practically applicable to the investigations of actual problems.*) Namely it attempted to extend the (semi-) microscopic models used successfully in light mass number region like as $\alpha+\alpha$ to be applicable to the cluster model studies in the *sd*-shell region where molecular states were observed to exist. In the latter project, there were pursued the theoretical identifications of the well-developed molecule-like structure in excited states and the problem of interplay between different kinds of structure —shell-model-like and molecule-like structure— in relatively low excitation energy region.

The accumulated studies under the above-mentioned two research projects enlarged greatly the applicability of the cluster model approach to various kinds of problems in the wide regions of mass number and excitation energy, and made possible to pursue the several themes in a comprehensive way which were proposed in P.T.P. Supplement No. 52 as future problems. These themes include (i) study of variety of the molecule-like structure in excited states, (ii) study of the structure-change between different kinds of structure, (iii) study of relation between cluster structure and the inter-cluster interaction, (iv) study of the interdependence of the cluster model and the accompanying effective two-nucleon interaction, (v) study of α-correlation in

*) A development of the technique to treat the inter-cluster interaction has been summarized in P.T.P. Supplement No. 62.[5]

fp-shell and heavier nuclei, and so forth. Based on the development of the above-mentioned two research projects, the joint project of the nuclear cluster theory was started by RIFP since 1974. This research project on "Alpha-Like Four Body Correlations and Molecular Structure in Nuclei" with the subtitle —Variety and Structure-Change— lasted two years until 1975.[6] Through this project the cluster model study proved to be indispensable for the comprehensive understanding of light nuclei. The research project in 1976~1978 on "Alpha-Like Four Body Correlations and Molecular Structure in Light Nuclei" was, therefore, organized by RIFP in order to advance actively the comprehensive study of light nuclei based on the molecular viewpoint, as is shown in the subtitle —Toward Comprehensive Understanding— of the projects in 1976~1978.[7),*)]

The cluster model researches under the above two projects during 1974~1978 were thus advanced by setting the main subjects on the study of variety of the molecule-like structure and on that of the dynamical aspects of structure-change between different kinds of structure. The cluster model studies covered wide range of mass number from the He or Li region to Ne or Mg region in fairly wide range of excitation energy where various kinds of molecule-like structure appear. Through the accumulation of successful results of these researches, the viewpoint of the molecule-like structure has been confirmed and developed in its dynamical facets and also deepened in its fundamental understanding. The cluster structure theory based on the viewpoint of the molecule-like structure is now giving us comprehensive understanding of the properties of the bound, quasi-bound and resonance levels of light nuclei in fairly wide range of excitation energy.

The purpose of the present article is to present the above-mentioned achievements of the cluster model studies on the structure of light nuclei which have been done after P.T.P. Supplement No. 52 based on the basic viewpoint of the molecule-like structure and the cluster correlation.**) In Chapter II the results of the studies are given on the self-conjugate $4n$ nuclei (α-nuclei) where the molecule-like structures due to the α-cluster correlations appear typically. Chapter III is devoted to the presentation of the results of studies on odd A and odd-odd nuclei (non-α-nuclei) where various clustering correlations and molecular aspects are also observed. The region of excitation energy treated in these two chapters is up to 10~15 MeV where the individual discrete levels are investigated and where the shell-model-like structure and

*) The project to publish the present Supplement article was started from 1978. The organizing members were Y. Abe, H. Horiuchi, K. Ikeda, M. Kamimura, K. Kato, S. Okabe, S. Saito and Y. Suzuki.

**) Fruitful results obtained by cluster model studies in this period of 1972~1979 can be also found in the proceedings of the conferences and symposia on clustering phenomena and composite particle interaction and in the review articles on the related subjects.[8)~15)]

some different kinds of molecule-like structure are observed to coexist and couple with each other. Structure studies of the state in highly excited energy region are reported in Chapter IV, where the so-called molecular resonances in the reaction processes between α-nuclei are discussed. Finally in Chapter V there are presented the results of the microscopic studies of inter-nucleus interaction which have been developed in close connection with the studies of the molecule-like structure in nuclei.

The α-nuclei of ^{12}C, ^{16}O and ^{20}Ne are those in which the molecule-like structures are clearly observed in excited states. These molecule-like structures have been recognized to be formed by the activation of α-cluster degree of freedom, like as 3α in ^{12}C, $^{12}\mathrm{C}+\alpha$ in ^{16}O and $^{16}\mathrm{O}+\alpha$ in ^{20}Ne. Similar feature is expected also in ^{24}Mg. This nucleus is noted to be the lightest nucleus which has the degree of freedom of relative motion between α-nuclei except for α-cluster one, that is, $^{12}\mathrm{C}+^{12}\mathrm{C}$. Progress of study has been made appreciably for these α-nuclei after P.T.P. Supplement No. 52 by the microscopic and semi-microscopic investigations based on the cluster model or molecular viewpoint. Attempts are to treat the dynamics of the structure change between the shell-model-like and molecule-like structures and between the different kinds of molecule-like structure, and then to obtain the comprehensive understanding of these α-nuclei in low excitation energy region up to $10\sim15$ MeV where there coexist the groups of levels with spatially compact shell-model-like structure and with different kinds of molecule-like structure. The clarification of dynamical aspects of the α-nuclei with strong individuality has been mainly achieved by studying the inter-cluster motion which is usually coupled with internal excitation modes of constituent clusters and by taking into account the essential effects of the Pauli principle and of the nuclear forces. The theoretical investigations have been developed not only by treating the two cluster structures of $^{12}\mathrm{C}$-α in ^{16}O and $^{16}\mathrm{O}$-α (and $^{12}\mathrm{C}$-^{8}Be) in ^{20}Ne but also by treating few body cluster system; they started from the study of 3α cluster structure in ^{12}C and extended to the model studies of $^{16}\mathrm{O}+2\alpha$ for ^{24}Mg, $^{12}\mathrm{C}+2\alpha$ for ^{20}Ne and so on. Results of the α-nucleus studies from the standpoint and treatments mentioned above have yielded the comprehensive understanding of the properties of almost all the levels of these α-nuclei in wide excitation energy below $10\sim15$ MeV. Chapter II is devoted to giving the summary of the results of the α-nucleus studies based on the α-clustering correlations and the molecular viewpoint, and to give the discussions on the implication of the results, from which it is known to obtain the firm confirmation of the basic molecular viewpoint and to deepen the understanding of the coexistence and coupling mechanism of the different kinds of structure in nuclei.

Studies of non-α-nuclei have also been developed on the basis of the viewpoint of cluster correlations. Variety of the structure is seen in non-α-

nuclei. A non-α-nucleus is composed of an α-nucleus-core and valence nucleons, which are correlated with each other in terms of the coupling between them. The α-nucleus-core can change its structure by the activation of the α-cluster degree of freedom, and the valence nucleons undergo various clustering correlations. These are combined with characteristic features of realistic nuclear forces and also with the effect of the Pauli principle acting in each situation, and thus there appear multi-phasic features of the molecule-like structure which could not be seen for the cases of α-nuclei. In Chapter III some of typical examples of the stuides on non-α-nuclei are presented, and the importance of the coexistence-and-coupling of different kinds of structure is emphasized. In detailed descriptions it is important to employ realistic effective nuclear potentials (abbreviated to RENP) and include non-central components: Such studies are presented for the N-α and d-α interactions regarded as the fundamentals of the cluster model for non-α-nuclei, and for the excited states of the ^{4}He nucleus. The studies are advanced to ^{9}Be, which is a prototype of the molecule-like structure in non-α-nuclei, and ^{10}Be (^{10}B) nuclei, in terms of the molecular-orbital model, and are extended to the extremely neutron-rich isotopes of Be. Then the results of the studies of ^{10}B and ^{11}B nuclei are presented from the viewpoint of three-cluster structure. Finally the development of the studies is described on various kinds of the molecule-like structure in the beginning of the *sd*-shell region. In these nuclear systems the coupling between the shell-model-like and molecule-like structures plays the fundamental role in explaining the experimental facts. Chapter III is arranged from the standpoint to enumerate the indispensable roles of the clustering correlations realized in the individual characteristics of these non-α-nuclei, which will make possible comprehensive understanding of non-α-nuclei.

The development of the studies on the molecule-like structure of nuclei is intimately connected with that of the experimental studies on heavy-ion reactions. Firstly the light heavy-ion reactions, especially the selective transfer reactions, clarify the existence of groups of the excited states with cluster structure and some facets indicating the fundamental properties of these states. The knowledge obtained from these heavy-ion reactions is indispensable for the development of the structure-studies discussed in Chapters II and III. Secondly, since the initial and/or final stages of heavy-ion collision processes are of cluster nature, the heavy-ion reaction phenomena provide the basic understanding of character of the inter-nucleus interaction. Stimulated by the development of the studies of the molecule-like structure in relatively low excitation energy region (which are presented in Chapters II and III) and based on the progress of detailed and systematic heavy-ion experiments, the studies of the molecule-like structure were able to be pursued in highly excited energy region in the research project of RIFP entitled "Molecule-like

Structure and Molecular Resonances in Highly Excited Energy Region" started since 1977. The main subjects of this project were to obtain the understandings on the molecule-like structure in high excitation energy region and on the interaction between α-nuclei.

The identification of the molecule-like structure in such high excitation energy region, where molecular (resonance) aspects are indicated by various kinds of so-called "Molecular resonance", is more difficult than that in the relatively low excitation energy region, due to (i) the existence of dense compound nucleus levels as background and (ii) the existence of very many open channels. Therefore in order to obtain the standard picture in this energy region, systematic analyses had to be made by a phenomenological model which treats at least the coupling to the inelastic channels of the collective excitation (rotational and vibrational motion) of colliding nuclei. The examination based on these analyses has given many affirmative bases for the existence of the di-nucleus structure. Chapter IV reports the present status of understanding on the molecule-like structure in the highly excited energy region, obtained from the correspondence of the theoretical analyses with experiments and from the compiled experimental information.

Study of the composite-particle interaction by microscopic models gives a basis for the study of the molecule-like structure discussed in Chapters II$\sim$IV. At the same time it gives an examination from the fundamental standpoint to the phenomenological and semi-phenomenological inter-nucleus interactions used for the heavy-ion reaction analyses. On the other hand the analyses of the well-developed di-nucleus structure presents a base camp for the study of the inter-nucleus interaction. Chapter V gives the results of the microscopic studies of the composite-particle interaction which have been developed with intimate relation to the cluster structure studies. In this chapter the main subjects of the microscopic studies are the detailed analyses of the role of the Pauli principle and that of the two-nucleon interaction to assure the saturation property, which are mainly done for the interaction between two closed-shell nuclei.

§ 2. Basic viewpoint of molecule-like structure in nuclei

In light nuclei, there has been clarified that a structure different in quality from the shell structure —that is the molecule-like structure as a well-developed cluster structure— appears systematically in the low-lying states near the ground state in the neighbourhood of ^{8}Be and also widely in the excited states of light nuclei. Appearance of the molecule-like structure, especially, the one with a fundamental unit of α-cluster, indicates that the clustering correlations, especially α-clustering correlations, are prominent correlations in the light nuclear system. Furthermore it suggests that the clustering correlations

appear in various forms from the polarization of clusters to the formation of the well-developed cluster structure.

The understanding of the basic properties of the clustering correlations has been developed by the early studies of 2α-cluster structure of ^{8}Be, together with the microscopic studies of α-α interaction, and by the later investigations of the cluster structure of light nuclei up to the mass region of the beginning of *sd*-shell. If we understand the fundamental characters of clustering correlations as to be the factors necessary for the formation of the molecule-like structure, they can be said to be "*strong internal cluster correlations and weak inter-cluster correlations*".[16),17)] This expresses firstly that the internal binding of constituent clusters is strong similarly as in their isolated situation and secondly that the inter-cluster interaction is weak in the region where the saturated clusters preserve their identity approximately. Since these characters of the correlations are generated in real nuclei, they are intimately connected with the action of the Pauli principle which gives strong restrictions to nuclear structure. In the case of the ground state of the usual nuclei which has a spatially compact configuration of nucleons, this Pauli principle acts as a healing function to the shell model orbits and brings about the independent-particle aspect. On the other hand in the molecule-like structure, the Pauli principle acts as an effective repulsive core force in the overlapping region of two α-nuclei and plays a role to sustain the molecule-like structure through strengthening the characteristics of the clustering correlations. These distinctive functions of the Pauli principle in two different kinds of structure (or phase) were called as the "*dual roles of the Pauli principle*".[16)]

The fact that the molecule-like structure appears generally in the excited states of nuclei means the occurrence of structure-change as a kind of phase-change of the nuclear many-body system due to the clustering correlations. One of the most important quantities related to this structure-change is the dissociation (break up) threshold energy into the constituent clusters. The relation between the dissociation threshold energy and the formation of the molecule-like structure was firstly noted as an empirical rule common to all kinds of the typical molecule-like structure known to us. Namely, the ground band states of ^{8}Be, the excited states of ^{12}C with the well-developed 3α molecular structure, the states of ^{16}O with $\alpha+^{12}$C structure and the ^{20}Ne states with $\alpha+^{16}$O structure, all of them exist near or above the respective threshold energies of the dissociation into saturating fragment nuclei (clusters).

The above-mentioned experimental regularity was recognized as a necessary condition for the formation of the molecule-like structure from the dynamical point of view for the self-sustaining nuclear many-body systems. This empirical rule regarded as a necessary condition for the formation of the molecule-like structure was called "*threshold rule*".[2),17)] We here discuss

briefly meanings of the threshold rule. Dissociation energy of a saturating nucleus into several saturating fragment nuclei is small due to the overall saturation property of the nuclear system. Therefore the appearance of the molecule-like structure near the threshold energy means that the structure-change into the molecule-like structure occurs by an input of a small amount of energy. The existence of it near the threshold energy indicates that the binding energy of the inter-cluster relative motion is almost zero or positive. This means that the binding strength of the inter-cluster interaction is only of the magnitude to confine the zero-point oscillation of the relative motion ($\alpha+\alpha$, $^{12}C+\alpha$) or to generate one-more oscillation to the relative motion ($^{16}O+\alpha$, $^{16}O+^{16}O$). When the molecule-like structure appears near the threshold energy (or near the top of the Coulomb barrier for heavy clusters), its inter-cluster wave function penetrates toward the outer region and the probability staying in the peripheral region increases. This gives advantage for keeping the identity of interacting clusters. The maintenance of the identity of interacting clusters brings about the repulsive core effect in the short distance part of the inter-cluster interaction due to the Pauli exclusion principle between clusters. Thus the occurrence of the structure-change is understood to be the result of the linked dynamical effect of the correlation characters and the Pauli principle.

Correlation characteristics premised in the above discussions (namely the strong internal cluster correlations and weak inter-cluster correlations) have been understood based on the fundamental properties of the realistic nuclear force.[19] For the understanding of the overall saturation from α-particle to heavy nuclei, the renormalization effect of the strong tensor force into the central force in ^3E*) was known to play one of the most important roles. Especially the renormalization effect is most strong in α-particle and decreases as the mass number increases. This mass number dependence is an indispensably important factor to ensure the saturation over the entire mass region, and therefore to provide the internal strong binding of clusters. On the other hand the following two properties of the meson-theoretical potential—namely (i) strong tensor force and (ii) the state dependence of the one-pion-exchange potential—are the important causes to weaken the interaction between spin-isospin saturated nuclei. What has to be stressed further about nuclear forces in connection with clustering aspects is the properties to favour the dissociation of a saturated α-nucleus into α-particle(s) and residual α-nucleus (α-nuclei).

The above-mentioned arguments about the combined effects of the Pauli principle and the nuclear force presented a basis to expect the possibility to observe various kinds of structure-change into the nuclear molecule-like structure in a wide range of energy and mass number. It was demanded to take

*) In this article, the two-nucleon states are abbreviated to the ^3E (triplet-even), ^1E (singlet-even), ^3O (triplet-odd) and ^1O (singlet-odd) states.

into account the dynamical aspects of the structure-change process in the understanding of the nuclear structure. In other words, we obtained a new scope to regard the nuclear systems, especially in light nuclei, as the many body systems, which possess such predisposition that a large structure-change is easily induced by a small amount of input energy into the dissociated systems composed of constituent nuclear clusters.

In order to show the possibility of various kinds of structure-change of nuclei, a diagram[2),18)] was presented for α-nuclei where the molecule-like structure appears prominently. This diagram displays, as a function of energy and mass number, patterns of all kinds of dissociations of α-nuclei into fragment α-nuclei at their respective threshold energies. We called this the "Ikeda diagram".[16)] It is shown in Fig. 1. Here we see that α-cluster plays a role of a fundamental unit of structure-change because α-particle is the smallest saturated nucleus. Also we see that the structure phase of $n\alpha$-clusters is situated as the upper limit in excitation energy of the diagram, and that the spatially compact shell-model-like structure near the ground state is located at the lower limit. Between these two phases of structure there are various kinds of combinations of element clusters expected to form molecule-like structure. Basic viewpoint on the molecule-like structure and on its structure-change expressed symbolically in this diagram was examined

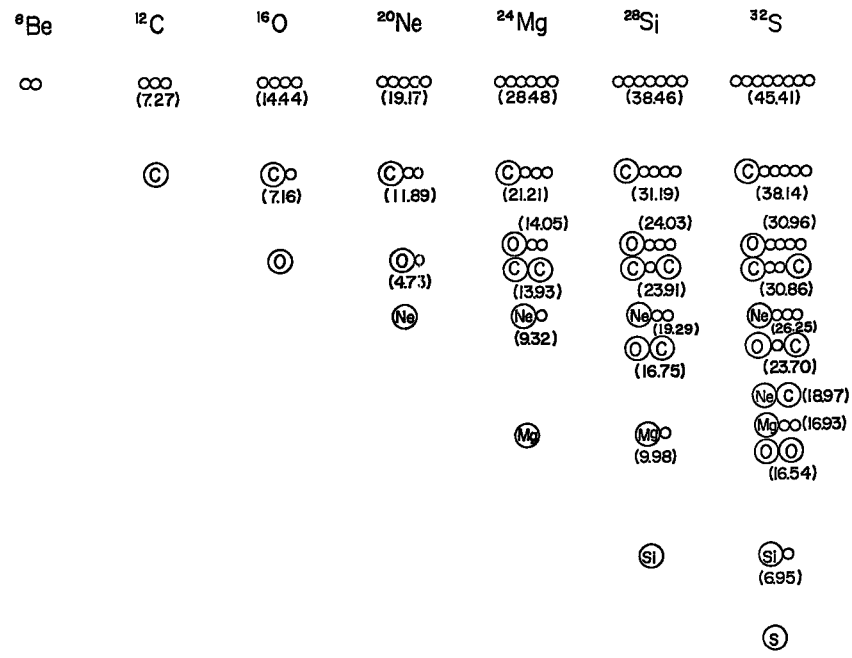

Fig. 1. The Ikeda diagram which shows the subunits of the possible molecule-like structures expected to appear near the respective threshold energies for the break-up into subunit nuclei. The threshold energies are written in parentheses.

in some typical examples and regions at the time of the P.T.P. Supplement No. 52.

What kinds of configuration are taken in the actual molecule-like structure and what mechanism of the structure-change acts depend on the action of the effective nuclear force and the Pauli principle. Furthermore since, except the ^{8}Be region, the structure-change occurs in the excited states, they depend also on the dynamical factors such as the effect of the interplay between different kinds of structure. One of the important developments of the cluster model studies after P.T.P. Supplement No. 52 consists in the investigations on the aspects of various changes of the molecule-like structure actually observed. They are done by taking into account the variety of the structure-change and through pursuing the dynamical process of the expected structure-change as will be stated in the following section.

§ 3. Comprehensive understanding on the structure of α-nuclei from the viewpoint of structure-change

The nuclides which are investigated in Chapter II are ^{12}C, ^{16}O, ^{20}Ne and ^{24}Mg. The first three nuclides, ^{12}C, ^{16}O and ^{20}Ne are the α-nuclei in which there were recognized the structure-change caused by the activation of α-clustering and the existence of the resultant molecule-like structure in the low excited states, from the experimental analyses and theoretical investigations at the time of the P.T.P. Supplement No. 52. The nucleus ^{24}Mg is an α-nucleus for which, besides the activation of the α-clustering degree of freedom, the existence of the di-nuclear molecule-like structure composed of two α-nuclei, ^{12}C-^{12}C, has been expected (see Chapter IV). In this section, we give a brief historical sketch of the study on α-nuclei performed by taking into account the structure change into the molecule-like structures, which will be described in details in Chapter II.

The existence of the molecule-like structure around the corresponding threshold energy means the structure-change caused by large rearrangement of the nuclear configuration in the lower excitation energy region. Important and interesting tasks for the nuclear theory are how to treat the structure possible to occur and how to elucidate the mechanism of structure-change. For the pursuit of these tasks, we need to take into account the fact that the scale of structure-change is such an extent for the degree of freedom of inter-cluster relative motion to become sufficiently well-developed. Under these requirements for the theory, it is natural that theoretical development has been made by using the models which can treat degree of freedom of the inter-cluster relative motion. Among many approaches the follwing three microscopic models have proven to give adequate description of the problems and have yielded many successful results: They are the resonating group method (RGM) proposed by Wheeler[20] (1937) and developed by Wildermuth[21]

(1958), the generator coordinate method (GCM)[22] which has been developed by generalizing the microscopic α-particle model by Brink[23] (1960), and the orthogonality condition model proposed by Saito[24] (1969) which is semi-microscopic and takes into account the essential effect of the Pauli principle.

The first success was obtained in the study of ^{20}Ne by the $^{16}\mathrm{O}+\alpha$ di-nucleus model. In the bound-state approximation for the ^{16}O-α relative motion by GCM, Nemoto and Bandō[25),43] predicted the existence of the "higher-nodal rotational band states" which have one more nodal points than the ground band states in their ^{16}O-α relative wave functions. This study was developed further by many people, which includes the OCM study by Matsuse and Kamimura[26] and the RGM study by Matsuse, Kamimura and Fukushima.[27] Important contributions to the cluster structure theory given by these studies were the reproduction of the ground band states with dominant shell-model-like structure and of the $K^\pi=0_1^-$ and 0_4^+ bands with the well-developed di-nucleus structure of ^{16}O-α.

Large progress of the culster model study next to the $^{16}\mathrm{O}+\alpha$ model of ^{20}Ne was made by the success of the study of ^{16}O structure by Suzuki[28] in the framework of the $^{12}\mathrm{C}+\alpha$ coupled channel OCM. This model treats the coupling of the rotational excitation mode of ^{12}C-cluster with the relative motion between ^{12}C and α. The model study succeeded on one side in reproducing the ground state with the dominant double-closed shell component and the negative-parity collective excited states with main component of $1p$-$1h$, and on the other side in reproducing many molecular states with the well-developed di-nucleus structure of ^{12}C-α. This OCM study satisfactorily explained all the levels with $T=0$ below about 13 MeV except the 0^- state at 10.95 MeV and clarified that the energy region up to about 13 MeV can be characterized as the coexistent region of two very different kinds of structure. A remarkable conclusion by this OCM study is that the number of levels with the ^{12}C-α molecule-like structure is much more larger than that with the shell-model-like one in this excitation energy region. As a higher rank molecule-like structure than ^{12}C-α di-nucleus structure, 4α molecule-like structure is expected to appear in higher excitation energy region. Considering the real occurrence of the structure-change of ^{12}C into 3α molecule-like structure which is stated below, one can have a reasoning for the structure-change of the ^{12}C-α structure into 4α one. Several studies on this structure-change problem are now in progress and are expected to advance further the present level of our understanding of the structure-change due to clustering correlations.

Structure-change of ^{12}C in its excited states was expected since early time to occur in the form of the change of the shell structure into 3α molecule-like one. This expectation was based on phenomenological and theoretical investigations on the excited positive parity states with anomalous properties.[18] Three-

body treatment of ^{12}C has been done by various models. Simultaneous reproduction of the ground rotational band and the excited 3α molecular states, together with the progress of the understanding of the structure-change mechanism, was initiated by a 3α-OCM study of ^{12}C by Horiuchi.[29] Then a full microscopic 3α-GCM study to ^{12}C, (which is equivalent to 3α-RGM study), was undertaken by Uegaki, Okabe, Abe and Tanaka[30] in an extensive way. Afterwards a 3α-RGM study was executed by Fukushima and Kamimura.[31] These studies are now the theoretical standards for the comprehensive understanding of ^{12}C system. These 3α approaches succeeded in reproducing all the $T=0$ levels of ^{12}C below the first $T=1$ level around 15 MeV except the 1^+ state at 12.71 MeV. Here it is noted that the identification of the 3α molecule-like structure with the strong activation of the α-clustering degree of freedom was done in a theoretical framework, which can simultaneously reproduce the levels with shell-model-like structure usually explained in the ordinary shell model scheme. Success of the comprehensive understanding of almost all the $T=0$ levels below about 15 MeV by the microscopic α-cluster model now can be said to have settled the picture of ^{12}C in this energy region; the ^{12}C system can be grasped by the growth and diminution of the 3α molecule-like structure.

A characteristic feature of ^{20}Ne is the presence of the strong interplay between the shell-model-like structure and the ^{16}O-α di-nucleus structure. Furthermore another characteristic feature is the appearance of the higher rank molecule-like structure of the type of ^{12}C-^{8}Be or ^{12}C-2α at rather low excitation energy region below and around 10 MeV. Former characteristic feature was successfully investigated by the α-^{12}C-α model with linear arrangement by Nemoto, Yamamoto, Horiuchi, Suzuki and Ikeda[32] which is based on the 5α model and also by the extended shell model of Tomoda and Arima[33] which is a hybridized model of the $(sd)^4$ shell model and the ^{16}O+α cluster model. The coupled channel OCM of (^{16}O-α) + (^{12}C-^{8}Be) by Fujiwara, Horiuchi and Tamagaki[34),35] was applied to explain both the former and latter characteristic features and led to the good reproduction of the observed data. What is distinctive in the cluster model studies of ^{20}Ne system is the successive enlargement of the model space which has step by step advanced our understanding of many levels with various properties. This reflects the real situation of ^{20}Ne system where the different kinds of structure appear successively with rather small energy interval as we go up to higher excitation energy region. At present almost all the rotational bands which start below 9 MeV are well explained from these model studies and further clarification of the mutual change of the qualities of the various kinds of structure in ^{20}Ne will be obtained by extending the present approaches including the investigations by the ^{12}C+2α model.[36]

Low excited states of ^{24}Mg are usually regarded to be well explained by

the shell model. Cluster model studies were applied to this nucleus since early time, but the successful reproduction of the observed data in low excitation energy region by the cluster model has been obtained only recently. And now the cluster model study is being extended to higher excitation energy region where the advantage of this model over other models is expected to be shown. As is shown in § 5 of Chapter II, the results of the systematic study of $^{16}O+2\alpha$ model done by Kato and Bandō[37] now presents us the basic understanding of ^{24}Mg from the standpoint of the cluster model. Cluster model study toward the higher excitation energy region has begun by using this $^{16}O+2\alpha$ model and other two-body cluster models. From the correspondence of the theoretical results hitherto obtained to the experimental data we can point out three excitation energy regions which are characterized by different kinds of structure. The first is the region up to the $^{20}Ne+\alpha$ threshold at 9.32 MeV where the shell model structure dominates, the second is the one from $^{20}Ne+\alpha$ threshold up to near the top of the Coulomb barrier of the $^{12}C+^{12}C$ channel and the third the one above the $^{12}C+^{12}C$ Coulomb barrier including the so-called molecular resonance region of $^{12}C+^{12}C$. In the first region the transient nature in some excited rotational bands toward the $^{20}Ne+\alpha$ cluster structure has been investigated. In the second region theoretical predictions of some characteristic levels have been obtained which are to be tested by experimental studies. Accumulation of the experimental and theoretical studies in the second region is a future problem together with the elucidation of the relation between the molecule-like structure expected in this second region and that of the third region.*)

In Chapter II we will present and explain the successful and fruitful achievements of the structure study on α-nuclei which take into account the dynamical aspects of the structure change between different kinds of the structures. These results have confirmed the molecular viewpoint symbolically expressed in the Ikeda diagram and furthermore have developed and deepened it in the dynamical facets. It is seen that the molecular states are neither anomalous nor exceptional but that the molecule-like structure is a fundamental one together with the shell structure.

§ 4. Various aspects of cluster correlations and cluster structure in non-α-nuclei

The structure study of non-α-nuclei (odd A and odd-odd nuclei) has also been progressed by taking into account the clustering correlations and the structure change into molecule-like structure. Through the systematic study on non-α-nuclei, there have been clarified, besides the aspects similar to α-

*) The molecule-like structures in the third region have been investigated extensively both experimentally and by phenomonological theories as is shown in Chapter IV.

nuclei, various facets of the cluster structure in non-α-nuclei. The results are summarized in details in Chapter III.

From the standpoint of the cluster model, non-α-nuclei can be regarded to be composed of α-nucleus cores and non-self-conjugate groups of nucleons. Nuclides which have been investigated by the cluster model till now belong either to the p-shell region or to the beginning of the sd-shell region. At the beginning of the p-shell region like as ^{6}Li, ^{7}Li and ^{9}Be, it has been regarded that the cluster structure constitutes the main feature of the levels around the ground states for which the cluster molecular models have been successfully applied since early time. In the region where the mass number exceeds 10, there appears the spatially compact structure for the levels around the ground states caused by the dissolution of α-cluster cores. Formerly for these levels around the ground states, there dominated the structure studies by the models which assume the one-center average field like the intermediate-coupling shell model and the deformed model. But even near the ground states of these non-α-nuclei, there exist universally levels reflecting the cluster structure and furthermore there have been found well-developed clustering aspects —molecule-like structure— in the excited states around the dissociation threshold into one or two α-particles and residual nucleus (or nucleon). It means that the structure-change in a single nuclear system occurs, as we go up to higher excitation energy, in a form more complex than the α-nuclei.

The following non-α-nucleus systems provide us with good examples where the cluster viewpoint works quite well as will be discussed in Chapter III. In the systems of ^{9}Be and neutron-rich Be, B isotopes, the valence nucleons are considered to move around in the two-center field produced by 2α core forming a molecule-like arrangement. The nuclei ^{6}Li and ^{7}Li show us the superiority of the cluster model description, where the structure of α-d and that of α-t are adopted, respectively. In the excited states of ^{11}B there appears the molecular aspect of $^{7}\mathrm{Li}+\alpha(t+\alpha+\alpha)$. In those of ^{10}B the persistency of the three-cluster model and d-cluster correlations outside the 2α core can be recognized in addition to the single particle the picture of the molecular orbitals of valence nucleons. At the beginning of the sd-shell, there appear the low lying excited states with the configuration of the so-called "$4p$-nh" and they coexist with the shell-model-like states usually with non-small mutual coupling. These anomalous states with the "$4p$-nh" configuration are regarded to have cluster structure of the type of $\alpha+$ (residual-core) and this cluster model description has proved to be very successful.

As seen from the studies on non-α-nuclei, the α-clustering correlations appear prominently also in non-α-nuclei, and we can say that the picture of α-cluster model constitutes a backbone for the structure of non-α-nuclei. One of the reasons for this fact is that in non-α-nuclei are also active the

basic factors realizing the α-cluster structure in α-nuclei. These factors are (i) strong α-clustering correlation enough to form a saturating α-cluster, (ii) effect of the Pauli principle which generates the inner repulsive core for the inter-cluster interaction and (iii) weak inter-α interaction. Another reason is the weakness of the α-nucleon (N) interaction and of the interaction between α-cluster and the hole state of ^{16}O core. Weakness of interactions of α-α, α-N and α-hole can be understood as coming from the basic properties of nuclear force and from the effect of the Pauli principle originating in the spin-isospin saturating property of α-cluster. Namely α-N interaction is effectively repulsive in S-wave due to the Pauli principle for the spin-isospin saturated α-cluster and is weakly attractive in P-wave. Thus α and N have only a quasi-bound state near the threshold energy. Inter α-α and α-hole interactions are weakened due to this property of α-N interaction, together with the effect of the Pauli principle.

Based on the recognition of the weak binding ability of the fundamental interactions between clusters, we can have a unified approximate picture for non-α-nuclei. This is to view the non-α-nuclei as systems with various kinds of coupling between α-nucleus core and valence particles where the α-nucleus core (including the molecule-like configuration) can change during the coupling process. In the case of the typical examples, the picture for ^{9}Be is to regard as a system composed of a particle moving in a field generated by the two-α core.[17] It was shown by Okabe, Abe and Tanaka[38] that the molecular orbital model proposed by Abe, Hiura and Tanaka[39] is quite successful for the unified description of the motion and coupling of valence nucleons and α-nucleus cores. This model was indicated by Seya, Kohno and Nagata,[40] to be applicable for the structure study of Be and B isotopes including the extremely neutron-rich isotopes, by which a new scope for the cluster model was obtained.

Increase of valence nucleons induces correlations among nucleons. Due to the weak coupling of valence nucleons with the core field, these correlations among nucleons are expected to appear as clustering correlations. On the other hand the Pauli principle tends to weaken the action of inter-nucleon force by restricting the available space for the motion of valence nucleons. This tendency gives very strong distortion to d-cluster and also strong distortion to t-cluster. Such a distortion effect in the $d+\alpha$ system has been well investigated by Kanada, Kaneko, Nishioka and Saito[41] by using realistic effective two-nucleon potentials (RENP) and by using the α-N interaction obtained from the study of ^{5}He system in the framework of OCM.

Validity of the cluster model of $d+\alpha$ for ^{6}Li and of $t+\alpha$ for ^{7}Li led to a very interesting subject, whether d- and t-clusters can be regarded to be good subunits in ^{10}B and ^{11}B, respectively. Study on ^{11}B structure with the elastic scattering of ^{7}Li$+\alpha$ was carried out by Nishioka, Saito and Yasuno[42]

as a three-cluster problem of $2\alpha+t$. (Note the appearance of the 3α-cluster structure in ^{12}C.) Since the model space of the three-cluster model contains the shell model configurations with maximum spatial symmetry, the description by the three-cluster model enables a unified treatment of the shell structure and the well-developed cluster structure including rearrangment channels. Thus just as in the case of α-nuclei, it is possible to derive simultaneously the shell and cluster structure and to understand the coupling and interplay between the two structures based on the three-cluster model. Analyses were made for ^{10}B and ^{11}B under the full use of these characteristics of the three-cluster model.

Activation of the α-clustering correlation generated by the excitation of the core nucleus and by the coupling among valence nucleons leads to the appearance of the "*4p-nh*" or α+residual-nucleus structure in the excited states.[48] The threshold energy of the breakup into α-residual-nucleus is not far from the ground state. Since in non-α-nuclei there exist valence nucleons outside the core, the coupling of the shell-model-like states formed by these valence nucleons with the cluster states of α-residual-nucleus structure becomes stronger than in the case of α-nuclei. Then, the coupling between them is very remarkable. A typical example was found in ^{18}F by Sakuda, Nagata and Nemoto[44] as a strong coupling of the $(sd)^2$ shell-model states with the cluster states of ^{14}N-α in low excitation energy region in the case of $T=0$. An introduction of the excited structure of the type of α-core enabled us to understand not only the properties of the 1^+ state at 1.7 MeV being difficult to be explained by the shell model but also the energy and other main properties of overall levels. At the beginning of *sd*-shell, cluster-structure picture successfully promoted the understanding of the ground and excited states in such a systematic manner: The overall level structure of ^{19}F was analyzed successively by Sakuda and Nemoto[45] in terms of the (^{16}O+t) and (^{15}N+α) coupled channel cluster model.

At the present stage, studies of non-α-nuclei have been done for typical examples with strong individuality. Contents of Chapter III are the summary reports of the cluster model studies of the typical non-α-nuclei, together with the presentation of the fundamental viewpoint about the clustering correlations. Variety of the distinctive dynamical aspects of the individual non-α-nuclei is made possible to be understood unifiedly by the introduction of the viewpoint of the *coexistence and coupling* of the shell-model-like and molecule-like structures and furthermore of the different kinds of cluster structure. Its deep background may be elucidated from more basic level of recognition, namely from the RENP and from manifold effects of the Pauli principle.

For the appearance of the various kinds of clustering correlations, role of the strong non-central force of the realistic nuclear force cannot be neglected. Effects of tensor force have been emphasized[19] in relation to the pro-

motion of the clustering correlations, while the role of the spin-orbit force which destroys the spatial symmetry is greatly related to the validity of the α-clustering picture. The RGM calculation for the N-α scattering carried out by Kanada, Kaneko, Nagata and Nomoto[46] showed that the tensor force plays quite a large role in the spin-orbit splitting. Such a study including tensor forces is expected to be fruitful in the study of the d- and t-cluster in nuclei. On the other hand consideration of break-up of an α-cluster is intimately related to the study of non-α-nuclei and that of non-central force. Decomposition of α particle is a very interesting subject in relation to the breaking down of spacial symmetry. The excited states of the $A=4$ system (^{4}H, ^{4}He, ^{4}Li) with $T=1$ were investigated by Furutani, Horiuchi and Tamagaki[47] to clarify the existence of a kind of cluster structure ^{3}He (or t) $+N$ and to examine the specific role of the non-central force. Structure-study by taking into account the break-up process of an α-cluster is left as a future problem at the present stage.

§ 5. Molecule-like structure in highly excited energy region

As is discussed in previous subsections, in the low excitation energy region up to 10$\sim$15 MeV, the existence of the molecule-like structure due to the activation of the α-clustering degree of freedom has been confirmed up to the mass region of ^{20}Ne. Interactions between α-α and α-(α-nucleus) are weakened by the exchange repulsion due to Pauli principle and there exists a possibility to have molecule-like structure near threshold energy of the dissociation into constituent nuclei. By similar reasons it was also expected[2),16] to have molecule-like structure composed of saturated α-nuclei other than α-particle. (See § 1.) As typical examples we can enumerate ^{12}C-^{12}C structure in ^{24}Mg, ^{16}O-^{12}C structure in ^{28}Si and ^{16}O-^{16}O structure in ^{32}S. (Threshold energies of respective structure are 13.93 MeV, 16.75 MeV and 16.54 MeV.)

By the progress of recent experimental studies and through the accumulation of data[48] there have been found many phenomena in excitation functions of heavy ion scatterings and reactions, which may be identified to be resonance states. However, there is no case where shape elastic ("quasi-molecular") resonance is confirmed clearly, contrary to the cases in relatively low excitation energy region discussed in Chapters II and III. We can indicate a few factors which bring about complexities of the situations in high excitation energy region: First the constituent nuclei are heavier α-nuclei other than α-particle. Secondly the threshold locates at about twice higher position than the threshold of α-decay. Thirdly there are very many open channels below the threshold. (Note that there are almost no open channel under the α-decay threshold.) Furthermore since the Coulomb barrier is high and thick, the effective separation energy becomes fairly large. (Height of the Coulomb barrier is about 6.5 MeV in ^{12}C$+^{12}$C, about 8.0 MeV in ^{12}C$+^{16}$O and about

11.0 MeV in $^{16}O+^{16}O$.)

The first factor mentioned above is due to the fact that the heavier α-nucleus different from α-particle has fairly low-lying excited states. Moreover, these excited states are due to various kinds of excitation mechanism including the collective excitation (like vibrational and rotational one) and the α clustering excitation clarified in Chapters II and III. The complexity caused by the second factor comes from the situation that at such high excitation energy region as 15~20 MeV the level density of compound nucleus becomes fairly dense. This means that a simple mode of excitation may disperse into many compound nuclear states with complicated configurations. The third factor means that the incident flux is dissipated into many other open channels by incooperating the second factor.

These factors show that, compared with the cluster model study in rather low excitation energy region, we need a systematic examination through analyses of various kinds of reaction processes with phenomenological models in order to obtain the recognition of the molecule-like structure in highly excited states.

An analysis which gives a standard for the study of molecule-like structure in highly excited states is the one by treating the coupling of the elastic channel with the inelastic channels of collective excitation modes. This picture was firstly introduced by Nogami and applied by Imanishi[49] to the quantitative analysis of the molecule-like structure near the Coulomb barrier of $^{12}C+^{12}C$. Later on, Kondo, Matsuse and Abe[50] reexamined the Nogami-Imanishi model by taking into account the mutual excitation as well as the single excitation into 2_1^+. They indicated its usefulness and showed the importance of the phenomenological analyses to understand the molecular resonance phenomena. Recently from the systematic coupled-channel analyses[51] on $^{12}C+^{12}C$ and $^{12}C+^{16}O$ in wide range of energy and spin, there have been found traces of the characteristic di-nucleus structure in the excited states.

Band-Crossing-Model proposed by Abe, Kondo and Matsuse[52] is a model which concisely represents the characteristics of di-nucleus molecular rotational bands by incorporating the coupling with the collective excitation modes. The content of this model is as follows: First it is assumed that there exists a di-nucleus rotational band without internal collective excitation of constituent nuclei. (It is called "an elastic rotational band".) The existence of the di-nucleus rotational band has been recognized in $\alpha+\alpha$ and $^{16}O+\alpha$, and theoretical basis is given for them and also for $^{16}O+^{16}O$ system. This elastic rotational band constitutes an yrast band for the low-spin molecular states. Next, when a collective excitation occurs, the energetically lowest rotational band in inelastic channel is given by the aligned band with a parallel coupling of spins of collective excitation mode and relative motion. At some high spin this aligned-band crosses the elastic rotational band. In high spin states

near this crossing, it is expected that the two rotational bands couple with one another even if the coupling strength is relatively weak. In higher spin states than crossing, the aligned band becomes an yrast band of the two-body nuclear molecular system. Thus it is necessary to take into account such crossing with the aligned band for the analyses of heavy ion elastic and inelastic scatterings.

When the crossing energy and the crossing angular momentum are near the grazing energy and the grazing angular momentum, respectively, in elastic or inelastic channel, it is expected that prominent features are observed as resonance peaks in the excitation function of elastic or inelastic scattering. As a complementary picture to this model there was proposed modified perturbed stationary state method,[53] based on the picture of the induced rotation mechanism.

Although above-mentioned qualitative considerations are now at the stage of the phenomenological models, their usefulness is tested, at present, by the degree in reproducing experiments by model calculations. Good correspondence with experiments has been obtained for the wide energy range of $^{12}C+^{12}C$, $^{12}C+^{16}O$ and $^{16}O+^{16}O$ systems by the coupled channel analyses incorporating inelastic channels of vibrational and rotational excitation. Some of observed "molecular resonances" in scatterings between two α-nuclei are considered to be due to this band crossing mechanism.

Through the development of theoretical studies and accumulation of experiments, there has been suggested the existence of the resonance states generated by different excitation mechanisms from vibrational and rotational ones. It is an important future subject to analyse scattering and reaction processes by taking into account the excitation mechanism due to the structure-change in the excited states like $^{12}C\rightarrow 3\alpha$, $^{16}O\rightarrow{}^{12}C\text{-}\alpha$ and $^{20}Ne\rightarrow{}^{16}O\text{-}\alpha$. There have been started already theoretical studies[54] which incorporate these processes, and there exist some experimental indications[55] on them. Furthermore it is considered for getting the comprehensive understanding on the realization of nuclear molecular phenomena to examine or modify these approaches by studying the reaction which gives the information in more interior region. An important understanding about them is mentioned in Chapter IV to be obtained by the investigation of a systematics of fusion cross section in light heavy ion reactions.

Chapter IV summarizes the present status of understanding on the molecular configurations at highly excited resonant states obtained by the phenomenological analyses based on the model like the band crossing one. The discussions are also given about the consistency of the model parameters by analyzing the various kinds of reaction data including the fusion cross sections.

§ 6. Interactions between complex nuclei as composite particles

The investigation of the interaction between complex nuclei as the composite particles was initially developed through the study of $\alpha+\alpha$ interaction[56),57)] by the microscopic model of Resonating Group Method (RGM) in close connection with the cluster structure study in Beryllium region.[17)] Recognition of the states with molecule-like structures in wide region of mass and excitation energy has been strong motivation to develop the microscopic investigation of the inter-cluster interaction. The progress of the investigation has enlightened the way to understand the molecule-like structure in terms of the interaction between nuclear clusters. Furthermore, the study of the interaction between complex nuclei has been needed to provide a basis of the model study on the heavy ion scattering and reaction phenomena, especially so-called "molecular resonance phenomena".

Microscopic study of the interaction between complex nuclei has clarified the essential effects of the Pauli principle and of the nuclear forces. The important features of the interaction is really determined by the interplay between them. We here notice the two essential characteristics of the interaction; one of them is the feature in the strongly overlap region where the Pauli principle plays a decisive role. Another is the one in the intermediate region where the strong interplay between the Pauli principle and the nuclear forces take place.

An important role of the Pauli principle in the strong overlap region was clearly recognized in the study of $\alpha+\alpha$ interaction. The repulsive core which appears in the phenomenological potentials[56)~60)] was shown to originate from the Pauli principle. In the study of RGM, the almost energy independent inner oscillation appears and the outermost nodal point of it just corresponds to the repulsive core radius. The amplitude of the inner oscillation is strongly damped in comparison with that of the wave function in the outside region. The nodal behavior of the inner oscillation is similar to the relative wave function of the cluster coupling shell model at the lowest state although the former amplitude is strongly damped. This linkage to the shell model description always remains in the microscopic description. The calcula tions[12),27),61)~63)] for the case of closed shell nuclei show that damping of the inner oscillation amplitudes is much strong for the heavier nucleus-nucleus cases. This inner oscillatory aspect originating from the stiffness of the internal structure represents the exchange repulsion and may be called "structure core". It is considered to be the universal character in the interaction between nuclei.

In the region outside outermost nodal point, there arises the strong interplay between the Pauli principle and the nuclear forces. Resulting interaction is generally expected not to be large in this interacting region. Because of

the overall nuclear saturation properties from light nuclei to the nuclear matter, the binding force between the nuclei cannot be so strong. If not so, the system composed of two nuclei would have the large binding energy which exceeds the energy of the ground state of the composed system. Based on the realistic nuclear force, the effective potential was examined for the scattering of $\alpha+\alpha$ system and resonances which has the short ranged attraction and confines scarecely the zero point oscillation. We note here that this weakness of the interaction guarantees also the damping of the inner oscillation. In the case of the $^{16}O+\alpha$,[27),62)] the effective potential for the bound and quasi-bound states was studied by taking into account the exchange effects which has the state and parity dependence reflecting their intrinsic structure. The optical potential in $^{16}O+^{16}O$ scattering can be taken to be shallow one.[64)] The investigation[63)] in the framework of RGM adopting the realistic effective nuclear potential (RENP) was shown to generate only a few ($\sim$two) rotational bands.

In the microscopic study of RGM, the intrinsic structures of the two fragment clusters are assumed not to change and are only affected by the Pauli principle. However this restriction of RGM is considered not to be serious for the general features of the interactions described by the inner oscillatory aspect and the weak binding ability. Since the outermost nodal point generally exceeds the half of the radius, the effects of the Pauli principle are expected to play a decisive role to maintain the nuclear molecule in the interaction region together with the cooperation with the weak binding interactions.

Microscopic analyses of the roles of the Pauli principle and RENP in the interacting region were done by Kamimura, Matsuse and Fukushima[27)] on $\alpha+^{16}O$ system in detail and by many authors on $^{16}O+^{16}O$ system which is a typical system as a door-way to heavier systems. Based on these analyses, a systematic investigation has been made by Andō, Ikeda and Tohsaki-Suzuki[43)] on the characters of the inter-nucleus interaction, putting a stress on the relation to the saturation property of nuclei. Many of the characteristics of the interaction obtained from the results of the above-mentioned studies on some typical systems are considered to be of general nature.

The orthogonality condition model (OCM) which takes into account concisely the basic elements of the inter-cluster interaction obtained by RGM was proposed by Saito[24)] in $\alpha+\alpha$ system with successful results and then was extended to $^{16}O+\alpha$ system[26)] showing its wide applicability. OCM was further extended to cover various sorts of problems; namely it was extended to the coupled channel problem which treats the coupling of the rotational excitation of clusters with the relative motion as in $^{12}C+\alpha$,[28)] to the three-cluster problem like a 3α problem[29)] and to the channel coupling problem including rearrangement process of clusters. (See Chapeters II and III.) OCM is a model where the inner oscillation of the relative wave function is represented by the orthogonality of the relative motion to the Pauli forbidden states, while the in-

teraction in the intermediate and surface region is represented by the effective potential obtained by renormalizing various effects into the folding (direct) potential. The reason why we retain the inner oscillation aspects without substituting the repulsive core potential in the inside region lies in that it makes possible to pursue the structure-change between different kinds of structure linked through the dynamical behaviour of the inner oscillation.

In order to examine the applicability of OCM in wider region, we need to compare OCM with RGM scheme which takes into account the antisymmetrization effect exactly, together with its test by analysing the reproduction of the observed data. Besides all the cases cited above (channel coupling with rotational excitation, multi-cluster problem, coupling with the rearrangement channels) we need further to examine the applicability of OCM to heavier systems and to higher energy region. For example it is interesting to examine whether OCM is capable to describe the characteristic featrues of interaction between heavy clusters like ^{16}O-^{16}O which includes the strong damping of the amplitude of the inner oscillation and the strong interplay between the Pauli principle and the nuclear force. Another interesting example is the comparison of OCM with RGM in the case of a three-cluster system from which we can obtain the knowledge about the exchange effect related simultaneously to the three clusters.

In Chapter V there are presented the results of the studies by the microscopic model (RGM) on the basic properties of the interaction between complex nuclei as composite particles. And there are given an argument about some recognition obtained by viewing the cluster structure from the standpoint of the inter-cluster interaction.

§ 7. Summary and future problems

The molecular viewpoint, which was recognized through the studies of groups of levels with anomalous properties in light nuclei, has been confirmed by the reproduction of almost all the levels up to 10∼15 MeV in excitation energy in the mass number region ranging from Li, Be∼Ne, Mg. By this, we have had a large development of the investigation on "the structure-change of the finite quantum many-body system, nucleus," which has become a central subject owing to the recognition of the variety of the clear existence-form of molecular aspects of nuclei.

(i) This investigation has been advanced from the standpoint of the many-body problem incorporating the molecular viewpoint. Namely the advance has been made through the clarification of the various dynamical aspects of individual systems which are brought about by the combined action of the realistic effective nuclear potentials and the Pauli principle. The recognition obtained by the investigation is on the characteristic features of individual systems and on the universality contained

in the above-mentioned viewpoint.

(ii) Structure studies done through the clarification of the dynamics of the cluster systems are on the analysis of the phenomena of coexistence, coupling and transition between structures with different aspects and on the pursuit of the mechanism underlying these phenomena. As a result we have recognized the universal existence of the levels with molecule-like structure. Especially in typical systems like ^{12}C and ^{16}O, we have found that the number of these levels is comparable with or even exceeds that of the shell-model-like levels in the present excitation energy region.

(iii) An important point in solving the many-body problems is the treatment of the cluster dynamics by taking account carefully the essential action of the Pauli principle for composite particles (clusters). Namely we have treated (a) two-cluster systems where the rotational excitations are involved besides the relative motion between clusters, (b) two-cluster systems where the couplings between rearranged channels are incorporated, (c) three-cluster systems, (d) and furthermore couplings of particle(s) and hole(s) with the cluster systems in the framework of LCMO and the hybrid model which connects the cluster model space with the shell model space.

A remarkable feature of the development of the cluster model study after P.T.P. Supplement No. 52 is the enlargement of the research subject and region around the theme "structure-change":

(i) Cluster model study which treats the α-nuclei from ^{8}Be to ^{24}Mg has been extended to that on the non-α-nuclei and furthermore to all the mass number region of light nuclei including neutron-rich nuclei.

(ii) The region of the excitation energy treated by the cluster model has been also extended from the ground state up to the threshold energy of the two-cluster break-up and furthermore to around the three-cluster break-up threshold energy.

(iii) Cluster structure treated by the cluster model has been extended from that composed of α-cluster(s) and α-nucleus cluster to that of α-nucleus clusters, which are observed as the so-called "molecular resonances". This has necessarily led to the study of the fairly high excited states.

(iv) The detailed study of ^{5}He, ^{6}Li and ^{7}Li nuclei as well as the study of excitations of ^{4}He has been advanced and has become to have a close connection with the above-mentioned cluster model studies. On the other side, the cluster model study is giving an impetus to the investigations of α-correlations in the region of Ca or Ti.

Through these studies the comprehensive understanding of nuclear structure in light nuclear system have been greatly progressed.

We summarize briefly below future problems, many of which are developing continuously along with the enlargement of the research subjects, the linkage of the research themes and the deepening of the research contents.

Structure-change, Coexistence and Transition

Under the quantitative studies of the various quality of structure woven out from the nuclear force and the Pauli principle, this theme given in the above title remains to be important. A characteristic point at present is the extension of the research subjects:

(i) Cluster units treated here include d, ^{3}He (^{3}H), α, ^{8}Be, ^{12}C, ^{16}O and others (like ^{14}N, ^{15}N).

(ii) Cluster systems composed of the above-cited cluster units are full of variety.

Therefore we are now able to pursue, widely and deeply in the whole region of light nuclei, various facets of the coexistence, coupling and transition of different kinds of cluster and shell structure including the rearrangement of cluster configurations.

The clarification of the coupling between different kinds of cluster structure with taking into account the effect of the Pauli principle prepares a basis for the study of the dynamical structure-change existent in the reaction process in the high excitation energy region.

Various Reflection of the Structure-change in the Highly Excited States

A typical example of the molecular aspects is the molecular resonance in the highly excited states. Its understanding has been advanced based on the phenomenological and semi-phenomenological analyses. This research subject on molecular resonances will be developed as an important one in understanding the reflection of the structure-change in the reaction processes. This expectation is based on the following facts: (a) We have continuous supply of the results of the systematic experimental studies with various quality. (b) Molecular aspects of the low excitation energy region are being made increasingly clear. (^{24}Mg system is a good example.) (c) Microscopic study of the composite particle interaction between fundamental clusters is developing. (d) Understanding of the "surface transparency" between nuclei is advancing. (e) Methods of the treatment of multi-cluster dynamics are also being developed.

Cluster Structure, Inter-Cluster Interaction and RENP (realistic effective nuclear potentials)

(i) For the investigation of cluster dynamics, it is indispensable to understand the inter-cluster interaction from the basic standpoint (namely from the microscopic level and in relation with the elementary strong interaction) in the wide-range systems including clusters with non-zero spin and heavy clusters.

The deep knowledge of the inter-cluster interaction is demanded not only for the bound and quasi-bound cluster systems but also for the scattering and reaction processes. We can expect continuous development of this research subject from allover aspects at various theoretical levels (semi-phenomenological, semi-microscopic and microscopic).

(ii) To clarify the mechanism of the formation and disintegration of the clusters in nuclei is one of the fundamental subjects in nuclear many-body system. For this purpose, it is necessary to study the inter-relation between the structure and the RENP. In order to develop this attempt of study, we need to elucidate not only the role of the RENP in the formation process of clusters but also the role of it in the disintegration process of clusters. Namely we need to clarify the synthetic roles of different characters in the RENP including the spin-orbit and tensor forces.

If we state the picture generated by our study in general words, the nuclear many-body systems, when supplied with energy, give rise to the formation of the orderly structure by saturated subunits (the one with the molecular aspects) in contrast to the disordered structure with thermal equilibrium (compound nucleus). When we go up from the low excitation energy region to high one, there open various cluster-breakup thresholds and there is expected the appearance of the corresponding cluster structures. In this excitation energy region, it is suggested that those structure-changes reflect their image in heavy ion reaction processes. Along the development of the experimental studies of heavy-ion reactions, it is expected that the theoretical studies develop on the structural problems in open nuclear systems incorporating the molecular viewpoint in wide facets.

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